Team Tunes

How We Hear

A mathematical model of the hearing process based on scientific research.

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Abstract

Hearing as a perception process consists of several stages, starting off from the outer ear going over the middle ear and ending up at the inner ear. The hearing process is like a hand over process, where each part of the ear hands over the detected sound wave to the proceeding part after processing it somehow. At the end of such process the sound is transformed form the mechanical form to the electrical form which the brain can deal with, in what is known as the transduction of sound, which is considered the most central event of the hearing process. The biological device which is responsible for such transduction is the cochlea, which contains a frequency analyzer in the form of membrane called the Basilar membrane. The task of the Basilar membrane is to identify the constituent frequencies of sound, and thus identify the sound. Such process is done by means of mechanical resonance, where each section on the membrane selectively resonates with specific frequency, and therefore gets a maximum amount of kinetic energy in the form of oscillation. The energy delivered to any section at the moment of its resonance is constant and sufficient to produce a shear force in the hair cells of the organ of Corti which mounts that section of membrane due to the action of friction. This shear force causes a change in the membrane potential which is transmitted through the auditory nerves directly to the brain where the interpretation is performed for different frequencies

Keyword:

Transduction, Cochlea, frequency analyzer, mechanical tuning, Basilar Membrane, Stapes, Oval Window, resonance action, average power, organ of Corti, hair cells, impulses

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1 Introduction

Hearing or auditory perception is a critical sense for all living creatures. And it kept being a questionable and research subject from the scientific audience throughout the ages. But why is that and what motivated us to search about it further?

Basically, the hearing ability contributes to our life in various ways. Starting with detecting the locations of the objects and ending with the normal conversation between people, hearing has a great role in all this. Imagine how many disasters could have been happened if we couldn't hear. Or how our mental health could have been if we live in an empty silent world. So, after all this, we thought that the hearing ability deserves to be studied in detail to understand and to try healing the diseases that can deprive someone of this blessing. And before we get to these diseases, we should understand the hearing process. And this will happen when we understand the anatomy of the hearing organ, we all call "**The Ear**" and the sound wave vibrations through the ear. So, in this report, we are going to explore the hearing process in form of mathematical models.

1.1 The Anatomy of the ear and the sound waves journey

The sound waves journey begins with the transducer that produce them. Sound waves are mechanical waves that need a medium to travel through until they reach the recipient which is the ear in our case.

At first, the sound waves reach the Pinna which is the external visible part of the ear that looks like a funnel. This pinna works to direct the sound waves through the auditory canal that ends with a membrane called "The Tympanic membrane" or "The Eardrum". The sound waves strike this membrane which makes it vibrate with the waves.

Secondly, according to the anatomy, the eardrum is in contact with three tiny bones called "**Hammer (Malleus)**", "**Anvil (Incus)**", and "**Stirrup (Stapes)**". These three bones are main parts of the middle ear. This last bone "The Stapes", which takes a stirrup shape, is in contact with the inner ear that have a very important part for the hearing process called "**The cochlea**". And the cochlea is snail-shaped liquid-filled tube that contains cilia, and it is divided into parts separated by the **basilar membrane** along with a cochlear duct called "**Scala Media**" and it is filled with endolymph. These parts are "**The Scala Vestibule**" and "**The Scala Tympani**".

The basilar membrane is a mechanical element that has a graded mass and stiffness properties over its length. And these properties help in separating the sound into frequency components that activates different cochlear regions.

The Stapes is lied on the oval window, which covers the opening the cochlea. And the vibration in the eardrum causes these bones to also vibrate. And this in turn vibrate the oval window which results in disturbance in the fluid inside the cochlea. And as the air strikes the hair of a child, the movements of the cochlear fluid cause the hair cells of the cochlea to bend. And these cells are attached to neurons, so when it bends these neurons start transmitting nerve impulses through the auditory nerve and then to the auditory cortex in the brain. Then the brain interprets these impulses to understandable words according to previous experiences.

Cochlea has about 16000 hair cells. Each of which holds a bundle of fibers known as cilia on its tip. This cilium is so sensitive to detect the movement that punches them.

Now, after we understood the ear's anatomy and the sound waves journey, we can talk about the diseases that can threatens this blessing.

- **Conductive hearing loss**
  That disease reduces the ability to transfer the vibrations from the external ear to the inner ear. And it happens due to a physical damage to the ear, mostly in the eardrum or ossicles which are the 3 bones in the middle ear.
- **Sensorineural hearing loss**
  This disease happens due to a damage to the cilia or the auditory nerve and mostly hits the old because the cilia get damaged by age and don't grow back.
- **Tinnitus (Ringing/Buzzing)**
  It happens due to a damage to the cilia. And it mostly hits those who are exposed to loud sounds like workers in factories with heavy machines or those who listens to loud music repeatedly.

The conductive hearing loss can be treated with hearing aids that amplify the incoming sound waves to compensate the reduction of vibrations. This solution cannot help much with the sensorineural loss.

Some patients need to have a cochlear implant which is device serves to bypass the hair cells by stimulating the auditory nerve cells directly [1].

1.2 Literature Review

Our project idea was triggered by a simple question: "How do we hear?" and of course, mathematics was part of this phenomenon as well. Have you ever wondered how sound makes its way from the source to your brain mainly the sound vibrations travel from the outer ear to the ear canal then the eardrum which vibrates tiny bones and then sent to the cochlea The Hungarian-American biophysicist Georg von Békésy made a theory by the name of Bekesy's theory which states that "Sound vibrations transmitted to the cochlear fluid by the round window triggered a traveling wave along the length of the basilar membrane" two-dimensional models of the mechanical tuning of the basilar membrane in the cochlea have been proposed The most representative models were developed by Peterson and Bogert, Ranke, Fletcher, Zwislocki, Lesser and Berkeley "**The model we covered in our**

project”. These models were proposed in the same way as the observations reported by Békésy.

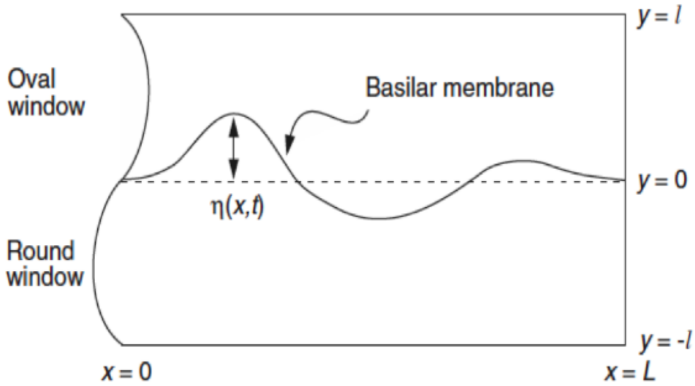
However, these models have disadvantages to represent the membrane behavior for high frequencies near the apex and for the low frequencies near the helicotrema.

Then numerical solutions for the two-dimensional models of the cochlea have been developed by Lesser and Berkley using the Fourier series “Numerical solution we covered using parameters of Neely” and Neely using the finite difference method to, and we also focused on the power analysis of the membrane in the cochlea by mechanical resonance using the Neely parameters

2 Mathematical Modelling

2.1 Lesser & Berkley Model

Despite of all two-dimensional models that have been developed, such model is one of the best models for the macro-mechanical response of the cochlea that has been developed by Lesser and Berkley, where each point of the basilar membrane is modelled as a simple damped harmonic oscillator with mass, damping, and stiffness that vary along the length of the membrane.



2.1.1 Assumptions

As usual, there are several assumptions about the cochlea structure and its fluid content (perilymph) for the sake of simplicity.

1. The 2D linear flow of the perilymph
2. The fluid potential controls the basilar membrane's motion (Φ)
3. The fluid has constant density, and irrotational flow, and is inviscid.
4. Each point of the basilar membrane acts as damped harmonic oscillator with mass, meaning that each of them is a vibrating system for which the amplitude of vibration decreases over time
5. The oval window's position is determined by the stapes which is a bone resting on the window.
6. The cochlea is a two-chamber model partitioned by the basilar membrane: the upper and lower part. The two parts are similar but to refer to the upper part, we will be using the subscript of 1. And we will be using the subscript of 2 for the lower part. For each part: [$0 \leq x \leq L$, $0 \leq y \leq l$]

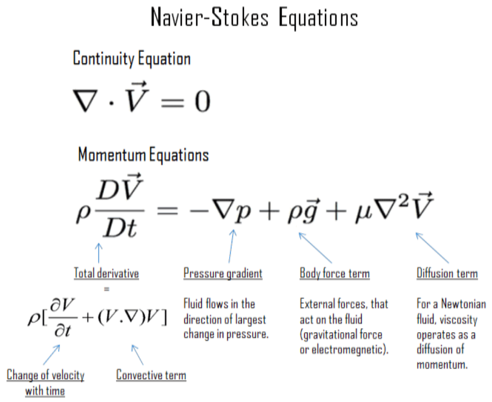
2.1.2 Variables

$\vec{u}$: velocity of fluid (2D)
η: y-displacement of BM (the Basilar Membrane)
ρ : fluid density (constant)
p: fluid pressure
Φ: fluid potential ($\nabla\Phi = \vec{u}$)

2.1.3 Derivation

- **Fluid equations:**

To describe the fluid system existent in the cochlea, we are going to use **Navier-Stokes Equations**, which are a set of equations which can describe the flow of any incompressible fluid.



By complying with the assumptions, we get a new set of equations

$$\nabla \cdot (\vec{u}) = 0 \qquad \text{............ (1)}$$

$$\rho \frac{\partial \vec{u}}{\partial t} + \nabla p = 0 \qquad \text{............ (2)}$$

Assuming that the fluid is irrotational. And mathematically, the curl of the gradient is zero, so we can define a scalar field suc$\nabla \Phi = \vec{u}$ gradient is equal to the velocity of the fluid Therefore, the first equation yields to:

$$\nabla^2 \phi = 0, \qquad \text{........... (3)}$$

- **Setting up the equation of the basilar membrane**
  According to the assumption no.4, the balance of force per unit area can be described as follows:

$$m(x)\eta_{tt} + r(x)\eta_t + \kappa(x)\eta = p_2 - p_1 \qquad \text{............ (4)}$$

Where: $m(x)$: mass per area
$r(x)$: resistance
$\kappa(x)$: stiffness

For better estimation of the membrane response, we are going to consider Neely parameters:

Parameter	Value	Units
$m(x)$	0,15	g/cm^2
$k(x)$	$10^9 e^{-2x}$	dyn/cm^3
$c(x)$	200	$dyn \cdot s/cm^3$

Table 1: Neely parameters

- **Boundary conditions:**

Combining the equations of the fluid mechanics and the basilar membrane, we can derive the system of partial differential equations with boundary conditions. Note that the functions in the lower chamber are odd in y to the functions in the upper chamber: (i.e., Φ2 = −Φ1 (−y, t), p2 = −p1 (−y, t)). The following are the boundary conditions for the upper chamber (y ∈ [0, l], x ∈ [0, L]).

As explained previously, the gradient of flow potential Φ is the flow velocity. So (∂Φ / ∂x) is the horizontal velocity and (∂Φ / ∂y) is the vertical velocity.

1. Vertical Velocity at $y = 0$
   The fluid must move like the membrane's motion since it is in contact with it. $$\frac{\partial \Phi}{\partial y} = \frac{\partial \eta}{\partial t}$$

2. Vertical Velocity at $y = l$
   We are assuming the velocity is equal to zero at the top of the chamber. $$\frac{\partial \Phi}{\partial y} = 0$$

3. Horizontal Velocity at $x = 0$
   From assumption no. 05, the horizontal velocity of the oval window at point y, determines the fluid's horizontal velocity at the beginning of the chamber. $$\frac{\partial \Phi}{\partial x} = \frac{\partial F(y,t)}{\partial t}$$

4. Horizontal Velocity at x = L
   At the end of the chamber, the fluid's horizontal velocity is assumed to be 0. $$\frac{\partial \Phi}{\partial x} = 0$$

- **Derivation of PDE and BC system for Flow Potential:**

Using a specific technique explained by lesser and Berkley, we can simplify the equations described before. First, the frequency-dependent functions F, Φ, p, and η will be expressed in analytic representation. Then, equations 2), 3), and 4) will be combined into one equation in Φ that results in a Laplace equation with four boundary conditions

Fluid Equations:

$$1)\ \Delta\Phi = 0$$
$$2)\ \rho\frac{\partial \Phi}{\partial t} + p = 0$$

Basilar Membrane balance of force:

$$3)\ m(x)\eta_{tt} + r(x)\eta_t + \kappa(x)\eta = p_2 - p_1 = -2p_1 = -2p$$

Note that $p2$ is odd to $p1$ in y.

Boundary Conditions:

$$\frac{\partial \Phi}{\partial y} = \frac{\partial \eta}{\partial t} \text{ at } (y=0) \qquad \frac{\partial \Phi}{\partial x} = \frac{\partial F(y,t)}{\partial t} \text{ at } (x=0)$$
$$\frac{\partial \Phi}{\partial y} = 0 \text{ at } (y=l) \qquad \frac{\partial \Phi}{\partial x} = 0 \text{ at } (x=L)$$

2.2 Analytical Solution

Lesser and Berkley examines steady-state response of the cochlea to the pure tone. Then for input has a single frequency $F(y,t) = \hat{F}(y)e^{i\omega t}$ where F is the displacement of the oval window, similarly for all the frequency-dependent function.

$$\Phi = \hat{\Phi}e^{iwt}, p = \hat{p}e^{iwt}, \eta = \eta_0 e^{iwt}$$

Equation 2, 3 yields to: $\Delta\hat{\Phi} = 0$
$iw\rho\hat{\Phi} + \hat{p} = 0$

Equation 4 yields to:

$$(-mw^2 + irw + \kappa)\hat{\eta} = -2\hat{p} \longrightarrow iwZ\hat{\eta} = -2\hat{p}$$

Where: $Z = miw + r + \frac{\kappa}{iw}$

Respectively, Boundary conditions 1 through 4 yields to:

$$\frac{\partial\hat{\Phi}}{\partial y} = \frac{2iw\rho\hat{\Phi}}{Z} \text{ at } (y=0) \qquad \frac{\partial\hat{\Phi}}{\partial y} = 0 \text{ at } (y=l) \qquad \frac{\partial\hat{\Phi}}{\partial x} = iw\hat{F} = U_0 \text{ at } (x=0)$$

$$\frac{\partial\hat{\Phi}}{\partial x} = 0 \text{ at } (x=L)$$

- **Setting up the system of Laplace equation with Neumann boundary conditions:**

By far, a system of Laplace Equation with Neumann Boundary Condition has been derived. Scaling x and y by L, Z by $iw\rho L$, and $\hat{\Phi}$ by $U_0 L$ and dropping the hats results in the following simplified system:

1)
$$\Delta\Phi = 0$$

2) at $y = 0$
$$\frac{\partial\Phi}{\partial y} = \frac{2\Phi}{Z}$$

3) at $(y = \frac{l}{L} = \sigma)$
$$\frac{\partial\Phi}{\partial y} = 0$$

4) at $(x = 0)$
$$\frac{\partial\Phi}{\partial x} = 1$$

5) at $(x = 1)$
$$\frac{\partial\Phi}{\partial x} = 0$$

The analytical solution of this problem can be found using standard Fourier series. The following solution of the fluid flow potential satisfies the equations 1), 3) 5)

$$\Phi = x\left(1 - \frac{1}{2}\right) - \sigma y\left(1 - \frac{y}{2\sigma}\right) + \sum_{n=0}^{\infty} A_n \cosh[n\pi(\sigma - y)]\cos(n\pi x) \qquad \text{........ (5)}$$

The complete solution can be found using the second equation (2) by determining the coefficient A_n , using Fast Fourier Transformation algorithm.

$$\sigma + \sum_{n=0}^{\infty} n\pi A_n \sinh(n\pi\sigma)\cos(n\pi x)$$
$$- \frac{2}{Z}\left[x(1-\frac{x}{2}) + \sum_{n=0}^{\infty} A_n \cosh(n\pi\sigma)\cos(n\pi x)\right] = 0.$$

Multiplying by $\cos(m\pi x)$, and integrating from 0 to 1, we find

$$A_m\alpha_m = f_m,$$

where

$$\alpha_m = \frac{1}{Z}\cosh(m\pi\sigma) - \frac{1}{2}n\pi\sinh(m\pi\sigma)$$

and

$$f_m = \sigma\delta_{m0} - \int_0^1 \frac{x(2-x)\cos(m\pi x)}{Z}dx = -\frac{2}{m^2\pi^2}.$$

2.3 Resonance & Power Analysis

This new solution has the advantage of determining the relationship between the excitation frequency of the system and the position along the basilar membrane where the average power is maximum, in addition, it has an advantage over the previous solution, that the function obtained depends only on the physical characteristics of mass per unit area, damping coefficient and stiffness per unit area along the basilar membrane without taking the cochlea structure into consideration.

When the Oval Window transmit single tone vibration into the cochlea, this excites the Basilar membrane by external force $Fe^{j\omega t}$

Equation no. 4 becomes: $m(x)\frac{\partial^2\eta}{\partial t^2} + R_m(x)\frac{\partial\eta}{\partial t} + k(x)\eta = Fe^{j\omega t}$ ………. (6)

Since the Force is periodic and complex, therefore it can be considered that the displacement η is also complex and then $\boldsymbol{\eta} = \mathbf{A}e^{j\omega t}$on of the differential equation is defined by displacement

By derivation and substitution in 6, we get: $\eta = \frac{1}{j\omega}\frac{Fe^{j\omega t}}{R_m(x) + j\left(\omega m(x) - \frac{k(x)}{\omega}\right)}$

The above equation can be expressed in a simpler form defining complex mechanical impedance as follows:

$$\mathbf{Z}_m(x) = R_m(x) + jX_m(x)$$

Where the mechanical reactance is defined by the following:

$$X_m(x) = \omega m(x) - \frac{k(x)}{\omega}$$

The mechanical impedance may also be expressed in polar form $\mathbf{Z}_m(x) = Z_m(x)e^{j\Theta(x)}$ an equation in terms of magnitude and one that determines the phase angle.

$$Z_m(x) = \sqrt{(R_m(x)^2 + X_m(x)^2)} \qquad \text{.......... (7)}$$

$$\Theta(x) = \tan^{-1}\frac{X_m(x)}{R_m(x)} \qquad \text{........... (8)}$$

From equations (7) and (8) we can write the displacement complex in the form.

$$\boldsymbol{\eta} = \frac{1}{j\omega}\frac{Fe^{j\omega t}}{Z_m(x)e^{j\Theta(x)}}$$

$$\boldsymbol{\eta} = \frac{1}{j\omega}\frac{F}{Z_m(x)}e^{j(\omega t - \Theta(x))}$$

Using Euler's identity, we get: $\boldsymbol{\eta} = \frac{1}{j\omega}\frac{F}{Z_m(x)}\left[\cos(\omega t - \Theta(x)) + j\sin(\omega t - \Theta(x))\right]$

Since the displacement of each membrane section is defined by the real part of the above equation, the equation yields to:

$$\eta = \frac{F}{\omega Z_m(x)}\sin(\omega t - \Theta(x)) \qquad \text{........... (9)}$$

Since $A = F/\omega Z_m(x)$, therefore we can express the amplitude algebraically in terms of mass, damping and stiffness, by the following expression

$$A = \frac{F/m(x)}{\sqrt{\left(\omega^2 - \frac{k(x)}{m(x)}\right)^2 + \omega^2\frac{R_m(x)^2}{m(x)^2}}} \qquad \text{.......... (10)}$$

The equation (10) shows that the amplitude for each section of the membrane depends on the frequency ω of the applied force. To get the maximum Amplitude at the specified distance, we derive equation (10) with respect to ω, then equates the denominator by zero, we get that the maximum amplitude coincides with ω is equal to $k(x)/m(x)$

$$A(x,f) = \frac{F/m(x)}{\sqrt{\left(4\pi^2 f^2 - \frac{k(x)}{m(x)}\right)^2 + 4\pi^2 f^2 \frac{R_m(x)^2}{m(x)^2}}} \quad \text{.. (11)}$$

Since ω = 2***πf,*** by substitution in Equation (10):

To obtain an equation to describe the power transfer which occurs along the membrane we must get the velocity of the membrane.

By derivation of equation (9), we get $v = \frac{F_0}{|Z|} cos(\omega_f t - \Theta)$ ………… (12)

By expressing the above equation in terms of the physical parameters of the membrane, we get:

$$A_v = \frac{F_0}{\sqrt{c(x)^2 + \left[m(x)2\pi f - \frac{k(x)}{2\pi f}\right]^2}} \quad \text{……………(13)}$$

Since, the power P in the model of the basilar membrane is defined as the multiplication of the equation of the velocity v (12) and the equation that defines the excitation force F which is given by $F_0 cos(\omega_f t)$

We get:

$$p = \frac{F_0}{|Z|} cos(\omega_f t - \Theta) \cdot F_0 cos(\omega_f t)$$

Using the trigonometric identities, we get: $p = \frac{{F_0}^2}{|Z|}\left(\frac{1}{2}cos\Theta + \frac{1}{2}cos2\omega_f t cos\Theta + \frac{1}{2}sin2\omega_f t sin\Theta\right)$

By averaging the power equation: $P = \frac{1}{T}\int_{t_0}^{t_0+T} p dt$

$$P = \frac{{F_0}^2}{|Z|}\frac{1}{2}cos\Theta + \frac{1}{T}\int_{t_0}^{t_0+T}\frac{1}{2}cos2\omega_f t cos\Theta dt + \frac{1}{T}\int_{t_0}^{t_0+T}\frac{1}{2}sin2\omega_f t sin\Theta dt$$

The expression yields to:

$$P = \frac{{F_0}^2}{2|Z|}cos\Theta$$

By expressing the above equation in terms of the physical parameters of the membrane, we get:

$$P = \frac{{F_0}^2}{2\sqrt{c(x)^2 + \left[m(x)2\pi f - \frac{k(x)}{2\pi f}\right]^2}} cos\Theta \qquad \text{....... (14)}$$

3 Experiment work

3.1 The Analytical Solution:

We are going to simulate the behavior of the Basilar membrane using the equation no. (5) over a wide range of single tone inputs so that we come out with our observations and then the conclusion.

Note: Keep in mind that we are just taking into consideration the distance at which there is a maximal oscillation in the cochlea's upper chamber, where the organ of Corti is located.

- **Algorithm Of the Analytical Solution**

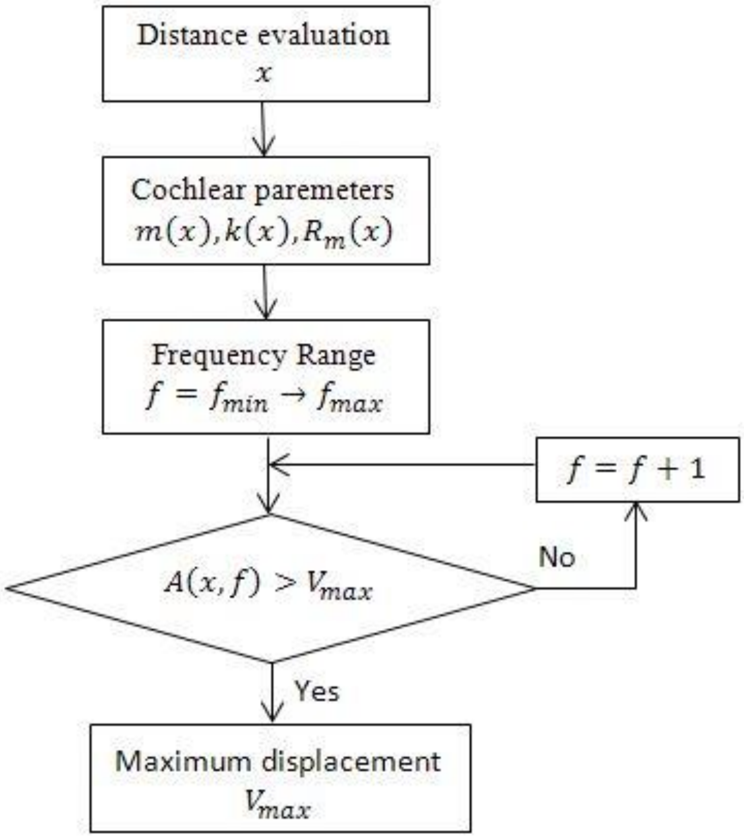
1) Enter the parameters that will be used in the equations (mass, damping, stiffness, $\boldsymbol{\sigma}$, frequency)
2) Calculate the impedance of the membrane for each point (Z)
3) Calculate α_{nm}
4) Calculate f_m
5) Calculate the coefficient A_n
6) Calculate Phi and the phasor
7) Plot the solution
8) Repeat for different frequencies

3.2 Resonance & Power Analysis

We are going to simulate the behaviour of Basilar membrane using equation no. (13) at specific point on the membrane over a wide range of frequencies, so that we can determine its resonance frequency which gives the highest amplitude of oscillation.

With the same approach, we are going simulate the average power delivered to the membrane using equation no. (14)

- **Flowchart for Mechanical Resonance**



4 Results & Discussion

4.1 Simulation

❖ Analytical Solution

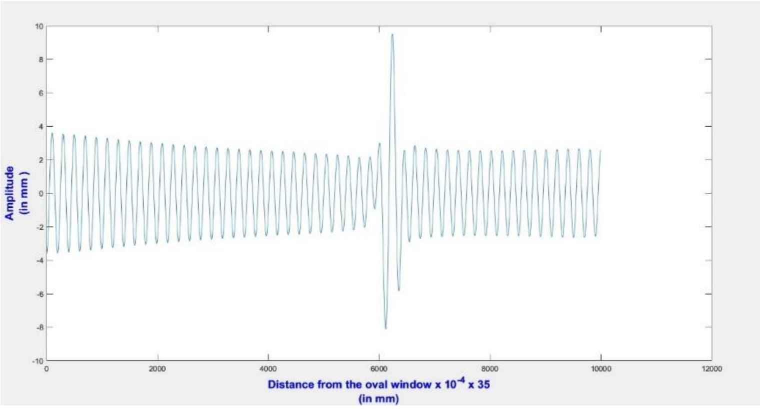


Figure 1: the response of the membrane at 7KHz

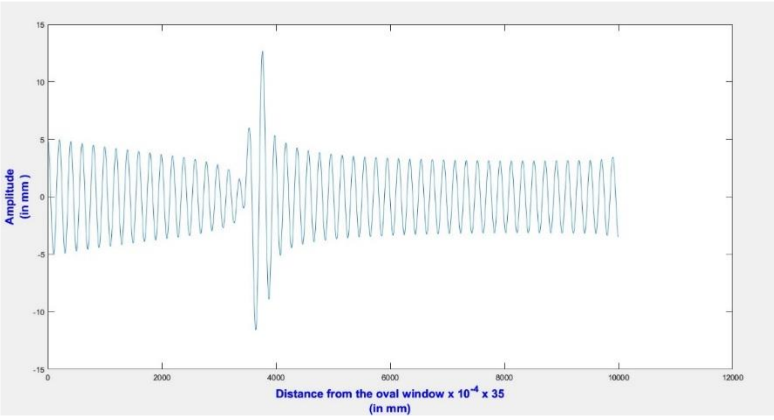


Figure 2:the response of the membrane at 9KHz

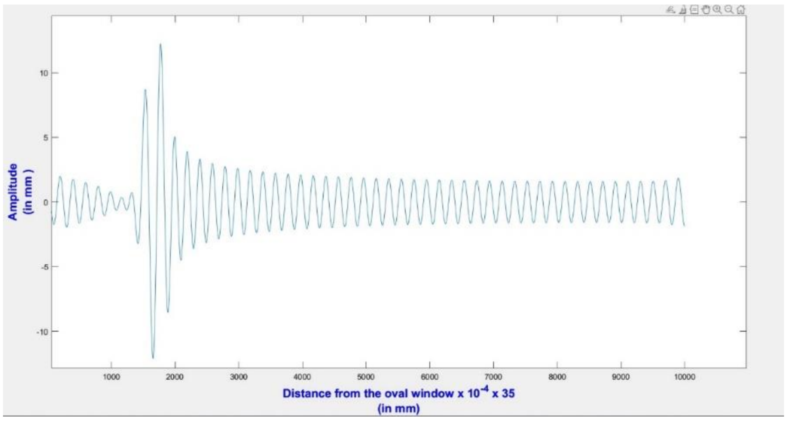


Figure 3: The response of the membrane at 11KHz

Frequency (kHz)	Distance (mm)
1	34.3
3	34.3
5	32.2
7	21.7
9	13.3
11	5.95
13	0.7
15	0.35
17	0.35

Table 2: Using Neely parameters

Observation

- Each response has one peak oscillation at some point on the membrane.
- The peak moves towards the oval window on increasing the frequency of the input tone.
- The response of the Basilar membrane at the extreme cases of input tones (either extreme high or low frequencies) may be steady, as it's shown in table (2), where there is nearly no change in the distance at which the resonance occurs.

❖ Resonance Analysis

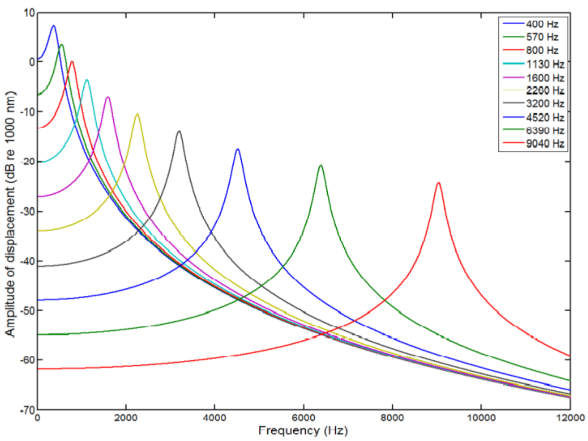


Figure 4: variation in amplitude at specific distance over a wide range of frequencies (using Neely parameters)

Observation

- Each point on the Basilar membrane oscillates with all the frequencies.
- The amplitude of oscillation increases gradually until specific point (resonance point)
- Beyond the resonance frequency, the amplitude of oscillation decreases gradually in the same manner.

❖ Power Analysis

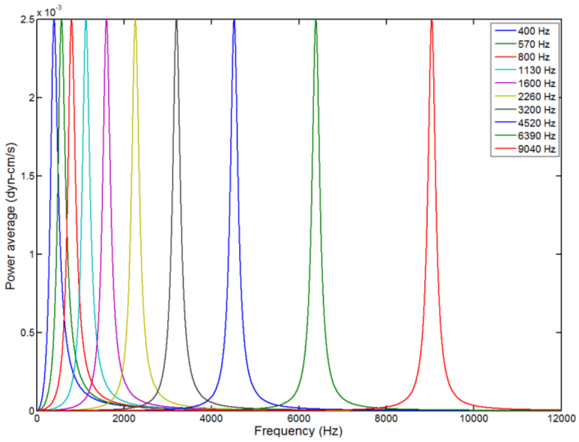


Figure 5: Average power delivered over a wide range of frequancies at specific distance (using Neely parameters)

Observation:

- At specific frequency the average power delivered to the membrane at specific point is maximum (resonance moment).
- At all the resonance moments the same amount of average power is delivered.

4.2 Comparison

1) Analytical Solution:

- ➢ Such solution depends on the physical characteristics of the Basilar Membrane besides the structure of the cochlea itself.
- ➢ It studies the response of the entire membrane at specific frequency input.

2) Resonance Analysis:

- ➢ Such solution depends completely on the physical characteristics of the Basilar Membrane.
- ➢ It studies the response of specific point on the membrane at a wide range of frequencies.

4.3 Several Parameter System

- We are going to compare the results obtained previously with Neely parameters with that of Lesser & Berkley

Parameter	Value	Units
$m(x)$	0,05	g/cm^2
$k(x)$	$10^9 e^{-3x}$	dyn/cm^3
$c(x)$	$3000e^{-1,5x}$	$dyn \cdot s/cm^3$

Table 3: Lesser & Berkley parameters

❖ Analytical Solution

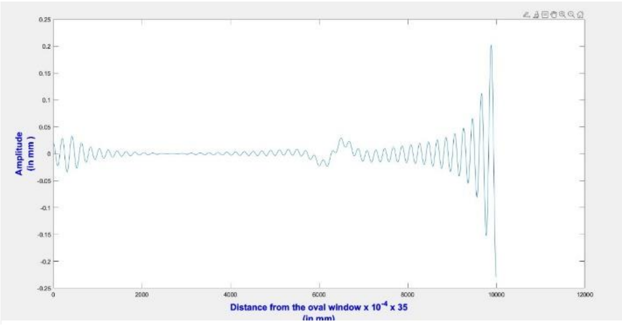


Figure 6: The response of the Basilar Membrane at 5KHz

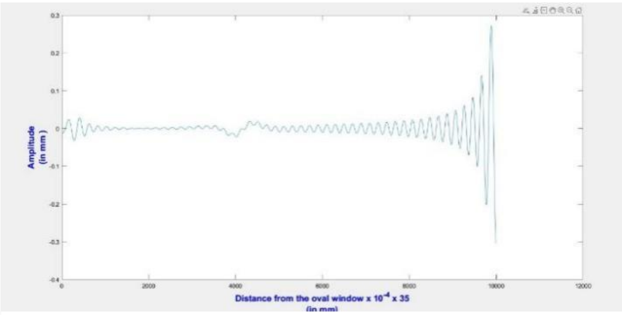


Figure 7: The response of the Basilar membrane at 7KHz

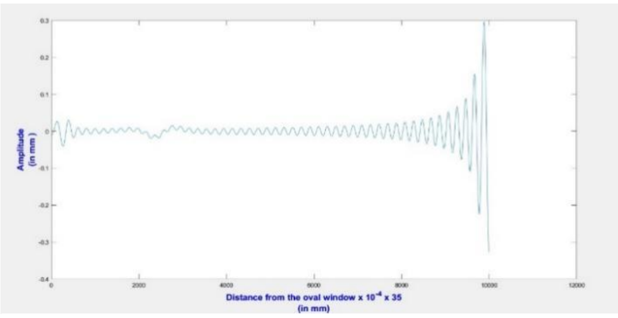


Figure 8: The response of the Basilar membrane at 11KHz

- By comparing the plots which we have got using Neely parameters with the above plots, we find that the peak is fixed at distance 34.4 mm and there is small ripple in the middle which in turn refers to the basilar membrane response, which is indeed unrealistic, therefore we have preferred to use Neely parameters.

❖ Resonance & power Analysis (using Lesser & Berkley parameters)

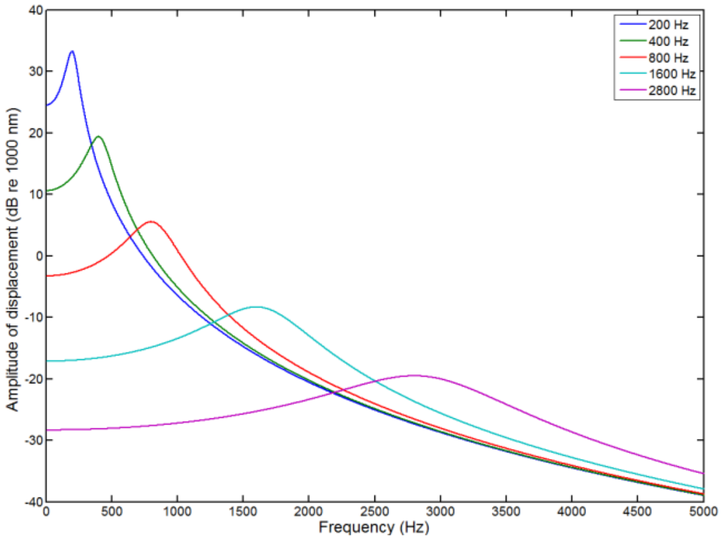


Figure 9: variation in amplitude at specific distance over a wide range of frequencies (using Lesser & Berkley parameters)

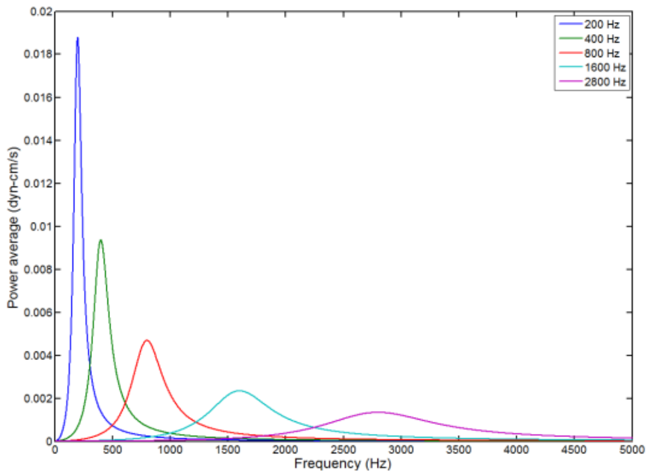


Figure 10: Average power delivered over a wide range of frequancies at specific distance (using Lesser &Berkley parameters)

- By comparing the results shown in figure (4) with that in figure (9), we find that there is slight difference which can be negligible.

- By comparing the results shown in figure (5) with that in figure (10), we find that there is contradiction with the maximum power theorem, where the delivered power in the moment of resonance is not constant.

5 Conclusion & Future Work

❖ Conclusion

From the results of the analytical solution and its experimental work, we can conclude that the membrane functions as a frequency analyser, with each point receiving the maximum power transfer and consequently the maximum oscillation due to the mechanical resonance achieved with a particular single tone input. Points of the membrane close to the apex resonate with low-frequency inputs, while points close to the oval window resonate with high-frequency inputs.

From the results of the resonance analysis and its experimental work, we can conclude that each point on the membrane oscillates with all the frequencies in different degrees, even though the input frequency is not the resonance frequency, but it gives the maximum oscillation when the input frequency is the resonance one.

From the results of the Power analysis and its experimental work, we can conclude that the value obtained of average power for each relationship between the excitation frequency and the distance along the basilar membrane is always constant. This validates that each position along of the basilar membrane is always excited with the same energy being this value the necessary for the activation of the hair cells.

❖ Future Work

Due to the limitation of time and resources, we would have oriented towards approximating the analytical solution further using the deep-water approximation method which assumes that the wavelength of the membrane waves is short compared to the depth of the cochlea, which gives more accurate response.

6 References

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