

Computational aspects of some simple statistical models on the Bayesian approach using STAN: basic concepts

https://github.com/clobos/Seminario_STAN_UFBA

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Section 1

What is Stan?

What is Stan?

Stan is a probabilistic programming language for specifying statistical models. As of version 2.2.0, Stan provides full Bayesian inference for continuous-variable models through Markov chain Monte Carlo methods such as the No-U-Turn sampler, an adaptive form of Hamiltonian Monte Carlo sampling. Penalized maximum likelihood estimates are calculated using optimization methods such as the Broyden-Fletcher-Goldfarb-Shanno algorithm.

Section 2

Introduction to Bayes Theorem

Bayes Theorem

Bayes Theorem

$$f(\theta|\text{data}) = \frac{f(\text{data}, \theta)}{f(\text{data})} = \frac{f(\text{data}|\theta)f(\theta)}{\int_{\theta \in \Theta} f(\text{data}|\theta)f(\theta)d\theta} \propto f(\text{data}|\theta)f(\theta) \quad (1)$$

where

- $f(\theta|\text{Data})$ **Posterior distribution**
- $f(\text{data}|\theta)$ **Likelihood function**
- $f(\theta)$ **Prior distribution**
- $f(\text{data})$ Normalizing constant

More about prior distribution

Bayesian Data Analysis by Andrew Gelman, John Carlin, Hal Stern, David Dunson, Aki Vehtari, and Donald Rubin. **Chapter 2: Single-parameter models** <http://www.stat.columbia.edu/~gelman/book/BDA3.pdf>, available for download for non-commercial purposes.

Beta posterior \propto Beta prior \times Binomial likelihood

<https://github.com/clobos/minicurso-seeb>

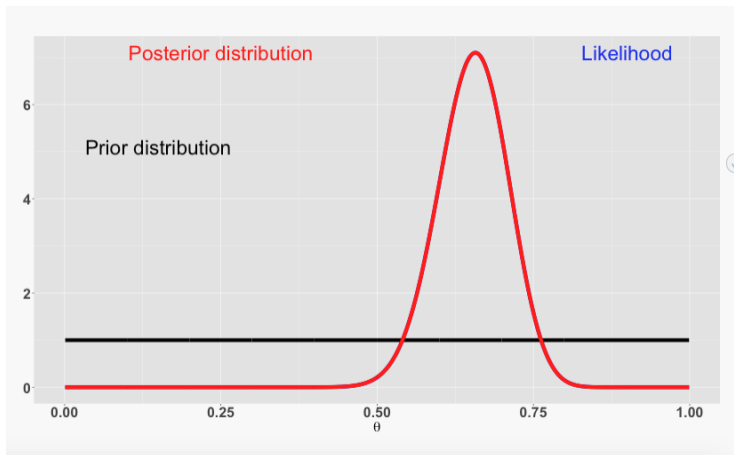


Figure 1: Beta(1,1) non-informative prior

Beta posterior \propto Beta prior \times Binomial likelihood

<https://github.com/clobos/minicurso-seeb>

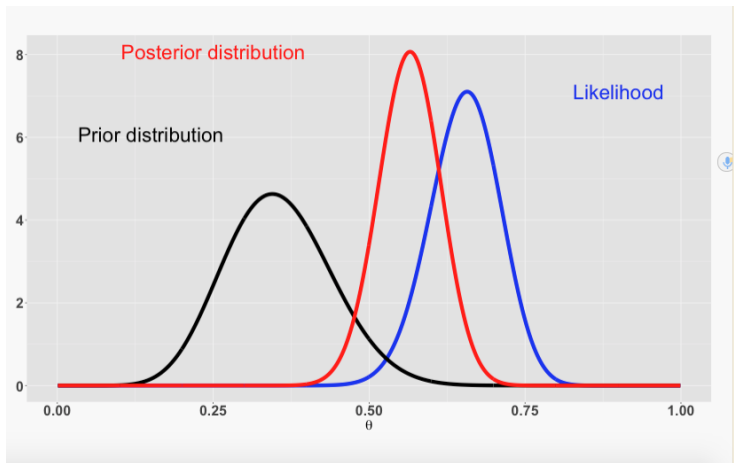


Figure 2: Beta(11,20) informative prior

Section 3

Beta prior + Binomial Likelihood

Beta posterior distribution based on: Beta prior \times Binomial Likelihood

Let $Y|\theta \sim \text{Binomial}(N, \theta)$ (**Likelihood**) and $\theta \sim \text{Beta}(a, b)$ (**Prior**) Then, $\theta|Y \sim \text{Beta}(a + y, b + N - y)$ (**Conjugate families**). We observe $y = 7$ successes out of $N = 10$ attempts.

Stan Code

```
beta_binomial2<-  
'data {  
  int<lower=0> N;  
  int<lower=0> y;  
}  
parameters {  
  real<lower=0,upper=1> theta;  
}  
model {  
  theta ~ beta(11,20); //Prior  
  y ~ binomial(N,theta); //Likelihood  
}  
'
```

Fit a model with Stan

```
fit_beta_binomial2 <- stan(model_code = beta_binomial2,  
  data = list(N = 10, y = 7),  
  chain = 3,  
  iter = 11000,  
  warmup = 1000,  
  thin = 10,  
  refresh=0)
```

Summary from the posterior distribution

```
fit_beta_binomial2
```

Inference for Stan model: b4b82b126fa209b8c37593acd50d81e7.

3 chains, each with iter=11000; warmup=1000; thin=10;

post-warmup draws per chain=1000, total post-warmup draws=3000.

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
theta	0.44	0.00	0.08	0.29	0.38	0.44	0.49	0.59	2636	1
lp__	-28.62	0.01	0.72	-30.66	-28.77	-28.34	-28.17	-28.11	2843	1

Samples were drawn using NUTS(diag_e) at Fri Aug 21 20:04:49 2020.

For each parameter, `n_eff` is a crude measure of effective sample size, and `Rhat` is the potential scale reduction factor on split chains (at convergence, `Rhat`=1).

MCMC diagnostics using the bayesplot package

```
traceplot(fit_beta_binomial2, pars = parameters,  
inc_warmup = TRUE)
```

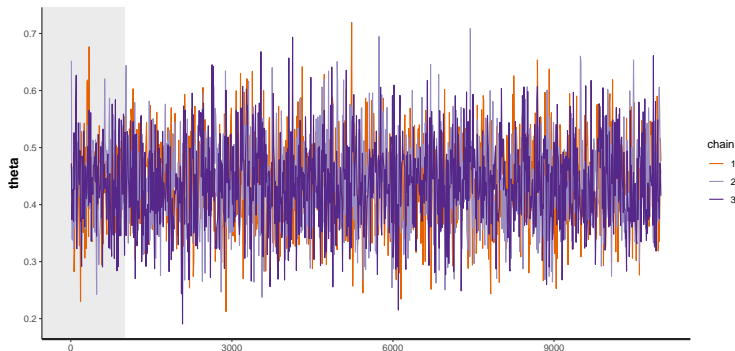


Figure 3: Traceplots for the Beta Binomial example

MCMC diagnostics using the bayesplot package

```
mcmc_combo(mcmc_chain2, pars = parameters,
            combo = c("hist", "dens"))
```

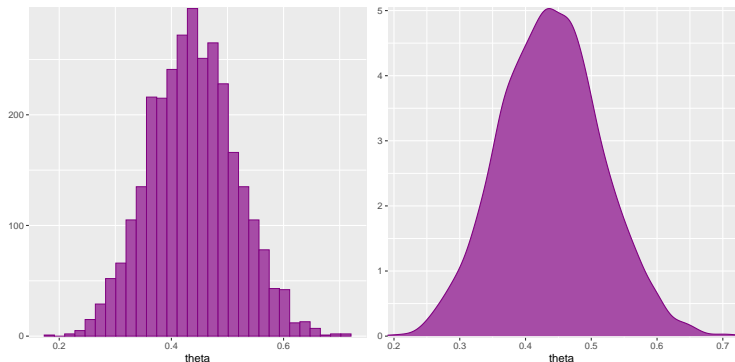


Figure 4: Posterior distributions and traceplots for the beta binomial example

Section 4

Bayesian Logistic Regression

Motivation (the proportion of dead beetles)

These are the number of adult flour beetles which died following a 5-hour exposure to gaseous carbon disulphide.

	ldose	n	y
1	1.691	59	6
2	1.724	60	13
3	1.755	62	18
4	1.784	56	28
5	1.811	63	52
6	1.837	59	53
7	1.861	62	61
8	1.884	60	60

Scatter plot of the proportion of dead beetles versus $\log(\text{Dose})$

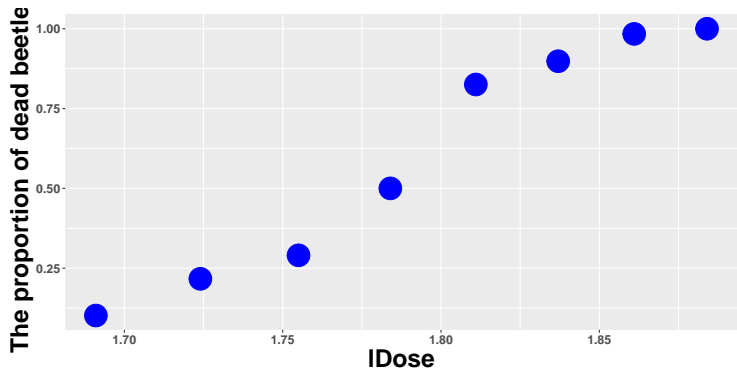


Figure 5: Scatter plot of the proportion of dead beetles versus $\log(\text{Dose})$

Bayesian approach

(Likelihood) $Y_i | \theta_i, x_i \sim \text{Bin}(n_i, \theta_i)$ ($i = 1, \dots, 8$), where

$$\text{logit}(\theta_i) = \log \left(\frac{\theta_i}{1 - \theta_i} \right) = \beta_1 + \beta_2 x_i \quad (2)$$

Prior distribution

- $\beta_1 \sim \text{Cauchy}(0, 10)$
- $\beta_2 \sim \text{Cauchy}(0, 2.5)$

(<http://www.stat.columbia.edu/~gelman/research/published/priors11.pdf>)

Stan code

```
logistic_example<- 'data {  
  int<lower=0> N;  
  vector[N] x;  
  int<lower=0> y[N];  
  int<lower=0> n[N];  
}  
parameters {  
  real beta1;  
  real beta2;  
}
```

Stan code

```
transformed parameters {
  real<lower=0, upper=1> prob[N];
  for (i in 1:N) {
    prob[i]=exp(beta1+beta2*x[i])/(exp(beta1+beta2*x[i])+1);
  }
  model {
    beta1 ~ cauchy(0,10);
    beta2 ~ cauchy(0,2.5);
    y ~ binomial_logit(n, beta1 + beta2 * x);
  }
}
```

Stan code

```
logistic_fit <- stan(model_code = logistic_example,  
  data = list(N = dim(beetleDat)[1],  
    n = beetleDat$n,  
    x = beetleDat$lDose,  
    y = beetleDat$y),  
  chain = 3,  
  iter = 11000,  
  warmup = 1000,  
  thin = 10,  
  refresh=0)
```


Summary from the posterior distribution

```
parameters<- c(paste('beta',1:2, sep=""))

CI_theta <- summary(logistic_fit,
                    pars = parameters,
                    probs = c(0.025, 0.975))$summary
print(round(CI_theta,3))
```

	mean	se_mean	sd	2.5%	97.5%	n_eff	Rhat
beta1	-59.687	0.104	5.097	-69.836	-50.036	2405.006	0.999
beta2	33.694	0.059	2.867	28.276	39.354	2396.101	0.999

MCMC diagnostics using the bayesplot package

```
traceplot(logistic_fit, pars = parameters,  
          inc_warmup = TRUE)
```

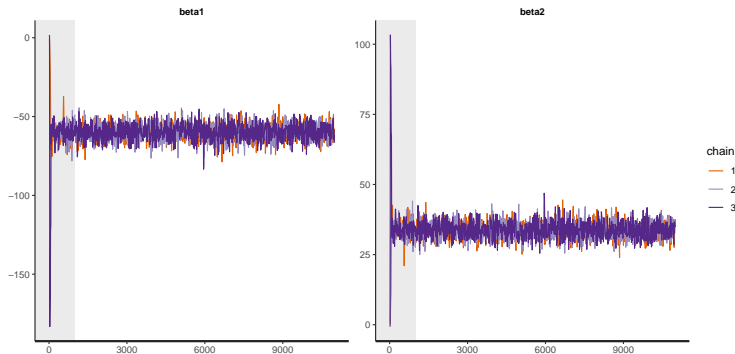


Figure 6: Traceplots for the bayesian logistic regression

MCMC diagnostics using the bayesplot package

```
mcmc_combo(mcmc_chain, pars = parameters,
            combo = c("hist", "dens"))
```

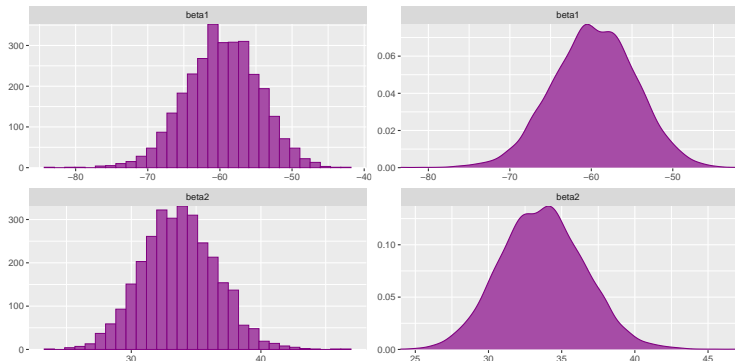


Figure 7: Posterior distributions and traceplots for the bayesian logistic regression

Fitted curve based on bayesian inference

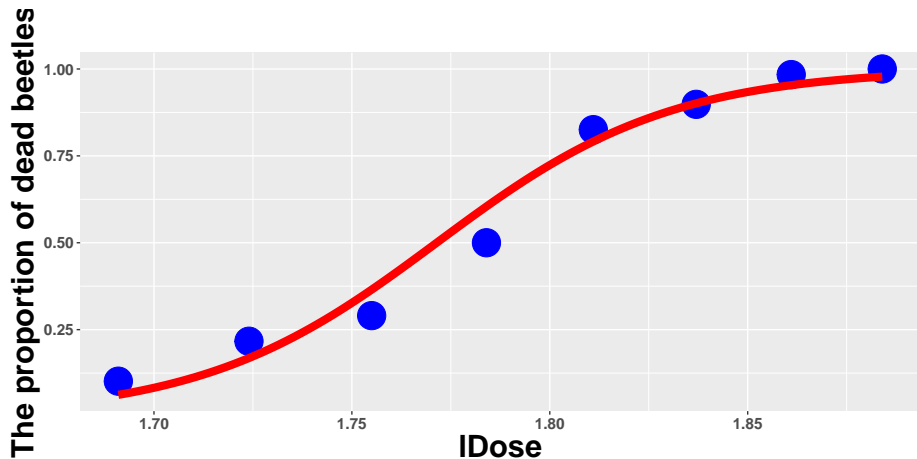


Figure 8: Fitted curve based on the bayesian logistic regression

Do I have more time for the shinystan r package?

```
rm(list=ls())  
load("logistic_fit1.Rdata")  
launch_shinystan(logistic_fit)
```

Section 5

More R packages based on Stan

More R packages based on Stan

- Bayesian Applied Regression Modeling via Stan: 'rstanarm' r package.
- Interactive Visual and Numerical Diagnostics and Posterior Analysis for Bayesian Models 'shinystan' r package.

Section 6

References

References

- Baptiste Auguie (2017). gridExtra: Miscellaneous Functions for "Grid" Graphics. R package version 2.3.
<https://CRAN.R-project.org/package=gridExtra>
- Jonah Gabry and Tristan Mahr (2020). bayesplot: Plotting for Bayesian Models. R package version 1.7.2.
<https://CRAN.R-project.org/package=bayesplot>
- Jonah Gabry (2018). shinystan: Interactive Visual and Numerical Diagnostics and Posterior Analysis for Bayesian Models. R package version 2.5.0. <https://CRAN.R-project.org/package=shinystan>
- Stan Development Team (2018). RStan: the R interface to Stan. R package version 2.18.2. <http://mc-stan.org/>.

References

- <http://www.stat.columbia.edu/~gelman/research/published/Stan-paper-aug-2015.pdf>
- https://mc-stan.org/docs/2_24/stan-users-guide/index.html
- https://mc-stan.org/docs/2_24/reference-manual/index.html
- https://mc-stan.org/docs/2_24/functions-reference/index.html
- <https://cran.r-project.org/web/views/Bayesian.html>
- <https://www.youtube.com/watch?v=uSjsJg8fcwY>