Theory of Infectious Disease

Homework

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Fall Term, 2017

Different forms of the SIR equations for a closed population.

In class, we derived the SIR equations for a closed population—one without births, deaths, or migration—in terms of the *numbers* of hosts in each of the S, I, and R compartments. We had

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -\beta\,\frac{X\,Y}{N}$$
$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \beta\,\frac{X\,Y}{N} - \gamma\,Y$$
$$\frac{\mathrm{d}Z}{\mathrm{d}t} = \gamma\,Y$$

where X, Y, and Z are, respectively, the numbers in each of the S, I, and R compartments, and $N = X+Y+Z$ is the total population size. Recall that β is called the *transmission rate* and γ, the *recovery rate*.

(a) Formally change variables to recast the equations in terms of $S = X/N$, $I = Y/N$, and $R = Z/N$.
(b) The above equations assume *frequency-dependent transmission*, i.e., $\lambda = \beta\,Y/N$. Write down the corresponding equations under the assumption of density-dependent transmission, and recast the equations, again, in terms of the fractional occupancy of each compartment.
(c) Compare the resulting equations with those you derived in part (a). Discuss.