

Theory of Infectious Disease

Homework

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Different forms of the SIR equations for a closed population.

In class, we derived the SIR equations for a closed population—one without births, deaths, or migration—in terms of the *numbers* of hosts in each of the S, I, and R compartments. We had

$$\begin{aligned}\frac{dX}{dt} &= -\beta \frac{XY}{N} \\ \frac{dY}{dt} &= \beta \frac{XY}{N} - \gamma Y \\ \frac{dZ}{dt} &= \gamma Y\end{aligned}$$

where X , Y , and Z are, respectively, the numbers in each of the S, I, and R compartments, and $N = X + Y + Z$ is the total population size. Recall that β is called the *transmission rate* and γ , the *recovery rate*.

- (a) Formally change variables to recast the equations in terms of $S = X/N$, $I = Y/N$, and $R = Z/N$.
- (b) The above equations assume *frequency-dependent transmission*, i.e., $\lambda = \beta Y/N$. Write down the corresponding equations under the assumption of density-dependent transmission, and recast the equations, again, in terms of the fractional occupancy of each compartment.
- (c) Compare the resulting equations with those you derived in part (a). Discuss.

Functional form of the force of infection

With respect to an infection you work on, describe verbally and/or mathematically the form of the force of infection. Explain your reasoning. In particular, you can make an argument for frequency- or density-dependent transmission, or something else.