Chapter 1

Library clodomir_lists

Capítulo 3 - Working with Structured Data (Lists)

Par Ordenado

```
Inductive natprod : Type :=
  | pair (x y : nat) : natprod.
```

```
Check (pair 3 5).
```

Projeção x

```
Definition fst (p : natprod) : nat :=
  match p with
  | pair x y ⇒ x
  end.
```

Projeção y

```
Definition snd (p : natprod) : nat :=
  match p with
  | pair x y ⇒ y
  end.
```

```
Notation "( x , y )" := (pair x y).
```

Projeção x

```
Definition fst' (p : natprod) : nat :=
  match p with
  | (x, y) ⇒ x
  end.
```

Projeção y

```
Definition snd' (p : natprod) : nat :=
  match p with
  | (x, y) ⇒ y
```

```
  end.
```

Troca de Componentes

```
Definition swap_pair (p : natprod) : natprod :=
  match p with
  | (x, y) => (y, x)
  end.
```

Sobrejetividade

```
Theorem surjective_pairing' : ∀ (x y : nat),
  (x, y) = (fst (x, y), snd (x, y)).

Theorem surjective_pairing : ∀ (p : natprod),
  p = (fst p, snd p).
```

Exercise: 1 star, standard (snd_fst_is_swap)

```
Theorem snd_fst_is_swap : ∀ (p : natprod),
  (snd p, fst p) = swap_pair p.
```

Exercise: 1 star, standard, optional (fst_swap_is_snd)

```
Theorem fst_swap_is_snd : ∀ (p : natprod),
  fst (swap_pair p) = snd p.
```

Lista de Naturais

```
Inductive natlist : Type :=
  | nil : natlist
  | cons (x : nat) (l : natlist) : natlist.
```

Notações

```
Notation "x :: l" := (cons x l) (at level 60, right associativity).

Notation "[ ]" := nil.

Notation "[ x ; .. ; y ]" := (cons x .. (cons y nil) ..).
```

Definições Equivalentes

```
Definition mylist1 := cons 1 (cons 2 (cons 3 nil)).

Print mylist1.

Definition mylist2 := 1 :: (2 :: (3 :: nil)).

Print mylist2.

Definition mylist3 := 1 :: 2 :: 3 :: nil.

Print mylist3.

Definition mylist4 := [1; 2; 3].

Print mylist4.
```

Função Repetir

```
Fixpoint repeat (x count : nat) : natlist :=
  match count with
  | O ⇒ nil
  | S count' ⇒ x :: (repeat x count')
  end.
```

Função Comprimento

```
Fixpoint length (l : natlist) : nat :=
  match l with
  | nil ⇒ O
  | h :: t ⇒ S (length t)
  end.
```

Função Concatenar

```
Fixpoint app (l1 l2 : natlist) : natlist :=
  match l1 with
  | nil ⇒ l2
  | h :: t ⇒ h :: (app t l2)
  end.
```

```
Notation "x ++ y" := (app x y) (right associativity, at level 60).
Example test_app1: [1; 2; 3] ++ [4; 5] = [1; 2; 3; 4; 5].
Example test_app2: nil ++ [4; 5] = [4; 5].
Example test_app3: [1; 2; 3] ++ nil = [1; 2; 3].
```

Função Cabeça

```
Definition hd (default : nat) (l : natlist) : nat :=
  match l with
  | nil ⇒ default
  | h :: t ⇒ h
  end.
```

```
Example test_hd1: hd 0 [1; 2; 3] = 1.
Example test_hd2: hd 0 [] = 0.
```

Função Cauda

```
Definition tl (l : natlist) : natlist :=
  match l with
  | nil ⇒ nil
  | h :: t ⇒ t
  end.
```

```
Example test_tl1: tl [1; 2; 3] = [2; 3].
```

Example $test\_tl2$: tl [1; 2; 3; 4] = [2; 3; 4].

Exercise: 2 stars, standard, recommended (list_funs)

```
Fixpoint nonzeros (l : natlist) : natlist :=
  match l with
  | nil ⇒ nil
  | O :: t ⇒ nonzeros t
  | h :: t ⇒ h :: (nonzeros t)
  end.
```

Example $test\_nonzeros$: $nonzeros$ [0; 1; 0; 2; 3; 0; 0] = [1; 2; 3].

```
Fixpoint oddmembers (l : natlist) : natlist :=
  match l with
  | nil ⇒ nil
  | h :: t ⇒ match (oddb h) with
    | true ⇒ h :: (oddmembers t)
    | false ⇒ oddmembers t
    end
  end.
```

Example $test\_oddmembers$: $oddmembers$ [0; 1; 0; 2; 3; 0; 0] = [1; 3].

```
Fixpoint countoddmembers (l : natlist) : nat :=
  match l with
  | nil ⇒ O
  | h :: t ⇒ if (oddb h) then 1 + (countoddmembers t) else countoddmembers t
  end.
```

Example $test\_countoddmembers1$: $countoddmembers$ [1; 0; 3; 1; 4; 5] = 4.

Example $test\_countoddmembers2$: $countoddmembers$ [0; 2; 4] = 0.

Example $test\_countoddmembers3$: $countoddmembers$ nil = 0.

Exercise: 3 stars, advanced (alternate)

```
Fixpoint alternate (l1 l2 : natlist) : natlist :=
  match l1, l2 with
  | nil, nil ⇒ nil
  | nil, yb ⇒ yb
  | xb, nil ⇒ xb
  | x :: xb, y :: yb ⇒ x :: y :: alternate xb yb
  end.
```

Example $test\_alternate1$: $alternate$ [1; 2; 3] [4; 5; 6] = [1; 4; 2; 5; 3; 6].

Example $test\_alternate2$: $alternate$ [1] [4; 5; 6] = [1; 4; 5; 6].

Example $test\_alternate3$: $alternate$ [1; 2; 3] [4] = [1; 4; 2; 3].

Example $test\_alternate4$: $alternate$ [] [20;30] = [20;30].

```
Definition bag := natlist.
```

Exercise: 3 stars, standard, recommended (bag_functions)

```
Fixpoint count (x : nat) (b : bag) : nat :=
  match b with
  | nil => O
  | h :: t => if eqb x h then S (count x t) else count x t
  end.

Example test_count1: count 1 [1; 2; 3; 1; 4; 1] = 3.

Example test_count2: count 6 [1; 2; 3; 1; 4; 1] = 0.

Definition sum : bag -> bag -> bag := app.

Example test_sum1: count 1 (sum [1; 2; 3] [1; 4; 1]) = 3.

Definition add (x : nat) (b : bag) : bag := x :: b.

Example test_add1: count 1 (add 1 [1; 4; 1]) = 3.

Example test_add2: count 5 (add 1 [1; 4; 1]) = 0.

Definition member (x : nat) (b : bag) : bool :=
  match (count x b) with
  | O => false
  | _ => true
  end.

Example test_member1: member 1 [1; 4; 1] = true.

Example test_member2: member 2 [1; 4; 1] = false.
```

Exercise: 3 stars, standard, optional (bag_more_functions)

```
Fixpoint remove_one (x : nat) (b : bag) : bag :=
  match b with
  | nil => nil
  | h :: t => if eqb x h then t else h :: remove_one x t
  end.

Example test_remove_one1: count 5 (remove_one 5 [2; 1; 5; 4; 1]) = 0.

Example test_remove_one2: count 5 (remove_one 5 [2; 1; 4; 1]) = 0.

Example test_remove_one3: count 4 (remove_one 5 [2; 1; 4; 5; 1; 4]) = 2.

Example test_remove_one4: count 5 (remove_one 5 [2; 1; 5; 4; 5; 1; 4]) = 1.

Fixpoint remove_all (x : nat) (b : bag) : bag :=
  match b with
  | nil => nil
  | h :: t => if eqb x h then remove_all x t else h :: remove_all x t
  end.

Example test_remove_all1: count 5 (remove_all 5 [2; 1; 5; 4; 1]) = 0.
```

```
Example test_remove_all2: count 5 (remove_all 5 [2; 1; 4; 1]) = 0.

Example test_remove_all3: count 4 (remove_all 5 [2; 1; 4; 5; 1; 4]) = 2.

Example test_remove_all4: count 5 (remove_all 5 [2; 1; 5; 4; 5; 1; 4; 5; 1; 4]) = 0.

Fixpoint subset (b1 : bag) (b2 : bag) : bool :=
  match b1 with
  | nil => true
  | h :: t => andb (member h b2) (subset t (remove_one h b2))
  end.

Example test_subset1: subset [1; 2] [2; 1; 4; 1] = true.

Example test_subset2: subset [1; 2; 2] [2; 1; 4; 1] = false.
```

Exercise: 2 stars, standard, recommended (bag_theorem)

```
Theorem bag_theorem : ∀ (b : bag), ∀ (x : nat),
  S (length b) = length (add x b).
```

Raciocínio sobre listas

```
Theorem nil_app : ∀ l : natlist,
  [] ++ l = l.

Theorem tl_length_pred : ∀ l : natlist,
  pred (length l) = length (tl l).

Theorem app_assoc : ∀ l1 l2 l3 : natlist,
  (l1 ++ l2) ++ l3 = l1 ++ (l2 ++ l3).

Fixpoint rev (l : natlist) : natlist :=
  match l with
  | nil => nil
  | h :: t => rev t ++ [h]
  end.

Example test_rev1: rev [1; 2; 3] = [3; 2; 1].

Example test_rev2: rev nil = nil.

Theorem app_length : ∀ l1 l2 : natlist,
  length (l1 ++ l2) = (length l1) + (length l2).

Theorem rev_length : ∀ l : natlist,
  length (rev l) = length l.
```

Exercise: 3 stars, standard (list_exercises)

```
Theorem app_nil_r : ∀ l : natlist,
  l ++ [] = l.

Theorem rev_app_distr: ∀ l1 l2 : natlist,
  rev (l1 ++ l2) = rev l2 ++ rev l1.

Theorem rev_involutive : ∀ l : natlist,
```

$rev\ (rev\ l) = l.$

Theorem $app\_assoc4 : \forall\ l1\ l2\ l3\ l4 : natlist,$
$l1 ++ (l2 ++ (l3 ++ l4)) = ((l1 ++ l2) ++ l3) ++ l4.$

Lemma $nonzeros\_app : \forall\ l1\ l2 : natlist,$
$nonzeros\ (l1 ++ l2) = (nonzeros\ l1) ++ (nonzeros\ l2).$

Exercise: 2 stars, standard (eqblist)

```
Fixpoint eqblist (l1 l2 : natlist) : bool :=
  match l1, l2 with
  | [], [] ⇒ true
  | _, [] ⇒ false
  | [], _ ⇒ false
  | h1 :: t1, h2 :: t2 ⇒ if eqb h1 h2 then eqblist t1 t2 else false
  end.
```

```
Example test_eqblist1 : (eqblist nil nil = true).
```

```
Example test_eqblist2 : eqblist [1; 2; 3] [1; 2; 3] = true.
```

```
Example test_eqblist3 : eqblist [1; 2; 3] [1; 2; 4] = false.
```

```
Theorem eqblist_refl : ∀ l : natlist,
  true = eqblist l l.
```

Exercise: 1 star, standard (count_member_nonzero)

```
Theorem count_member_nonzero : ∀ (b : bag),
  leb 1 (count 1 (1 :: b)) = true.
```

```
Theorem leb_x_Sx : ∀ x,
  leb x (S x) = true.
```

Exercise: 3 stars, advanced (remove_does_not_increase_count)

```
Theorem remove_does_not_increase_count: ∀ (b : bag),
  leb (count 0 (remove_one 0 b)) (count 0 b) = true.
```

Exercise: 3 stars, standard, optional (bag_count_sum)
Falta fazer
Exercise: 4 stars, advanced (rev_injective)

```
Theorem rev_injective : ∀ (l1 l2 : natlist),
  rev l1 = rev l2 → l1 = l2.
```

Opções

```
Fixpoint xth_bad (l : natlist) (x : nat) : nat :=
  match l with
  | nil ⇒ 42  arbitrário    | a :: l' ⇒ match eqb x O with
    | true ⇒ a
    | false ⇒ xth_bad l' (pred x)
```

```
    end
  end.

Inductive natoption : Type :=
  | Some (x : nat) : natoption
  | None : natoption.

Fixpoint xth_error (l : natlist) (x : nat) : natoption :=
  match l with
  | nil ⇒ None
    | a :: l' ⇒ match eqb x O with
      | true ⇒ Some a
      | false ⇒ xth_error l' (pred x)
    end
  end.

Example test_xth_error1 : xth_error [4; 5; 6; 7] 0 = Some 4.

Example test_xth_error2 : xth_error [4; 5; 6; 7] 3 = Some 7.

Example test_xth_error3 : xth_error [4; 5; 6; 7] 9 = None.

Fixpoint xth_error' (l : natlist) (x : nat) : natoption :=
  match l with
  | nil ⇒ None
  | a :: l' ⇒ if eqb x O then Some a else xth_error' l' (pred x)
  end.

Definition option_elim (x : nat) (o : natoption) : nat :=
  match o with
  | Some n' ⇒ n'
  | None ⇒ x
  end.
```

Exercise: 2 stars (hd_error)

```
Definition hd_error (l : natlist) : natoption :=
  match l with
  | [] ⇒ None
  | h :: _ ⇒ Some h
  end.

Example test_hd_error1 : hd_error [] = None.

Example test_hd_error2 : hd_error [1] = Some 1.

Example test_hd_error3 : hd_error [5; 6] = Some 5.
```

Exercise: 1 star, optional (option_elim_hd)

```
Theorem option_elim_hd : ∀ (l : natlist) (default : nat),
  hd default l = option_elim default (hd_error l).
```

Partial Maps

```
Inductive id : Type :=
  | Id (x : nat) : id.
```

```
Definition eqb_id (x1 x2 : id) :=
  match x1, x2 with
    | Id n1, Id n2 ⇒ eqb n1 n2
  end.
```

Exercise: 1 star (eqb_id_refl)

```
Theorem eqb_id_refl : ∀ x, true = eqb_id x x.
```

```
Inductive partial_map : Type :=
  | empty : partial_map
  | record (i : id) (x : nat) (m : partial_map) : partial_map.
```

```
Definition update (m : partial_map) (i : id) (x : nat) : partial_map :=
  record i x m.
```

```
Fixpoint find (i : id) (m : partial_map) : natoption :=
  match m with
  | empty ⇒ None
  | record x y m' ⇒ if eqb_id i x then Some y else find i m'
  end.
```

Exercise: 1 star (update_eq)

```
Theorem update_eq : ∀ (m : partial_map) (i : id) (x: nat),
    find i (update m i x) = Some x.
```

Exercise: 1 star (update_neq) `Theorem update_neq : ∀ (m : partial_map) (x y : id) (n: nat),`

```
    eqb_id x y = false → find x (update m y n) = find x m.
```

Exercise: 2 stars (baz_num_elts)

```
Inductive baz : Type :=
  | Baz1 (x : baz) : baz
  | Baz2 (y : baz) (b : bool) : baz.
```

How *many* elements does the type *baz* have?
Falta fazer