Maude's Prover. System G3c (Classical Logic)

April 23, 2020

1 Cut-Elimination

Cases $\rightarrow_R - \rightarrow_R$

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\begin{array}{c} \frac{\text{h13}:\Delta11,\text{F8},\text{F18} \vdash \Delta10,\text{F9}}{\text{h13}\bullet:\Delta11,\text{F18} \vdash \Delta10,\text{F8} \to \text{F9}} \to_{R} & \frac{\text{h13}:\Delta14,\text{F15} \vdash \Delta17,\text{F16},\text{F18}}{\text{h13}\bullet:\Delta14 \vdash (\Delta17,\text{F15} \to \text{F16}),\text{F18}} & \xrightarrow{R} \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\text{F8} \to \text{F9},\Delta17,\text{F15} \to \text{F16}} \\ & \xrightarrow{} \\ \hline \frac{\text{h13}:\Delta11,\text{F18},\text{F8} \vdash \Delta10,\text{F9}}{\text{h13}\bullet:\Delta11,\text{F18} \vdash \Delta10,\text{F9}} \to_{R} & \frac{\text{ax}}{\text{h13}:\Delta14,\text{F15} \vdash \Delta17,\text{F16},\text{F18}} \\ \hline & \xrightarrow{} \\ \hline -:\Delta11,\Delta14,\text{F15} \vdash \Delta10,\Delta17,\text{F16},\text{F8} \to \text{F9}} \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F15} \to \text{F16},\text{F8} \to \text{F9}} & \xrightarrow{} \\ \hline \\ \hline \frac{\text{h13}:\Delta11,\text{F8},\text{F15} \to \text{F16} \vdash \Delta10,\text{F9}}{\text{-}:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F15} \to \text{F16},\text{F8} \to \text{F9}} & \xrightarrow{} \\ \hline \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F15} \to \text{F16},\text{F8} \to \text{F9}} \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F15} \to \text{F16},\text{F8} \to \text{F9}} & \xrightarrow{} \\ \hline \\ \hline \\ \hline \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F15} \to \text{F16},\text{F8} \to \text{F9}} \\ \hline \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F17},\Delta10,\text{F8} \to \text{F9}} \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline -:\Delta11,\Delta14 \vdash \Delta10,\text{F8} \to \text{F9}} \\ \hline \\ \hline \\ \hline \\ \hline -:\Delta11,\Delta14,\text{F15} \to \text{F16} \vdash \Delta10,\text{F9}} \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline -:\Delta11,\Delta14,\text{F8} \vdash \Delta10,\Delta17,\text{F9}} \\ \hline \\ \hline \\ \hline \\ \hline -:\Delta11,\Delta14,\text{F8} \vdash \Delta10,\Delta17,\text{F9}} \\ \hline \\ \hline \\ \hline \\ \hline -:\Delta11,\Delta14,\text{F8} \vdash \Delta10,\Delta17,\text{F9}} \\ \hline \end{array}
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Cases \rightarrow_R - \wedge_R

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\frac{\text{h}13 : \Delta 11, \text{F8}, \text{F18} \vdash \Delta 10, \text{F9}}{\text{h}13 \bullet : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \to \text{F9}} \to_{R} \frac{\text{h}13 : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18}}{\text{h}13 \bullet : \Delta 14 \vdash (\Delta 17, \text{F15} \land \text{F16}), \text{F18}} \text{Cut}} \land_{R}
\frac{-: \Delta 11, \Delta 14 \vdash \Delta 10, \text{F8} \to \text{F9}, \Delta 17, \text{F15} \land \text{F16}}{\text{h}13 \bullet : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18}} \text{ax}} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18}} \text{Az}} \uparrow_{R}
\frac{\text{h}13 : \Delta 11, \text{F18}, \text{F8} \vdash \Delta 10, \text{F9}}{\text{h}13 \cdot \Delta 11, \text{F18}, \text{F8} \vdash \Delta 10, \text{F9}}} \text{ax} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18}} \text{ax}} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18}}} \uparrow_{R}
\frac{\text{h}13 : \Delta 11, \text{F18}, \text{F15} \land \text{F16} \vdash \Delta 10, \text{F9}}{\text{h}13 \cdot \Delta 11, \text{F15}, \text{F16} \vdash \Delta 10, \text{F9}}} \rightarrow_{R} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F16}}} \uparrow_{R}
\frac{\text{h}13 : \Delta 11, \text{F8}, \text{F15} \land \text{F16} \vdash \Delta 10, \text{F9}}{\text{h}13 \cdot \Delta 11, \text{F15}, \text{F16} \vdash \Delta 10, \text{F8} \to \text{F9}}} \rightarrow_{R} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F16}}} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F16}}} \uparrow_{R}
\frac{\text{h}13 : \Delta 11, \text{F15}, \text{F16} \vdash \Delta 10, \text{F8} \to \text{F9}}{\text{h}13 \cdot \Delta 14 \vdash \Delta 17, \text{F15}}} \rightarrow_{R} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F16}}} \uparrow_{h13 : \Delta 14 \vdash \Delta 17, \text{F16}}} \uparrow_{R}
\frac{\text{h}13 : \Delta 11, \text{F8}, \text{F15} \land \text{F16} \vdash \Delta 10, \text{F9}}}{\text{h}13 \cdot \Delta 14 \vdash \Delta 17, \text{F15}}} \xrightarrow{\text{h}13 : \Delta 14 \vdash \Delta 17, \text{F16}}} \uparrow_{R} \uparrow_{R}
\frac{\text{h}13 : \Delta 11, \text{F8}, \text{F15} \land \text{F16} \vdash \Delta 10, \text{F9}}}{\text{h}13 \cdot \Delta 14 \vdash \Delta 17, \text{F15}}} \xrightarrow{\text{h}13 : \Delta 14 \vdash \Delta 17, \text{F16}}} \uparrow_{R} \uparrow_{
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Cases \rightarrow_R - \vee_R

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\frac{ \frac{\text{h13} : \Delta 11, \text{F8}, \text{F18} \vdash \Delta 10, \text{F9}}{\text{h13} \bullet : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \rightarrow \text{F9}} }{-: \Delta 11, \text{L14} \vdash \Delta 10, \text{F8} \rightarrow \text{F9}} \xrightarrow{} R \xrightarrow{ \frac{\text{h13} : \Delta 14}{\text{h13} \bullet : \Delta 14} \vdash (\Delta 17, \text{F15}, \text{F16}, \text{F18}}{\text{h13} \bullet : \Delta 14} \vdash (\Delta 17, \text{F15} \lor \text{F16}), \text{F18}}} \xrightarrow{} \text{Cut} \\ \frac{-: \Delta 11, \Delta 14}{\text{L13} : \Delta 11, \text{F18}, \text{F8} \vdash \Delta 10, \text{F9}} \xrightarrow{\text{ax}} \xrightarrow{\frac{\text{h13} : \Delta 14} \vdash \Delta 17, \text{F15}, \text{F16}, \text{F18}}{\text{h13} \bullet : \Delta 14} \xrightarrow{} \text{hCut}} \xrightarrow{} \frac{\text{ax}}{\text{hCut}} \\ \frac{-: \Delta 11, \Delta 14, \text{F8} \vdash \Delta 10, \Delta 17, \text{F9}, \text{F15} \lor \text{F16}}{-: \Delta 11, \Delta 14} \xrightarrow{} \text{hCut}} \xrightarrow{} \frac{\text{hCut}}{\text{hCut}}
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\frac{\mathsf{h}13:\Delta11,\mathsf{F}8,\mathsf{F}15\vee\mathsf{F}16\vdash\Delta10,\mathsf{F}9}{\mathsf{h}13\underbrace{\bullet}:\Delta11,\mathsf{F}15\vee\mathsf{F}16\vdash\Delta10,\mathsf{F}8\to\mathsf{F}9} \ \to_R \ \frac{\mathsf{h}13:\Delta14\vdash\Delta17,\mathsf{F}15,\mathsf{F}16}{\mathsf{h}13\bullet:\Delta14\vdash\Delta17,\mathsf{F}15\vee\mathsf{F}16} \ \vee_R \text{Cut}
                                                                                                                                                                                                                                                                                                                 -: \Delta 11, \Delta 14 \vdash \Delta 17, \Delta 10, F8 \rightarrow F9
                                                                                                                                                                                                                               \frac{1}{2} \frac{1}
                                                                                                                                                                                                                                                                           \frac{-:\Delta 11,\Delta 14,F8\vdash \Delta 10,\Delta 17,F9}{-:\Delta 11,\Delta 14\vdash \Delta 10,\Delta 17,F8\to F9}\to_R
Cases \rightarrow_R - \perp_R
                                                                                                                                                                                                                                                    \begin{array}{l} \frac{\text{h13}:\Delta11,\text{F8},\bot\vdash\Delta10,\text{F9}}{\text{h13}\bullet:\Delta11,\bot\vdash\Delta10,\text{F8}\to\text{F9}} \to_{R} & \frac{\text{h13}:\Delta14\vdash\Delta15}{\text{h13}\bullet:\Delta14\vdash\Delta15,\bot} \\ & -:\Delta11,\Delta14\vdash\Delta15,\Delta10,\text{F8}\to\text{F9} \end{array} \xrightarrow{L_{R}} \text{Cut} 
                                                                                                                                                                                                                                                                                                                      \frac{}{-:\Delta 11,\Delta 14\vdash \Delta 10,\Delta 15,F8\to F9} \quad \text{ax}
                                                                                                                                                                                                                                  \frac{\text{h13}:\Delta 11, \text{F8}, \text{F16} \vdash \Delta 10, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F16} \vdash \Delta 10, \text{F8} \rightarrow \text{F9}} \rightarrow_{R} \frac{\text{h13}:\Delta 14 \vdash \Delta 15, \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash (\bot, \Delta 15), \text{F16}} \stackrel{\bot_{R}}{\xrightarrow{\text{opt}}}
                                                                                                                                                                                                                                                         -:\Delta11,\Delta14\vdash\Delta10, F8\to F9,\bot,\Delta15
                                                                                                                                                                                                                                                   \boxed{ \frac{}{\mathtt{h}13:\Delta11,\mathtt{F}1}6,\mathtt{F}8 \vdash \Delta10,\mathtt{F}9} \quad \text{ax} \quad
                                                                                                                                                                                                                                          \begin{array}{c} \frac{\text{h13}:\Delta11,\text{F16},\text{F8}\vdash\Delta10,\text{F9}}{\text{h13}\bullet:\Delta11,\text{F16}\vdash\Delta10,\text{F8}\to\text{F9}} & \rightarrow_{R} \\ \\ \frac{\text{h13}:\Delta14\vdash\bot,\Delta15,\text{F16}}{\text{hCut}} & \text{ax} \\ \\ \text{hCut} & \text{hCut} \end{array}
                                                                                                                                                                                                                                                                                                             -: \Delta 11, \Delta 14 \vdash \bot, \Delta 10, \Delta 15, F8 \rightarrow F9
Cases \to_R - \top_R
                                                                                                                                                                                                                                                      \frac{\text{h13}:\Delta11,\text{F8},\top\vdash\Delta10,\text{F9}}{\text{h13}\bullet:\underline{\Delta}11,\top\vdash\Delta10,\text{F8}\to\text{F9}} \to_{R} \frac{}{\text{h13}\bullet:\Delta14\vdash\Delta15,\top} \xrightarrow{}_{\text{Cut}}
                                                                                                                                                                                                                                                                                         -:\Delta11,\Delta14\vdash\Delta15,\Delta10,\mathsf{F8}\to\mathsf{F9}
                                                                                                                                                                                                                                                                    \frac{\frac{1}{\text{b13}: \top, \Delta 11, \text{F8} \vdash \Delta 10, \text{F9}} \xrightarrow{\text{ax}} \frac{}{\text{b13}\bullet: \Delta 14 \vdash \top, \Delta 15}}{\frac{-: \Delta 11, \Delta 14, \text{F8} \vdash \Delta 10, \Delta 15, \text{F9}}{-: \Delta 11, \Delta 14 \vdash \Delta 10, \Delta 15, \text{F8} \rightarrow \text{F9}} \xrightarrow{} R} \xrightarrow{\text{hCut}}
                                                                                                                                                                                                                                           h13 : \Delta11, F8, F16 \vdash \Delta10, F9
                                                                                                                                                                                                                                  \frac{\text{h13}: \Delta11, \text{F8}, \text{F16} \vdash \Delta10, \text{F9}}{\text{h13}\bullet: \Delta11, \text{F16} \vdash \Delta10, \text{F8} \rightarrow \text{F9}} \xrightarrow{} \xrightarrow{}_{R} \frac{1}{\text{h13}\bullet: \Delta14 \vdash (\top, \Delta15), \text{F16}} \leftarrow \text{Cut}
                                                                                                                                                                                                                                                                                                            \frac{}{-:\Delta 11,\, \Delta 14 \vdash \top,\, \Delta 10,\, \Delta 15, \mathrm{F8} \, \rightarrow \mathrm{F9}} \ ^{\top}R
Cases \rightarrow_R - \rightarrow_L
                                                                                                                                                            \frac{\frac{\text{h13}:\Delta11,\text{F8},\text{F17}\vdash\Delta10,\text{F9}}{\text{h13}\bullet:\Delta11,\text{F17}\vdash\Delta10,\text{F8}\to\text{F9}}}{-:\Delta11,\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F14}} \xrightarrow{PA} \frac{\frac{\text{h13}:\Delta18\vdash\Delta16,\text{F14},\text{F17}}{\text{h13}\bullet:\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F17}}}{\frac{\text{h13}:\Delta18,\text{F15}\vdash\Delta16,\text{F17}}{\text{h13}\bullet:\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F17}}} \xrightarrow{\text{Cut}} \xrightarrow{\text{Cut}}
                                                                                                                                                                    Cases \to_R - \wedge_L
                                                                                                                                                                                                                    \frac{\frac{1}{\text{h13}:\Delta11,\text{F17},\text{F8}\vdash\Delta10,\text{F9}}}{\frac{-:\Delta11,\Delta18,\text{F14},\text{F15}\vdash\Delta16,\text{F17}}{-:\Delta11,\Delta18,\text{F8},\text{F14}\wedge\text{F15}\vdash\Delta10,\Delta16,\text{F9}}} \xrightarrow{\text{hx}} \frac{\wedge L}{\text{hcut}}
Cases \rightarrow_R - \vee_L
                                                                                                                                                              \frac{\frac{\text{h}13:\Delta11,\text{F}8,\text{F}17} \vdash \Delta10,\text{F}9}{\text{h}13\bullet:\Delta11,\text{F}17} \vdash \Delta10,\text{F}8 \to \text{F}9}{-:\Delta11,\Delta18,\text{F}14} \to R} \xrightarrow{\rightarrow} R \frac{\text{h}13:\Delta18,\text{F}14} \vdash \Delta16,\text{F}17}{\text{h}13\bullet:\Delta18,\text{F}14} \lor \text{F}15} \vdash \Delta16,\text{F}17}_{-:\Delta11,\Delta18,\text{F}14} \lor \text{F}15} \vdash \Delta16,\Delta10,\text{F}8 \to \text{F}9} \\ \xrightarrow{\text{Cut}} \text{Cut}
                                                                                                                                                                     \frac{\frac{\rightarrow}{\text{h13}:\Delta11,\text{F17},\text{F8}\vdash\Delta10,\text{F9}} \text{ax}}{\frac{-1}{\text{h13}:\Delta18,\text{F14}\vdash\Delta16,\text{F17}} \text{ax}} \frac{\frac{\rightarrow}{\text{h13}:\Delta18,\text{F15}\vdash\Delta16,\text{F17}}}{\text{h13}:\Delta18,\text{F15}\vdash\Delta16,\text{F17}} \text{ax}} \\ \frac{-:\Delta11,\Delta18,\text{F8},\text{F14}\vee\text{F15}\vdash\Delta10,\Delta16,\text{F9}}{-:\Delta11,\Delta18,\text{F14}\vee\text{F15}\vdash\Delta10,\Delta16,\text{F9}} \\ \rightarrow R} \text{hCut}
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Cases \rightarrow_R - \perp_L
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \frac{ \underset{1}{\underline{\mathsf{h}}13 \bullet} : \Delta 11, \mathsf{F8}, \mathsf{F15} \vdash \Delta 10, \mathsf{F9}}{\underline{\mathsf{h}}13 \bullet} \to_{R} \frac{}{\underline{\mathsf{h}}13 \bullet} : \Delta 11, \mathsf{F15} \vdash \Delta 10, \mathsf{F8} \to \mathsf{F9}} \xrightarrow{}_{R} \frac{}{\underline{\mathsf{h}}13 \bullet} : \bot, \Delta 16 \vdash \Delta 14, \mathsf{F15}} \underbrace{}_{\mathsf{Cut}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         -: \Delta 11, \perp, \Delta 16 \vdash \Delta 14, \Delta 10, f8 \rightarrow f9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Cases \rightarrow_R - I
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \frac{\text{h13}:\Delta 11, \text{F8}, \text{F15} \vdash \Delta 10, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F15} \vdash \Delta 10, \text{F8} \rightarrow \text{F9}} \xrightarrow{} R \xrightarrow{\text{h13}\bullet:\Delta 17, \text{p16} \vdash (\Delta 14, \text{p16}), \text{F15}} \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -:\Delta11,\Delta17,\mathtt{p}16\vdash\Delta10,\mathtt{f}8\to\mathtt{f}9,\Delta14,\mathtt{p}16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{}{-:\Delta 11,\Delta 17,\mathtt{p}16 \vdash \Delta 10,\Delta 14,\mathtt{p}16,\mathtt{f}8 \to \mathtt{f}9} \quad iI
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      \frac{\text{h13}:\Delta11, \text{F8}, \text{p15}\vdash\Delta10, \text{F9}}{\text{h13}\bullet:\Delta11, \text{p15}\vdash\Delta10, \text{F8}\to \text{F9}} \to_{R} \frac{}{\text{h13}\bullet:\Delta16, \text{p15}\vdash\Delta14, \text{p15}} \stackrel{\text{Cut}}{}{}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -:\Delta11,\Delta16, \mathtt{p}15 \vdash \Delta14,\Delta10, \mathtt{F}8 \to \mathtt{F}9 \\ \to
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \frac{ \overline{-:\Delta11,\Delta16, F8, p15 \vdash \Delta10, \Delta14, F9}}{-:\Delta11,\Delta16, p15 \vdash \Delta10,\Delta14, F8 \rightarrow F9} \overset{\text{ax}}{\rightarrow}_{R}
Cases \to_R - \top_L
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \frac{\underset{\bullet}{\text{h}13} : \Delta11, \text{F8}, \text{F15} \vdash \Delta10, \text{F9}}{\underset{-}{\text{h}13} \bullet : \Delta11, \text{F15} \vdash \Delta10, \text{F8} \to \text{F9}} \xrightarrow{} A \quad \frac{\underset{\bullet}{\text{h}13} : \Delta16 \vdash \Delta14, \text{F15}}{\underset{\bullet}{\text{h}13} \bullet : \top, \Delta16 \vdash \Delta14, \text{F15}}}{\underset{-}{\text{cut}}} \xrightarrow{} \Upsilon_L \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{13:\Delta11,F15,F8\vdash\Delta10,F9}{126:\Delta10} \text{ ax} \rightarrow
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \begin{array}{c} \overbrace{\text{h13}:\Delta\text{11},\text{F15},\text{F8}\vdash\Delta\text{10},\text{F9}}^{-1} \xrightarrow{}_{R} \\ \underline{\text{h13}\bullet:\Delta\text{11},\text{F15}\vdash\Delta\text{10},\text{F8}\to\text{F9}}^{-1} \xrightarrow{}_{R} \\ -:\top,\Delta\text{11},\Delta\text{16}\vdash\Delta\text{10},\Delta\text{14},\text{F8}\to\text{F9} \end{array} \quad \begin{array}{c} \text{ax} \\ \text{hCut} \end{array}
   Cases \wedge_R - \rightarrow_R
                                                                                                                                                                                                                                                                                                                                                                                                                           \frac{-:\Delta 11,\Delta 14,F15\vdash\Delta 10,\Delta 17,F16,F8\wedge F9}{-:\Delta 11,\Delta 14\vdash\Delta 10,\Delta 17,F15\to F16,F8\wedge F9}\to R
                                                                                                                                                                                                                                                                                                                                                                          \frac{\frac{\text{h13}:\Delta11,\text{F15}\to\text{F16}\vdash\Delta10,\text{F8}\quad\text{h13}:\Delta11,\text{F15}\to\text{F16}\vdash\Delta10,\text{F9}}{\frac{\text{h13}\bullet:\Delta11,\text{F15}\to\text{F16}\vdash\Delta10,\text{F8}\land\text{F9}}{-:\Delta11,\Delta14\vdash\Delta17,\Delta10,\text{F8}\land\text{F9}}} \; \land_{R} \; \frac{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}{\text{h13}\bullet:\Delta14\vdash\Delta17,\text{F15}\to\text{F16}} \; \xrightarrow{\mathcal{C}_{\text{R}}} \; \frac{\partial}{\partial \mathcal{C}_{\text{R}}} \; \frac{\partial}{\partial \mathcal{C}_{\text{R}}
                                                                                        \frac{1}{13:\Delta 11,F15\rightarrow F16\vdash \Delta 10,F8} = \frac{1}{13:\Delta 14,F15\vdash \Delta 17,F16} = \frac{1}{13:\Delta 14,F15\vdash \Delta 17,F16} = \frac{1}{13:\Delta 14,F15\vdash \Delta 17,F16} = \frac{1}{13:\Delta 11,F15\rightarrow F16\vdash \Delta 10,F9} = \frac{1}{13:\Delta 14,F15\vdash \Delta 17,F16} = \frac{1}{13:\Delta 14\vdash \Delta 17,F15\rightarrow F16} = \frac{1}{13:\Delta 11,F15\rightarrow F16\vdash \Delta 10,F9} = \frac{1}{13:\Delta 11,F15\rightarrow F16\vdash \Delta 10,F15\rightarrow F16\vdash \Delta
                                                                                                                                                                                                                                                                                                                                                    Cases \wedge_R - \wedge_R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{ \underbrace{ \text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \quad \text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F9} }_{\text{h13} \bullet} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{h13} \bullet} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F16}, \text{F16}, \text{F16}, \text{F16}, \text{F18} }_{\text{out}} \quad \wedge_{R} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{
                 \frac{\text{h13} \bullet : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{-: \Delta 11, \Delta 14 \vdash \Delta 10, \text{F8} \land \text{F9}} \qquad \text{h13} \bullet : \Delta 14 \vdash (\Delta 17, \text{F15} \land \text{F16}), \text{F18}}{-: \Delta 11, \Delta 14 \vdash \Delta 10, \text{F8} \land \text{F9}, \Delta 17, \text{F15} \land \text{F16}} \text{ cut}}
\frac{\text{h13} \bullet : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}}{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}} \qquad \text{ax} \qquad \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}}{\wedge R} \qquad \text{ax} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}}{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}} \qquad \text{ax} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}}{\wedge R} \qquad \frac{\text{ax}}{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}} \qquad \frac{\text{ax}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \land \text{F9}}{\wedge R} \qquad \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta
                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{\text{h13}:\Delta 11, \text{F15} \wedge \text{F16} \vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F15} \wedge \text{F16} \vdash \Delta 10, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F15} \wedge \text{F16} \vdash \Delta 10, \text{F8} \wedge \text{F9}} \quad \wedge_{R} \quad \frac{\text{h13}:\Delta 14 \vdash \Delta 17, \text{F15} \quad \text{h13}:\Delta 14 \vdash \Delta 17, \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \wedge_{R} \quad \frac{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \wedge_{R} \quad \frac{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \wedge_{R} \quad \frac{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \wedge_{R} \quad \frac{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \\ \frac{\text{h13}\cdot\Delta 11, \text{F15} \wedge \text{F16} \vdash \Delta 10, \text{F8} \wedge \text{F9}}{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \frac{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}}{\text{h13}\cdot\Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \quad \\ \frac{\text{h13}\cdot\Delta 11, \text{H13}\cdot\Delta 11, \text{H13
                     \frac{-:\Delta 11,\Delta 14}{\text{h}13:\Delta 11,F15 \wedge F16 \vdash \Delta 10,F8} \xrightarrow{\text{ax}} \frac{-:\Delta 11}{\text{h}13:\Delta 14 \vdash \Delta 17,F15} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F15}{\text{h}13:\Delta 14 \vdash \Delta 17,F15} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F15}{\text{h}13:\Delta 14 \vdash \Delta 17,F15} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F15}{\text{h}13:\Delta 11,F15 \wedge F16 \vdash \Delta 10,F9} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F15 \wedge F16}{\text{h}Cut} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F15 \wedge F16}{\text{h}Cut} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 10,\Delta 17,F9}{\text{h}Cut} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F15 \wedge F16}{\text{h}Cut} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 10,\Delta 17,F9}{\text{h}Cut} \xrightarrow{\text{ax}} \frac{-:\Delta 11,\Delta 14 \vdash \Delta 17,F9}
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 $-: \Delta 11, \Delta 14 \vdash \Delta 10, \Delta 17, F8 \land F9$

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Cases \wedge_R - \vee_R
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\frac{\texttt{h}13:\Delta11,\texttt{F}18\vdash\Delta10,\texttt{F}8\quad\texttt{h}13:\Delta11,\texttt{F}18\vdash\Delta10,\texttt{F}9}{\underbrace{\frac{\texttt{h}13\bullet:\Delta11,\texttt{F}18\vdash\Delta10,\texttt{F}9}{\texttt{h}13\bullet:\Delta11,\texttt{F}18\vdash\Delta10,\texttt{F}8\land\texttt{F}9}}} \quad \land_R \quad \frac{\texttt{h}13:\Delta14\vdash\Delta17,\texttt{F}15,\texttt{F}16,\texttt{F}18}{\texttt{h}13\bullet:\Delta14\vdash(\Delta17,\texttt{F}15\lor\texttt{F}16),\texttt{F}18}} \quad \lor_R \quad \text{Out}
                                                                                                                                                                                                                                                     -:\Delta 11,\Delta 14\vdash \Delta 10,\mathsf{F8}\wedge\mathsf{F9},\Delta 17,\mathsf{F15}\vee\mathsf{F16}
                                                                                                                                                                                 : \Delta 11, \Delta 14 \vdash \Delta 10, \Delta 17, \texttt{F8} \land \texttt{F9}, \texttt{F15} \lor \texttt{F16}
                                                                                                                                                              \frac{\text{h13}:\Delta 11,\text{F15}\vee \text{F16}\vdash \Delta 10,\text{F8}\quad \text{h13}:\Delta 11,\text{F15}\vee \text{F16}\vdash \Delta 10,\text{F9}}{\underline{\text{h13}\bullet}:\Delta 11,\text{F15}\vee \text{F16}\vdash \Delta 10,\text{F8}\wedge \text{F9}} \quad \wedge_{R} \quad \frac{\text{h13}:\Delta 14\vdash \Delta 17,\text{F15},\text{F16}}{\text{h13}\bullet:\Delta 14\vdash \Delta 17,\text{F15}\vee \text{F16}} \quad \vee_{R} \quad \text{Cut}
                                                   Cases \wedge_R - \perp_R
                                                                                                                                                                                                                            \frac{\text{h13} : \Delta11, \bot \vdash \Delta10, \text{F8} \quad \text{h13} : \Delta11, \bot \vdash \Delta10, \text{F9}}{\text{h13} \bullet : \Delta11, \bot \vdash \Delta10, \text{F8} \land \text{F9}} \quad \land_{R} \quad \frac{\text{h13} : \Delta14 \vdash \Delta15}{\text{h13} \bullet : \Delta14 \vdash \Delta15, \bot} \quad \underset{\text{Cut}}{\overset{\bot}{R}} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                               \frac{ \begin{array}{l} \mathbf{h}13:\Delta11, \mathbf{F}16 \vdash \Delta10, \mathbf{F}8 \quad \mathbf{h}13:\Delta11, \mathbf{F}16 \vdash \Delta10, \mathbf{F}9 \\ \hline \\ \underline{ \begin{array}{l} \mathbf{h}13 \bullet : \Delta11, \mathbf{F}16 \vdash \Delta10, \mathbf{F}8 \land \mathbf{F}9 \\ \hline \\ -:\Delta11,\Delta14 \vdash \Delta10, \mathbf{F}8 \land \mathbf{F}9, \bot, \Delta15 \\ \hline \end{array} \\ \end{array} \begin{array}{l} \boldsymbol{L}_{R} \\ \mathbf{cut} \end{array}
                                                                                                                                                                                                   -: \Delta 11, \Delta 14 \vdash \bot, \Delta 10, \Delta 15, F8 \land F9
Cases \wedge_R - \top_R
                                                                                                                                                                                                                              \frac{\frac{}{\text{h13}:\top,\Delta11\vdash\Delta10,\text{F8}}}{\frac{-:\Delta11,\Delta14\vdash\Delta10,\text{F8}}{\text{tot}}} \overset{\text{ax}}{\text{h13}\bullet:\Delta14\vdash\top,\Delta15} + \frac{}{\text{hCut}} \xrightarrow{} \frac{}{\text{hCut}} \frac{}{\frac{}{\text{h13}:\top,\Delta11\vdash\Delta10,\text{F9}}} \overset{\text{ax}}{\text{ax}} \frac{}{\text{h13}\bullet:\Delta14\vdash\top,\Delta15} \xrightarrow{} \frac{}{\text{hCut}} \frac{}{\text{hCut}}
                                                                                                                                                                                                                                                                                                                                                        -: \Delta 11, \Delta 14 \vdash \Delta 10, \Delta 15, F8 \land F9
                                                                                                                                                                                                   \frac{ \frac{\text{h13} : \Delta11, \text{F16} \vdash \Delta10, \text{F8} \quad \text{h13} : \Delta11, \text{F16} \vdash \Delta10, \text{F9}}{\text{h13} \bullet : \Delta11, \text{F16} \vdash \Delta10, \text{F8} \land \text{F9}} \quad \wedge_{R} \quad \frac{\text{h13} \bullet : \Delta14 \vdash (\top, \Delta15), \text{F16}}{\text{h13} \bullet : \Delta14, \text{F16} \vdash \Delta10, \text{F8} \land \text{F9}, \top, \Delta15} \quad \text{Cut} \\ \\ - : \Delta11, \Delta14 \vdash \Delta10, \text{F8} \land \text{F9}, \top, \Delta15
                                                                                                                                                                                                                                                                                                                                               \overline{-: \Delta 11, \Delta 14 \vdash \top, \Delta 10, \Delta 15, F8 \wedge F9} \top_R
Cases \wedge_R - \rightarrow_L
                                                                                                                                                                                                               \frac{ \frac{\text{h13} : \Delta11, \text{F17} \vdash \Delta10, \text{F8} \quad \text{h13} : \Delta11, \text{F17} \vdash \Delta10, \text{F9}}{\text{h13} \cdot \Delta11, \text{F17} \vdash \Delta10, \text{F8} \land \text{F9}} \quad \wedge_{R} \quad \frac{\text{h13} : \Delta18 \vdash \Delta16, \text{F14}, \text{F17} \quad \text{h13} : \Delta18, \text{F15} \vdash \Delta16, \text{F17}}{\text{h13} \bullet : \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}} \quad \rightarrow_{L} \quad \frac{\text{h13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{h13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{h13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta16, \text{F17}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F17} \rightarrow \text{H13}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F17} \rightarrow \text{H13}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{F17} \rightarrow \text{H13}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot \Delta18, \text{H13}}{\text{Cut}} \quad \rightarrow_{L} \quad \frac{\text{H13} \cdot
       \frac{1}{13:\Delta 11,F17+\Delta 10,F8} \times \frac{1}{13:\Delta 18+\Delta 16,F14,F17} \times \frac{1}{13:\Delta 18,F15+\Delta 16,F17} \times \frac{1}{13:\Delta 18,F15+\Delta 16,F17} \times \frac{1}{13:\Delta 18,F15+\Delta 16,F17} \times \frac{1}{13:\Delta 11,F17+\Delta 10,F8} \times \frac{1}{13:\Delta 11,F17+\Delta 10,F8} \times \frac{1}{13:\Delta 18,F14+\Delta 15,F15+\Delta 16,F17} \times \frac{1}{13:\Delta 11,F17+\Delta 10,F8} \times \frac{1}{13:\Delta 11,F17+\Delta 10,F18} \times \frac{1}{13:\Delta
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       -: \Delta 11, \Delta 18, F14 \rightarrow F15 \vdash \Delta 10, \Delta 16, F9 \land R
                                                                                                         -:\Delta11,\Delta18, F14 \rightarrow F15 \vdash \Delta10,\Delta16, F8
                                                                                                                                                                                                                                                                                                                                                                        -: \Delta 11, \Delta 18, F14 \rightarrow F15 \vdash \Delta 10, \Delta 16, F8 \land F9
 Cases \wedge_R - \wedge_L
                                                                                                                                                                                      \frac{\text{h13}:\Delta 11,\text{F17}\vdash\Delta 10,\text{F8}\quad \text{h13}:\Delta 11,\text{F17}\vdash\Delta 10,\text{F9}}{\underline{\text{h13}\bullet}:\Delta 11,\text{F17}\vdash\Delta 10,\text{F9}} \quad \wedge_{R} \quad \frac{\text{h13}:\Delta 18,\text{F14},\text{F15}\vdash\Delta 16,\text{F17}}{\text{h13}\bullet:\Delta 18,\text{F14}\wedge\text{F15}\vdash\Delta 16,\text{F17}} \quad \wedge_{L} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                   -:\Delta 11,\Delta 18,\mathsf{F}14\wedge\mathsf{F}15\vdash\Delta 16,\Delta 10,\mathsf{F}8\wedge\mathsf{F}9
                                                                                                                                                                                    \frac{-:\Delta 11,\Delta 18,F14 \land F15 \vdash \Delta 10,F6 \land F5}{\rightarrow} \text{ ax} \frac{-:\Delta 11,\Delta 18,F14 \land F15 \vdash \Delta 10,F6 \land F5}{\rightarrow} \text{ ax} \frac{-:\Delta 11,F17 \vdash \Delta 10,F8 \land F9}{\rightarrow} \text{ ax} \frac{-:\Delta 11,F17 \vdash \Delta 10,F8 \land F9}{\rightarrow} \text{ ax} \frac{-:\Delta 11,\Delta 18,F14,F15 \vdash \Delta 16,F17}{\rightarrow} \text{ ax}
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 $\frac{-:\Delta 11,\Delta 18,F14,F15 \vdash \Delta 10,\Delta 16,F8 \land F9}{-:\Delta 11,\Delta 18,F14 \land F15 \vdash \Delta 10,\Delta 16,F8 \land F9} \land L$

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Cases \wedge_R - \vee_L
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\frac{ \frac{\text{h13} : \Delta11, \text{F17} \vdash \Delta10, \text{F8}}{\text{h13} \bullet : \Delta11, \text{F17} \vdash \Delta10, \text{F9}}{\text{h13} \bullet : \Delta11, \text{F17} \vdash \Delta10, \text{F9}}}{-: \Delta11, \Delta18, \text{F14}} \wedge_{R} \\ \frac{\text{h13} \bullet : \Delta18, \text{F14} \vdash \Delta16, \text{F17}}{\text{h13} \bullet : \Delta18, \text{F14} \lor \text{F15} \vdash \Delta16, \text{F17}}}{\text{cut}} \vee_{L} \\ \frac{\text{L} \times \Delta11, \text{L} \times
       \frac{1}{13:\Delta 11,F17\vdash\Delta 10,F8} = \frac{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}}{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}} = \frac{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}}{\frac{1}{13:\Delta 18,F14\lorF15\vdash\Delta 16,F17}} = \frac{\frac{1}{13:\Delta 11,F17\vdash\Delta 10,F9}}{\frac{1}{13:\Delta 11,F17\vdash\Delta 10,F9}} = \frac{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}}{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}} = \frac{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}}{\frac{1}{13:\Delta 18,F14\lorF15\vdash\Delta 16,F17}} = \frac{\frac{1}{13:\Delta 18,F14\vdash\Delta 16,F17}}{\frac{1}{13:\Delta 18,F14\lorF15\vdash\Delta 10,\Delta 16,F17}} = \frac{\frac{1}{13:\Delta 18,F14\lorA16,F17}}{\frac{1}{13:\Delta 18,F14\lorF15\vdash\Delta 10,\Delta 16,F17}}} = \frac{\frac{1}{13:\Delta 18,F14\lorA16,F17}}{\frac{1}{13:\Delta 18,F14\lorA16,F17}} = \frac{\frac{1}{13:\Delta 18,F14\lorA16,F17}}{\frac{1}{13:\Delta 18,F14\lorA16,F17}}} = \frac{\frac{1}{13:\Delta 18,F
                                                                                                                     Cases \wedge_R - \perp_L
                                                                                                                                                                                                                        \frac{\text{h13}:\Delta11, \text{F15}\vdash\Delta10, \text{F8} \quad \text{h13}:\Delta11, \text{F15}\vdash\Delta10, \text{F9}}{\text{h13}\bullet:\Delta11, \text{F15}\vdash\Delta10, \text{F8}\wedge\text{F9}} \quad \land_{R} \quad \frac{\text{h13}\bullet:\bot,\Delta16\vdash\Delta14, \text{F15}}{\text{h13}\bullet:\bot,\Delta16\vdash\Delta14, \text{F15}} \quad \frac{\bot_{L}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                            \frac{}{-:\bot,\Delta 11,\Delta 16\vdash \Delta 10,\Delta 14,\mathsf{F8}\wedge\mathsf{F9}}\ ^{\bot}L
Cases \wedge_R - I
                                                                                                                                                                                                     \frac{\text{h13}:\Delta 11, \text{F15} \vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F15} \vdash \Delta 10, \text{F9}}{\text{h13} \bullet :\Delta 11, \text{F15} \vdash \Delta 10, \text{F9}} \quad \wedge_{R} \quad \frac{}{\text{h13} \bullet :\Delta 17, \text{p16} \vdash (\Delta 14, \text{p16}), \text{F15}} \quad I \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                       -:\Delta11,\,\Delta17,\mathtt{p}16\vdash\Delta10,\mathtt{f}8\wedge\mathtt{f}9,\,\Delta14,\mathtt{p}16
                                                                                                                                                                                                                                                                                                                                                              \frac{}{-:\Delta 11,\,\Delta 17,\,\mathtt{p}16\vdash\Delta 10,\,\Delta 14,\,\mathtt{p}16,\,\mathtt{f}8\wedge\mathtt{f}9}\quad iI\quad
                                                                                                                                                                                                                          \frac{\text{h13}:\Delta 11, \text{p15}\vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{p15}\vdash \Delta 10, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{p15}\vdash \Delta 10, \text{F8} \wedge \text{F9}} \quad \wedge_{R} \quad \frac{\text{h13}\bullet:\Delta 16, \text{p15}\vdash \Delta 14, \text{p15}}{\text{Cut}} \quad I \quad \text{Cut}
                                                                                                                                                                                                                                                                                                 -\,:\Delta11,\Delta16,\mathtt{p}15\vdash\Delta14,\Delta10,\mathtt{F}8\wedge\mathtt{F}9
                                                                                                                                                                                                                                                                \frac{-:\Delta 11, \Delta 16, p15 \vdash \Delta 10, \Delta 14, F8}{-:\Delta 11, \Delta 16, p15 \vdash \Delta 10, \Delta 14, F9} \xrightarrow{\text{ax}} \frac{-:\Delta 11, \Delta 16, p15 \vdash \Delta 10, \Delta 14, F9}{-:\Delta 10, \Delta 14, F8 \land F9} \xrightarrow{\text{ax}} \wedge_{R}
Cases \wedge_R - \top_L
                                                                                                                                                                                                                          \frac{\text{h13}:\Delta 11, \text{F15}\vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F15}\vdash \Delta 10, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F15}\vdash \Delta 10, \text{F8} \wedge \text{F9}} \quad \wedge_{R} \quad \frac{\text{h13}:\Delta 16\vdash \Delta 14, \text{F15}}{\text{h13}\bullet:\top,\Delta 16\vdash \Delta 14, \text{F15}} \quad \top_{L} \\ -:\Delta 11, \top,\Delta 16\vdash \Delta 14,\Delta 10, \text{F8} \wedge \text{F9} \\ \longrightarrow \quad \text{Cut}
                                                                                                                                                                                                                  Cases \vee_R - \rightarrow_R
                                                                                                                                                                                                                                                         \frac{\text{h13}:\Delta 11, \text{F18}\vdash\Delta 10, \text{F8}, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F18}\vdash\Delta 10, \text{F8}\vee \text{F9}} \quad \vee_{R} \quad \frac{\text{h13}:\Delta 14, \text{F15}\vdash\Delta 17, \text{F16}, \text{F18}}{\text{h13}\bullet:\Delta 14\vdash(\Delta 17, \text{F15}\to \text{F16}), \text{F18}} \quad \rightarrow_{R} \quad \text{Out}
                                                                                                                                                                                                                                                                                                                                         -:\Delta11,\Delta14\vdash\Delta10, \mathsf{F8}\vee\mathsf{F9},\Delta17, \mathsf{F}15\to\mathsf{F}16
                                                                                                                                                                                                                                                                             \frac{-:\Delta 11,\Delta 14,F15\vdash\Delta 10,\Delta 17,F16,F8\vee F9}{-:\Delta 11,\Delta 14\vdash\Delta 10,\Delta 17,F15\to F16,F8\vee F9}\to_R
                                                                                                                                                                                                                                                        \frac{\text{h13}:\Delta11,\text{F15}\rightarrow\text{F16}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta1}\underbrace{}_{1},\text{F15}\rightarrow\text{F16}\vdash\Delta10,\text{F8}\vee\text{F9}} \quad \vee_{R} \quad \frac{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}{\text{h13}\bullet:\Delta14\vdash\Delta17,\text{F15}\rightarrow\text{F16}} \quad \underset{\text{Cut}}{\rightarrow_{R}}
                                                                                                                                                                                                                                                                                                                                                                          -: \Delta 11, \Delta 14 \vdash \Delta 17, \Delta 10, F8 \lor F9
                                                                                                                                                                                                                                                                      \frac{1}{13:\Delta 11,F15 \rightarrow F16 \vdash \Delta 10,F8,F9} = \frac{1}{13:\Delta 14,F15 \vdash \Delta 17,F16} \xrightarrow{ax} \frac{1}{13:\Delta 14 \vdash \Delta 17,F15 \rightarrow F16} \xrightarrow{hCut} hCut
                                                                                                                                                                                                                                                                                                                                                                              \frac{-:\Delta 11,\Delta 14\vdash \Delta 10,\Delta 17,F8,F9}{-:\Delta 11,\Delta 14\vdash \Delta 10,\Delta 17,F8\vee F9} \vee_{R}
Cases \vee_R - \wedge_R
                                                                                                                                                                                                     \frac{\text{h13}: \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}, \text{F9}}{\text{h13} \bullet : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \vee \text{F9}} \ \lor_{R} \ \frac{\text{h13}: \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \ \text{h13}: \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18}}{\text{h13} \bullet : \Delta 14 \vdash (\Delta 17, \text{F15} \wedge \text{F16}), \text{F18}} \ \land_{R} \ \frac{\text{h13}}{\text{Cut}} \ \xrightarrow{\text{Cut}}
                                                                                                                                                                                                                                                                                                                         -:\Delta 11,\Delta 14\vdash \Delta 10, \texttt{f8} \lor \texttt{f9},\Delta 17, \texttt{f15} \land \texttt{f16}
                                                                                                                                                                                                        \frac{1}{\text{h13}:\Delta 11,\text{F18}\vdash\Delta 10,\text{F8},\text{F9}} \text{ ax } \frac{\frac{\neg}{\text{h13}:\Delta 14\vdash\Delta 17,\text{F15},\text{F18}} \text{ ax}}{\text{h13}:\Delta 14\vdash\Delta 17,\text{F16},\text{F16}} \frac{\text{h13}:\Delta 14\vdash\Delta 17,\text{F16},\text{F18}}{\text{h13}:\Delta 14\vdash\Delta 17,\text{F16},\text{F16}} \frac{\text{ax}}{\land R}
                                                                                                                                                                                                                                                                                                      \frac{-:\Delta 11, \Delta 14 \vdash \Delta 10, \Delta 17, F8, F9, F15 \land F16}{-:\Delta 11, \Delta 14 \vdash \Delta 10, \Delta 17, F15 \land F16, F8 \lor F9} \lor_{R}
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 \begin{array}{l} \frac{\text{h13}:\Delta11,\text{F15}\wedge\text{F16}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta11,\text{F15}\wedge\text{F16}\vdash\Delta10,\text{F8}\vee\text{F9}} & \vee_{R} & \frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15} & \text{h13}:\Delta14\vdash\Delta17,\text{F16}}{\text{h13}\bullet:\Delta14\vdash\Delta17,\text{F15}\wedge\text{F16}} \\ & -:\Delta11,\Delta14\vdash\Delta17,\Delta10,\text{F8}\vee\text{F9} \end{array} \\ & \wedge_{R} & \frac{\text{h23}:\Delta14\vdash\Delta17,\text{F15}\wedge\text{F16}}{\text{h23}\bullet:\Delta14\vdash\Delta17,\text{F15}\wedge\text{F16}} \\ & \wedge_{R} & \frac{\text{h23}:\Delta14\vdash\Delta17,\text{F15}\wedge\text{F16}}{\text{h23}\bullet:\Delta14\vdash\Delta17,\text{F15}\wedge\text{F16}} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} \\ & -\frac{1}{2} & \frac{1}{2} &
                                                                                                                                                                                  \underbrace{ \frac{\rightarrow}{\text{h13}:\Delta11,\text{F15} \land \text{F16} \vdash \Delta10,\text{F8},\text{F9}}}_{\text{h13}:\Delta14 \vdash \Delta17,\text{F15}} \underbrace{ \frac{\rightarrow}{\text{h13}:\Delta14 \vdash \Delta17,\text{F15}}}_{\text{h13}\bullet:\Delta14 \vdash \Delta17,\text{F15} \land \text{F16}} \underbrace{ \frac{\rightarrow}{\text{hCut}}}_{\text{hCut}} \underbrace{ \frac{-:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F8},\text{F9}}{-:\Delta11,\Delta14 \vdash \Delta10,\Delta17,\text{F8} \lor \text{F9}}}_{\text{C}} \vee_{R} 
Cases \vee_R - \vee_R
                                                                                                                                                                                                                         \frac{\text{h13}:\Delta 11, \text{F18} \vdash \Delta 10, \text{F8}, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \vee \text{F9}} \quad \vee_{R} \quad \frac{\text{h13}:\Delta 14 \vdash \Delta 17, \text{F15}, \text{F16}, \text{F18}}{\text{h13}\bullet:\Delta 14 \vdash (\Delta 17, \text{F15} \vee \text{F16}), \text{F18}} \quad \vee_{R} \quad \text{Cut}
                                                                                                                                                                                                                                                                                           -:\Delta11,\Delta14\vdash\Delta10,\texttt{f8}\vee\texttt{f9},\Delta17,\texttt{f15}\vee\texttt{f16}
                                                                                                                                                                                                                                   :\Delta11,\Delta14\vdash\Delta10,\Delta17,\texttt{F}15\lor\texttt{F}16,\texttt{F}8\lor\texttt{F}9
                                                                                                                                                                                                                         \frac{\text{h13}:\Delta 11, \text{F15}\vee \text{F16}\vdash \Delta 10, \text{F8}, \text{F9}}{\text{h13}\bullet:\Delta 11, \text{F15}\vee \text{F16}\vdash \Delta 10, \text{F8}\vee \text{F9}} \quad \vee_{R} \quad \frac{\text{h13}:\Delta 14\vdash \Delta 17, \text{F15}, \text{F16}}{\text{h13}\bullet:\Delta 14\vdash \Delta 17, \text{F15}\vee \text{F16}} \quad \vee_{R} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                            -: \Delta 11, \Delta 14 \vdash \Delta 17, \Delta 10, F8 \lor F9
                                                                                                                                                                                                                                     \frac{-:\Delta 11,\Delta 14 \vdash \Delta 10,\Delta 17,F8,F9}{-:\Delta 11,\Delta 14 \vdash \Delta 10,\Delta 17,F8 \vee F9} \vee_{R}
Cases \vee_R - \perp_R
                                                                                                                                                                                                                                                               \begin{array}{c|c} \frac{\text{h13}:\Delta11,\bot\vdash\Delta10,F8,F9}{\text{h13}\bullet:\Delta11,\bot\vdash\Delta10,F8\vee F9} \lor_{R} & \frac{\text{h13}:\Delta14\vdash\Delta15}{\text{h13}\bullet:\Delta14\vdash\Delta15,\bot} \\ & -:\Delta11,\Delta14\vdash\Delta15,\Delta10,F8\vee F9 \\ & \xrightarrow{} & \text{ax} \end{array} \quad \overset{\bot}{\text{cut}}
                                                                                                                                                                                                                                                                                                                                 \frac{}{-:\Delta 11,\Delta 14\vdash \Delta 10,\Delta 15,F8\vee F9} \quad \text{ax}
                                                                                                                                                                                                                                            \frac{\text{h13}:\Delta11,\text{F16}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta11,\text{F16}\vdash\Delta10,\text{F8}\vee\text{F9}} \quad \forall_R \quad \frac{\text{h13}:\Delta14\vdash\Delta15,\text{F16}}{\text{h13}\bullet:\Delta14\vdash(\bot,\Delta15),\text{F16}} \quad \bot_R \quad \frac{\bot_R}{}
                                                                                                                                                                                                                                                                                          -:\Delta11,\Delta14\vdash\Delta10, \mathsf{F8}\vee\mathsf{F9},\bot,\Delta15
                                                                                                                                                                                                                                                         h13:\Delta 11, F16 \vdash \Delta 10, F8, F9
                                                                                                                                                                                                                                                     \begin{array}{l} \frac{\text{h13}:\Delta11,\text{F16}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}:\Delta11,\text{F16}\vdash\Delta10,\text{F8}\vee\text{F9}} \ \ \vee_{R} \ \ \frac{\text{h13}:\Delta14\vdash\bot,\Delta15,\text{F16}}{\text{h13}:\Delta14\vdash\bot,\Delta15,\text{F16}} \end{array} \begin{array}{l} \text{ax} \\ -:\Delta11,\Delta14\vdash\bot,\Delta10,\Delta15,\text{F8}\vee\text{F9} \end{array} \end{array}
Cases \vee_R - \top_R
                                                                                                                                                                                                                                                                 \frac{\text{h13}:\Delta11,\top\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta11,\top\vdash\Delta10,\text{F8}\vee\text{F9}} \quad \vee_{R} \quad \frac{}{\text{h13}\bullet:\Delta14\vdash\Delta15,\top} \quad \overset{\top}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                        -: \Delta 11, \Delta 14 \vdash \Delta 15, \Delta 10, F8 \lor F9
                                                                                                                                                                                                                                                                             \underbrace{\frac{1}{\text{h13}: \top, \Delta 11 \vdash \Delta 10, F8, F9}}_{\text{h13}\bullet: \Delta 14 \vdash \top, \Delta 15} \underbrace{\xrightarrow{}}_{\text{hCut}}^{\top}_{\text{RCut}} 
                                                                                                                                                                                                                                                                                                                      \frac{-:\Delta 11,\Delta 14 \vdash \Delta 10,\Delta 15,F8,F9}{-:\Delta 11,\Delta 14 \vdash \Delta 10,\Delta 15,F8 \lor F9} \lor_{R}
                                                                                                                                                                                                                                           \begin{array}{c|c} \underline{\mathtt{h}13} : \Delta 11, \mathtt{F}16 \vdash \Delta 10, \mathtt{F}8, \mathtt{F}9 \\ \underline{\mathtt{h}13} \bullet : \Delta 11, \mathtt{F}16 \vdash \Delta 10, \mathtt{F}8 \vee \mathtt{F}9 \end{array} \vee_{R} \begin{array}{c} \vee_{R} \\ \underline{\mathtt{h}13} \bullet : \Delta 14 \vdash (\top, \Delta 15), \mathtt{F}16 \end{array} \begin{array}{c} \top_{R} \\ \underline{\mathtt{cut}} \end{array} 
                                                                                                                                                                                                                                                                                                                     -:\Delta 11,\Delta 14 \vdash \Delta 10, \texttt{F8} \lor \texttt{F9}, \top, \Delta 15
                                                                                                                                                                                                                                                                                                                      \frac{\rightarrow}{-:\Delta11,\Delta14 \vdash \top,\Delta10,\Delta15, \texttt{F8} \lor \texttt{F9}} \ \top_{R}
Cases \vee_R - \rightarrow_L
                                                                                                                                                                      \frac{\frac{\text{h13}:\Delta11,\text{F17}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta11,\text{F17}\vdash\Delta10,\text{F8}\vee\text{F9}}}{-:\Delta11,\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F14},\text{F17}} \vee_{R} \frac{\text{h13}:\Delta18,\text{F15}\vdash\Delta16,\text{F17}}{\text{h13}\bullet:\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F17}}}{\text{h13}\bullet:\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F17}} \xrightarrow{\text{Cut}} \rightarrow_{L}
                                                                                                                                                                        \frac{1}{\frac{\text{h13}:\Delta 11,\text{F17}\vdash\Delta 10,\text{F8},\text{F9}}{\text{c}}} \xrightarrow{\text{ax}} \frac{\frac{\neg}{\text{h13}:\Delta 18\vdash\Delta 16,\text{F14},\text{F17}}} \xrightarrow{\text{ax}} \frac{\text{h13}:\Delta 18,\text{F15}\vdash\Delta 16,\text{F17}}{\text{h13}\bullet:\Delta 18,\text{F14}\to\text{F15}\vdash\Delta 16,\text{F17}}} \xrightarrow{\text{hCut}} \xrightarrow{\text{ac}} \frac{-:\Delta 11,\Delta 18,\text{F14}\to\text{F15}\vdash\Delta 10,\Delta 16,\text{F8},\text{F9}}{-:\Delta 11,\Delta 18,\text{F14}\to\text{F15}\vdash\Delta 10,\Delta 16,\text{F8}\vee\text{F9}}} \vee_{R}
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Cases \vee_R - \wedge_L

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 \begin{array}{l} \frac{\text{h13}:\Delta11,\text{F17}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta11,\text{F17}\vdash\Delta10,\text{F8}\vee\text{F9}} \\ -:\Delta11,\Delta18,\text{F14}\wedge\text{F15}\vdash\Delta16,\text{F17} \\ -:\Delta11,\Delta18,\text{F14}\wedge\text{F15}\vdash\Delta16,\Delta10,\text{F8}\vee\text{F9} \end{array} \\ \begin{array}{l} \wedge_L \\ \text{cut} \\ \end{array} 
                                                                                                                                                                                                                                                                                                                                                                     \frac{\rightarrow}{\text{h13}:\Delta11,\text{F17}\vdash\Delta10,\text{F8},\text{F9}} \text{ ax} \\ \frac{\rightarrow}{\text{h13}:\Delta18,\text{F14},\text{F15}\vdash\Delta16,\text{F17}} \\ \text{h13}\bullet:\Delta18,\text{F14}\land\text{F15}\vdash\Delta16,\text{F17}} \\ \uparrow \Delta10\bullet\Delta16,\text{F8},\text{F0} \\ \text{hOut} \\ \text{hCut}
                                                                                                                                                                                                                                                                                                                                                                                                                                    \frac{-:\Delta 11,\Delta 18, F14 \wedge F15 \vdash \Delta 10,\Delta 16, F8, F9}{-:\Delta 11,\Delta 18, F14 \wedge F15 \vdash \Delta 10,\Delta 16, F8 \vee F9} \vee_{R}
Cases \vee_R - \vee_L
                                                                                                                                                                                                                                                                   \frac{\text{h13} : \Delta 11, \text{F17} \vdash \Delta 10, \text{F8}, \text{F9}}{\text{h13} \bullet : \Delta 11, \text{F17} \vdash \Delta 10, \text{F8} \vee \text{F9}} \quad \vee_{R} \quad \frac{\text{h13} : \Delta 18, \text{F14} \vdash \Delta 16, \text{F17}}{\text{h13} \bullet : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}} \quad \vee_{L} \quad \frac{\text{h13} \bullet : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{h13} \bullet : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{F14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{H14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{H14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{H14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{H14} \vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H13} \circ : \Delta 18, \text{H14} \vee \text{F15}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14} \vee \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L} \quad \frac{\text{H14} \circ : \Delta 18, \text{H14}}{\text{Cut}} \quad \vee_{L}
                                                                                                                                                                                                                                                                                                                                                                                                                         -: \Delta 11, \Delta 18, F14 \vee F15 \vdash \Delta 16, \Delta 10, F8 \vee F9
                                                                                                                                                                                                                                                                     \frac{\frac{-}{\text{h13}:\Delta11,\text{F17}\vdash\Delta10,\text{F8},\text{F9}}}{\frac{-}{\text{ex}}} = \frac{\frac{-}{\text{h13}:\Delta18,\text{F14}\vdash\Delta16,\text{F17}}}{\frac{\text{h13}\bullet:\Delta18,\text{F14}\lor\Delta16,\text{F17}}} = \frac{\text{ax}}{\text{h13}:\Delta18,\text{F15}\vdash\Delta16,\text{F17}}}{\frac{-}{\text{ex}}:\Delta11,\Delta18,\text{F14}\lor\text{F15}\vdash\Delta10,\Delta16,\text{F8},\text{F9}}}{\frac{-}{\text{ex}}:\Delta11,\Delta18,\text{F14}\lor\text{F15}\vdash\Delta10,\Delta16,\text{F8}\lor\text{F9}}} = \frac{\text{ACut}}{\text{hCut}}
Cases \vee_R - \perp_L
                                                                                                                                                                                                                                                                                                                                                                                  Cases \vee_R - I
                                                                                                                                                                                                                                                                                                                                                         \begin{array}{l} \frac{\text{h13}:\Delta11, \text{F15} \vdash \Delta10, \text{F8}, \text{F9}}{\text{h13}\bullet:\Delta11, \text{F15} \vdash \Delta10, \text{F8} \vee \text{F9}} \\ & -:\Delta11, \Delta17, \text{p16} \vdash \Delta10, \text{F8} \vee \text{F9} \end{array} \\ & -:\Delta11, \Delta17, \text{p16} \vdash \Delta10, \text{F8} \vee \text{F9}, \Delta14, \text{p16} \\ \end{array} \begin{array}{l} I \\ \text{Cut} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \frac{}{-:\Delta 11,\,\Delta 17,\,\mathtt{p}16\vdash\Delta 10,\,\Delta 14,\,\mathtt{p}16,\,\mathtt{F8}\vee\mathtt{F9}}\quad iI
                                                                                                                                                                                                                                                                                                                                                                                \frac{ \frac{ \text{h}13 : \Delta11, \text{p}15 \vdash \Delta10, \text{F8}, \text{F9} }{ \text{h}13 \bullet : \Delta11, \text{p}15 \vdash \Delta10, \text{F8} \vee \text{F9} } }{ - : \Delta11, \Delta16, \text{p}15 \vdash \Delta14, \Delta10, \text{F8} \vee \text{F9} } \  \, \frac{ I}{ \text{Cut} }
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{-:\Delta11,\Delta16,\mathtt{p}15 \vdash \Delta10,\Delta14,\mathtt{f}8,\mathtt{f}9}{-:\Delta11,\Delta16,\mathtt{p}15 \vdash \Delta10,\Delta14,\mathtt{f}8 \vee \mathtt{f}9} \overset{\mathsf{ax}}{\vee}_R
Cases \vee_R - \top_L
                                                                                                                                                                                                                                                                                                                                                                                    \frac{\underset{\bullet}{\text{h}13} \cdot \Delta 11, \text{F}15 \vdash \Delta 10, \text{F}8, \text{F}9}{\underset{-}{\text{h}13} \cdot \Delta 11, \text{F}15 \vdash \Delta 10, \text{F}8 \vee \text{F}9}} \quad \vee_{R} \quad \frac{\underset{\bullet}{\text{h}13} \cdot \Delta 16 \vdash \Delta 14, \text{F}15}{\underset{\bullet}{\text{h}13} \cdot \top, \Delta 16 \vdash \Delta 14, \text{F}15}}{\underset{-}{\text{cut}}} \quad \top_{L}
                                                                                                                                                                                                                                                                                                                                                                                                  \frac{1}{\text{h13}:\Delta11,\text{F15}\vdash\Delta10,\text{F8},\text{F9}} \quad \text{ax}
                                                                                                                                                                                                                                                                                                                                                                                          \frac{\text{h13}:\Delta11,\text{F15}\vdash\Delta10,\text{F8},\text{F9}}{\text{h13}\bullet:\Delta11,\text{F15}\vdash\Delta10,\text{F8}\vee\text{F9}} \ \ \vee_{R} \quad \frac{\text{h13}:\top,\Delta16\vdash\Delta14,\text{F15}}{\text{hCut}} \quad \text{ax} \\ \text{hCut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -:\top,\Delta 11,\Delta 16 \vdash \Delta 10,\Delta 14, \texttt{F8} \lor \texttt{F9}
Cases \perp_R - \rightarrow_R
                                                                                                                                                                                                                                                                                                                                                                      \frac{ \text{h11} : \Delta 9, \text{F16} \vdash \Delta 8}{\text{h11} \bullet : \Delta 9, \text{F16} \vdash \bot, \Delta 8} \quad \bot_R \quad \frac{ \text{h11} : \Delta 12, \text{F13} \vdash \Delta 15, \text{F14}, \text{F16}}{\text{h11} \bullet : \Delta 12 \vdash (\Delta 15, \text{F13} \rightarrow \text{F14}), \text{F16}} \quad \xrightarrow{\text{Cut}} \quad \text{Cut} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                -:\Delta 9,\Delta 12\vdash\bot,\Delta 8,\Delta 15,\mathsf{F} 13\to\mathsf{F} 14
                                                                                                                                                                                                                                                                                                                                                                                        -: \overline{\Delta 12, \Delta 9 \vdash \bot, \Delta 15, \Delta 8, F13 \rightarrow F14}
                                                                                                                                                                                                                                                                                                                                                                      \underbrace{\begin{array}{c} \text{h}11: \Delta 9, \text{F}13 \rightarrow \text{F}14 \vdash \Delta 8 \\ \text{h}11 \bullet : \Delta 9, \text{F}13 \rightarrow \text{F}14 \vdash \bot, \Delta 8 \end{array}}_{\text{h}11: \Delta 12, \text{F}13 \vdash \Delta 15, \text{F}14} \xrightarrow{} R \underbrace{\begin{array}{c} \text{h}11: \Delta 12, \text{F}13 \vdash \Delta 15, \text{F}14 \\ \text{h}11 \bullet : \Delta 12 \vdash \Delta 15, \text{F}13 \rightarrow \text{F}14 \end{array}}_{\text{Cut}} \xrightarrow{} R \underbrace{\begin{array}{c} \text{Cut} \\ \text{Cut} \end{array}}_{\text{Cut}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                -: \Delta 9, \Delta 12 \vdash \Delta 15, \perp, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                               \frac{1}{111:\Delta9, 113 \rightarrow 114 \vdash \Delta8} \text{ ax } \frac{1}{111:\Delta12, 113 \vdash \bot, \Delta15, 114} \xrightarrow{\text{ax}} R
111:\Delta12, 113 \vdash \bot, \Delta15, 113 \rightarrow 114
111\bullet:\Delta12, 113 \vdash \bot, \Delta15, 113 \rightarrow 114
111\bullet:\Delta12, 113 \vdash \bot, \Delta15, 113 \rightarrow 114
111:\Delta12, 113 \vdash \bot, \Delta15, 113 \vdash \bot, \Delta1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -: \Delta 12, \Delta 9 \vdash \bot, \Delta 15, \Delta 8
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\frac{\frac{1}{\text{hl1}:\Delta9,\text{Fl6}\vdash\Delta8}}{-:\Delta12,\Delta9} \text{ ax} \quad \frac{\text{hl1}:\Delta12\vdash\bot,\Delta15,\text{Fl3},\text{Fl6}}{\text{hl1}\bullet:\Delta12\vdash\bot,\Delta15,\text{Fl6},\text{Fl3}\wedge\text{Fl4}} \quad \frac{\text{ax}}{\wedge R} \\ -:\Delta12,\Delta9\vdash\bot,\Delta15,\Delta8,\text{Fl3}\wedge\text{Fl4}} \quad \frac{\text{hCut}}{\wedge R}
                                                                                                                                                                                                                                                                                                           \frac{ \underbrace{ \text{h}11 : \Delta 9, \text{F}13 \wedge \text{F}14 \vdash \Delta 8 }_{\text{h}11 \bullet : \Delta 9, \text{F}13 \wedge \text{F}14 \vdash \bot, \Delta 8} \quad \bot_{R} \quad \underbrace{ \frac{ \text{h}11 : \Delta 12 \vdash \Delta 15, \text{F}13 \quad \text{h}11 : \Delta 12 \vdash \Delta 15, \text{F}13 \wedge \text{F}14 }_{\text{h}11 \bullet : \Delta 9, \text{F}13 \wedge \text{F}14 \vdash \bot, \Delta 8} \quad \land_{R} }_{\text{Cut}} \quad \land_{R}
                                                                                                                                                                                                                                                                                          \frac{\frac{}{\text{h11}:\Delta 9,\text{F13}\wedge\text{F14}\vdash\Delta 8}}{\frac{}{\text{h11}:\Delta 9,\text{F13}\wedge\text{F14}\vdash\Delta 8}} \overset{\text{ax}}{\text{ax}} \frac{\frac{}{\text{h11}:\Delta 12\vdash\bot,\Delta 15,\text{F13}}}{\text{h11}\bullet:\Delta 12\vdash\bot,\Delta 15,\text{F13}\wedge\text{F14}}}{\text{h11}\bullet:\Delta 12\vdash\bot,\Delta 15,\text{F13}\wedge\text{F14}} \overset{\text{ax}}{\wedge}_{R}
Cases \perp_R - \vee_R
                                                                                                                                                                                                                                                                                                                                                                       \frac{\text{h11}:\Delta9,\text{F16}\vdash\Delta8}{\text{h11}\bullet:\Delta9,\text{F16}\vdash\bot,\Delta8} \quad \bot_R \quad \frac{\text{h11}:\Delta12\vdash\Delta15,\text{F13},\text{F14},\text{F16}}{\text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13}\vee\text{F14}),\text{F16}} \quad \lor_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                        -: \Delta 9, \Delta 12 \vdash \bot, \Delta 8, \Delta 15, F13 \lor F14
                                                                                                                                                                                                                                                                                                                                                                                              \frac{1}{\underbrace{\text{h11}:\Delta 9,\text{F16}\vdash\Delta 8}} \text{ ax } \frac{\underbrace{\text{h11}:\Delta 12\vdash\bot,\Delta 15,\text{F13},\text{F14},\text{F16}}}{\text{h11}\bullet:\Delta 12\vdash\bot,\Delta 15,\text{F16},\text{F13}\vee\text{F14}} \overset{\text{ax}}{\vee}_{R}
                                                                                                                                                                                                                                                                                                                                                                                                                                                              -: \Delta 12, \Delta 9 \vdash \bot, \Delta 15, \Delta 8, F13 \lor F14
                                                                                                                                                                                                                                                                                                                                                                            \underbrace{\frac{\text{h11} : \Delta 9, \text{F13} \vee \text{F14} \vdash \Delta 8}{\text{h11} \bullet : \Delta 9, \text{F13} \vee \text{F14} \vdash \bot, \Delta 8}}_{\text{h11} \bullet : \Delta 12 \vdash \Delta 15, \text{F13} \vee \text{F14}} \xrightarrow{\text{V}_R} \underbrace{\frac{\text{h11} \bullet : \Delta 12 \vdash \Delta 15, \text{F13} \vee \text{F14}}{\text{h11} \bullet : \Delta 12 \vdash \Delta 15, \text{F13} \vee \text{F14}}}_{\text{Cut}} \underbrace{\frac{\text{V}_R}{\text{Cut}}}_{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: \Delta 9, \Delta 12 \vdash \Delta 15, \perp, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                                        \frac{1}{111:\Delta 9,F13\vee F14\vdash \Delta 8} \text{ ax } \frac{\overrightarrow{\rightarrow} \\ \frac{111:\Delta 12\vdash \bot,\Delta 15,F13,F14}{111\bullet:\Delta 12\vdash \bot,\Delta 15,F13\vee F14} \vee_{\text{RCut}} \\ + \frac{1}{111}\underbrace{\rightarrow} \\ + \frac
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               -: \Delta 12, \Delta 9 \vdash \bot, \Delta 15, \Delta 8
Cases \perp_R - \perp_R
                                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{\frac{\text{h}11:\Delta9,\bot\vdash\Delta8}{\text{h}11\bullet:\Delta9,\bot\vdash\bot,\Delta8}}{-:\Delta9,\bot\vdash\bot,\Delta8} \ ^\bot_R \quad \frac{\text{h}11:\Delta12\vdash\Delta13}{\text{h}11\bullet:\Delta12\vdash\Delta13,\bot} \quad \frac{^\bot_R}{\text{cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \underbrace{ \begin{array}{c} \text{h}11: \Delta 9, \text{F}14 \vdash \Delta 8 \\ \text{h}11 \bullet : \Delta 9, \text{F}14 \vdash \bot, \Delta 8 \end{array}}_{\text{h}11: \Delta 12} \perp_{R} \underbrace{ \begin{array}{c} \text{h}11: \Delta 12 \vdash \Delta 13, \text{F}14 \\ \text{h}11 \bullet : \Delta 12 \vdash (\bot, \Delta 13), \text{F}14 \end{array}}_{\text{Cut}} \perp_{R} \underbrace{ \begin{array}{c} \bot_{R} \\ \text{Cut} \end{array}}_{\text{Cut}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -: \Delta 9, \Delta 12 \vdash \bot, \Delta 8, \bot, \Delta 13
                                                                                                                                                                                                                                                                                                                                                                                                                          \underbrace{\frac{\rightarrow}{\text{h}11:\Delta9,\text{F}14\vdash\Delta8}}_{\text{h}11:\Delta12\vdash\bot,\Delta13,\text{F}14} \overset{\text{ax}}{\rightarrow} \underbrace{\frac{\rightarrow}{\text{h}11:\Delta12\vdash\bot,\Delta13,\text{F}14}}_{\text{h}11\bullet:\Delta12\vdash\bot,\bot,\Delta13,\text{F}14} \overset{\text{ax}}{\rightarrow} \underbrace{\frac{\rightarrow}{\text{h}11:\Delta12\vdash\bot,\Delta13,\text{F}14}}_{\text{h}\text{Cut}} \underbrace{\frac{\rightarrow}{\text{h}11:\Delta12}\vdash\bot,\Delta13,\text{F}14}_{\text{h}\text{Cut}}}_{\text{h}\text{Cut}} \underbrace{\frac{\rightarrow}{\text{h}11:\Delta12}\vdash\bot,\Delta13,\text{F}14}_{\text{h}\text{Cut}}}_{\text{h}\text{Cut}}
Cases \perp_R - \top_R
                                                                                                                                                                                                                                                                                                                                                                                                                                      \underbrace{\frac{\text{h}11:\Delta9,\,\top\vdash\Delta8}{\text{h}11\bullet:\Delta9,\,\top\vdash\bot,\,\Delta8}}_{\text{H}11\bullet:\Delta12\vdash\Delta13,\,\top} \ \ \underset{\text{Cut}}{\top_{R}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        -:\Delta 9,\Delta 12\vdash \Delta 13,\bot,\Delta 8
                                                                                                                                                                                                                                                                                                                                                                                                                                                  \underbrace{\frac{111:\top,\Delta9\vdash\Delta8}_{\text{h110}}}_{\text{h210}} \underbrace{\frac{}{\text{h110}}:\Delta12\vdash\bot,\top,\Delta13}_{\text{hCut}} \underbrace{\frac{}{\text{hCut}}}_{\text{hCut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -: \Delta 12, \Delta 9 \vdash \bot, \Delta 13, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                                                                       \mathtt{h11}:\Delta9,\mathtt{F14}\vdash\Delta8
                                                                                                                                                                                                                                                                                                                                                                                                      \frac{\mathsf{h}\mathsf{1}\mathsf{1} \bullet : \Delta9, \mathsf{F}\mathsf{1} 4 \vdash \Delta8}{\mathsf{h}\mathsf{1} \bullet : \Delta9, \mathsf{F}\mathsf{1} 4 \vdash \bot, \Delta8} \perp_{R} \frac{\mathsf{h}\mathsf{1}\mathsf{1} \bullet : \Delta12 \vdash (\mathsf{T}, \Delta13), \mathsf{F}\mathsf{1} 4}{\mathsf{h}\mathsf{1} \mathsf{1} \bullet : \Delta9, \Delta12 \vdash \bot, \Delta8, \mathsf{T}, \Delta13} \xrightarrow{\mathsf{T}_{R}} \mathsf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{}{-:\Delta 12,\Delta 9\vdash \bot,\top,\Delta 13,\Delta 8} \ \top_{R}
Cases \perp_R - \rightarrow_L
                                                                                                                                                                                                                                                                                        \frac{111:\Delta 9, F15 \vdash \Delta 8}{-111:\Delta 16, \Delta 9, F15 \vdash \Delta 8} \xrightarrow{\text{ax}} \frac{\frac{}{\text{h11}:\Delta 16 \vdash \bot, \Delta 14, F15}}{\text{h11} \bullet : \Delta 16, F12 \rightarrow F13 \vdash \bot, \Delta 14, F15}} \xrightarrow{\text{ax}} \frac{\text{h11}:\Delta 16, F13 \vdash \bot, \Delta 14, F15}}{-111} \xrightarrow{\text{hCut}} \frac{\text{ax}}{\text{hCut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                8
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 $\frac{\mathsf{h}11 : \Delta 9, \mathsf{F}16 \vdash \Delta 8}{\mathsf{h}11 \bullet : \Delta 9, \mathsf{F}16 \vdash \bot, \Delta 8} \quad \bot_R \quad \frac{\mathsf{h}11 : \Delta 12 \vdash \Delta 15, \mathsf{F}13, \mathsf{F}16 \quad \mathsf{h}11 : \Delta 12 \vdash \Delta 15, \mathsf{F}14, \mathsf{F}16}{\mathsf{h}11 \bullet : \Delta 12 \vdash (\Delta 15, \mathsf{F}13 \land \mathsf{F}14), \mathsf{F}16} \quad \bot_R \quad - : \Delta 9, \Delta 12 \vdash \bot, \Delta 8, \Delta 15, \mathsf{F}13 \land \mathsf{F}14} \quad \mathsf{Cut}$

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Cases \perp_R - \wedge_L
                                                                                                                                                                                                                                                                                                                                               \begin{array}{c|c} \frac{\text{h11}:\Delta9,\text{F15}\vdash\Delta8}{\text{h11}\bullet:\Delta9,\text{F15}\vdash\bot,\Delta8} & \bot_R & \frac{\text{h11}:\Delta16,\text{F12},\text{F13}\vdash\Delta14,\text{F15}}{\text{h11}\bullet:\Delta16,\text{F12}\land\text{F13}\vdash\Delta14,\text{F15}} & \land_L \\ & -:\Delta9,\Delta16,\text{F12}\land\text{F13}\vdash\Delta14,\bot,\Delta8 & \text{Cut} \end{array} 
                                                                                                                                                                                                                                                                                                                                                         \frac{1}{111:\Delta9,F15\vdash\Delta8} = \frac{1}{111:\Delta16,F12,F13\vdash\bot,\Delta14,F15} = \frac{1}{111} \times \frac{1}{111
                                                                                                                                                                                                                                                                                                                                                                                                                                        -: \Delta 16, \Delta 9, F12 \wedge F13 \vdash \bot, \Delta 14, \Delta 8
Cases \perp_R - \vee_L
                                                                                                                                                                                                                                                                   \frac{ \underbrace{ \begin{array}{l} \text{h}11: \Delta 9, \text{F}15 \vdash \Delta 8 \\ \text{h}11 \bullet: \Delta 9, \text{F}15 \vdash \bot, \Delta 8 \end{array}}_{-: \Delta 9, \text{F}15 \vdash \bot, \Delta 8} \ \bot_{R} \ \frac{ \text{h}11: \Delta 16, \text{F}12 \vdash \Delta 14, \text{F}15 \quad \text{h}11: \Delta 16, \text{F}13 \vdash \Delta 14, \text{F}15}_{-: \Delta 14, \text{F}15} \ \times_{L} \\ \underbrace{ \begin{array}{l} \text{h}11 \bullet: \Delta 9, \text{F}15 \vdash \bot, \Delta 8 \end{array}}_{-: \Delta 9, \Delta 16, \text{F}12 \lor \text{F}13 \vdash \Delta 14, \bot, \Delta 8} \ \text{Cut} \end{array}}_{-: \Delta 9, \Delta 16, \text{F}12 \lor \text{F}13 \vdash \Delta 14, \bot, \Delta 8} 
                                                                                                                                                                                                                                                                                                                                                                                                -: \Delta 9, \Delta 16, F12 \lor F13 \vdash \Delta 14, \bot, \Delta 8
                                                                                                                                                                                                                                                     \frac{1}{11:\Delta 9, F15 \vdash \Delta 8} \text{ ax } \frac{1}{11:\Delta 16, F12 \vdash \bot, \Delta 14, F15} \text{ ax } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ ax } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{ both } \frac{1}{11:\Delta 16, F13 \vdash \bot, \Delta 14, F15} \text{
Cases \perp_R - \perp_L
                                                                                                                                                                                                                                                                                                                                                                      \frac{\rightarrow}{-: \bot, \Delta 14, \Delta 9 \vdash \bot, \Delta 12, \Delta 8} \bot_L
Cases \perp_R - I
                                                                                                                                                                                                                                                                                                                                                             \mathtt{h}11:\Delta9,\mathtt{F}13 \vdash \Delta8
                                                                                                                                                                                                                                                                                                                                               \begin{array}{c|c} \frac{\text{h11}: \Delta 9, \text{F13} \vdash \Delta 8}{\text{h11}\bullet: \Delta 9, \text{F13} \vdash \bot, \Delta 8} & \bot_R & \overline{\text{h11}\bullet: \Delta 15, \text{p14} \vdash (\Delta 12, \text{p14}), \text{F13}} & I \\ \hline & -: \Delta 9, \Delta 15, \text{p14} \vdash \bot, \Delta 8, \Delta 12, \text{p14} \\ & \to & \end{array} \quad \text{Cut} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                       \frac{\rightarrow}{-:\Delta15,\Delta9,\mathtt{p}14\vdash\bot,\Delta12,\Delta8,\mathtt{p}14} \ iI
                                                                                                                                                                                                                                                                                                                                                                  \begin{array}{c} \frac{\text{h11}:\Delta 9,\text{p13}\vdash\Delta 8}{\text{h11}\bullet:\Delta 9,\text{p13}\vdash\bot,\Delta 8} \stackrel{\bot}{-} R \quad \frac{\text{h11}\bullet:\Delta 14,\text{p13}\vdash\Delta 12,\text{p13}}{-:\Delta 9,\Delta 14,\text{p13}\vdash\Delta 12,\bot,\Delta 8} \quad \text{Cut} \\ \xrightarrow{\qquad \qquad ---} \text{av} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \overline{-: \Delta 14, \Delta 9, p13 \vdash \bot, \Delta 12, \Delta 8} ax
Cases \perp_R - \top_L
                                                                                                                                                                                                                                                                                                                                                                        \frac{\text{h11}:\Delta9,\text{F13}\vdash\Delta8}{\text{h11}\bullet:\Delta9,\text{F13}\vdash\bot,\Delta8} \quad \bot_R \quad \frac{\text{h11}:\Delta14\vdash\Delta12,\text{F13}}{\text{h11}\bullet:\top,\Delta14\vdash\Delta12,\text{F13}} \quad \top_L \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 9, \top, \Delta 14 \vdash \Delta 12, \bot, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                                 -: \top, \Delta 14, \Delta 9 \vdash \bot, \Delta 12, \Delta 8
Cases \top_R - \rightarrow_R
                                                                                                                                                                                                                                                                                                                                -: \Delta 9, \Delta 12 \vdash \top, \Delta 8, \Delta 15, F13 \rightarrow F14
\rightarrow
                                                                                                                                                                                                                                                                                                                                                                                                                                           \frac{}{-:\Delta 12, \Delta 9 \vdash \top, \Delta 15, \Delta 8, F13 \rightarrow F14} \vdash^{R} 
                                                                                                                                                                                                                                                                                                                               \frac{1}{\mathsf{h}11\bullet:\Delta9,\mathsf{F}13\to\mathsf{F}14\vdash\top,\Delta8} \quad \top_R \quad \frac{\mathsf{h}11:\Delta12,\mathsf{F}13\vdash\Delta15,\mathsf{F}14}{\mathsf{h}11\bullet:\Delta12\vdash\Delta15,\mathsf{F}13\to\mathsf{F}14} \quad \underset{\mathsf{Cut}}{\to} R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 9, \Delta 12 \vdash \Delta 15, \top, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{}{-:\Delta 12,\Delta 9 \vdash \top,\Delta 15,\Delta 8} \quad \top_{R}
Cases \top_R - \wedge_R
                                                                                                                                                                                                                                                                 \frac{\frac{1}{\text{hil}\bullet:\Delta9,\text{Fi6}\vdash\top,\Delta8}}{-:\Delta9,\Delta12\vdash\top,\Delta8,\Delta15,\text{Fi3},\Delta14} \xrightarrow{\text{T}_{R}} \frac{\frac{\text{hil}:\Delta12\vdash\Delta15,\text{Fi4},\text{Fi6}}{\text{hil}\bullet:\Delta12\vdash(\Delta15,\text{Fi3}\wedge\text{Fi4}),\text{Fi6}}}{-:\Delta9,\Delta12\vdash\top,\Delta8,\Delta15,\text{Fi3}\wedge\text{Fi4}} \xrightarrow{\text{Cut}} \wedge_{R}
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 $\overline{-: \Delta 12, \Delta 9 \vdash \top, \Delta 15, \Delta 8, F13 \land F14}$ \top_R

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\frac{\frac{}{\text{hll}\bullet:\Delta9,\text{Fl3}\wedge\text{Fl4}\vdash\top,\Delta8}}{-:\Delta9,\text{Fl3}\wedge\text{Fl4}\vdash\top,\Delta8}\top_{R} \frac{\text{hll}:\Delta12\vdash\Delta15,\text{Fl3}\quad\text{hll}:\Delta12\vdash\Delta15,\text{Fl3}\wedge\text{Fl4}}{\text{hll}\bullet:\Delta12\vdash\Delta15,\text{Fl3}\wedge\text{Fl4}} \\ -:\Delta9,\Delta12\vdash\Delta15,\top,\Delta8 \\ \text{Cut} \\ \wedge_{R}
                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{}{-:\Delta 12,\Delta 9 \vdash \top,\Delta 15,\Delta 8} \top_R
Cases \top_R - \vee_R
                                                                                                                                                                                                                                                                                                        \frac{\frac{1}{\text{h}11\bullet:\Delta9,\text{F}16\vdash\top,\Delta8}}{-:\Delta9,\Delta10\vdash\top,\Delta8} \ \top_{R} \ \frac{\text{h}11:\Delta12\vdash\Delta15,\text{F}13,\text{F}14,\text{F}16}{\text{h}11\bullet:\Delta12\vdash(\Delta15,\text{F}13\vee\text{F}14),\text{F}16}}{-:\Delta9,\Delta12\vdash\top,\Delta8,\Delta15,\text{F}13\vee\text{F}14} \ \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                      \frac{\rightarrow}{-:\Delta 12, \Delta 9 \vdash \top, \Delta 15, \Delta 8, \mathtt{F}13 \vee \mathtt{F}14} \ \top_{R}
                                                                                                                                                                                                                                                                                                         \frac{\overbrace{\text{h11}\bullet:\Delta9,\text{F13}\vee\text{F14}\vdash\top,\Delta8}^{\text{h11}\bullet:\Delta12}\vdash\Delta15,\text{F13},\text{F14}}_{-:\Delta9,\Delta12\vdash\Delta15,\top,\Delta8} \ \ \begin{matrix} \vee_R \\ \hline \\ \text{h11}\bullet:\Delta12\vdash\Delta15,\text{F13}\vee\text{F14} \end{matrix} \\ & \begin{matrix} \vee_R \\ \\ \text{Cut} \end{matrix} 
                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{}{-:\Delta 12,\Delta 9 \vdash \top,\Delta 15,\Delta 8} \ \top_{R}
Cases \top_R - \bot_R
                                                                                                                                                                                                                                                                                                                                                            \underbrace{\frac{\mathbf{h}11\bullet:\Delta9,\bot\vdash\top,\Delta8}{\mathbf{h}11\bullet:\Delta12\vdash\Delta13,\bot}}_{-:\Delta9,\Delta12\vdash\Delta13,\top,\Delta8} \ \mathbf{\uparrow}_{R} \ \frac{\mathbf{h}11:\Delta12\vdash\Delta13,\bot}{\mathbf{h}11\bullet:\Delta12\vdash\Delta13,\bot}_{\mathbf{Cut}} \ \mathbf{\downarrow}_{R} 
                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{\neg}{-:\Delta 12, \Delta 9 \vdash \top, \Delta 13, \Delta 8} \; \top_{R}
                                                                                                                                                                                                                                                                                                                               \frac{ \frac{ \mathsf{h}11 \bullet : \Delta 9, \mathsf{F}14 \vdash \top, \Delta 8}{ \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11} \\ - : \Delta 9, \Delta 12 \vdash \top, \Delta 8, \bot, \Delta 13}{ - } \quad \frac{\mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11}{ \mathsf{h}11 \cdot \mathsf{h}111 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}111 \cdot \mathsf{h}11 \cdot \mathsf{h}111 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}11 \cdot \mathsf{h}
                                                                                                                                                                                                                                                                                                                                                                                                                                \overline{-: \Delta 12, \Delta 9 \vdash \bot, \top, \Delta 13, \Delta 8} \top_R
Cases \top_R - \top_R
                                                                                                                                                                                                                                                                                                                                                            \frac{\overline{\text{h11}\bullet}:\Delta9,\top\vdash\top,\Delta8}{-:\Delta9,\Delta12\vdash\Delta13,\top,\Delta8} \ \frac{\top_{R}}{\text{h11}\bullet:\Delta12\vdash\Delta13,\top} \ \frac{\top_{R}}{\text{Cut}} 
                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{\rightarrow}{-:\Delta 12, \Delta 9 \vdash \top, \Delta 13, \Delta 8} \ \top_{R}
                                                                                                                                                                                                                                                                                                                                  \frac{\overline{\text{h11}\bullet}:\Delta9,\text{F14}\vdash\top,\Delta8}{-:\Delta9,\Delta12\vdash\top,\Delta8,\top,\Delta13} \xrightarrow{\top_{R}} \frac{\top_{R}}{\text{cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                \frac{}{-:\Delta 12,\Delta 9 \vdash \top, \top, \Delta 13, \Delta 8} \ \top_{R}
Cases \top_R - \rightarrow_L
                                                                                                                                                                                                                                       \frac{\frac{}{\text{h}11\bullet:\Delta9,\text{F15}\vdash\top,\Delta8}}{-:\Delta9,\text{A16},\text{F12}\rightarrow\text{F13}\vdash\Delta14,\text{F12}} \xrightarrow{\text{h}11:\Delta16,\text{F13}\vdash\Delta14,\text{F15}} \frac{}{\text{h}11\bullet:\Delta9,\text{F15}\vdash\top,\Delta8} \xrightarrow{\text{Cut}} \frac{}{}
                                                                                                                                                                                                                                                                                                                                                                                                   Cases \top_R - \wedge_L
                                                                                                                                                                                                                                                                                                                \frac{\underbrace{\text{h}11\bullet:\Delta9,\text{F}15\vdash\top,\Delta8}}{-:\Delta9,\Delta10,\Delta10} \xrightarrow{\top_R} \frac{\text{h}11:\Delta16,\text{F}12,\text{F}13\vdash\Delta14,\text{F}15}}{\text{h}11\bullet:\Delta16,\text{F}12,\Delta13\vdash\Delta14,\text{F}15}} \xrightarrow{\wedge_L} \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                        \overline{-: \Delta 16, \Delta 9, F12 \wedge F13 \vdash \top, \Delta 14, \Delta 8} \top_R
Cases \top_R - \vee_L
                                                                                                                                                                                                                                          \frac{\frac{}{\mathtt{h}11\bullet:\Delta9,\mathtt{F}15\vdash\top,\Delta8}}{-:\Delta9,\mathtt{F}15\vdash\top,\Delta8} \ \ \frac{\mathtt{h}11:\Delta16,\mathtt{F}12\vdash\Delta14,\mathtt{F}15}{\mathtt{h}11\bullet:\Delta16,\mathtt{F}13\vdash\Delta14,\mathtt{F}15} \ \ \mathsf{Cut}}{-:\Delta9,\Delta16,\mathtt{F}12\vee\mathtt{F}13\vdash\Delta14,\top,\Delta8} \ \ \frac{}{\hookrightarrow} \ \ \mathsf{Cut}
```

 $\frac{}{-:\Delta 16,\Delta 9,\mathtt{F}12 \vee \mathtt{F}13 \vdash \top,\Delta 14,\Delta 8} \ \top_{R}$

Cases \top_R - \bot_L

```
-:\Delta 9,\bot,\Delta 14\vdash\Delta 12,\top,\Delta 8
                                                                                                                                                                                                                                                                                                                                                                                                                                            \frac{\rightarrow}{-:\bot,\Delta 14,\Delta 9\vdash\top,\Delta 12,\Delta 8} \;\; \top_{R}
Cases \top_R - I
                                                                                                                                                                                                                                                                                                                            \frac{\overbrace{\text{h11}\bullet:\Delta9,\text{F13}\vdash\top,\Delta8}^{} \quad T_{R}}{-:\Delta9,\Delta15,\text{p14}\vdash\top,\Delta8,\Delta12,\text{p14}} \quad \underbrace{I}_{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{}{-:\Delta 15,\Delta 9,\mathtt{p}14 \vdash \top,\Delta 12,\Delta 8,\mathtt{p}14} \;\; \top_{R}
                                                                                                                                                                                                                                                                                                                                              -: \Delta 9, \Delta 14, p13 \vdash \Delta 12, \top, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                                                                                        Cases \top_R - \top_L
                                                                                                                                                                                                                                                                                                                                                  \frac{}{\mathsf{h}11\bullet}:\underline{\Delta9,\mathsf{F}13\vdash\top,\Delta8}\quad \top_{R}\quad \frac{\mathsf{h}11:\Delta14\vdash\Delta12,\mathsf{F}13}{\mathsf{h}11\bullet}:\underline{\top,\Delta14\vdash\Delta12,\mathsf{F}13}\quad \top_{L}\quad \mathsf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                        -:\Delta 9, \top, \Delta 14 \vdash \Delta 12, \top, \Delta 8
                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{\rightarrow}{-: \top, \Delta 14, \Delta 9 \vdash \top, \Delta 12, \Delta 8} \top_{R}
Cases \to_L - \to_R
                                                                                                                                                                                                          \frac{\text{h13} : \Delta11, \text{F18} \vdash \Delta10, \text{F8} \quad \text{h13} : \Delta11, \text{F9}, \text{F18} \vdash \Delta10}{\text{h13} \bullet : (\Delta11, \text{F8} \to \text{F9}), \text{F18} \vdash \Delta10} \\ - : \Delta14, \Delta11, \text{F8} \to \text{F9} \vdash \Delta10, \Delta17, \text{F15} \to \text{F16}) \\ - : \Delta14, \Delta11, \text{F8} \to \text{F9} \vdash \Delta10, \Delta17, \text{F15} \to \text{F16}} \\ - : \Delta14, \Delta11, \text{F8} \to \text{F9} \vdash \Delta10, \Delta17, \text{F15} \to \text{F16}} \\ \text{Cut}
                                                                                                                                                                                                                   -: \Delta 11, \Delta 14, F8 \rightarrow F9 \vdash \Delta 10, \Delta 17, F15 \rightarrow F16
                                                                                                                                                                                 \frac{\text{h13}:\Delta 11,\text{F15}\rightarrow \text{F16}\vdash \Delta 10,\text{F8}}{\text{h13}\bullet:(\Delta 11,\text{F8}\rightarrow \text{F9}),\text{F15}\rightarrow \text{F16}\vdash \Delta 10} \rightarrow_{L} \frac{\text{h13}:\Delta 14,\text{F15}\vdash \Delta 17,\text{F16}}{\text{h13}\bullet:\Delta 14\vdash \Delta 17,\text{F15}\rightarrow \text{F16}} \rightarrow_{R} \frac{\text{Cut}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                            -:\Delta14,\Delta11, F8 \rightarrow F9 \vdash \Delta10,\Delta17
                                            \frac{\frac{1}{\text{h13}:\Delta11,\text{F15}\rightarrow\text{F16}\vdash\Delta10,\text{F8}}}{\frac{1}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}} \overset{\text{ax}}{\Rightarrow} \frac{\frac{1}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}} \overset{\text{ax}}{\Rightarrow} \frac{
                                                                                                                                                                                                        \frac{\mathbf{h}14:\Delta11\vdash\Delta10,F12\quad\mathbf{h}14:\Delta11,F13\vdash\Delta10}{\mathbf{h}14\bullet:\Delta11,F12\to F13\vdash\Delta10}\to_{L} \quad \frac{\mathbf{h}14:\Delta15,F16\vdash\Delta18,F17,F12\to F13}{\mathbf{h}14\bullet:\Delta15\vdash(\Delta18,F16\to F17),F12\to F13} \to_{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                       -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \rightarrow F17
                                                                                                                                                                                                                    \frac{ \frac{1}{\text{h14}:\Delta11\vdash\Delta10,\text{F12}} \text{ ax}}{\frac{\text{h14}\circ:\Delta11,\text{F12}\rightarrow\text{F13}\vdash\Delta10}{}} \xrightarrow{\text{h14}:\Delta11,\text{F13}\vdash\Delta10}} \xrightarrow{\text{ax}} \xrightarrow{\text{h14}:\Delta15,\text{F16}\vdash\Delta18,\text{F17},\text{F12}\rightarrow\text{F13}}} \xrightarrow{\text{hCut}} \frac{\frac{-:\Delta11,\Delta15,\text{F16}\vdash\Delta10,\Delta18,\text{F17}}{}}{-:\Delta11,\Delta15\vdash\Delta10,\Delta18,\text{F16}\rightarrow\text{F17}}} \xrightarrow{R} 
                                                                                                                                                                                                                                                        \frac{ \frac{\mathtt{h}12:\Delta11\vdash\Delta10,\mathsf{F}14\quad\mathsf{h}12:\Delta11,\mathsf{F}15\vdash\Delta10}{\mathtt{h}12\bullet:\Delta11,\mathsf{F}14\to\mathsf{F}15\vdash\Delta10} \to_L \quad \frac{\mathtt{h}12:\Delta13,\mathsf{F}14\vdash\Delta16,\mathsf{F}15}{\mathtt{h}12\bullet:\Delta13\vdash\Delta16,\mathsf{F}14\to\mathsf{F}15} \to_R \\ -:\Delta11,\Delta13\vdash\Delta10,\Delta16 \quad \quad Cut 
                                                                                                                                             \frac{ \begin{array}{c} -: \Delta 11, \Delta 13 \vdash \Delta 10, \Delta 16 \\ \to \\ -: \Delta 11, \Delta 13, F14, F15 \vdash \Delta 10, \Delta 16 \end{array} }{ \begin{array}{c} -: \Delta 11, \Delta 13, F14 \vdash \Delta 10, \Delta 16 \\ \to : \Delta 11, \Delta 13, F14 \vdash \Delta 10, \Delta 16 \end{array} } \xrightarrow{\text{ax}} \\ \frac{ -: \Delta 11, \Delta 13, F14 \vdash \Delta 10, \Delta 16}{ \begin{array}{c} -: \Delta 11, \Delta 13 \vdash \Delta 10, \Delta 16 \\ \end{array} } \xrightarrow{\text{sCut}} \xrightarrow{\text{---} \Delta 11, \Delta 13 \vdash \Delta 10, \Delta 16, F14} \xrightarrow{\text{ax}} \xrightarrow{\text{sCut}} 
Cases \to_L - \land_R
     \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \quad \text{h13} : \Delta 11, \text{F18} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \rightarrow \text{F9}), \text{F18} \vdash \Delta 10} \rightarrow_{L} \frac{\text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}, \text{F18}}{\text{h13} \bullet : \Delta 14 \vdash (\Delta 17, \text{F15} \land \text{F16}), \text{F18}} \text{Cut}} \land_{R} \\ \hline -: \Delta 14, \Delta 11, \text{F18} \rightarrow \text{F9} \vdash \Delta 10, \Delta 17, \text{F15} \land \text{F16}} \\ \hline \frac{\text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8}}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{h13} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{A1} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{A1} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \frac{1}{\text{A1} : \Delta 11, \text{F18}, \text{F9} \vdash \Delta 10} \xrightarrow
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\frac{\text{h13} : \Delta 11, \text{F15} \land \text{F16} \vdash \Delta 10, \text{F8} \quad \text{h13} : \Delta 11, \text{F9}, \text{F15} \land \text{F16} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \to \text{F9}), \text{F15} \land \text{F16} \vdash \Delta 10} \\ \xrightarrow{\text{h13} \bullet : (\Delta 11, \text{F8} \to \text{F9}), \text{F15} \land \text{F16} \vdash \Delta 10} \\ \xrightarrow{\text{h13} \bullet : \Delta 14 \vdash \Delta 17, \text{F15} \land \text{F16}} \\ \xrightarrow{\text{Cut}} \\ \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 14, \Delta 11, F8 \rightarrow F9 \vdash \Delta 10, \Delta 17
            \frac{1}{13:\Delta 11,F15 \wedge F16 \vdash \Delta 10,F8} = \frac{1}{13:\Delta 14 \vdash \Delta 17,F15} = \frac{1}{13:\Delta 14 \vdash \Delta 17,F15} = \frac{1}{13:\Delta 14 \vdash \Delta 17,F15} = \frac{1}{13:\Delta 14} = \frac{1}{13
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -:\Delta11,\Delta14, \texttt{f8} \rightarrow \texttt{f9} \vdash \Delta10,\Delta17
                                                                                                                                                                                                                                                                                          \frac{\text{h14} : \Delta 11 \vdash \Delta 10, \text{F12} \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}{\text{h14} \bullet : \Delta 11, \text{F12} \to \text{F13} \vdash \Delta 10} \to L \quad \frac{\text{h14} : \Delta 15 \vdash \Delta 18, \text{F16}, \text{F12} \to \text{F13} \quad \text{h14} : \Delta 15 \vdash \Delta 18, \text{F17}, \text{F12} \to \text{F13}}{\text{h14} \bullet : \Delta 15 \vdash (\Delta 18, \text{F16} \land \text{F17}), \text{F12} \to \text{F13}} \quad \text{Cut}} \quad \land_R = \frac{\text{h14} : \Delta 15 \vdash \Delta 10, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F16} \land \text{F17}}{\text{h14} \bullet : \Delta 15 \vdash (\Delta 18, \text{F16} \land \text{F17}), \text{F12} \to \text{F13}} \quad \text{Cut}}{\text{h14} \bullet : \Delta 15 \vdash \Delta 10, \Delta 18, \text{F16} \land \text{F17}}
          \frac{1}{14:\Delta11\vdash\Delta10,F12} \xrightarrow{ax} \frac{1}{14:\Delta11,F13\vdash\Delta10} \xrightarrow{ax} \frac{1}{14:\Delta11,F13\vdash\Delta10} \xrightarrow{ax} \xrightarrow{h14:\Delta11,F13\vdash\Delta10} \xrightarrow{ax} \xrightarrow{h14:\Delta11,F13\vdash\Delta10} \xrightarrow{ax} \xrightarrow{h14:\Delta11,F12\toF13\vdash\Delta10} \xrightarrow{h14:\Delta11,F12\toF13\vdash\Delta10} \xrightarrow{h14:\Delta15\vdash\Delta10,\Delta18,F16,F12\toF13} \xrightarrow{hCt} \xrightarrow{hCt} \xrightarrow{h14:\Delta11,F12\toF13\vdash\Delta10} \xrightarrow{ax} \xrightarrow{h14:\Delta11,F12\toF13\vdash\Delta10} \xrightarrow{ax} \xrightarrow{h14:\Delta11,F12\toF13\vdash\Delta10} \xrightarrow{hCt} \xrightarrow{hCt
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \land F17
Cases \rightarrow_L - \vee_R
                                                                                                                                                                                                                                                                \frac{ \underbrace{ \text{h13} : \Delta 11, \text{F18} \vdash \Delta 10, \text{F8} \quad \text{h13} : \Delta 11, \text{F9}, \text{F18} \vdash \Delta 10}_{\text{h13} \bullet} \quad \rightarrow_{L} \quad \frac{ \text{h13} : \Delta 14 \vdash \Delta 17, \text{F15}, \text{F16}, \text{F18}}_{\text{h13} \bullet} : \Delta 14 \vdash (\Delta 17, \text{F15} \lor \text{F16}), \text{F18}} \quad \vee_{R} \quad \text{Cut}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                  (\Delta 11, F8 \rightarrow F9), F16 \vdash \Delta 10
-: \Delta 14, \Delta 11, F8 \rightarrow F9 \vdash \Delta 10, \Delta 17, F15 \lor F16
\rightarrow
                                                                                                                                                                                                                                                                      \frac{ \frac{}{\text{h13}:\Delta11,\text{F18}\vdash\Delta10,\text{F8}} \text{ ax} }{ \frac{\text{h13}:\Delta11,\text{F18},\text{F9}\vdash\Delta10}{\text{h13}:\Delta11,\text{F18},\text{F9}\vdash\Delta10}} \xrightarrow{\text{ax}} \frac{}{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16},\text{F18}} \text{ ax}} \\ \frac{\frac{\text{h13}\bullet:\Delta11,\text{F18},\text{F8}\to\text{F9}\vdash\Delta10}{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16},\text{F18}}}{\text{-}:\Delta11,\Delta14,\text{F8}\to\text{F9}\vdash\Delta10,\Delta17,\text{F15},\text{F16}}} \xrightarrow{\text{V}_{R}} \text{ hCut} 
                                                                                                                                                                                                                                      \frac{\mathbf{h}13:\Delta11,\mathbf{F}15\vee\mathbf{F}16\vdash\Delta10,\mathbf{F}8\quad\mathbf{h}13:\Delta11,\mathbf{F}9,\mathbf{F}15\vee\mathbf{F}16\vdash\Delta10}{\underbrace{\mathbf{h}13\bullet:(\Delta11,\mathbf{F}8\to\mathbf{F}9),\mathbf{F}15\vee\mathbf{F}16\vdash\Delta10}_{-:\Delta14,\Delta11,\mathbf{F}8\to\mathbf{F}9\vdash\Delta10,\Delta17}} \to_{L} \quad \frac{\mathbf{h}13:\Delta14\vdash\Delta17,\mathbf{F}15,\mathbf{F}16}{\mathbf{h}13\bullet:\Delta14\vdash\Delta17,\mathbf{F}15\vee\mathbf{F}16}}_{-:\Delta14,\Delta11,\mathbf{F}8\to\mathbf{F}9\vdash\Delta10,\Delta17} \quad \mathbf{cut}
                                                                           \frac{\frac{1}{\text{h13}:\Delta11,\text{F15}\vee\text{F16}\vdash\Delta10,\text{F8}}}{\frac{1}{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}} = \frac{\frac{1}{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}} + \frac{\frac{1}{\text{h23}}}{\text{h23}} + \frac{\frac{1}{\text{h23}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}}{\text{h24}} = \frac{\frac{1}{\text{h23}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}} + \frac{\frac{1}{\text{h23}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}}{\text{h24}}} + \frac{\frac{1}{\text{h23}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}} +
                                                                                                                                                                                                                                                              \frac{ \underbrace{ \text{h}14:\Delta11\vdash\Delta10,\text{F}12\quad\text{h}14:\Delta11,\text{F}13\vdash\Delta10}_{\text{h}14\bullet:\Delta11,\text{F}13\vdash\Delta10} \rightarrow_{L} \quad \underbrace{ \text{h}14:\Delta15\vdash\Delta18,\text{F}16,\text{F}17,\text{F}12\rightarrow\text{F}13}_{\text{h}14\bullet:\Delta15\vdash\Delta18,\text{F}16\vee\text{F}17),\text{F}12\rightarrow\text{F}13} }_{\text{Cut}} \vee_{R}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \lor F17
                                                                                                                                                                                                                                                                     \frac{\frac{\text{h14}:\Delta11\vdash\Delta10,\text{F12}}{\text{h14}:\Delta11,\text{F13}\vdash\Delta10}}{\text{h14}:\Delta11,\text{F13}\vdash\Delta10} \xrightarrow{\text{ax}} \frac{\text{ax}}{\text{h14}:\Delta15\vdash\Delta18,\text{F16,F17,F12}\to\text{F13}}}{\text{hCut}} \xrightarrow{\text{hCut}} \frac{\text{ax}}{\text{hCut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \frac{-:\Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16, F17}{-:\Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \lor F17} \lor_{R}
Cases \to_L - \bot_R
                                                                                                                                                                                                                                                                                                                                 -: \Delta 11, \Delta 14, F8 \rightarrow F9 \vdash \Delta 10, \Delta 15  ax
                                                                                                                                                                                                                                                                                               \frac{\mathbf{h}13:\Delta 11, \mathbf{F}16 \vdash \Delta 10, \mathbf{F}8 \quad \mathbf{h}13:\Delta 11, \mathbf{F}9, \mathbf{F}16 \vdash \Delta 10}{\mathbf{h}13 \bullet : (\Delta 11, \mathbf{F}8 \to \mathbf{F}9), \mathbf{F}16 \vdash \Delta 10} \to_{L} \quad \frac{\mathbf{h}13:\Delta 14 \vdash \Delta 15, \mathbf{F}16}{\mathbf{h}13 \bullet : \Delta 14 \vdash (\bot, \Delta 15), \mathbf{F}16} \\ -:\Delta 14,\Delta 11, \mathbf{F}8 \to \mathbf{F}9 \vdash \Delta 10,\bot,\Delta 15} \quad \quad_{\mathbf{Cut}}
                                                                                                                                                                                                                                                                                            \frac{\overbrace{\text{h13}:\Delta11,\text{F16}\vdash\Delta10,\text{F8}}^{\text{ax}} \quad \frac{}{\text{h13}:\Delta11,\text{F16},\text{F9}\vdash\Delta10}}{\underbrace{\frac{\text{h13}\bullet:\Delta11,\text{F16},\text{F8}\rightarrow\text{F9}\vdash\Delta10}{}}^{\text{ax}} \quad \frac{}{\rightarrow}L \quad \frac{}{\text{h13}:\Delta14\vdash\bot,\Delta15,\text{F16}}}_{\text{hCut}} \quad \frac{\text{ax}}{\text{hCut}}
                                                                                                                                                                                                                                                                                            \mathtt{h}14:\Delta15\vdash\Delta16,\mathtt{F}12\to\mathtt{F}13
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Cases \rightarrow_L - \top_R
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\frac{\text{h13}:\Delta11,\top\vdash\Delta10,\text{F8}\quad\text{h13}:\Delta11,\text{F9},\top\vdash\Delta10}{\frac{\text{h}13\bullet:(\Delta11,\text{F8}\to\text{F9}),\top\vdash\Delta10}{-:\Delta14,\Delta11,\text{F8}\to\text{F9}\vdash\Delta10,\Delta15}} \quad \frac{}{\text{h}13\bullet:\Delta14\vdash\Delta15,\top} \quad \frac{}{\text{Cut}}
                                                                                                                                                                                                              \frac{\frac{\text{h13}: \top, \Delta 11 \vdash \Delta 10, \text{F8}}{\text{ax}} \xrightarrow[\text{h13}\bullet: \Delta 14 \vdash \top, \Delta 15]{\text{hCut}}}{\frac{-: \Delta}{\text{111}}, \Delta 14 \vdash \Delta 10, \Delta 15, \text{F8}} \xrightarrow[\text{hCut}]{\text{T}} \frac{\uparrow}{\text{R}} \xrightarrow[\text{hCut}]{\text{H13}: \top, \Delta 11, \text{F9} \vdash \Delta 10} \xrightarrow[\text{ax}]{\text{h13}\bullet: \Delta 14 \vdash \top, \Delta 15}}{-: \Delta 11, \Delta 14, \text{F9} \vdash \Delta 10, \Delta 15} \xrightarrow[\text{hCut}]{\text{T}} \frac{\uparrow}{\text{hCut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 11, \Delta 14, F8 \rightarrow F9 \vdash \Delta 10, \Delta 15
                                                                                                                                                                                                                                                                                                           \frac{\text{h13}:\Delta 11, \text{F16} \vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F9}, \text{F16} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \to \text{F9}), \text{F16} \vdash \Delta 10} \quad \rightarrow_{L} \quad \frac{}{\text{h13} \bullet :\Delta 14 \vdash (\top, \Delta 15), \text{F16}} \quad \top_{R} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -:\Delta14,\Delta11, \texttt{F8} \rightarrow \texttt{F9} \vdash \Delta10, \top, \Delta15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \frac{\text{h}14:\Delta11\vdash\Delta10,\text{F}12\quad\text{h}14:\Delta11,\text{F}13\vdash\Delta10}{\text{h}14\bullet:\underline{\Delta11,\text{F}12}\rightarrow\text{F}13\vdash\Delta10} \ \rightarrow_{L} \ \frac{}{\text{h}14\bullet:\Delta15\vdash(\top,\Delta16),\text{F}12\rightarrow\text{F}13} \ }_{\text{Cut}} \ \ \frac{\top_{R}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       -: \Delta 11, \Delta 15 \vdash \Delta 10, \top, \Delta 16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 11, \Delta 15 \vdash \top, \Delta 10, \Delta 16 \top_R
Cases \rightarrow_L - \rightarrow_L
                                                                                                                                                                                                                                                                                                                               \frac{\text{h13}:\Delta 11, \text{F17} \vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F9}, \text{F17} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8} \to \text{F9}), \text{F17} \vdash \Delta 10} \to_{L} \\ \frac{\text{h13}:\Delta 18 \vdash \Delta 16, \text{F14}, \text{F17} \quad \text{h13}:\Delta 18, \text{F15} \vdash \Delta 16, \text{F17}}{\text{h13}\bullet:\Delta 18, \text{F14} \to \text{F15} \vdash \Delta 16, \text{F17}} \\ -:\Delta 11, \text{F8} \to \text{F9}, \Delta 18, \text{F14} \to \text{F15} \vdash \Delta 10, \Delta 16} \\ \to \\ \to \\ \text{Out}
           \frac{\text{h13} : \Delta 11, \text{F17} \vdash \Delta 10, \text{F8}}{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \xrightarrow{\text{ax}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F9} \rightarrow \Delta 10} \xrightarrow{\text{b13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{b13} : \Delta 18} \xrightarrow{\text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h10} : \Delta 16, \text{F14}, \text{F17}} \xrightarrow{\text{h0xt}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{ax}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 18, \text{F15} \vdash \Delta 16, \text{F17}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 18, \text{F15} \vdash \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F17}, \text{F18} \rightarrow \text{F19} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F18} \rightarrow \text{F19} \vdash \Delta 10} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F19} \rightarrow \text{F19}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F19} \rightarrow \text{F19}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F19} \rightarrow \text{H10}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F19}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F19} \rightarrow \text{H10}} \xrightarrow{\text{h13} : \Delta 11, \text{F17}, \text{F19} \rightarrow \text{H10}} \xrightarrow{\text{h13} : \Delta 11, \text{H10}} 
                                                                                                                                                                                                                                                                                  \frac{-:\Delta 11,\Delta 18,F8\to F9\vdash \Delta 10,\Delta 16,F14}{-:\Delta 11,\Delta 18,F14\to F15,F8\to F9\vdash \Delta 10,\Delta 16}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  -: \Delta 11, \Delta 18, F15, F8 \rightarrow F9 \vdash \Delta 10, \Delta 16 \rightarrow_L
                                                                                                                                                                                                                                                                                                      \frac{\frac{\text{h14} : \Delta11 \vdash \Delta10, F12 \quad \text{h14} : \Delta11, F13 \vdash \Delta10}{\text{h14} \bullet : \Delta11, F12 \to F13 \vdash \Delta10} \to_{L} \quad \frac{\text{h14} : \Delta18 \vdash \Delta17, F15, F12 \to F13 \quad \text{h14} : \Delta18, F16 \vdash \Delta17, F12 \to F13}{\text{h14} \bullet : \Delta18, F15 \to F16 \vdash \Delta17, F12 \to F13} \quad \text{Cut} \\ & - : \Delta11, \Delta18, F15 \to F16 \vdash \Delta10, \Delta17} \quad \text{Cut}
           \frac{\frac{1}{\text{h14}:\Delta11+\Delta10,\text{F12}}}{\frac{\text{h14}\bullet:\Delta11,\text{F13}\to\Delta10}}{\frac{\text{h14}:\Delta11,\text{F13}\to\Delta10}} \xrightarrow{\Delta L} \xrightarrow{\text{h14}:\Delta18+\Delta17,\text{F15},\text{F12}\to\text{F13}} \xrightarrow{\text{hCut}} \xrightarrow{\text{hCut}} \xrightarrow{\text{h14}\bullet:\Delta11,\text{F12}} \xrightarrow{\text{h14}\bullet:\Delta11,\text{F13}\to\Delta10} \xrightarrow{\text{hCut}} \xrightarrow{\text{h14}:\Delta11,\text{F13}\to\Delta10} \xrightarrow{\text{hCut}} \xrightarrow{\text{hCut
Cases \to_L - \land_L
                                                                                                                                                                                                                                                                                  \frac{\text{h13}:\Delta11,\text{F17}\vdash\Delta10,\text{F8}\quad\text{h13}:\Delta11,\text{F9},\text{F17}\vdash\Delta10}{\text{h13}\bullet:(\Delta11,\text{F8}\to\text{F9}),\text{F17}\vdash\Delta10} \ \to_L \ \frac{\text{h13}:\Delta18,\text{F14},\text{F15}\vdash\Delta16,\text{F17}}{\text{h13}\bullet:\Delta18,\text{F14}\land\text{F15}\vdash\Delta16,\text{F17}} \ \land_L
                                                                                                                                                                                                                                                                                                                                                                              -: \Delta 11, F8 \rightarrow F9, \Delta 18, F14 \land F15 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                                                                                \frac{-:\Delta 11, \text{F17} \rightarrow \text{F29}, \Delta 129, \text{F12} \land 110 \land 120}{\rightarrow} \xrightarrow{\text{A13}} \frac{13:\Delta 11, \text{F17}, \text{F9} \rightarrow \text{F20}}{\text{A13}} \xrightarrow{\text{A13}} \frac{\text{A13}}{\rightarrow} L
\frac{\text{h13} \bullet :\Delta 11, \text{F17}, \text{F8} \rightarrow \text{F9} \vdash \Delta 10}{\rightarrow} \xrightarrow{\text{A13}} \frac{\text{A13}}{\rightarrow} L \xrightarrow{\text{h13}} \frac{\text{A14}, \text{F15} \vdash \Delta 16, \text{F17}}{\text{hOut}} \xrightarrow{\text{hOut}} \frac{\text{A15}}{\rightarrow} L
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \frac{-:\Delta 11, \Delta 18, F14, F15, F8 \to F9 \vdash \Delta 10, \Delta 16}{-:\Delta 11, \Delta 18, F8 \to F9, F14 \land F15 \vdash \Delta 10, \Delta 16} \land_L
                                                                                                                                                                                                                                                                                  \frac{ \underbrace{ \text{h14} : \Delta 11 \vdash \Delta 10, \text{F12} \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}_{\text{h14} \bullet : \Delta 11, \text{F12} \to \text{F13} \vdash \Delta 10} \quad \rightarrow_{L} \quad \underbrace{ \underbrace{ \text{h14} : \Delta 18, \text{F15}, \text{F16} \vdash \Delta 17, \text{F12} \to \text{F13}}_{\text{h14} \bullet : \Delta 18, \text{F15} \land \text{F16} \vdash \Delta 17, \text{F12} \to \text{F13}}_{\text{Cut}} \quad \wedge_{L} \quad \underbrace{ \text{Cut}}_{\text{Cut}} \quad \text{Cut} 
                                                                                                                                                                                                                                                                                                                                                                                                                                      -:\Delta 11,\Delta 18,\mathsf{F}15\wedge\mathsf{F}16\vdash\Delta 10,\Delta 17
                                                                                                                                                                                                                                                                             \frac{\overbrace{\frac{\text{h14}:\Delta11\vdash\Delta10,\text{F12}}{\text{h}}}^{\text{ax}}}{\underbrace{\frac{\text{h14}:\Delta11,\text{F12}\to\Delta10}{\text{h}}}^{\text{ax}}} \xrightarrow{\frac{\text{ax}}{\text{h}}}^{\text{h}} \underbrace{\frac{\text{h14}:\Delta11,\text{F13}\vdash\Delta10}{\text{h}}}^{\text{ax}} \xrightarrow{\frac{\text{ax}}{\text{h}}}^{\text{h}} \underbrace{\frac{\text{h14}:\Delta18,\text{F15},\text{F16}\vdash\Delta17,\text{F12}\to\text{F13}}{\text{h}}}^{\text{ax}}}_{\text{hCut}} \underbrace{\frac{-:\Delta11,\Delta18,\text{F15},\text{F16}\vdash\Delta10,\Delta17}{\text{-}:\Delta11,\Delta18,\text{F15},\text{F16}\vdash\Delta10,\Delta17}}_{\text{-}} \land_{L}}^{\text{A}}
Cases \rightarrow_L - \vee_L
                                                                                                                                                                                                                                                                                                                        \frac{\text{h13} : \Delta11, \text{F17} \vdash \Delta10, \text{F8} \quad \text{h13} : \Delta11, \text{F9}, \text{F17} \vdash \Delta10}{\text{h13} \bullet : (\Delta11, \text{F8} \to \text{F9}), \text{F17} \vdash \Delta10} \to L \quad \frac{\text{h13} : \Delta18, \text{F14} \vdash \Delta16, \text{F17} \quad \text{h13} : \Delta18, \text{F15} \vdash \Delta16, \text{F17}}{\text{h13} \bullet : \Delta18, \text{F14} \lor \text{F15} \vdash \Delta16, \text{F17}} \quad \vee_L \\ - : \Delta11, \text{F8} \to \text{F9}, \Delta18, \text{F14} \lor \text{F15} \vdash \Delta10, \Delta16} \quad \text{Cut} 
             \frac{-:\Delta 11,F8 \to F9,\Delta 10,F15 \vee LUC}{\frac{1}{13}:\Delta 18,F14 \vdash \Delta 16,F17} \xrightarrow{ax} \xrightarrow{h13:\Delta 18,F14 \vdash \Delta 16,F17} \xrightarrow{ax} \xrightarrow{h13:\Delta 18,F15 \vdash \Delta 16,F17} \xrightarrow{ax} \xrightarrow{c} \xrightarrow{b13:\Delta 11,F17 \vdash \Delta 10,F8} \xrightarrow{ax} \xrightarrow{c} \xrightarrow{b13:\Delta 11,F17 \vdash \Delta 10,F18} \xrightarrow{ax} \xrightarrow{h13:\Delta 18,F14 \vdash \Delta 16,F17} \xrightarrow{ax} \xrightarrow{h13:\Delta 18,F15 \vdash \Delta 16,F17} \xrightarrow{hCut} \xrightarrow{c} \xrightarrow{hCut} \xrightarrow
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    -: \Delta 11, \Delta 18, F8 \rightarrow F9, F14 \lor F15 \vdash \Delta 10, \Delta 16
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\frac{\text{h14} : \Delta 11 \vdash \Delta 10, \text{F12} \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}{\text{h14} \bullet : \Delta 11, \text{F12} \to \text{F13} \vdash \Delta 10} \to L \quad \frac{\text{h14} : \Delta 18, \text{F15} \vdash \Delta 17, \text{F12} \to \text{F13} \quad \text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \to \text{F13}}{\text{h14} \bullet : \Delta 18, \text{F15} \lor \text{F16} \vdash \Delta 17, \text{F12} \to \text{F13}} \quad \vee_L \to \Delta 11, \Delta 18, \Delta
          \frac{1}{\underbrace{\text{h14}:\Delta11+\Delta10,\text{F12}}} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta11,\text{F13}+\Delta10} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta11,\text{F13}+\Delta10} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta18,\text{F15}+\Delta17,\text{F12}} \xrightarrow{\text{F13}} \xrightarrow{\text{h04}} \frac{1}{\text{h04}} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta11,\text{F13}+\Delta10} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta11,\text{F13}+\Delta10} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta11,\text{F13}+\Delta10} \xrightarrow{\text{ax}} \frac{1}{\text{h14}:\Delta18,\text{F16}+\Delta17,\text{F12}\to\text{F13}} \xrightarrow{\text{h04}} \frac{1}{\text{h04}} \xrightarrow{\text{h04}} \frac{1}{\text{h04}:\Delta11,\text{F12}\to\text{F13}+\Delta10} \xrightarrow{\text{h04}:\Delta11,\text{F16}+\Delta10,\Delta17} \xrightarrow{\text{h14}:\Delta18,\text{F16}+\Delta17,\text{F12}\to\text{F13}} \xrightarrow{\text{h04}:\Delta11,\text{F12}\to\text{F13}+\Delta10} \xrightarrow{\text{h04}:\Delta11,\text{F16}\to\text{h04}} \xrightarrow{\text{h04}:\Delta11,\text{h04}:\Delta11,\text{F16}\to\text{h04}} \xrightarrow{\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta11,\text{h04}:\Delta
Cases \rightarrow_L - \perp_L
                                                                                                                                                                                                                                                                                               \frac{\text{h13}:\Delta 11, \text{F15}\vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F9}, \text{F15}\vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\rightarrow \text{F9}), \text{F15}\vdash \Delta 10} \rightarrow_{L} \frac{\text{h13}\bullet:\bot,\Delta 16\vdash \Delta 14, \text{F15}}{-:\Delta 11, \text{F8}\rightarrow \text{F9},\bot,\Delta 16\vdash \Delta 10,\Delta 14} \xrightarrow{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -: \bot, \Delta 11, \Delta 16, F8 \to F9 \vdash \Delta 10, \Delta 14  ^{\bot}L
                                                                                                                                                                                                                                                                                               \frac{ \underbrace{ \text{h14} : \Delta 11 \vdash \Delta 10, \text{F12} \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}_{\text{h14} \bullet : \Delta 11, \text{F12} \rightarrow \text{F13} \vdash \Delta 10} \rightarrow_{L} \underbrace{ \underbrace{ \text{h14} \bullet : \bot, \Delta 16 \vdash \Delta 15, \text{F12} \rightarrow \text{F13}}_{\text{h14} \bullet : \bot, \Delta 16 \vdash \Delta 10, \Delta 15} }_{\text{Cut}} \xrightarrow{L} \underbrace{ \text{Cut}}_{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{}{-:\bot,\Delta11,\Delta16\vdash\Delta10,\Delta15} \perp_{L}
Cases \rightarrow_L - I
                                                                                                                                                                                                                                                                      \frac{\text{h13} : \Delta11, \text{F15} \vdash \Delta10, \text{F8} \quad \text{h13} : \Delta11, \text{F9}, \text{F15} \vdash \Delta10}{\text{h13} \bullet : (\Delta11, \text{F8} \to \text{F9}), \text{F15} \vdash \Delta10} \\ - : \Delta11, \text{F8} \to \text{F9}, \Delta17, \text{p16} \vdash \Delta10, \Delta14, \text{p16}} \quad \frac{I}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \frac{}{-:\Delta11,\Delta17,\mathtt{p}16,\mathtt{f}8\to\mathtt{f}9\vdash\Delta10,\Delta14,\mathtt{p}16}^{}-iI
                                                                                                                                                                                                                                                                                               \frac{ \underbrace{ \text{h13} : \Delta 11, \text{p15} \vdash \Delta 10, \text{F8} \quad \text{h13} : \Delta 11, \text{F9}, \text{p15} \vdash \Delta 10}_{ - 13, - 14, - 15, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12, - 12,
                                                                                                                                                                                                                                                                                                                                                   \begin{array}{c|c} & \xrightarrow{\quad -: \ \Delta 11, \ \Delta 16, \ p15 \ \vdash \ \Delta 10, \ \Delta 14, \ F8} & \text{ax} & \xrightarrow{\quad -: \ \Delta 11, \ \Delta 16, \ F9, \ p15 \ \vdash \ \Delta 10, \ \Delta 14} & \text{ax} \\ & \xrightarrow{\quad -: \ \Delta 11, \ \Delta 16, \ p15, \ F8 \ \rightarrow \ F9 \ \vdash \ \Delta 10, \ \Delta 14} & \xrightarrow{\quad \bot}_{L} \end{array}
                                                                                                                                                                                                                                                                      \frac{\frac{\text{h14}:\Delta 11 \vdash \Delta 10, \text{F12} \quad \text{h14}:\Delta 11, \text{F13} \vdash \Delta 10}{\text{h14}\bullet:\Delta 11, \text{F12} \rightarrow \text{F13} \vdash \Delta 10} \rightarrow_{L} \frac{}{\text{h14}\bullet:\Delta 17, \text{p16} \vdash (\Delta 15, \text{p16}), \text{F12} \rightarrow \text{F13}} I \text{ Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 11, \Delta 17, p16 \vdash \Delta 10, \Delta 15, p16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{\cdot}{-:\Delta11,\Delta17,\mathtt{p}16\vdash\Delta10,\Delta15,\mathtt{p}16} \ iI
Cases \to_L - \top_L
                                                                                                                                                                                                                                                                                               \frac{\text{h13}:\Delta 11, \text{F15}\vdash \Delta 10, \text{F8} \quad \text{h13}:\Delta 11, \text{F9}, \text{F15}\vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\rightarrow \text{F9}), \text{F15}\vdash \Delta 10} \rightarrow_{L} \frac{\text{h13}:\Delta 16\vdash \Delta 14, \text{F15}}{\text{h13}\bullet:\top,\Delta 16\vdash \Delta 14, \text{F15}} \quad \tau_{L} \\ -:\Delta 11, \text{F8}\rightarrow \text{F9}, \top,\Delta 16\vdash \Delta 10,\Delta 14} \xrightarrow{} \text{Cut}
                                                                                                                                                                                                                                                                                         \frac{ \begin{array}{c} \mathbf{h} \mathbf{14} : \Delta \mathbf{11} \vdash \Delta \mathbf{10}, \mathbf{F} \mathbf{12} \quad \mathbf{h} \mathbf{14} : \Delta \mathbf{11}, \mathbf{F} \mathbf{13} \vdash \Delta \mathbf{10} \\ \\ \underline{ \begin{array}{c} \mathbf{h} \mathbf{14} \bullet : \Delta \mathbf{11}, \mathbf{F} \mathbf{12} \rightarrow \mathbf{F} \mathbf{13} \vdash \Delta \mathbf{10} \\ \\ - : \Delta \mathbf{11}, \top, \Delta \mathbf{16} \vdash \Delta \mathbf{10}, \Delta \mathbf{15} \\ \\ \end{array} \\ \end{array} \begin{array}{c} \boldsymbol{\tau}_L \\ \mathbf{h} \mathbf{14} \bullet : \top, \Delta \mathbf{16} \vdash \Delta \mathbf{15}, \mathbf{F} \mathbf{12} \rightarrow \mathbf{F} \mathbf{13} \\ \mathbf{Cut} \\ \end{array} \\ \mathbf{Cut}
                                                                                                                                                                                                                                                                                         Cases \wedge_L - \rightarrow_R
                                                                                                                                                                                                                                                                                                                                         \underbrace{\begin{array}{l} \text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F18} \vdash \Delta 10 \\ \text{h13} \underbrace{\bullet: (\Delta 11, \text{F8} \land \text{F9}), \text{F18} \vdash \Delta 10} \\ \end{array}}_{\text{Cut}} \quad \bigwedge_{L} \quad \underbrace{\begin{array}{l} \text{h13}:\Delta 14, \text{F15} \vdash \Delta 17, \text{F16}, \text{F18} \\ \text{h13} \underbrace{\bullet: (\Delta 14 \vdash (\Delta 17, \text{F15} \rightarrow \text{F16}), \text{F18}} \\ \end{array}}_{\text{Cut}} \quad \xrightarrow{\text{Cut}} \quad \underbrace{\begin{array}{l} \text{Cut} \\ \text{Cut} \\ \text{Cut} \\ \end{array}}_{\text{Cut}} 
                                                                                                                                                                                                                                                                                                                                                                                                                -:\Delta 14,\Delta 11, \mathsf{F8} \wedge \mathsf{F9} \vdash \Delta 10,\Delta 17, \mathsf{F15} \to \mathsf{F16}
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\frac{\text{h13} : \Delta 11, \text{F8}, \text{F9}, \text{F15} \rightarrow \text{F16} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \land \text{F9}), \text{F15} \rightarrow \text{F16} \vdash \Delta 10} \quad \wedge_L \quad \frac{\text{h13} : \Delta 14, \text{F15} \vdash \Delta 17, \text{F16}}{\text{h13} \bullet : \Delta 14 \vdash \Delta 17, \text{F15} \rightarrow \text{F16}} \quad \xrightarrow{\text{Cut}} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -: \Delta 14, \Delta 11, F8 \land F9 \vdash \Delta 10, \Delta 17
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \frac{1}{\text{h13}:\Delta11,\text{F8},\text{F9},\text{F15}\rightarrow\text{F16}\vdash\Delta10} \xrightarrow{\text{ax}} \frac{1}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}} \xrightarrow{\text{ax}} \xrightarrow{\rightarrow}_{R} \frac{1}{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\rightarrow\text{F16}} \xrightarrow{\text{hCut}} \frac{1}{\text{hCut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{-:\Delta 11,\Delta 14, \texttt{F8}, \texttt{F9} \vdash \Delta 10,\Delta 17}{-:\Delta 11,\Delta 14, \texttt{F8} \land \texttt{F9} \vdash \Delta 10,\Delta 17} \land_{L}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{\text{h14}:\Delta 11,\text{F12},\text{F13}\vdash\Delta 10}{\text{h14}\bullet:\Delta 11,\text{F12}\land\text{F13}\vdash\Delta 10} \quad \land_L \quad \frac{\text{h14}:\Delta 15,\text{F16}\vdash\Delta 18,\text{F17},\text{F12}\land\text{F13}}{\text{h14}\bullet:\Delta 15\vdash(\Delta 18,\text{F16}\rightarrow\text{F17}),\text{F12}\land\text{F13}} \quad \rightarrow_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \rightarrow F17
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \boxed{ \texttt{h}14:\Delta11,\texttt{F}12,\texttt{F}13\vdash\Delta10} \quad \texttt{ax}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \begin{array}{c|c}  & \text{h14} : \Delta 11, \text{F12}, \text{F13} \vdash \Delta 10 \\ \hline & \text{h14} \underbrace{\bullet} : \Delta 11, \text{F12} \land \text{F13} \vdash \Delta 10 \\ \hline & \bullet : \Delta 11, \text{F12} \land \text{F13} \vdash \Delta 10 \\ \end{array} \, \stackrel{\wedge}{\wedge} L \quad \overline{\text{h14} : \Delta 15, \text{F16} \vdash \Delta 18, \text{F17}, \text{F12} \land \text{F13}} \quad \text{ax} \\ \text{hCut} \quad \text{hCut} \quad \overline{\text{hCut}} \quad \overline{\text{hCut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{-:\Delta11,\Delta15,\mathtt{F}16\vdash\Delta10,\Delta18,\mathtt{F}17}{-:\Delta11,\Delta15\vdash\Delta10,\Delta18,\mathtt{F}16\to\mathtt{F}17}\to_{R}
Cases \wedge_L - \wedge_R
                                                                                                                                                                                                                                                                                                                                                                                                      \frac{\text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F18} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8} \land \text{F9}), \text{F18} \vdash \Delta 10} \quad \wedge_L \quad \frac{\text{h13}:\Delta 14 \vdash \Delta 17, \text{F15}, \text{F18} \quad \text{h13}:\Delta 14 \vdash \Delta 17, \text{F16}, \text{F18}}{\text{h13}\bullet:\Delta 14 \vdash (\Delta 17, \text{F15} \land \text{F16}), \text{F18}} \quad \wedge_R \quad \wedge_R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -:\Delta 14,\Delta 11,\mathsf{F8}\wedge\mathsf{F9}\vdash\Delta 10,\Delta 17,\mathsf{F15}\wedge\mathsf{F16}
                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{1}{113} \underbrace{: \Delta 11, F18, F8, F9 \vdash \Delta 10}_{} \quad \text{ax} \quad \frac{\rightarrow}{113 : \Delta 14 \vdash \Delta 17, F15, F18} \quad \frac{\rightarrow}{ax} \quad \frac{\rightarrow}{113 : \Delta 14 \vdash \Delta 17, F16, F18} \quad \frac{\rightarrow}{h \times 1} \quad \frac{\rightarrow}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \frac{-:\Delta 11,\Delta 14,\mathsf{F8},\mathsf{F9}\vdash\Delta 10,\Delta 17,\mathsf{F15}\wedge\mathsf{F16}}{-:\Delta 11,\Delta 14,\mathsf{F8}\wedge\mathsf{F9}\vdash\Delta 10,\Delta 17,\mathsf{F15}\wedge\mathsf{F16}} \ \land_L
                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{ \frac{\text{h13} : \Delta 11, \text{F8}, \text{F9}, \text{F15} \wedge \text{F16} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \wedge \text{F9}), \text{F15} \wedge \text{F16} \vdash \Delta 10}}{-: \Delta 14, \Delta 11, \text{F8} \wedge \text{F9} \vdash \Delta 10, \Delta 17}  \wedge_L \frac{\text{h13} : \Delta 14 \vdash \Delta 17, \text{F15} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}}{\text{h13} \bullet : \Delta 14 \vdash \Delta 17, \text{F15} \wedge \text{F16}} \text{Cut}} \wedge_R 
                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{1}{113} \cdot \Delta 11, F8, F9, F15 \wedge F16 \vdash \Delta 10 \qquad \text{ax} \qquad \frac{1}{113 \cdot \Delta 14 \vdash \Delta 17, F15} \quad \frac{\text{ax}}{113 \cdot \Delta 14 \vdash \Delta 17, F15 \wedge F16} \quad \frac{\text{ax}}{\wedge R}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \frac{-:\Delta11,\Delta14,\texttt{F8},\texttt{F9}\vdash\Delta10,\Delta17}{-:\Delta11,\Delta14,\texttt{F8}\land\texttt{F9}\vdash\Delta10,\Delta17} \land_L
                                                                                                                                                                                                                                                                                                                                                     \frac{\text{h14}:\Delta 11,\text{F12},\text{F13} \vdash \Delta 10}{\text{h14}\bullet:\Delta 11,\text{F12}\wedge\text{F13} \vdash \Delta 10} \ \wedge_L \ \frac{\text{h14}:\Delta 15 \vdash \Delta 18,\text{F16},\text{F12}\wedge\text{F13} \ \text{h14}:\Delta 15 \vdash \Delta 18,\text{F17},\text{F12}\wedge\text{F13}}{\text{h14}\bullet:\Delta 15 \vdash (\Delta 18,\text{F16}\wedge\text{F17}),\text{F12}\wedge\text{F13}} \ \wedge_R \ \frac{\text{h14}\cdot\Delta 15 \vdash \Delta 10}{\text{h14}\bullet:\Delta 15 \vdash (\Delta 18,\text{F16}\wedge\text{F17}),\text{F12}\wedge\text{F13}} \ \text{Out}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \land F17
                                                                                 \frac{\overbrace{\text{h14} : \Delta 11, \text{F12}, \text{F13} \vdash \Delta 10}^{\text{h14} : \Delta 11, \text{F12}, \text{F13} \vdash \Delta 10}^{\text{ax}} \land \underbrace{\frac{\text{h14} : \Delta 11, \text{F12}, \text{F13} \vdash \Delta 10}{\text{h14} : \Delta 11, \text{F12}, \text{F13} \vdash \Delta 10}^{\text{h2}} \land \underbrace{\frac{\text{h14} : \Delta 11, \text{F12}, \text{F13} \vdash \Delta 10}{\text{h24}}^{\text{h2}} \land \underbrace{\frac{\text{h14} : \Delta 15 \vdash \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}}}^{\text{ax}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}, \text{F12} \land \text{F13}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \text{h25} \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, \text{F17}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}}}^{\text{h24}} \land \underbrace{\frac{\text{h24} : \Delta 11, \Delta 11,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           \frac{\text{h12}:\Delta 11,\text{F14},\text{F15}\vdash\Delta 10}{\text{h12}\bullet:\Delta 11,\text{F14}\wedge\text{F15}\vdash\Delta 10} \wedge_{L} \frac{\text{h12}:\Delta 13\vdash\Delta 16,\text{F14}}{\text{h12}\bullet:\Delta 13\vdash\Delta 16,\text{F14}\wedge\text{F15}} \wedge_{R}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 11, \Delta 13 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                                                                       Cases \wedge_L - \vee_R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{\text{h13}:\Delta11,\text{F8},\text{F9},\text{F18}\vdash\Delta10}{\text{h13}\bullet:(\Delta11,\text{F8}\land\text{F9}),\text{F18}\vdash\Delta10} \quad \wedge_L \quad \frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16},\text{F18}}{\text{h13}\bullet:\Delta14\vdash(\Delta17,\text{F15}\vee\text{F16}),\text{F18}} \quad \vee_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 14, \Delta 11, F8 \land F9 \vdash \Delta 10, \Delta 17, F15 \lor F16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \frac{1}{12} \frac{1}{12}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      \frac{\text{h13}: \Delta11, \text{F18, F8, F9} \vdash \Delta10}{\text{h13} \bullet: \Delta11, \text{F18, F8} \land} \frac{}{\text{F9} \vdash \Delta10} \quad \land_L \quad \frac{}{\text{h13}: \Delta14 \vdash \Delta17, \text{F15, F16, F18}} \quad \text{ax} \quad \text{hcut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \frac{-:\Delta 11,\Delta 14,F8 \wedge F9 \vdash \Delta 10,\Delta 17,F15,F16}{-:\Delta 11,\Delta 14,F8 \wedge F9 \vdash \Delta 10,\Delta 17,F15 \vee F16} \vee_{R}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{\text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F15}\vee \text{F16}\vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\wedge \text{F9}), \text{F15}\vee \text{F16}\vdash \Delta 10} \  \  \, \wedge_{L} \  \  \, \frac{\text{h13}:\Delta 14\vdash \Delta 17, \text{F15}, \text{F16}}{\text{h13}\bullet:\Delta 14\vdash \Delta 17, \text{F15}\vee \text{F16}} \  \  \, \vee_{R} \  \  \, \text{Cut}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: \Delta 14, \Delta 11, F8 \wedge F9 \vdash \Delta 10, \Delta 17
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{\frac{\text{h13}:\Delta11,\text{F8},\text{F9},\text{F15}\vee\text{F16}\vdash\Delta10}{-:\Delta11,\Delta14,\text{F8},\text{F9}\vdash\Delta10,\Delta17}}{\frac{-:\Delta11,\Delta14,\text{F8},\text{F9}\vdash\Delta10,\Delta17}{-:\Delta11,\Delta14,\text{F8}\wedge\text{F9}\vdash\Delta10,\Delta17}} \wedge_L \overset{\text{ax}}{\overset{\text{cx}}{\sim}} \frac{\text{dx}}{\text{hCut}}
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\frac{\mathsf{h}14:\Delta11,\mathsf{F}12,\mathsf{F}13\vdash\Delta10}{\mathsf{h}14\bullet:\Delta11,\mathsf{F}12}\land\mathsf{F}13\vdash\Delta10} \land_L \quad \frac{\mathsf{h}14:\Delta15\vdash\Delta18,\mathsf{F}16,\mathsf{F}17,\mathsf{F}12\land\mathsf{F}13}{\mathsf{h}14\bullet:\Delta15\vdash(\Delta18,\mathsf{F}16\lor\mathsf{F}17),\mathsf{F}12\land\mathsf{F}13}} \lor_R \quad \mathsf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                        -:\Delta11,\Delta15\vdash\Delta10,\Delta18,\mathsf{F}16\vee\mathsf{F}17
                                                                                                                                                                                                                                                                                                                                    \frac{1}{\text{h}14:\Delta11,\text{F}12},\text{F}13\vdash\Delta10} \quad \text{ax}
                                                                                                                                                                                                                                                                                                                        \frac{-:\Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16, F17}{-:\Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \lor F17} \lor_{R}
Cases \wedge_L - \perp_R
                                                                                                                                                                                                                                                                                                                                                                       \begin{array}{c|c} \underline{\mathbf{h}13:\Delta11,\mathsf{F8},\mathsf{F9},\bot\vdash\Delta10} \\ \underline{\mathbf{h}13\bullet:(\Delta11,\mathsf{F8}\wedge\mathsf{F9}),\bot\vdash\Delta10} \end{array} \wedge_{L} \begin{array}{c|c} \underline{\mathbf{h}13:\Delta14\vdash\Delta15} \\ \underline{\mathbf{h}13\bullet:\Delta14\vdash\Delta15,\bot} \end{array} \xrightarrow{\mathsf{Cut}} \mathbf{Cut} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 14, \Delta 11, F8 \land F9 \vdash \Delta 10, \Delta 15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                         -: \Delta 11, \Delta 14, F8 \wedge F9 \vdash \Delta 10, \Delta 15 ax
                                                                                                                                                                                                                                                                                                                                           \begin{array}{l} \frac{\text{h13}:\Delta11,\text{F8},\text{F9},\text{F16}\vdash\Delta10}{\text{h13}\bullet:(\Delta11,\text{F8}\land\text{F9}),\text{F16}\vdash\Delta10} & \wedge_L & \frac{\text{h13}:\Delta14\vdash\Delta15,\text{F16}}{\text{h13}\bullet:\Delta14\vdash(\bot,\Delta15),\text{F16}} \\ -:\Delta14,\Delta11,\text{F8}\land\text{F9}\vdash\Delta10,\bot,\Delta15 & \text{cut} \end{array} \right. \\ \\ \frac{\text{cut}}{\text{cut}} 
                                                                                                                                                                                                                                                                                                                                                                     \frac{}{\texttt{h}13:\Delta11,\texttt{F}16,\texttt{F}8,\texttt{F}9}\vdash\Delta10 \quad \text{ax}
                                                                                                                                                                                                                                                                                                                                                              \begin{array}{l} \text{h13} : \Delta 11, \text{F16}, \text{F8}, \text{F9} \vdash \Delta 10 \\ \\ \underline{\text{h13} \bullet} : \Delta 11, \text{F16}, \text{F8} \land \text{F9} \vdash \Delta 10 \end{array} \  \, \begin{array}{l} \land L \\ \\ \hline - : \Delta 11, \Delta 14, \text{F8} \land \text{F9} \vdash \bot, \Delta 10, \Delta 15 \end{array} \  \, \begin{array}{l} \text{ax} \\ \text{hCut} \end{array} 
                                                                                                                                                                                                                                                                                                                                    -: \Delta 11, \Delta 15 \vdash \Delta 10, \bot, \Delta 16
                                                                                                                                                                                                                                                                                                                                                       \frac{1}{\text{h}14:\Delta11,\text{F}12,\text{F}13} \vdash \Delta10 \quad \text{ax}
                                                                                                                                                                                                                                                                                                                                              \begin{array}{c} \frac{\text{h14}:\Delta11,\text{F12},\text{F13} \vdash \Delta10}{\text{h14}\bullet:\Delta11,\text{F12} \land \text{F13} \vdash \Delta10} & \wedge_L & \\ \hline & \frac{\text{h14}\cdot\Delta15 \vdash \bot,\Delta16,\text{F12} \land \text{F13}}{-:\Delta11,\Delta15 \vdash \bot,\Delta10,\Delta16} & \text{ax} \\ \hline & & \text{hCut} \end{array} 
Cases \wedge_L - \top_R
                                                                                                                                                                                                                                                                                                                                                                      \begin{array}{c|c} \underline{\mathbf{h}13:\Delta11,\mathbf{F8},\mathbf{F9},\top\vdash\Delta10} \\ \underline{\mathbf{h}13}\underline{\bullet:(\Delta11,\mathbf{F8}\wedge\mathbf{F9}),\top\vdash\Delta10} \end{array} \wedge_{L} \quad \underline{\underline{\mathbf{h}13}\underline{\bullet:\Delta14\vdash\Delta15,\top}} \quad \top_{R} \\ \underline{\mathbf{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: \Delta 14, \Delta 11, F8 \wedge F9 \vdash \Delta 10, \Delta 15
                                                                                                                                                                                                                                                                                                                                                                                             \frac{\underset{1}{\underbrace{\text{h13}: \top, \Delta 11, \text{F8}, \text{F9} \vdash \Delta 10}} \overset{\rightarrow}{\text{ax}} \overset{\rightarrow}{\text{h13}\bullet: \Delta 14 \vdash \top, \Delta 15}} \\ \frac{-: \Delta 11, \Delta 14, \text{F8}, \text{F9} \vdash \Delta 10, \Delta 15}{-: \Delta 11, \Delta 14, \text{F8} \land \text{F9} \vdash \Delta 10, \Delta 15} \overset{\wedge}{\wedge}_{L} 
                                                                                                                                                                                                                                                                                                                                          \frac{\text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F16} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8} \land \text{F9}), \text{F16} \vdash \Delta 10} \quad \wedge_L \quad \frac{}{\text{h13}\bullet:\Delta 14 \vdash (\top,\Delta 15), \text{F16}} \quad \top_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                    -: \Delta 14, \Delta 11, F8 \land F9 \vdash \Delta 10, \top, \Delta 15
                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{}{-:\Delta 11,\Delta 14,\mathsf{F8}\wedge\mathsf{F9}\vdash\top,\Delta 10,\Delta 15}\quad\top_{R}
                                                                                                                                                                                                                                                                                                                                   \begin{array}{c|c} \underline{\mathbf{h}} 14: \Delta 11, \mathbf{F} 12, \mathbf{F} 13 \vdash \Delta 10 \\ \underline{\mathbf{h}} 14 \bullet : \Delta 11, \mathbf{F} 12 \underline{\wedge} \mathbf{F} 13 \vdash \Delta 10 \end{array} \  \, \wedge_{L} \quad \underline{\mathbf{h}} 14 \bullet : \Delta 15 \vdash (\top, \Delta 16), \mathbf{F} 12 \wedge \mathbf{F} 13 \\ \mathbf{Cut} \\ \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 11, \Delta 15 \vdash \Delta 10, \top, \Delta 16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \overline{-:\Delta 11,\Delta 15} \vdash \top,\Delta 10,\Delta 16 \top R
Cases \wedge_L - \rightarrow_L
                                                                                                                                                                                                                                   \frac{ \begin{array}{l} \text{h13}:\Delta11, \text{F8}, \text{F9}, \text{F17} \vdash \Delta10 \\ \hline \\ \underline{\text{h13} \bullet : (\Delta11, \text{F8} \land \text{F9}), \text{F17} \vdash \Delta10} \end{array} \\ \wedge_L \quad \frac{ \text{h13}:\Delta18 \vdash \Delta16, \text{F14}, \text{F17} \quad \text{h13}:\Delta18, \text{F15} \vdash \Delta16, \text{F17} \\ \hline \\ -:\Delta11, \text{F8} \land \text{F9}, \Delta18, \text{F14} \rightarrow \text{F15} \vdash \Delta10, \Delta16} \end{array} \\ \rightarrow_L \quad \frac{ \text{h13}:\Delta18 \vdash \Delta16, \text{F17} \quad \text{h13}:\Delta18, \text{F15} \vdash \Delta16, \text{F17} \\ }{ \text{Cut}} \quad \rightarrow_L \quad \frac{ \text{Cut}}{ \text{Cut}} \\ \rightarrow_L \quad \frac{ \text{Cut}}{ \text{Cut}} \quad \frac{ \text{Cut}}{ \text{Cut}} \\ \rightarrow_L \quad \frac{ \text
                                                                                                                                                                                                                                             \frac{\frac{}{\text{h13}:\Delta11,\text{F17},\text{F8},\text{F9}\vdash\Delta10}}{\frac{\text{h13}:\Delta11,\text{F17},\text{F8},\text{F9}\vdash\Delta10}{\text{ex}}} \xrightarrow{\text{ax}} \frac{\frac{}{\text{h13}:\Delta18\vdash\Delta16,\text{F14},\text{F17}}} \xrightarrow{\text{ax}} \frac{\text{h13}:\Delta18,\text{F15}\vdash\Delta16,\text{F17}}{\text{h13}\bullet:\Delta18,\text{F14}\to\text{F15}\vdash\Delta16,\text{F17}}} \xrightarrow{\text{hCut}} \frac{-:\Delta11,\Delta18,\text{F8},\text{F9},\text{F14}\to\text{F15}\vdash\Delta10,\Delta16}}{-:\Delta11,\Delta18,\text{F14}\to\text{F15},\text{F8}\wedge\text{F9}\vdash\Delta10,\Delta16}} \wedge L
                                                                                                                                                                                                      \frac{\mathsf{h}14:\Delta11,\mathsf{F}12,\mathsf{F}13\vdash\Delta10}{\mathsf{h}14\bullet:\Delta11,\mathsf{F}12\land\mathsf{F}13\vdash\Delta10} \ \land_L \quad \frac{\mathsf{h}14:\Delta18\vdash\Delta17,\mathsf{F}15,\mathsf{F}12\land\mathsf{F}13\quad \mathsf{h}14:\Delta18,\mathsf{F}16\vdash\Delta17,\mathsf{F}12\land\mathsf{F}13}{\mathsf{h}14\bullet:\Delta18,\mathsf{F}15\to\mathsf{F}16\vdash\Delta17,\mathsf{F}12\land\mathsf{F}13} \quad \mathsf{Cut} \\
                                                                                                                                                                                                                                                                                                                                                         -1 h14•: Δ18, F15
-1 : Δ11, Δ18, F15 → F16 +1 Δ10, Δ17
                                                 \frac{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F15, F12, F13}}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}}{\text{hCut}}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}}{\text{-} : \Delta 11, \Delta 18, \text{F16} \vdash \Delta 10, \Delta 17}} \overset{\text{ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}}{\text{-} : \Delta 11, \Delta 18, \text{F16} \vdash \Delta 10, \Delta 17}} \overset{\text{ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}} \overset{\text{ax}}{\wedge} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}}_{\text{-} \rightarrow 1} \overset{\text{ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}}_{\text{-} \rightarrow 10} \overset{\text{ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \land \text{F13}}_{\text{-} \rightarrow 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F12, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F13, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14} : \Delta 11, \text{F13, F13} \vdash \Delta 10}}_{\text{-} \rightarrow 10} \overset{\text{Ax}}{\rightarrow} \underbrace{ \frac{1}{\text{h14}
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Cases \wedge_L - \wedge_L
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\frac{\text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F17}\vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\land \text{F9}), \text{F17}\vdash \Delta 10} \quad \wedge_L \quad \frac{\text{h13}:\Delta 18, \text{F14}, \text{F15}\vdash \Delta 16, \text{F17}}{\text{h13}\bullet:\Delta 18, \text{F14}\land \text{F15}\vdash \Delta 16, \text{F17}} \quad \wedge_L \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                               -: \Delta 11, F8 \land F9, \Delta 18, F14 \land F15 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                                                                                                                                                                                                                   : \Delta 11, \Delta 18, F14 \wedge F15, F8 \wedge F9 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                                                                                                                                                                                         \frac{\text{h14}:\Delta 11,\text{F12},\text{F13}\vdash\Delta 10}{\text{h14}\bullet:\Delta 11,\text{F12}\land\text{F13}\vdash\Delta 10} \quad \land_L \quad \frac{\text{h14}:\Delta 18,\text{F15},\text{F16}\vdash\Delta 17,\text{F12}\land\text{F13}}{\text{h14}\bullet:\Delta 18,\text{F15}\land\text{F16}\vdash\Delta 17,\text{F12}\land\text{F13}} \quad \land_L \quad \text{Out}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -: \Delta 11, \Delta 18, F15 \land F16 \vdash \Delta 10, \Delta 17
                                                                                                                                                                                                                                                                                                                                                                                                  Cases \wedge_L - \vee_L
                                                                                                                                                                                                                                                                                                  \frac{\text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F17} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \land \text{F9}), \text{F17} \vdash \Delta 10} \quad \land_L \quad \frac{\text{h13}:\Delta 18, \text{F14} \vdash \Delta 16, \text{F17} \quad \text{h13}:\Delta 18, \text{F15} \vdash \Delta 16, \text{F17}}{\text{h13} \bullet :\Delta 18, \text{F14} \lor \text{F15} \vdash \Delta 16, \text{F17}} \quad \lor_L \quad \text{Out} \quad \lor_L \quad \bot_L \quad \bot_L
                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -: \Delta 11, F8 \wedge F9, \Delta 18, F14 \vee F15 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                                                                                                            \frac{\frac{1}{\text{h13}:\Delta11,\text{F17},\text{F8},\text{F9} \vdash \Delta10}}{\frac{-1}{\text{h13}:\Delta18,\text{F18} \vdash \Delta10,\text{F17}}} \xrightarrow{\text{ax}} \frac{\frac{1}{\text{h13}:\Delta18,\text{F15} \vdash \Delta16,\text{F17}}}{\text{h13}:\Delta18,\text{F15} \vdash \Delta16,\text{F17}}} \xrightarrow{\text{ax}} \frac{\text{cx}}{\text{cyc}} \times \frac{1}{\text{cyc}} \times \frac{1}{
                                                                                                                                                                                                                                                            \frac{\frac{\text{h14} : \Delta11, \text{F12}, \text{F13} \vdash \Delta10}{\text{h14} \bullet : \Delta11, \text{F12} \land \text{F13} \vdash \Delta10}}{-: \Delta11, \text{F12} \land \text{F13} \vdash \Delta10} \land L \\ \frac{\frac{\text{h14} : \Delta18, \text{F15} \vdash \Delta17, \text{F12} \land \text{F13}}{\text{h14} \bullet : \Delta18, \text{F16} \vdash \Delta17, \text{F12} \land \text{F13}}}{\text{h14} \bullet : \Delta18, \text{F16} \vdash \Delta17, \text{F12} \land \text{F13}}} \\ -: \Delta11, \Delta18, \text{F15} \lor \text{F16} \vdash \Delta10, \Delta17} \\ \text{Cut}
                                                         \frac{1}{14 \cdot \triangle 11, F12, F13 \vdash \triangle 10} \xrightarrow{\text{ax}} \xrightarrow{\text{h14} \cdot \triangle 11, F12, F13 \vdash \triangle 10} \xrightarrow{\wedge_L} \xrightarrow{\text{h14} \cdot \triangle 13, F15 \vdash \triangle 17, F12 \land F13} \xrightarrow{\text{ax}} \xrightarrow{\text{h14} \cdot \triangle 11, F12, F13 \vdash \triangle 10} \xrightarrow{\wedge_L} \xrightarrow{\text{h14} \cdot \triangle 11, F12, F13 \vdash \triangle 10} \xrightarrow{\wedge_L} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\wedge_L} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\wedge_L} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 10} \xrightarrow{\text{h14} \cdot \triangle 11, F12 \land F13 \vdash \triangle 
                                                                                                                                                                                                                                    -: \Delta 11, \Delta 18, F16 \vdash \Delta 10, \Delta 17 \\ \lor_L
Cases \wedge_L - \perp_L
                                                                                                                                                                                                                                                                                                                                                                                                                                  \begin{array}{c} \text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F15} \vdash \Delta 10 \\ \hline \text{h13}\bullet:(\Delta 11, \text{F8} \land \text{F9}), \text{F15} \vdash \Delta 10 \\ \hline -:\Delta 11, \text{F8} \land \text{F9}, \bot, \Delta 16 \vdash \Delta 10, \Delta 14 \\ \end{array} \stackrel{\wedge_L}{\longrightarrow} \begin{array}{c} \bot_L \\ \text{Cut} \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   \frac{\rightarrow}{-:\bot,\Delta11,\Delta16,F8\wedge F9\vdash\Delta10,\Delta14} \quad \bot_L
                                                                                                                                                                                                                                                                                                                                                                                                                         \begin{array}{l} \frac{\text{h14}:\Delta11,\text{F12},\text{F13}\vdash\Delta10}{\text{h14}\bullet:\Delta11,\text{F12}\land\text{F13}\vdash\Delta10} & \wedge_L & \\ \hline -:\Delta11,\bot,\Delta16\vdash\Delta10,\Delta15 & \\ \end{array} \begin{array}{l} \bot_L \\ \text{Out} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{}{-: \bot, \Delta 11, \Delta 16 \vdash \Delta 10, \Delta 15} \bot_L
Cases \wedge_L - I
                                                                                                                                                                                                                                                                                                                                                                                                     \frac{\text{h13}:\Delta 11, \text{F8}, \text{F9}, \text{F15} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \land \text{F9}), \text{F15} \vdash \Delta 10} \quad \wedge_L \quad \frac{}{\text{h13} \bullet : \Delta 17, \text{p16} \vdash (\Delta 14, \text{p16}), \text{F15}} \quad I \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -:\Delta11, \texttt{F8} \land \texttt{F9}, \Delta17, \texttt{p}16 \vdash \Delta10, \Delta14, \texttt{p}16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \mathtt{h13}:\Delta11, \mathtt{F8}, \mathtt{F9}, \mathtt{p15} \vdash \Delta10
                                                                                                                                                                                                                                                                                                                                                                                                                               \frac{\text{h13} : \Delta \text{I1}, \text{F8}, \text{F9}, \text{p15} \vdash \Delta \text{I0}}{\text{h13} \bullet : (\Delta \text{I1}, \text{F8} \land \text{F9}), \text{p15} \vdash \Delta \text{10}} \land L \qquad \frac{\text{h13} \bullet : \Delta \text{16}, \text{p15} \vdash \Delta \text{14}, \text{p15}}{\text{-} : \Delta \text{11}, \text{F8} \land \text{F9}, \Delta \text{16}, \text{p15} \vdash \Delta \text{10}, \Delta \text{14}} \qquad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \frac{ \overline{-:\Delta11,\Delta16, \mathsf{F8}, \mathsf{F9}, \mathsf{p15} \vdash \Delta10,\Delta14} \ \ ^{\mathsf{ax}} }{-:\Delta11,\Delta16, \mathsf{p15}, \mathsf{F8} \land \mathsf{F9} \vdash \Delta10,\Delta14} \ \ ^{\mathsf{bx}} \land_L
                                                                                                                                                                                                                                                                                                                                                                                         \frac{\text{h}14:\Delta11,\text{F}12,\text{F}13\vdash\Delta10}{\text{h}14\bullet:\Delta11,\text{F}12\land\text{F}13\vdash\Delta10} \quad \land_L \quad \frac{\text{h}14\bullet:\Delta17,\text{p}16\vdash(\Delta15,\text{p}16),\text{F}12\land\text{F}13}{\text{Cut}} \quad I \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          -: \Delta 11, \Delta 17, p16 \vdash \Delta 10, \Delta 15, p16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \frac{}{-:\Delta11,\Delta17,\mathtt{p}16\vdash\Delta10,\Delta15,\mathtt{p}16}\quad iI
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Cases \wedge_L - \top_L
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\frac{\frac{\text{h13}:\Delta11,\text{F8},\text{F9},\text{F15}\vdash\Delta10}{\text{h13}\bullet:(\Delta11,\text{F8}\land\text{F9}),\text{F15}\vdash\Delta10} \land_L \quad \frac{\text{h13}:\Delta16\vdash\Delta14,\text{F15}}{\text{h13}\bullet:\top,\Delta16\vdash\Delta14,\text{F15}}}{-:\Delta11,\text{F8}\land\text{F9},\top,\Delta16\vdash\Delta10,\Delta14} \quad \overset{\top}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                  -: \top, \Delta 11, \Delta 16, F8 \wedge F9 \vdash \Delta 10, \Delta 14
                                                                                                                                                                                                                                                                                                                                                                                                                            \frac{\text{h14}:\Delta 11,\text{F12},\text{F13}\vdash \Delta 10}{\text{h14}\bullet :\Delta 11,\text{F12}\land \text{F13}\vdash \Delta 10} \quad \land_L \quad \frac{\text{h14}:\Delta 16\vdash \Delta 15,\text{F12}\land \text{F13}}{\text{h14}\bullet :\top,\Delta 16\vdash \Delta 15,\text{F12}\land \text{F13}} \quad \top_L \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -: \Delta 11, \top, \Delta 16 \vdash \Delta 10, \Delta 15
                                                                                                                                                                                                                                                                                                                                                                                                                                             \frac{140 + \Delta 11, F12, F13 \vdash \Delta 10}{140 + \Delta 11} \xrightarrow{ax}
                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{\mathsf{h}\mathsf{1}4 : \Delta \mathsf{1}\mathsf{1}, \mathsf{F}\mathsf{1}2, \mathsf{F}\mathsf{1}3 \vdash \Delta \mathsf{1}0}{\mathsf{h}\mathsf{1}4 \bullet : \Delta \mathsf{1}\mathsf{1}, \mathsf{F}\mathsf{1}2 \wedge \mathsf{F}\mathsf{1}3 \vdash \Delta \mathsf{1}0} \wedge_L \quad \frac{\mathsf{h}\mathsf{1}4 : \mathsf{T}, \Delta \mathsf{1}6 \vdash \Delta \mathsf{1}5, \mathsf{F}\mathsf{1}2 \wedge \mathsf{F}\mathsf{1}3}{\mathsf{h}\mathsf{Cut}} \xrightarrow{\mathsf{ax}} \quad \text{hCut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -: \top, \Delta 11, \Delta 16 \vdash \Delta 10, \Delta 15
Cases \vee_L - \rightarrow_R
                                                                                                                                                                                                                                                                                       \frac{\text{h13} : \Delta11, \text{F8}, \text{F18} \vdash \Delta10 \quad \text{h13} : \Delta11, \text{F9}, \text{F18} \vdash \Delta10}{\text{h13} : (\Delta11, \text{F8} \lor \text{F9}), \text{F18} \vdash \Delta10} \quad \vee_{L} \quad \frac{\text{h13} : \Delta14, \text{F15} \vdash \Delta17, \text{F16}, \text{F18}}{\text{h13} \bullet : \Delta14 \vdash (\Delta17, \text{F15} \to \text{F16}), \text{F18}} \\ \quad - : \Delta14, \Delta11, \text{F8} \lor \text{F9} \vdash \Delta10, \Delta17, \text{F15} \to \text{F16}} \quad \text{Out} 
                                                                                                                                                                                                                                                                                                      \frac{\frac{1}{\text{h13}:\Delta11,\text{F18},\text{F8}\vdash\Delta10}}{\frac{\text{h13}\bullet:\Delta11,\text{F18},\text{F8}\vdash\Delta10}{\text{ax}}} \xrightarrow[\text{h13}:\Delta11,\text{F18},\text{F9}\vdash\Delta10]{\text{h13}:\Delta11,\text{F18},\text{F9}\vdash\Delta10}} \xrightarrow[\text{h13}:\Delta11,\text{F18},\text{F9}\vdash\Delta10]{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16},\text{F18}}} \xrightarrow[\text{hCut}]{\text{hCut}} \frac{\frac{\text{h13}\bullet:\Delta11,\text{F18},\text{F9}\vdash\Delta10}{\text{hCut}}}{\text{-}:\Delta11,\Delta14,\text{F15},\text{F8}\vee\text{F9}\vdash\Delta10,\Delta17,\text{F16}}} \xrightarrow[\text{hCut}]{\text{A}} \xrightarrow[\text{hCut}]{\text{hCut}}
                                                                                                                                                                                                                                                   \frac{\text{h13}:\Delta 11, \text{F8}, \text{F15} \rightarrow \text{F16} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8} \lor \text{F9}), \text{F15} \rightarrow \text{F16} \vdash \Delta 10} \\ -:\Delta 14, \Delta 11, \text{F8} \lor \text{F9} \vdash \Delta 10, \Delta 17} \\ \xrightarrow{} \\ \text{Cut} \\ \xrightarrow{} \\ \text{Cut}
                                                              \frac{1}{\frac{\text{h13}:\Delta11,\text{F8},\text{F15}\rightarrow\text{F16}\vdash\Delta10}{\text{ax}}} \underbrace{\frac{\frac{1}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}}{\text{h13}\bullet:\Delta14\vdash\Delta17,\text{F15}\rightarrow\text{F16}}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{\frac{1}{\text{h13}:\Delta11,\text{F9},\text{F15}\rightarrow\text{F16}\vdash\Delta10}}{\text{ax}}}_{\text{h13}\bullet:\Delta14\vdash\Delta17,\text{F15}\rightarrow\text{F16}}^{\text{ax}} \underbrace{\frac{2}{\text{h13}:\Delta11,\text{F9},\text{F15}\rightarrow\text{F16}\vdash\Delta10}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{1}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{2}{\text{h13}:\Delta11,\text{F9},\text{F15}\rightarrow\text{F16}\vdash\Delta10}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{1}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{2}{\text{hCut}}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{2}{\text{h13}:\Delta11,\text{F9},\text{F15}\rightarrow\text{F16}\vdash\Delta10}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{2}{\text{h13}:\Delta14,\text{F15}\vdash\Delta17,\text{F16}}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{2}{\text{hCut}}}_{\text{hCut}}^{\text{ax}} \underbrace{\frac{2}{\text{hCut}}}_{\text{
                                                                                                                                                                                                                                                                                            \frac{\mathbf{h}14:\Delta 11,\mathbf{F}12\vdash\Delta 10\quad\mathbf{h}14:\Delta 11,\mathbf{F}13\vdash\Delta 10}{\mathbf{h}14\bullet:\Delta 11,\mathbf{F}12\vee\mathbf{F}13}\vdash\Delta 10}\quad\vee_{L}\quad\frac{\mathbf{h}14:\Delta 15,\mathbf{F}16\vdash\Delta 18,\mathbf{F}17,\mathbf{F}12\vee\mathbf{F}13}{\mathbf{h}14\bullet:\Delta 15\vdash(\Delta 18,\mathbf{F}16\to\mathbf{F}17),\mathbf{F}12\vee\mathbf{F}13}\quad\rightarrow_{R}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -:\Delta11,\Delta15\vdash\Delta10,\Delta18,\texttt{F}16\to\texttt{F}17
                                                                                                                                                                                                                                                                                                           Cases \vee_L - \wedge_R
                                                                                                                                                                                                                                                                                                                                                        \frac{\text{h13} : \Delta11, \text{F8}, \text{F18} \vdash \Delta10 \quad \text{h13} : \Delta11, \text{F9}, \text{F18} \vdash \Delta10}{\text{h13} \bullet : (\Delta11, \text{F8} \lor \text{F9}), \text{F18} \vdash \Delta10} \quad \vee_L \quad \frac{\text{h13} : \Delta14 \vdash \Delta17, \text{F15}, \text{F18} \quad \text{h13} : \Delta14 \vdash \Delta17, \text{F16}, \text{F18}}{\text{h13} \bullet : \Delta14 \vdash (\Delta17, \text{F15} \land \text{F16}), \text{F18}} \quad \wedge_R \quad \wedge_
              \frac{1}{13:\Delta 11,F18,F8+\Delta 10} = \frac{1}{13:\Delta 11,F18,F9+\Delta 10} = 
                                                                                                                                                                                                                                                                                                             -:\Delta 11,\Delta 14, \texttt{F8} \vee \texttt{F9} \vdash \Delta 10,\Delta 17, \texttt{F15}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 11, \Delta 14, F8 \lor F9 \vdash \Delta 10, \Delta 17, F16 \land R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -:\Delta11,\Delta14, \texttt{F8} \lor \texttt{F9} \vdash \Delta10,\Delta17, \texttt{F15} \land \texttt{F16}
                                                                                                                                                                                                                                                                                                           \frac{\text{h13} : \Delta 11, \text{F8}, \text{F15} \land \text{F16} \vdash \Delta 10 \quad \text{h13} : \Delta 11, \text{F9}, \text{F15} \land \text{F16} \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \lor \text{F9}), \text{F15} \land \text{F16} \vdash \Delta 10} \quad \lor_{L} \quad \frac{\text{h13} : \Delta 14 \vdash \Delta 17, \text{F15} \quad \text{h13} : \Delta 14 \vdash \Delta 17, \text{F16}}{\text{h13} \bullet : \Delta 14 \vdash \Delta 17, \text{F15} \land \text{F16}} \quad \land_{R} \\ - : \Delta 14, \Delta 11, \text{F8} \lor \text{F9} \vdash \Delta 10, \Delta 17} \quad \to \quad \land_{R} \\ \text{Cut}
```

 $-: \Delta 11, \Delta 14, F8 \lor F9 \vdash \Delta 10, \Delta 17$

 $\frac{1}{13:\Delta 11,F8,F15 \wedge F16 \vdash \Delta 10} = \frac{1}{13:\Delta 14 \vdash \Delta 17,F15} = \frac{1}{13:\Delta 14} = \frac{1}{13:\Delta 14$

```
\frac{\text{h14}:\Delta 11, \text{F12} \vdash \Delta 10 \quad \text{h14}:\Delta 11, \text{F13} \vdash \Delta 10}{\text{h14}\bullet:\Delta 11, \text{F13} \vdash \Delta 10} \quad \vee_L \quad \frac{\text{h14}:\Delta 15 \vdash \Delta 18, \text{F16}, \text{F12} \vee \text{F13} \quad \text{h14}:\Delta 15 \vdash \Delta 18, \text{F17}, \text{F12} \vee \text{F13}}{\text{h14}\bullet:\Delta 15 \vdash (\Delta 18, \text{F16} \wedge \text{F17}), \text{F12} \vee \text{F13}} \quad \wedge_R \quad \wedge_R \cap_{L_{2}} \left( \frac{\text{h14}:\Delta 11}{\text{h14}\bullet:\Delta 11} + \frac{\text{h14}\cdot\Delta 11}{\text{h14}\bullet:\Delta 11} + \frac{\text{h1
                                                                                                                                                                                                                                                                                                                                                                                                             \frac{\sum_{14:\Delta11,F12+\Delta10}^{1}\sum_{14:\Delta11,F12+\Delta10}^{1}\sum_{14:\Delta11,F13+\Delta10}^{1}\sum_{14:\Delta11,F13+\Delta10}^{1}\sum_{14:\Delta15+\Delta18,F16,F12\vee F13}^{1}\sum_{14:\Delta15+\Delta18,F16,F12\vee F13}^{1}\sum_{14:\Delta11,F12+\Delta10}^{1}\sum_{14:\Delta11,F12\vee F13+\Delta10}^{1}\sum_{14:\Delta11,F13+\Delta10}^{1}\sum_{14:\Delta15+\Delta18,F17,F12\vee F13}^{1}\sum_{14:\Delta11,F12\vee F13+\Delta10}^{1}\sum_{14:\Delta15+\Delta18,F17,F12\vee F13}^{1}\sum_{14:\Delta15+\Delta18,F17,F12\vee F13}^{1}\sum_{14:\Delta11,F12\vee F13+\Delta10}^{1}\sum_{14:\Delta15+\Delta18,F17,F12\vee F13}^{1}\sum_{14:\Delta15+\Delta18,F17,F12\vee F13}^{1}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      - : \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F17 \land R
                                                                                                                                                                                                                                                                                                                                           -:\Delta11,\Delta15\vdash\Delta10,\Delta18,\mathrm{F}16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \land F17
Cases \vee_L - \vee_R
                                                                                                                                                                                                                                                                                                      \frac{\text{h13}:\Delta 11, \text{F8}, \text{F18} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F18} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\vee \text{F9}), \text{F18} \vdash \Delta 10} \quad \vee_{L} \quad \frac{\text{h13}:\Delta 14 \vdash \Delta 17, \text{F15}, \text{F16}, \text{F18}}{\text{h13}\bullet:\Delta 14 \vdash (\Delta 17, \text{F15}\vee \text{F16}), \text{F18}} \quad \vee_{R} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  -:\Delta 14,\Delta 11, \texttt{f8} \vee \texttt{f9} \vdash \Delta 10,\Delta 17, \texttt{f15} \vee \texttt{f16}
                                                                                                                                                                                                                                                                                                           \frac{ \mathbf{h}13 : \Delta 11, \mathbf{F}8, \mathbf{F}15 \vee \mathbf{F}16 \vdash \Delta 10 \quad \mathbf{h}13 : \Delta 11, \mathbf{F}9, \mathbf{F}15 \vee \mathbf{F}16 \vdash \Delta 10}{\mathbf{h}13 \bullet : (\Delta 11, \mathbf{F}8 \vee \mathbf{F}9)}, \mathbf{F}15 \vee \mathbf{F}16 \vdash \Delta 10} \quad \forall_L \quad \frac{\mathbf{h}13 : \Delta 14 \vdash \Delta 17, \mathbf{F}15, \mathbf{F}16}{\mathbf{h}13 \bullet : \Delta 14 \vdash \Delta 17, \mathbf{F}15 \vee \mathbf{F}16} \quad \forall_R \in \mathcal{A}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        -:\Delta 14,\Delta 11, \texttt{F8} \vee \texttt{F9} \vdash \Delta 10,\Delta 17
                                                                                     \frac{\frac{1}{\text{h13}:\Delta11,\text{F8},\text{F15}\vee\text{F16}\vdash\Delta10}}{\frac{1}{\text{h13}:\Delta14,\text{F8}\vdash\Delta10,\Delta17}} \overset{\text{av}}{\text{av}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h}3:\Delta11,\text{F9},\text{F15}\vee\text{F16}\vdash\Delta10}{\text{hcut}}} \overset{\text{av}}{\text{av}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15},\text{F16}}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta11,\text{F9},\text{F15}\vee\text{F16}\vdash\Delta10}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta11,\text{F9},\text{F15}\vee\text{F16}\vdash\Delta10}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}{\text{hcut}}} \overset{\text{av}}{\text{hcut}} \xrightarrow{\frac{\text{h13}:\Delta14\vdash\Delta17,\text{F15}\vee\text{F16}}{\text{hcut}}} \overset{\text{hcut}}{\text{hcut}} \xrightarrow{\frac{\text{hcut}}{\text{hcut}}} \xrightarrow{
                                                                                                                                                                                                                                                                                                           \frac{\mathbf{h} 14 : \Delta 11, \mathbf{f} 12 \vdash \Delta 10 \quad \mathbf{h} 14 : \Delta 11, \mathbf{f} 13 \vdash \Delta 10}{\mathbf{h} 14 \bullet \underbrace{: \Delta 11, \mathbf{f} 12 \lor \mathbf{f} 13 \vdash \Delta 10}} \quad \vee_{L} \quad \frac{\mathbf{h} 14 : \Delta 15 \vdash \Delta 18, \mathbf{f} 16, \mathbf{f} 17, \mathbf{f} 12 \lor \mathbf{f} 13}{\mathbf{h} 14 \bullet \underbrace{: \Delta 15 \vdash (\Delta 18, \mathbf{f} 16 \lor \mathbf{f} 17), \mathbf{f} 12 \lor \mathbf{f} 13}} \quad \vee_{R} \quad \mathbf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          -: \Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \lor F17
                                                                                                                                                                                                                                                                                                                 \frac{\underbrace{^{\text{h14}:\Delta11,\text{F12}\vdash\Delta10}}_{\text{h14}\bullet:\Delta11,\text{F12}\lor\text{F13}\vdash\Delta10} \overset{\text{ax}}{\overset{\text{h14}:\Delta11,\text{F13}\vdash\Delta10}} \overset{\text{ax}}{\overset{\text{v}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}{\overset{\text{h}}}{\overset{\text{h}}{\overset{\text{h}}}{\overset{\text{h}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}{\overset{\text{h}}}}{\overset{\text{h}}}{\overset{\text{h}}}}{\overset{\text{h}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h}}}}{\overset{\text{h
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \frac{-:\Delta 10}{-:\Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16, F17} \\ -:\Delta 11, \Delta 15 \vdash \Delta 10, \Delta 18, F16 \lor F17} \lor_{R}
                                                                                                                                                                                                                                                                                                                                                                   \frac{\text{h12}:\Delta 11, \text{F14} \vdash \Delta 10 \quad \text{h12}:\Delta 11, \text{F15} \vdash \Delta 10}{\text{h12}\bullet:\Delta 11, \text{F15} \vdash \Delta 10} \quad \vee_L \quad \frac{\text{h12}:\Delta 13 \vdash \Delta 16, \text{F14}, \text{F15}}{\text{h12}\bullet:\Delta 13 \vdash \Delta 16, \text{F14} \vee \text{F15}} \quad \vee_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              -: \Delta 11, \Delta 13 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                    \begin{array}{c} \xrightarrow{} & \xrightarrow{} & \xrightarrow{} & \xrightarrow{} \\ -: \Delta 11, \ \Delta 13, F14 \vdash \Delta 10, \ \Delta 16 \end{array} \\ \xrightarrow{} = : \Delta 11, \ \Delta 13, F15 \vdash \Delta 10, \ \Delta 16, F14 \ \text{ax} \\ & \xrightarrow{} -: \Delta 11, \ \Delta 13 \vdash \Delta 10, \ \Delta 16, F14 \ \text{sCut} \\ & \xrightarrow{} -: \Delta 11, \ \Delta 13 \vdash \Delta 10, \ \Delta 16 \end{array} \\ \xrightarrow{} \text{sCut} 
Cases \vee_L - \perp_R
                                                                                                                                                                                                                                                                                                                                                                                  \frac{\text{h13}:\Delta 11, \text{F8}, \bot \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \bot \vdash \Delta 10}{\underbrace{\text{h13} \cdot (\Delta 11, \text{F8} \vee \text{F9}), \bot \vdash \Delta 10}}_{} \quad \forall_L \quad \frac{\text{h13}:\Delta 14 \vdash \Delta 15}{\text{h13} \bullet :\Delta 14 \vdash \Delta 15, \bot}}_{\text{Cut}} \quad \bot_R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -: \Delta 14, \Delta 11, F8 \lor F9 \vdash \Delta 10, \Delta 15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \frac{\text{h13}:\Delta 11, \text{F8}, \text{F16} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F16} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\vee \text{F9}), \text{F16} \vdash \Delta 10} \quad \vee_{L} \quad \frac{\text{h13}:\Delta 14 \vdash \Delta 15, \text{F16}}{\text{h13}\bullet:\Delta 14 \vdash (\bot,\Delta 15), \text{F16}} \quad \bot_{R} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \begin{array}{c} -: \Delta 14, \Delta 11, \texttt{F8} \lor \texttt{F9} \vdash \Delta 10, \bot, \Delta 15 \\ \rightarrow \end{array}
                                                                                                                                                                                                                                                                                                                                      \frac{\overline{\text{h13}:\Delta11,\text{F16},\text{F8}\vdash\Delta10}}{\underbrace{\frac{\text{h13}\bullet:\Delta11,\text{F16},\text{F8}\vee\text{F9}\vdash\Delta10}{-:\Delta11,\Delta14,\text{F8}\vee\text{F9}\vdash\bot,\Delta10,\Delta15}}}^{\text{ax}} \times \frac{}{\frac{\text{h13}:\Delta14\vdash\bot,\Delta15,\text{F16}}{\text{hCut}}}}
                                                                                                                                                                                                                                                                                                                                           \frac{ \frac{\mathtt{h}14:\Delta11,\mathtt{F}12 \vdash \Delta10 \quad \mathtt{h}14:\Delta11,\mathtt{F}13 \vdash \Delta10}{\mathtt{h}14 \bullet : \Delta11,\mathtt{F}12 \vee \mathtt{F}13 \vdash \Delta10} \quad \vee_{L} \quad \frac{\mathtt{h}14:\Delta15 \vdash \Delta16,\mathtt{F}12 \vee \mathtt{F}13}{\mathtt{h}14 \bullet : \Delta15 \vdash (\bot,\Delta16),\mathtt{F}12 \vee \mathtt{F}13} \quad _{\mathtt{Cut}}^{\bot_{R}} \quad \\ \frac{\mathtt{h}14 \bullet : \Delta11,\mathtt{F}12 \vee \mathtt{F}13 \vdash \Delta10}{-:\Delta11,\Delta15 \vdash \Delta10,\bot,\Delta16} \quad _{\mathtt{Cut}}^{\bot_{R}}
                                                                                                                                                                                                                                                                                                                                           -: \Delta 11, \Delta 15 \vdash \bot, \Delta 10, \Delta 16
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Cases \vee_L - \top_R
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\frac{\text{h13}:\Delta 11, \text{F8}, \top \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \top \vdash \Delta 10}{\text{h13} \bullet : (\Delta 11, \text{F8} \vee \text{F9}), \top \vdash \Delta 10} \quad \vee_{L} \quad \frac{}{\text{h13} \bullet : \Delta 14 \vdash \Delta 15, \top} \quad \top_{R} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                        -:\Delta 14,\Delta 11, \texttt{F8} \lor \texttt{F9} \vdash \Delta 10,\Delta 15
                                                                                                                                                             -: \Delta 11, \Delta 14, F8 \vee F9 \vdash \Delta 10, \Delta 15
                                                                                                                                                                                                                                       \frac{\text{h13}:\Delta 11, \text{F8}, \text{F16} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F16} \vdash \Delta 10}{\text{h13}\bullet:\underbrace{(\Delta 11, \text{F8}\vee \text{F9}), \text{F16} \vdash \Delta 10}} \quad \vee_L \quad \underbrace{\frac{1}{\text{h13}\bullet:\Delta 14 \vdash (\top,\Delta 15), \text{F16}}}_{\text{Cut}} \quad \underbrace{\top_R}_{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                           -:\Delta 14,\Delta 11, \texttt{F8} \lor \texttt{F9} \vdash \Delta 10, \top, \Delta 15
                                                                                                                                                                                                                                                                                                                                                                                                           \frac{\mathbf{h}14:\Delta 11,\mathbf{F}12\vdash\Delta 10\quad\mathbf{h}14:\Delta 11,\mathbf{F}13\vdash\Delta 10}{\mathbf{h}14\bullet} \cdot \underline{\Delta 11,\mathbf{F}12\lor\mathbf{F}13\vdash\Delta 10} \quad \lor_{L} \quad \frac{}{\mathbf{h}14\bullet:\Delta 15\vdash(\top,\Delta 16),\mathbf{F}12\lor\mathbf{F}13} \quad \overset{\top}{\mathbf{cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                    -: \Delta 11, \Delta 15 \vdash \Delta 10, \top, \Delta 16
                                                                                                                                                                                                                                                                                                                                                                                                                                         \overline{-: \Delta 11, \Delta 15 \vdash \top, \Delta 10, \Delta 16} \top_R
Cases \vee_L - \rightarrow_L
                                                                                                                                                                                                                                                   \frac{\text{h13}:\Delta 11, \text{F8}, \text{F17} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F17} \vdash \Delta 10}{\text{h13}:\Delta 11, \text{F9}, \text{F17} \vdash \Delta 10} \quad \vee_{L} \quad \frac{\text{h13}:\Delta 18 \vdash \Delta 16, \text{F14}, \text{F17} \quad \text{h13}:\Delta 18, \text{F15} \vdash \Delta 16, \text{F17}}{\text{h13}\bullet:\Delta 18, \text{F14} \rightarrow \text{F15} \vdash \Delta 16, \text{F17}} \quad \text{Cut} \\ -:\Delta 11, \text{F8} \vee \text{F9}, \Delta 18, \text{F14} \rightarrow \text{F15} \vdash \Delta 10, \Delta 16} \quad \rightarrow
        \frac{1}{\text{h13} : \Delta 11, \text{F17}, \text{F8} \vdash \Delta 10} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11}{\text{h17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F8} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F8} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F8} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}, \text{F9} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h13} : \Delta 11, \text{F17}
                                                                                                                                                                                                                    -: \Delta 11, \Delta 18, F15, F8 \vee F9 \vdash \Delta 10, \Delta 16 \rightarrow_L
                                                                                                                                                                                                                                \frac{\frac{\text{h14} : \Delta 11, \text{F12} \vdash \Delta 10 \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}{\text{h14} \bullet : \Delta 11, \text{F13} \vdash \Delta 10} \quad \vee_{L} \quad \frac{\text{h14} : \Delta 18 \vdash \Delta 17, \text{F15}, \text{F12} \vee \text{F13} \quad \text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \vee \text{F13}}{\text{h14} \bullet : \Delta 18, \text{F15} \rightarrow \text{F16} \vdash \Delta 17, \text{F12} \vee \text{F13}} \quad \underset{\text{Cut}}{} \rightarrow_{L} \quad \xrightarrow{\text{Cut}}
        \frac{1}{14 : \Delta 11, F12 \vdash \Delta 10} \xrightarrow{ax} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{ax} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{ax} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{ax} \frac{1}{h14 : \Delta 11, F12 \vdash \Delta 10} \xrightarrow{ax} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{ax} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash \Delta 10} \xrightarrow{back} \frac{1}{h14 : \Delta 11, F13 \vdash
Cases \vee_L - \wedge_L
                                                                                                                                                                                                                      \frac{\text{h13}:\Delta11,\text{F8},\text{F17}\vdash\Delta10}{\text{h13}\bullet:(\Delta11,\text{F8}\vee\text{F9}),\text{F17}\vdash\Delta10} \ \lor_{L} \ \frac{\text{h13}:\Delta18,\text{F14},\text{F15}\vdash\Delta16,\text{F17}}{\text{h13}\bullet:\Delta18,\text{F14}\wedge\text{F15}\vdash\Delta16,\text{F17}} \ \land_{L}
                                                                                                                                                                                                                                                                                                    -: \Delta 11, F8 \lor F9, \Delta 18, F14 \land F15 \vdash \Delta 10, \Delta 16
                                                                                                                                                                                                                  \frac{-:\Delta 11, F0 \vee F9, \Delta 18, F14 \wedge F15 \vdash \Delta 10, \Delta 16}{\to \\ \frac{13:\Delta 11, F17, F8 \vdash \Delta 10}{\bullet} \xrightarrow{ax} \frac{\to \\ h13:\Delta 11, F17, F9 \vdash \Delta 10}{} \xrightarrow{ax} \\ \frac{-:\Delta 11, \Delta 18, F14, F15, F8 \vee F9 \vdash \Delta 10, \Delta 16}{-:\Delta 11, \Delta 18, F14, F15, F8 \vee F9 \vdash \Delta 10, \Delta 16} \wedge_{L} \xrightarrow{ax} \xrightarrow{hCut}
                                                                                                                                                                                                                        \frac{\mathbf{h} \mathbf{14} : \Delta \mathbf{11}, \mathbf{F} \mathbf{12} \vdash \Delta \mathbf{10} \quad \mathbf{h} \mathbf{14} : \Delta \mathbf{11}, \mathbf{F} \mathbf{13} \vdash \Delta \mathbf{10}}{\mathbf{h} \mathbf{14} \bullet : \Delta \mathbf{11}, \mathbf{F} \mathbf{12} \lor \mathbf{F} \mathbf{13} \vdash \Delta \mathbf{10}} \quad \vee_{L} \quad \frac{\mathbf{h} \mathbf{14} \bullet : \Delta \mathbf{18}, \mathbf{F} \mathbf{15}, \mathbf{F} \mathbf{16} \vdash \Delta \mathbf{17}, \mathbf{F} \mathbf{12} \lor \mathbf{F} \mathbf{13}}{\mathbf{h} \mathbf{14} \bullet : \Delta \mathbf{18}, \mathbf{F} \mathbf{15} \land \mathbf{F} \mathbf{16} \vdash \Delta \mathbf{17}, \mathbf{F} \mathbf{12} \lor \mathbf{F} \mathbf{13}} \quad \wedge_{L} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 11, \Delta 18, F15 \wedge F16 \vdash \Delta 10, \Delta 17
                                                                                                                                                                                                                      Cases \vee_L - \vee_L
                                                                                                                                                                                                                                                         \frac{\text{h13}:\Delta 11, \text{F8}, \text{F17} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F17} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\vee \text{F9}), \text{F17} \vdash \Delta 10} \quad \vee_{L} \quad \frac{\text{h13}:\Delta 18, \text{F14} \vdash \Delta 16, \text{F17} \quad \text{h13}:\Delta 18, \text{F15} \vdash \Delta 16, \text{F17}}{\text{h13}\bullet:\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F15} \vdash \Delta 16, \text{F17}}{\text{cut}} \quad \vee_{L} \quad \frac{\text{h13}\cdot\Delta 18, \text{F14}\vee \text{F17}}{\text{cut}} \quad \frac{\text{h13}\cdot\Delta 18, \text{F17}}{\text{cut}} \quad 
        -: \Delta 11, \Delta 18, F15, F8 \lor F9 \vdash \Delta 10, \Delta 16
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\frac{ \frac{\text{h14} : \Delta 11, \text{F12} \vdash \Delta 10 \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}{\text{h14} \bullet : \Delta 11, \text{F12} \lor \text{F13} \vdash \Delta 10} \quad \vee_{L} \quad \frac{\text{h14} : \Delta 18, \text{F15} \vdash \Delta 17, \text{F12} \lor \text{F13} \quad \text{h14} : \Delta 18, \text{F16} \vdash \Delta 17, \text{F12} \lor \text{F13}}{\text{h14} \bullet : \Delta 18, \text{F15} \lor \text{F16} \vdash \Delta 17, \text{F12} \lor \text{F13}} \quad \vee_{L} \quad \vee_{L} \quad \frac{\text{h14} : \Delta 18, \text{F15} \lor \text{F16} \vdash \Delta 10, \Delta 17}{\text{h14} \bullet : \Delta 18, \text{F15} \lor \text{F16} \vdash \Delta 17, \text{F12} \lor \text{F13}} \quad \vee_{L} \quad
              \frac{\frac{1}{\text{h14} : \Delta 11, \text{F12} \vdash \Delta 10}}{\frac{1}{\text{h14} : \Delta 11, \text{F12} \lor \text{F13} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{b14} : \Delta 11, \text{F13} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{bx}}{\overset{\text{bx}}{\lor}}} \underbrace{\frac{1}{\text{h14} : \Delta 18, \text{F15} \vdash \Delta 17, \text{F12} \lor \text{F13}}} \overset{\text{ax}}{\overset{\text{h14} : \Delta 11, \text{F12} \vdash \Delta 10}} \overset{\text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}{\overset{\text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}} \overset{\text{ax}}{\overset{\text{h14} : \Delta 11, \text{F13} \vdash \Delta 10}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{ax}}{\overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash \Delta 10, \Delta 11}} \overset{\text{h14} : \Delta 11, \text{F15} \vdash 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     -: \Delta 11, \Delta 18, F15 \vee F16 \vdash \Delta 10, \Delta 17
Cases \vee_L - \perp_L
                                                                                                                                                                                                                                                                                                                                                    \frac{\text{h13}:\Delta 11, \text{F8}, \text{F15} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F15} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\vee \text{F9}), \text{F15}\vdash \Delta 10} \\ -:\Delta 11, \text{F8}\vee \text{F9}, \bot, \Delta 16 \vdash \Delta 10, \Delta 14} \\ \xrightarrow{-:\Delta 11, \text{F8}\vee \text{F9}, \bot, \Delta 16 \vdash \Delta 10, \Delta 14} \\ \xrightarrow{-:\Delta 11, \text{F8}\vee \text{F9}, \bot, \Delta 16} \\ \xrightarrow{-:\Delta 11, \text{F9}\vee \text{F9}, \bot, \Delta 16} \\ \xrightarrow{-:\Delta 11, \text
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      -: \bot, \Delta 11, \Delta 16, F8 \lor F9 \vdash \Delta 10, \Delta 14 \bot_L
                                                                                                                                                                                                                                                                                                                                                    \frac{ \underbrace{ \text{h14} : \Delta 11, \text{F12} \vdash \Delta 10 \quad \text{h14} : \Delta 11, \text{F13} \vdash \Delta 10 }_{ - : \Delta 11, \text{F13} \vdash \Delta 10} \  \  \, \bigvee_{L} \quad \underbrace{ \underbrace{ \text{h14} \bullet : \bot, \Delta 16 \vdash \Delta 15, \text{F12} \lor \text{F13} }_{ \text{h14} \bullet : \bot, \Delta 16 \vdash \Delta 10, \Delta 15} \  \  \, \underbrace{ \text{Cut} }_{ - : \Delta 11, \bot, \Delta 16 \vdash \Delta 10, \Delta 15}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{}{-:\bot,\Delta 11,\Delta 16\vdash\Delta 10,\Delta 15}\quad {}^{\bot}L
Cases \vee_L - I
                                                                                                                                                                                                                                                                                                                    \frac{\text{h13} : \Delta11, \text{F8}, \text{F15} \vdash \Delta10 \quad \text{h13} : \Delta11, \text{F9}, \text{F15} \vdash \Delta10}{\text{h13} \bullet : (\Delta11, \text{F8} \lor \text{F9}), \text{F15} \vdash \Delta10} \quad \vee_{L} \quad \frac{\text{h13} \bullet : \Delta17, \text{p16} \vdash (\Delta14, \text{p16}), \text{F15}}{\text{-} : \Delta11, \text{F8} \lor \text{F9}, \Delta17, \text{p16} \vdash \Delta10, \Delta14, \text{p16}} \quad I \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \frac{}{-:\Delta11,\Delta17,\mathrm{p}16,\mathrm{F}8\vee\mathrm{F}9\vdash\Delta10,\Delta14,\mathrm{p}16} \quad iI
                                                                                                                                                                                                                                                                                                                                                  \frac{\text{h13}:\Delta 11, \text{F8}, \text{p15} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{p15} \vdash \Delta 10}{\frac{\text{h13}\bullet:(\Delta 11, \text{F8}\vee \text{F9}), \text{p15} \vdash \Delta 10}{-:\Delta 11, \text{F8}\vee \text{F9}, \Delta 16, \text{p15} \vdash \Delta 10, \Delta 14}} \vee_{L} \frac{1}{\text{h13}\bullet:\Delta 16, \text{p15} \vdash \Delta 14, \text{p15}}} \quad I \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                   \begin{array}{c|c} & \xrightarrow{} & \xrightarrow{} & \xrightarrow{} \\ -:\Delta11,\Delta16,F8,p15 \vdash \Delta10,\Delta14 & \text{ax} & \xrightarrow{} & \xrightarrow{} -:\Delta11,\Delta16,F9,p15 \vdash \Delta10,\Delta14 & \text{ax} \\ & & \vee_L \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 -: \Delta 11, \Delta 16, p15, f8 \lor f9 \vdash \Delta 10, \Delta 14
                                                                                                                                                                                                                                                                                                                         \frac{\mathbf{h}14:\Delta11,\mathbf{F}12\vdash\Delta10}{\mathbf{h}14\bullet:\Delta11,\mathbf{F}12}\vee \mathbf{F}13\vdash\Delta10} \quad \forall L \quad \frac{\mathbf{h}14\bullet:\Delta17,\mathbf{p}16\vdash(\Delta15,\mathbf{p}16),\mathbf{F}12\vee\mathbf{F}13}{\mathbf{h}14\bullet:\Delta17,\mathbf{p}16\vdash(\Delta15,\mathbf{p}16),\mathbf{F}12\vee\mathbf{F}13} \quad I \quad \mathbf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           -: \Delta 11, \Delta 17, \mathtt{p}16 \vdash \Delta 10, \Delta 15, \mathtt{p}16
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Cases \vee_L - \top_L
                                                                                                                                                                                                                                                                                                                                                    \frac{\text{h13}:\Delta 11, \text{F8}, \text{F15} \vdash \Delta 10 \quad \text{h13}:\Delta 11, \text{F9}, \text{F15} \vdash \Delta 10}{\text{h13}\bullet:(\Delta 11, \text{F8}\vee \text{F9}), \text{F15} \vdash \Delta 10} \\ -:\Delta 11, \text{F8}\vee \text{F9}, \top, \Delta 16 \vdash \Delta 10, \Delta 14} \\ \xrightarrow{-:\Delta 11, \text{F8}\vee \text{F9}, \top, \Delta 16 \vdash \Delta 10, \Delta 14} \\ \xrightarrow{} \text{Cut}
                                                                                                                                                                                                                                                                                                                                             \frac{ \frac{\text{h14} : \Delta11, \text{F12} \vdash \Delta10 \quad \text{h14} : \Delta11, \text{F13} \vdash \Delta10}{\text{h14} \cdot : \Delta11, \text{F12} \lor \text{F13} \vdash \Delta10} \quad \vee_{L} \quad \frac{\text{h14} : \Delta16 \vdash \Delta15, \text{F12} \lor \text{F13}}{\text{h14} \bullet : \top, \Delta16 \vdash \Delta15, \text{F12} \lor \text{F13}} \quad \top_{L} \quad \text{Cut}}{-: \Delta11, \top, \Delta16 \vdash \Delta10, \Delta15} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                               \frac{\overbrace{^{\text{h}14:\Delta11,\,\text{F}12}\vdash\Delta10}^{\text{h}14:\Delta11,\,\text{F}12\vdash\Delta10}^{\text{ax}}} \xrightarrow{\overset{\bullet}{\text{h}14:\Delta11,\,\text{F}13\vdash\Delta10}^{\text{h}14:\Delta11,\,\text{F}13\vdash\Delta10}^{\text{ax}}} \xrightarrow{\text{v}_{L}} \frac{\downarrow}{\text{h}14:\top,\,\Delta16\vdash\Delta15,\,\text{F}12\vee\text{F}13}^{\text{h}2}} \xrightarrow{\overset{\text{ax}}{\text{h}\text{Cut}}}
Cases \perp_L - \rightarrow_R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \frac{\frac{100 \bullet : \Delta 9, \bot \vdash \Delta 8}{-100} \ ^{\bot}L}{-100} \ \frac{\frac{100 \circ : \Delta 11, F12 \vdash \bot, \Delta 14, F13}{100 \circ : \Delta 11 \vdash (\Delta 14, F12 \to F13), \bot}}{-1000} \ \xrightarrow{Cut} Cut
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \begin{array}{c} -: \Delta 9, \Delta 11 \vdash \Delta 8, \Delta 14, F12 \rightarrow F13 \\ & \rightarrow \\ \hline \underline{110 \bullet : \bot \vdash *} & \bot L & \rightarrow \\ \hline \underline{-: \Delta 11, \Delta 9, F12 \vdash \Delta 14, \Delta 8, F13} \\ -: \Delta 11, \Delta 9 \vdash \Delta 14, \Delta 8, F12 \rightarrow F13 \\ \hline -: \Delta 11, \Delta 9 \vdash \Delta 14, \Delta 8, F12 \rightarrow F13 \\ \hline \end{array} \rightarrow_{R} \text{ acc}
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\frac{\underbrace{\text{h11}\bullet:(\bot,\Delta9),\text{F16}\vdash\Delta8}}{-:\Delta12,\bot,\Delta9\vdash\Delta8,\Delta15,\text{F13}\rightarrow\text{F14}),\text{F16}} \xrightarrow{} \frac{\text{h11}:\Delta12,\text{F13}\vdash\Delta15,\text{F14},\text{F16}}}{\text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13}\rightarrow\text{F14}),\text{F16}} \xrightarrow{} Cut
                                                                                                                                                                                                                                                              -: \bot, \Delta 12, \Delta 9 \vdash \Delta 15, \Delta 8, F13 \rightarrow F14  ^{\bot}L
                                                                                                                                                   \frac{\frac{1}{\text{h}11 \bullet : (\bot, \Delta 9), \text{F}13 \rightarrow \text{F}14 \vdash \Delta 8} \perp_{L} \quad \frac{\text{h}11 : \Delta 12, \text{F}13 \vdash \Delta 15, \text{F}14}{\text{h}11 \bullet : \Delta 12 \vdash \Delta 15, \text{F}13 \rightarrow \text{F}14}}{-: \Delta 12, \bot, \Delta 9 \vdash \Delta 8, \Delta 15} \quad \xrightarrow{Cut}

\begin{array}{c}
-1.5, -1.5, -1.5, -1.5, -1.5 \\
\rightarrow \\
-1.5, -1.5, -1.5, -1.5, -1.5, -1.5
\end{array}

                                                                                                                             \frac{\frac{}{\mathtt{h}10\bullet:\Delta9,\bot\vdash\Delta8}}{} \; \perp_{L} \; \; \frac{\mathtt{h}10:\Delta11\vdash\bot,\Delta14,\mathtt{F}12}{\mathtt{h}10\bullet:\Delta11\vdash(\Delta14,\mathtt{F}12\land\mathtt{F}13),\bot}}_{\mathsf{h}10\bullet:\Delta11\vdash(\Delta14,\mathtt{F}12\land\mathtt{F}13),\bot} \; \; \mathsf{Cut}
                                                                                                                                                                                                                                                 -: \Delta 9, \Delta 11 \vdash \Delta 8, \Delta 14, F12 \land F13
\frac{\overbrace{\text{h10}\bullet:\bot\vdash\ast}^{+} \stackrel{\bot_L}{} \stackrel{\text{h10}:\Delta11,\Delta9\vdash\bot,\Delta14,\Delta8,F12}{} \stackrel{\text{ax}}{} \stackrel{\text{hCut}}{} \stackrel{\text{hCut}}{} \stackrel{\text{h10}:\bot\vdash\ast}^{+} \stackrel{\bot_L}{} \stackrel{\text{h10}:\Delta11,\Delta9\vdash\bot,\Delta14,\Delta8,F13}{} \stackrel{\text{ax}}{} \stackrel{\text{hCut}}{} \stackrel{\text{hCut}}{} \stackrel{\text{hCut}}{} \stackrel{\text{h10}:\bot\vdash\ast}^{+} \stackrel{\bot_L}{} \stackrel{\text{h10}:\Delta11,\Delta9\vdash\bot,\Delta14,\Delta8,F13}{} \stackrel{\text{hCut}}{} \stackrel{\text{hCut}
                                                                                             \frac{\underbrace{\frac{1}{\text{hll}\bullet:(\bot,\Delta9),\text{Fl}6}\vdash\Delta8}}{-:\Delta12,\bot,\Delta9\vdash\Delta8,\Delta15,\text{Fl}3,\text{Fl}6}\underbrace{\frac{\text{hll}:\Delta12\vdash\Delta15,\text{Fl}4,\text{Fl}6}{\text{hll}\bullet:\Delta12\vdash(\Delta15,\text{Fl}3,\text{Fl}4),\text{Fl}6}}_{\text{Cut}} \land_{R}
                                                                                                                                                                                                                                                                   -: \bot, \Delta 12, \Delta 9 \vdash \Delta 15, \Delta 8, F13 \land F14 \bot_L
                                                                                                        \frac{\frac{}{\mathsf{h}11 \bullet : (\bot, \Delta 9), \mathsf{F}13 \wedge \mathsf{F}14 \vdash \Delta 8}}{\mathsf{h}11 \bullet : (\bot, \Delta 9), \mathsf{F}13 \wedge \mathsf{F}14 \vdash \Delta 8} \stackrel{\bot}{}L \frac{\mathsf{h}11 : \Delta 12 \vdash \Delta 15, \mathsf{F}13}{\mathsf{h}11 \bullet : \Delta 12 \vdash \Delta 15, \mathsf{F}13 \wedge \mathsf{F}14}}{\mathsf{h}11 \bullet : \Delta 12 \vdash \Delta 15, \mathsf{F}13 \wedge \mathsf{F}14} \underset{\mathsf{Cut}}{\mathsf{Cut}} \wedge_{R}

\begin{array}{c}
14 \vdash \Delta 8 \\
-: \Delta 12, \perp, \Delta 9 \vdash \Delta 8, \Delta 15 \\
\rightarrow
\end{array}

                                                                                                                                                                                                                                                                                                      \overline{-:\bot,\Delta 12,\Delta 9 \vdash \Delta 15,\Delta 8} \ ^{\perp}L
                                                                                                                                                                                    \frac{\frac{10\bullet:\Delta9,\bot\vdash\Delta8}{-:\Delta9,\Delta11\vdash\Delta8,\Delta11\vdash\bot,\Delta14,\mathrm{F}12,\mathrm{F}13}}{-:\Delta9,\Delta11\vdash\Delta8,\Delta14,\mathrm{F}12\vee\mathrm{F}13}}_{-:\Delta9,\Delta11\vdash\Delta8,\Delta14,\mathrm{F}12\vee\mathrm{F}13}_{-:\Delta9}
                                                                                                                                                                                              \begin{array}{c|c} \underline{\frac{1}{100 \bullet : \bot \vdash *}} & ^{\bot}L & \xrightarrow{} \underline{\frac{1}{10} : \Delta11, \Delta9 \vdash \bot, \Delta14, \Delta8, F12, F13}} \\ \underline{\frac{-: \Delta11, \Delta9 \vdash \Delta14, \Delta8, F12, F13}{-: \Delta11, \Delta9 \vdash \Delta14, \Delta8, F12 \vee F13}} & ^{\lor}R \end{array} \quad \text{hCut} 
                                                                                                                                                              \frac{}{\underbrace{\text{h11}\bullet:(\bot,\Delta9),\text{F16}\vdash\Delta8}} \;\; \bot_L \;\; \underbrace{\begin{array}{l} \text{h11}:\Delta12\vdash\Delta15,\text{F13},\text{F14},\text{F16} \\ \text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13}\vee\text{F14}),\text{F16} \end{array}}_{\text{Cut}} \;\; \lor_R
                                                                                                                                                                                                                                                             -: \Delta 12, \bot, \Delta 9 \vdash \Delta 8, \Delta 15, F13 \lor F14
\rightarrow
                                                                                                                                                                                                                                                                   \overline{-:\bot,\Delta 12,\Delta 9 \vdash \Delta 15,\Delta 8,\mathtt{F} 13 \vee \mathtt{F} 14}^{\quad \bot}_{L}
                                                                                                                                                              \frac{1}{\underbrace{\text{h11}\bullet:(\bot,\Delta9),\text{F13}\vee\text{F14}\vdash\Delta8}} \;\; \bot_L \quad \frac{\text{h11}:\Delta12\vdash\Delta15,\text{F13},\text{F14}}{\text{h11}\bullet:\Delta12\vdash\Delta15,\text{F13}\vee\text{F14}} \;\; \lor_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                    -: \Delta 12, \perp, \Delta 9 \vdash \Delta 8, \Delta 15
                                                                                                                                                                                                                                                                                                      \frac{}{-:\bot,\Delta 12,\Delta 9\vdash \Delta 15,\Delta 8} \ ^{\bot}L
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Cases \perp_L - \perp_R

Cases \perp_L - \vee_R

Cases $\perp_L - \wedge_R$

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Cases \perp_L - \top_R
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Cases $\perp_L - \rightarrow_L$

$$\frac{\frac{10\bullet : \Delta 9, \bot \vdash \Delta 8}{} \bot_{L} \quad \frac{\text{h}10 : \Delta 14 \vdash \bot, \Delta 13, \text{F}11 \quad \text{h}10 : \Delta 14, \text{F}12 \vdash \bot, \Delta 13}{\text{h}10\bullet : \Delta 14, \text{F}11 \rightarrow \text{F}12 \vdash \Delta 13, \bot} \text{Cut}} \rightarrow_{L} \\ \frac{\frac{10\bullet : \bot \vdash *}{} \bot_{L} \quad \frac{1}{\text{h}10 : \Delta 14, \Delta 9 \vdash \bot, \Delta 13, \Delta 8, \text{F}11}}{\text{out}} \quad \frac{\text{ax}}{\text{h}0\text{ut}} \quad \frac{\text{ax}}{\text{h}0\text{ut}} \quad \frac{1}{} \bot_{L} \quad \frac{\text{h}10 : \Delta 14, \Delta 9, \text{F}12 \vdash \bot, \Delta 13, \Delta 8}{\text{h}0\text{ut}}} \quad \frac{\text{ax}}{\text{h}0\text{ut}} \quad \frac{\text{h}10\bullet : \bot \vdash *}{\text{-} : \Delta 14, \Delta 9, \text{F}12 \vdash \bot, \Delta 13, \Delta 8}} \rightarrow_{L} \\ \frac{\frac{1}{} \bot_{L} \quad \frac{1}{} \bot_{L} \quad \frac{\text{h}11 : \Delta 16 \vdash \Delta 14, \text{F}12, \text{F}15}}{\text{h}11 : \Delta 16, \text{F}13 \vdash \Delta 14, \text{F}15}} \rightarrow_{L} \\ \frac{\frac{1}{} \bot_{L} \quad \frac{1}{} \bot_{L} \quad \frac{\text{h}11 : \Delta 16, \text{F}12 \rightarrow \text{F}13 \vdash \Delta 14, \text{F}15}}{\text{h}11 : \Delta 16, \text{F}13 \vdash \Delta 14, \text{F}15}} \rightarrow_{L} \\ \frac{1}{} \bot_{L} \quad \frac{1}{} \bot_{L} \quad$$

Cases $\perp_L - \wedge_L$

Cases \perp_L - \vee_L

$$\frac{1}{10 \bullet : \Delta 9, \bot \vdash \Delta 8} \xrightarrow{\bot_{L}} \frac{10 \circ : \Delta 14, F11 \vdash \bot, \Delta 13 \quad h10 : \Delta 14, F12 \vdash \bot, \Delta 13}{110 \bullet : \Delta 14, F11 \lor F12 \vdash \Delta 13, \bot} \xrightarrow{Cut} \lor_{L}$$

$$\frac{1}{10 \bullet : \bot \vdash *} \xrightarrow{\bot_{L}} \frac{\bot_{L}}{10 : \Delta 14, \Delta 9, F11 \vdash \bot, \Delta 13, \Delta 8} \xrightarrow{Ax} \xrightarrow{hCut} \frac{\bot_{L}}{10 \bullet : \bot \vdash *} \xrightarrow{\bot_{L}} \frac{\bot_{L}}{10 : \Delta 14, \Delta 9, F12 \vdash \bot, \Delta 13, \Delta 8} \xrightarrow{Ax} \lor_{L}$$

$$\frac{- : \Delta 14, \Delta 9, F11 \vdash \Delta 13, \Delta 8}{- : \Delta 14, \Delta 9, F11 \vdash \Delta 13, \Delta 8} \xrightarrow{- : \Delta 14, \Delta 9, F12 \vdash \Delta 13, \Delta 8} \lor_{L}$$

$$\frac{h11 \bullet : (\bot, \Delta 9), F15 \vdash \Delta 8}{- : \bot, \Delta 9, \Delta 16, F12 \lor F13 \vdash \Delta 8, \Delta 14} \xrightarrow{L} \xrightarrow{L} \frac{h11 : \Delta 16, F12 \lor F13 \vdash \Delta 14, \Delta 8} \xrightarrow{L} \xrightarrow{L}$$

Cases \perp_L - \perp_L

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\frac{\rightarrow}{-:\bot,\bot,\Delta14,\Delta9\vdash\Delta12,\Delta8} \perp_L
Cases \perp_L - I
                                                                                                                                                                                    \begin{array}{c|c} \overline{\mathbf{h}10 \bullet : \Delta 9, \bot \vdash \Delta 8} & \bot L & \overline{\mathbf{h}10 \bullet : \Delta 13, \mathbf{p}12 \vdash (\Delta 11, \mathbf{p}12), \bot} & I \\ \hline - : \Delta 9, \Delta 13, \mathbf{p}12 \vdash \Delta 8, \Delta 11, \mathbf{p}12 & \to \\ \hline - : \Delta 13, \Delta 9, \mathbf{p}12 \vdash \Delta 11, \Delta 8, \mathbf{p}12 & iI \end{array} \quad \mathbf{Cut} 
                                                                                                                                                                       \frac{\overline{\mathtt{h}11 \bullet} : (\bot, \Delta9), \mathtt{p}13 \vdash \Delta8}{- : \bot, \Delta9, \Delta14, \mathtt{p}13 \vdash \Delta8, \Delta12} \quad I \\ \\ \underline{- : \bot, \Delta9, \Delta14, \mathtt{p}13 \vdash \Delta8, \Delta12} \quad \mathsf{Cut} 
                                                                                                                                                                                                                                     Cases \perp_L - \top_L
                                                                                                                                                                                                 \frac{\overbrace{\mathbf{h}10\bullet:\Delta9,\bot\vdash\Delta8}^{}~\bot_{L}~~\underbrace{\mathbf{h}10:\Delta12\vdash\bot,\Delta11}_{}~\top_{L}}{-:\Delta9,\top,\Delta12\vdash\Delta8,\Delta11}~~\top_{L}
                                                                                                                                                                                                \frac{}{\frac{\text{h}10 \bullet : \bot \vdash *}{\text{h}10 \cdot \vdash \bot}} \, \, \frac{\bot_L}{\text{h}10 : \top, \Delta 12, \Delta 9 \vdash \bot, \Delta 11, \Delta 8}} \, \, \frac{\text{ax}}{\text{hCut}} \\ - : \top, \Delta 12, \Delta 9 \vdash \Delta 11, \Delta 8} 
                                                                                                                                                                                 \frac{\frac{\mathsf{h}11 \bullet : (\bot, \Delta9), \mathsf{F}13 \vdash \Delta8}{- : \bot, \Delta9, \top, \Delta14 \vdash \Delta8, \Delta12} \  \, \begin{matrix} \bot_L \\ \hline \\ \hline \\ \hline \\ \hline \end{matrix} \frac{\mathsf{h}11 \bullet : \top, \Delta14 \vdash \Delta12, \mathsf{F}13}{\mathsf{h}11 \bullet : \top, \Delta14 \vdash \Delta12, \mathsf{F}13} \\ \hline \\ \mathsf{Cut} \\ \hline \end{matrix} 
                                                                                                                                                                                                                                       Cases I - \rightarrow_R
                                                                                                                                                                 \frac{\frac{1}{\text{h}11 \bullet : \Delta 10, \text{p}16 \vdash \Delta 8, \text{p}16}}{-: \Delta 10, \text{p}16 \vdash \Delta 8, \text{p}16} I \xrightarrow{\text{h}11 : \Delta 12, \text{F}13 \vdash \Delta 15, \text{F}14, \text{p}16} \text{h}11 \bullet : \Delta 12 \vdash (\Delta 15, \text{F}13 \rightarrow \text{F}14), \text{p}16} \\ -: \Delta 10, \Delta 12 \vdash \Delta 8, \text{p}16, \Delta 15, \text{F}13 \rightarrow \text{F}14} \xrightarrow{\text{Cut}} Cut
                                                                                                                                                                                                                 \frac{\boxed{-:\Delta10,\Delta12,\mathrm{F}13\vdash\Delta15,\Delta8,\mathrm{F}14,\mathrm{p}16}}{-:\Delta10,\Delta12\vdash\Delta15,\Delta8,\mathrm{p}16,\mathrm{F}13\to\mathrm{F}14}}\overset{\mathrm{ax}}{\to}_{R}
                                                                                                                                                                                                                                                                                                   \mathtt{h12}:\Delta\mathtt{13},\mathtt{F14}\vdash\Delta\mathtt{16},\mathtt{F15},\mathtt{F17}
                                                                                                                                                           \frac{ \underbrace{ \begin{array}{c} \mathbf{h} 12 \bullet : (\Delta 10, \mathbf{p}9), \mathbf{F} 17 \vdash \Delta 8, \mathbf{p}9 \\ - : \Delta 13, \Delta 10, \mathbf{p}9 \vdash \Delta 8, \mathbf{p}9 \end{array} I \begin{array}{c} \mathbf{n} 12 : \Delta 13, \mathbf{F} 14 \vdash \Delta 16, \mathbf{F} 15, \mathbf{F} 17 \\ \mathbf{h} 12 \bullet : \Delta 13 \vdash (\Delta 16, \mathbf{F} 14 \rightarrow \mathbf{F} 15), \mathbf{F} 17 \\ - : \Delta 13, \Delta 10, \mathbf{p}9 \vdash \Delta 8, \mathbf{p}9, \Delta 16, \mathbf{F} 14 \rightarrow \mathbf{F} 15 \\ - \\ - \end{array} \begin{array}{c} \mathbf{Cut} \end{array} } \\ \mathbf{Cut} 
                                                                                                                                                                                                                  \frac{}{-:\Delta 10,\,\Delta 13,\,\mathtt{p9}\vdash \Delta 16,\,\Delta 8,\,\mathtt{p9},\,\mathtt{F}14\to\mathtt{F}15}\quad iI
                                                                                                                                                          \frac{ \frac{ \text{h12} \bullet : (\Delta 10, \text{p9}), \text{F14} \rightarrow \text{F15} \vdash \Delta 8, \text{p9} }{ - : \Delta 13, \Delta 10, \text{p9} \vdash \Delta 16, \Delta 8, \text{p9} } I \quad \frac{ \text{h12} : \Delta 13, \text{F14} \vdash \Delta 16, \text{F15} }{ \text{h12} \bullet : \Delta 13 \vdash \Delta 16, \text{F14} \rightarrow \text{F15} } \quad \frac{\rightarrow}{\text{Cut}} 
                                                                                                                                                                                                                                       \frac{}{-:\Delta 10,\Delta 13,\mathtt{p9}\vdash \Delta 16,\Delta 8,\mathtt{p9}}\quad iI
Cases I - \wedge_R
                                                                                                                                  \frac{\frac{1}{\text{h11}\bullet:\Delta 10,\text{p16}\vdash\Delta 8,\text{p16}}}{-:\Delta 10,\text{p16}\vdash\Delta 8,\text{p16}} I \frac{\text{h11}:\Delta 12\vdash\Delta 15,\text{F13},\text{p16} \quad \text{h11}:\Delta 12\vdash\Delta 15,\text{F14},\text{p16}}{\text{h11}\bullet:\Delta 12\vdash(\Delta 15,\text{F13}\wedge\text{F14}),\text{p16}} \cap \mathcal{A}_{R}
                                                                                                                                                         \frac{-:\Delta 10,\Delta 12\vdash \Delta 15,\Delta 8,F13,p16}{-:\Delta 10,\Delta 12\vdash \Delta 15,\Delta 8,F14,p16} \xrightarrow{\text{ax}} \frac{\rightarrow}{-:\Delta 10,\Delta 12\vdash \Delta 15,\Delta 8,F14,p16} \xrightarrow{\text{ax}} \wedge_{R}
                                                                                                                          \frac{\frac{}{\text{h12}\bullet:(\Delta10,p9),\text{F17}\vdash\Delta8,p9}}{-:\Delta13,\Delta10,p9\vdash\Delta8,p9} \begin{array}{c} I & \frac{\text{h12}:\Delta13\vdash\Delta16,\text{F14},\text{F17} & \text{h12}:\Delta13\vdash\Delta16,\text{F15},\text{F17}}{\text{h12}\bullet:\Delta13\vdash(\Delta16,\text{F14}\land\text{F15}),\text{F17}} \\ & -:\Delta13,\Delta10,p9\vdash\Delta8,p9,\Delta16,\text{F14}\land\text{F15} \end{array} \begin{array}{c} \text{Cut} \end{array}
                                                                                                                                                                                                                    \frac{}{-:\Delta 10,\Delta 13,p9\vdash \Delta 16,\Delta 8,p9,\mathtt{F}14\wedge\mathtt{F}15}\quad iI
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\frac{\frac{}{\text{h12}\bullet:(\Delta10,p9),\text{F14}\wedge\text{F15}\vdash\Delta8,\text{p9}}{-:\Delta13,\Delta10,\text{p9}\vdash\Delta16,\Delta8,\text{p9}}} I - \frac{\text{h12}:\Delta13\vdash\Delta16,\text{F14} \quad \text{h12}:\Delta13\vdash\Delta16,\text{F15}}{\text{h12}\bullet:\Delta13\vdash\Delta16,\text{F14}\wedge\text{F15}} \\ -:\Delta13,\Delta10,\text{p9}\vdash\Delta16,\Delta8,\text{p9}} \text{Cut} - \frac{\text{Cut}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                       \frac{}{-:\Delta 10,\Delta 13,\mathtt{p9}\vdash \Delta 16,\Delta 8,\mathtt{p9}} \quad iI
Cases I - \vee_R
                                                                                                                                                                                                                                                                                                                \frac{\frac{\text{h}11 \bullet : \Delta10 \text{, p}16 \vdash \Delta8 \text{, p}16}{-: \Delta10, \Delta12 \vdash \Delta8, p}I \quad \frac{\text{h}11 : \Delta12 \vdash \Delta15, F13, F14, p}16}{\text{h}11 \bullet : \Delta12 \vdash (\Delta15, F13 \vee F14), p}16} \quad \frac{\vee_R}{\text{cut}}
                                                                                                                                                                                                                                                                                                                                                                                                          \frac{\frac{}{\text{h12}\bullet:(\Delta10,p9),\text{F17}\vdash\Delta8,\text{p9}}}{-:\Delta13,\Delta10,\text{p9}\vdash\Delta8,\text{p9}} \cdot I \quad \frac{\text{h12}:\Delta13\vdash\Delta16,\text{F14},\text{F15},\text{F17}}{\text{h12}\bullet:\Delta13\vdash(\Delta16,\text{F14}\vee\text{F15}),\text{F17}} \\ -:\Delta13,\Delta10,\text{p9}\vdash\Delta8,\text{p9},\Delta16,\text{F14}\vee\text{F15}} \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                      \frac{}{-:\Delta 10,\Delta 13,\mathtt{p9}\vdash \Delta 16,\Delta 8,\mathtt{p9},\mathtt{F}14\vee\mathtt{F}15}\quad iI
                                                                                                                                                                                                                                                                                                     \frac{\frac{1}{\text{h12} \bullet : (\Delta 10, p9), F14 \vee F15 \vdash \Delta 8, p9}}{-: \Delta 13, \Delta 10, p9 \vdash \Delta 16, \Delta 8, p9} I \xrightarrow{\text{h12} \bullet : \Delta 13 \vdash \Delta 16, F14 \vee F15} \text{Cut}
\frac{-: \Delta 13, \Delta 10, p9 \vdash \Delta 16, \Delta 8, p9}{-: \Delta 10, \Delta 13, p9 \vdash \Delta 16, \Delta 8, p9} iI
Cases I - \perp_R
                                                                                                                                                                                                                                                                                                                                       \frac{}{-:\Delta 10,\Delta 12\vdash \bot,\Delta 13,\Delta 8,\mathtt{p} 14}\quad \mathtt{ax}\quad
                                                                                                                                                                                                                                                                                                                                                     \frac{\frac{\frac{1}{2\bullet}:(\Delta10,\mathbf{p9}),\bot\vdash\Delta8,\mathbf{p9}}{-:\Delta13,\Delta10,\mathbf{p9}\vdash\Delta14,\Delta8,\mathbf{p9}}}{I}\frac{\frac{1}{\bullet12\bullet}:\Delta13\vdash\Delta14}{\bullet12\bullet}:\Delta13\vdash\Delta14,\bot}{\overset{-}{\bullet}}\underbrace{\frac{\bot_{R}}{\cot}}_{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         \mathtt{h}12:\Delta13\vdash\Delta14,\mathtt{F}15
                                                                                                                                                                                                                                                                                                                            \frac{\mathbf{h}12\bullet:(\Delta10,\mathbf{p}9),\mathbf{F}15\vdash\Delta8,\mathbf{p}9}{-:\Delta13,\Delta10,\mathbf{p}9\vdash\Delta8,\mathbf{p}9,\bot,\Delta14} \stackrel{\mathbf{n}12\bullet:\Delta13\vdash\Delta14,\mathbf{F}15}{\subset} \underbrace{}^{\perp}R
                                                                                                                                                                                                                                                                                                                                                                                                                                Cases I - \top_R
                                                                                                                                                                                                                                                                                                                                        \frac{\overline{\mathbf{h}11\bullet}:\Delta10,\mathbf{p}14\vdash\Delta8,\mathbf{p}14}{-:\Delta10,\Delta12\vdash\Delta8,\mathbf{p}14} \quad I \quad \frac{\mathbf{h}11\bullet:\Delta12\vdash(\top,\Delta13),\mathbf{p}14}{-:\Delta10,\Delta12\vdash\Delta8,\mathbf{p}14,\top,\Delta13} \quad \mathbf{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                  \frac{}{-:\Delta 10,\Delta 12 \vdash \top,\Delta 13,\Delta 8,\mathtt{p} 14} \ \top_{R}
                                                                                                                                                                                                                                                                                                                                                      \frac{\overline{\text{h12}\bullet:(\Delta10,p9),\top\vdash\Delta8,p9}}{-:\Delta13,\Delta10,p9\vdash\Delta14,\Delta8,p9}} \begin{array}{c} I \\ \text{h12}\bullet:\Delta13\vdash\Delta14,\top \\ \hline \\ \text{Cut} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                           \overline{-:\Delta 10,\Delta 13, \mathsf{p9} \vdash \Delta 14,\Delta 8,\mathsf{p9}} \quad iI
                                                                                                                                                                                                                                                                                                                              \frac{\frac{\text{h12}\bullet:(\Delta10,p9),\text{F15}\vdash\Delta8,p9}{-:\Delta13,\Delta10,p9\vdash\Delta8,p9,\top,\Delta14}}{-:\Delta13,\Delta10,p9\vdash\Delta8,p9,\top,\Delta14} \overset{\top_{R}}{\text{Cut}}
                                                                                                                                                                                                                                                                                                                                                                                                                             Cases I - \rightarrow_L
                                                                                                                                                                                                                                               \frac{\underbrace{\text{h11}\bullet:\Delta10,\text{p15}\vdash\Delta8,\text{p15}}_{-:\Delta10,\text{p15}\vdash\Delta8,\text{p15}} I \quad \frac{\text{h11}:\Delta16\vdash\Delta14,\text{F12},\text{p15}}{\text{h11}\bullet:\Delta16,\text{F12}\rightarrow\text{F13}\vdash\Delta14,\text{p15}}_{-:\Delta10,\Delta16,\text{F12}\rightarrow\text{F13}\vdash\Delta14,\Delta8,\text{p15}} \quad \underbrace{\text{Cut}}_{-:\Delta10,\Delta16,\text{F12}\rightarrow\text{F13}\vdash\Delta14,\Delta8,\text{p15}}^{\text{h11}\bullet:\Delta16,\text{F13}\vdash\Delta14,\text{p15}}_{-:\Delta10,\Delta16,\text{F12}\rightarrow\text{F13}\vdash\Delta14,\Delta8,\text{p15}}^{\text{Cut}}
                                                                                                                                                                                                                                                                                      \underbrace{ -: \Delta 10, \Delta 16 \vdash \Delta 14, \Delta 8, F12, p15}_{} \quad \text{ax} \quad \underbrace{ \xrightarrow{} \quad }_{} \quad -: \Delta 10, \Delta 16, F13 \vdash \Delta 14, \Delta 8, p15}_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \Delta 10, \Delta 16, F13 \vdash \Delta 14, \Delta 8, p15}_{} \quad \underbrace{ \xrightarrow{} \quad }_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{} \quad \text{ax} \quad \xrightarrow{}_{} \quad \xrightarrow{}_{
                                                                                                                                                                                                                                                                                                                                                                                                  -:\Delta10,\Delta16,\mathtt{F}12\to\mathtt{F}13\vdash\Delta14,\Delta8,\mathtt{p}15
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\frac{1}{\text{h12}\bullet: (\Delta10, p9), F16 \vdash \Delta8, p9} I \xrightarrow{\text{h12}: \Delta17 \vdash \Delta15, F13, F16} \text{h12}: \Delta17, F14 \vdash \Delta15, F16} \rightarrow_{L}
                                                                                                                                                                     -:\Delta10, \mathtt{p9}, \Delta17, \mathtt{F}13 \to \mathtt{F}14 \vdash \Delta15, \Delta8, \mathtt{p}9
                                                                                                                                                                                        \frac{}{-:\Delta 10,\Delta 17,p9,F13\rightarrow F14\vdash \Delta 15,\Delta 8,p9} \quad iI
Cases I - \wedge_L
                                                                                                                                                     \frac{1}{\mathtt{h}11\bullet:\Delta10,\mathtt{p}15\vdash\Delta8,\mathtt{p}15} \quad I \quad \frac{\mathtt{h}11:\Delta16,\mathtt{F}12,\mathtt{F}13\vdash\Delta14,\mathtt{p}15}{\mathtt{h}11\bullet:\Delta16,\mathtt{F}12\wedge\mathtt{F}13\vdash\Delta14,\mathtt{p}15} \quad \wedge_L
                                                                                                                                                                                            -:\Delta10,\Delta16,\mathtt{F}12\wedge\mathtt{F}13\vdash\Delta14,\Delta8,\mathtt{p}15
                                                                                                                                                                                           \frac{-:\Delta 10,\Delta 16,\mathsf{F}12,\mathsf{F}13\vdash \Delta 14,\Delta 8,\mathsf{p}15}{-:\Delta 10,\Delta 16,\mathsf{F}12,\wedge \mathsf{F}13\vdash \Delta 14,\Delta 8,\mathsf{p}15} \  \, \stackrel{\mathsf{ax}}{\wedge}_L
                                                                                                                                               \frac{1}{\text{h}12 \bullet : (\Delta 10, \text{p}9), \text{F}16 \vdash \Delta 8, \text{p}9} \quad I \quad \frac{\text{h}12 : \Delta 17, \text{F}13, \text{F}14 \vdash \Delta 15, \text{F}16}{\text{h}12 \bullet : \Delta 17, \text{F}13 \land \text{F}14 \vdash \Delta 15, \text{F}16} \quad \land L \quad \text{Cut}} \quad \text{Cut}
                                                                                                                                                                                      -: \Delta 10, p9, \Delta 17, F13 \land F14 \vdash \Delta 15, \Delta 8, p9
\longrightarrow
                                                                                                                                                                                          \overline{-:\Delta10,\Delta17,\mathtt{p9},\mathtt{F}13\wedge\mathtt{F}14\vdash\Delta15,\Delta8,\mathtt{p9}}\quad i\,\mathbf{1}
Cases I - \vee_L
                                                                                                                  \frac{\frac{}{\text{h}11 \bullet : \Delta 10, \text{p}15 \vdash \Delta 8, \text{p}15}}{-: \Delta 10, \text{p}15 \vdash \Delta 10, \Delta 16, \text{F}12 \lor \text{F}13 \vdash \Delta 14, \text{p}15}} \frac{I}{\text{h}11 : \Delta 16, \text{F}12 \lor \text{F}13 \vdash \Delta 14, \text{p}15}} \\ \frac{\text{h}11 \bullet : \Delta 16, \text{F}12 \lor \text{F}13 \vdash \Delta 14, \text{p}15}}{-: \Delta 10, \Delta 16, \text{F}12 \lor \text{F}13 \vdash \Delta 14, \Delta 8, \text{p}15}} \text{ Cut} \\ \downarrow L
                                                                                                                                       \frac{-:\Delta 10, \Delta 16, F12 \vdash \Delta 14, \Delta 8, p15}{-:\Delta 10, \Delta 16, F12 \vdash \Delta 14, \Delta 8, p15} \xrightarrow{\text{ax}} \frac{-:\Delta 10, \Delta 16, F13 \vdash \Delta 14, \Delta 8, p15}{-:\Delta 10, \Delta 16, F12 \lor F13 \vdash \Delta 14, \Delta 8, p15} \xrightarrow{\text{ax}} \bigvee_{L} 
                                                                                                            \frac{1}{\text{h12} \bullet : (\Delta 10, \text{p9}), \text{F16} \vdash \Delta 8, \text{p9}} \quad I \quad \frac{\text{h12} : \Delta 17, \text{F13} \vdash \Delta 15, \text{F16} \quad \text{h12} : \Delta 17, \text{F14} \vdash \Delta 15, \text{F16}}{\text{h12} \bullet : \Delta 17, \text{F13} \vee \text{F14} \vdash \Delta 15, \text{F16}} \quad \vee_{L} \quad \text{out} \quad \vee_{L} \quad \text{out}
                                                                                                                                                                      -:\Delta10, \mathtt{p}9, \Delta17, \mathtt{F}13 \vee \mathtt{F}14 \vdash \Delta15, \Delta8, \mathtt{p}9
                                                                                                                                                                                         \frac{}{-:\Delta 10,\Delta 17,\mathrm{p9},\mathrm{F}13\vee\mathrm{F}14\vdash\Delta 15,\Delta 8,\mathrm{p9}}\quad iI
Cases I - \perp_L
                                                                                                                                                                \frac{\overbrace{\mathtt{h}11\bullet:\Delta10,\mathtt{p}13\vdash\Delta8,\mathtt{p}13}^{I} \quad \overbrace{\mathtt{h}11\bullet:\bot,\Delta14\vdash\Delta12,\mathtt{p}13}^{I} \quad \underbrace{\bot_{L}}_{\mathtt{Cut}}}{-:\Delta10,\bot,\Delta14\vdash\underbrace{\Delta12,\Delta8,\mathtt{p}13}} \quad \underbrace{\Box_{L}}_{\mathtt{Cut}}
                                                                                                                                                                                                      \frac{\rightarrow}{-:\bot,\Delta 10,\Delta 14\vdash\Delta 12,\Delta 8,\mathtt{p}13}\ \bot_{L}
                                                                                                                                                          Cases I - I
                                                                                                                                                     \overline{\text{h11}\bullet:\Delta10,\text{p13}\vdash\Delta8,\text{p13}}\quad I\quad \overline{\text{h11}\bullet:\Delta15,\text{p14}\vdash(\Delta12,\text{p14}),\text{p13}}\quad I\quad \text{Cut}
                                                                                                                                                                                          \begin{array}{c} -: \Delta 10, \Delta 15, \mathtt{p}14 \vdash \Delta 8, \mathtt{p}13, \Delta 12, \mathtt{p}14 \\ \longrightarrow \\ \hline -: \Delta 10, \Delta 15, \mathtt{p}14 \vdash \Delta 12, \Delta 8, \mathtt{p}13, \mathtt{p}14 \end{array} \ iI \\
                                                                                                                                                                 \cfrac{ \cfrac{ \underbrace{ \text{h11} \bullet : \Delta 10, \text{p13} \vdash \Delta 8, \text{p13} }_{- : \Delta 10, \Delta 14, \text{p13} \vdash \Delta 12, \text{p13} } \overbrace{\text{Cut}}_{-} }_{- : \Delta 10, \Delta 14, \text{p13} \vdash \Delta 12, \Delta 8, \text{p13}} \cfrac{I}{\text{Cut}} 
                                                                                                                                                                                                      \frac{}{-:\Delta 10,\Delta 14,\mathtt{p}13\vdash \Delta 12,\Delta 8,\mathtt{p}13}\quad iI
                                                                                                                                                \frac{ \overbrace{ \mathbf{h} 12 \bullet : (\Delta 10, \mathbf{p}9), \mathbf{F} 14 \vdash \Delta 8, \mathbf{p}9 } }{-: \Delta 10, \mathbf{p}9, \Delta 16, \mathbf{p}15 \vdash \Delta 8, \mathbf{p}9, \Delta 13, \mathbf{p}15), \mathbf{F}14} } \quad \underbrace{ \begin{matrix} I \\ \text{Cut} \\ \end{matrix} }_{\text{Cut}} 
                                                                                                                                                                                             \frac{}{-:\Delta 10,\Delta 16,\mathtt{p} 15,\mathtt{p} 9 \vdash \Delta 13,\Delta 8,\mathtt{p} 15,\mathtt{p} 9} \quad iI
                                                                                                                                                          \frac{1}{\text{h12} \bullet : (\Delta 10, \text{p9}), \text{p14} \vdash \Delta 8, \text{p9}} I \frac{1}{\text{h12} \bullet : \Delta 15, \text{p14} \vdash \Delta 13, \text{p14}} I \\ - : \Delta 10, \text{p9}, \Delta 15, \text{p14} \vdash \Delta 13, \Delta 8, \text{p9}} \text{Cut}
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Cases I - \top_L

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 \frac{\overline{\text{h11}\bullet:\Delta10,\text{p13}\vdash\Delta8,\text{p13}}}{I \xrightarrow[]{\text{h11}:\Delta14\vdash\Delta12,\text{p13}}} I \xrightarrow[]{\text{h11}\bullet:\top,\Delta14\vdash\Delta12,\text{p13}} Cut \\ \xrightarrow[]{\text{cut}}
                                                                                                                                                                                                                                                                                                                       \overline{-: \top, \Delta 10, \Delta 14 \vdash \Delta 12, \Delta 8, p13} ax
                                                                                                                                                                                                                                           \frac{\frac{}{\mathtt{h}12\bullet:(\Delta10,\mathtt{p}9),\mathtt{F}14\vdash\Delta8,\mathtt{p}9}}{-:\Delta10,\mathtt{p}9,\top,\Delta15\vdash\Delta13,\mathtt{p}9} \ I \ \frac{\mathtt{h}12:\Delta15\vdash\Delta13,\mathtt{F}14}{\mathtt{h}12\bullet:\top,\Delta15\vdash\Delta13,\mathtt{F}14} \ \tau_L
                                                                                                                                                                                                                                                                                                                 \frac{}{-:\top,\Delta 10,\Delta 15,p9\vdash \Delta 13,\Delta 8,p9} iI
Cases \top_L - \to_R
                                                                                                                                                                                                                                           \begin{array}{c|c} \underline{\mathbf{h}10: \Delta9 \vdash \Delta8} \\ \underline{\mathbf{h}10 \bullet : \Delta9, \top \vdash \Delta8} \end{array} \ \top_L \quad \underline{\mathbf{h}10 \bullet : \Delta11, F12 \vdash \top, \Delta14, F13} \\ \underline{\mathbf{h}10 \bullet : \Delta9, \top \vdash \Delta8} \end{array} \ \top_L \quad \underline{\mathbf{h}10 \bullet : \Delta11 \vdash (\Delta14, F12 \rightarrow F13), \top}_{\mathbf{Cut}} \quad \underline{\mathbf{Cut}} 
                                                                                                                                                                                                                                                                                                      -: \Delta 9, \Delta 11 \vdash \Delta 8, \Delta 14, F12 \rightarrow F13
                                                                                                                                                                                                                                                                                                               \frac{}{-:\Delta 11,\,\Delta 9\vdash \Delta 14,\,\Delta 8,\mathtt{F}12\rightarrow\mathtt{F}13}\quad \text{ax}\quad
                                                                                                                                                                                                                        \frac{\text{h11}:\Delta9,\text{F16}\vdash\Delta8}{\text{h11}\bullet:(\top,\Delta9),\text{F16}\vdash\Delta8} \quad \top_L \quad \frac{\text{h11}:\Delta12,\text{F13}\vdash\Delta15,\text{F14},\text{F16}}{\text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13}\rightarrow\text{F14}),\text{F16}} \quad \xrightarrow{\sim}_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                                -:\Delta 12, \top, \Delta 9 \vdash \Delta 8, \Delta 15, \mathtt{F} 13 \to \mathtt{F} 14
                                                                                                                                                                                                                                          \underbrace{\frac{1}{\text{h11}: \Delta 9, \text{F16} \vdash \Delta 8}}_{\text{h11}: \Gamma, \Delta 12, \text{F13} \vdash \Delta 15, \text{F14}, \text{F16}} \xrightarrow{\text{ax}} \underbrace{\frac{1}{\text{h11} \cdot \Gamma, \Delta 12, \text{F13} \vdash \Delta 15, \text{F14}, \text{F16}}}_{\text{h11}\bullet: \Gamma, \Delta 12 \vdash \Delta 15, \text{F16}, \text{F13} \rightarrow \text{F14}} \xrightarrow{\text{hOut}} \underbrace{\frac{1}{\text{hOut}}}_{\text{hout}} 
                                                                                                                                                                                                                                                                                                -: \top, \Delta 12, \Delta 9 \vdash \Delta 15, \Delta 8, F13 \rightarrow F14
                                                                                                                                                                                                                      \underbrace{\frac{\text{h11} : \Delta 9, \text{F13} \rightarrow \text{F14} \vdash \Delta 8}{\text{h11} \bullet : (\top, \Delta 9), \text{F13} \rightarrow \text{F14} \vdash \Delta 8}}_{\text{h11} \bullet : \Delta 12 \vdash \Delta 15, \text{F13} \rightarrow \text{F14}} \xrightarrow{} R \underbrace{\frac{\text{h11} \bullet : \Delta 12 \vdash \Delta 15, \text{F13} \rightarrow \text{F14}}{\text{h11} \bullet : \Delta 12 \vdash \Delta 15, \text{F13} \rightarrow \text{F14}}}_{\text{Cut}} \xrightarrow{} Cut 
                                                                                                                                                                                                                                                                                                                        -: \Delta 12, \top, \Delta 9 \vdash \Delta 8, \Delta 15
                                                                                                                                                                                                                                  -: \top, \Delta 12, \Delta 9 \vdash \Delta 15, \Delta 8
Cases \top_L - \wedge_R
                                                                                                                                                                                                   -: \Delta 9, \Delta 11 \vdash \Delta 8, \Delta 14, F12 \land F13
                                                                                                                                                                                                                                                                                                                 \frac{}{-:\Delta 11,\Delta 9\vdash \Delta 14,\Delta 8,\mathtt{F}12\wedge\mathtt{F}13}\quad \mathtt{ax}
                                                                                                                                                                             \frac{\text{h11}:\Delta9,\text{F16}\vdash\Delta8}{\text{h11}\bullet:(\top,\Delta9),\text{F16}\vdash\Delta8} \quad \top_L \quad \frac{\text{h11}:\Delta12\vdash\Delta15,\text{F13},\text{F16}}{\text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13},\text{F14},\text{F16}} \\ \quad \frac{\text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13},\text{F14}),\text{F16}}{\text{Cut}} \quad \land_R
                                                                                                                                                                                                                                                                             -: \Delta 12, \top, \Delta 9 \vdash \Delta 8, \Delta 15, F13 \land F14
                                                                                                                                                                        \frac{1}{11:\Delta 9, F16 \vdash \Delta 8} = \frac{1}{\text{ax}} \frac{
                                                                                                                                                                                   \frac{\text{h11}:\Delta9,\text{F13}\wedge\text{F14}\vdash\Delta8}{\text{h11}\bullet:(\top,\Delta9),\text{F13}\wedge\text{F14}\vdash\Delta8} \quad \top_L \quad \frac{\text{h11}:\Delta12\vdash\Delta15,\text{F13} \quad \text{h11}:\Delta12\vdash\Delta15,\text{F14}}{\text{h11}\bullet:\Delta12\vdash\Delta15,\text{F13}\wedge\text{F14}} \quad \wedge_R \\ -:\Delta12,\top,\Delta9\vdash\Delta8,\Delta15 \\ \rightarrow \quad \rightarrow \quad \text{Cut} \quad \rightarrow
                                                                                                                                                                               \frac{1}{11:\Delta 9,F13 \wedge F14 \vdash \Delta 8} \text{ ax } \frac{1}{\text{h11}:\top,\Delta 12 \vdash \Delta 15,F13} \text{ ax } \frac{1}{\text{h11}:\top,\Delta 12 \vdash \Delta 15,F14} \text{ ax } \frac{\text{h11}:\top,\Delta 12 \vdash \Delta 15,F14}{\text{h11}\bullet:\top,\Delta 12 \vdash \Delta 15,F13 \wedge F14} \wedge_{R}
Cases \top_L - \vee_R
                                                                                                                                                                                                                                                \underbrace{ \frac{\text{h}10: \Delta9 \vdash \Delta8}{\text{h}10 \bullet : \Delta9, \top \vdash \Delta8}}_{\text{h}10 \bullet : \Delta9, \top \vdash \Delta8} \ \top_L \ \underbrace{ \frac{\text{h}10: \Delta11 \vdash \top, \Delta14, F12, F13}{\text{h}10 \bullet : \Delta11 \vdash (\Delta14, F12 \lor F13), \top}}_{\text{Cut}} \ \lor_R 
                                                                                                                                                                                                                                                                                                          \overline{-: \Delta 11, \Delta 9 \vdash \Delta 14, \Delta 8, F12 \lor F13} ax
                                                                                                                                                                                                                            \frac{\text{h11}:\Delta9,\text{F16}\vdash\Delta8}{\text{h11}\bullet:(\top,\Delta9),\text{F16}\vdash\Delta8} \quad \top_L \quad \frac{\text{h11}:\Delta12\vdash\Delta15,\text{F13},\text{F14},\text{F16}}{\text{h11}\bullet:\Delta12\vdash(\Delta15,\text{F13}\vee\text{F14}),\text{F16}} \quad \vee_R \quad \text{Cut}
                                                                                                                                                                                                                                                                                      -: \Delta 12, \top, \Delta 9 \vdash \Delta 8, \Delta 15, F13 \vee F14
                                                                                                                                                                                                                                                                                                                                                                           h11: T, \Delta 12 \vdash \Delta 15, F13, F14, F16 ax
                                                                                                                                                                                                                                            \frac{\text{h11} : \Delta 9, \text{F16} \vdash \Delta 8}{\text{h11} : \Delta 9, \text{F16} \vdash \Delta 8} \xrightarrow{\text{ax}} \frac{\text{h11} : \top, \Delta 12 \vdash \Delta 15, \text{F13}, \text{F14}, \text{F16}}{\text{h11} \bullet : \top, \Delta 12 \vdash \Delta 15, \text{F16}, \text{F13} \vee \text{F14}} \bigvee_{\text{hCut}} V_{\text{R}}
                                                                                                                                                                                                                                                                                        -\,:\,\top,\,\Delta 12,\,\Delta 9\,\vdash\,\Delta 15,\,\Delta 8,\mathsf{F} 13\,\vee\,\mathsf{F} 14
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\frac{\text{h11}:\Delta9,\text{F13}\vee\text{F14}\vdash\Delta8}{\text{h11}\bullet:(\top,\Delta9),\text{F13}\vee\text{F14}\vdash\Delta8} \ \top_L \quad \frac{\text{h11}:\Delta12\vdash\Delta15,\text{F13},\text{F14}}{\text{h11}\bullet:\Delta12\vdash\Delta15,\text{F13}\vee\text{F14}} \quad \vee_R \\ -:\Delta12,\top,\Delta9\vdash\Delta8,\Delta15 \quad \text{Cut}
                                                                                                                                                                         \begin{array}{c} \begin{array}{c} - \cdot \Delta 12, \ \cdot, \ \Sigma \\ \\ \rightarrow \\ \\ \hline \underline{ 111: \Delta 9, F13 \vee F14 \vdash \Delta 8} \end{array} \text{ ax } \\ \begin{array}{c} \\ \rightarrow \\ \hline \underline{ 111: T, \Delta 12 \vdash \Delta 15, F13, F14} \\ \hline \underline{ 110: T, \Delta 12 \vdash \Delta 15, F13 \vee F14} \end{array} \begin{array}{c} \mathbf{ax} \\ \vee \\ \mathbf{hOut} \end{array} 
                                                                                                                                                                                                                            -: \top, \Delta 12, \Delta 9 \vdash \Delta 15, \Delta 8
Cases \top_L - \bot_R
                                                                                                                                                                                         \frac{\text{h}10:\Delta9\vdash\Delta8}{\text{h}10\bullet:\Delta9,\top\vdash\Delta8} \ \top_L \ \frac{\text{h}10:\Delta11\vdash\top,\Delta12}{\text{h}10\bullet:\Delta11\vdash(\bot,\Delta12),\top} \ \bot_R
                                                                                                                                                                                                                                    -: \Delta 9, \Delta 11 \vdash \Delta 8, \bot, \Delta 12
                                                                                                                                                                                                                                           \overline{-:\Delta11,\Delta9\vdash\bot,\Delta12,\Delta8}\quad\text{ax}\quad
                                                                                                                                                                                          \begin{array}{c|c} \underbrace{\frac{\text{n11}:\Delta 9, \perp \vdash \Delta 8}{\text{h11}\bullet:(\top,\Delta 9), \perp \vdash \Delta 8}}_{\text{L}} & \underbrace{\frac{\text{h11}:\Delta 12 \vdash \Delta 13}{\text{h11}\bullet:\Delta 12 \vdash \Delta 13, \perp}}_{\text{Cut}} & \underbrace{\frac{\perp_{R}}{\text{Cut}}}_{\text{Cut}} \end{array}
                                                                                                                                                                                                                                           \overline{-:\top,\Delta 12,\Delta 9\vdash \Delta 13,\Delta 8}\quad \text{ax}\quad
                                                                                                                                                                          \frac{\frac{\text{h}11:\Delta9,\text{F}14\vdash\Delta8}{\text{h}11\bullet:(\top,\Delta9),\text{F}14\vdash\Delta8}}{-:\Delta12,\top,\Delta9),\text{F}14\vdash\Delta8} \ \top_L \quad \frac{\text{h}11:\Delta12\vdash\Delta13,\text{F}14}{\text{h}11\bullet:\Delta12\vdash(\bot,\Delta13),\text{F}14}}{-:\Delta12,\top,\Delta9\vdash\Delta8,\bot,\Delta13} \quad \overset{\bot_R}{\text{cut}}
                                                                                                                                                                                       \frac{\underset{111:\Delta9,F14\vdash\Delta8}{\underbrace{h11:\top,\Delta12\vdash\Delta13,F14}}}{\underset{-:\top,\Delta12,\Delta9\vdash\bot,\Delta13,\Delta8}{\underbrace{h11:\top,\Delta12\vdash\bot,\Delta13,F14}}} \overset{\text{ax}}{\underset{\text{hCut}}{\underbrace{\vdash}}} \frac{\bot_R}{\underset{\text{hCut}}{\underbrace{\vdash}}}
Cases \top_L - \top_R
                                                                                                                                                                                                    \frac{ \underbrace{ \mathsf{h}10 : \Delta 9 \vdash \Delta 8 }_{\mathsf{h}10 \bullet} : \Delta 9, \top \vdash \Delta 8 } {} \top_L \quad \underbrace{ \underbrace{ \mathsf{h}10 \bullet : \Delta 11 \vdash \Delta 12, \top }_{\mathsf{h}10 \bullet} : \Delta 9, \top \vdash \Delta 8 }_{\mathsf{Cut}} \quad \top_R
                                                                                                                                                                                                                                                \overline{-:\Delta11,\Delta9\vdash\Delta12,\Delta8}\quad\text{ax}\quad
                                                                                                                                                                                          \begin{array}{c|c} \frac{\text{h11}:\Delta9,\top\vdash\Delta8}{\text{h11}\bullet:(\top,\Delta9),\top\vdash\Delta8} & \top_L & \frac{}{\text{h11}\bullet:\Delta12\vdash\Delta13,\top} & \top_R \\ \hline & -:\Delta12,\top,\Delta9\vdash\Delta8,\Delta13 & \text{Cut} \\ \hline & & \rightarrow & \text{ax} \end{array}
                                                                                                                                                                                                   \mathtt{h11}:\Delta9,\top\vdash\Delta8
                                                                                                                                                                                                                                           \overline{-: \top, \Delta 12, \Delta 9 \vdash \Delta 13, \Delta 8} ax
                                                                                                                                                                                     \mathtt{h}11:\Delta9,\mathtt{F}14 \vdash \Delta8
                                                                                                                                                                           \frac{\mathsf{h}\mathsf{1}\mathsf{1} : \Delta9, \mathsf{F}\mathsf{1} \mathsf{4} \vdash \Delta8}{\mathsf{h}\mathsf{1}\mathsf{1} \bullet : (\top, \Delta9), \mathsf{F}\mathsf{1} \mathsf{4} \vdash \Delta8} \quad \top_L \quad \frac{\mathsf{h}\mathsf{1}\mathsf{1} \bullet : \Delta12 \vdash (\top, \Delta13), \mathsf{F}\mathsf{1} \mathsf{4}}{-: \Delta12, \top, \Delta9 \vdash \Delta8, \top, \Delta13} \quad \top_R \quad \mathsf{Cut}
                                                                                                                                                                                                                                 Cases \top_L - \to_L
                                                                                                                                            \frac{\frac{\text{h}10:\Delta9\vdash\Delta8}{\text{h}10\bullet:\Delta9\vdash\Delta8}}{-:\Delta9,\top\vdash\Delta8} \ \top_L \ \frac{\text{h}10:\Delta14\vdash\top,\Delta13,\text{F}11 \ \text{h}10:\Delta14,\text{F}12\vdash\top,\Delta13}{\text{h}10\bullet:\Delta14,\text{F}11\to\text{F}12\vdash\Delta13,\top} \ \text{Cut}}{-:\Delta9,\Delta14,\text{F}11\to\text{F}12\vdash\Delta8,\Delta13} \ Cut
                                                                                                                                                                                                                             -: \top, \Delta 9, \Delta 16, F12 \rightarrow F13 \vdash \Delta 8, \Delta 14
                                                                                                                          \frac{1}{\text{h}11:\Delta 9,\text{F15} \vdash \Delta 8} \text{ ax } \frac{\frac{1}{\text{h}11:\top,\Delta 16 \vdash \Delta 14,\text{F12},\text{F15}} \text{ ax}}{\text{h}11\bullet:\top,\Delta 16,\text{F13} \vdash \Delta 14,\text{F15}} \text{ ax}}{\text{h}11:\top,\Delta 16,\text{F13} \vdash \Delta 14,\text{F15}} \xrightarrow{\text{hCut}} \xrightarrow{\text{hCut}}
                                                                                                                                                                                           -: \mathsf{T}, \Delta 16, \Delta 9, \mathsf{F}12 \to \mathsf{F}13 \vdash \Delta 14, \Delta 8
Cases \top_L - \wedge_L
                                                                                                                                                                                \begin{array}{c} \frac{\text{h10}:\Delta9\vdash\Delta8}{\text{h10}\bullet:\Delta9,\top\vdash\Delta8} & \top_L & \frac{\text{h10}:\Delta14,\text{F11},\text{F12}\vdash\top,\Delta13}{\text{h10}\bullet:\Delta14,\text{F11}\land\text{F12}\vdash\Delta13,\top} \\ & -:\Delta9,\Delta14,\text{F11}\land\text{F12}\vdash\Delta8,\Delta13 \\ & \rightarrow \end{array} & \stackrel{\wedge}{\leftarrow} \text{Cut} \\ \end{array}
                                                                                                                                                                                                                             \frac{}{-:\Delta 14,\Delta 9,F11\wedge F12\vdash \Delta 13,\Delta 8} \quad \text{ax}
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\frac{\text{h11} : \Delta 9, \text{F15} \vdash \Delta 8}{\text{h11} \bullet : (\top, \Delta 9), \text{F15} \vdash \Delta 8} \quad \top_L \quad \frac{\text{h11} : \Delta 16, \text{F12}, \text{F13} \vdash \Delta 14, \text{F15}}{\text{h11} \bullet : \Delta 16, \text{F12} \land \text{F13} \vdash \Delta 14, \text{F15}} \quad \land_L \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                               -: \top, \Delta 9, \Delta 16, \mathtt{F} 12 \wedge \mathtt{F} 13 \vdash \Delta 8, \Delta 14
                                                                                                                                                                                                                                                                                                                                                           \begin{array}{c} \xrightarrow{} \\ \frac{1}{\text{h11}:\Delta 9,\text{F15}\vdash \Delta 8} \end{array} \text{ ax } \begin{array}{c} \xrightarrow{} \\ \frac{\text{h11}:\mathsf{T},\Delta 16,\text{F12},\text{F13}\vdash \Delta 14,\text{F15}}{\text{h11}\bullet:\mathsf{T},\Delta 16,\text{F12}\wedge\text{F13}\vdash \Delta 14,\text{F15}} \end{array} \begin{array}{c} \text{ax} \\ \wedge_L \\ \text{hCut} \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                              -: \top, \Delta 16, \Delta 9, F12 \wedge F13 \vdash \Delta 14, \Delta 8
Cases \top_L - \vee_L
                                                                                                                                                                                                                                                                                             \frac{\frac{\text{h10}:\Delta9\vdash\Delta8}{\text{h10}\bullet:\Delta9,\top\vdash\Delta8}}{-:\Delta9,\top\vdash\Delta8} \ \top_L \ \frac{\text{h10}:\Delta14,\text{F11}\vdash\top,\Delta13}{\text{h10}\bullet:\Delta14,\text{F11}\lor\text{F12}\vdash\Delta13,\top}}{\text{h10}\bullet:\Delta14,\text{F11}\lor\text{F12}\vdash\Delta13,\top} \ \lor_L \\ -:\Delta9,\Delta14,\text{F11}\lor\text{F12}\vdash\Delta8,\Delta13} \ \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \frac{}{-:\Delta 14,\Delta 9,\mathtt{F}11\vee\mathtt{F}12\vdash\Delta 13,\Delta 8}\quad\mathtt{ax}
                                                                                                                                                                                                                                                             \frac{ \frac{\text{h11} : \Delta 9, \text{F15} \vdash \Delta 8}{\text{h11} \bullet : (\top, \Delta 9), \text{F15} \vdash \Delta 8}}{-: \top, \Delta 9, \Delta 16, \text{F12} \lor \text{F13} \vdash \Delta 14, \text{F15}} \frac{\text{h11} : \Delta 16, \text{F13} \vdash \Delta 14, \text{F15}}{\text{h11} \bullet : \Delta 16, \text{F12} \lor \text{F13} \vdash \Delta 14, \text{F15}} \\ -: \top, \Delta 9, \Delta 16, \text{F12} \lor \text{F13} \vdash \Delta 8, \Delta 14} \\ \text{Cut} \\ & \\ \vee_L
                                                                                                                                                                                                                                                         \frac{1}{111:\Delta 9,F15\vdash \Delta 8} \text{ ax } \frac{1}{11:T,\Delta 16,F12\vdash \Delta 14,F15} \text{ ax } \frac{1}{11:T,\Delta 16,F13\vdash \Delta 14,F15} \text{ ax } \frac{1}{11:T,\Delta 16,F13\vdash \Delta 14,F15} \text{ ACUTED TO SET OF A STATE OF A STAT
                                                                                                                                                                                                                                                                                                                                                                                            -: \top, \Delta 16, \Delta 9, F12 \lor F13 \vdash \Delta 14, \Delta 8
Cases \top_L - \bot_L
                                                                                                                                                                                                                                                                                                                                                                                               \begin{array}{l} \underline{\mathbf{h}10:\Delta9\vdash\Delta8}\\ \underline{\mathbf{h}10\bullet:\Delta9,\top\vdash\Delta8} & \top_L & \underline{\mathbf{h}10\bullet:\bot,\Delta12\vdash\Delta11,\top}\\ -:\Delta9,\bot,\Delta12\vdash\Delta8,\Delta11 & \mathbf{Cut} \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \frac{\rightarrow}{-: \perp, \Delta 12, \Delta 9 \vdash \Delta 11, \Delta 8} \perp_{L}
                                                                                                                                                                                                                                                                                                                                                                     \begin{array}{c} \text{h11}: \Delta 9, \text{F13} \vdash \Delta 8 \\ \text{h11} \bullet : (\top, \Delta 9), \text{F13} \vdash \Delta 8 \end{array} \quad \top_L \quad \begin{array}{c} \bot_L \\ \text{h11} \bullet : \bot, \Delta 14 \vdash \Delta 12, \text{F13} \end{array} \quad \begin{array}{c} \bot_L \\ \text{Cut} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       -: \top, \Delta 9, \bot, \Delta 14 \vdash \Delta 8, \Delta 12
                                                                                                                                                                                                                                                                                                                                                                                                                                                                             Cases \top_L - I
                                                                                                                                                                                                                                                                                                                                                                      \begin{array}{c} \frac{\text{h}10:\Delta9\vdash\Delta8}{\text{h}10\bullet:\Delta9,\top\vdash\Delta8} \; \top_L \quad \\ \frac{\text{h}10\bullet:\Delta9,\top\vdash\Delta8}{-:\Delta9,\Delta13,\text{p}12\vdash\Delta8,\Delta11,\text{p}12} \; \stackrel{I}{\leftarrow} \text{Cut} \\ \\ \frac{-:\Delta9,\Delta13,\text{p}12\vdash\Delta8,\Delta11,\text{p}12}{-:\Delta9,\Delta13,\text{p}12\vdash\Delta8,\Delta11,\text{p}12} \\ \\ \frac{\text{h}10\bullet:\Delta9\vdash\Delta8}{-:\Delta9,\Delta13,\text{p}12\vdash\Delta8,\Delta11,\text{p}12} \\ \\ \frac{\text{h}10\bullet:\Delta9}{-:\Delta9,\Delta13,\text{p}12\vdash\Delta8,\Delta11,\text{p}12} \\ \\ \frac{\text{h}10\bullet:\Delta9}{-:\Delta9}(-:\Delta9,\Delta12,\text{p}12) \\ \\ \frac{\text{h}10\bullet:\Delta9}(-:\Delta9,\Delta12,\text{p}12) \\ \\ \frac{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \overline{-:\Delta 13,\Delta 9,\mathtt{p}12\vdash \Delta 11,\Delta 8,\mathtt{p}12}\quad iI\quad
                                                                                                                                                                                                                                                                                                                                                                 \mathtt{h}11:\Delta9,\mathtt{F}13\vdash\Delta8
                                                                                                                                                                                                                                                                                                                                             \begin{array}{c|c} \underline{\mathbf{h}11: \Delta 9, F13 \vdash \Delta 8} \\ \underline{\mathbf{h}11 \bullet : (\top, \Delta 9), F13 \vdash \Delta 8} & \top_L & \underline{\mathbf{h}11 \bullet : \Delta 15, p14 \vdash (\Delta 12, p14), F13} \\ \underline{- : \top, \Delta 9, \Delta 15, p14 \vdash \Delta 8, \Delta 12, p14} & \mathbf{Cut} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                            \mathtt{h}11:\Delta9,\mathtt{p}13\vdash\Delta8
                                                                                                                                                                                                                                                                                                                                                                   \overline{-:\top,\Delta 14,\Delta 9,\mathtt{p} 13\vdash \Delta 12,\Delta 8}\quad \text{ax}\quad
Cases \top_L - \top_L
                                                                                                                                                                                                                                                                                                                                                                                                \begin{array}{l} \frac{\text{h}10:\Delta9\vdash\Delta8}{\text{h}10\bullet:\Delta9,\top\vdash\Delta8} \ \top_L \ \frac{\text{h}10:\Delta12\vdash\top,\Delta11}{\text{h}10\bullet:\top,\Delta12\vdash\Delta11,\top} \ \top_L \\ -:\Delta9,\top,\Delta12\vdash\Delta8,\Delta11 \end{array} \text{ Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            \overline{-:\top,\Delta 12,\Delta 9 \vdash \Delta 11,\Delta 8} \quad \text{ax} \quad
                                                                                                                                                                                                                                                                                                                                                                   \frac{\text{h11}:\Delta 9,\text{F13}\vdash\Delta 8}{\text{h11}\bullet:(\top,\Delta 9),\text{F13}\vdash\Delta 8} \quad \top_L \quad \frac{\text{h11}:\Delta 14\vdash\Delta 12,\text{F13}}{\text{h11}\bullet:\top,\Delta 14\vdash\Delta 12,\text{F13}} \quad \top_L \quad \text{Cut}
                                                                                                                                                                                                                                                                                                                                                                                                                            -: \top, \Delta 9, \top, \Delta 14 \vdash \Delta 8, \Delta 12
                                                                                                                                                                                                                                                                                                                                                                                    \frac{\frac{1}{\text{h11}:\Delta9,\text{F13}\vdash\Delta8}\text{ ax}}{-:\text{T,T,}\Delta14,\Delta9}\text{ ax} \quad \frac{\frac{1}{\text{h11}:\text{T,}\Delta14\vdash\Delta12,\text{F13}}}{\text{h11}\bullet:\text{T,T,}\Delta14\vdash\Delta12,\text{F13}}\text{ ax}} \\ -:\text{T,T,}\Delta14,\Delta9\vdash\Delta12,\Delta8} \\ \text{ hCut}
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