Nuclear Structure Assignment 4

Charles Loelius

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1 Exercise 9

In this case we want to make a table of all possible partial waves with isospin T, projection T_z, spin S, orbital momentum L and total spin J. We limit the cases to J$\leq$ 2.

Isospin(T)	Projection (T_z)	Spin(S)	Orbital (L)	Total Spin(J)	Spectroscopic Notation
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$^2S_{\frac{1}{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$^2P_{\frac{1}{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$^2P_{\frac{3}{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{3}{2}$	$^2D_{\frac{3}{2}}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{5}{2}$	$^2D_{\frac{5}{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$^2S_{\frac{1}{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$^2P_{\frac{1}{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$^2P_{\frac{3}{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{3}{2}$	$^2D_{\frac{3}{2}}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{5}{2}$	$^2D_{\frac{5}{2}}$
1	1	0	0	0	$^1S_0$
1	1	0	1	1	$^1P_1$
1	1	0	2	2	$^1D_2$
1	0	0	0	0	$^1S_0$
1	0	0	1	1	$^1P_1$
1	0	0	2	2	$^1D_2$
1	0	1	0	0	$^3S_1$
1	0	1	1	0	$^3P_0$
1	0	1	1	1	$^3P_1$
1	0	1	1	2	$^3P_2$
1	0	1	2	1	$^3D_1$
1	0	1	2	2	$^3D_2$
1	0	1	2	3	$^3D_3$
1	-1	0	0	0	$^1S_0$
1	-1	0	1	1	$^1P_1$
1	-1	0	2	2	$^1D_2$
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$^2S_{\frac{1}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$^2P_{\frac{1}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$^2P_{\frac{3}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{3}{2}$	$^2D_{\frac{3}{2}}$
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{5}{2}$	$^2D_{\frac{5}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$^2S_{\frac{1}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$^2P_{\frac{1}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$^2P_{\frac{3}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{3}{2}$	$^2D_{\frac{3}{2}}$
$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{5}{2}$	$^2D_{\frac{5}{2}}$

Table 1: Table of Allowed Partial Waves

Here the table includes all possible l, j states less than j=2 for the case where there is one particle (Isospin=$\frac{1}{2}$). Now in the case of two particles, $T = 1$. If the projection is ± 1 we cannot have both spins the same, which limits the total spin to zero. However if they are different particles then $T_z = 0$ and any spin combination is possible. This explains the much larger number of possible states for $T_z = 0$.

However, in the case of three particles there is only one form the system can take, namely that two particles are of one isospin projection and the other is different. This leads to the two that are the same having the opposite spin(and so canceling out their spin effects). Thus the only degrees of freedom are in which isospin has two particles, and the angular momentum l.

2 Exercise 10: Spin Orbit Force

We note that here the spin orbit force becomes

$$U_{so} = AS \cdot L \tag{1}$$

We then note that

$$J^2 = (L+S)^2 = L^2 + S^2 + 2L \cdot S \tag{2}$$

Now we then note that if we take the expectation value we have

$$\langle J^2 \rangle = \langle L^2 \rangle + \langle S^2 \rangle + 2 \langle L \cdot S \rangle \tag{3}$$

Hence we have that

$$\langle L \cdot S \rangle = \frac{J(J+1) - L(L+1) - S(S+1)}{2} \tag{4}$$

And so we have that

$$U_{so} = \frac{A}{2} \left(J(J+1) - L(L+1) - S(S+1) \right) \tag{5}$$

2.1 S waves have no spin-orbit force

Now we then note that if $L = 0$ it follows that $J = S$ and so

$$U_{so} = \frac{A}{2} \left(J(J+1) - L(L+1) - S(S+1) \right) = \frac{A}{2} \left(J(J+1) - J(J+1) \right) = 0 \tag{6}$$