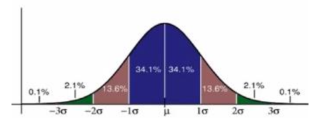
Session 3: Aggregation

3.1 – Aggregation

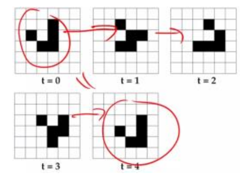
- **Simple Models to Understand Aggregation**

- **Phil Anderson – Nobel Winner:** *"More is Different"* paper – wetness is a property of a group of molecules, not a single molecule of H_2O. Brain as a collection of neurons but has cognition.
- **Emergent properties of a system,** as opposed to behavior of individual parts.
- **Aggregation**
  - **Actions**
  - **Single Rule**
  - **Family of Rules**
  - **Preferences** - how do you add up preferences
- **Module 3:** Understand how to aggregate
- **Module 4:** Understand the individual automata

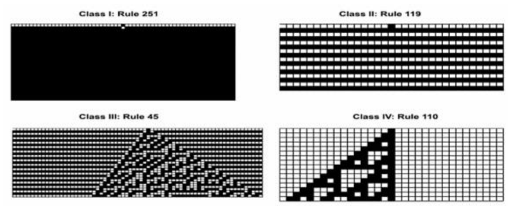
- **Using models:**
  - **Predict Points**



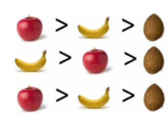
  - **Understand Data**
    Game of Life
    time -> change pattern



  - **Understand Class of Outcome**



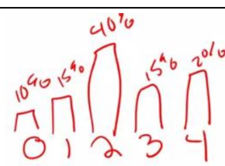
Stable/Equilibrium – Cyclical/Pattern – Random – Complex



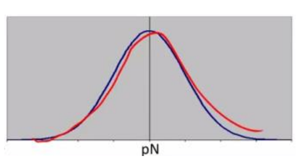
  - **Preferences** (thru logic)
    Preferences don't just add.
    Source of political challenges to satisfy electorate.

3.2 – Central Limit Theorem

- **Probability Distribution:** people going for a walk



- Central Limit Theorem: Add large number of independent events.
  Flip a Coin 4 Times and note the P(1H), P(2H), P(3H), and P(4H). Extending to N tosses results in the binomial distribution that approached the Gaussian (bell curve) distribution with average number of heads



  mean $\boldsymbol{\mu} = \boldsymbol{pN}$ where $\quad = \boldsymbol{P(X = H)}$

Note:
the probability of k heads in n coin tosses is

$$\boldsymbol{P(X = k) = \binom{n}{k} p^k (1 - \quad)^{n-k}}$$

with $\boldsymbol{\mu = \ pN}$ and $\boldsymbol{\sigma^2 = np(1 - \quad)}$

and $\sigma = \sqrt{\boldsymbol{np(1 - \quad)}}$

Quiz: At a college with 2000 students, each student wears a baseball hat with probability 0.2. Assume that their decisions to wear hats are independent. What's the expected number of students wearing baseball hats on any given day? (a) 400, (b) 200, (c) 300, (d) 1000

Analysis: from $\mu = \ pN$ where $p = 0.2$ and $N = 2000$.

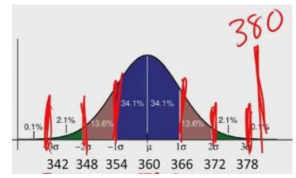
Ans: (a) 400

- **Standard Deviation (σ):**
  ±1 σ = 68%, ±2 σ = 95%, ±3 σ = 99.75%.

  Also recall raw score is **$z=(x-\mu)/\sigma$**

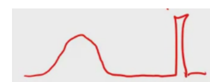
- **Binomial Distribution with p = 0.5 = ½**
  <u>mean</u> = $\mu = pN = N/2$
  &
  <u>Std Dev</u> = $\sigma = \sqrt{p(1-p)N} = \sqrt{\tfrac{1}{2}\tfrac{1}{2}N} = \tfrac{1}{2}\sqrt{N}$

- **Example: Airline Seats**
  - Boeing 747 with 380 seats and 90% show up rate, sell 400 tickets.
  Thus mean flyers
  $\mu = 0.9(400) = 360$ and
  $\sigma = \sqrt{(0.9)(0.1)400} = 6$ thus
  P(Flyers>400) occurs at z-score = (380-360)/6 = 3.33 thus won't overbook more than 99.75% of the time.



<u>**Quiz:**</u> If 10,000 people each wear blue jeans independently with probability 0.8, what's the standard deviation of the resulting distribution of people wearing blue jeans? (a) 400, (b) 20, (c) 40, (d) 20
<u>**Ans:**</u> (c) 40, from $\sigma = \sqrt{p(1-p)N} = \sqrt{0.8(0.2)10{,}000} = \sqrt{1600} = 40$

- <u>**Central Limit Theorem:**</u> The sum of random variables that are ***independent with finite variance*** is characterized by a normal distribution.
- <u>**Bimodal Distribution:**</u> not common in normal everyday events with people making independent decisions.

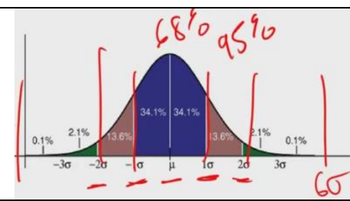


3.3 – Six Sigma

- **Six standard deviations**. z-score of ± 6 where $|Z|=|x-\mu|/\sigma$ and thus $x = \mu \pm \sigma|Z|$ = ± 6σ
- **Example:** Average banana sales of 500 lbs, with S.D.=10 lbs. Then if one buys 560 lbs of bananas then will have 6 sigma probability of not running out of bananas.
- **Example:** Required metal thickness [500-560] mm. Have a measured mean of 530 mm. What is the six-sigma standard deviation? Note that the specification is 6 =(560-530)/σ so must drive σ to 30/6= 5.
  Note: standard normal z-scores = [-6, +6].

Probability of being outside ±6 sigma = $1.973\ (10^{-9})$,

Wikipedia



<u>**Quiz:**</u> Suppose that a production process creates tires with an average diameter of 20 inches and a standard deviation of 0.1 inch. What is the six sigma range? (a) [20,26], (b) [19.4,20.6], (c) [14,26], (d) [20,20.6]

<u>**Analysis:**</u> from $6\sigma = 6(0.1) = 0.6\ in \Rightarrow [19.4, 20.6]$

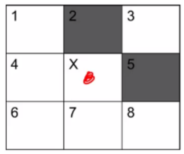
<u>**Ans:**</u> (b) [19.4, 20.6],

3.4 – Game of Life

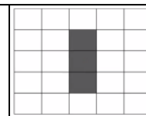
Toy model to explain Aggregation.
Why hard to understand micro behavior from macro behavior.

- **John Conroy -** Go Board for Game – works similar to Schelling's model, the 9-square neighbor model.

- **Black=alive, White=dead**
- **Rules:**
  - if OFF, turn ON if 3 neighbors are ON
  - if ON, stay ON if 2 or 3 neighbors are ON



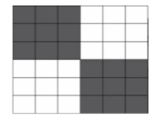
- **Blinker**



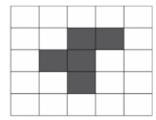
Net logo used to model

- **Beacon**

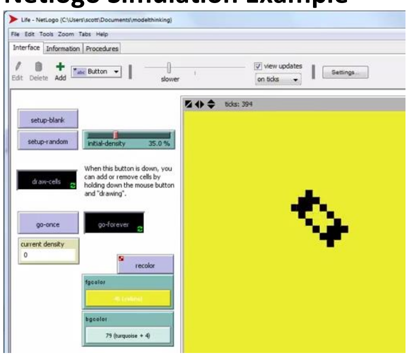
- **Figure 8**

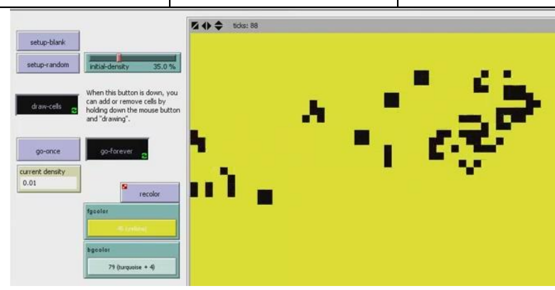


- **F-Pimento**



Netlogo Simulation Example

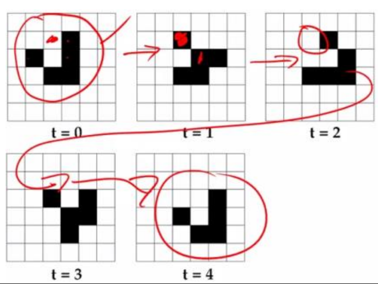




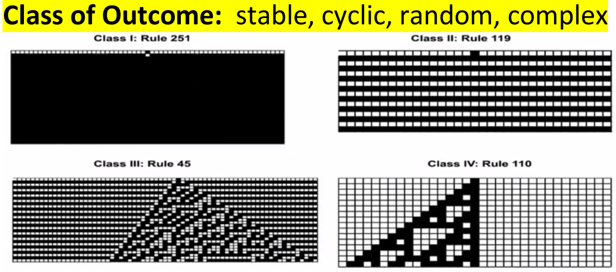
Quiz: What happens in the game of life if we begin with only two live cells adjacent to each other? (a) they blink, (b) the configuration is stable, (c) Both die, (d) Nothing happens

Ans: (c) Both die, from Rule 2, if ON – stay ON if 2 or 3 neighbors are ON, but only 1 neighbor is ON so both die and from Rule 1, if OFF, turn ON iff 3 neighbors are ON, but no neighbors are ON so stay OFF.

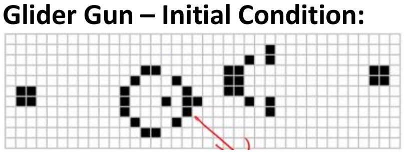
Glider: reconstructs shifted diagonally



Class of Outcome: stable, cyclic, random, complex



Glider Gun – Initial Condition:



Summary –

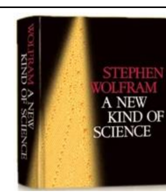
- **Self Organization:** Patterns appear without a designer.
- **Emergence:** Functionalities appear: gliders, glider guns, counters, computers
- **Logic Right:** Simple rules produce incredible phenomena

3.5 – Cellular Automata

Question:

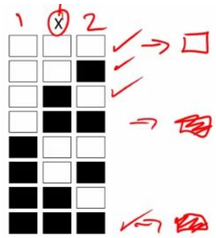
- **Kind of outcomes –** equilibrium, patterns, chaotic, complex
- **John Von Neuman** – 1 Dimension Cellular Automata

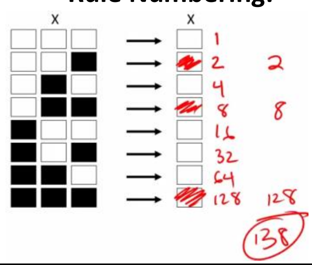
Stephen Wolfram – A New Kind of Science



- **In depth exploration of automata:** computational inductive way of looking at the world.
- **1-Dimensional-** each cell only has 2 neighbors.

- **1D Rules –** There are eight states and thus 2^8 = 256 different mapping templates.
- Set rule for next state per assignment
- **Rule Numbering:**

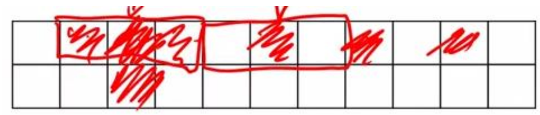




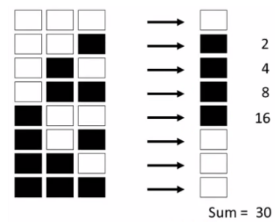
Classes of Outcome: fixed point (stable), alternating (cyclic), random, complex

- **<u>Each *Row* is a time step</u>:** Each cell checks his neighbors, applies the template to decide its next state:

T1
T2



- **Rule 30:**

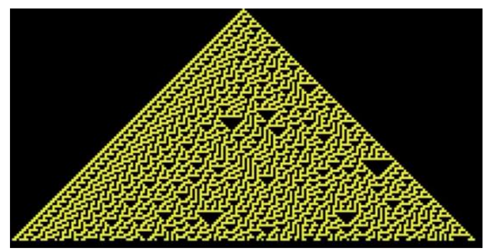


<u>Quiz:</u> Using Rule 30 above, what happens in the second period if the automata begins with just two side-by-side cells that are on? (a) four consecutive cells are ON, (b) the configuration is stable, (c) Two consecutive cells are ON, one OFF, and then one ON, (d) Two cell are ON separated by one cell OFF.

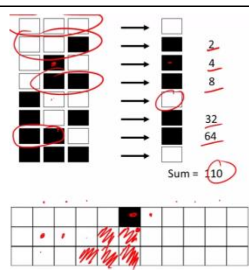
<u>Ans:</u> (c) Two consecutive cells are ON, one OFF, and then one ON, from the table below

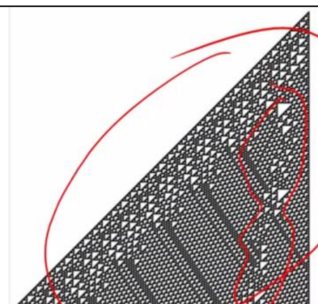
T1			■	■		
T2		■	■		■	

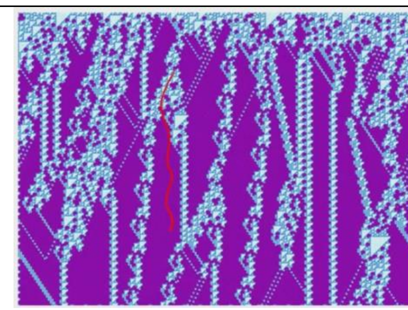
Rule 30 produces a random time sequence along the originating ON cell. This rule produces perfect randomness.



- **Rule 110:** Example of Complex (Class 4) behavior.





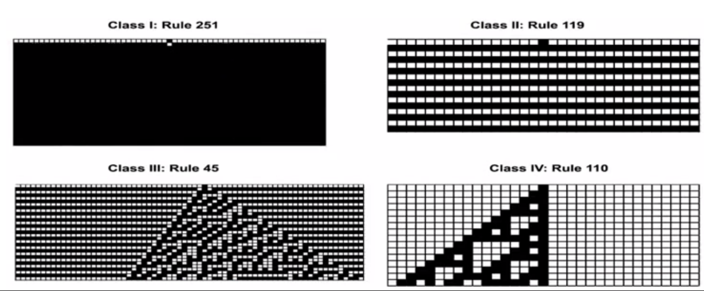


John Wheeler " It from Bit"

Simple rules underlie all.

"... every it--every particle, every field of force,even the spacetime continuum itself--derives its function, its meaning, its very existence entirely--even if in some contexts indirectly--from the apparatus-elicited answers to yes-or-no questions, binary choices, _bits_."

Examples:
Class 1 – Stable – Rule 251,
Class II – Cyclic – Rule 119,
Class III – Random – Rule 45,
Class IV – Complex – Rule 110



Langton's λ:

- Rule bits, fraction of bits ON, e.g., for Rule 73, λ=3/8; for Rule 30, λ=4/8, for Rule 0, λ=0/8,

Rule 73 :

Rule 0: Dies out, λ=0/8
Rule 1: [10000000]
Blinks, λ=1/8

Rule 30:

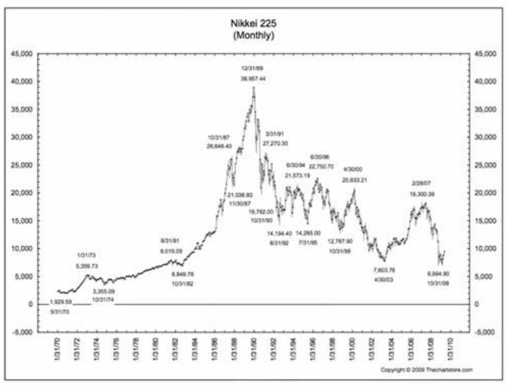
Rule 30: [01111000]
Chaotic, λ=4/8
Rule 110: [01110110]
Complex, λ=5/8

Langton's λ Table:

λ∈ [2, 6] interesting

λ	ALL Rules	Class III (chaos)	Class IV (complex)
0	1	0	0
1	8	0	0
2	28	2	0
3	56	4	1
4	70	20	4
5	56	4	1
6	38	2	0
7	8	0	0
8	1	0	0
	256	32	6

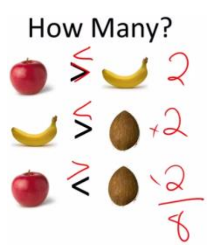
- **More interdependecy (stock markets) => more complexity**



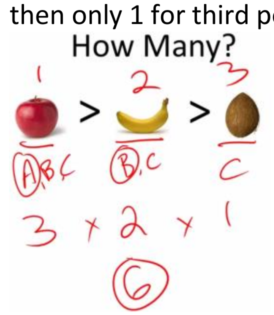
- **Lessons:**
  - **Simple rules combine to form anything!**
  - **"it from bit"**
  - **Complexity and randomness require some interdependency**
- **Observation: If we see complex behaviors in the world, it is likely the result of strong interdependencies – albeit they can derive from quite simple rules, e.g., swarming fish.**

3.6 – Preference Aggregation

- **Preferences & Preference Ordering**
- **Non-Transitive Ordering – irrational preferences:**
- **How many preference orderings.** Count ordering where position matters.



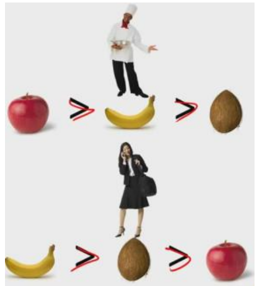
- **Transitive Ordering – rational preferences**
- **How to count given choices: Three item example:** 3 for first, then 2 for second, and then only 1 for third position.



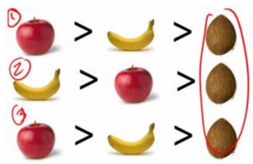
- More sophisticated models could include equality.

Collective Preferences Problem

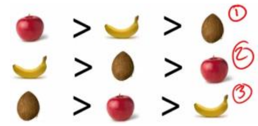
- Aggregation to for a 'group' preference,



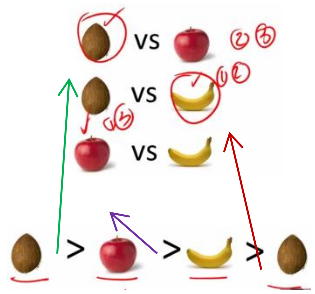
- Example with persons 1, 2, 3
  Easy by multivoting assuming equal weighting: coconuts last & apples first implies Apples>Bananas>Coconuts



Complex Example



- **Pairwise analysis: 2 & 3 prefer C to A.
  1 & 2 prefer B to C. But 1 & 3 prefer A to B**



- **Individual Rational, Group Result is irrational**

Condorcet Paradox:

Each ***Person is RATIONAL*** **but the** ***Collective is IRRATIONAL.*** The consequence is that in social policy, voting, and group decisions their is ***opportunity to have strategic preferences to bias the 'collective preferences'.***

Aggregation Summary:

Looked at aggregation in several forms –

a) numbers via the central limit theorem,
b) rules via game of life and 1 D automata (Wolfram) , and
c) orderings via individual to collective.