Session 15 – Randomness & Brownian Motion

15.1 Randomness & Random Walk Models

Will discuss randomness and random walks. Will cover distinguishing Skill and Luck as influences on performance. Then cover: (a) binary random walk model (+1or -1), (b) Normal walk (step follows a Gaussian distribution) – efficient market hypothesis, (c) Finite Memory random walk (n-past steps remembered).

15.2 Sources of Randomness

Probability Distribution: uniform, 'Bell' curve (Gaussian), exponential.
Manifestation: variable X plus some variation ε
Sources of randomness (ε): (a) noise - measurement noise, (b) error - process inaccuracies, (c) uncertainty - estimates versus reality, (d) complexity – (recall system types; stable, periodic, complex, random), (e) capriciousness – people are hard to predict.

Quiz: Capriciousness leads to randomness. This statement is based on the assumption of a ____ model of human action. (Recall lecture 5, Thinking Electrons). (a) rational, (b) behavioral, (c) rule-based, (d) markov.
Ans: (b) behavioral
Explanation: *People being capricious means that they are hard to predict. The agents in our Rational Actor model are not hard to predict because they maximize utility given a certain goal. Rule-based agents follow a set of simple rules. The correct answer, Behavioral, accepts that agents are people who make some unpredictable decisions.*

15.3 Skill and Luck

Can say: outcome = skill + luck
Michael Mauboussin – leggmason – Book, model: **Outcome = a (luck) + (1-a) (skill)** with a in (0, 1)
Use performance data to determine a.

Quiz: The Montreal Canadiens are a professional hockey team that had very much success during the late 1950s and early 60s. During a span of 5 years between 1955 and 1960, their percentages of games won were as follows: 68.6%, 65%, 65.7%, 65.7%, 70%. Manny wants to know whether these outcomes are due to skill or luck, so he thinks of a Skill Luck Model. What is the more likely value of the variable a in Manny's model? (a) 0.92, (b) 0.08
Ans: (b) 0.08
Explanation: *Remember that in our Skill Luck Model, Outcome = a*Luck + (1-a)*Skill. If a=.92, we would expect winning percentages that fluctuate wildly year-to-year, because luck would be so influential according to our equation. If a=.08, however, we would expect to see outcomes in a tight range of percentages that reflect the skill of the team.*

Why?:
(a) Assess outcomes, (b) anticipate reversion to the mean (if luck), (c) give good discerning feedback, (d) fair allocation of resources (skill or luck(fairness).

The Paradox of Skill (Maubossin): When you have the very best competing, the differences in their skill levels may be close. So the winner will be determined by <u>luck!</u>

Player	Skill	Luck	Outcome	
1	60	6	66	
2	50	5	55	2
3	40	9	49	3

wide variation in skill yields expected outcome rankings.

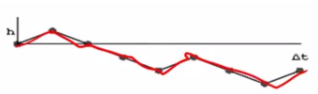
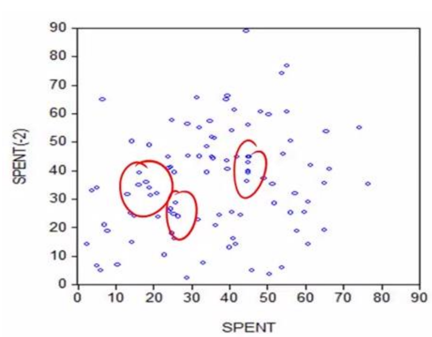
Player	Skill	Luck	Outcome
1	61	6	67
2	60	5	65
3	59	9	68

Olympic athletes have luck as a larger determinant of winner.

American Leage Batting Title: (1) Cabrera (DET) 0.344, (2) Gonzales (BOS) 0.338, (3) Young (TEX) 0.338, (4) Martinez (DET) 0.330. Skill got Cabrera to the top four, luck got him into first place in all likelihood. Another example: Swimming Olympics: Michael Phelps – milliseconds of finger touch difference.

Summary: (a) some cases are skill and luck that can be ferretted out with data, and (b) the paradox of skill in that when competitors are very close in skill, luck can play a major role in their winning.

Session 15.4 Random Walks

Binary Random Walk Model: model – coin flip each period with outcome P(H) = P(T) = ½ . Let winnings be X = 0 initially. Each period flip a fair coin. X = X +1 if Heads (H), or X-1 if Tails (T)  Odds: P(H) n times in a row =(½)^n. Caution if 100,000 people play coin flipping 16 times, there is the expectation that one players gets a 16 heads streak. ~100,000/2^{16} ~ 1.5	**Results:** (1) after 2N flips (N a real number) the expected value of $X = 0$. (2) for any number K, a random walk will pass both –K and +K an infinite number of times for an infinite step random walk! ➔ always bounded with $(X) = 0$. (3) for any number K, a random walk will have a streak of K heads (and K tails) an infinite number of times. So beware of steaks. There is always the opposite streak in your future.
Regression to the Mean: A group that did well for a short time, should be average in the long run.	**Hot Hand Study:** 1980-81 Boston Celtics – • miss first free throw: make second 75% of the time. • make first free throw: make second 75% of the time. ➔ both are just examples of random walk with P(2^{nd} made) = 0.75.
Jim Collins: *Good to Great*- humble leaders, right people on the bus, confront facts, focus (money, ability, fire), rinsing your cottage cheese (avoid fat), technology, superadditivity (combine good diverse ideas). But the Collins great companies still seem to regress to the mean. Explanation – could be good heuristics in 2001 were not in 2010. Or maybe, in my mind, these were diluted or swamped over time.	**Great companies 2001 -2010** Abbott Laboratories Stock up 0% Circuit City Bankrupt Fannie Mae Placed in conservatorship Gillette bought by P&G Kimberly-Clark Stock up 1% Kroger Stock up 0% Nucor stock up 4-fold Philip Morris Stock down 20% Pitney Bowes Stock down 20% Walgreens Stock up 0% Wells Fargo Stock up 0% S&P up about 0% during that time.
Clusters: randomness can create appearance of clusters. 	**Example:** Say given a 1000 x 1000 checkerboard with each square filled in with probability 0.1 ➔ We can expect 100,000 squares are filled in. How many 5 x 5 clusters might there be? Ans. 996 x 996 clusters of (how to calculate?) that size. And could easily find a majority of those cells filled to appear as a cluster. Or take a ten cell row. There are 990 x 1000 of these rows. Again these may appear as a cluster but are just random.

Quiz: In any study, the appearance of a cluster on a spatial graph does not necessarily signify a causal linkage. The big idea here is which of the following? (a) performance in any field over time depends more on skill than on luck, (b) spatial graphs are generally misleadings, (c) there is no way to determine what makes a company or team “successful”, (d) we can use a random walk model to determine whether or not the cluster signifies a causal linkage (highly susceptible to luck).
Ans: (d)
Explanation: *The big idea of Random Models here is that we can determine whether a cluster (or streak) is luck or skill. On the other hand, the model DOES NOT imply that performance is based on luck, that spatial graphs are misleading, or that there is no way to measure skill.*

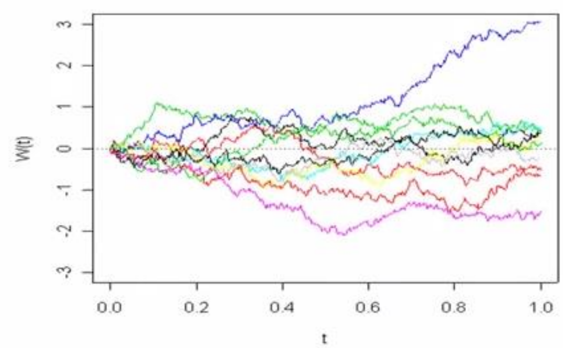
Summary: An outcome may be a series of random events and thus we should expect to see some big winners and some big losers. So we cannot expect past performance to be a good indicator of future performance. Thus key question is – is this a random walk process – or not? If so, expect regression to the mean.

Session 15.5 Random Walks and Wall Street (Normal RW)

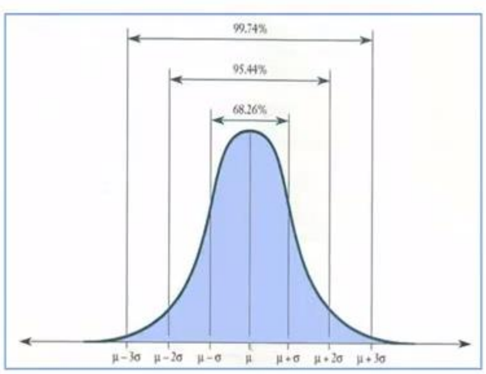
Normal Random Walks:

Gaussian and efficient market hypothesis. Let winnings be X = 0 initially. Each period draw a number from a normal distribution with a mean = 0 and standard deviation σ = 1 with X = X + sample number.

Normal Random Walks:

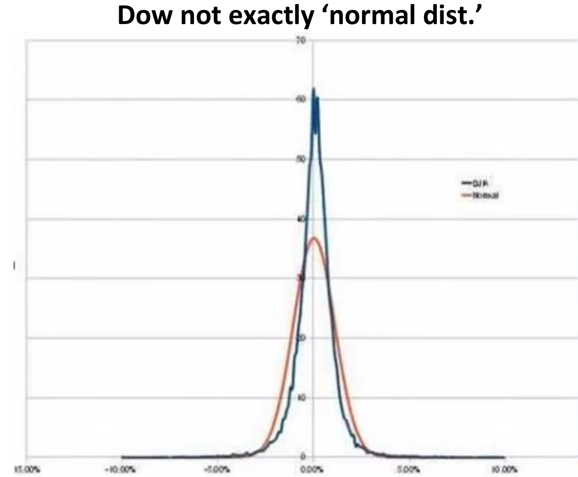


Normal Distribution



Note Dow distribution to the right has more days near mean (peak) and more days (tails) where there are really big events.

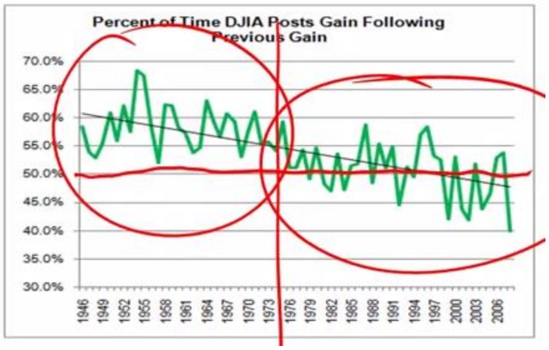
Dow not exactly ‘normal dist.’



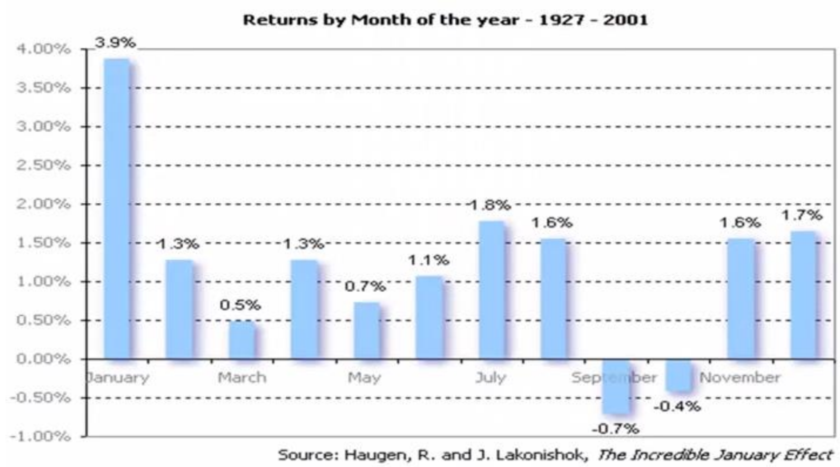
Quiz: Going back for a moment, we can say that the expected result of a Normal Random Walk model is as follows: small walks with be frequent, larger walks will not be rare, and huge walks will be ______? (a) rare, (b) not rare, (c) non-existent, (d) frequent.
Ans: (a) rare

Explanation: *The walks follow a normal distribution, meaning that small walks - values close to the mean of our bell curve - will be frequent. Larger walks - say, those between one and two standard deviations from the mean - will not be rare. Large jumps, however, such as those that fall outside of three standard deviations from the mean, will only occur 0.26% of the time. However, we still expect some huge jumps to occur (rarely).*

Book: *A Random Walk Down Wall Street* - Burton G. Malkiel – markets after 1970 do appear to have become more efficient – seeming to vary around the mean of 50%.



Efficient Market Hypothesis: prices reflect all relevant information. Therefore, it's impossible to beat the market. *"The efficient market hypothesis is associated with the idea of a 'random walk,' which is a term loosely used in the finance literature to characterize a price series where all subsequent price changes represent random departures from previous prices. The logic of the random walk idea is that if the flow of information is unimpeded and information is immediately reflected in stock prices, the tomorrow's price change will reflect only tomorrow's news and will be independent of the price changes today. But news is by definition unpredictable and, thus, resulting price changes must be unpredictable and random.* Burton Malkiel – Journal of Economic Perspectives 2003.

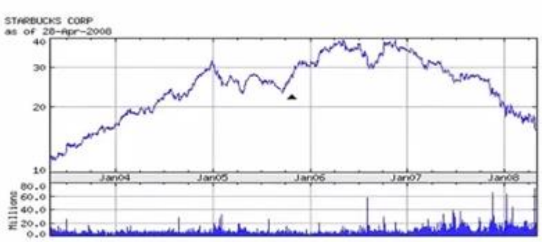


stock prices should be random –

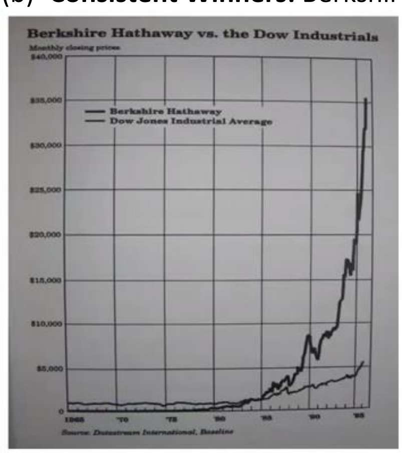
BUT note that prices are higher in January, BUT once trends are noticed in the market, others react to mitigate those trends and thus revert to the random walk.

Critiques of the Efficient Market Hypothesis:

(a) Starbucks stock – too much variation



(b) **Consistent Winners:** Berkshire Hathaway



Summary: EMH helps explain although there may be some deviations / challenges as noted in the critiques.

Session 15.6 Finite Memory Random Walks

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Finite Memory Random Walks: key idea: instead of the value V (X in binary random walk notation) being the sum of all previous steps, the value at time T is: $V_T = X_T + X_{T-1} + X_{T-2} + X_{T-3} + X_{T-4}$

e.g., $V_{10} = X_{10} + X_9 + V_8 + X_7 + X_6$ and

$V_{11} = X_{11} + X_{10} + X_9 + X_8 + X_7$, dropping X_6

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Apply to ball team players, new (add) and retired (drop):

28 Teams, $V_T = X_T + X_{T-1} + X_{T-2} + X_{T-3} + X_{T-4}$ with champion (highest V_T) and run model for 28 years. Can use model to predict a number of things. (a) number of distinct champions,

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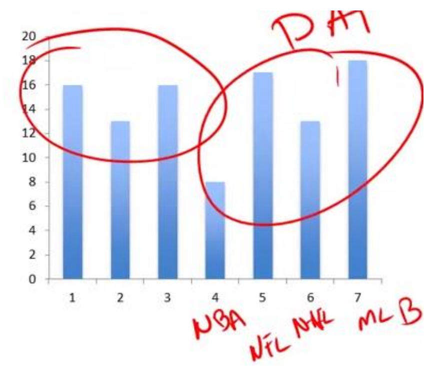
 # Distinct Champions:



model (1,2,3), Data (4-NBA, 5-NFL, 6- NHL, 7-MLB) – ‘very similar’

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Most Championships:
model (1,2,3), Data (4-NBA, 5-NFL, 6- NHL, 7-MLB) – ‘very similar’

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Winning Streaks:

model (1,2,3), Data (4-NBA, 5-NFL, 6- NHL, 7-MLB)
Not to be expected due to regression to the mean

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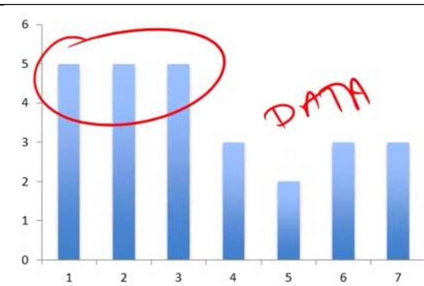

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Summary:

1. (1) Don’t confuse skill and luck.
2. (2) Paradox of Skill – winners among skilled peers are often the beneficiaries of luck.
3. (3) Do math on streaks/clusters – distinguish between statistically random phenomena and causal influences.
4. (4) Appreciate Regression to the Mean – no free lunch, past performance does not necessarily predict future performance – successful heuristics may be temporary and apply only to current, not future contexts.
5. (5) Wall Street’s ‘sort of’ random walk. Participants adapt but the delay in the adaptation process (imperfect information) distorts a pure random walk (my interpretation).
6. (6) Finite Memory Walk – Can capture the essence of many phenomena and help explain/predict a number of questions/answers.

Relation to Path Dependence:

1. (1) A <u>Random Walk is not Path Dependence</u>.
2. (2) <u>Markov Processes is not a random walk-</u> Markov depends upon current state.

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