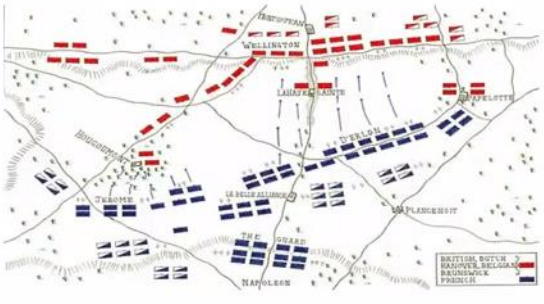
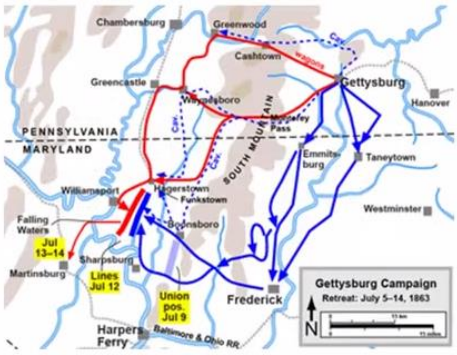
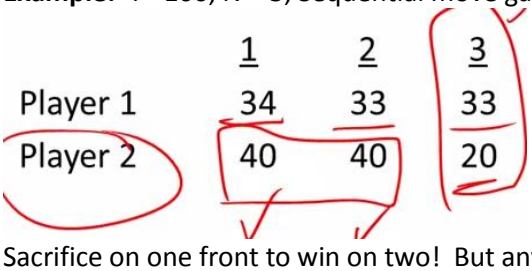
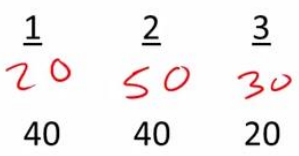
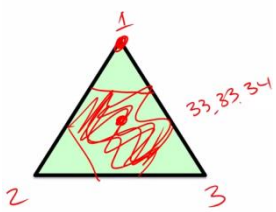
Session 16 – Colonel Blotto

16.1 Colonel Blotto Game

Two games to affect policy: (a) ***Prisoner's Dilemma*** and (b) ***Colonel Blotto***.	**Colonel Blotto:** Trying to find advantages through ***strategic mismatch***. Very fertile model applicable in a number of settings.
Waterloo: How to position troops? 	**Gettysburg:** How to position troops? 
Quiz: Colonel Blotto is a model about strategic mismatch and allocation of resources. (a)True, (b) False **Ans:** (a) True	
Will cover: (a) ***basic game***, (b) ***one side advantaged*** (more troops), (c) ***competition*** models comparison	

16.2 Blotto: No Best Strategy

How it works: **Strategic Mismatch** – Two Players with T troops and N fronts with (T >> N) ***Actions***: allocate troops across the fronts ***Payoffs***: Number of Fronts won	**This is a zero sum game: win or lose.** *Best to avoid zero sum games if you can*. Try for net growth games – but ***Colonel Blotto is zero sum***. **Note <u>any strategy can be beaten! (if you know your opponents strategy). So no obvious best strategy</u>**.
Quiz: Which of the two scenarios below is a zero sum game? (a) A trading of surpluses - as when one farmer trades milk to another in exchange for vegetables. (b) A chess match. **Ans:** (b) a chess match **Explanation:** *In a zero sum game, one party's gain necessitates another party's equivalent loss. This is the case in chess, in which one player wins and the other loses. Trading of surpluses is a positive-sum game, in which an action between two or more parties leads to a net gain.*	
Example: T= 100, N = 3, Sequential move game  Sacrifice on one front to win on two! But any strategy can be beaten in sequential move game.	**Another (better) strategy:**  But P2 could note P1's strategy and shift to 30 20 50 and so on. Note info on opponent strategy required. *AND not all 100 troops required to win!*
Quiz: Suppose that you playing a game of Blotto on three fronts. Your opponent allocates her 100 troops as follows: 40 on front A, 40 on front B, and 20 on front C. If you knew how she was allocated her troops, how many troops would you need to defeat her? **Ans:** 62 **Explanation:** *You need only 62 troops. You can put 41 on front A, 0 on front B, and 21 on front C.*	

What structure is the game? Equilibrium, Cycle, Complexity, Chaos? It's a ***mixture of equilibrium and random*** (chaos). **Colonel Blotto Solved:** – Vertices of Δ are fronts (1, 2, 3). Center is strategy position (33, 33, 34), vertices are strategy position (0, 0, 100) where 100 is at your vertex of choice. ***Optimum*** is a ***mixed strategy*** where troop allocations are randomly selected within the central 'hexagon.' Expected value is 0 for either player.	Vertex 1 selected = [1:100, 2:0,3:0] Middle selected = [1:33,2:33,3:34] Best is random selection in center hexagon Equilibrium in sense both players choose random strategy in center (hexagon). Winner will be random and both get expected payoff of 0, or 1:1 win-loss ratio. 
Is Winning at Blotto Skill (or luck)? **Troops:** number of troops is akin to ability, **Allocating Troops**: is strategic ability save for this case where you choose a Mixed strategy (random center position) being the best for skilled players indicates winning is a matter of luck. For differently skilled players, then skill comes more to the forefront for winning. In short, any strategy can be beaten, and confusing the other player is best.	
Quiz: The idea that the outcome of a Blotto game between two talented players depends on luck recalls which concept from the previous lecture on randomness? (a) The Paradox of Skill, (b) Efficient Market Hypothesis, (c) Predicting Others, (d) Zero Sum Game **Ans:** (a) The Paradox of Skill	

16.3 Applications of Colonel Blotto

Why Construct Models: They are extremely fertile and help in understanding competitive contexts. **Recall Strategic Mismatch:** 2 Players with T troops and N fronts with (T >> N) ***Actions***: allocate troops across the fronts ***Payoffs***: Number of Fronts won Does it apply elsewhere? **YES!** ➔ ➔ ➔	**Electoral College as Colonel Blotto Game:** where to put my money, and campaign time? Win most states. And CA is too democratic for the republicans (no troops worthwhile there!) 
Quiz: In a Blotto game, different fronts can have different values (or in other words, winning one front may have a greater payoff than winning another front). (a) True, (b) False **Ans:** (a) True **Explanation:** *See electoral vote map, each state has a different value you can win on that front.*	
Terrorism: **Where to protect?** *Too many vulnerabilities for government, only one target necessary for impact (but will that win the war – not likely – just sustain it).*	**Trials:** **Players:** *Lawyers,* **Fronts:** *Lines of Defense,* **Resources:** *Time spend preparing for each line of defense.*
Quiz: If a legal case is a game of blotto, who has the advantage: the prosecution who goes first, or the **defense** who goes second? (a) prosecution, (b) defense? (b) defense as 2nd mover has advantage **Ans:** (b) defense **Explanation:** *If Blotto is played sequentially, whoever goes second has a large advantage. So the defense has the advantage. The defense could still lose though if the other has more troops.* *As mentioned in the question, there is a lot more at play than the sequence of arguments. The point is that Blotto is a useful model because it gives us some hints at how to strategize in a competition such as this.*	

Hiring: Players: *Applicants*, **Fronts:** *relevant skills and abilities,* **Resources:** *level of each skill or number of people with each skill*	**Sports: Players:** *Tennis Players,* **Fronts:** *serve, return of serve, ground strokes, net play, etc.,* **Resources:** *ability in each area of the game.*
Summary: (a) many areas of application, **(b)** any position can be defeated, **(c)** don't need all of your troops to win. Next, what if you are 'troop' disadvantaged?	

16.4 Blotto: Troop Advantages.

First, let's review the basics: In Blotto, you won't need all of your troops to win, and any strategy can be defeated. Generally speaking, what's a good troop allocation if you don't know your opponents strategy? (a) Allocate an even number of troops on each front. (b) Be confusing - randomize your troop allocation

Ans: (b) Be Confusing, randomize

What does it mean to have a troop advantage? If only three fronts, weaker player at a significant disadvantage. Consider

1	2	3	
60	60	60	= 180 troops vs.
70	30	0	= 100 troops

Claim:
As the number of fronts increases, the country needs a larger relative resource advantage to guarantee victory. In short, as number of fronts increases you need more than expected to win! Why? Consider a perfect mismatch that favors the troop disadvantaged player. That player can win in some scenarios.

Three fronts: A has significant advantage

A = 150, B = 96

	1	2	3
Player A	**50**	**50**	**50**
Player B	48	48	0

Five fronts: B can prevail sometimes.

A = 150, B = 96

	1	2	3	4	5
Player A	30	30	30	**30**	**30**
Player B	**32**	**32**	**32**	0	0

Thus: the larger army needs relatively more troops to provide the same dominance in the game as the number of fronts increase.

Insight: ***Weaker player improves opportunities by opening more fronts (dimensions).*** Example: differentiation in products, terrorists attack in unexpected places.

Quiz: Country A has Troops 44 and Country B has 70 troops. How many fronts are needed so that B cannot guarantee at least a tie. (a) 5, (b) 7, (c) 9, (d) 3 ?

Analysis: B has to guarantee to win if fronts are odd number, or to guarantee a tie if an even number of fronts. All options are odd, so a win is (a) is 3 of 5, (b) is 4 of 7, (c) is 5 of 9, and (d) is 2 of 3. In (d) with 3 fronts B can do (23, 23, 24) and A cannot win more than 1. In (a) with 5 fronts, B can do (14, 14, 14, 14, 14) and A can counter with (15, 15, 14, 0, 0) which is two wins for both and a tie, so B can guarantee at least a tie with (a). The next largest option is (b) 7 so at that level, B is no longer guaranteed to at least a tie.

Ans. (b) 7 fronts.

Explanation: *If B allocates her troops evenly across the seven fronts, then player A could win by placing eleven troops on each of four fronts. If there are only 5 fronts, player B can put 14 troops on each of the fronts and player A can put 15 on two fronts and 14 on one, resulting in a tie. With three fronts, B can place 23 on two fronts and 24 on the third, so A cannot win on three fronts regardless of who goes first and how many turns each player has.*

What about multiple (>2) players? 5 fronts and 20 troops, play pairwise, [P1:P2], [P2:P3]

Player 1	4	6	2	6	2
Player 2	5	0	8	7	0
Player 3	6	5	1	5	3

Thus Blotto is akin to **Rock, Paper, Scissors –** note whether sequenced play or simultaneous play.

Analysis: P1 > P3 > P2 > P1 ➔ cycles
In this game with each player having 20 'troops' we have a ***cycle of dominances***!!!

As opposed to the skill-luck model (no cycles) in Blotto-like games, expect to find cycles.

Summary: (a) ***zero-sum game***, (b) ***two-player***: stronger player advantages, weaker player should open more fronts to better chances of winning, (c) ***multiplayer (>2)*** Blotto-like games are *likely to exhibit cycles* of dominance, (d) ***Blotto-like games*** offer some sense of the ***structure for winners,*** different from what we've seen in other models (?).

16.5 Blotto and Competition

Competition in general & applying our models:
Consider two types:
(a) Competition for market share (e.g., cars, brands and market share.
(b) win – loss records (sports teams)

Can models help us make sense of these two types?

Quiz: Would competing for market share be a zero-sum game, a positive-sum game, or a negative-sum game (in which actions provide a net loss)? (a) Zero-sum, (b) Positive-sum, (c) Negative-sum, (d) None of the above
Ans: (a) zero-sum
Explanation: *As one firm gains market share, other firms lose the equivalent amount of that market share. So gains and losses are equal and opposite.*

Many models: (a) random, (b) skill plus luck, (c) finite memory random, (d) Blotto model.

Random:
Should get-
(a) equal wins and
(b) no time dependency.

Skill Plus Luck: should get- ***(a) unequal wins, (b) semi-consistent rankings, and (c) no time dependency.*** Remember paradox of skill, better competitors rank higher but may depend upon luck among equally skilled peers. Reasonable to apply to industry market shares and sports

Finite Memory Random Walk:
should get –
(a) unequal wins,
(b) semi-consistent rankings,
(c) time dependent (regression to the mean), and
(d) more movement from top to bottom.

Compare Skill + Luck, $(\alpha L + (1-\alpha)S)$ to
$X_t + X_{t-1} + X_{t-2} + X_{t-3}$ ➔ regression to the mean

Blotto – Equal Troops: (a) Outcomes the same as Random, (b) lots of maneuvering (mixed strategies)
Blotto – Unequal Troops: (a) outcomes the same as Skill + Luck, (b) lots of maneuvering (weaker player should increase fronts).
Blotto – Unequal Troops – Limited Movement: (a) outcomes same as Finite Memory Random Walk, (b) lots of maneuvering, (c) P1 > P3 > P2 > P1 cycles become evident.

Quiz: Which two classes of outcomes are produced by Blotto competition? (a) Equilibrium, (b) Cycles, (c) Complexity, (d) Randomness **Ans:** (a) and (d)
Explanation: *Col. Blotto leads to a mixed strategy equilibrium within which players select random strategies.*

Blotto Game or Skill_Luck Game: (a) ***Dimensionality*** – if players are making high dimensional strategic decisions, then it's more Blotto like, (b) ***Zero Sum*** – if actions are only good relative to other actions (invade from east vs. invade from north) then it may be more Blotto like. Notes: ***Blotto is zero-sum and mismatching resources (high dimensional sports like football & Tennis) whereas the 100 meter dash, marathons, and javelin throwing (low dimensional?) are more like Skill-Luck.***

Quiz: Which would be the best model for capturing competition in the shot put -- where people want to toss a 16 pound ball as far as possible? (a) Blotto, (b) Skill Luck, (c) Random Walk, (d) Finite Memory Random Walk?
Ans. (b) Skill Luck (low dimensionality). **Explanation:** *The winner will be the person who puts the shot the furthest. That will depend on skill and luck -- wind, a good release point, etc. It's not really like Blotto. Nor is it combining variables like the finite memory random walk or the random walk.*

Presidential Election: (a) ***Random:*** economic shocks shape incumbent's image, (b) ***Luck_Skill:*** candidates vary in their skill, (c) ***Random Walk:*** a sequence of events drive outcome, (d) ***Blotto:*** winning requires allocating resources across 'fronts'. These various models are lens on the influences that drive election outcomes. There is no single model that addresses all of the factors that can impact the outcomes of an election. In a Presidential Election, all apply.