Session 8: Economic Growth

8.1 – Introduction to Growth

Economic Growth: Why some countries are rich and some are poor.

Models:

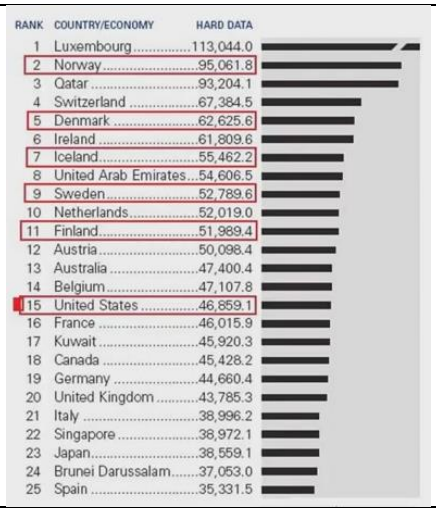
(a) Exponential Growth (compounding growth)

(b) Economic Growth (limits – without innovation growth stops)

(c) Solow's Growth Model (innovation leverage)

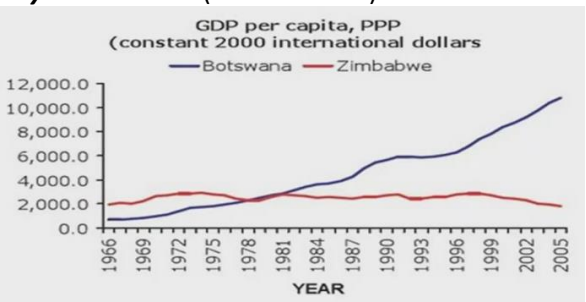
(d) Extensions

Gross Domestic Product (GDP):
Total market value of all goods and services

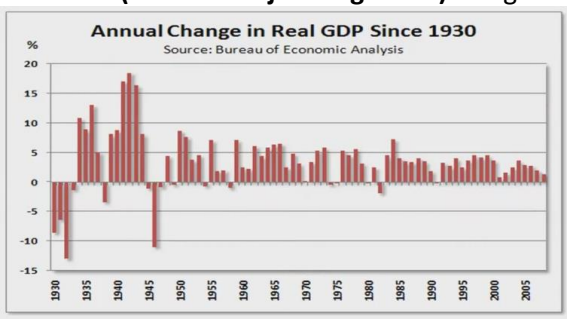


RANK	COUNTRY/ECONOMY	HARD DATA
1	Luxembourg	113,044.0
2	Norway	95,061.8
3	Qatar	93,204.1
4	Switzerland	67,384.5
5	Denmark	62,625.6
6	Ireland	61,809.6
7	Iceland	55,462.2
8	United Arab Emirates	54,606.5
9	Sweden	52,789.6
10	Netherlands	52,019.0
11	Finland	51,989.4
12	Austria	50,098.4
13	Australia	47,400.4
14	Belgium	47,107.8
15	United States	46,859.1
16	France	46,015.9
17	Kuwait	45,920.3
18	Canada	45,428.2
19	Germany	44,660.4
20	United Kingdom	43,785.3
21	Italy	38,996.2
22	Singapore	38,972.1
23	Japan	38,559.1
24	Brunei Darussalam	37,053.0
25	Spain	35,331.5

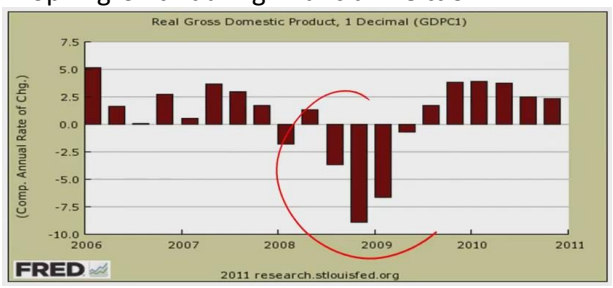
Why Nations Fail (Robinson & ?)



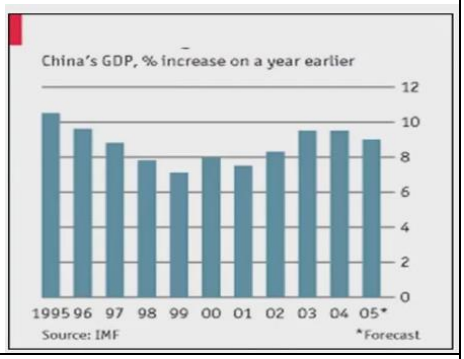
Real GDP (inflation adjusted growth) – avg ~3-4%



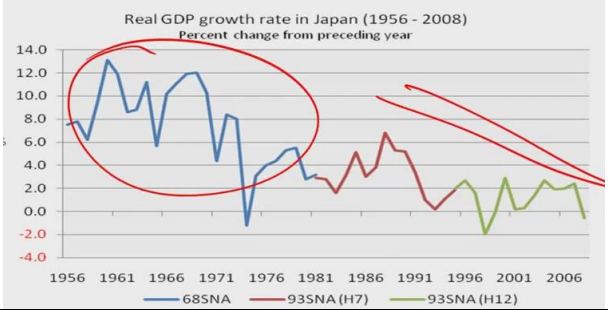
2008-2010 Recession:
Drop in growth during financial meltdown



China's Growth Rate:
very high relative to developed countries

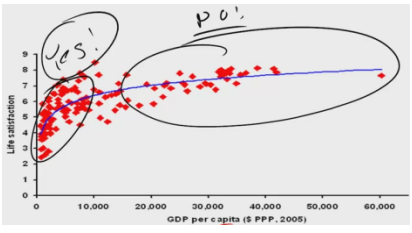


Japan's Growth:



Exponential Growth: Put money in the bank and it just grows and grows; $FV = PV\,(1+i)^n$

Economic Growth: Start fast and then begins to tail off; Life Satisfaction vs. GDP. Note above $30K, NO. Below $10K, YES.



8.2 – Exponential Growth

Key Topics:
- **Compounding**
- **Exponential Growth**
- **Rule of 72**

Compounding:
PV dollars @ r% interest for n years
PV = present value, FV = future value
$FV = PV\,(1+r)^n$

Example: Growth rate impact on national wealth

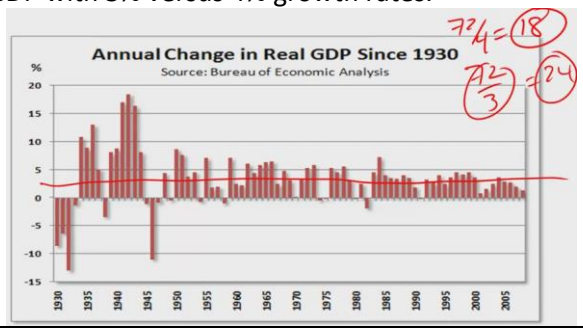
Year	2%	6%
0	1000	1000
1	1020	1060
10	1219	1791
35	2000	7686
100	7245	339,302

Rule of 72:
Divide the growth rate (%/year) into 72 for an estimate of the number of years for GDP to DOUBLE.

Interest	Years
2 %	~ 36
6 %	~ 12
8 %	~ 9
12 %	~ 6

Quiz: Let's use our growth model to compare two countries: Country 1 has a growth rate of 8% while Country 2 has a growth rate of 2%. How many more years does it take Country 2 to double its GDP, relative to Country 1? (Hint: use the rule of 72). (a) 9 years, (b) 36 years, (c) 27 years, (d) 50 years
Anaylsis: $y_1 = {}^{72}/_{8} = 9$ and : $y_2 = {}^{72}/_{2} = 36$; $y_2 - y_1 = 36 - 9 = 27$
Ans: (c) 27
Explanation: *First, find the number of years it takes each country to double its GDP: Country 1: 72/8 = 9 Country 2: 72/2 = 36; now find the difference; 36 – 9 = 27.*
It takes 27 more years for Country 2 to double its GDP than it takes Country 1.

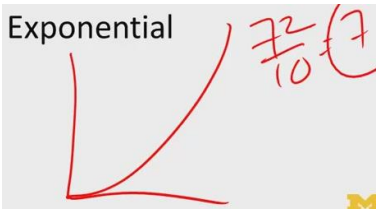
US Basic Growth Model: Impact of time to double GDP with 3% versus 4% growth rates.



Continuous Compounding:
Over *t* years
Annually: $FV = PV\,(1+r)^t$
Monthly: $FV = PV\,(1+\frac{r}{12})^{12t}$
Daily: $FV = PV\,(1+\frac{r}{365})^{365t}$
Continuously: $FV = PV\,e^{rt}$
Recall $\lim_{k\to\infty}\left(1-\frac{r}{k}\right)^{kt} = e^{rt}$; exponential

Linear:

Exponential:



Summary: Now understand that compounding growth is important to grow GDP exponentially and that a few differences in interest rates make a large difference over time.

8.3 – Basic Growth Model

Basic Model:

- **Workers**
- **Coconuts** (can eat or use to build machines)
- **Picking Machines** (help pick coconuts faster)
- **Machines wear out** (degrade over time)

Parameters:

$L_t =$ Workers at time t
$M_t =$ Machines at time t
$O_t =$ Output of Coconuts at time t
$E_t =$ Number Eaten Consumed at time t
$I_t =$ Number invested in Machines at time t
$s =$ Savings rate
$d =$ Depreciation rate

Assumptions:

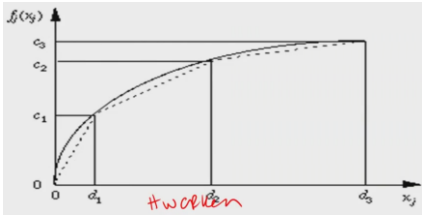
(1) Output is increasing and concave in labor and machines. $O_t = \sqrt{L_t}\sqrt{M_t}$

(2) Output is consumed or invested
$O_t = E_t + I_t$ and $I_t = sO_t$

(3) Machines can be built but depreciate
$M_{t+1} = M_t + I_t - dM_t$

Diminishing Returns to Scale:

Concave means that the first machine is worth more than the 2nd, the 2nd more than the 3rd , and the 3rd more than the 4th.



Simplification:

- $L_t = 100$, workers
- $O_t = 10\sqrt{M_t}$

Example:

$d = 0.25, s = 0.3$, depreciation & savings rates
Year 1: 4 machines
$Output = 10\sqrt{4} = 20$
$Investment = sO_t = 20(0.3) = 6$
$Eaten = E_t = O_t - I_t = 20 - 6 = 14$
$Depreciation = dM_t = 0.25(4) = 1$
Year 2:
$Machines = M_{t+1} = M_t + I_t - dM_t$=4+6-1=9

Example (Cont.):

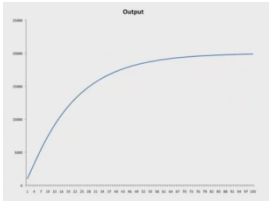
Year 2: 9 machines
$Output = 10\sqrt{9} = 30$
$Investment = sO_t = 30(0.3) = 9$
$Eaten = E_t = O_t - I_t = 30 - 9 = 21$
$Depreciation = dM_t = 0.25(9) = 2.25 \sim 2$
Year 3:
$Machines = M_{t+1} = M_t + I_t - dM_t$=9+9-2=16
$Output = 10\sqrt{16} = 40$
$Investment = sO_t = 40(0.3) = 12$
$Eaten = E_t = O_t - I_t = 40 - 12 = 28$
$Depreciation = dM_t = 0.25(16) = 4$

What happens in time?:

Year N: 400 machines
$Output = 10\sqrt{400} = 200$
$Investment = sO_t = 200(0.3) = 60$
$Eaten = E_t = O_t - I_t = 30 - 9 = 21$
$Depreciation = dM_t = 0.25(400) = 100$
Year N+1
$Machines = M_t + I_t - dM_t$=200+60-100=360
Note: Number of Machines in N+1 is less than in Year N

Growth Maximum at Equilibrium:

Occurs when
Investment = Depreciation



Example:

$Output = 10\sqrt{M}$
$Investment = 0.3(10\sqrt{M})$
$Depreciation = 0.25M$
$0.3(10\sqrt{M}) = 0.25M$ implies $12 = \sqrt{M}$ and $M = 144$

Check Equilibrium: Year K: 144 machines
$Output = 10\sqrt{144} = 120$
$Investment = sO_t = 120(0.3) = 36$
$Depreciation = dM_t = 0.25(144) = 36$
Year K+1
$Machines = M_t + I_t - dM_t$=144+36-36=144

Quiz: The economies of two small countries operate with the same output equation: Output= 5√M. Both countries have the same depreciation rate: d=0.10. However, Country 1 is made up of very patient people who are able to sustain a high savings rate: s=0.90. Country 2, on the other hand, is made up of very impatient people with a low savings rate: s=0.10. What is the difference in long run equilibrium output between Country 1 and Country 2? (a) Country 1 has 2000 more units of output. (b) Country 1 has 200 more units of output. (c) Country 1 has 225 more units of output. (d) Country 1 has 1025 more units of output.

Analysis: Equilibrium and *investment = depreciation*. $d = 0.1,\ s_1 = 0.9,\ s_2 = 0.1$. Compare $sO_t = dM_t$ for both. $0.9(5\sqrt{M_1}) = 0.1M_1$ ➔ $M_1 = 2025$ and $O_1 = 5\sqrt{2025} = 225$. For the second country, $0.1(5\sqrt{M_2}) = 0.1M_2$ ➔ $M_2 = 25$ and $O_2 = 5\sqrt{25} = 25$

Ans: (a) 2000, Question really wants difference in machines produced, not overall output. See Explanation.

Explanation: *Equilibrium occurs when Investment = Depreciation. Formally, this means:*
*Savings Rate * Output = Depreciation Rate * M.*
To find the equilibria, we solve for M: Country 1:(5√M)(0.90)=(0.10)(M) With algebra you find that M=2025
Country 2: (5√M)(0.10)=(0.10)(M), In this case, M=25
Now find the difference between these two results: 2025−25=2000
Therefore, Country 1 has 2000 more units of output at long run equilibrium than does Country 2.

Growth Model Irony: There is a limit to growth!	**Extending Growth:**
Why? *Depreciation is Linear*. *Output is Concave!*	*Innovation* required to extend growth.

Summary: Simple model shows growth is limited. Sustained growth requires new innovation.

8.4 – Solow Growth Model

One variable Enables Growth:
Bob Solow (MIT Economist) – need multiple models to understand the economy.

Parameters:
$L_t =$ Workers at time t
$K_t =$ Capital at time t
$O_t =$ Output at time t
$A_t =$ Technology at time t

Equation:

$$O_t = A_t K_t^{\beta} L_t^{1-\beta}$$

Interpretation:
$\beta = 0.5$ then $O_t = A_t K_t^{0.5} L_t^{0.5}$
β is an indicator of the importance of capital in the economy being larger for more capital intensive industry.

Recall Example: Note: M(machines)=K(capital)
$Output = 10\sqrt{M}$
$Investment = 0.3(10\sqrt{M})$
$Depreciation = 0.25M$
$0.3\left(10\sqrt{M}\right) = 0.25M$ ➔ $12 = \sqrt{M}$ and $M = 144$
$Output = 10\sqrt{M} = 120$

Add Innovation Parameter: $A_t = 2$
$Output = 2(10\sqrt{M})$
$Investment = 0.3(20\sqrt{M})$
$Depreciation = 0.25M$
$0.3\left(20\sqrt{M}\right) = 0.25M$ ➔ $24 = \sqrt{M}$ and $M = 576$
$Output = 20\sqrt{M} = 20(24) = 480$

Observation:
Productivity doubled (2x) and Equilibrium point quadrupled (4x)
.
Why?
(1) making more productive hence more stuff,
(2) getting more stuff encourages investing in more machines.

Innovation Multiplier:
- Labor and capital is more productive
- Incentive is then to invest in more capital

Additive or Multiplicative Effect?:
- Output quadrupled from doubling $A_t K_t^{0.5} L_t^{0.5}$
- Is it 2+2 or 2^2?

Innovation Parameter: $A_t = 3$
$Output = 3(10\sqrt{M})$
$Investment = 0.3\left(30\sqrt{M}\right) = 9\sqrt{M}$
$Depreciation = 0.25M$
$9\sqrt{M} = 0.25M$ ➔ $36 = \sqrt{M}$ and $M = 1296$
$Output = 30\sqrt{M} = 30(36) = 1080$
Output tripled! ➔ $\boldsymbol{Output \propto A_t^2}$

Quiz: A country's economy originally operates according to the following output equation: Output= 5√M. This country has a depreciation rate of d=0.10, and savings rate of s=0.40. However, an innovation allows this country to double its output, so that its new output equation is: Output= 10√M. Following this innovation, how much does equilibrium output increase? (The answer will be in terms of units) (a) 1200 units, (b) 400 units, (c) 2 units, (d) 200 units

Analysis: Equilibrium and *investment = depreciation*. $d = 0.1,\ s = 0.4$. Compare $sO_t = dM_t$ for both.

$0.4(5\sqrt{M_1}) = 0.1M_1$ ➔ $M_1 = 400$ and $O_1 = 5\sqrt{400} = 100$. For the second country,

$0.4(10\sqrt{M_2}) = 0.1M_2$ ➔ $M_2 = 1600$ and $O_2 = 5\sqrt{1600} = 200$

Ans: (a) 1200 units. Again not that this is the number of machines not GDP.

Explanation: *First find the equilibrium without innovation: (5√M)(0.4)=(0.1)(M) ; M=400.*
After the innovation, the equilibrium equation is: (10√M)(0.4)=(0.1)(M) ; Now M=1600.Tthe difference between the equilibrium outputs before and after the innovation: 1600−400=1200
So, following the innovation, the equilibrium output increases by 1200 units.

Solow Model: Growth can continue with changes to technology (increases equilibrium level) **Endogenous Growth:** Labor can go to output or technology/idea creation	**Growth ceases without innovation**, so nearly everyone should be pro innovation.

Quiz: Source of each quote (Obama or Reagan)?

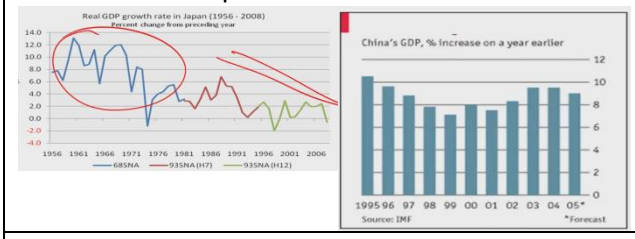
(a) "In America, innovation doesn't just change our lives. It is how we make our living."

(b) "The remarkable thing is that although basic research does not begin with a particular practical goal, when you look at the results over the years, it ends up being one of the most practical things government does. . . "

Ans: (a) by Obama, (b) by Reagan

8.5 – Will China Continue to Grow?

Recall Japan & China Growth Rates:
Reasonable to expect China's Growth Rate to slow.



Analysis: Year K: 3600 machines

$d = 0.1, s = 0.2$; depreciation & savings rates

$Labor = 10{,}000$

$Output = 100\sqrt{3600} = 6000$ (*per capita income*)

$Investment = I_t = sO_t = 6000(0.2) = 1200$

$Depreciation = dM_t = 0.1(3600) = 360$

Year K+1

$Machines = M_t + I_t - dM_t$=4440

Analysis (cont.): Year K+1: 4400 machines

$Output = 100\sqrt{4400} \cong 6700$ (*per capita income*)

$Investment = I_t = sO_t \cong 6700(0.2) = 1340$

$Depreciation = dM_t = 0.1(4440) \cong 440$

Year K+2

$Machines = M_t + I_t - dM_t$=4440+1340-440=5340

$Growth = \frac{O_2}{O_1} \cong \frac{6700}{6000} = 1.117 \rightarrow 11.7\%$

Analysis (cont.): Year K+2: 5340 machines

$Output = 100\sqrt{5340} \cong 7300$ (*per capita income*)

$Investment = I_t = sO_t \cong 7300(0.2) = 1460$

$Depreciation = dM_t = 0.1(5340) \cong 530$

Year K+3

$Machines = M_t + I_t - dM_t$=5340+1460-530=6270

$Growth = \frac{O_2}{O_1} \cong \frac{7300}{6700} = 1.0896 \rightarrow 8.96\%$

Analysis (cont.): Year K: 10,000 machines

$Output = 100\sqrt{10{,}000} \cong 10{,}000$

$Investment = I_t = sO_t \cong 10{,}000(0.2) = 2000$

$Depreciation = dM_t = 0.1(10{,}000) \cong 1000$

$Machines = M_t + (I_t - dM_t)$=11,000

$$Growth = \frac{O_{K+1}}{O_K} \cong \frac{100\sqrt{11{,}000}}{10{,}000} = 1.0488 \rightarrow 4.88\%$$

Analysis (cont.): Year P: 22,500 machines

$Output = 100\sqrt{22500} \cong 15{,}000$

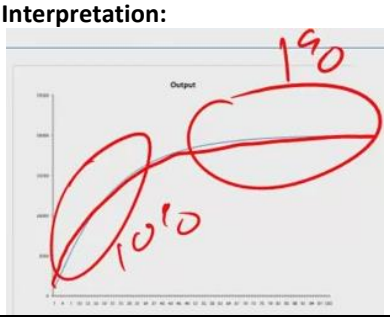
$Investment = I_t = sO_t \cong 15{,}000(0.2) = 3000$

$Depreciation = dM_t = 0.1(22{,}500) \cong 2250$

$Machines = M_t + (I_t - dM_t)$=23,250

$$Growth = \frac{O_{P+1}}{O_P} \cong \frac{100\sqrt{23{,}2500}}{15{,}000} = 1.0165 \rightarrow 1.65\%$$

Graphical Interpretation:



Equation:

$$O_t = A_t K_t^{\beta} L_t^{1-\beta}$$

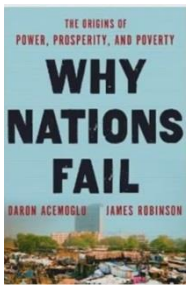
- For China to continue to grow, they will have to leverage innovation (A_t) but there has never been innovation that sustains 8-9% growth rate.
- China is most likely to follow slowing growth rates such as Japan.

Summary: Two types of growth drivers, capital investment and innovation. Once capital has reached a certain level and growth rate has slowed, then innovation must be introduced in order to mitigate natural decline in growth rates.

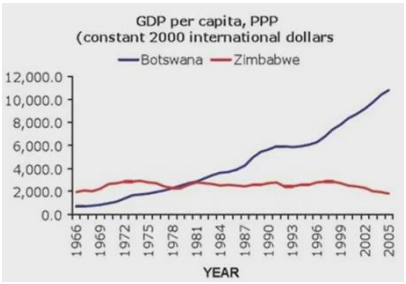
8.6 – Why Do Some Countries Not Grow?

What has not been considered?:

- Equality, Culture, etc.
- Why Nations Fail insights.



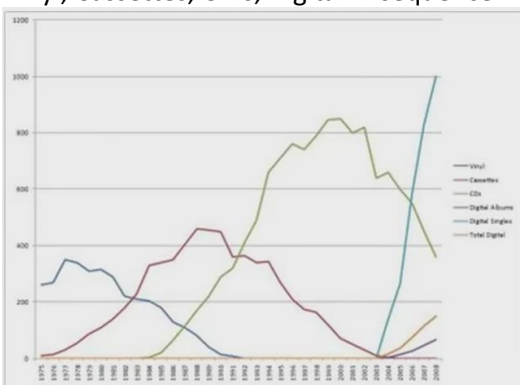
Recall Botswana & Zimbabwe:



Lessons:

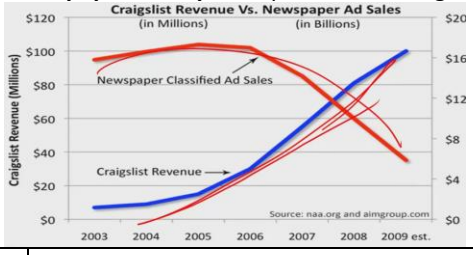
- Growth requires a strong central government to protect capital and investment
- BUT that government cannot be controlled by a select few. Extraction (by protected government supporters) limits growth by lowering investment and capital.
- Increases in innovation means less labor is required.
- Growth requires creative destruction

Creative Destruction in the Music Industry:
Vinyl, Cassettes, CD's, Digital in sequence

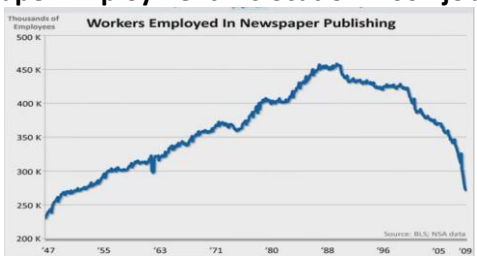


Quiz: How we watch movies has changed several times. From VHS to DVDs to Blu-rays, each new technology improves sound and picture quality. Today, VHS tapes are no longer produced. This is likely a result of Creative Destruction. (a) True, (b) False
Ans: (a) True

Newspaper Example: Paper ads vs. Craigslist



Newspaper Employment Dislocation: 200K jobs lost!



But they're not all going to Online Jobs!:

But Internet does not absorb many of them. Note only 23 employees at Craigslist!

pages	employees	company
1	10000	Yahoo!
2	90000	TimeWarner
3	10000	Google
4	70000	Microsoft
5	50000	News Corp
6	12000	eBay
7	23	craigslist
8	25000	BBC
9	130000	Disney
10	12000	Amazon

Fertility of the Model:	**Summary:**
• **Personal Application –** at some point personal innovation and reinvention is necessary to not be displaced.	• Growth rates are supported by innovation in the long run. • Need strong central government that doesn't extract too much which starves capital and innovation • Creative destruction is essential to innovation that is necessary to sustain growth rates.