# Session 18 – Mechanism Design

# C18.1 Mechanism Design

**One Course Objective:** How to design better institutions. Here we formalize an institution into actions people can take and the payoffs from those actions. In short: *a) rules, b) actions, c) payoffs.* One challenge (hidden motivations, information, and actions), you only observe the outcomes. You want the payoffs to incentivize the actions you want.

**Quiz:** What three things do we define when designing a mechanism? (a) The rules of the game (b) The logic behind the rules. (c) The actions people can take (d) Payoffs associated with the actions **Ans:** (a) rules, (c) actions, and (d) payoffs.

How to **design incentive structures** to overcome: **(a) Hidden Actions**,

**(b) Hidden Information** (how to ferret out this information, what types people are, e.g., high or low ability, risky or not risky driver)

Imagine how difficult physics would be if electrons could think. Murray Gell-Mann Recall people models: a) rational choice, b) psychological, and c) rule-based

**First application is to Auctions:** as a metaphor for mechanisms. How to get people to reveal their types and preferences.

**Public Goods:** what we want as a group resource but have the issue of free riders, (non-rival?)

**Methodology:** Start with assuming rational actors in laying out the mechanism. Then will extend to irrational actors (psychologically irrational or simple rule-based).

**Outline:** 1) basics of mechanism design, 2) talk about hidden action information, 3) move on to auctions, and 4) conclude with discussion of public good games.

#### C18.2 Hidden Action and Hidden Information

**HIDDEN ACTION** Incentive structures to get what we want. Employee contract structure, auction – get people to reveal their information. Core problems: 1) dealing with hidden action, 2) dealing with hidden information. Sometimes called moral hazard problems (people could cheat, slack off). How to overcome?

**Model for Hidden Action:** Incentives to influence hidden actions

Moral Hazard Evaluation: Want to induce effort =

1

Action (hidden): effort of [0, 1] Outcome = {good, bad}

Probability, P(Good | effort=1) = 1

Probability, P(Good | effort=0) = p

Cost of effort (to worker) = c

## **Incentive Compatible solution:**

Contract: Pay M if Good, 0 if Bad so that it makes sense for the worker to put in the effort.

Worker Payoffs: Effort of  $1 \rightarrow M - c$ , Effort of 0

→ pM

Where p = P(Good | effort=0) . Thus Worker incentive is to have  $M-c \ge pM$  or M-c/(1-p) Implication is that owner has to pay workers more than c/(1-p) in order to assure a good outcome.

**Quiz:** True or False: In words, the equation  $M-c \ge pM$  means: It makes sense for a worker to put in effort if the amount of money received minus the cost of effort must be greater than or equal to the probability that if they slack off, the outcome will be good anyways. (a) True, (b) False **Ans:** (a) True

Comparative Statics: Given  $M \ge c/(1-p)$  what are the influence (sensitivity) of p and c on M?

c: if c (the cost to worker of effort) increases, M must increase.

p: if p (P(Good | effort=0) increases then M must also increase to preserve the  $M \ge c/(1-p)$  relationship.

<u>Bottom line:</u> it costs **M** to convert a hidden action into an incentive where a rational player will choose to put forth effort to get a Good product. Thus a hidden action becomes 'visible'.

Now know action that was hidden at cost M.

**Quiz:** Molly is a graphic designer who makes advertisements. She can either make high quality advertisements or poor quality advertisements. If she puts forth effort e=1, she makes high quality advertisements with probability=0.75. If she puts forth effort e=0, she makes high quality advertisements with probability=0.25. The cost of 1 unit of effort =0.5. She is paid M if the advertisement is high quality and N if the advertisement is poor quality. For which of the following M and N will she put forth effort e=1? (a) M=1/2; N=1/4, (b) M=0.8; N=0, (c) M=2; N=1/2, (d) M=0;N=0.8

#### Analysis: Cost of Effort

P(G|0) = p = 0.25, P(NG|0) = (1-p) = 0.75P(G|1) = q = 0.75, P(NG|1) = (1-q) = 0.25

Worker indifferent at  $U_0 = U_1$ :

Mp + N(1-p) = Mq + N(1-q) - c

M(0.25) + N(0.75) = M(0.75) + N(0.25) - 0.5

0.5N = 0.5M - 0.5 or N = M - 1

Thus at  $M \ge N + 1$  the worker puts for effort 1 Of the four answers, only (c) meets this inequality.

Ans: (c) M=2, N=1/2

#### **Explanation:**

0.75(M-0.5) + 0.25(N-0.5) > 0.25(M) + 0.75(N). To make Molly put in effort of 1, her payoff with e=1 must be greater than her payoff with e=0. Plug in values of M & N given to find out when this is true.

*Note difference in equations:* 

Mp + N(1-p) = (M-c)q + N(1-q) =

0.25M + 0.75N = 0.75(M - 0.5) + 0.25N

0.5M + 0.375 = 0.5N or M + 0.75 = N and  $M \ge N + 0.75$ 

# **HIDDEN INFORMATION**

Don't know if driver is high risk (H) or is low risk (L), or

Don't know if worker ability is high (H) or low (L) H type: their perceived cost per hour of effort is C L type: their perceived cost per hour of effort is C where  $\mathcal{C} > c$ 

Assume trial period of productivity assessment – Contract: will pay M but first you must work K hours

**Incentive Compatible:** H-type: M > Kc, L-type: M < kC

Where Kc and KC are the number of hours worked times the worker's personal perceived cost of working versus what the employer will pay for K hours of their work (M). The employer should choose  $K = M/C + \varepsilon$  in order to discourage L-types from applying as the L-type will perceive the return (M) as less than their value (KC).

# **Comparative Statics**: for $K \ge M/C$

M: increasing M requires K (number of trial period hours worked) to increase.

C: increasing C allows for reducing K (the number of trial period hours worked).

Sometimes called **Costly Signaling** Models: The high ability workers had to signal by working K hours that they are H-type. This reveals hidden information because the H-types will take the job whereas the L-types will not. So ability information is no longer hidden.

# C18.3 Auctions

In Session 18 we address: Why Model? To design laws, rules, and institutions.

Let's look at auctions. Objective to get as much value as possible. Buyer (as low as reasonable), Seller (as high as reasonable).

Three types of auctions: (a) Ascending price, (b), second price, Each person submit, (c) sealed bid Three ways to think about bidding: (a) rational, (b) psychological, (c) rule-following.

# **Ascending Price Auction:**

Definition: individuals call out bids until no one

bids a higher price. Pays his high bid. Rational: Only bid up to your value

Psychological: Some bias to bid over value to

'win"

Rule Following: Some heuristic but still likely only to bid up to value.

# Second Price Auction (truth revealing):

<u>Definition</u>: Each person submits a bid. Highest bidder gets it at the second highest bid.

Example: P1 value=\$80, P1 bid = \$80

Highest		
Other Bid	Your Bid	Net
60	80	20
75	80	5
85	80	0

bidders motivated only to bid up to their value.

Ascending Price Auction Outcome: Highest value bidder gets it at the second highest value as he will stop before reaching his higher valuation when the competition stops at a lower value!

#### **Second Price Auction Outcome:**

Highest value bidder wins at the second highest value.

<u>Rational</u>: should bid your true value <u>Psychological</u>: doesn't influence your best bid <u>Rule Following</u>: also doesn't influence your best bid

**Quiz:** When talking about auctions, the "net gain" refers to which of the following? (a) The amount you end up paying, (b) The amount of money you win, (c) The amount you bid minus the amount you end up paying. (d) The difference between the amount you're willing to pay and the amount you actually end up paying. **Ans:** (d) The difference between the amount you're willing to pay and the amount you actually end up paying.

#### **Sealed Bid Auction:**

<u>Definition</u>: Each person submits a bid. Highest bidder gets it at the highest bid.

Example: 2 bidders – Assume uniform distribution of bids:

Probability(Bid < B) = B

\$0 B \$100

for P1
V = Value, B=Bid
(V-B) = profit, B=P(win)
E{profit} = B(V-B)
Maximize expected profit
dE{profit}/dB = 0 = V - 2B

→ Optimal Bid: B = V/2

Rational: Should bid V(n-1)/n {from game theory class}, shade bid by (n-1)/n where n is # of bidders.

<u>Psychological</u>: doesn't influence your best bid <u>Rule Following</u>: might shade by some percentage.

#### **Sealed Bid Auction Outcome:**

Highest value bidder wins the auction at half of their value.

Note: All about the same, ascending and second price win at second highest value. Sealed wins at (n-1)/n V which for 2 bidders is V/2 =  $E[V_{B2}]$ 

**Quiz:** If all bidders are rational, then the bidder with the highest value will win, regardless of the type of auction (Ascending Price, Second Price, or Sealed Bid). (a) True, (b) False

Ans: (a) True

**Explanation:** In an Ascending Price auction, you bid until the price is above your value. Therefore, the winner - who is still in after everyone else has stopped bidding - must have the highest value.

In a Second Price auction, you bid your true value (weakly dominant strategy). Therefore the highest bidder will be the person with the highest value, since B=V. In a Sealed Bid auction, you will never pay over your value. We showed that you would bid half your value. If everyone bids half their value, you still bid half your value (see lecture), so B=V/2. Whoever has the highest value wins again.

#### **Summary:**

- Sealed Bid (First Price Auction): Highest value bidder wins at (n-1)/n of their value

  Note: this is the same as the expected value of the second highest bidder!
- Ascending Bid: Highest value bidder wins at second highest value
- Second Price: Highest value bidder wins at second highest value

# **Revenue Equivalence Theorem:**

With <u>rational bidders</u> in a wide class of auction mechanisms including:

- (a) sealed bid,
- (b) second price,
- (c) ascending bid

Produce identical expected outcomes.

Roger Myerson, "Optimal Auction Design", Mathematics of Operations Research 1981

# **Bottom Line:**

Auction Type doesn't matter if all bidders are rational!

# What about unsophisticated bidders?

Sealed bid and second price auctions may be confusing to psychological and rule-following bidders.

Thus: Ascending auctions might generate more revenue as psychology will play a bigger role in assessing their values against their biases and thus bids.

**Quiz:** In which of the following auctions might it make sense for you to shade your bid (that is, bid lower or higher than your true bid) assuming all players are rational? (a) Second Price Auction, (b) Ascending Price Auction, (c) Sealed (First Price) Auction, (d) Never shade

Ans: (c)

**Explanation:** In the lecture, we showed that in both Second Price and Ascending Price auctions you are always better off bidding your true value. In the Sealed Bid auction, on the other hand, it might make sense to bid below your value. This is because you are basing your bid off of an expected distribution of other bidders. You need to bid above the expected value of the next highest bid, which may not be as high as your own value. Revisit the lecture to understand the math.

# **C18.4 Public Projects**

# Key Question: Whether to Fund Public Projects? Hidden information example: →

The question comes down to incentive compatibility

**Pivot Mechanism:** to reveal hidden information: **Idea:** Pay the minimal amount you would have to contribute for the project to be viable. That is, your cost is only your *marginal amount* (amount remaining after others pay their claimed value) so you are more inclined to reveal your value.

Copy Machine Example: Value to P1=\$40, P2=\$50, P3=\$20 → sum of \$110 vs. a cost of say \$80. But these values are hidden information so is unknown. Challenge is to design to encourage people to reveal their hidden information.

# Clarke Groves Vickery Pivot Mechanism:

Each person claims value: V1, V2, V3 (may not necessarily be their true value)

If V1+V2+V3 > Cost → DO THE PROJECT

P1 pays max{cost -V2 -V3, 0}

Note the 0, P1 may not have to pay anything!

**Quiz:** Is the pivot mechanism a way to overcome hidden actions or hidden information? (a) Hidden Action, (b) Hidden Information

Ans: (b) Hidden Information

#### **Example:**

P1 = \$40, P2=\$50, P3=\$20 with cost=\$80. Now P1 + P2 + P3 = \$110 so do the project.

P1 Pays = \$(80 - 50 - 20) = \$10 Should P1 Cheat?

Case 1 – baseline, P1 bids true value \$40 P1 will pay as shown in 'Pay' column with P1 net

in 4<sup>th</sup> column.

Sum of	Claimed		
<u>Others</u>	<u>Value</u>	<u>Pay</u>	<u>Net</u>
30	40		
40	40	40	0
45	40	35	5
90	40	0	40

Case 2 - true value = \$40,

P1 shades to claimed value = \$30, and P1 is less well off. Project doesn't get done until others=\$50 and P1 misses some scenarios [40 & 45].

Claimed		
<u>Value</u>	<u>Pay</u>	<u>Net</u>
30	0	
30	0	
30	U	
30	0	40
	<u>Value</u> 30 30 30	<u>Value</u> 30

Case 2 – true value = \$40, P1 over claims value @ \$50

P1 risks overpaying for his true value.

Sum of	Claimed		
<b>Others</b>	<u>Value</u>	Pay	<u>Net</u>
30	50	50	-10
40	50	40	0
45	50	35	5
90	50	0	40

**Note** P1 does not gain by under-claiming P1's value.

Also P1 does not gain by over-claiming P1;s value. Thus P1 should state his true value and not shade or overstate.

Clarke groves Vickery Pivot Mechanism is an Incentive Compatible Mechanism: Each person has an incentive to reveal their true value.

**Review:** What is the purpose of the pivot mechanism? You may select more than one answer (a) .To get people to reveal their true value in a public project or joint purchase. (b) To reveal hidden actions. (c) To reveal hidden information. (d) To get people to reduce their claimed value.

**Ans:** (a) and (c)

Incentive Compatible Downside:	P1 = \$30	P1 = \$40
	P2 = \$30	P2 = \$50
BUT does not guarantee projects will be funded.	P3 = \$30	P3 = \$20
	$\Sigma$ = \$90 & each pays	$\Sigma$ = \$110
That depends upon the stakeholder values and	\$30 and project is	But P1 pays \$20, P2
what they add up to. Consider the \$80 Cost	done!	pays \$30, P3 pays \$0
project in the right colums.	Note:	and net payments =
	each pays difference	\$50 and project is not
	(Cost - ∑others)	done!

**Quiz:** Three workers are deciding whether to buy a new couch for the lounge. The couch would cost \$700. Person 1 values a new couch at (P1) \$100, Person 2 values a new couch at (P2) \$500 and Person 3 values a new couch at (P3) \$250. Using the Clarke-Groves-Vickrey mechanism, do they buy the couch? How much does each person pay? (a) No, they don't buy it. Person 1 would only pay \$100, Person 2 would only pay \$500 and Person 3 would pay 0. Since the \$600 they'd raise is less than the \$700 cost, they would not buy it (end up paying 0). (b) Yes, they would buy it. Person 1 would pay \$0, person 2 would pay \$450, person 3 would pay \$250. They would exactly be able to cover their costs. (c) No, they don't buy it. Person 1 would pay \$0, Person 2 would pay \$350, Person 3 would pay \$100. (d) Yes, they would buy it. Person 1 would pay \$100, Person 2 would pay \$500 and Person 3 would pay \$100.

**Analysis**: Sum of values (V1+V2+V3) = 100+500+250 = \$850 > \$700 cost. Therefore: P1 pays \$0 according to the explanation and P2 pays (700 - P1 - P3) = \$350, and P3 pays (700 - P1 - P2) = \$100 but then P1<sub>a</sub> + P2<sub>a</sub> + P3<sub>a</sub> = 0 + 350 + 100 = \$400 which is less than \$700. So they don't buy the couch!

Ans: (c) they don't buy the couch

**Explanation**: Remember, each person does not necessarily pay her value. What we're concerned with, instead, is each person's "claimed value" given the values of the other two people.

Lawyer 1 sees that the two others (2 & 3) are willing to pay a sum of \$750. Since the couch costs \$700, Lawyer 1 claims a value of \$0.

Lawyer 2 sees that the two others (1 & 3) are willing to pay a sum of \$350. So Lawyer 2 claims a value of \$350. Lawyer 3 see that the two others (1 & 2) are willing to pay a sum of \$600. So Lawyer 3 claims a value of \$100. Because each person only wants to pay as much as is necessary to ensure the couch is paid for, they each pay less than their true value.

They raise a total of \$0+\$350+\$100=\$450, and don't have enough to buy the \$700 couch.

#### The impossible task:

Getting (a) efficiency, (b) always incented join, (c) incentive compatible, and (d) balanced (can pay for the project).

Only option is tradeoffs in these parameters. The Clarke groves Vickery Pivot Mechanism sacrifices *Balance*.

Other mechanisms sacrifice other restrictions.

# **Objective:**

Design mechanisms to uncover hidden actions (induce people to take the right actions) and hidden information (public purchase case (do if values high enough) and auction case (highest bidder wins) where want people to reveal their true values).

These mechanisms start by assuming people are rational actors:

Evolve these to include realities of people having psychological biases and perhaps heuristic rules that influence their values.

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