

Session 4: Decision Models

4.1 – Introduction to Decision Making

Rationale:

- a) **Normative** – make us better at deciding,
 - b) **Positive** – predict the behavior of others.
- **Normative Examples: How to make better personal choices.** School? Job? Insurance? Drive or Fly? Computer? Investments? Wedding? House?
 - **Positive: Answer the question, Why?** Policy choices, Nominations, Platforms, Investments, Technology choice

Models: (a) Multi-Criterion, (b) Probabilistic

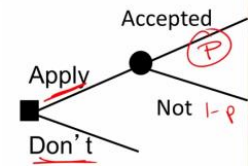
- **Multi-criteria: Car purchase choice**
 - a) 1D flat criteria (sum the winning features)
 - or
 - spatial (ND) nearest distance preferences

FORD	CHEVY
X	X
X	
	X
X	
	X



b)

- **Probabilistic: Decision Trees as an example**
 - **Value of Information (expected value)**



4.2 – Multi-Criterion Decision Making

Qualitative and Quantitative:

- **Qualitative:** House Purchase

Also may have other subjective criteria that could be considered.

- **Qualitative:** Policy Preference, binary, vote for one in each row.

Criteria	House 1	House 2
Square Feet	2000	1800
# Bedrooms	4	3
# Bathrooms	1 1/2	3
Lot Size	1/4	1/2
Location	10	15
Condition	6x	6000
TOTAL	4	2

DECISION SUPPORT FOR REAL: THE MCRI

We now want to use this same decision support worksheet to help you make a decision on the MCRI. Before you'll use a list of six dimensions we discussed. On each of these dimensions, fill in your personal experience – which side has the stronger argument and how important each dimension is to you – that you've already written in this booklet. When finished, you'll have a table like the one above that you can use to help you make an informed and considered decision. We've even included space for additional arguments if, in thinking through this issue, you have come up with new dimensions of your own. We hope that you have.

	Support MCRI	Oppose MCRI
EQUALITY	✓	
QUALITY OF EDUCATION	✓	
HEALTH/SAFE WORKING & ENVIRONMENT	✓	
BUSINESS GROWTH/CHANCE	✓	
ENVIRONMENTAL IMPACT	✓	
APPROPRIATE LAND USE	✓	
INAPPROPRIATE RESTRICTIONS		✓
TOTAL	4	2

Quantitative: Refine the value of each by weighting each criteria.

Recall:

"Don't want the models to tell us what to do, they are to help us make better choices."

Scott E Page

- **House Example (weighted values):** Quantitative method with weighting of each variable. Note one has to be careful in evaluating true needs and wants.

Criteria	Wgt	H1	H2
Square Feet	1	✓	
# Bedrooms	1	✓	
# Bathrooms	2		✓
Lot Size	2		✓
Location	1	✓	
Condition	1	✓	
Basement	2		✓
TOTAL		4	6

Summary:

1. can use to help us make decisions and
2. (b) help us understand why others made decisions the way they did.

Quiz: Joanne is considering two cameras. Here are the relevant specifications: Camera A: 8 oz, 12 MB memory, 5X zoom, 2" screen. Camera B: 6 oz, 10 MB memory, 4X zoom, 1.5" screen. If Joanne chooses camera B, what can we say about her decision-making approach? (a) she uses quantitative methods and cares about zoom, (b) she uses qualitative multi-criterion, decision making, (c) she's irrational, (d) she uses a quantitative approach and cares about having a light camera.

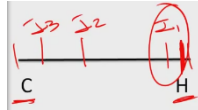
Ans: (d) she uses a quantitative approach and cares about having a light camera. Camera A is qualitatively better than B in three respects so by qualitative weighting would imply 'choose A'. As Joanne chooses B despite this, we can infer that she's using a quantitative model and cares particularly about less weight.

	Camera A	Camera B
weight	8 oz	6 oz
memory	12 MB	10 MB
zoom	5x	4x
screen	2"	1.5"

4.3 – Spatial Choice Models

- Finding my ideal point in a range of options.**

Started as geographic choice, George Hoteling: which ice cream vendor (near or far).

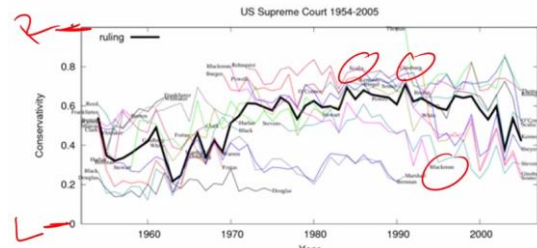


- Indian Food Preference:** degree of spiciness

- Voter Example:** Democrat, Republican, Voter picks necessary



- Supreme Court History:** Andrew Gelman (Left or Right Leaning, by justice and 'ruling' of the court). Note ordinate: Presidential jurist preference by party. Ideology of the court over time.



- Spatial in multidimensions:** Ideal vs Big Mac vs Whopper, 6 DoF: calculate difference → 5 vs 2 [vector length]

Attribute	IDEAL	Big Mac	DIFF
Cheese	2	2	0
Patties	2	2	0
Tomatoes	2	0	2
Ketchup	4	3	1
Mayo	4	4	0
Pickles	4	6	2

Attribute	IDEAL	BKW	DIFF
Cheese	2	2	0
Patties	2	1	1
Tomatoes	2	2	0
Ketchup	4	3	1
Mayo	4	4	0
Pickles	4	4	0

- Spatial inferences:** Use choices to infer preferences – if select Big Mac → like patties or pickles or not tomatoes, or some weighted combination of them.

Attribute	IDEAL	Big Mac	BKW
Cheese	?	2	2
Patties	?	2	1
Tomatoes	?	0	2
Ketchup	?	3	3
Mayo	?	4	4
Pickles	?	6	4

Attribute	IDEAL	Big Mac	BKW
Patties	?	2	1
Tomatoes	?	0	2
Pickles	?	6	4

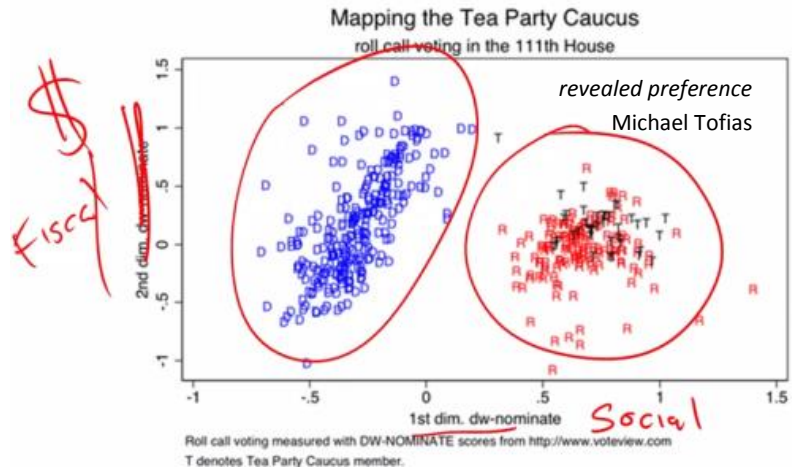
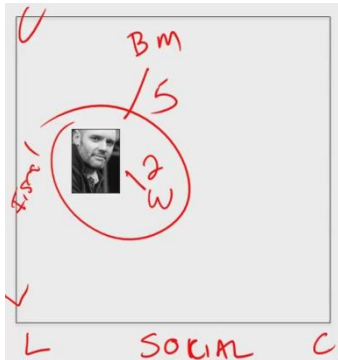
Quiz: My friend Jim uses this spatial preference idea to pick out an ice cream cone. Jim cares about two attributes: the height of the cone and the number of scoops it holds. Jim chooses a 4" cone with 3 scoops (4, 3); turning down two others: a 7" cone with 2 scoops (7,2), and a 2" cone with 4 scoops (2,4). Thinking in terms of spatial preferences, which of the following points(s) could be Jim's ideal point? (Assume that he computes the distance from each option to his ideal point). (a) 5" cone with 3 scoops, (b) 8" cone with 1 scoop, (c) 4" cone with 2 scoops, (d) 3" cone with 11 scoops

Ans: 5" cone with 3 scoops and 4" cone with 2 scoops as they have the shortest distance of any of the alternatives.

Ideal\Options	(4,3)	(7,2)	(2,4)
(5,3)	1.00	2.24	3.16
(8,1)	4.47	1.41	6.71
(4,2)	1.00	3.00	2.83
(3,11)	8.06	9.85	7.07

Example of Mapping Spatial

Preferences: Shows ideological preferences: Liberal/Conservative on Fiscal and Social matters



4.4 – Probability: The Basics

Axioms:

1. probabilities range from zero to one, $[0, 1]$
2. sum of all possible outcomes equals one (1)
3. A is a subset of B, then $P(B) \geq P(A)$

Quiz: Paul wants to predict who will win the Best Actor Award in this year's Oscars. He knows that the nominees will be Clooney, Pitt, Brody, Cusack, and Penn. Paul estimates the probability that each actor will win, as follows: $p(\text{Clooney}) = 0.4$, $p(\text{Pitt}) = 0.3$, $p(\text{Brody}) = 0.15$, $p(\text{Cusack}) = 0.05$, $p(\text{Penn}) = 0.2$. But there's something wrong with these probabilities. Which axiom did Paul forget? Was it 1, 2, 3, or all of the above?

Ans: Axiom 2

Types of Probability

1. **Classical** (calculate as in roll of perfect die)
2. **Frequency** (measure or observe)
3. **Subjective** (axiom-driven estimation)
On this approach must be careful that axioms are followed and use models to help with consistency – (MLO –Bayesian)

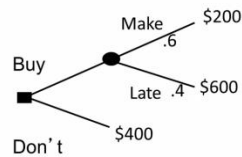
Subjective Probability Example: Shelly majored in political science and was very involved in the college Republicans. Estimate probabilities for the following events: (Event A) Flight Attendant -5% , (Event B) Blogger - 10% , (Event C) Flight Attendant while finishing her MBA - 10%, (Event D) Medical Field - 15%. Note error in Flight Attendant Events. $C \subset A$

Question: Will housing prices go up next year? Probability (classical, frequency, or subjective) or model. Modeling is preferable over and improves upon subjective estimates.

4.5 – Decision Trees

Rationale: (a) make better decisions, (b) infer other people's thinking/preferences, (c) learn about our own thinking/preferences.

- Example:** Can buy 3 pm ticket for \$200 with 40% chance won't make it. Or buy 4 pm ticket for \$400.



Do you buy?

$$E(\text{Buy}) = 0.6(200) + 0.4(600) = \$360$$

$$E(\text{Don't}) = \$400$$

Smallest cost then is to buy for 3pm train.

Note: box = decision (choice $\rightarrow \max[x, y]$), oval = chance event, expected value, $E(Ap+B(p-1))$.

Example: Scholarship \$5000. 200 applicants, 2pg essay required, 10 finalists with 10pg essay req'd. Assume costs: 2pg = \$20, 10pg = \$40

- Steps: (1)**

Draw Tree,

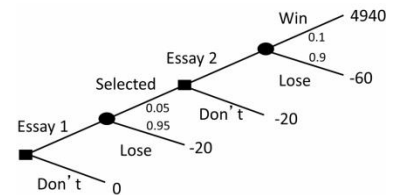
(2) Write

down

payoffs and

probabilities,

(3) solve backwards.



$$\text{Back 1: } E(4) = 0.1(4940) - 0.9(60) = \$440$$

$$\text{Back 2: } C(3) = \max[\$440, -\$20] = \$440$$

$$\text{Back 3: } E(2) = 0.05(440) - 0.95(20) = 22 - 19 = \$3$$

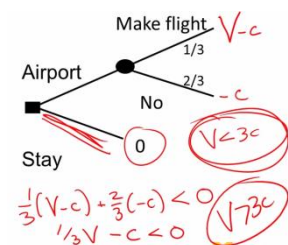
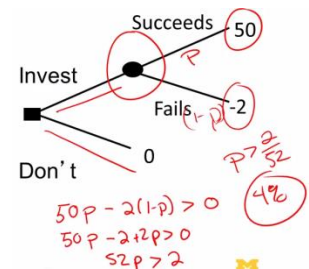
$$\text{Back 4: } C(1) = \max[\$3, \$0] = \$3$$

Quiz: In the previous example, suppose that 250 people are invited to submit essays. There are still 10 finalists. Does it make sense to write the essay? (a) Yes, (b) No

Ans: (b) No because at $E(2) = 0.04(440) - 0.96(20) = 17.6 - 19.2 = -\1.6

Insights of Others:

Investment Opportunity: Your friend tells you about a risky investment opportunity that will pay \$50,000 on a \$2000 investment. She's in. Do you invest? You can infer their assessment of the probability of success. **For success $p > 2/52 \sim 4\%$.** Can infer friend's assessment of $P(\text{success})$. Do I believe $P(\text{success}) > 0.04$.



Visit your Parents (how motivated are you?): You

have a Standby chance to go visit your parents. The airline says that there is a one third chance that you'll make the flight. You decide not to go the airport and to stay on campus. How much did you want to see your parents? Let V be your 'value' to see your parents and c be the cost to drive to and from the airport. Then as the diagram shows, $V < 3c$, you value a visit with your parents at less than 3 times the driving costs.

4.6 – Value of Information

- How much is information worth? Roulette**

38 slots, $P(n) = 1/38$

- What if you know you will win?**

Say \$100 is what you win, then $E(n) = \$100/38$. The value of knowing which number will win is the amount you would win, \$100.

- Value of Information Calculation Steps:**

(1) Calculate value without the information

(2) Calculate value with the information

(3) Calculate the difference



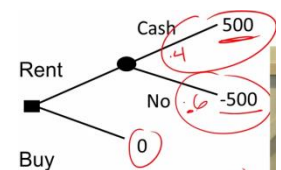
- Example- Step 1:**

Calculating value with the information

(Option A) buy a car

now, (Option B) rent

for \$500. But there is a 40% chance that the car company offers \$1000 cash back starting next month.



$$E(2) = 0.4(1000 - 500) + 0.6(0 - 500) = -\$100$$

$$C(1) = \max[E(2), 0] = \$0,$$

in this case then – Buy Now

• **Example- Step 2:**

Calculate Value with the information assumed.
Calculate from E(1) with P(rebate)=0.4 and P(no rebate)=0.6.

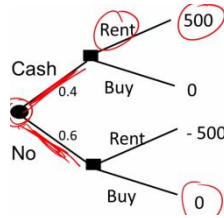
Note: redrawn decision tree

$$C(2,1)=\max[500, 0] = \$500,$$

$$C(2,2)=\max[-500,0] = \$0$$

$$E(1) = 0.4(500)+0.6(0) = \$200$$

in this case then – Rent first, then Buy



• **Example- Step 3:** Calculate the difference

The value of information:

$$\text{Step 2} - \text{Step 1} = \$200 - \$0 = \$200$$

1. Calculate value without the information

2. Calculate value with the information

3. Calculate the difference

Quiz: If the probability of this cash-back program occurring were 60% instead of 40%, what would be the value of information? Assume all payoffs remain the same.

Analysis: Step 1 ($\max[\$100, \$0] = \$100$), Step 2 ($0.6(500)+0.4(0) = \300), Value = Step 2 - Step 1 = \$200

Ans: \$200

Summary:

Can repurpose the Decision Tree model from helping to decide in the presence of uncertainty to finding the value of information.

Value of Information steps:

(1) calculate value without the information

(2) calculate value with the information (change tree, solve as if you knew the answer)

(3) calculate the difference