Session 13: Path Dependence

13.1 Path Dependence

Path Dependence:

- What happened along the path determines what happens now. History matters!
- In contrast to Markov Processes where path history does not affect the equilibrium (statistical) state. History does not matter.

Urn Models:

- A class of models to help describe path dependence.
- Flesh out the logic what is path dependence, what causes path dependence, distinguishing different types of path dependence.
- Equilibrium, what happens in the long run.
- Distribution over all possible outcomes could be path dependent

Path Dependence Example – Typewriter Keyboard:

- Today the keyboard is QWERTY layout (next to top row of keys – letters)
- Lots of initial layouts proposed but QWERTY emerged in a ***'process of increasing returns'***. It became a standard as more using caused more to want to use.

Path Dependence:

- Outcome ***probabilities*** depend upon the sequence of past outcomes.
- History matters (in a probabilistic sense).

Examples: 1) technology - e.g., AC vs DC, gas vs. electric cars, 2) common law – e.g., precedence where past law influences present law, 3) institutional choices – single-payer vs multi-payer healthcare system, pensions (defined benefits or defined contributions), 4) economic success.

Economic Success Example:

- Ann Arbor: Largest Public University
- Jackson: Largest four-walled prison
- Both in Michigan, 50 miles apart, with past choices influencing present success.
- Note the influence of crime in the 1920's which grew Jackson, then the war which reduced both, and then post WWII GI bill grew Ann Arbor.

Jackson			Ann Arbor		
1910	31,433	24.8%	1910	14,817	2.1%
1920	48,374	53.9%	1920	19,516	31.7%
1930	55,187	14.1%	1930	26,944	38.1%
1940	49,656	−10.0%	1940	29,815	10.7%
1950	51,088	2.9%	1950	48,251	61.8%
1960	50,720	−0.7%	1960	67,340	39.6%
1970	45,484	−10.3%	1970	100,035	48.6%
1980	39,739	−12.6%	1980	107,969	7.9%
1990	38,303	−3.6%	1990	109,592	1.5%
2000	36,316	−5.2%	2000	114,024	4.0%

Increasing Returns:

- Ann Arbor hosts a University. Then hospitals, law schools, other educational things emerge to reinforce growth.
- Virtuous cycle – good building on good.

Increasing Returns ≠ Path Dependence:

- Logically completely separate concepts

Chaos ≠ Path Dependence: ESTIC – Extreme Sensitivity to Initial Conditions. Chaos deals with initial points. PD on the path.

Markov Processes:

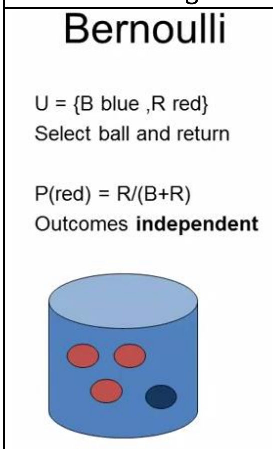
Recall: 1) Starting point doesn't matter, 2) Path doesn't matter, 3) History doesn't matter

- What is violated is the 'fixed state transition probabilities'.
- In the URN models, transition probabilities do change and that is why history matters.

13.2 – Urn Models

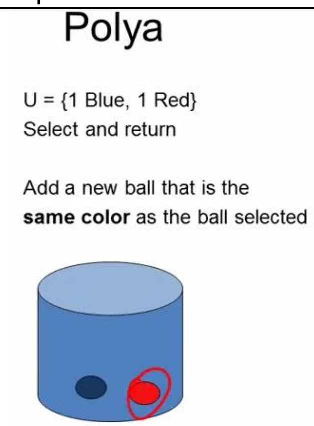
Path Dependence (PD):

- Use Urn models to describe ***path-dependent*** versus ***phat-dependent*** (order doesn't matter but the set of things matter – [recall "n choose k" from discrete mathematics]). (≠PATH) = PHAT
- Also help us distinguish process outcomes in a given period that are path dependent and processes that have equilibriums – a long run distribution over a set of outcomes, that are path dependent.
- Will distinguish between path-dependent outcomes and path-dependent equilibrium.

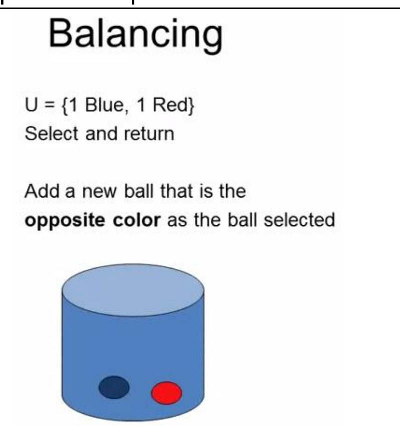


Fixed # of balls, describes roulette, blackjack

Used to describe PD. Simplest: Polya & Balancing



selected ball & new ball both back to urn, $P(R)_n$ changes & is PD

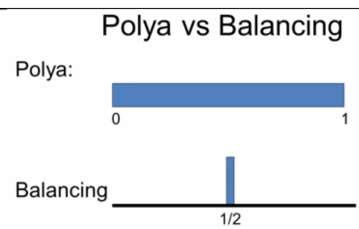


selected ball & new ball both back to urn, $P(R)_n$ changes & is NOT PD

Basic Urn Model: Urn contains balls of various colors. The outcome is the color of the ball selected.

Result 1: Any probability of red balls is in equilibrium and is equally likely. We could get 4%, 23%, or 99% red balls. Shirt choice example illustrates Polya process and history dependent.

Result: The balancing process converges to equal percentages of the two colors of balls. That is long run is 50% red and 50% blue so is NOT PATH DEPENDENT



percentage

Uniform versus 50% long run

Result 2: Any history of B blue and R red balls is equally likely. P(RRRB)=P(BRRR) ➔ frequency of occurrence says nothing about order.
Observation:
Period Outcomes: color of ball in a given period

Application: Keeping political constituencies happy. Where to host the Olympic games.

Observation: Equilibrium percentage of red balls in the long run.

Path Dependent Perspective:

- **Path Dependent Outcomes**: color of ball in a ***given period*** depends on the path, or
- **Path Dependent Equilibrium**: percentage of red balls in the ***long run*** depends on the path.

	Path Dependent Equilibria	**Path Dependent Outcomes**
Polya	Yes	YES
Balancing	½	YES

History can matter at each moment but not have any impact in the long run. Depends on the underlying process that is in effect. Examples: Manifest Destiny: (growth of US). Railroads: tracks may follow one another but may also have been defined by economic routes. In short, if there is an equilibria in the long-run, then the process is path independent. ???

PHAT Dependent: Outcome probabilities depend upon past outcomes <u>but not their order.</u>
Polya is PHAT – Outcome probabilities do not depend upon the order in which outcomes occurred. They only depend on the set of outcomes that occurred. RRBR ←→ BRRR

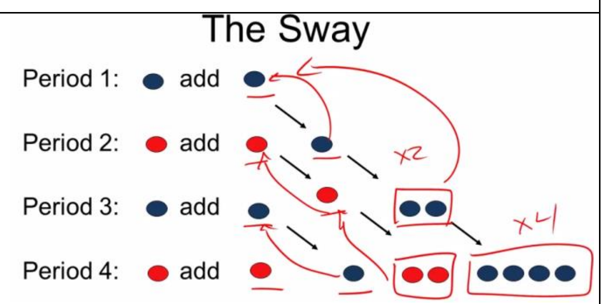
Why the concern: exponential growth in path possibilities vs. linear growth in outcomes.
e.g., thirty rounds of 2 choices → 2^{30} possible outcomes vs. just 31 sets (Scott's logic, any order of ball selection that has the same ratio of red/blue balls). Permutations [2^n] vs combinations [n choose k].

Two Questions:
1. Could something actually be path-dependent, depend on all this going different paths?
2. If there is one, can I construct it using my simple urn model?

Ans: Yes, there is a simple urn model called 'the sway'

The Sway: U = {1 Blue, 1 Red}, Select and Return
In period t, add a ball of the same color as the selected ball and add $2^{t-s} - 2^{t-s-1}$ of color chosen in each period s<t.

Period 1 – pick a blue ball →add a blue ball.
Period 2 – pick a red ball → add a red ball, and add a blue ball for the blue ball picked in Period 1.
Period 3 – assume picked a blue ball →add a red ball again for the red ball picked in Period 2. Now add two blue balls, (multiplying times two the blue ball picked in Period 1.
Period 4 – assume pick a red ball, →add a red ball.
Add a blue ball for Period 3. Add two red balls for Period 2. Add four (2x2), blue balls for Period 1. Thus for each ball picked, I'm adding 1, then 2, then 4, then 8, then 16, then 32, as I move through time. In short as move back in time decisions take on more and more weight.



If the past takes on more weight over time then you can get full path dependence.
e.g., first-movers, law, etc.

Summary:

Urn models can model some concepts of path vs. set dependence and the influence of ecosystem development (my words) where the outcomes of the past can compound current and future outcomes.

We learned how to construct a very simple path-dependent process, called the Polya process; we constructed a balancing process, i.e., can have path-dependent outcomes in each period, but not have path-dependent equilibrium. You can go to a particular thing each time. And constructed the sway process – full path dependence and that showed us one way you can get full path dependence and not just PHAT dependence is by having the weight of history increase over time, which is something intuitively I think that a lot of historians and a lot of scholars of technology have felt was true.

13. 3 The Proofs:

Will prove both results: Will start with Result 2 and use to prove Result 1.

Result 2: Any history of B blue and R red balls is equally likely.

Example 1:

$P(RB\underline{B}B) = \frac{1}{2}\,\frac{1}{3}\,\frac{2}{4}\,\frac{3}{5}$ $\frac{1\cdot1\cdot2\cdot3}{2\cdot3\cdot4\cdot5}$

$P(BBBR) = \frac{1}{2}\,\frac{2}{3}\,\frac{3}{4}\,\frac{1}{5}$ $\frac{1\cdot2\cdot3\cdot1}{2\cdot3\cdot4\cdot5}$

Example 2:

$P(RBRB) = \frac{1}{2}\,\frac{1}{3}\,\frac{2}{4}\,\frac{2}{5} = \frac{1\cdot1\cdot2\cdot2}{2\cdot3\cdot4\cdot5}$

$P(BBRR) = \frac{1}{2}\,\frac{2}{3}\,\frac{1}{4}\,\frac{2}{5} = \frac{1\cdot2\cdot1\cdot2}{2\cdot3\cdot4\cdot5}$

Note: the denominator increases by 1 on each draw-replace-add same cycle. The numerator follows the number of red (on R draw) or blue(on B draw) that are in the urn at that time, starting with one R and one B on first draw. Observe p = ½ on every first draw.

Note: Example 2 Numerator – the order doesn't matter as these have the same numbers one, two, one, two multiplies in different orders. This follows from the number of the color when the color is drawn. Check with Example 1 as well.

Quiz: If we want to find the probability of choosing a red ball, another red, then two blues, or in other words P(R,R,B,B), what would be the numerators (in order) of the probability fraction?

[1,2,2,1]; **[1,2,1,2]**; [2,1,2,1]; [1,1,2,2]

Ans: [1,2,1,2]

Explanation: Any answer beginning with a 2 in the numerator must be incorrect, because there are only two balls to start with. So that probability of choosing one - regardless of what color - must equal 1/2. Since we want the second ball to be the same color as the first one chosen, the probability will be 2/3. In the third stage, we want the a ball the color that has not been chosen yet; there is only one, so the probability is 1/4. The probability of another blue is now 2/5.

Result 1: Any probability of red balls is an equilibrium and is equally likely. We could get 4%, 23%, or 99% red balls.

$P(BBBB) = \cancel{\frac{1}{2}}\,\cancel{\frac{2}{3}}\,\cancel{\frac{3}{4}}\,\cancel{\frac{4}{5}} = \frac{1}{5} = RRRR$

$P(BBB\underline{R}) = \frac{1}{2}\,\frac{2}{3}\,\frac{3}{4}\,\frac{1}{5} = \frac{1}{20}$ $\times 4$ $\frac{1}{5} = BBBR$

BBRB
BRBB
RBBB

Argument is that p(4 B) = 1/5 but that p(1R & 3B) in any order is 1/20 times 4 ways to get (1R & 3B) = 1/5. Similarly p(4R) = 1/5 and p(1B&3R) = 1/5 and then p(2R&2B), the only remaining combination is 1 – (1/5 + 1/5 + 1/5 + 1/5) = 1/5.

Note: the denominator increases by 1 on each draw-replace-add same cycle. The numerator follows the number of red (on R draw) or blue(on B draw) that are in the urn at that time, starting with one R and one B on first draw. Observe p = ½ on every first draw.

$P(50B) = \cancel{\frac{1}{2}}\,\cancel{\frac{2}{3}}\,\cancel{\frac{3}{4}} \cdots \cancel{\frac{49}{50}}\,\cancel{\frac{50}{51}} = \frac{1}{51} = P(50R)$

$P(49B\ 1R) = \cancel{\frac{1}{2}}\,\cancel{\frac{2}{3}}\,\cancel{\frac{3}{4}} \cdots \cancel{\frac{49}{50}} \cdot \frac{1}{51} = \frac{1}{50\cdot51} \times 50 = \frac{1}{51}$

$P(47B\ \underline{3R}) = \cancel{\frac{1}{2}}\,\cancel{\frac{2}{3}} \cdots \cancel{\frac{47}{48}}\,\frac{1}{49}\,\frac{2}{50}\,\frac{3}{51}$

$3\times2\times1$
12 17 16
17 16 12
16 17 12

$\frac{\cancel{1\cdot2\cdot3}}{48\cdot49\cdot\cancel{50}\cdot51}$ $\frac{\cancel{50\cdot49\cdot48}}{\cancel{3\cdot2\cdot1}}$ $\frac{1}{51}$

Shows logic of how any sequence has the same probability. Note the use of combination counting in the p(47B 3R) case to account for all of the ways one can draw 3 balls, (3x2x1) and there are 50 possible locations for the first R ball, 49 locations then for the second R ball, and 48 places for the third R ball. But the three balls are indistinguishable, hence / (3x2x1).

Note: started with the idea that maybe the number of purchases of a red shirt is driven by the number that have previously been purchased. Any equilibium is equally likely. Any history of R and B shirt purchases are equally likely.

Summary:
Given a simple model that the proportion of, the probability of somebody buying something is in proportion to the number of previous people that had bought it, like the blue and red shirts. This formal model yields two really interesting results: 1) any equilibrium is equally likely [ratio of red to blue shirts] and 2) any history, with the same number of Blue shirt purchases and Red shirt purchases, is also equally likely [order doesn't matter, (n chose k)].

13.4 Path Dependence & Chaos

Path Dependent: Outcome probabilities depend upon the sequence of past outcomes.

Markov Process. A1: Finite States, A2: Fixed transition probabilities, A3: Can eventually get from one state to any other, A4: Not a simple cycle. Markov Convergence Theorem – Given A1 to A4, a Markov process converges to an equilibrium distribution which is UNIQUE.

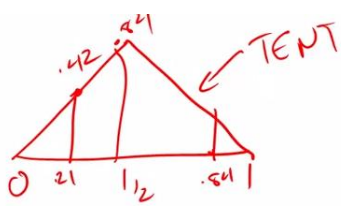
Note: the Markov model is not path dependent because of A2 – fixed transition probabilities. Thus history matters if the transition probabilities change.

Relating to Chaos – Recursive Function: Outcome at time t, x(t) & Outcome function F: X→ X
Example: F(X) = X+2 generates 1, 2, 5, 7, 9, 11, . . . or F(X_1, X_2, X_3) = X_4 or $F(X_t) = X_{t-1}$

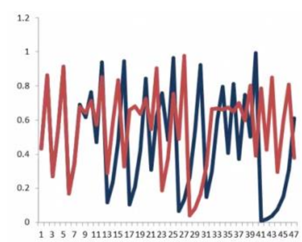
Choas: Extreme Sensitivity to Initial Conditions (ESTIC): If initial points x and x' differ by even a tiny amount after many iterations of the outcome function, they differ by arbitrary amounts.

Tent Map (function):
X in (0, 1) with
$$F(X) = \begin{cases} 2X & if\ X < 1/2 \\ 2X - 1 & if\ X > 1/2 \end{cases}$$
The tent map is chaotic.

ESTIC but not Path Dependent



X(1)=.4321	X(1)=.4322
0.4321	0.4322
0.8642	0.8644
0.2716	0.2712
0.5432	0.5424
0.9136	0.9152
0.1728	0.1696
0.3456	0.3392
0.6912	0.6784
0.6176	0.6432
0.7648	0.7136
0.4704	0.5728
0.9408	0.8544
0.1184	0.2912
0.2368	0.5824



Quiz: Two men go for a walk. They are next-door neighbors named Roland and Micah, and they both set out from their own doorstep. They both decide that they will walk for 20 houses and then take a break. If they stop in front of a house with an even numbered address, they will stop walking; if, on the other hand, they stop in front of a house with an odd numbered address, they will walk another 20 houses, with a maximum total distance of one mile. Assume addresses are numbered consecutively from 0 on to 100. When both men have stopped walking, Roland is a mile from home, and Micah is just down the street from his front door. This result is an example of: (a) ESTIC (extreme sensitivity to initial conditions), (b) Path Dependence

Analysis: This is recursive with $F(X) = \begin{cases} X + 20 & if\ X\ even \\ X & if\ X\ odd \end{cases}$ and house are numbered in sequence. So if start at even number will continue to the end at one mile. If odd, go 20 houses reach another odd and stop.

Ans: (a) ESTIC

Explanation: ESTIC means that the process - in this case, walking 20 houses at a time - is determined solely by the starting point of the process. In this example, houses are numbered consecutively, and Roland and Micah walk 20 houses at a time. This means that whoever lives in an odd-numbered house will stop each time at an odd-numbered house, and thus have to continue walking until he reaches the limit of one mile. We can infer, since Roland walks a mile, that Roland lives in the odd-numbered house. Micah, who lives in the even-numbered

house stops after his first set of 20 houses.
The point is that the initial condition - whether each man lived in an odd- or even-numbered house, determines the entire process. There are no factors along the way that alter either man's path (other than that initial condition), meaning this isn't a path dependent process.

Independent (probabilistic): outcome doesn't depend on the starting point or what happens along the way.	**Initial Conditions:** Outcome depends on the starting state, e.g., chaotic
Path Dependent: Outcome probabilities depend upon the sequence of past outcomes.	**PHAT dependent:** Outcome probabilites depend upon past outcomes but not their order.

Historians: it's really about PATH or PHAT. Why Path over PHAT? A set of events over time in different orders don't make sense as an explanation of today. And a history determined by a few intial choices in an underterministic process doesn't make any sense either.

Path versut Fact: PATH says that sequence matters, Fact says the set Matters. Why do they think it's the path and not the set? Consider this hard to imagine set of events.

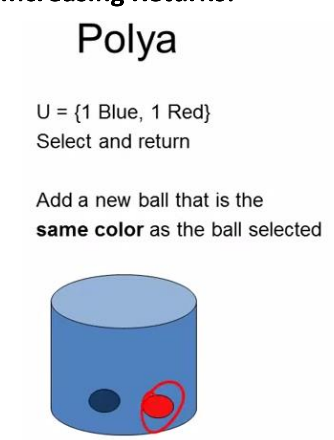
1814: Give women right to vote
1823: Fight civil war ending slavery
1880: Finish transcontinental railroad
1894: Find Gold in California
1923: Buy Midwest from France
1967: Defeat British in War of Independence

When we think about these ideas. Path dependence. Fact dependence. Independence. Sensitivity to initial conditions or chaos. What we see is these simple models help us organize our thinking about the world might look like. And we understand why a lot of historians focus so much on path dependence. Because it seems the most reasonable. We also see why people who study gambling in casinos consider independence. And we see why a lot of physicists are interested in things like chaos, because there are actual physical recursive that produce this extreme sensitivity to initial conditions. the URN model, in particular, helps us draw bright lines between path dependence, PHAT dependence and Independence.

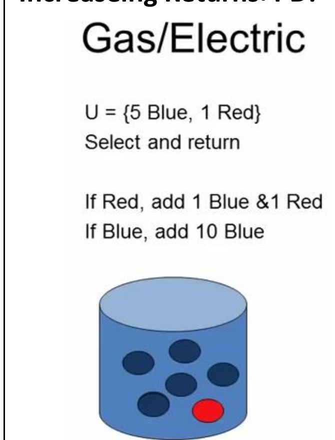
13.5 Path Dependence & Increasing Returns

Illustrate that path dependence and increasing returns are related but are not the same thing. Both Sway process and Polya process generate increasing returns [positive feedback]. Balancing process does not produce path dependence, not increasing returns, in fact, decreasing returns [negative feedback].

Increasing Returns:



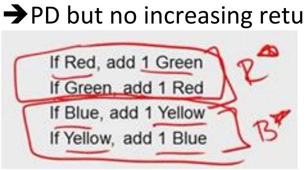
Increaseing Returns≠PD:

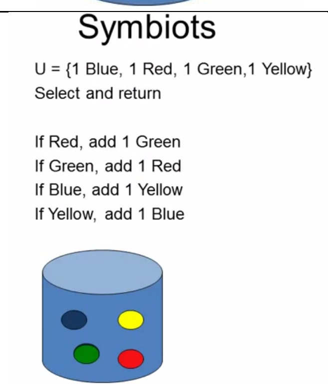


Question: Is increasing Returns equivalent to path dependent equilibrium?
Early gas car versus an electric car evolution. With the rule shown at the left, it is evident that the increasing returns of the B (gas) balls will overwhelm the R (electric) balls. <u>But increasing returns themselves do not imply path dependence.</u> **Increasing returns but not path dependent. P(B) > P(R) ➔ not path dependent where P(nR)=P(kR) from Polya model.**

Polya model is an example of increasing returns. More red balls I pick, the more likely I will pick more red balls in the future.

Symbiots to Polya: Create two groups from four as shown ➔PD but no increasing returns





Question: Could you get path dependence without increasing returns? YES!
(See Symbiots to Polya): This symbiosis model. Consider as a ***R-G set***, & a ***B-Y set*** with add R if R-G member is drawn and add B if B-Y member is drawn ➔ a R-B Polya experiment, path dependent but not increasing returns. (not enough gain, so to speak.) **Path dependent but not increasing returns.**
Gas-Electric Model: IR but no PD
Symbiots Model: PD but no IR

Increasing returns is logically distinct of PD. Completely separate concepts. But can occur together.

Externalities: Another driver of path dependence. My choices affect you, positive or negative.

Public projects are BIG. They likely BUMP into each other. They CREATE externalities. Examples include: Roads, Universities, Sewer Systems, etc.

Model: Projects {A, B, C, D, E}, Each has a value of 10, each creates externalities.

Externality Matrix: Diagonal is the value of the project. Below diagonal is the externality between (i, j) projects. Consider project and order:
Bottom line: Starting with A gives different path dependencies of projects than starting with B, e.g., AC vs BD.

Start: A = 10, would do
AB = 10 + 10 – 20 = 0 ➔ not
AC = 10 + 10 + 5 = **25**
ACD = 10 + 10 + 5 + 10 – 10 ➔ D?
Start: B = 10
AB = 0 ➔ not
BC = 10 + 10 – 10 ➔ C not
BD = 10 + 10 + 30 = **50**

A big cause of path dependence is externalities. Dissertation: More positive externalities ➔ less PD.

	A	B	C	D	E
A	10				
B	-20	10			
C	5	-10	10		
D	-10	30	0	10	
E	10	-10	0	0	10

More negative externalities create more adverse dependencies that hinder project adoption.

13. 6 Path Dependent or Tipping Point?

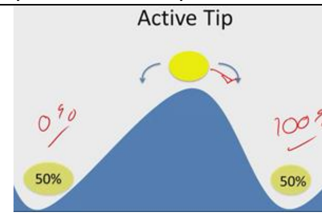
Path Dependent: Outcome probabilities depend upon the sequence of past outcomes.

Path Dependent Outcomes: e.g., color of ball in a given period depends on the path.

Path Dependent Equilibrium: e.g., percentage of red balls in the long run depends on the path.

Tipping Points: Direct (Active) Tip and Contextual Tips (forest fire)
Note that an ***active tip*** has an association with ***path dependence*** since and unstable node will go one way or the other, once started it will continue to the next stable position.

Recall in tips there is an abrupt change in the outcome.



Measuring Tips: (measure of uncertainty) (a) diversity index (number of stable positions available), (b) entropy (measure of information/uncertainty.) Recall $DI = \left(\sum P_i^2\right)^{-1} = 1/\left(\sum P_i^2\right)$

Polya Process: Does this represent a tip system? Check diversity index for drawing four balls from the urn. From previous calculations (shown below) could get 0, 1, 2, 3, 4 Red balls. So our diversity calculation is $1/\{\sum(P_i)^2$ which equals 5 in this case. What happens after we pick the first ball which, say, is Red? The similar calculations shows p(4R) = 0.4, p(3R)=0.3, p(2R)=0.2, p(1R)=0.1 and $1/\{\sum(P_i^2)\} = 3.33$

Recall Polya Result 1:

Any number of Red balls is an ***equilibrium*** and is ***equally likely***. Therefore find probability of each equilibrium.

Five possible outcomes {0,1,2,3,4}

0 1 2 3 4
1/5 1/5 1/5 1/5 1/5

$\frac{1}{\sum P_i^2}$

$\frac{1}{(1/5)^2+(1/5)^2+(1/5)^2+(1/5)^2+(1/5)^2} = \frac{1}{5/5^2} = \frac{1}{1/5} = 5$

Diversity Index

$\frac{2}{3}\frac{3}{4}\frac{4}{5} = 2/5$ 4R

$\frac{2}{3}\frac{3}{4}\cdot\frac{1}{5} = \frac{1}{10}\times 3 = 3/10$ 3R

$\frac{2}{3}\frac{1}{4}\frac{2}{5} = \frac{1}{15}\times 3 = 2/10$ 2R

$\frac{1}{3}\frac{2}{4}\frac{3}{5} = 1/10$

4R	3R	2R	1R
4/10	3/10	2/10	1/10

First Ball Red

Suppose first ball chosen is red ball. → Start with 2 Red & 1 Blue balls. Possible different outcomes are? 1) RRR red balls in the next three periods → total four reds. $P(RRRR) = (\frac{2}{3})(\frac{3}{4})(\frac{4}{5}) = \frac{2}{5}$. 2) RRB or RBR or BRR in next three periods → $\binom{3}{2}P(RRB) = \binom{3}{2}\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{1}{5}\right) = \frac{3}{10}$
3) RBB or BRB or BBR → $\binom{3}{2}P(RBB) = \binom{3}{2}\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)\left(\frac{2}{3}\right) = \frac{2}{10}$
4) BBB → $P(BBB) = \left(\frac{1}{3}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{10}$. ***Note not equally likely!***

Initially
DI = 5
DI = 3 1/3

4/10, 3/10, 2/10, 1/10

$\frac{1}{(\frac{4}{10})^2+(3/10)^2+(2/10)^2+(1/10)^2}$

$\frac{1}{\frac{16}{100}+\frac{9}{100}+\frac{4}{100}+\frac{1}{100}} = \frac{1}{\frac{30}{100}} = \frac{100}{30}$

Diversity Index

Initial diversity index was 5 from Result 1. Based on drawing four balls from the urn now computes to 3.33. Suggests something happened as we moved along the path. But it is slow relative to the rapid change seen in tipping phenomena.

So the Polya model does not represent a tipping process.

The difference between path dependence and different [tipping] points is one of degree. When you think of path dependence, what we mean is things along the path change where we're likely to go. So we move from, you know, this set of things to this other set of things but in a gradual way. Tipping points mean that there's an abrupt change, there were a bunch of things that could occur. Now we're likely to move to something entirely different, or something that was unexpected, or that uncertainty got resolved.