Session 19 – Learning Models: Replicator Dynamics

C19.1 Replicator Dynamics

Psychology –learning Economics – populations of people learning Ecology – model evolution Replicator Dynamics helps us understand the process of successful types evolution. Expect worse types to start copying the better types.	**Model:** Set of types {1, 2, 3, 4, 5, 6, 7, …N} Payoff for each type π(i) Proportion of each type Pr(i)
LEARNING Model: People using different strategies (S1:40%-5, S2:40%-4, S3:20%-6) Percentage of people (y) using with payoffs (x) (y%-x) for each Si. What Strategy to Use? If follow the more Common Si – standing ovation, contagion, conformity, or the more Rational Actor (follow the one with the highest payoff). Involves a combination of models: (1) copying dominant strategies and (2) choosing highest payoff strategies.	**EVOLUTION (fitness) Model**: Can use learning model to think about evolution. Think Phenotypes (Pi) in place of Strategies (Si), e.g., tongue length of frogs, % & fitness (P1:50%-5, P2:30%-4, P3:20%-6) Evolution is doing the same tradeoffs. Higher proportions have a reproductive advantage but the more fit species do as well. Expect % to change over time.
Quiz: In terms of either learning or evolution, what two pieces of information do we need to know about each strategy or phenotype? (a) Proportion and Six Sigma, (b) Proportion and Payoff (fitness), (c) Proportion and distribution, (d) Payoff and fitness **Ans:** (b) Proportion and Payoff (fitness)	
Fisher's Theorem: Rate of adaptation of a population is proportional to the variation of the population – variation in the types. Convergence to more fit phenotypes is faster if there is more variation.	**6-Sigma:** Fisher's Theorem tends to run counter to 6-sigma error reduction concept which focuses on convergence to a static (local?) maximum. That is, more variation is worse. (Or maybe not, different purposes)

C19.2 The Replicator Equation

Replicator Dynamics in the context of LEARNING Set of types {1, 2, 3, 4, 5, 6, 7, …, N} These are actions or strategies **Payoff** for each type $\pi(i)$ How well each type is doing **Proportion** (or ***probability***) of each type $Pr(i)$ <u>Fraction</u> of each type <u>or probability</u> of (i)	**Strategies:** **Rule-Based** (Sociological): Copy someone else (choose highest proportion) or **Hill climb** **Rational Actor**: Choose the highest payoff
Quiz: The replicator dynamics model will seek to tell us something about the relationship between (1) copying what other agents are doing, and (2) doing whatever has the highest payoff. (a)True, (b) False. **Ans:** (a) True	
Replicator Dynamics tries to **'balance'** these two strategies **(payoff vs proportion).** Weight = π(i) + Pr(i) vs. $\boldsymbol{Weight = \pi(i) \bullet Pr(i)}$ The latter (product) is preferred because it deals with Pr(i) = 0 more rationally. Won't use former.	**Replicator Equation** Numerator: Probability (i) plays strategy Si times its payoff. Denominator = $\sum_{all}$ Prob payoffs $Pr_{t+1}(i) = \frac{Pr_t(i)\pi(i)}{\sum_{j=1}^{N} Pr_t(j)\pi(j)}$

Example (Period t):
Payoffs: (2, 4, 5) & Proportions (1/3,1/6,1/2)
W1 = 2(1/3) =4/6
W2 = 4(1/6) = 4/6
W3 = 5(1/2) =15/6
Sum = 23/6

Next Period (t+1) strategy then is:
Pr(1) =(4/6)/(23/6) = 4/23
Pr(2) =(4/6)/(23/6) = 4/23
Pr(3) =(15/6)/(23/6) = 15/23
Sum = 23/23 = 1

Quiz: There are three strategies (S1,S2,S3). The payoffs for those strategies are (6, 9, 12). The proportions of people using each strategy at time t are (1/3, 1/3, 1/3). What will be the proportion of people using S1 at time t+1? (a) 3/27, (b) 6/3, (c) 1/3, (d) 6/27

Analysis: Find $\sum_{j=1}^{N} Pr_t\ j)\pi\ j) = \sum_{j=1}^{3} Pr_t\ j)\pi\ j) = \left(6\left(\frac{1}{3}\right)+9\left(\frac{1}{3}\right)+12\left(\frac{1}{3}\right)\right) = \ 2+3+4) = 9$ then find $Pr_t\ i)\pi(i) = 6\left(\frac{1}{3}\right) = 2,$ then calculate $Pr_{t+1}\ i) = \frac{Pr_t\ i)\pi(i)}{\sum_{j=1}^{N} Pr_t\ j)\pi\ j)} = \frac{2}{9} = \frac{6}{27}$

Ans: (d) 6/27

Explanation: *Remember the replicator equation:* $Pr_{t+1}\ i) = \frac{Pr_t\ i)\pi(i)}{\sum_{j=1}^{N} Pr_t\ j)\pi(j)}$,
Then calculate the weight of each strategy by multiplying its proportion by its payoff:
*The weight of S1 is 1/3*6=6/3*
*The weight of S2 is 1/3*9=9/3*
*The weight of S3 is 1/3*12=12/3*
So the denominator in our replicator equation will be 6/3+9/3+12/3=27/3
The proportion using S1 at time t+1 is (6/3)/(27/3)=6/27.

Shake – Bow Game:

P2

	S	B
S	2,2	0,0
B	0,0	1,1

$P(S,B) = Pr(½\ ,½\ )$
π_{P1} (S, B) = (**2**(½) + **0**(½), **0**(½) + **1**(½)) = (1, ½)

$W_S = \pi(i)Pr(i) = 1(½) = ½$

$W_B = (½)(½) = ¼$

Next Period Shakers and Bowers then is:
$Pr_S = W_S / (W_S + W_B)$ = ½ / (½ + ¼) = 2/3
$Pr_B = W_B / (W_S + W_B)$ = ¼ / (½ + ¼) = 1/3
Note: started with ½ : ½ but have moved to 2/3 : 1/3
Due to increased payoff (reminds me of a particle filter). In a large number of trials then it will migrate to [1 : 0] just as rational actors would do so all become Shakers (S).

SUV/ Compact Game:

	S	C
S	2,2	2,0
C	0,2	3,3

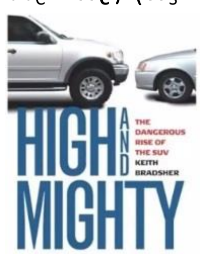
Can't see around, Don't feel safe. Note: 2,2 & 3,3 are NE.

P(S, C) = Pr(½ , ½)
π_{P1} (S, C) = (**2**(½) + **2**(½), **0**(½) + **3**(½)) = (2, 3/2)

$W_S = 2(½) = 1$

$W_C = (3/2)(½) = 3/4$

Next Period SUV and Compact then is:
$Pr_S = W_S / (W_S + W_C)$ = 1 / (1 + 3/4) = 4/7
$Pr_C = W_C / (W_S + W_C)$ = 3/4 / (1 + 3/4) = 3/7



Note: started with [½ : ½] but have moved to [4/7 : 3/7] Replicator dynamics moves us to 2,2 and not 3,3. RD leads us to non-optimal [2,2].
Book: *High and Mighty,* Keith Bradshoe

Quiz: Which of the following concepts from Section 17 better describes the Compact / SUV situation? (a) Common Resource, (b) Collective Action
Ans: (b) Collective Action

Summary: Learned one way to predict what people do is to build a replicator model. This captures two phenomena – that people are both copiers of other people and rational actors. And that these can lead to suboptimal equilibria (Compact/SUV game).

C19.3 Fisher's Theorem

Reminder:

$$Pr_{t+1}\ i) = \frac{Pr_t\ i)\pi\ i)}{\sum_{j=1}^{N} Pr_t\ j)\pi\ j)}$$

As applied to ***Ecology***

Replicator Dynamics in the context of LEARNING

Set of types {1, 2, 3, 4, 5, 6, 7, …N}

These are actions or strategies

Fitness of each type π(i)

How well each type is doing

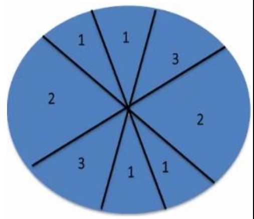
Proportion of each type Pr(i)

Fraction of each type or probability of (i)

Fitness Wheel:

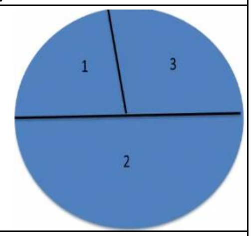
Spin the wheel. Stops. *Size of a slice* is ***proportional to fitness***.

Number of slices for that fitness is the population of that type.



Alternative Fitness Wheel view:

result of Pr(i) π(i)



Quiz: In a fitness wheel, the size of the slices of "pie" represents which one of the following? (a) Proportion of each type, (b) Fitness level of each type

Ans: (b) Fitness level of each type

Fisher's Theorem allows us to combine a number of models that we have already used:

Model 1: There is no cardinal

(lots of variation in a species)

Model 2: Rugged Landscape

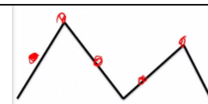
(a function with min – max)

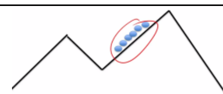
Model 3: Replicator Dynamics

(predict future of populations)

Fisher combines all of these models into one to give insight to role variation plays in adaptation.

Cardinals:  Genetic and phenotypic variation in the population of cardinals.

Rugged Landscape: (fitness)  different cardinals in ecosystem

Replicator Dynamics: sorts cardinals  sort themselves per RD over time

Fisher's Theorem (idea): Higher variance in a population increases the rate of adaptation.

Quiz: Which three models does Fisher's Theorem combine? (a) Rugged Landscape, (b) Prisoners' Dilemma, (c) There is no cardinal, (d) Replicator Dynamics **Ans: (a), (c), (d)**

Population 1 (Low Variation): (Pr : fitness) (1/3 : 3), (1/3 : 4), (1/3 : 5), **Average = 4** W1 = 3(1/3) = 1, W2 = 4(1/3) = 4/3, W3 = 5(1/3) = 5/3 Sum = 12/3 = 4	**Next Period:** Pr(1) = (3/3)/sum = 1/4 Pr(2) = (4/3)/sum = 1/3 Pr(3) = (5/3)/sum = 5/12	**New Avg Fitness:** 3(1/4) + 4(1/3) + 5(5/12) = 50/12 = **4 1/6**
Population 2 (Medium Variation): (Pr : fitness) (1/3 : 2), (1/3 : 4), (1/3 : 6), **Average = 4** W1 =2(1/3) = 2/3, W2 = 4(1/3) = 4/3, W3 = 6(1/3) = 6/3 Sum = 12/3 = 4	**Next Period:** Pr(1) = (2/3)/sum = 2/12 Pr(2) = (4/3)/sum = 4/12 Pr(3) = (6/3)/sum = 6/12	**New Avg Fitness:** 2(2/12) + 4(4/12) + 6(6/12) = 56/12 = **4 2/3**
Population 3 (High Variation): (Pr : fitness) (1/3 : 0), (1/3 : 4), (1/3 : 8), **Average = 4** W1 =0(1/3) = 0/3, W2 = 4(1/3) = 4/3, W3 = 8(1/3) = 8/3 Sum = 12/3 = 4	**Next Period:** Pr(1) = (0/3)/sum = 0/12 Pr(2) = (4/3)/sum = 4/12 Pr(3) = (8/3)/sum = 8/12	**New Avg Fitness:** 0(0/12) + 4(4/12) + 8(8/12) = 80/12 = **6 2/3**

Fitness Comparison:	**Fitness of Population:**	**Gain in Avg Fitness:**
Low Variation	(3, 4, 5)	1/6
Medium Variation	(2, 4, 6)	4/6
High Variation	(0, 4, 8)	2 4/6

Computing Variation of Population Fitness:	**Gain : Variation**	**Note** that Variation / Gain = 2 and is thus proportional per Fisher's Theorem "change in average fitness due to selection will be proportial to the variance."
$(f1 - avg)^2 + (f2\text{-}avg)^2 + (f3\text{-}avg)^2 =$	normalized	
$(3\text{-}4)^2 + (4\text{-}4)^2 + (5 - 4)^2 = 1 + 0 + 1 = 2$	**1 : 2**	
$(2\text{-}4)^2 + (4\text{-}4)^2 + (6 - 4)^2 = 4 + 0 + 4 = 8$	**4 : 8**	
$(0\ \text{-}4)^2 + (4 - 4)^2 + (8 - 4)^2 = 16 + 0 + 16 = 32$	**16 : 32**	

Quiz: The idea behind Fisher's Theorem is that higher variation increases the rate of adaptation. (a) True, (b) False
Ans: (a) True

Quiz: There are two populations. Population 1 consists of 1/4 individuals with fitness 5, 1/4 with 10, 1/4 with 15, and 1/4 with 20. Population 2 consists of 1/4 with fitness 11, 1/4 with 12, 1/4 with 13, and 1/4 with 14. Which population will have the greatest average fitness in the next round? (a) Population 1, (b) Population 2
Analysis: Both populations have same average fitness of 12.5. However Population 1 has larger variation so will move the Average Fitness more than Population 2. **Ans:** (a) Population 1
Explanation: *The average fitness for both populations begins at 12.5. Since there is greater variance in population 1, we know based on Fisher's Fundamental Theorem that there will be a greater increase in fitness in population 1. Doing the math, we find that the average fitness in the next round for population 1 is 13, while the average fitness next round for population 2 is 12.6.*

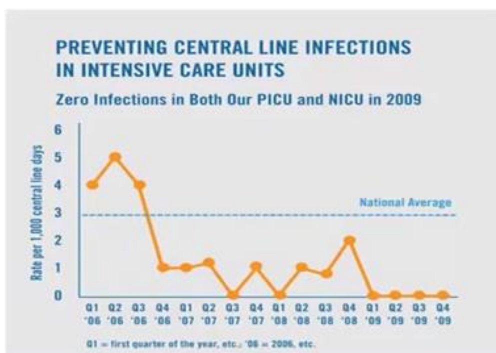
Summary: There is a rub, this "increased variation is good for adaptation" contrasts with the six-sigma view of reducing errors by reducting variation. (Impication: maybe errors are good for moving towards better solutions – let's see).

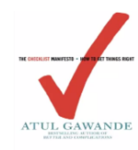
C19.4 Variation or Six Sigma

Replicator Dynamics:
more variation ➔ faster adaptation

Opposite Proverbs:
Both are true but have opposite proverbs:
"You're never too old to learn. You can't teach an old dog new tricks."
"Don't change horses in midstream. Variety is the spice of life."
"The pen is mightier than the sword. Actions speak louder than words."
Models have assumptions but *proverbs don't*. And context matters when it's appropriate to use which model 'type'.

Six Sigma: Reduce variation around 'desired' point, in this case the 'highest' value results in net value (sum) greater than the net value with higher variation.





Context landscape:
At a peak (or equilibrium world): ➔ want to coalesce around that peak ➔ six sigma
If not at peak (or context is not static): ➔ RD as more variation will find the peak faster.

Fixed Landscape: get to peak quickly and use then use six sigma to optimize. Assumes peak will be static. E.g., pilot checklists
Dancing Landscape: maintain variation, typical of ecological or competitive business environments

Quiz: If you are working with a fixed landscape, Fisher's Theorem will likely be a better model than six sigma. (a) True, (b) False. **Ans:** (b) False