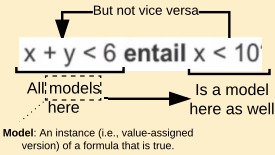


WEEK 1: Entailment and other concepts

ENTAILMENT IN ARITHMETIC FORMULAS

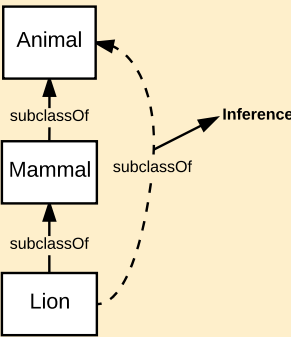


ENTAILMENT IN CONCEPT HIERARCHIES

Concept: An abstraction or generalization arrived through experiences or transformation of existing ideas.

Lion subclassOf Mammals

Everything that is true for mammals must be true for all lions, but not everything true for lions is true for all mammals.



Universe: {object1, object2, object3, ...}

Knowledge base: {axiom1, axiom2, axiom3, ...}

Axiom: Lion subclassOf Mammal

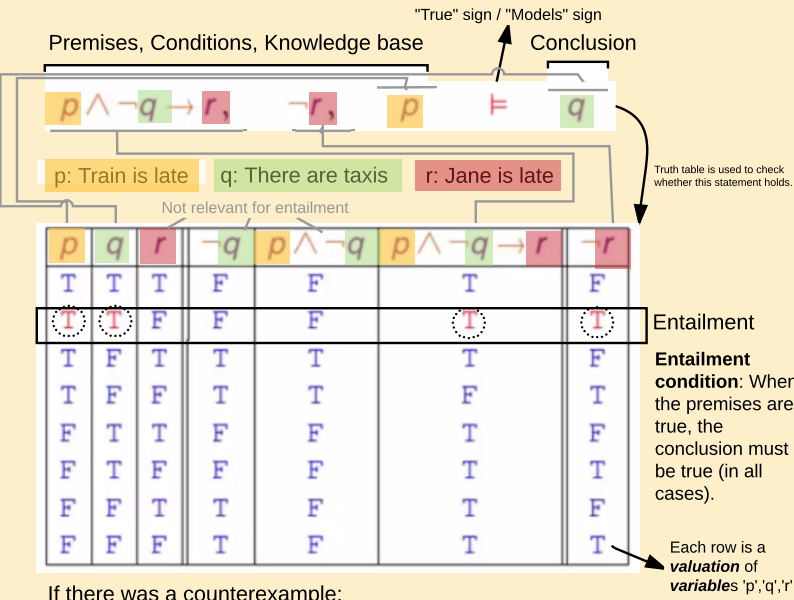
This statement becomes a model of the axiom only if the model is true,.

Model: lionNamedX subclassOf Mammals

Not a model (and not a counterexample): snakeNamedY subclassOf Mammals

This statement is not true; therefore, it is not a model. If it was true, then it would have been a counterexample, but it is not.

ENTAILMENT IN PROPOSITIONAL LOGIC



If there was a counterexample:

T	F				T	T
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COUNTEREXAMPLE

$\phi_1, \dots, \phi_n \neq \psi$

Does  $p \vee q$  entail  $p \rightarrow q$ ?

p	q	$p \vee q$	$p \rightarrow q$	
T	T	T	T	✓ Entailment condition met
T	F	T	F	✗ Cond. not met (Counterexam.)
F	T	T	T	✓ Entailment condition met
F	F	F	T	Irrelevant, as premise is F

TRUTH TABLE

Not all lines in a truth table make sense. Establishing validity is to ascertain which ones do.

p	q	r	$\neg q$	$p \vee \neg q$	$p \vee \neg q \rightarrow r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	F

e.g., Train is late (p), and there are no taxis ( $\neg q$ ), but in one case Jane is late (r) and in the other she is not late.

EQUIVALENCE

$p \rightarrow q \equiv \neg p \vee q$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Same

EXCLUSIVE 'OR'

$\phi$	$\psi$	$\phi \oplus \psi$
T	T	F
T	F	T
F	T	T
F	F	F

DISJUNCTION ('OR')

$\phi$	$\psi$	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

TAUTOLOGY

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

IMPLICATION

$\phi$	$\psi$	$\phi \rightarrow \psi$
T	T	T
T	F	F
F	T	T
F	F	T

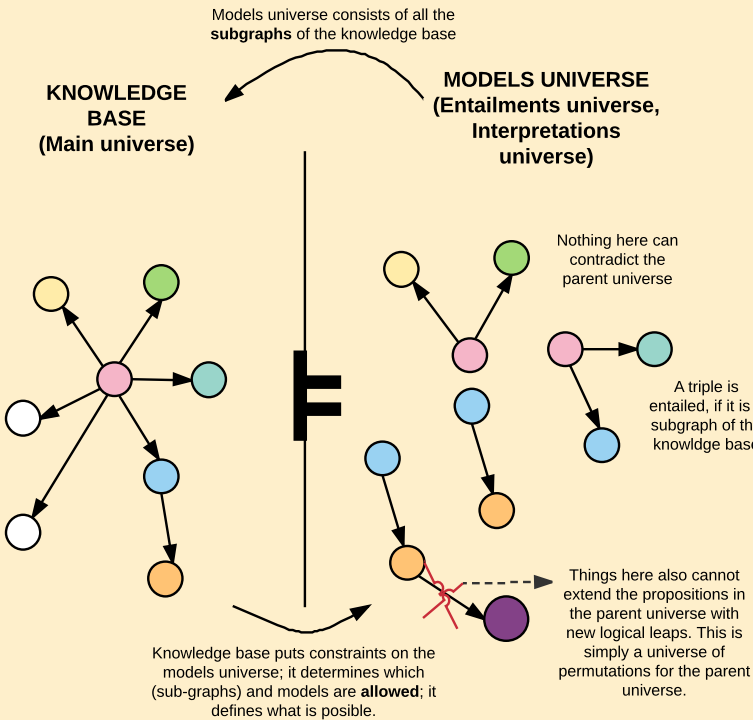
If you start out with a true premise, then the implication should be true only when the conclusion is also true. (This corresponds to the scenario in when  $\phi$  is true, the truth of the implication is the same as the truth of  $\psi$ .)

If you start out with a false premise, then, as far as implication is concerned, you are free to conclude anything. (This corresponds to the scenario in when  $\phi$  is false, the implication  $\phi \rightarrow \psi$  is true no matter what  $\psi$  is.)

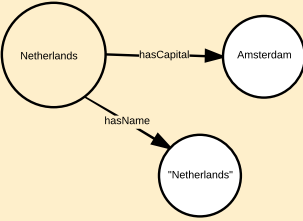
CONTRADICTION

p	q	$p \rightarrow q$	$\neg q$	$p \wedge \neg q$	$(p \rightarrow q) \wedge (p \wedge \neg q)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

ENTAILMENT IN SIMPLE GROUNDED GRAPHS



KNOWLEDGE BASE (Main universe)



Knowledge Base = { (Netherlands has\_name "Netherlands") (Netherlands has\_capital Amsterdam) }

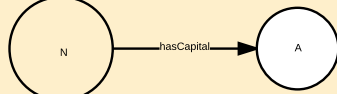
MODELS UNIVERSE (Entailments universe, Interpretations universe)

Model:



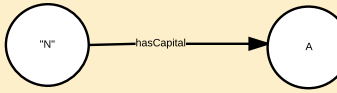
Interpretation(hasCapital)={Netherlands, Amsterdam}

Alternative model:



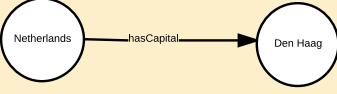
altInterpretation = { @, \$, % }, Interpretation("Netherlands") = "N" Interpretation("Netherlands") = "N" Interpretation("Amsterdam") = "A" Interpretation(hasCapital)={N, A}

Not a model:



Interpretation(hasCapital)={"Netherlands", Amsterdam} This statement establishes a relationship between Netherland's name and Amsterdam, and therefore, it is not a model of the knowledge base.

Not a model:



Interpretation(hasCapital)={Netherlands, Den Haag} This statement establishes a relationship between Netherland's name and Amsterdam, and therefore, it is not a model of the knowledge base.