Time-lagged Independent Component Analysis

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Instantaneous mixing model

- **Problem**: m independent sources s_m are mixed into n signals x_n

$$x_n = As_n \tag{1}$$

where A the mixing matrix and s_m are unknown.

- **Solution**: to recover the sorces s_m from this model we estimate a separating matrix W, which is inverse to A under the condition that all sources are stochastically independent and $n \geq m$

$$s_m = Wx_n \tag{2}$$

Whitening

- Whitening the data $x(t)$ is a preprocessing step of TICA. It is used to decorrelate the data x(t) to achieve independence after this step.

- **Attention**: When $x(t)$ is uncorrelated it doesn't mean $x(t)$ is independent.

Whitening

- The first step is subtracting the mean of $x(t)$.

$$\tilde{x} = x(t) - mean(x(t)) \tag{3}$$

- Step two is computing the covariance matrix of $x(t)$.

$$C = cov(x(t)) = E\{x_i(t)x_j(t)\} \tag{4}$$

Whitening

- Step three is the diagonalization of C.

$$C = V \Lambda V^T \tag{5}$$

- With step four we receive the principal components

$$y(t) = V^T \tilde{x}(t) \tag{6}$$

Whitening

- The last step is to normalize the principal components $y(t)$, such that $C = I$
- We obtain whitened(normalized) data $z(t)$ with

$$z(t) = \Lambda^{-\frac{1}{2}} y(t) \tag{7}$$

TICA

- To achieve stochastical independence for data $x(t)$ we use TICA for whitened $z(t)$. For that we are using a time lag τ.

- The first step is to compute a time-lagged covariance matrix

$$C_\tau^z = E\{z(t)z(t-\tau)^T\} \tag{8}$$

TICA

- The second step is making the covarience matrix symmetric.

$$\tilde{C}_{\tau}^{z} = \frac{1}{2}[C_{\tau}^{z} + (C_{\tau}^{z})^{T}] \tag{9}$$

- Step three is the diagonalization of the symmetric covariance matrix.

$$\tilde{C}_{\tau}^{z} = V \Lambda V^{T} \tag{10}$$

TICA

- With the the last step we receive the separating matrix W and the independent sources $s(t)$.

$$W = V^T \tag{11}$$

$$s(t) = Wx(t) \tag{12}$$

The AMUSE algorithm

- **Step on:** Whiten the data $x(t)$ to obtain $z(t)$.
- **Step two:** Apply TICA to $z(t)$.