ECON 100A - Section Notes
September 16, 2025

GSI: Clotaire Boyer

Quick reminders on the course notions

Budget set. Prices $\boldsymbol{p} = (p_1, p_2) \in \mathbb{R}^2_{++}$, income $m > 0$:

$$B(\boldsymbol{p}, m) = \{(x_1, x_2) \in \mathbb{R}^2_+ : \ p_1 x_1 + p_2 x_2 \le m\}, \qquad \text{budget line: } p_1 x_1 + p_2 x_2 = m.$$

Preferences, ICs, MRS. A utility $u : \mathbb{R}^2_+ \to \mathbb{R}$ represents (nice) preferences; indifference curves are level sets $\{(x_1, x_2) : u = \bar{u}\}$. Marginal utilities $MU_i = \partial u / \partial x_i$, and the marginal rate of substitution (good 1 for 2) is

$$\mathrm{MRS}_{12}(\boldsymbol{x}) = \frac{MU_1}{MU_2} \quad (\text{IC slope} = -\mathrm{MRS}_{12}).$$

Tangency method (interior optimum). With monotone & strictly convex preferences, the interior optimum $\boldsymbol{x}^\star$ solves

$$\mathrm{MRS}_{12}(\boldsymbol{x}^\star) = \frac{p_1}{p_2}, \qquad p_1 x_1^\star + p_2 x_2^\star = m.$$

Steps: (i) compute MU_1, MU_2 and MRS; (ii) set MRS $= p_1/p_2$; (iii) combine with the budget; (iv) check corners if needed.

Slutsky decomposition (I define the notion in words below: this is enough if you understand this). A price change does two conceptually distinct things:

- **Substitution effect (SE):** holding the original purchasing power (utility) fixed, the consumer re-optimizes at the new *relative* prices and slides along the same indifference curve. Graphically: draw a budget line *parallel to the new one* that passes through the *old bundle*; the move from the old choice to this compensated choice is the SE.
- **Income effect (IE):** with the actual (uncompensated) budget, purchasing power changes; the move from the compensated choice to the new Marshallian choice is the IE.

SE captures pure relative-price substitution; IE captures how real income shifts change demand.

Slutsky identity (math: you do not need to get it perfectly right). Let $\boldsymbol{x}(\boldsymbol{p}, m)$ be Marshallian demand and $h(\boldsymbol{p}, \bar{u})$ Hicksian demand. Then, for small changes:

$$\frac{\partial x_i}{\partial p_j} = \underbrace{\frac{\partial h_i}{\partial p_j}}_{\text{substitution (compensated)}} - \underbrace{x_j \frac{\partial x_i}{\partial m}}_{\text{income}}.$$

I am re-defining below in words this time :) WARP and IIA and try to give you some motivations on each; abstract set theory will not fully get you what each notion matters for economists so you might find what follows more interesting than last week's notes!

WARP (what you test in standard consumer choice). Deterministic, budget-based consistency. If bundle $\boldsymbol{x}$ is chosen from a budget $B = \{\boldsymbol{p}\cdot \leq m\}$ while some was affordable, then there is no other budget B' where both $\boldsymbol{x}$ and are affordable and the consumer chooses instead of $\boldsymbol{x}$. *Takeaway for 100A:* WARP is the minimal "no direct cycles" condition that makes observed choices rationalizable by *some* utility. It's about consistency across **budgets**, not about how close substitutes share demand.

IIA (relevant in *probabilistic* discrete choice, e.g., Logit). A modeling *assumption* on how choice *probabilities* react to menu changes: adding or removing other options does not change the odds of picking i over j. In words, the presence of a third alternative should not tilt the relative likelihood of i vs. j. *Where it comes from:* IIA holds exactly in the multinomial logit (MNL) model:

$$\frac{P(i \mid S)}{P(j \mid S)} = \exp\bigl(V_i - V_j\bigr),$$

so the ratio depends only on i and j, not on what else is in S. In sum, "proportional substitution" i.e. if you add a near-duplicate of i, it steals share *proportionally* from all alternatives, not mainly from the closest substitute (the classic red-bus/blue-bus IO issue).

Practice

1. Tangency method: well-behaved utilities

In each case, *find the optimal choice using the tangency method* and show intermediate steps.
a) $u = \ln x_1 + 4\ln x_2, \quad p_1 = 3,\ p_2 = 2,\ m = 30.$
b) $u = x_1 + \sqrt{x_2}, \quad p_1 = 8,\ p_2 = 2,\ m = 12.$
c) $u = \ln x_1 + 2\ln x_2, \quad p_1 = 2,\ p_2 = 1,\ m = 30.$

2. Not well-behaved: solve by picture

Sketch the budget set and two or three indifference curves; use geometry to identify the optimum.
a) $u = \min\{x_1, x_2\}, \quad p_1 = 3,\ p_2 = 2,\ m = 20.$
b) $u = \max\{x_1, x_2\}, \quad p_1 = 2,\ p_2 = 1,\ m = 10.$

3. Slutsky decomposition on a diagram

I will draw a budget line and a decision maker's optimal choice x_o from the budget set it bounds.
a) The price of good 1 increases. Add to the diagram an example of how the consumer's new budget line might look, and how the 'hypothetical' budget line would look in a Slutsky decomposition of the effect of the price change. Explain the position of each of the additions to the diagram.

b) For the decision maker's choices to be consistent with the weak axiom of revealed preference, when we perform the Slutsky decomposition, what must be true about the size of the substitution effect? Carefully explain your answer with reference to the diagram.