ECON 100A - Section Notes
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Monopoly: a one–page refresher (tools for problem solving)

Setup. Let inverse demand be linear: $P(Q) = a - bQ$ with $a > 0$, $b > 0$. Let cost be $C(Q) = F + cQ$ (fixed cost $F \geq 0$, constant marginal cost $c \geq 0$). Then

$$\mathrm{TR}(Q) = P(Q)Q = (a - bQ)Q, \quad \mathrm{MR}(Q) = \frac{d\,\mathrm{TR}}{dQ} = a - 2bQ, \quad \mathrm{MC} = c.$$

Monopoly choice (uniform price). A profit-maximizing monopolist chooses Q_M with $MR(Q_M) = MC$ (provided $P(Q_M) \geq 0$); price $P_M = P(Q_M)$; profit $\pi_M = P_M Q_M - cQ_M - F$. For linear demand and constant $MC = c$:

$$Q_M = \frac{a-c}{2b}, \qquad P_M = \frac{a+c}{2}, \qquad \pi_M = \frac{(a-c)^2}{4b} - F.$$

Surplus and deadweight loss (DWL). The efficient (first-best) quantity solves $P(Q^{\text{eff}}) = MC$, i.e.

$$Q^{\text{eff}} = \frac{a-c}{b}, \qquad P^{\text{eff}} = c.$$

Under uniform monopoly pricing:

$$\mathrm{CS}_M = \frac{1}{2}\,(a - P_M)\,Q_M = \frac{(a-c)^2}{8b}, \quad \mathrm{PS}_M = \text{revenue} - \text{variable cost} = (P_M - c)Q_M = \frac{(a-c)^2}{4b}.$$

Total surplus at the efficient allocation (net of variable cost) is

$$\mathrm{TS}^{\text{eff}} = \int_0^{Q^{\text{eff}}} \big(P(q) - c\big)\,dq = \frac{(a-c)^2}{2b}.$$

Hence the standard DWL triangle is

$$\mathrm{DWL} = \mathrm{TS}^{\text{eff}} - \big(\mathrm{CS}_M + \mathrm{PS}_M\big) = \frac{(a-c)^2}{8b}.$$

(Notes: Producer surplus excludes F; profit is $\pi = \mathrm{PS} - F$.)

First-degree price discrimination (perfect PD). The firm sets individualized prices and serves all buyers with $P(q) \geq c$, so $Q^{\text{PD}} = Q^{\text{eff}}$. It extracts *all* surplus above MC:

$$\pi^{\text{PD}} = \int_0^{Q^{\text{eff}}} \big(P(q) - c\big)\,dq - F = \frac{(a-c)^2}{2b} - F, \qquad \mathrm{CS}^{\text{PD}} = 0, \quad \mathrm{DWL}^{\text{PD}} = 0.$$

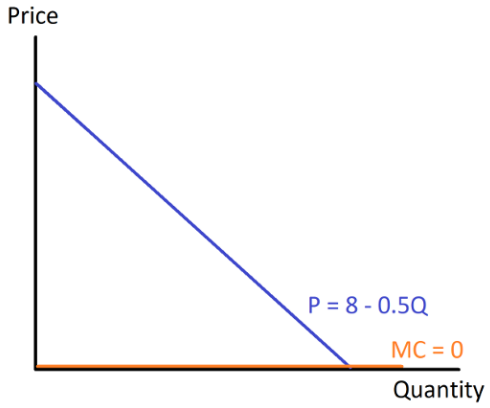
Value of perfect PD (relative to uniform monopoly) is

$$\pi^{\text{PD}} - \pi_M = \frac{(a-c)^2}{4b}.$$

Section Exercises

Take $\sim$ 15 minutes to work on these exercises individually, then turn to your neighbors and discuss your responses in small groups for a few minutes. We will then come together to work through them as a class, with groups sharing their progress/responses.

1. A profit-motivated monopolist produces a product with inverse demand curve $P = 8 - 0.5Q$. The monopolists only production cost is a fixed cost of \$20. The diagram below illustrates the inverse demand curve and marginal cost.



   (a) Say that the monopolist must set a constant per-unit price for each unit of its output. What price and what quantity of output will they choose? How much profit do they earn?

   (b) What price per unit would maximize the sum of consumer and producer surplus? What quantity would be traded? How much producer surplus and how much profit would the monopolist earn?

   (c) Instead of choosing their preferred constant per-unit price as in a), say that the firm could buy detailed data on all their potential customers that would enable them to use first degree price discrimination. What's the most that the monopolist would be willing to pay for this data? Show your work and explain what you did to calculate your answer.

Discussion Prompts

Break into groups of 3 (different from your section exercise groups). Discuss the prompt for $\sim$ 5 minutes and prepare a (written) summary of your discussion to share with the class. We will then come together and discuss both prompts.

1. *Market power is nothing to worry about new competitors could come along at any moment.* Agree or disagree?