ECON 100A - Section Notes
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Two-Good Exchange Economies: Quick Recap

Primitives

Two agents $i \in \{A, B\}$; two goods $g \in \{1, 2\}$. Endowments: $\omega_i = (\omega_{1,i}, \omega_{2,i}) \in R^2_+$; aggregate $\Omega = \omega_A + \omega_B$. Feasible allocations (x_A, x_B) satisfy $x_A, x_B \in R^2_+$ and $x_A + x_B = \Omega$. Preferences $\succeq_i$ (often continuous, convex, locally non-satiated) admit a utility representation $u_i(x_i)$.

Edgeworth Box

Rectangle of size $\Omega_1 \times \Omega_2$ with opposite origins for A and B. Endowment point $\omega = (\omega_A, \omega_B)$. Indifference curves: convex toward each agent's origin.

Pareto Efficiency (PE)

An allocation (x_A, x_B) is PE if no feasible (y_A, y_B) makes both agents weakly better off and at least one strictly better off. *Interior FOC (differentiable, LNS):* if $x_A \gg 0$ and $x_B \gg 0$, then

$$\mathrm{MRS}_A(x_A) \; = \; \frac{MU_{1,A}}{MU_{2,A}} \; = \; \frac{MU_{1,B}}{MU_{2,B}} \; = \; \mathrm{MRS}_B(x_B),$$

i.e. indifference curves are tangent (the *contract curve*). Corner cases obey standard KKT inequalities.

Competitive (Walrasian) Equilibrium (CE)

Prices $p \in R^2_{++}$ **and allocation** (x^*_A, x^*_B) such that :
(i) $x^*_i \in \arg\max\{u_i(x_i) : p \cdot x_i \leq p \cdot \omega_i,\ x_i \geq 0\}$ for $i \in \{A, B\}$;
(ii) $x^*_A + x^*_B = \Omega$.

Interior characterization: $\mathrm{MRS}_A(x^*_A) = p_1/p_2 = \mathrm{MRS}_B(x^*_B)$ and $p \cdot x^*_i = p \cdot \omega_i$. Existence holds under continuity, convexity, and LNS.

Welfare Theorems:

First: Any CE allocation is PE (under LNS).
Second: Any PE allocation can be decentralized as a CE after lump-sum endowment transfers (under convexity and continuity).

CE method (2 goods, 2 agents)

1. **Normalize prices:** set $p_1 = 1$, write $p_2 = p > 0$.

2. **Incomes from endowments:** $m_i = \omega_{1,i} + p\,\omega_{2,i}$.

3. **Individual demand:** solve

$$x_i^*(p, m_i) \in \arg\max_{x_i \geq 0} u_i(x_i) \text{ s.t. } x_{1,i} + p\,x_{2,i} = m_i.$$

   (Interior: $\text{MRS}_i = 1/p$ + budget; otherwise check corners.)

4. **Clear one market (Walras' Law):**

$$x_{1,A}^*(p, m_A) + x_{1,B}^*(p, m_B) = \Omega_1 \quad \Rightarrow \quad p^*.$$

5. **Compute and verify:** $x_i^* = x_i^*(p^*, m_i^*)$, check $x_i^* \geq 0$ and $x_A^* + x_B^* = \Omega$.

Section Exercises

Take $\sim$ 15 minutes to work on these exercises individually, then turn to your neighbors and discuss your responses in small groups for a few minutes. We will then come together to work through them as a class, with groups sharing their progress/responses.

1. An exchange economy has two people, Biniam (B) and Jasper (J), and two goods, steak (s) and candy (c). Preferences for Biniam are represented by $u_B = s_B - c_B$ and for Jasper by $u_J = s_J$, where s_B is the amount of steak Biniam gets, and so on. Biniam's endowment is 3 steak and 2 candy; Jasper's 1 steak and 2 candy.

   (a) Sketch an Edgeworth box to represent this economy; make sure to properly label the axes. Mark the endowment point on your diagram.

   (b) On your diagram from a), sketch an indifference curve for each person that passes through the endowment point. Indicate in what area, if any, we would find allocations that are Pareto improvements over the endowment point, and explain your answer in simple terms.

   (c) Is there a competitive equilibrium in this economy with prices for each good $p_S > 0$ and $p_C > 0$? If so, find it. If not, explain why not.

2. Consider a two-person, two-good exchange economy. Each person has well-behaved preferences represented by utility functions $u_A = x_{1,A} + \ln x_{2,A}$ and $u_B = \ln x_{1,B} + x_{2,B}$, respectively. Endowments are $\omega_A = (2, 4)$ and $\omega_B = (4, 1)$. Normalize the price of good 1 to 1 and let the price of good 2 be p. It can be shown that, as a function of p, A's demand functions are $x_{1,A}^* = 1 + \frac{4}{p}$ and $x_{2,A}^* = 1$; B's are $x_{1,B}^* = p$ and $x_{2,B}^* = \frac{4}{p}$.

   (a) Find and write down a competitive equilibrium in this economy. In a couple of sentences, explain in simple terms why your answer is a competitive equilibrium.

(b) The conditions of the First theorem of welfare economics are satisfied in this economy, and so we know that the competitive equilibrium you found in a) has a particular property. State that property, prove mathematically that it holds true at the equilibrium from a), and explain (precisely but with as little jargon as possible) how your math proves it.

Discussion Prompts

Break into groups of 3 (different from your section exercise groups). Discuss the prompt for $\sim$ 10 minutes and prepare a (written) summary of your discussion to share with the class. We will then come together and discuss both prompts.

1. Does general equilibrium theory offer any guidance about how we could reduce inequality in society?

2. What, if anything, do the two fundamental theorems of welfare economics teach us about economic policy?