ECON 100A - Section Notes
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Production & Firm Behavior

Technology. A (single–input) production function $y = f(x)$ or (two–input) $y = f(x_1, x_2)$ maps inputs to maximal output. Standard assumptions: f is increasing in inputs; often concave.

Marginal product (MP) & diminishing returns. $\text{MP}_x = \frac{\partial f}{\partial x}$, $\quad \text{MP}_i = \frac{\partial f}{\partial x_i}$. *Diminishing marginal returns*: $\frac{\partial^2 f}{\partial x_i^2} < 0$.

Returns to scale (RTS). For $t > 0$, compare $f(t\mathbf{x})$ to $tf(\mathbf{x})$:

$$\begin{cases} f(t\mathbf{x}) > tf(\mathbf{x}) & \text{IRS,} \\ f(t\mathbf{x}) = tf(\mathbf{x}) & \text{CRS,} \\ f(t\mathbf{x}) < tf(\mathbf{x}) & \text{DRS.} \end{cases}$$

Isoquants & MRTS. Isoquant for output $\bar{y}$: $\{\mathbf{x} : f(\mathbf{x}) = \bar{y}\}$. Marginal Rate of Technical Substitution (of x_1 for x_2):

$$\text{MRTS}_{12} \equiv -\frac{dx_2}{dx_1}\bigg|_y = \frac{\text{MP}_1}{\text{MP}_2}.$$

Cost minimization (given w_1, w_2).

$$\min_{x_1, x_2} \; w_1 x_1 + w_2 x_2 \quad \text{s.t. } f(x_1, x_2) \geq y.$$

FOC for a smooth interior optimum: $\dfrac{\text{MP}_1}{\text{MP}_2} = \dfrac{w_1}{w_2}$. Let $h_i(w_1, w_2, y)$ be the conditional (cost-min) demands; the cost function is $C(w_1, w_2, y) = w_1 h_1 + w_2 h_2$. Average and marginal cost: $\text{AC}(y) = \frac{C(y)}{y}$, $\quad \text{MC}(y) = C'(y)$.

Single input shortcut. If $y = f(x)$ and input price is w, then

$$C(w, y) = w\,x(y), \quad \text{MC}(y) = \frac{w}{\text{MP}_x(y)} \quad \text{since } \frac{dy}{dx} = \text{MP}_x.$$

Profit maximization under price-taking. With output price p, profit is $\pi(y) = py - C(w, y)$. FOC (interior): $\quad p = \text{MC}(y^*)$. *Single input shortcut*: choose x^* so that

$$p \cdot \text{MP}_x(x^*) = w \quad \Rightarrow \quad y^* = f(x^*), \;\; \pi^* = py^* - wx^*.$$

Isoprofit lines. In (x, y) space with one input at price w and output price p:

$$\pi = py - wx \quad \iff \quad y = \frac{w}{p}\,x + \frac{\pi}{p}.$$

Slope $= \frac{w}{p}$; intercept $= \frac{\pi}{p}$. Break-even ($\pi = 0$) passes through the origin.

Perfect complements (Leontief–type). If $y = a \cdot \min\{g_1(x_1), g_2(x_2)\}$, optimal cost-min bundles satisfy the *kink condition* $g_1(x_1) = g_2(x_2) = y/a$ (no interior tangency). Substitute these equalities into $w_1x_1 + w_2x_2$ to get $C(w, y)$; then compare p to unit cost $C(w, 1)$ to decide $\arg\max_y \, py - C(w, y)$.

Section Exercises

1. A producer makes and sells output y using a single input x according to a production function $y(x) = 6\sqrt{x}$. All markets are perfectly competitive; y has a price 10 per unit and x has a price 10 per unit. There are no production costs other than those incurred to hire x.

   (a) If this producer is profit-motivated, find their optimal choice of x and y and how much profit they make.

   Explain in simple terms why the producer would earn less profit if they hired a little more of the input x than the amount you found.

   (b) Make a rough sketch of the producer's production function. Then we'll do two things.

   (i) Mark the profit-maximizing plan from a). Sketch the isoprofit line on which it lies (including its slope and intercept).

   (ii) Say that instead of being profit-motivated, the producer wanted to produce the most possible output without making negative profit. Sketch the isoprofit line on which their optimal choice in this case lies (including its slope and intercept). Mark the position of their optimal production plan in this case (no need to find it numerically, just mark the position on the diagram.)

2. Jim Corp. produces lecture slides y using two inputs, coffee (x_1, measured in cups) and labor (x_2, measured in hours). Its production function is $y = 10min\{\sqrt{x_1}, \sqrt{x_2}\}$. That is, it needs one cup of coffee and one hour of labor to achieve any output; coffee is useless without time to work, and time to work is useless without coffee. Jim Corp. sells lecture slides on the thriving black market for economics for a price of \$12 per unit. Coffee costs \$2 per cup and labor costs \$10 per hour.

   Does Jim Corp.s production function display increasing, decreasing, or constant returns to scale? Assuming that Jim Corp. maximizes profits, what is its optimal choice of production plan?

3. Consider a model of a producer in a perfectly competitive industry. Explain, in simple terms, why (under typical assumptions) marginal revenue equals marginal cost

describes the quantity of output that maximizes profit, but cannot tell us how much profit would be made if the producer chose that quantity.