1. "$0 \leq r \leq m$"
2. "$0 \leq r \leq n$"
3. "m - r"
4. "When d lies in the column space of A^t (the row space of A)"
5. "When the nullspace of A^t contains only the zero vector, i.e., when r = m"
6. "I don't know"
7. "I don't know"
8. "For a rank 2 matrix A in the space of 3×3 matrices, the nullspace is 1-dimensional. The basis consists of all non-zero vectors x satisfying $A \cdot x = 0$."
9. "The determinant of B^tB is 0."
10. "The rank of B is 2."
11. "The eigenvalues of A are 1, 4, and 6."
12. "The eigenvalues of B are √3, -√3, and 2."
13. "The eigenvalues of C are 0 (with multiplicity 2) and 6."
14. "Subtract 3 times the first equation from the second equation to eliminate x."
15. "The pivots are 2 and 6."
16. "The upper triangular matrix U is:[[2, 3],[0, 6]]"
17. "y = -1/2"
18. "I don't know"
19. "X is any non-zero vector in the nullspace of A. Since the nullspace is 1-dimensional, X can be written as $X = \alpha \cdot x_o$, where α is a scalar and x_o is a basis vector."
20. "Matrices of the form $A \cdot X$ have rank at most 2 and their columns are linear combinations of the columns of A."
21. "The nullspace of $A \cdot X$ is 1-dimensional, and the column space is 2-dimensional."
22. "The determinant of B^tB is 0."
23. "The eigenvalues of matrix A are 4 and 2."
24. "The eigenvalue associated with such a Markov matrix is $\lambda = 1$."
25. "The determinant of A - I is 0."
26. "The eigenvalues of C are 0 (with multiplicity 2) and 6."
27. "To ensure the Haar wavelet basis vectors are orthonormal, divide each vector by its length."
28. "A basis consists of the matrices [[1, 0], [0, 0]], [[0, 1], [0, 0]], [[0, 0], [1, 0]], and [[0, 0], [0, 1]]."
29. "The rank of matrix A is 2."
30. "The special solutions are [[-23/4, -1/4, 1, 0]] and [[-1/4, -7/4, 0, 1]]."
31. "The span of $S \cup T$ is S + T, the set of all sums of vectors from S and T."
32. "A 4×4 symmetric matrix has 10 independent entries that can be chosen freely."
33. "There are 6 independent entries in a 4×4 skew-symmetric matrix."
34. "The nullspace of C is the intersection of the nullspaces of A and B; that is, $N(C) = N(A) \cap N(B)$."
35. "The matrix E is [[1, 0, 0], [-2, 1, 0], [-2, 3, 1]]."
36. "The row-reduced form is [[1, 0, 23/4, 1/4], [0, 1, 1/4, 7/4], [0, 0, 0, 0]]."
37. "The steady-state vector is proportional to [9, 4]."
38. "The eigenvalues are 1 and -0.3; the corresponding eigenvectors are [9, 4] and [1, -1], respectively."
39. "Vectors on the plane have the form [x, y, z] = [12 + 3y + z, y, z]."
40. "The determinant of P is 1."
41. "Four pivots are required for A to satisfy A = LU with full rank."
42. "The independent variables are x3 and x4."
43. "The nullspace of C is the intersection of the nullspaces of A and B; that is, $N(C) = N(A) \cap N(B)$."