ECE447 - Homework 4

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A. Thought Experiment

Three orthogonal and degenerate energy valleys in bulk silicon have one electron each. The electric field is applied along the y-direction to which all three electrons respond giving rise to a current I_1. Now, some amount of tensile strain is applied in the material causing all three electrons located in the transverse valleys and producing a current flow of I_2. Given that $I \propto 1/m^*$ and $m_l^* \propto 4m_t^*$ what will be the ratio of I_2/I_1?

Solution
For the first current I_1, in the presence of the electric field $\vec{E} = \hat{y}E$, we will get the k_y electron along the semimajor axis of its energy valley, and the k_x and k_z electrons will move along the semiminor axis of their respective energy valleys. This tells us to use m_l^* for k_y, and m_t^* for k_x and k_z.
We can then find the average effective mass m_1^*:

$$\begin{aligned} m_1^* &= \frac{1}{3}(m_l^* + 2m_t^*) \\ &= \frac{1}{3}(4m_t^* + 2m_t^*) \\ &= 2m_t^* \end{aligned}$$

Then we can do the same, finding the average effective mass, for when all of the electrons are located in transverse valleys.

$$\begin{aligned} m_2^* &= \frac{1}{3}(3m_t^*) \\ &= m_t^* \end{aligned}$$

Using the inverse relationship between current and effective mass:

$$\boxed{\frac{I_2}{I_1} \propto \frac{1/m_t^*}{1/2m_t^*} = 2}$$

So, from adding tensile strain and forcing our electrons into their transverse valleys, we can effectively **double** the current.

B. Negative Differential Resistance

Explain the origin (from the bandstructure point of view) of negative differential resistance (NDR) is GaAs or GaN? How could you use this phenomenon in device applications?

Solution
For a free electron, its energy as a function of its wavenumber k, or momentum p, resembles a parabola. Which says that in the conduction band, as the electron gains momentum in the positive or negative direction, then the electron will also gain energy.

However, the periodic potential U_c from a semiconductor material destroys this perfect picture. Instead the energy in the bands fluctates aperiodically depending on the speed and direction of the particle. For an electron in GaAs or GaN, the crystal lattice does not look the same in all directions, so depending on where it is travelling in the crystal, it will behave differently.

From the derivation of effective mass, we found the first derivative of energy with respect to wavenumber was directly proportional to the velocity of the electron/particle.

$$\frac{dE}{dk} \propto v$$

If we imagine the slopes of the tangent lines along the conduction band for GaAs from the textbook, we find that the derivative in the lower energy valley is larger than the derivative in the higher energy valley. **The electron slows down as energy increases.**

The energy the electron feels is from our external voltage bias. So, as voltage increases, the conduction electrons lose speed.

$$I = qnAv$$

From the model for current in a conductor, if the electron's velocity drops, so does the current in the semiconductor.

For positive changes in voltage, the GaAs or GaN device will experience negative changes in current, hence negative differential resistance.

$$\frac{\Delta v}{\Delta i} < 0$$

Since we have a differential resistance, devices utilizing this dynamic range can be amplifiers—the basis of small-signal analysis is dependent on the differential conductance of a device ($r \propto 1/g$). Also, a device with NDR can be used an oscillator without the need for a feedback loop (like an oscillating op-amp circuit) since we can leverage the small decreases in current due to small increases in the bias voltage.

C. E-k Diagram Characterization

E-k diagram of a new semiconductor material (possibly MoS_2) is shown. Identify (and comment on) a few important parameters/features of this material.

Solution
Assuming $T = 300K$, the bandgap of this material is is over half an electronvolt larger than the bandgap for Silicon. Larger bandgaps allow engineers more play in the devices they can build.

The valence band's curvature is much shallower than the curvature of the conduction bands. This tells us that:

$$m_p^* > m_n^*$$

Current being inversely proportional to effective mass would mean for this material, holes do not contribute to current as much as electrons. However, the valence band experiences nearly zero degeneracy, but the conduction bands are much more energetically alike. This will make it much harder to place the electrons in the conduction band exactly as planned.

We also see that the valence band peak happens at Γ, but the conduction band minimum occurs at K. Since the bandgap is not local to the same direction in MoS_2, this device will not be good for optoelectronics, as deenergizing electrons have to drastically change their momentum to easily reach the valence band.

Problem 3.12

Plot $E_g = E_g(0) - \frac{\alpha T^2}{(\beta + T)}$

Solution
I implemented the following plot for $E_g(T)$ using python's library *matplotlib.pyplot*.

```
import matplotlib.pyplot as plt
import numpy as np
import math

# IMPORTANT CONSTANTS
Eg0 = 1.17          # Si bandgap at T=0K [eV]
alpha = 4.73e-4     # [eV/K]
beta = 636          # [K]

# define Eg(T) function
def bandgapTemp(T):
    """
    Calculate Si bandgap energy for a given temperature in K.
    Parameters:
    T = temp in K.
    """
    Eg = Eg0 - ( ( alpha * T**2 ) / (beta + T) )
    return Eg


##  Plotting ##
```

```
# Set up x-vals and input into Eg(T)
x = np.linspace(0, 600, 6000)
y = bandgapTemp(x)

markers_on = [3000]

# Create plot of Eg(T)
plt.plot(x, y, '-go', markevery=markers_on, label = "Eg(T)")

EgROOM = bandgapTemp(300)
plt.text((300 + 20), EgROOM, "(300, %.4f)" % EgROOM, fontsize = 12)

# Labels and Titles
plt.xlabel('Temperature (K)')
plt.ylabel('Bandgap Energy (eV)')
plt.title('Si Bandgap Energy v. Temperature')

# Axis Formatting
plt.xlim(0,600)

# Show plot
plt.legend()
plt.show()
```

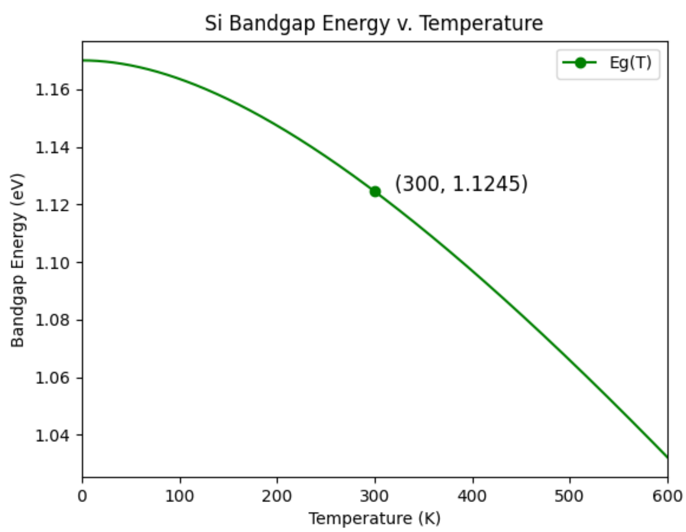

Figure 1: E_G versus T

For $T = 300K$, we see that the bangap energy of Silicon is 1.1245 eV, which is the textbook bandgap for Silicon at room temperature.

ECE 447

HW4
TEXTBOOK PROBS

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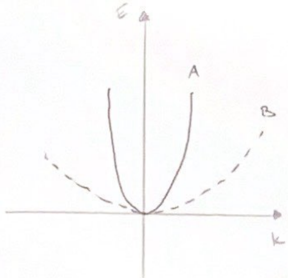
3.13 State which band will result in heavier electron mass. Why?



A and B are both positive parabolas

$\Rightarrow A = c_1 k^2$ $\quad c_1, c_2 \in \mathbb{R}$

$B = c_2 k^2$

Visually, $c_1 > c_2$.

Taking $\frac{d^2A}{dk^2}$ and $\frac{d^2B}{dk^2}$

$\Rightarrow \frac{d^2A}{dk^2} = c_1$ and $\frac{d^2B}{dk^2} = c_2$

This means $\frac{d^2A}{dk^2} > \frac{d^2B}{dk^2}$

We know that $\frac{1}{\hbar}\frac{d^2E}{dk^2} = \frac{1}{m^*}$

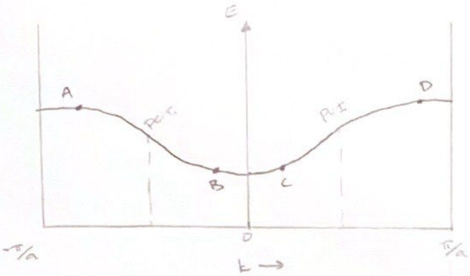
$\Rightarrow \frac{1}{\hbar}\frac{d^2A}{dk^2} > \frac{1}{\hbar}\frac{d^2B}{dk^2}$

$\Rightarrow \frac{1}{m_A^*} > \frac{1}{m_B^*}$

$\Rightarrow$ $\boxed{m_A^* < m_B^* \quad \therefore \text{ B has the heavier electron mass}}$

3.15 E versus k. Determine: (a) the sign of the effective mass

(b) direction of velocity for the particle



(A) Visually, the points A and D are located in concave down regions, and B and C are located in concave up regions

If CCDN, $\frac{d^2E}{dk^2} \propto \frac{1}{m^*} < 0$. If CCUP, $\frac{d^2E}{dk^2} \propto \frac{1}{m^*} > 0$.

This means $\boxed{\begin{array}{l} \text{A and D have negative effective mass} \\ \text{B and C have positive effective mass} \end{array}}$

(B) $p = mv = \hbar k \Rightarrow \underline{\underline{k = \frac{mv}{\hbar}}}$, so $\underline{\underline{k \propto v}}$.

On the positive k-axis we find $\boxed{\text{C and D} \therefore \text{positive velocity}}$ (+x-dir)

On the negative k-axis we find $\boxed{\text{A and B} \therefore \text{negative velocity}}$ (−x-dir)

$\boxed{3.20}$ For Si in the [100]-direction, the energy in this one-dimensional direction can be approximated as:

k_0 is k @ minimum energy).

$$E = E_0 - E_1 \cos(\alpha(k-k_0)) \quad ?$$ unclear...

Determine the effective mass of the particle @ $k = k_0$.

$$\frac{d}{dk}(E) = \frac{d}{dk}\left(E_0 - E_1 \cos\alpha(k-k_0)\right)$$

$$= +E_1 \sin(\alpha k - \alpha k_0)(\alpha)$$

$$\Rightarrow \frac{dE}{dk} = \alpha E_1 \sin(\alpha(k-k_0)) \qquad \left(\frac{dE}{dk} \propto v\right)$$

$$\frac{d}{dk}\left(\frac{dE}{dk}\right) = \frac{d}{dk}\left(\alpha E_1 \sin\alpha(k-k_0)\right)$$

$$\Rightarrow \frac{d^2E}{dk^2} = \alpha^2 E_1 \cos\alpha(k-k_0)$$

$$\Rightarrow \frac{1}{\hbar}\frac{d^2E}{dk^2} = \frac{\alpha^2 E_1}{\hbar}\cos\alpha(k-k_0) = \frac{1}{m^*}$$

@ $k = k_0$

$$\Rightarrow \frac{\alpha^2 E_1}{\hbar}\cancelto{1}{\cos\alpha(k_0-k_0)} = \frac{1}{m^*}$$

$$\Rightarrow \boxed{m^* = \frac{\hbar}{\alpha^2 E_1}}$$

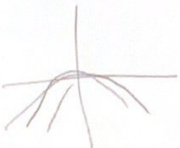
3.22 Heavy and light holes exist in GaAs w/ $m_{hh} = 0.45 m_0$ and $m_{\ell h} = 0.082 m_0$.

Determine: (a) the density-of-states effective mass

(b) the conductivity effective mass

(A) 3D Density of States is given by: (for holes)

$$g_{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2 m_{p,dos}^*}{\hbar^2}\right)^{3/2}\sqrt{E_V - E}$$



$g(E) \propto$ Volume $\propto k^3 \propto (m_{p,dos}^*)^{3/2}$ — (1)
(in k space)

The curves are spherically symmetric in k-space.

$\Rightarrow$ Total Volume $\propto (m_{hh}^*)^{3/2} + (m_{\ell h}^*)^{3/2}$ — (2)

Equating (1) and (2):

$$(m_{p,dos}^*)^{3/2} = (m_{hh}^*)^{3/2} + (m_{\ell h}^*)^{3/2}$$

$$\Rightarrow m_{p,dos}^* = \left[(m_{hh}^*)^{3/2} + (m_{\ell h}^*)^{3/2}\right]^{2/3}$$

$$\Rightarrow m_{p,dos}^* = \left[(.45 m_0)^{3/2} + (0.082 m_0)^{3/2}\right]^{2/3}$$

$$= \left[0.3254\, m_0^{3/2}\right]^{2/3}$$

$$\boxed{m_{p,dos}^* = 0.473\, m_0}$$

(B)

From textbook,

$$m_{cp}^* = \frac{(m_{hh}^*)^{3/2} + (m_{\ell h})^{3/2}}{(m_{hh}^*)^{1/2} + (m_{\ell h}^*)^{1/2}}$$

$$= \frac{(0.45 m_0)^{3/2} + (0.082 m_0)^{3/2}}{(0.45 m_0)^{1/2} + (0.082 m_0)^{1/2}}$$

$$= \frac{(\;)}{(\;)} m_0$$

$$\boxed{m_{cp}^* = 0.3399 m_0}$$

3.26 (a) Determine the total number (#/cm³) of energy states in Si between E_c and $E_c + 2kT$ @ (i) T = 300K
(ii) T = 400K

(b) Repeat for GaAs

$E_{g_{Si}} = 1.12eV$

$E_{g_{GaAs}} = 1.42eV$

$k = 8.6173 \times 10^{-5}$ eV/K

*note
$E_c + 2kT - E_c = +2kT$
can skip int step.

(a) Si
(i) T = 300K,

$$N = \int_{E_c}^{E_c+2kT} \frac{4\pi(2m_{n,dos}^*)^{3/2}}{h^3}\sqrt{E-E_c}\,dE$$

$$= \frac{4\pi(2m_{n,dos}^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(E-E_c)^{3/2}\Big|_{E_c}^{E_c+2kT}$$

$\times 1.6\times10^{-19}$

$$= \frac{4\pi(2\cdot1.08\cdot9.11\times10^{-31})^{3/2}}{(6.626\times10^{-34})^3}\cdot\frac{2}{3}(2\cdot8.6173\times10^{-5}\times300)^{3/2}$$

$$\boxed{N = 5.98\times10^{25}\ \frac{\#}{cm^{-3}}}$$

Integration bounds only change...

@ T = 400K

$$N = \frac{4\pi(2\cdot1.08\cdot9.11\times10^{-31})^{3/2}}{(6.626\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2\cdot8.6173\times10^{-5}\times400)^{3/2}$$

$$\boxed{N = 9.21\times10^{25}\ \frac{\#}{cm^{-3}}}$$

GaAs
(b) $m_{n,dos}^* = 0.067m_0$

@ T = 300K

$$N = \frac{4\pi(2\cdot0.067\cdot9.11\times10^{-31})^{3/2}}{(6.626\times10^{-34})^{3/2}}\cdot\frac{2}{3}(2\cdot8.6173\times10^{-5}\times300)^{3/2}$$

$$\boxed{N = 9.24\times10^{23}\ \#/cm^3}$$

@ T = 400K

Same but = 400K

$$\boxed{N = 1.42\times10^{24}\ \#/cm^3}$$