

ECE355L Project 4

Fourier Series

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Introduction

For this project, we are exploring the Fourier Series with MATLAB. Specifically, we are using the trigonometric Fourier Series:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t, \quad t_1 \leq t \leq t_1 + T_0 \quad (1)$$

Where:

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t) dt \quad (2)$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t dt \quad (3)$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \sin n\omega_0 t dt \quad (4)$$

Exercise I

Modify the example code to plot the partial sums for $n = 10, 20, 50$, and 100 . (For $y(t) = t$)

Solution.

```
1 % Chase Lotito - ECE355L
2 % Exercise 1
3
4 clc
5 clear all
6 close all
7 syms t k L n % Initialize symbolic variables
8 evalin(symengine,'assume(k,Type::Integer)'); % Let matlab know that the
   variable k is an integer
9 a = @(f,t,k,L) int(f*cos(k*pi*t)/L,t,-L,L); % create kth cosine
   coefficient a
10 b = @(f,t,k,L) int(f*sin(k*pi*t)/L,t,-L,L); % create kth sine
   coefficient b
11 fs = @(f,t,n,L) a(f,t,0,L)/2 + ...
12 symsum(a(f,t,k,L)*cos(k*pi*t/L) + b(f,t,k,L)*sin(k*pi*t/L),k,1,n); %
   generate the nth partial sum
13 f = t; % Original function
14 ezplot(fs(f,t,2,1),-1,1) % Plotting the functions and the partial sum
15 hold on
```

```

16 ezplot(fs(f,t,10,1),-1,1) % n=10
17 hold on
18 ezplot(fs(f,t,20,1),-1,1) % n=20
19 hold on
20 ezplot(fs(f,t,50,1),-1,1) % n=50
21 hold on
22 ezplot(fs(f,t,100,1),-1,1) % n=100
23 hold on
24 ezplot(f,-1,1)
25 hold off
26 title('Partial Sums n=2, 10, 20, 50, 100'),xlabel('Time'),ylabel('
    Amplitude')
27 legend('n=2', 'n=10', 'n=20', 'n=50', 'n=100','Original')

```

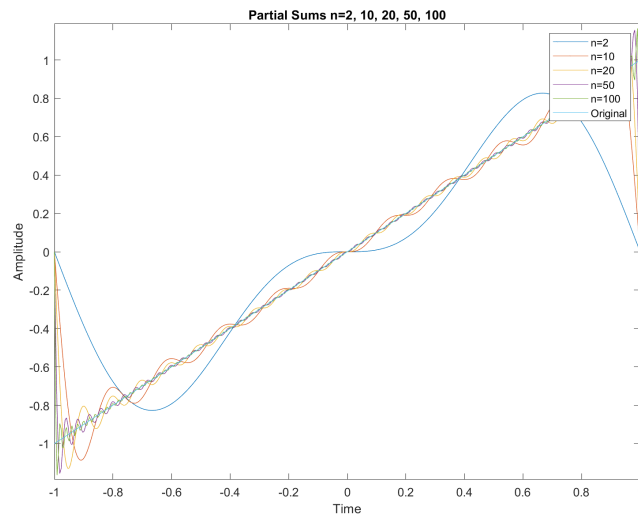


Figure 1: Trigonometric Fourier Series for $y(t) = t$ and $n = 10, 20, 50, 100$

Exercise II

Plot the partial sums for $n = 10, 20, 50, 100$, and 1000, for $f(t) = u(t)$.

Solution.

```

1 % Chase Lotito - ECE355L
2 % Exercise 2
3

```

```

4 clc
5 clear all
6 close all
7 syms t k L n % Initialize symbolic variables
8 evalin(symengine,'assume(k,Type::Integer)'); % Let matlab know that the
    variable k is an integer
9 a = @(f,t,k,L) int(f*cos(k*pi*t)/L,t,-L,L); % create kth cosine
    coefficient a
10 b = @(f,t,k,L) int(f*sin(k*pi*t)/L,t,-L,L); % create kth sine
    coefficient b
11 fs = @(f,t,n,L) a(f,t,0,L)/2 + ...
12 symsum(a(f,t,k,L)*cos(k*pi*t/L) + b(f,t,k,L)*sin(k*pi*t/L),k,1,n); %
    generate the nth partial sum
13 f = heaviside(t); % Original Function --> Unit Step
14 ezplot(fs(f,t,2,1),-1,1) % Plotting the functions and the partial sum
15 hold on
16 ezplot(fs(f,t,10,1),-1,1) % n=10
17 hold on
18 ezplot(fs(f,t,20,1),-1,1) % n=20
19 hold on
20 ezplot(fs(f,t,50,1),-1,1) % n=50
21 hold on
22 ezplot(fs(f,t,100,1),-1,1) % n=100
23 hold on
24 ezplot(fs(f,t,1000,1),-1,1) % n=1000
25 hold on
26 ezplot(f,-1,1)
27 hold off
28 title('Partial Sums n=2, 10, 20, 50, 100, 1000'),xlabel('Time'),ylabel(
    'Amplitude')
29 legend('n=2', 'n=10', 'n=20', 'n=50', 'n=100', 'n=1000','Original')

```

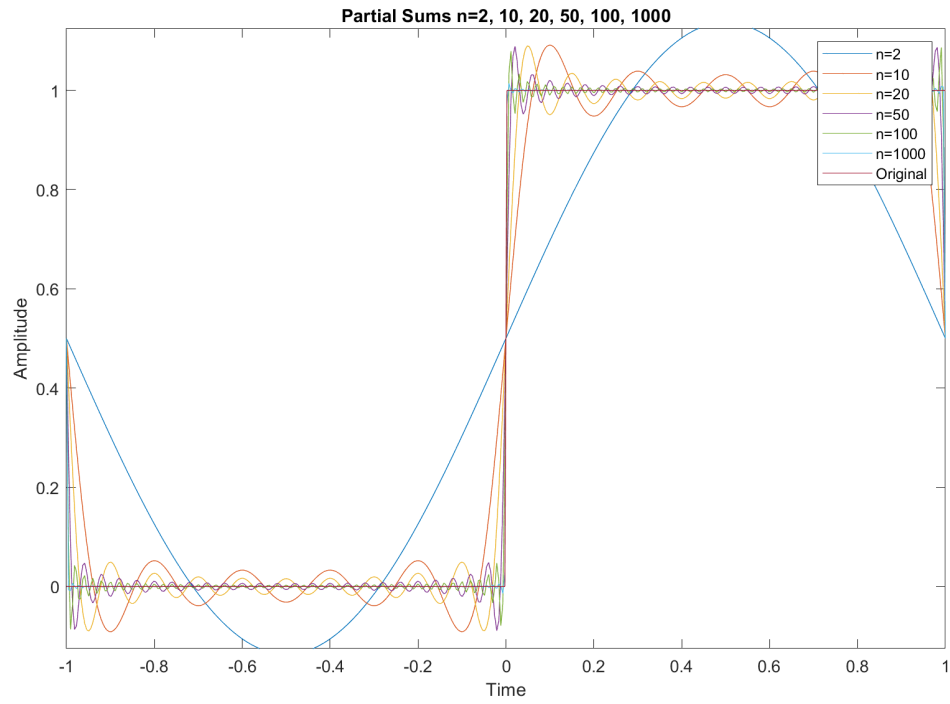


Figure 2: Trigonometric Fourier Series for $y(t) = u(t)$
and $n = 10, 20, 50, 100, 1000$