ELE447 | HW3 | CHASE LOTTO

e^-: electron shorthand

Q1

ⓐ Why do we calculate $g(E)$ for e^-?

The key proportion is $i \propto nv$. From this, we know that the current in a device is directly proportional to the electron concentration n. We also know:

$$n = g(E)f(E).$$

So to engineer n, we first need to understand $g(E)$, the density-of-states function for an e^-.

ⓑ Derive $g(E)$ for free electrons in a 2D and 1D system.

L: confinement, $n \in \mathbb{N}$

From the Schrodiger Eq, we know the wavenumber is quantized

$$k_n = \frac{n\pi}{L}$$

Given this and the fact that e^- can have negative momentum (where $k \propto p$), the size of a 1-D electron energy state is:

$$\Delta k_n = \frac{2\pi}{L}.$$

For the 1-D case, we only have k_x, so

$$\Delta k_{n,1D} = \frac{2\pi}{L} \quad \leftarrow \text{size of the electronic hotel room.}$$

Because of the Pauli Exclusion Principle, each state can accomodate $2e^-$.

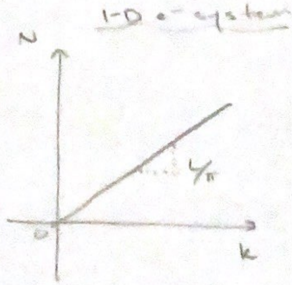
In the k-space, the 1-D system can have a width of k: in

How many electrons can it hold compared to $2e^-$ in $\frac{2\pi}{L}$?

$$\frac{N}{k} = \frac{2}{2\pi/L}$$

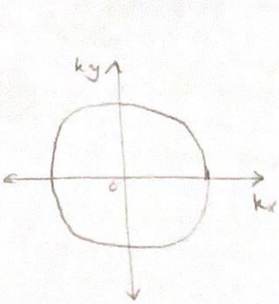
$$\Rightarrow \boxed{N = \frac{L}{\pi}k}$$

So, larger constraints allow more electrons.



For the 2D case, our e^- can now be found in energy states of size $2\pi/L$ in the k_x and k_y direction.

The size of the 2D energy state then is:



$$\Delta k_n = \left(\frac{2\pi}{L}\right)\left(\frac{2\pi}{L}\right) = \left(\frac{2\pi}{L}\right)^2$$

Again, given the Pauli Exclusion Principle, only $2e^-$ can be found for $\left(\frac{2\pi}{L}\right)^2$ k-space.

For a symmetric area with radius k, how many electrons can it hold?

The area:

$$\pi k^2$$

Setting a proportion to find N electrons for a given value of k.

$$\frac{N}{\pi k^2} = \frac{2}{\left(\frac{2\pi}{L}\right)^2}$$

$$\Rightarrow N = 2 \cdot \frac{L^2}{4\pi^2} \cdot \pi k^2$$

$$N = \frac{L^2}{2\pi} k^2$$

Now that we know how many electrons can be found within the 1D and 2D-systems for a given k (or p or E), we can find the density of states.

$$g(E) \equiv \text{states/volume/energy} \quad \text{or} \quad g(E) = \frac{1}{\text{volume}} \frac{dN}{dE}$$

volume in 1D → length

volume in 2D → area.

In the 1D case, the volume will be the L constraint and that is constant.

$$g(E)_{1D} = \frac{1}{L}\frac{dN}{dE} = \frac{1}{L}\frac{dN}{dk}\cdot\frac{dk}{dE}$$

N is a function of k and k is a function of E ∴ chain rule.

$$* \ \frac{\text{states}}{\text{energy}} \equiv \frac{dN}{dE}$$

$$\left(\frac{dN}{dE} = \frac{dN}{dk}\cdot\frac{dk}{dE}\right)$$

$$k = \frac{\sqrt{2m_0E}}{\hbar}$$

$$\frac{dN}{dk} = \frac{d}{dk}\left(\frac{L}{\pi}k\right) = \frac{L}{\pi}$$

$$\Rightarrow g(E)_{1D} = \frac{1}{L}\cdot\frac{L}{\pi}\cdot\frac{dk}{dE} = \frac{1}{\pi}\left(\frac{dE}{dk}\right)^{-1}$$

$$\frac{dE}{dk} = \frac{d}{dk}\left(\frac{\hbar^2k^2}{2m_0}\right) = \frac{\hbar^2}{m_0}k$$

$$\Rightarrow g(E)_{1D} = \frac{1}{\pi}\cdot\frac{m_0}{\hbar^2 k} = \frac{m_0}{\pi\hbar^2}\cdot\frac{\hbar}{\sqrt{2m_0E}} = \boxed{\frac{1}{\pi\hbar}\cdot\sqrt{\frac{m_0}{2}}\cdot\frac{1}{\sqrt{E}} = g(E)_{1D}}$$

the density drops off as $1/\sqrt{E}$.

Now for the 2-D density of states which occupies an area in physical space of L^2.

$*$ using $k = \frac{\sqrt{2m_0 E}}{\hbar}$

$$g(E)_{2D} = \frac{1}{L^2} \cdot \frac{dN}{dE}$$

$$= \frac{1}{L^2} \cdot \frac{d}{dE}\left(\frac{L^2}{2\pi} k^2\right)$$

$$= \frac{1}{L^2} \frac{d}{dE}\left(\frac{L^2}{2\pi} \cdot \frac{2m_0 E}{\hbar^2}\right)$$

$$= \frac{1}{L^2} \cdot \frac{m_0 L^2}{\pi\hbar^2} \cdot 1$$

$$\boxed{g(E)_{2D} = \frac{m_0}{\pi\hbar^2}}$$ CONSTANT

Here is the trend. $g(E)_{1D} \propto \frac{1}{\sqrt{E}}$, $g(E)_{2D}$ constant for all E, $g(E)_{3D} \propto \sqrt{E}$.

Luckily we don't live in 1D or 2D worlds since increasing current by increasing energy would be either counterproductive or pointless.

(c) For a free e^- in 3D. Calc density of quantum states ($\#/cm^3$) over $0 \le E \le 2.0\,eV$ and $1 \le E \le 2.0\,eV$. Comment on results.

$0 \le E \le 2(1.6\times10^{-19})\,J$

$1.6\times10^{-19}\,J \le E \le 2(1.6\times10^{-19})\,J$

$$g(E)_{3D} = \frac{1}{2\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$0 \le E \le 2eV$

$$g(0 \le E \le 2eV) = \frac{1}{2\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\int_{0eV}^{2eV} E^{1/2}\,dE = \frac{1}{3\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2} E^{3/2}\Big|_0^{3.2\times10^{-19}J} = \frac{1}{3\pi^2}\left(\frac{2(1.607\times10^{-31})}{(1.056\times10^{-34})^2}\right)^{3/2}(3.2\times10^{-19})^{3/2}$$

$$\Rightarrow \boxed{g(0 \le E \le 2eV) = 9.398\times10^{44}\ \#/cm^3}$$

$$g(1 \le E \le 2eV) = \frac{1}{2\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\int_{1.6\times10^{-19}}^{3.2\times10^{-19}} E^{1/2}\,dE = \frac{1}{3\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\left((3.2\times10^{-19})^{3/2} - (1.6\times10^{-19})^{3/2}\right)$$

$$\Rightarrow \boxed{g(1eV \le E \le 2eV) = 6.075\times10^{44}\ \#/cm^3}$$

So nearly 65% of the states are in the more energetic half. Which makes sense as $g(E)_{3D}$ is increasing $\forall E \in k$.

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# Chase Lotito - SIUC - ECE447 HW 3 - Q2: Tunneling
Probability Graphs

# We wish to write a script that will plot the tunneling
probability of an electron T as a function of the
electron's energy E.

# We have a conduction electron with an effective mass
of 0.067m, potential barrier thickness of 15A, and
potential barrier height of 0.3eV

import matplotlib.pyplot as plt
import numpy as np
import math

# IMPORTANT CONSTANTS
q = 1.6e-19                 # fundamental charge / eV-
to-J conversion factor
h = 6.63e-34                # Planck's constant [J*s]
hbar = h / ( 2 * math.pi )  # Reduced Planck's Constant
mfe = 9.8e-31               # mass of free electron
me = 0.067 * mfe            # effective mass of electron
a1 = 15e-10                 # potential barrier
thickness
a2 = 5e-10                  # second potential barrier
thickness
v0_eV = 0.3                 # potential barrier height
in eV           # energy must be less than this for
real solutions
v0_J = v0_eV * q            # potential barrier height
in Joules
```

```python
# Tunneling Probability Function
def tunnelProb(x, a):
    # Find the energy of the electron
    energy = x * q       # making sure to convert eV to J
    energyOverPotential = energy / v0_J
    k =  ( 2 * me * (v0_J - energy) )**0.5 / hbar  # second wavenumber

    exponentialTerm = np.exp(-1 * 2 * k * a)

    return (16 * energyOverPotential * ( 1 - energyOverPotential ) * exponentialTerm)

# Ranges for graph
x = np.linspace(0,4,4000)                 # Gives a range of 0 to 4 with steps of 0.001
y1 = tunnelProb(x, a1)                    # Evals tunneling prob of 15A barrier thickness
y2 = tunnelProb(x, a2)                    # Does this again with 5A barrier thickness

# Plot the graphs
plt.plot(x, y1, label = '15A')
plt.plot(x, y2, label = '5A')

# Find the maximums of the functions!
max_x = np.argmax(tunnelProb(x, a1))
max_y = tunnelProb(float(max_x), a1)
print(f"The maximum of the function occurs at x = {x[max_x]} with a value of {max_y}")
```

```
# Plot the maximums of our functions!
# plt.plot(max_x, 5e-6, marker="o", markersize=2,
markeredgecolor="orange", markerfacecolor="orange")

# Console log some values for import debug
print('[15A] E = ' + '{:e}'.format(0.2 * q) + ', T = ' +
'{:e}'.format(tunnelProb(0.2,a1)))
print('[5A] E = ' + '{:e}'.format(0.2 * q) + ', T = ' +
'{:e}'.format(tunnelProb(0.2,a2)))


'''
ai = a1
for i in range(6):
    y = tunnelProb(x, ai)
    plt.plot(x, y, label = '%d A' % (ai * 1e10))
    ai = ai - 2e-10
'''

# Labels and Titles
plt.xlabel('Energy (eV)')
plt.ylabel('Tunneling Coefficient')
plt.title('Tunneling Coefficient for 5A and 15A Well')

# Axis formatting
plt.xlim(0,4)

# Show the plot
plt.legend()
plt.show()
```

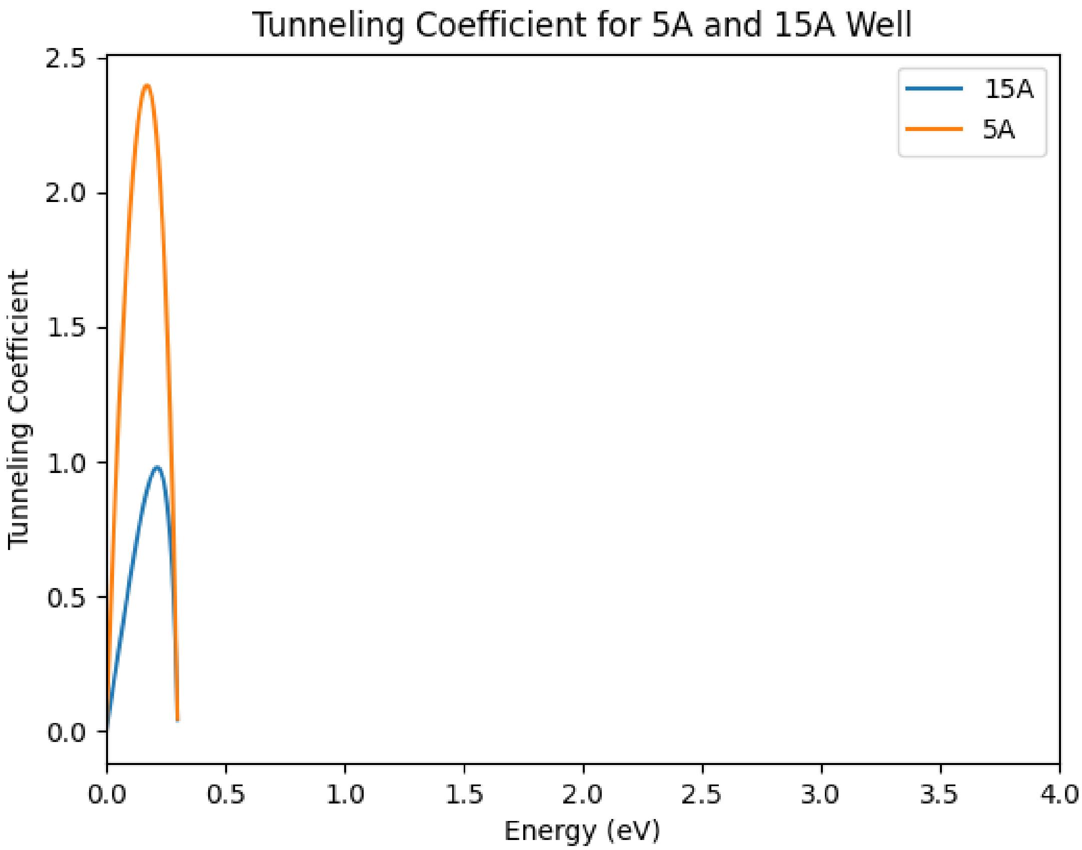

Figure 1: Tunneling Coefficient for 5A and 15A

The Tunneling Coefficient equation approximation is:

$$T \approx 16(E/V)(1 - E/V)exp\left(-2\frac{\sqrt{2m^*(V-E)}}{\hbar}L\right)$$

E is the energy of the electron, V is the value of the potential well (V=0.3eV), m* is the effective mass of the electron ($m^*=0.067m_0$), and L is the size of the potential well (1.5nm or 0.5nm).

The exponential term defines the domain of this as a function of the electron energy E. So, we sweep the graph from 0 to 4eV, but we only have real solutions for the tunneling coefficient for values of $E \in (0, 0.3eV)$.

We see that the electron's tunneling coefficient peaks, or the electron is most likely to tunnel through the potential barrier at around 0.2eV.

But, as we shrink our well, from 15Å to 5Å, the tunneling coefficient doubles.

Inside of a nanoscale device, the more the device shrinks, the higher probability we have of electrons tunneling through barriers. Through the gate terminal, this could lead to power loss in leakage current, as electrons will have a high chance of tunneling from the channel region through the oxide layer.