Exam 1:

- Time: 03/07/2024, 8:00am – 9:15am (D0033)
- Close books and notes
- Score from this exam will determine 20% of your final grade.
- 50 minutes
- Bring a regular calculator
- It covers handouts 2 – 12.

Schedule

Data	Note
02/22 (Thursday)	Homework 3 assignment
02/29 (Thursday)	Exam 1 review Homework 3 due, Quiz 3 Onine
03/05 (Tuesday)	No class. Preparing exam 1.
03/07 (Thursday)	Exam 1 (8am - 9:15am)

What to study?

- The content of the exam will be covered by notes and homework (Lab notes is not required).
- Lecture notes
  – Example problems worked
- Homework problems are important.
- Quiz problems are also important.

Introduction to Signals and Systems

- **Classification of Signals**
- **Classification of Systems**
- **Useful Signal Operations**
- **Useful Signal Functions and Models**

Time-Domain Analysis of Continuous-Time Systems

- **LTIC Systems**
- **Zero-input Response**
- **Impulse Response**
- **Convolution Integral and Properties**

Appendix: Some equations may be used in the Exam 1.

Convolution Integral: $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau$

Impulse response *h(t)*: $h(t) = b_n \delta(t) + [P(D) y_n(t)] u(t)$

Initial Conditions for $y_n(t)$: $\boldsymbol{n = 1,\ y_n(0) = 1}$

$\boldsymbol{n = 2,\ y_n(0) = 0\ and\ Dy_n(0) = 1}$

Multiplying Properties: $f(t)\delta(t) = f(0)\delta(t)$

$f(t-T)\delta(t) = f(T)\delta(t)$

Sampling Properties: $\int_{-\infty}^{\infty} f(t)\delta(t)\, dt = f(0)$

$$\int_{-\infty}^{\infty} f(t-T)\delta(t)\, dt = f(T)$$

Shift Property: $f_1(t) * f_2(t) = c(t)$

$f_1(t-T_1) * f_2(t) = c(t-T_1)$

$f_1(t-T_1) * f_2(t-T_2) = c(t-T_1-T_2)$

Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\frac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=o}^{n}\frac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\frac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\sum_{j=0}^{m}\frac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{n}\frac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Trig Identity Table

$e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$
$\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
$\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$
$\cos(\theta \pm \pi/2) = \mp\sin(\theta)$
$\sin(\theta \pm \pi/2) = \pm\cos(\theta)$
$2\sin(\theta)\cos(\theta) = \sin(2\theta)$
$\sin^2(\theta) + \cos^2(\theta) = 1$
$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$
$\cos^2(\theta) = \frac{1}{2}[1 + \cos(2\theta)]$
$\sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$
$\cos^3(\theta) = \frac{1}{4}[3\cos(\theta) + \cos(3\theta)]$
$\sin^3(\theta) = \frac{1}{4}[3\sin(\theta) - \sin(3\theta)]$
$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\sin(A)\sin(B) = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
$\cos(A)\cos(B) = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$
$\sin(A)\cos(B) = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$
$a\cos(\theta) + b\sin(\theta) = C\cos(\theta + \phi)$ $C = \sqrt{a^2 + b^2} \quad \phi = \tan^{-1}(-b/a)$

Integral Table

$\int u\,dv = uv - \int v\,du$
$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$
$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$
$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$
$\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$
$\int \cos^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$
$\int x\sin(ax)dx = \frac{1}{a^2}\left[\sin(ax) - ax\cos(ax)\right]$
$\int x\cos(ax)dx = \frac{1}{a^2}\left[\cos(ax) + ax\sin(ax)\right]$
$\int x^2\sin(ax)dx = \frac{1}{a^3}\left[2ax\sin(ax) + 2\cos(ax) - a^2x^2\cos(ax)\right]$
$\int x^2\cos(ax)dx = \frac{1}{a^3}\left[2ax\cos(ax) - 2\sin(ax) + a^2x^2\sin(ax)\right]$
$\int \sin(ax)\sin(bx)dx = \frac{1}{2(a-b)}\sin((a-b)x) - \frac{1}{2(a+b)}\sin((a+b)x)$ $a^2 \neq b^2$
$\int \sin(ax)\cos(bx)dx = -\frac{1}{2(a-b)}\cos((a-b)x) - \frac{1}{2(a+b)}\cos((a+b)x)$ $a^2 \neq b^2$
$\int \cos(ax)\cos(bx)dx = \frac{1}{2(a-b)}\sin((a-b)x) + \frac{1}{2(a+b)}\sin((a+b)x)$ $a^2 \neq b^2$
$\int e^{ax}dx = \frac{1}{a}e^{ax}$
$\int xe^{ax}dx = \frac{1}{a^2}e^{ax}(ax-1)$
$\int x^2e^{ax}dx = \frac{1}{a^3}e^{ax}(a^2x^2 - 2ax + 2)$
$\int e^{ax}\sin(bx)dx = \frac{1}{a^2+b^2}e^{ax}(a\sin(bx) - b\cos(bx))$
$\int e^{ax}\cos(bx)dx = \frac{1}{a^2+b^2}e^{ax}(a\cos(bx) + b\sin(bx))$

Examples

True or False:

1) Any signal can be written as a combination of an even and odd signal.
   True

2) An analog signal must be a discrete time signal.
   False

3) A signal $f(t) = t^2 e^t$ is an odd signal.
   False

Examples

Multiple Choice question (1):

In the following signals, which one is an **even** signal

$(A) \qquad f(t) = t^2 - 4$

$(B) \qquad f(t) = e^{-t}u(t)$

$(C) \qquad f(t) = 1/t - 5$

$(D) \qquad f(t) = t^2 + 10t - 5$

Examples

Calculation:

An LTIC system is specified by: $(D^2+5D+6)y(t)=(D-1)f(t)$

1. Find the unit impulse response *h(t)* of the system

$$h(t)=b_n\delta(t)+[P(D)y_n(t)]u(t)$$

Examples

Calculation:

Using the convolution table, determine following convolutions:

$$e^{-(t-1)}u(t-1) * (2e^{-3t} - e^{-2t})u(t)$$

$$= 2e^{-(t-1)}u(t-1) * e^{-3t}u(t) - e^{-(t-1)}u(t-1) * e^{-2t}u(t)$$

$$2e^{-t}u(t) * e^{-3t}u(t) - e^{-t}u(t) * e^{-2t}u(t)$$

$$= 2\frac{e^{-t} - e^{-3t}}{2}u(t) - \frac{e^{-t} - e^{-2t}}{1}u(t)$$

$T_1 = 1$ $\qquad$ $T_1 = 1$

$$= [e^{-2t} - e^{-3t}]u(t)$$

$$= [e^{-2(t-1)} - e^{-3(t-1)}]u(t-1)$$