# ECE355L / BME355L Signals and Systems Lab

## Project 5: Filters Design

## Report due: 04/25/2024

**Objective:** Design digital filters using the signal processing toolbox in MATLAB

1. **IIR Filters (***Note the difference in generation of analog and digital filter)*

- **Butterworth Filter**

  [N, Wn]=buttord(Wp, Ws, Rp, Rs)

  [B,A]=butter(N, Wn,'type')

- **Type 1 Chebyshev Filter**

  [N, Wn]=cheb1ord(Wp, Ws, Rp, Rs)

  [B,A]=cheby1(N, Rp,Wn,'type')

- **Type 2 Chebyshev Filter**

  [N, Wn]=cheb2ord(Wp, Ws, Rp, Rs)

  [B,A]=cheby2(N, Rs,Wn,'type')

- **Elliptic Filter**

  [N, Wn]=ellipord(Wp, Ws, Rp, Rs)

  [B,A]=ellip(N, Rp, Rs,Wn,'type')

2. **FIR Filters**

- **FIRPMORD** Parks-McClellan optimal equiripple FIR order estimator.

  [N,$F_o$,$A_o$,W] =firpmord(F,A,DEV,$F_s$)

  It finds the approximate order N, normalized frequency band edges $F_o$, frequency band magnitudes $A_o$ and weights $W$ to be used by the **FIRPM** function as follows:

  B = firpm(N,$F_o$,$A_o$,W)

  The resulting filter will approximately meet the specifications given by the input parameters *F, A,* and *DEV*. *F* is a vector of cutoff frequencies in Hz, in ascending order between 0 and half the sampling frequency $F_s$. If you do not specify $F_s$, it defaults to 2. *A* is a vector specifying the desired function's amplitude on the bands defined by *F*. The length of *F* is twice the length of *A*, minus 2 (it must therefore be even). The first frequency band always starts at zero, and the last always ends at $F_s/2$. It is not necessary to add these elements to the *F* vector.

*DEV* is a vector of maximum deviations or ripples allowable for each band. *DEV* must have the same length as *A*.

$$DEV=[\delta_1,\delta_2]=\left[\begin{array}{cc}\dfrac{10^{\left(\frac{R_p}{20}\right)}-1}{10^{\left(\frac{R_p}{20}\right)}+1} & 10^{\frac{-Rs}{20}}\end{array}\right]$$

**CAUTION 1:** The order *N* is often underestimated. If the filter does not meet the original specifications, a higher order such as *N+1* or *N+2* will.

**CAUTION 2:** Results are inaccurate if cutoff frequencies are near zero frequency or the Nyquist frequency.

- **REMEZ** : Parks-McClellan optimal equiripple FIR filter design.
  B=remez(N,F,A)
  It returns a length *N+1* linear phase (real, symmetric coefficients) FIR filter which has the best approximation to the desired frequency response described by *F* and *A*. *F* is a vector of frequency band edges in pairs, in ascending order between 0 and 1. 1 corresponds to the Nyquist frequency or half the sampling frequency. *A* is a real vector the same size as *F*, which specifies the desired amplitude of the frequency response of the resultant filter *B*. The desired response is the line connecting the points *(F(k),A(k))* and *(F(k+1),A(k+1))* for odd *k*. **REMEZ** treats the bands between *F(k+1)* and *F(k+2)* for odd *k* as "transition bands" or "don't care" regions. Thus the desired amplitude is piecewise linear with transition bands. The maximum error is minimized.

  B=remez(N,F,A,W)
  It uses the weights in *W* to weight the error. *W* has one entry per band (so it is half the length of *F* and *A*) which tells **REMEZ** how much emphasis to put on minimizing the error in each band relative to the other bands.

3. **Frequency Response**:

- **FREQZ:** Digital filter frequency response**.**
  [H,W] = FREQZ(B,A,N)
  It returns the *N*-point complex frequency response vector *H* and the *N*-point frequency vector *W* in radians/sample of the filter:

$$H(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_Pz^{-P}}{a_0+a_1z^{-1}+a_2z^{-2}+\cdots+a_Qz^{-Q}}$$

given numerator and denominator coefficients in vectors *B* and *A*. The frequency response is evaluated at *N* points equally spaced around the upper half of the unit circle. If *N* isn't specified, it defaults to 512.

[H,W] = FREQZ(B,A,N,'whole')
It uses N points around the whole unit circle.

H = FREQZ(B,A,W)
It returns the frequency response at frequencies designated in vector *W*, in radians/sample (normally between 0 and pi).

[H,F] = FREQZ(B,A,N,Fs) and [H,F] = FREQZ(B,A,N,'whole',Fs)
They both return frequency vector *F* (in Hz), where $F_s$ is the sampling frequency (in Hz).

H = FREQZ(B,A,F,Fs)
It returns the complex frequency response at the frequencies designated in vector *F* (in Hz), where $F_s$ is the sampling frequency (in Hz).

FREQZ(B,A,...) with no output arguments plots the magnitude and unwrapped phase of the filter in the current figure window.

## Lab Procedure:

**Task 1:** Design a minimum-order lowpass FIR filter with a 500 Hz passband cutoff frequency and 600 Hz stopband cutoff frequency, with a sampling frequency of 2000 Hz, at least 40 dB attenuation in the stopband, and less than 3 dB of ripple in the passband. Plot 1024-point magnitude and gain-response of the filter.

**Sample Codes:**

```
clear all;
close all;
clc;

% Step 1: Write Filter specifications
Rp = 3;                    %%  Passband ripple
Rs = 40;                   %%  Stopband ripple
Ft = 2000;                 %%  Sampling frequency
Fp = 500;                  %%  Passband cutoff frequency
Fs = 600;                  %%  Stopband cutoff frequency

f = [Fp Fs];               % % Passband and Stopband Cutoff frequencies
a = [1 0];                  % % Desired amplitudes , 1 for passband, 0 for stopband

% Step 2: Compute deviations
```

```
dev = [(10^(Rp/20)-1)/(10^(Rp/20)+1) , 10^(-Rs/20)];

% Parks-McClellan optimal equiripple FIR order estimator
[n,fo,ao,w] = firpmord(f,a,dev,Ft);

%use either firpm() or remez() for Parks-McClellan optimal equiripple FIR filter design
B = remez(n,fo,ao,w);           %% Numerator of transfer function
A = 1;                          %% Denominator of transfer function

[H,W]=freqz(B,A,1024);          % Frequency response of the filter
figure(1),subplot(211),plot((W/pi)*(Ft/2),abs(H))
xlabel('Frequency in Hz'),ylabel('Magnitude'),title('Frequency Response')
subplot(212),plot((W/pi)*(Ft/2),20*log10(abs(H)))
xlabel('Frequency in Hz'),ylabel('Gain in dB'),title('Gain Response')
```

**Task2:** Design an IIR filter with specifications: passband edge Fp=800 Hz, stopband edge Fs=1.5 kHz, passband ripple of 1 dB, minimum stopband attenuation of 80dB, and sampling rate FT=8kHz. Please design different types of IIR filter as follows:

a) Butterworth
b) Type 1 Chebyshev 1

(Note: You can use following sample codes for Butterworth and Chebyshev1 IIR filters)

**Sample Codes for Butterworth IIR filter:**

```
%% Butterworth lowpass digital filter
clear all;
close all;
clc

% Step 1: Write Filter specifications
Fp=800;                         % Passband frequency in Hz
Fs=1500;                        % Stopband frequency in Hz
Ft=8000;                        % sampling frequency in Hz
Rp=1;                           % Passband ripple in dB
Rs=80;                          % Stopband ripple in dB

% Wp and Ws are respectively the passband and stopband edge frequencies of the filter,
Wp=Fp/4000;                     % Normalized Passband frequency = fp/fn = fp/(ft/2)
Ws=Fs/4000;                     % Normalized Stopband frequency = fs/fn

% Step 2: Find filter order and cutoff frequency
[N_b, Wn_b]=buttord(Wp, Ws, Rp, Rs);
disp('Filter Order'),N_b
disp('Cut-off Frequency'),Wn_b

% Step 3: Find filter coefficients
```

```
[num,den]=butter(N_b,Wn_b);

% Step 4: 512-pint Frequency response of the filter.
[H_b,W_b]=freqz(num,den,512);                    % Frequency response of digital filter
                                                 %W_b is the frequency vector
subplot(2,1,1),plot(W_b,abs(H_b),'b')
xlabel('Frequency in Hz'),ylabel('Magnitude of H'),title('Frequency repsonse')
subplot(2,1,2); semilogx(W_b,20*log10(abs(H_b)),'b');
xlabel('Frequency in Hz'),ylabel('Magnitude of H in dB'),title('Gain repsonse')
```

**Note:**

*Wp* and *Ws* are the passband and stopband edge frequencies, normalized from 0 to 1 (where 1 corresponds to pi radians/sample).

For examples,

Lowpass: *Wp = 0.1,* *Ws = 0.2*

Highpass: *Wp = 0.2,* *Ws = 0.1*

Bandpass: *Wp = [0.2 0.7], Ws = [0.1 0.8]*

Bandstop: *Wp = [0.1 0.8], Ws = [0.2 0.7]*

The 3-dB cutoff frequency $W_n$ must be $0.0 < W_n < 1.0$, with 1.0 corresponding to half the sample rate.

Wp=Fp/(Ft/2);

Ws=Fs/(Ft/2);

Rp = Passband attenuation in db

Rs = Stopband attenuation in db

Fp = Passband frequency in Hz

Fs = Stopband frequency in Hz

Ft = Sampling frequency in Hz

Sample Codes for Chebyshev 1 IIR filter:

```
%% Chebysev1 lowpass digital filter
clear all;
close all;
clc

% Step 1: Write Filter specifications
Fp=800;                        % Passband frequency in Hz
Fs=1500;                       % Stopband frequency in Hz
Ft=8000;                       % sampling frequency in Hz
Rp=1;                          % Passband ripple in dB
Rs=80;                         % Stopband ripple in dB

Wp=Fp/4000;                    % Normalized Passband frequency = fp/(ft/2) = fp/fn
```

```
Ws=Fs/4000;                  % Normalized Stopband frequency = fs/(ft/2) = fs/fn

% Step 2: Find filter order and cutoff frequency
[N_c1, Wn_c1]=cheb1ord(Wp, Ws, Rp, Rs);          % Chebyshev1 lowpass analog filter
disp('Filter Order'),N_c1
disp('Cut-off Frequency'),Wn_c1

% Step 3: Find filter coefficients
[num,den]=cheby1(N_c1,Rp,Wn_c1,'low');

% Step 4: Find zeros, poles and gain
[z_c1,p_c1,k_c1] = cheb1ap(N_c1,Rp);
disp('zeros'),z_c1
disp('poles'),p_c1
disp('gain'),k_c1

% Step 5: Frequency response of the chebysev1 digital filter
[H_c1,W_c1]=freqs(num,den,512);          % Frequency response of analog filter
subplot(2,1,1),plot(W_c1,abs(H_c1),'r')
xlabel('Frequency in Hz'),ylabel('Magnitude of H'),title('Frequency repsonse')
subplot(2,1,2); semilogx(W_c1,20*log10(abs(H_c1)),'r');
xlabel('Frequency in Hz'),ylabel('Magnitude of H in dB'),title('Gain repsonse')
```

## Project Report:

The report is due on .04/25/2024 There is no specific requirement of the detailed format of your report (e.g. font size, line space, pages, etc.). The report should contain the following:

1. **Experimental Results:**
   1) submit the plots of designed FIR filter in Task 1. (MATLAB codes are not needed.)
   2) submit the plots of the Butterworth IIR and the Chebyshev 1 IIR filters in Task 2. (MATLAB codes are not needed.)
2. **Discussion:** Please discuss your understanding of the digital filter design with considerations of public health, safety, and welfare, as well as global, cultural, social, environmental, and economic factors. (This discussion could be in general and should not be limited to the tasks of IIR and FIR filters design).
3. **References.**