ECE 447 HOMEWORK 5 CHASE LOTTO

① E_g characterization. Given the $\log(n_i)$ vs. $1000/T$ characteristics of Si, Ge, and GaAs, extract the bandgaps. ($kT = 0.026\,eV$ @ $T = 300K$)

$$n_i = \sqrt{N_c N_v}\exp\left(\frac{-E_g}{2kT}\right)$$

The curves are marked @ $3.\bar{3} \rightarrow \frac{1000}{T} = 3.\bar{3} \Rightarrow T = 300K$

From the plot

$n_{i,Si} \approx 1\times10^{10}\ cm^{-3}$

$n_{i,Ge} \approx 1.5\times10^{13}\ cm^{-3}$

$n_{i,GaAs} \approx 1.5\times10^{6}\ cm^{-3}$

* values for N_c and N_v are found in Table 4.1 in text for Si, Ge, and GaAs.

Solve for E_g

$$\frac{n_i}{\sqrt{N_c N_v}} = e^{-E_g/2kT} \Rightarrow \ln\left(\frac{n_i}{\sqrt{N_c N_v}}\right) = \frac{-E_g}{2kT} \Rightarrow E_g = -2kT\ln\left(\frac{n_i}{\sqrt{N_c N_v}}\right)$$

@ 300K

$$\Rightarrow E_{g,Si} = -2(0.026)\ln\left(\frac{1\times10^{10}}{\sqrt{2.8\times10^{19}\cdot1.04\times10^{19}}}\right)$$

$\boxed{E_{g,Si} = 1.1054\,eV}$ ← 1.3% off the canonical 1.12eV for Si @ T=300K.

$$\Rightarrow E_{g,Ge} = -2(0.026)\ln\left(\frac{1.5\times10^{13}}{\sqrt{1.04\times10^{19}\cdot6\times10^{18}}}\right)$$

$\boxed{E_{g,Ge} = 0.685\,eV}$

$$\Rightarrow E_{g,GaAs} = -2(0.026)\ln\left(\frac{1.5\times10^{6}}{\sqrt{4.7\times10^{17}\cdot7.0\times10^{18}}}\right)$$

$\boxed{E_{g,GaAs} = 1.447\,eV}$

all of these values are close to accepted values for these SC materials

(2) Si @ 300K is doped w/ Arsenic atoms s.t. $n_0 = 5\times10^{17}\ cm^{-3}$

(a) Find $E_c - E_F$

(b) Determine $E_F - E_v$

(c) Calculate p_0

(d) Minority carrier?

(e) Find $E_F - E_i$

From Table 4.1

Si | $N_c = 2.8\times10^{19}\ cm^{-3}$
| $N_v = 1.04\times10^{19}\ cm^{-3}$

(A)

@ T = 300K , for Si, $n_i = 1.5\times10^{10}\ cm^{-3}$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$\Rightarrow \ln\left(\frac{n_0}{N_c}\right) = \frac{-1}{kT}(E_c - E_F) \Rightarrow E_c - E_F = -kT\ln\left(\frac{n_0}{N_c}\right)$$

$$\Rightarrow E_c - E_F = -(0.026)\ln\left(\frac{5\times10^{17}}{2.8\times10^{19}}\right) = \boxed{0.105\,eV = E_c - E_F}$$

(B)

$E_g = 1.12\,eV = E_c - E_v$

$\Rightarrow 1.12\,eV = E_c - E_v + E_F - E_F$

$\Rightarrow 1.12 = \underbrace{(E_c - E_F)}_{0.105} + (E_F - E_v)$

$\Rightarrow 1.12 - 0.105 = E_F - E_v$

$\boxed{E_F - E_v = 1.015\,eV}$

(C) $n_0 p_0 = n_i^2$

$\Rightarrow p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{5\times10^{17}} = 450$

$\boxed{p_0 = 450\ cm^{-3}}$

(D) Since $n_0 > p_0$, the majority carriers in this SC are $\boxed{\text{electrons.}}$

(E) Fermi level & Intrinsic Fermi level

$E_F - E_i = kT\ln(n_0/n_i)$

$= (0.026)\ln\left(\frac{5\times10^{17}}{1.5\times10^{10}}\right)$

$\boxed{E_F - E_i = 0.45\,eV}$

↑ the Fermi level is above the intrinsic Fermi level.

This E_i may not be @ the center of E_g if $m_n^* \neq m_p^*$

```python
'''
Chase Lotito - SIUC - SP2024
ECE447 - HW6
Q3: A sample of silicon doped N_a = 0 & N_d = 10E15 cm^-
3

--> Plot the majority carrier concentration
    versus temperature over the range
    100 <= T <= 600 K
'''

# NOTES:
# --> the SC is doped with DONORS => n-type (e majority)
# Na = 0
# -> n_0 = 0.5N_d + sqrt((0.5N_d)**2 + n_i**2)
# --------------------------------------------------

import matplotlib.pyplot as plt
import numpy as np
import math

# IMPORTANT CONSTANTS
q = 1.6e-19                # fundamental charge / eV-
to-J conversion
h = 6.63e-34               # Planck's constant [J*s]
hbar = h / ( 2 * math.pi ) # Reduced Planck's Constant
m_0 = 9.8e-31              # mass of free electron
k_b = 8.617e-5             # Boltzmann constant [eV/K]
m_nEff = 1.08 * m_0        # electron DOS effective
mass in Si
m_pEff = 0.56 * m_0        # hole DOS effective mass in
Si
```

```
N_d = 10e+15                    # donor concentration [cm^-
3]
n_i = 1.5e+10                   # intrinsic carrier
Concentration for Si at 300K
E_g0 = 1.17                     # Si bandgap at T=0K [eV]
alpha = 4.73e-4                 # [eV/K]
beta = 636                      # [K]

def majCarrierConcentration(T):
    """
    This function evaluates the majority carrier
    concentration in Si w.r.t. temperature.
    """

    # Effective Density of States Function in Conduction
Band
    # --> n_c = 2 ( 2pi m* kT / h^2)^3/2
    N_c = 2 * ( 2 * math.pi * m_nEff * k_b * T /
h**2)**(3/2)
    N_v = 2 * ( 2 * math.pi * m_pEff * k_b * T /
h**2)**(3/2)

    # Intrinsic Carrier Density:
    # --> use temperature dependent bandgap equation
    # --> n_i = sqrt(N_c * N_v) exp(-E_g / 2kT)
    E_g = E_g0 - ( ( alpha * T**2 ) / (beta + T) )
    n_i = ( (N_c * N_v)**0.5 * np.exp(-1 * E_g / (2 *
k_b * T)) )**0.5

    # -> n_0 = 0.5N_d + sqrt((0.5N_d)**2 + n_i**2)
    n_0 = 0.5 * N_d + ((0.5 * N_d)**2 + n_i)**0.5
    return n_0
```

```python
# Ranges for graph
x = np.linspace(100, 600, 5000)
y = majCarrierConcentration(x)

# Plot the graphs
plt.plot(x, y)

# Labels and Titles
plt.xlabel('Temperature (K)')
plt.ylabel('Majority Carrier Concentration (cm^-3)')
plt.title('Majority Carrier Concentration w.r.t. 
Temperature')

# Axis formatting
plt.xlim(0,700)

# Show the plot
plt.legend()
plt.show()
```

Plot:

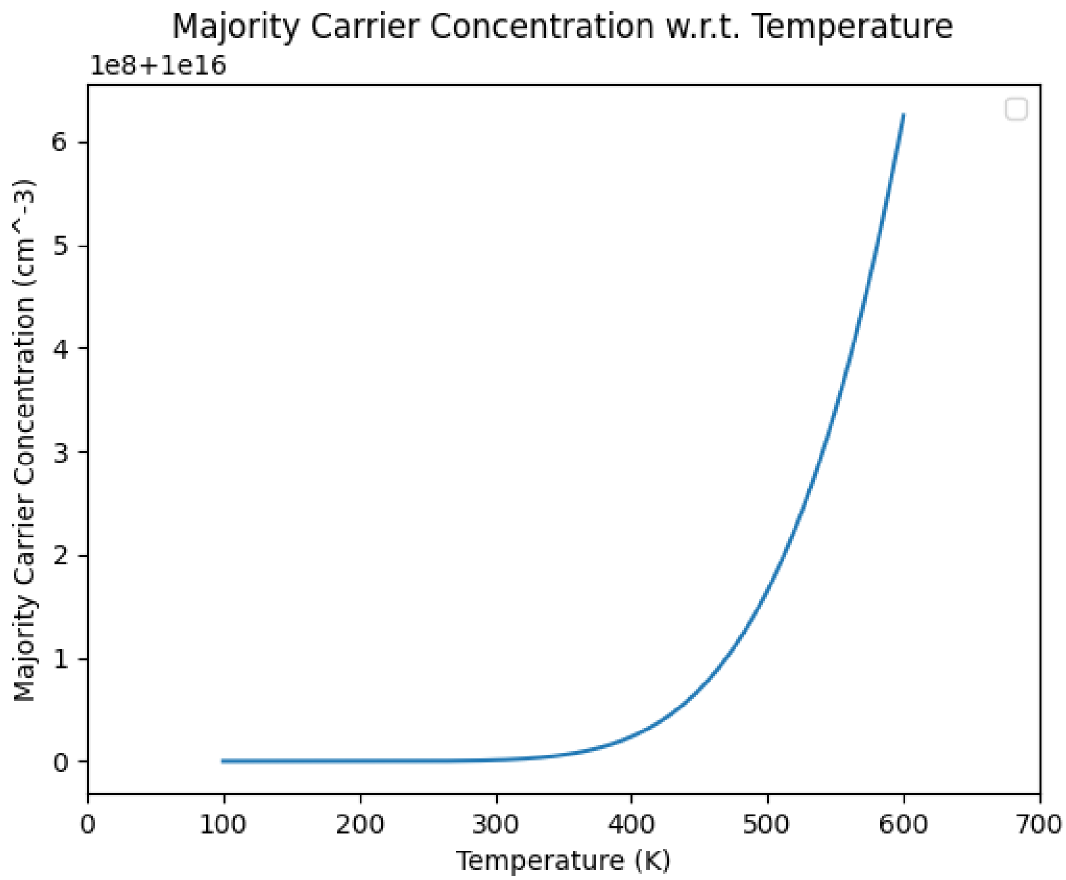

Figure 1: Majority Carrier Concentration plot for Question 3

From the plot, it is hard to see, but at T=100K, n_0=1e+16 cm^{-3}. But, as we expect, the majority carrier concentration is constant for most values, and then it begins to mimic the behavior of the intrinsic concentration after we reach high temperatures.

(4) New SC material to be "designed": p-type and doped w/ $5\times10^{15}\,cm^{-3}$ acceptor atoms. Assume complete ionization and $N_d=0$. $N_c=1.2\times10^{19}\,cm^{-3}$ and $N_v=1.8\times10^{19}\,cm^{-3}$, at $T=300K$ and vary as T^2. A device from this material requires the hole concentration be no greater than $5.08\times10^{15}\,cm^{-3}$ @ $T=350K$. What is the minimum E_g for this material?

p-type

- $N_a = 5\times10^{15}\,cm^{-3}$

$N_d = 0$

@ $T=300K$ $\begin{cases} N_c = 1.2\times10^{19}\,cm^{-3} \\ N_v = 1.8\times10^{19}\,cm^{-3} \end{cases}$

@ $T=350K$, $p_0 \le 5.08\times10^{15}\,cm^{-3}$

complete ionization, so all acceptors are active in carrier transport.

BRAINSTORM

$$p_0 = N_v \exp\left[\frac{E_v - E_F}{kT}\right], \quad n_0 p_0 = N_c N_v \exp\left[\frac{-E_g}{kT}\right] = n_i^2 \quad (1)$$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \overset{N_d=0}{=} \frac{1}{2}N_a + \sqrt{\frac{1}{4}N_a^2 + n_i^2}$$

$$\Rightarrow p_0 = \frac{1}{2}N_a + \sqrt{\frac{1}{4}N_a^2 + n_i^2} \quad (2)$$

INTRINSIC CONCENTRATION

(2)

$$\Rightarrow \left(p_0 - \tfrac{1}{2}N_a\right)^2 = \tfrac{1}{4}N_a^2 + n_i^2$$

$p_0 = 5.08\times10^{15}$ @ 350K

$$\Rightarrow \sqrt{\left(p_0 - \tfrac{1}{2}N_a\right)^2 - \tfrac{1}{4}N_a^2} = n_i$$

This is independent of temperature

$$\Rightarrow n_i = \sqrt{\left(\left(5.08\times10^{15} - \tfrac{1}{2}(5\times10^{15})\right)\right)^2 - \tfrac{1}{4}(5\times10^{15})^2}$$

$$n_i = 6.375\times10^{14}\,cm^{-3} \text{ or } n_i^2 = 4.06\times10^{29}\,cm^{-3}$$

EFF. DOS @ 350K

Now we have n_i^2, but we now need N_c and N_v @ $T=350K$.

Since they vary as T^2, we can construct an equation for $N(T)$.

$N(T) = N_{300K}\left(T/300K\right)^2$, this gives the original 300K values, and scales higher temps accordingly

$$N_v(350K) = (1.8\times10^{19})\left(\tfrac{350}{300}\right)^2 = 2.45\times10^{19}\,cm^{-3}$$

$$N_c(350K) = (1.2\times10^{19})\left(\tfrac{350}{300}\right)^2 = 1.63\times10^{19}\,cm^{-3}$$

EQ 1

$$n_i^2 = N_c N_v e^{-E_g/kT} \Rightarrow \ln\left(\frac{n_i^2}{N_c N_v}\right) = \frac{-E_g}{kT} \Rightarrow E_g = -kT\ln\left(\frac{n_i^2}{N_c N_v}\right)$$

$$\Rightarrow E_g\big|_{350K} = -(8.617\times10^{-5})(350)\ln\left(\frac{4.06\times10^{29}}{(1.63\times10^{19})(2.45\times10^{19})}\right)$$

$$\Rightarrow \boxed{E_g\big|_{T=350K} = 0.625\,eV}$$