Question bank

Selection of questions (the ones in **bold** are most important):

A. Define:

1. Dual space, covector, k-form
2. Inner product (scalar product). (Non-degenerate, symmetric, Euclidean, pseudo-Euclidean)
3. Contraction of vector and an exterior form (e.g., 2-form, 3-form, etc.)
4. Smooth map between manifolds, diffeomorphism
5. Manifold, Riemannian manifold
6. **Tangent vector** , tangent space, tangent bundle
7. **Exterior derivative** d
8. Smooth function, smooth curve, smooth vector field
9. Induced maps: pull-back of functions and forms, push-forward of vectors
10. **Exterior derivative** d
11. Lie bracket of vector fields
12. Lie algebra
13. **Curl, div, grad** in a three dimensional manifold. Properties. (Mandala)
14. Chain, boundary, integrals of forms

B. Calculate:

1. $\langle \alpha \mid \mathbf{v} \rangle$, $X \lrcorner \omega$, $d\omega$, Xf, $\alpha \wedge \beta$, $[\mathbf{A}, \mathbf{B}]$, ... (trivial stuff)
2. Rank of a given form.
3. **Curl, div, grad** given g and η
4. Induced fields (function, differential form, inner product), given a map between manifolds
5. Integral of a form over a chain directly
6. Integral of a form over a chain using Stokes theorem to make it effortless

C. Applications:

1. **Gravitation:** Distribution of masses given a gravitational field. Check if a field is possible. Calculate work along a curve.
2. **Electromagnetism:** draw and understand the 3 "mandalas" [only final]
3. **Minowski space:** interpret the basic S-T diagrams [only final]

D. Prove/show

1. L^* is a vector space and $\dim L^* = \dim L$ (if L is a vector space)
2. $\dim T_p M = \dim M$ (hint: find basis)
3. Jacobi identity for vector fields
4. Stokes theorem (at least the case of 2D simple chain)
5. Gradient theorem, divergence theorem, curl theorem
   (by reduction of the general Stokes theorem to a particular dimension)

Know the meaning of these symbols: T_pM, $\mathscr{X}M$, $\Lambda^k M$, ∂_x, dx , df, C_kM, φ^*, φ_*.