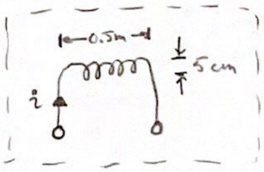
ECE 385 | HW1 | CHASE LOTTO

## PROBLEM 1

THE LONG SOLENOID COIL HAS 250 TURNS. THE FIELD IS UNIFORM INSIDE THE COIL. NEGLECT THE FIELD OUTSIDE.

(A) DETERMINE THE FIELD INTENSITY (H) AND THE FLUX DENSITY (B). ($i = 100A$)

(B) DETERMINE THE INDUCTANCE OF THE COIL.




(A)

$N = 250$ turns

$\oint H \cdot dl = Ni$

$\hookrightarrow Hl = Ni$

$\hookrightarrow H = \frac{Ni}{l}$ mean path ∴ $l = 0.5m$ ...

$$H = \frac{(250 \text{ turns})(100\text{ A})}{(0.5\text{ m})} = \boxed{5\times10^4 \text{ At/m} = H}$$

$\Rightarrow B = \mu H$, since no core, $\mu = \mu_0 = 4\pi\times10^{-7}$ H/m

$$\Rightarrow B = (4\pi\times10^{-7})(5\times10^4 \text{ At/m}) = \boxed{0.0628 \text{ Wb/m}^2 = B}$$

(B) $$L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{NBA}{i} = \frac{NB\pi r^2}{i} = \frac{NB\pi(\frac{D}{2})^2}{i} = \frac{(250)(0.0628)\pi(\frac{5\times10^{-2}}{2})^2}{100} = \boxed{308.3\ \mu H = L}$$

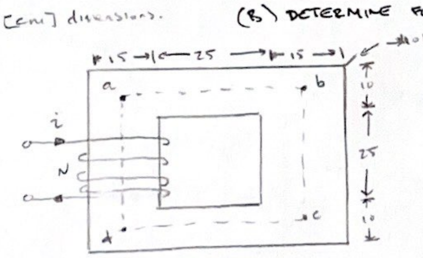
## PROBLEM 2

MAGNETIC SYSTEM. CORE DEPTH IS 10cm, $\mu_r = 2000$ H/m for the core, $N = 300$ turns, coil current $i = 1A$.

(A) DETERMINE FLUX IN CORE

(B) DETERMINE FLUX DENSITIES IN THE PARTS OF THE CORE.

[cm] dimensions.




(A)

mmf relation $\Rightarrow Ni = \Phi R$ ∴ $\Phi_c = \frac{Ni}{R_c} = ?$

$R_c$ has components since core has different thicknesses.

$\Rightarrow R_c = R_{ab} + R_{bc} + R_{cd} + R_{da}$

via symmetry,

$$\Rightarrow R_c = 2R_{ab} + 2R_{bc} = 2(R_{ab} + R_{bc}) = 2\left(\frac{l_{ab}}{\mu_c A_{ab}} + \frac{l_{bc}}{\mu_c A_{bc}}\right)$$

$$\hookrightarrow = \frac{2}{\mu_c}\left(\frac{l_{ab}}{A_{ab}} + \frac{l_{bc}}{A_{bc}}\right) = \frac{2}{2000 \cdot 4\pi\times10^{-7}}\left(\frac{(15\times10^{-2}+25\times10^{-2})}{(10\times10^{-2})(10\times10^{-2})} + \frac{(10\times10^{-2}+25\times10^{-2})}{(15\times10^{-2})(10\times10^{-2})}\right)$$

$\hookrightarrow = 50399.1$ At/Wb

$$\Rightarrow \Phi_c = \frac{(300)(1)}{(50399.1)} = \boxed{5.95\times10^{-3} \text{ Wb}}$$

(B) $B = \Phi/A$

$$\rightarrow B_{thick} = \frac{5.95\times10^{-3}}{(10\times10^{-2})(15\times10^{-2})} = \boxed{0.399 \text{ Wb/m}^2 = B_{thick}}$$

$A_{thin}$: areas for $R_{ab}$, $R_{dc}$

$A_{thick}$: areas for $R_{da}$, $R_{bc}$

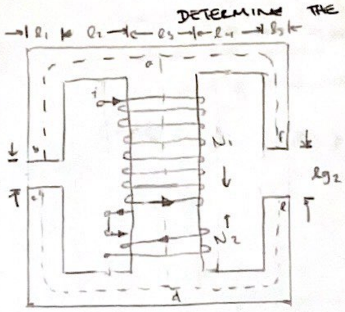
$$\rightarrow B_{thin} = \frac{5.95\times10^{-3}}{(10\times10^{-2})(10\times10^{-2})} = \boxed{0.595 \text{ Wb/m}^2 = B_{thin}}$$

## PROBLEM 3

TWO AIR GAPS. $N_1 = 700$; $N_2 = 200$, CONNECTED IN SERIES, BOTH CARRY $i = 0.5A$.

NEGLECT LEAKAGE FLUX & THE RELUCTANCE OF IRON ($\therefore \mu_{iron} = \infty$) & FRINGING @ AIR GAPS.

DETERMINE THE FLUX AND FLUX DENSITY IN THE AIR GAPS.




$N_1 = 700$

$N_2 = 200$

$i = 0.5A$

$\mu = \infty$

$l_{g_1} = 0.05cm$, $l_{g_2} = 0.1cm$

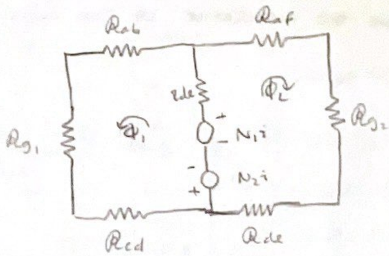
$l_1 = l_2 = l_4 = l_5 = 2.5cm$

$l_3 = 5cm$

depth of core $= 2.5cm = d$

The flux in $N_2$ counteracts the flux in $N_1$.

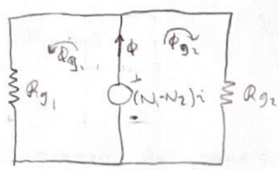
Make a "circuit" to model:




By symmetry, $R_{ab} = R_{af} = R_{cd} = R_{de} = \frac{l}{\mu_c A} = \frac{l}{(\infty) A} = 0$. Also, $R_{de} = \frac{l}{\mu A} = 0$.

In our "circuit", those become shorts.

$\Rightarrow$




$$R_{g_1} = \frac{l_{g_1}}{\mu_0 A_{g_1}} = \frac{l_{g_1}}{\mu_0 l_1 d} = \frac{0.05 \times 10^{-2}}{(4\pi \times 10^{-7})(2.5 \times 10^{-2})(2.5 \times 10^{-2})} = 6.366 \times 10^5 \text{ At/Wb}$$

$$R_{g_2} = \frac{l_{g_2}}{\mu_0 A_{g_2}} = \frac{l_{g_2}}{\mu_0 l_5 d} = \frac{0.1 \times 10^{-2}}{(4\pi \times 10^{-7})(2.5 \times 10^{-2})(2.5 \times 10^{-2})} = 1.273 \times 10^6 \text{ At/Wb}$$

$$(N_1 - N_2) i = (700 - 200)(0.5) = 250 \text{ At}$$

$$\Rightarrow Ni = \Phi R \therefore \phi = \frac{N_i}{R} \Rightarrow \phi_{g_1} = \frac{250}{6.366 \times 10^5} = 393 \mu Wb$$

$$\phi_{g_2} = \frac{250}{1.273 \times 10^6} = 196 \mu Wb$$

$\mu Wb = 10^{-6} Wb$

$$\phi = BA \Rightarrow B = \phi / A \Rightarrow B = \frac{\phi}{l_1 d}$$

$$B_{g_1} = \frac{393 \times 10^{-6}}{(2.5 \times 10^{-2})(2.5 \times 10^{-2})} = 0.628 T$$

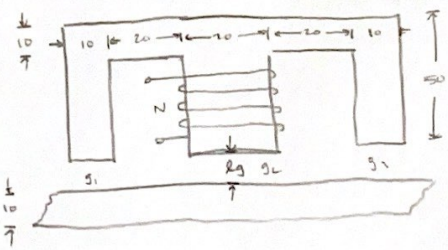
$$B_{g_2} = \frac{196 \times 10^{-6}}{(2.5 \times 10^{-2})(2.5 \times 10^{-2})} = 0.314 T$$

$\therefore$

$$\boxed{\begin{aligned} \phi_{g_1} &= 393 \mu Wb \\ \phi_{g_2} &= 196 \mu Wb \\ B_{g_1} &= 0.628T \\ B_{g_2} &= 0.314T \end{aligned}}$$

## PROBLEM 4

ELECTROMAGNET TO LIFT STEEL STRIP. COIL W/ 500 TURNS. THE MAGNETIC MATERIAL HAS NEGLIGIBLE RELUCTANCE @ B = 1.4T. DETERMINE THE MAXIMUM AIRGAP FOR WHICH B = 1.4T CAN BE ESTABLISHED W/ i = 20A. NEGLECT MAGNETIC LEAKAGE & FRINGING @ AIRGAP.




$$\mathcal{R}_{core} = 0. \quad \therefore \mu_{core} = \infty.$$

$$\mathcal{R}_{g_1} = \mathcal{R}_{g_3} = \frac{l_g}{\mu_0 A}$$

$$\mathcal{R}_{g_2} = \frac{l_g}{\mu_0 2A} = \frac{1}{2}(\mathcal{R}_{g_1})$$

$$Ni = \phi\mathcal{R} = \phi(\mathcal{R}_{g_1} \| \mathcal{R}_{g_2} \| \mathcal{R}_{g_3})$$

$$\mathcal{R}_{g_1} = \mathcal{R}_{g_3} = 2\mathcal{R}_{g_2}$$

so,

$$Ni = \phi(\mathcal{R}_{g_1} \| \tfrac{1}{2}\mathcal{R}_{g_1} \| \mathcal{R}_{g_1}) = \phi \frac{\frac{1}{2}\mathcal{R}_{g_1}^3}{5/2\,\mathcal{R}_{g_1}} = \phi\left(\tfrac{1}{5}\right)(\mathcal{R}_{g_1})^2$$

X

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Different Approach...

$$Ni = \phi\mathcal{R} = Hl \Rightarrow H = \frac{Ni}{l} \Rightarrow B/\mu = \frac{Ni}{l} \Rightarrow B = \frac{\mu Ni}{l}$$

Rearrange,

$$l_g = \frac{\mu_0 Ni}{B_g} = \frac{4\pi \times 10^{-7} \cdot 500 \cdot 20}{1.4} = 8.98 \times 10^{-3}\ m \quad \therefore \boxed{\max\ l_g = 8.98\ mm}$$

## Problem 5

Coil wound on magnetic core is excited by the following voltages:

(a) 100V, 50Hz $\rightarrow E_{max} = 100$

(b) 110V, 60Hz $\rightarrow E_{max} = 110$

Calculate hysteresis & eddy current losses. For hysteresis, n=2.

$e$ or $v = N\frac{d\phi}{dt}$

$\Rightarrow N d\phi = e\,dt$

flux link change = volt-time product

$$P_h = K_H B_{max}^n f = K_H B_{max}^2 f \quad , \quad B_{max} = \frac{\phi_{max}}{A}, \quad E_{max} = N\omega\phi_{max}$$

$$P_e = K_e B_{max}^2 f^2 \qquad\qquad = 2\pi f N \phi_{max}$$

$$\Rightarrow \phi_{max} = \frac{E_{max}}{2N\pi f} \quad \therefore B_{max} = \frac{E_{max}}{2NA\pi f}$$

$$\Rightarrow P_h = K_H\left(\frac{E_{max}}{2NA\pi f}\right)^2 f = \frac{K_H E_{max}^2}{4N^2A^2\pi^2 f}$$

same but $f^2/f^2 = 1$ $\Rightarrow P_e = \frac{K_e E_{max}^2}{4N^2A^2\pi}$

For Hysteresis losses,

$$P_h(50) = \frac{K_H(100)^2}{4N^2A^2\pi^2(60)} \;;\; P_h(60) = \frac{K_H(110)^2}{4N^2A^2\pi^2(50)} \overset{\text{ratio}}{\Longrightarrow} \frac{P_h(60)}{P_h(50)} = \frac{\frac{K_H(110)^2}{4N^2A^2\pi^2(60)}}{\frac{K_H(100)^2}{4N^2A^2\pi^2(50)}} = \frac{(110)^2(50)}{(100)^2(60)} = 1.008$$

$$\therefore \boxed{\frac{P_h(60Hz)}{P_h(50Hz)} = 1.008}$$

$\therefore$ more losses in 60Hz source...

For Eddy current losses,

$$P_e(50) = \frac{K_e(100)^2}{4N^2A^2\pi^2} \;;\; P_e(60) = \frac{K_e(110)^2}{4N^2A^2\pi^2}$$

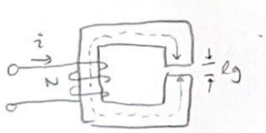
$$\overset{\text{ratio}}{\Longrightarrow} \frac{P_e(60)}{P_e(50)} = \frac{K_e(110)^2}{4N^2A^2\pi^2}\cdot\frac{4N^2A^2\pi^2}{K_e(100)^2} = \frac{(110)^2}{(100)^2} = 1.21 \quad \therefore \boxed{\frac{P_e(60Hz)}{P_e(50Hz)} = 1.21}$$

but, Eddy current losses increased b/c of voltage... but independent of frequency.

## Problem 6

Magnetic circuit. N = 100 turns, $A_c = A_g = 5cm^2$, $\mu_c = \infty$. Determine airgap length $l_g$ to provide coil inductance of 10mH.

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{NBA_g}{i} = \frac{\mu_0 NHA_g}{i} \quad -(1)$$




But we don't have coil current $i$...

$$Ni = \phi R \Rightarrow i = \frac{\phi R_g}{N} = \frac{\mu_0 HA_g \cdot \frac{l_g}{\mu_0 A_g}}{N} = \frac{l_g H}{N} \quad (2)$$

Combine (1)+(2)

$$\Rightarrow L = \frac{\mu_0 NHA_g}{l_g \frac{H}{N}} = \frac{\mu_0 N^2 A_g}{l_g} \quad (*)$$

$$\Rightarrow l_g = \frac{\mu_0 N^2 A_g}{L} = \frac{(4\pi\times10^{-7})(100^2)(5\times10^{-4})}{10\times10^{-3}} = 0.628\,mm$$

$$\Rightarrow \boxed{l_g = 0.628\,mm}$$