MATH450: Basic Preliminaries

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Vector Space Axioms

A **vector space** (a finite dimensional linear space) V is a group of objects called *vectors* which follow the following rules. Given $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$ and $a, b \in \mathbb{R}$:

1. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
3. $\mathbf{v} + \mathbf{0} = \mathbf{v}$
4. $\mathbf{v} + -\mathbf{v} = \mathbf{0}$
5. $a(b\mathbf{v}) = (ab)\mathbf{v}$
6. $1\mathbf{v} = \mathbf{v}$
7. $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
8. $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$

Note again, a linear space L follows the same axioms, but it can be infinite dimensional.

Abstract Algebra Topics

The **Cartesian Product**, for the sets A and B, is the set of ordered pairs:

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

The cartesian product is not associative and it is not commutative.

Manifolds

A **manifold** M is a smooth/differentiable space that is locally Euclidean.