MATH 450
HOMEWORK 3

Exercise 1. Here are some differential forms on $M \cong \mathbf{R}^4$:

$$\alpha = x^2\, dx + z\, dy$$
$$\beta = dy + 3dz + t\, dt$$
$$\gamma = dx \wedge dy + 2y\, dx \wedge dt$$
$$\delta = dx \wedge dy \wedge dz + x\, dy \wedge dz \wedge dt$$

Calculate $\alpha\wedge\beta$, $\beta\wedge\gamma$, $\gamma\wedge\gamma$, $\alpha\wedge\delta$, $\beta\wedge\delta$

Exercise 2. Calculate the exterior derivatives of the following exterior differential forms on a manifold with coordinates $\{x, y, z, t\}$:

$$f = xy + e^{zt}$$
$$\alpha = x\, dx + y\, dy$$
$$\beta = x\, dy + y\, dx$$
$$\gamma = x\, dy - y\, dx$$
$$\delta = xyzt\, dx \wedge dy$$
$$\mu = x\, dx \wedge dy \wedge dt + xyz\, t\, dy \wedge dz \wedge dt$$
$$\sigma = (1 + yx + yzx)\, dx + (x + xz)\, dy + x^2 y\, dz$$
$$\omega = (x^2 + y^2 + z^2 + t^2)\, dx \wedge dy \wedge dz \wedge dt$$

Exercise 3. Consider the following most general form on a 3-dimensional manifold with coordinates $\{x^1, x^2, x^3\}$.

0-form	$f = f(x^1, x^2, x^3)$
1-form	$\alpha = a_1\, dx^1 + a_2\, dx^2 + a_3\, dx^3$
2-form	$\beta = A_1\, dx^2 \wedge dx^3 + A_2\, dx^3 \wedge dx^1 + A_3\, dx^1 \wedge dx^2$
3-form	$\omega = g\ dx^1 \wedge dx^2 \wedge dx^3$

where f, g, a_i and A_i are arbitrary unspecified differentiable functions.
Calculate

$$df, \quad d\alpha, \quad d\beta, \quad d\omega.$$

Also, **calculate** ddf to see how it vanishes.

Exercise 4. Let $\alpha = x dy + y dx$, and $\omega = dx \wedge dy + dz \wedge dt$. Calculate:

$$\alpha\wedge\alpha, \quad \alpha\wedge\omega, \quad \text{and} \quad \omega\wedge\omega$$