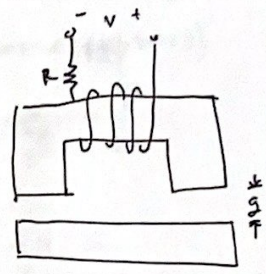
ELE385 HW4 CHASE LOTITO

PROBLEM 1



THE ELECTROMAGNET IS USED TO LIFT A SHEET OF STEEL.
THE COIL HAS $N = 500$ Turns, $R_{coil} = 5\Omega$. R_c IS NEGLIGIBLE.

$A_c = (5cm)(5cm)$. Airgap $g = 1mm$. $f_{avg} = 550N$ is required to lift the steel.

a) FOR DC SUPPLY

(i) DETERMINE THE DC SOURCE VOLTAGE.

COIL CURRENT

$$f_m = \frac{\partial}{\partial g}\left(\frac{B_g^2}{2\mu_0} A_g \cdot 2g\right)$$ where its $2g$ since 2 airgaps.

$$= \frac{2B_g^2}{2\mu_0}A_g = \frac{A_g B_g^2}{\mu_0}.$$

Here $B_g = \frac{\mu_0 N i}{g}$

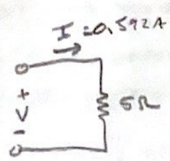
$$\Rightarrow f_m = \frac{A_g}{\mu_0}\frac{\mu_0^2 N^2 i^2}{g^2} = \frac{A_g \mu_0 N^2 i^2}{g^2}$$

Rearrange for current in coil i.

$$\Rightarrow i = \sqrt{\frac{f_m g^2}{A_g \mu_0 N^2}} = \sqrt{\frac{550(1\times10^{-3})^2}{(2(5\times10^{-2})(5\times10^{-2}))(4\pi\times10^{-7})(500)^2}} = \sqrt{0.35} = \underline{\underline{0.592A = i}}$$

COIL IMPEDANCE

For DC, $\omega = 0$, so $Z_{coil} = 5 + j(0)L = 5\Omega$

$\Rightarrow$ 

$V = (0.592A)(5\Omega)$

$\boxed{V_{DC} = 2.96V}$

b) (ii) DETERMINE ENERGY STORED IN MAGNETIC FIELD.

$$Wf = \frac{B_g^2}{2\mu_0} \times Vol_g = \frac{1}{2\mu_0}\left(\frac{\mu_0 N i}{g}\right)^2 \cdot Vol_g = \frac{\mu_0 N^2 i^2 A_g}{2g} = \frac{4\pi\times10^{-7}(500)^2(0.592)^2(5cm)(5cm)}{2(1mm)}$$

$$\Rightarrow \boxed{Wf = 0.550 J}$$

makes sense as $W = Fd$
$= (550N)(0.001m)$
$= 0.550J$

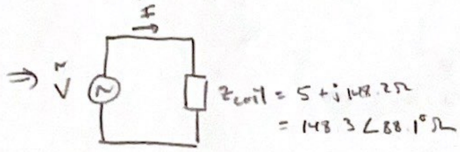
(b) For AC supply @ 60Hz, determine supply voltage.

$|\tilde{I}| = 0.592A$ but now we have to worry about coil inductance.

$$L = \frac{N^2}{\mathcal{R}_g} = \frac{N^2 \mu_0 A_g}{g} = \frac{(500)^2(4\pi\times10^{-7})((5cm)^2)}{2.1mm} = 0.393H$$

$$\Rightarrow X = j\omega L = j(2\pi)(60)(0.393) = j148.2\Omega$$

$$\therefore Z_{coil} = 5 + j148.2\Omega.$$

$\Rightarrow$ 

Using Ohm's we can find $|\tilde{V}|$.

$$|\tilde{V}| = |\tilde{I}||Z_{coil}| = (0.592)(148.3)$$
$$= 87.8V_{AC}$$

$$\Rightarrow \boxed{V_{AC} = 87.8V_{AC} \text{ @ } 60Hz}$$ ← much more power needed to supply the same current.

Problem 2 The rotating machine has the following parameters.

$$\begin{cases} L_{ss} = 0.15H \\ L_{rr} = 0.06H \\ L_{sr} = \underbrace{0.08}_{M}\cos\theta \ H \end{cases}$$

(A) The rotor is driven @ 3600 rpm. If the stator winding carries a current of $5A_{rms}$, @ 60Hz, determine the instantaneous voltage and rms voltage induced in the rotor coil. Determine the frequency of the rotor induced voltage.

$i_s = I_{sm}\cos\omega_s t$

$i_r = I_{rm}\cos(\cdot\theta)$

Lenz's Law: $V = -\frac{d\lambda}{dt}$, where $\lambda = Li$

Rotor position: $\theta = \omega_m t + \delta$

Find rotor position θ

$$\theta = \omega_m t + \delta, \text{ where } \omega_m \Rightarrow \frac{3600 \text{ rpm}}{60 \text{ s/min}} = 60 \frac{rot}{sec} = f_{mech}$$

$$\Rightarrow \omega_m = 2\pi f_{mech} = 120\pi \text{ rad/sec}.$$

$$\therefore \underline{\theta = 120\pi t + \delta}$$

Find ω_s $\omega_s = 2\pi(60Hz) = 120\pi.$

Find max stator current ✓ $I_{sm} = \sqrt{2}\, I_{s,rms} = \sqrt{2}\cdot 5 = 5\sqrt{2}A.$

Flux linkage λ $\lambda = L_{sr}i_s = 0.08\cos\theta \cdot I_{sm}\cos\omega_s t$

$$\Rightarrow \lambda = 0.08\cos(120\pi t + \delta)\cdot 5\sqrt{2}\cos(120\pi t)$$

* coincidence $\omega_m t = \omega_s t$. (synchronous).

↳ cont.

INDUCED VOLTAGE

$$V = \frac{d\lambda}{dt} = \frac{d}{dt}\left[0.4\sqrt{2}\cos(120\pi t+\delta)\cos(120\pi t)\right]$$

$$= 0.4\sqrt{2}\left(-120\pi\sin(120\pi t+\delta)\cos(120\pi t) - 120\pi\cos(120\pi t+\delta)\sin(120\pi t)\right)$$

$$= -48\pi\sqrt{2}\left(\sin(120\pi t+\delta)\cos(120\pi t) + \cos(120\pi t+\delta)\sin(120\pi t)\right)$$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= -48\pi\sqrt{2}\sin(120\pi t+\delta+120\pi t)$$

$$= -48\pi\sqrt{2}\sin(240\pi t+\delta)$$

$-\sin(\theta) = \sin(-\theta)$

$$= 48\pi\sqrt{2}\sin(-(240\pi t+\delta))$$

$\cos(\theta-90^\circ) = \sin(\theta)$

$$= 48\pi\sqrt{2}\cos(-(240\pi t+\delta)-90^\circ)$$

$$= 48\pi\sqrt{2}\cos(-(240\pi t+\underbrace{\delta+90^\circ}))$$

δ absorbs $\therefore \delta + 90^\circ = \delta$.

$$= 48\pi\sqrt{2}\cos(-(240\pi t+\delta))$$

$\cos(\theta)$ is even $\therefore \cos(-\theta) = \cos\theta$

$$= \underbrace{48\pi\sqrt{2}}_{\cong 213.3}\cos(240\pi t+\delta)$$

RMS

$$\Rightarrow \boxed{V_{ab} = 213.3\cos(240\pi t+\delta)\,V}$$

The RMS would be $\frac{213.3}{\sqrt{2}}$. $\Rightarrow$ $V_{rms} = 150.8\cos(240\pi t+\delta)\,V_{rms}$

FREQUENCY $\quad \omega_r t = 240\pi t \quad \therefore \quad \omega_r = 240\pi \Rightarrow f_r = \frac{240\pi}{2\pi} = \boxed{120\,Hz}$

(B) SUPPOSE THE STATOR & ROTOR COILS ARE CONNECTED IN SERIES, AND A 5Arms, 60Hz CURRENT IS PASSING THROUGH THEM. DETERMINE SPEEDS FOR NON-ZERO AVERAGE TORQUE. DETERMINE MAX TORQUE @ EACH SPEED.

Series $\therefore i_s = i_r = 5\sqrt{2}\cos\omega t = 5\sqrt{2}\cos(2\pi(60)t) = 5\sqrt{2}\cos(120\pi t)$

$$T = i_s i_r \frac{dL_{sr}}{d\theta} = (5\sqrt{2})^2\cos^2(120\pi t)\frac{d}{d\theta}(0.08\cos\theta) = 50\cos^2(120\pi t)\frac{d}{d\theta}(0.08\cos(\omega_m t+\delta))$$

Half Angle

$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

$$= 50\cos^2(120\pi t)\cdot -0.08\sin(\omega_m t+\delta)$$

$$= -4\cos^2(120\pi t)\cdot\sin(\omega_m t+\delta)$$

$$= -4(1+\cos(240\pi t))\sin(\omega_m t+\delta)$$

$$= -2\sin(\omega_m t+\delta) - 2\sin(\omega_m t+\delta)\cos(240\pi t)$$

$$= -2(\sin(\omega_m t+\delta) + \sin(\omega_m t+\delta)\cos(240\pi t))$$

Product Identity

$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

$$= -2\left(\sin(\omega_m t+\delta) + \frac{1}{2}\sin(\omega_m t+240\pi t+\delta) + \frac{1}{2}\sin(\omega_m t-240\pi t+\delta)\right)$$

cont.

$\Rightarrow T = -2\,(\sin(\omega_m t+\delta) + \frac{1}{2}\sin(\omega_m t + 240\pi t + \delta) + \frac{1}{2}\sin(\omega_m t - 240\pi t + \delta))$

$= -2\,(\sin(\omega_m t+\delta) + \frac{1}{2}\sin((\omega_m + 240\pi)t + \delta) + \frac{1}{2}\sin((\omega_m - 240\pi)t + \delta))$

→ We get nonzero avg torque if we can get the t terms inside any of the sinusoids to cancel.

So we have $\omega_m t$, $(\omega_m + 240\pi)t$, & $(\omega_m - 240\pi)t$

Set to 0.

$\omega_m t = 0 \Rightarrow \boxed{\omega_m = 0}$

$(\omega_m + 240\pi)t = 0 \Rightarrow \boxed{\omega_m = -240\pi}$

$(\omega_m - 240\pi)t = 0 \Rightarrow \boxed{\omega_m = +240\pi}$

MAX TORQUE

For $\omega_m = 0$, $T = -2\sin(\delta)$ $\therefore$ $\boxed{|T_{max}|\big|_{\omega_m=0} = 2\ N\cdot m}$

For $\omega_m = \pm 240\pi$, $T = -2\left(\frac{1}{2}\sin(\delta)\right)$ $\therefore$ $\boxed{|T_{max}|\big|_{\omega_m = \pm 240\pi} = 1\ N\cdot m}$

PROBLEM 3 — 3φ, 5HP, 208V, 60Hz INDUCTION MOTOR RUNS @ 1746 RPM WHEN DELIVERING RATED OUTPUT POWER.

(A) DETERMINE NUMBER OF POLES.

$n_s = \frac{60 f_s}{P/2} \Rightarrow P = \frac{2\cdot 60 \cdot f_s}{n_s} = \frac{2\cdot 60\cdot(60\text{Hz})}{1746} \approx \boxed{4\text{ poles}}$

(B) DETERMINE SLIP @ FULL LOAD.

$n_m = 1746$ rpm @ FULL-LOAD. $s = 1 - \frac{n_m}{n_s}$

$f_s = 60$Hz, so $n_s = \frac{120\cdot 60}{4} = 1800$ RPM IN IDEAL CONDITIONS.

$\Rightarrow s = 1 - \frac{1746}{1800} = \boxed{0.03 = s}$

(C) DETERMINE FREQUENCY OF ROTOR CURRENT.

$f_r = s f_s = (0.03)(60\text{Hz}) = \boxed{1.8\text{Hz} = f_r}$

(D) DETERMINE SPEED OF ~~the~~ ROTOR FIELD WRT (i) STATOR (ii) STATOR FIELD.

(i) $\boxed{1800\text{ RPM}}$

(ii) $\boxed{0\text{ RPM}}$

PROBLEM 4 3ϕ, 460V, 100HP, 60Hz, SIX-POLE INDUCTION MACHINE OPERATES AT +3% SLIP AT FULL-LOAD.

(A) DETERMINE SPEEDS OF ~~[illegible]~~ MOTOR AND ITS DIRECTION RELATIVE TO THE ROTATING FIELD.

SINCE SLIP IS POSITIVE, THE DIRECTION OF MOTOR ROTATION IS IN THE SAME DIRECTION OF THE ROTATING FIELD, BUT 3% SLOWER.

$$n_s = \frac{120 f_s}{P} = \frac{120\,(60\,Hz)}{6} = \boxed{1200\,rpm = n_s}$$

$$n_m = (1-s)n_s = (1-0.03)(1200) = \boxed{1164\,rpm}$$

(B) DETERMINE ROTOR FREQUENCY & SPEED OF STATOR FIELD.

$$f_r = s f_s = (0.03)(60) = \boxed{1.8\,Hz}$$

& Speed of stator field $= n_s = 1200\,rpm$

(C) DETERMINE SPEED OF AIRGAP FIELD

$$\boxed{n_g = 1200\,rpm}$$

(D) SPEED OF ROTOR FIELD RELATIVE TO

(i) ROTOR STRUCTURE

$$n_{slip} = s n_s = (0.03)(1200) = \boxed{36\,rpm}$$

(ii) STATOR STRUCTURE

$$\boxed{1200\,rpm}$$

(iii) STATOR FIELD.

$$\boxed{0\,rpm}$$