Vectors on Manifolds

Chase Lotito

Southern Illinois University Carbondale

November 9, 2024

What is a tangent vector?

DEF: A tangent vector at a point $p \in M$ is a map from smooth functions on your manifold to scalars,

$$v_p : \mathcal{F}M \to \mathbb{R} \tag{1}$$

such that, for f, $g \in \mathcal{F}M$ and $c \in \mathbb{R}$:

1. $v_p(f+g) = v_p(f) + v_p(g)$
2. $v_p(cf) = cv_p(f)$
3. $v_p(fg) = v_p(f)g(p) + f(p)v_p(g)$