Homework 08

ECE 478/ECE 570 – Principles of Communication Systems

Posted date: 04/21/2025
Due by: 11.59 PM – 05/02/2025
Section: Baseband and bandpass digital transmissions
Number of problems: 06
Policy: Late submissions will not be accepted.
(Q.3) and (Q.5) are optional for undergraduate students.

Question (01): Matched filtering [20 marks]

Consider the pulse signal $s(t)$ given in Fig. 1.

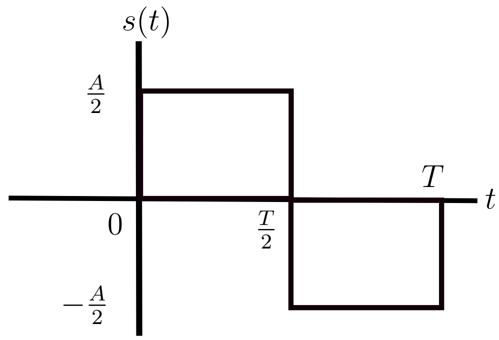

Figure 1: Pulse signal $s(t)$

(a.) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.

(b.) Suppose that the pulse signal $s(t)$ in Fig. 1 is passed through the optimal filter in part (a). Determine and plot the matched filter output as a function of time.

(c.) Determine the peak value of the output and hence find the best sampling instance.

Question (02): Raise cosine spectrum [20 marks]

(a). A raised cosine spectrum with roll-off factor, which can be used to eliminate inter-symbol interference (ISI), is given by

$$P(f) = \begin{cases} \frac{1}{2W}, & 0 \le |f| < f_1 \\ \frac{1}{4W}\left\{1 - \sin\left(\frac{\pi[|f|-W]}{2W-2f_1}\right)\right\}, & f_1 \le |f| < 2W - f_1 \\ 0, & |f| \ge 2W - f_1, \end{cases}$$

where the frequency parameter f_1 and bandwidth W are related by

$$\alpha = 1 - \frac{f_1}{W}$$

Suppose that a binary PAM wave is to be transmitted over a baseband channel with an absolute maximum bandwidth of 75 kHz. The bit duration is 10 μs.

(a.1) Find a raised cosine spectrum that satisfy these requirements.

[Hint: You just need to find the corresponding values for the roll-off α and frequency f_1 for above $P(f)$. Then substitute these values back to $P(f)$ to obtain the desired raised cosine pulse.]

(a.2) The raised cosine pulse spectrum is not the only one that satisfies Nyquist's criteria for ISI free baseband digital transmission. Consider the trapezoidal pulse spectrum shown in Fig. 2. Justify that this trapezoidal spectrum too satisfy the zero ISI condition.

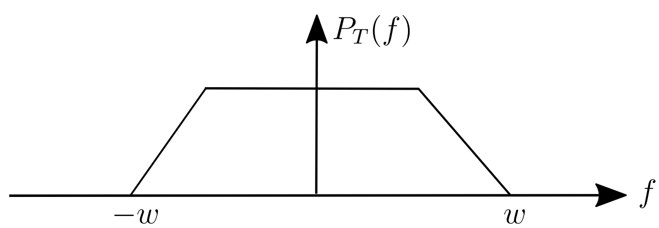

Figure 2: Trapezoidal pulse spectrum

(a.3) Suggest another pulse spectrum that satisfies Nyquist's criterion.

Question (03): Probability of error for M-ary baseband PAM [20 marks]

(a). Binary signaling provides the greatest immunity to noise for a given signal-to-noise ratio because it has only two amplitude levels. Multilevel M-ary signaling requires more signal power but less transmission bandwidth because the signaling rate will be smaller than the bit rate of an equivalent binary signal. Consequently, M-ary signaling suits applications such as digital transmission over voice channels where the available bandwidth is limited and the signal-to-noise ratio is relatively large.

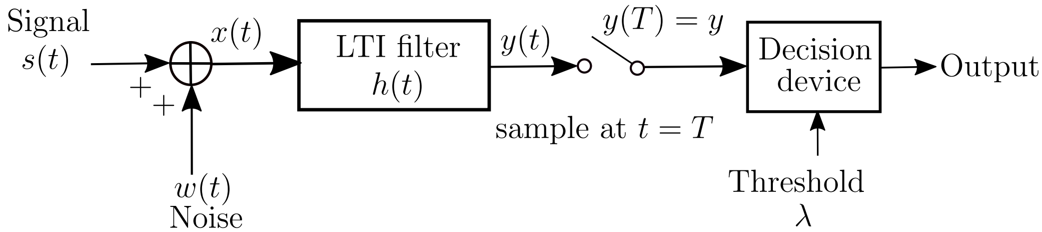

Figure 3: M-ary PAM receiver

Consider a baseband M-ary system using M discrete amplitude levels. The receiver model is as shown in Fig. 3. Suppose the following conditions are satisfied by the receiver.

- The most common case of polar signaling with an even number of equispaced levels are considered as follows:

$$a_k = \pm A/2, \pm 3A/2, \cdots, \pm (M-1)A/2$$

- The M levels are equiprobable and the symbols transmitted in adjacent time-slots are statistically independent.
- The noise $w(t)$ at the receiver input is white and Gaussian with zero mean and power spectral density $N_0/2$.
- The low-pass filter is ideal with bandwidth $B = 1/(2T)$.
- The threshold levels used in the decision device are $0, \pm A, \cdots, \pm(M-3)A/2$.

The average probability of symbol error in this system is defined as

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{A}{2\sigma}\right)$$

where σ is the standard deviation of the noise at the input of the decision device.

(a.1) Demonstrate the validity of the aforementioned P_e by deriving the probability of symbol error for $M = 4$.

Question (04): Probability of error for binary ASK and PSK [20 marks]

(a.) In the on-off version of an amplitude shift keying (ASK) system, symbol 1 is represented by transmitting a sinusoidal carrier of amplitude $\sqrt{2E_b/T_b}$ where E_b is the signal energy per bit and T_b is the bit duration. Symbol 0 is represented by switching off the carrier. Assume that symbol 1 and symbol 0 occur with equal probability. The noise is modeled as additive white Gaussian noise with mean zero and power spectral density $N_0/2$.

(a.1) Provide a block diagram for a coherent receiver for this ASK signal.

(a.2) Determine the average probability of error for this ASK system with coherent reception.

(a.3) Suppose symbol 1 occurs with probability 2/3 and symbol 0 occurs with probability 1/3. How would the receiver design and probability of error change if the objective is to minimize the overall probability of error?

(b.) A binary phase shift keying (PSK) signal is applied to a coherent receiver supplied with a phase reference that lies within ϕ radians of the exact carrier phase.

(b.1) Provide a block diagram for a coherent receiver for this PSK signal.

(b.2) Determine the average probability of error for this PSK system with coherent reception.

Question (05): Probability of error for binary FSK [20 marks]

(a.) In a coherent frequency shift keying (FSK) system, the signals $s_1(t)$ and $s_2(t)$ representing symbol 1 and symbol 0, respectively, are defined by

$$\begin{aligned} s_1(t) &= A_c \cos\left[2\pi\left(f_c + \frac{\Delta f}{2}\right)\right], \quad 0 \le t \le T_b \\ s_2(t) &= A_c \cos\left[2\pi\left(f_c - \frac{\Delta f}{2}\right)\right], \quad 0 \le t \le T_b \end{aligned}$$

(a.1) Assuming that $f_c > \Delta f$, show that the correlation co-efficient of the signals $s_1(t)$ and $s_2(t)$ defined by

$$\rho = \frac{\int_0^{T_b} s_1(t)s_2(t)dt}{\sqrt{\int_0^{T_b} s_0^2(t)dt \int_0^{T_b} s_1^2(t)dt}}$$

can be approximated as follows:

$$\rho \approx \mathrm{sinc}(2\Delta f T_b)$$

(a.2) What is the minimum value of frequency shift Δf for which the signals $s_1(t)$ and $s_2(t)$ are orthogonal?

(a.3) What is the value of Δf that minimizes the average probability of symbol error?

(a.4) For the value of Δf obtained in part (a.3), determine the increase in E_b/N_0 required so that this coherent FSK receiver has the same noise performance as a coherent binary PSK receiver.

(b.) A binary FSK signal with discontinuous phase is generated by using the incoming binary data stream to switch between two independent sinusoidal oscillators:

$$\begin{aligned} \text{Symbol 1:} \quad s_1(t) &= \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t + \theta_1) \\ \text{Symbol 0:} \quad s_2(t) &= \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t + \theta_2) \end{aligned}$$

(b.1) Show that this BFSK signal may be expressed as the sum of two independent BASK signals.

(b.2) For $\theta_1 = 30^0$and $\theta_2 = 45^0$, plot the BFSK waveform for the binary data stream 01101001.