ECE 448/ECE 544 (Spring 2025): Homework 2

1. *(Exercise 1.4-1) Special Forms of the Ray-Transfer Matrix.* Consider the following situation in which one of the four elements of the ray-transfer matrix vanishes:

   a. Show that $A = 0$ represents a focusing system, in which all rays entering the system at a particular angle, whatever this position, leave at a single position.
   b. Show that $B = 0$ represents an imaging system, in which all rays entering the system at a particular position, whatever this angle, leave at a single position.
   c. What are the special features of a system for which $C = 0$ or $D = 0$?

   You should commit these relationships to memory because they will be very important—both in this homework and beyond.

2. *(Based on Exercise 1.4-2) A Set of Parallel Transparent Plates.* Consider a set of two parallel planar transparent plates of refractive indices n_1 and n_2 and thicknesses d_1 and d_2, placed in air (with $n = 1$) normal to the z axis.

   a. Show that the ray-transfer matrix is

$$M = \begin{bmatrix} 1 & \frac{d_1}{n_1} + \frac{d_2}{n_2} \\ 0 & 1 \end{bmatrix}$$

   b. How are the height y_i and angle θ_i of an incident ray related to the height y_t and angle θ_t of a transmitted ray? Do their transmitted values depend on the order of the plates?

3. *(Based on Exercise 1.4-4) Imaging with a Thin Lens.*

   a. Derive an expression for the ray-transfer matrix of a system comprised of free space (of thickness d_1) followed by a thin lens (of focal length f) followed by free space (of thickness d_2).
   b. Show that if the imaging condition $1/d_1 + 1/d_2 = 1/f$ is satisfied, all rays originating from a single point y_1 in the input plane reach the output plane at the single point y_2, regardless of their angles.
   c. Show that if the focusing condition $d_2 = f$ is satisfied, all parallel incident rays are focused by the lens onto a single point in the output plane.

4. *(Based on Exercise 1.4-6) Ray-Transfer Matrix of a Lens System.*

   a. Determine the ray-transfer matrix for a thin lens system made of a thin convex lens of focal length f and a thin concave lens of focal length $-f$ separated by a distance f.

b. Determine the ray-transfer matrix for an optical system consisting of free space (of thickness d_1) followed by the thin lens system from the previous part followed by free space (of thickness d_2).
c. If all rays originating from a single point in the input plane arrive at a single point in the output plane, determine an expression for d_2 in terms of d_1. If the rays originate at a height y_1 above the optical axis and arrive at a height y_2 above the optical axis, determine an expression for y_2 in terms of y_1.