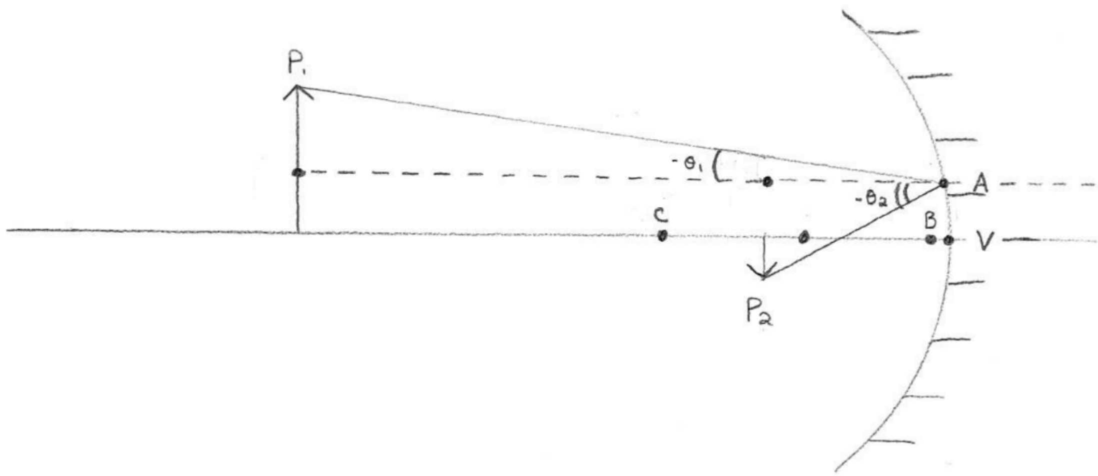
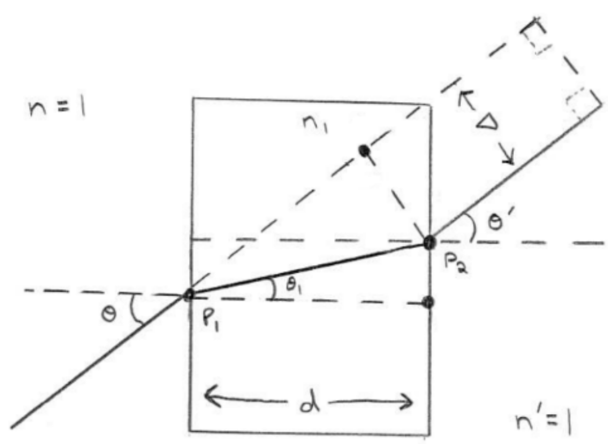
ECE 448/ECE 544/BME 531 (Spring 2025): Homework 1

1. *(Based on Exercise 1.2-1) Image Formation by a Spherical Mirror.* Show that, under the small-angle approximation, rays originating from a point $P_1 = (y_1, z_1)$ are reflected by a spherical mirror of focal length f to a point $P_2 = (y_2, z_2)$ where $1/z_1 + 1/z_2 = 1/f$ and $y_2 = -y_1 z_2/z_1$. This means that rays from each point in the plane $z = z_1$ meet at a single corresponding point in the plane $z = z_2$ so that the mirror acts as an image-formation system with magnification $-z_2/z_1$. Negative magnification means that the image is inverted.



2. *(Based on Exercise 1.2-7) Transmission through Planar Plates.*

   a. Use Snell's Law to show that a ray entering a planar plate of thickness d and refractive index n_1 (placed in air with refractive index $n \approx 1$) emerges parallel to its initial direction (that is, $\theta = \theta'$). Then, derive an expression for the ray's displacement Δ as a function of the angle of incidence θ.



b. If the plate instead comprises a stack of N parallel layers stacked against each other with thicknesses $d_1, d_2, \ldots, d_N$ and refractive indices $n_1, n_2, \ldots, n_N$, show that the transmitted ray is parallel to the incident ray. If θ_m is the angle of the ray in the mth layer, show that $n_m \sin\theta_m = \sin\theta$ for $m = 1,2, \ldots$.

3. An object of height 20 mm lies a distance 60 mm from a concave mirror of radius 40 mm.

    a. Using a graphical approach (i.e., a ray-tracing diagram), determine the distance of the image with respect to the mirror and the height of the image above the optical axis. Additionally, determine the magnification of the image with respect to the object. (To help with this task, a to-scale system diagram is provided in the following pages.)
    b. Using a computational approach (i.e., the Gaussian imaging equations), determine the distance of the image with respect to the mirror and the height of the image above the optical axis. Additionally, determine the magnification of the image with respect to the object. (Note that the answer to this part should match up pretty well to the answer from the last part.)
    c. Is the image real or virtual? Explain.

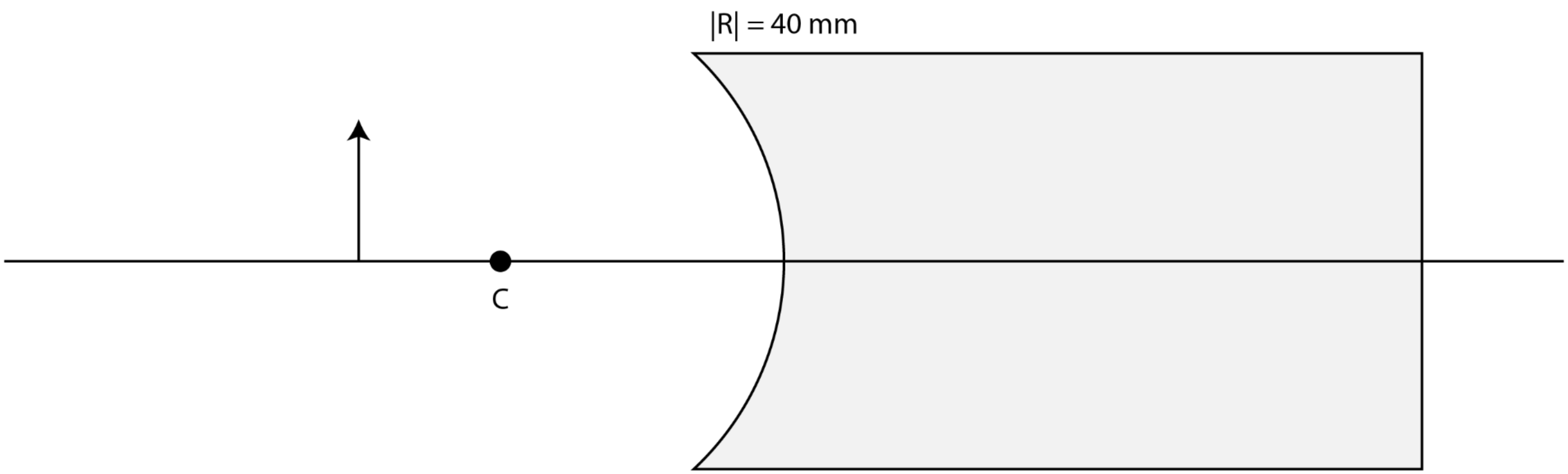
4. Repeat the previous problem, replacing the mirror with a convex mirror.

5. An object of height 10 mm lies a distance 50 mm from a lens L1 which in turn lies a distance 115 mm from a lens L2. Lens L1 (with focal length 30 mm) is biconvex with surface radii 61.32 mm, lens L2 (with focal length 20 mm) is biconvex with surface radii 40.88 mm, and both lens L1 and lens L2 are made of high-index LASF35 optical glass with refractive index 2.022. Assume that the surrounding medium is air.

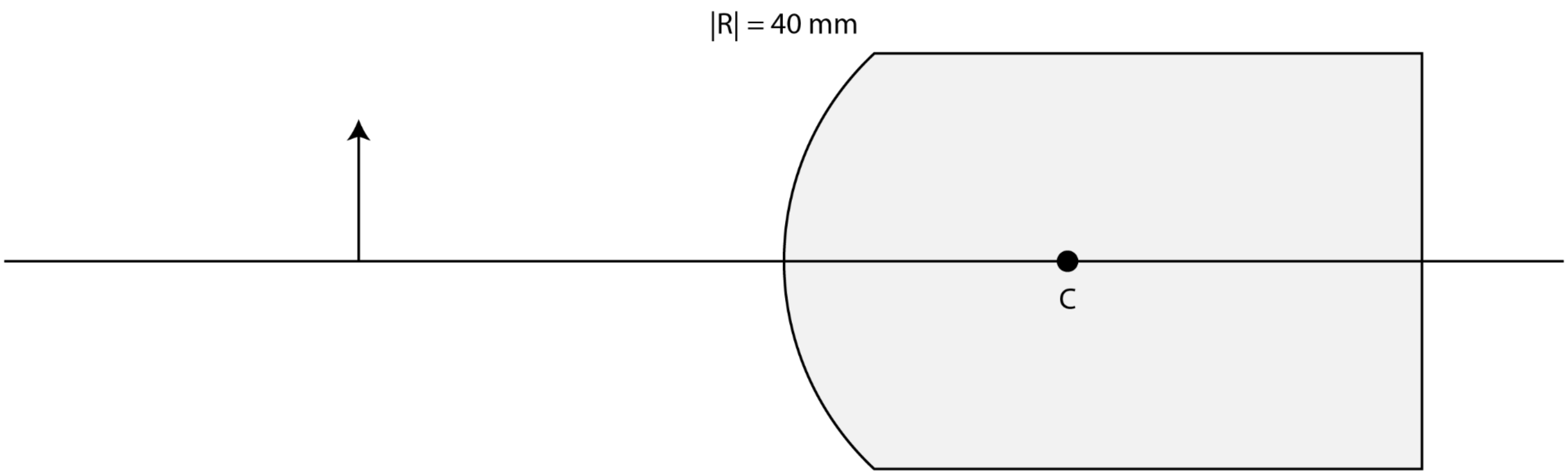
    a. Using a graphical approach (i.e., ray-tracing diagrams), determine the distance of the image with respect to lens L2 and the height of the image above the optical axis. Additionally, determine the magnification of the image with respect to the object. (To help with this task, a to-scale system diagram is provided in the following pages.)
    b. Using a computational approach (i.e., equations), determine the distance of the image with respect to lens L2 and the height of the image above the optical axis. Additionally, determine the magnification of the image with respect to the object, and verify that this value, the combined magnification of lens L1 and lens L2, equals the product of the individual magnifications from lens L1 and lens L2. (Note that the answer to this part should match up pretty well to the answer from the last part.)

6. Repeat the previous problem, moving the object to a distance 18 mm from lens L1.

To-Scale System Diagram for Problem 3:



To-Scale System Diagram for Problem 4:



To-Scale System Diagram for Problem 5:

To-Scale System Diagram for Problem 6: