Homework 04

ECE 478/ECE 570 – Principles of Communication Systems

SIU Southern Illinois University CARBONDALE

Posted date: 03/05/2025
Due by: 11.59 PM – 03/19/2025
Section: Angle Modulation
Number of problems: 04
Policy: Late submissions will not be accepted.
(Q.4) is optional for undergraduate students.

Question (01): Frequency Modulation [25 marks] SO1-SOLVE: KPI - 1.1 (complexity), 1.2 (formulate) and 1.3 (solve)

A sinusoidal carrier wave with amplitude A_c and frequency f_c is frequency modulated (FM) using a sinusoidal information-bearing signal with frequency f_m and amplitude A_m. Assume that $f_c \gg f_m$. Then, the resulting wide-band FM signal is given by

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t], \tag{1}$$

where $J_n(\cdot)$ is the Bessel function of the first kind and nth order. The frequency deviation of FM signal is given by $\Delta f = k_f A_m$, where k_f is the frequency sensitivity of the modulator. The modulation index of this FM modulator is defined as $\beta = \Delta f / f_m$.

(a) [SO1.1] Determine the values of the modulation index β for which the carrier component of the FM signal is reduced to zero for the first, second, third and fourth times by identifying the complexity of amplitude changes associated with the FM signal in (1), and recognizing the known and unknown variables. The underlying problem-solving strategy must be clearly specified when computing the corresponding modulation indexes.

(b) [SO1.2] In a certain experiment conducted with $f_m = 2\,\text{kHz}$ and increasing A_m starting from 0 volts, it is observed that the carrier component of the FM signal is reduced to zero for the first time when $A_m = 4$ volts. Formulate an analytical expression for the frequency sensitivity of the modulator in terms of the modulation index, frequency and amplitude of the information-bearing signal. Thereby, numerically quantify the frequency sensitivity k_f of the modulator.

(c) [SO1.3] Write an analytical expression to find the amplitude of the information-bearing signal (A_m) for which the carrier component reduces to zero. By using this expression, solve for A_m for which the carrier component is reduced to zero for the second, third and fourth times.

Question (02): Frequency Modulation [25 marks]

Consider the wide-band FM signal given by

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

(b.1) Determine the spectrum $S(f)$ of the above FM signal by taking the Fourier transform of $s(t)$.

(b.2) Assuming the modulation index $\beta = 1$ and $n \in \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$, plot the truncated amplitude spectrum $|S(f)|$ of the above FM signal.

[hint: $J_n(x) = (-1)^n J_{-n}(x)$.]

(b.3) Assume that the above FM signal is transmitted through an ideal band-pass filter in Fig. 1 with mid-frequency f_c and bandwidth $7f_m$, where f_c is the carrier frequency and f_m is the frequency of the sinusoidal message signal. Determine the amplitude spectrum of the filter output.

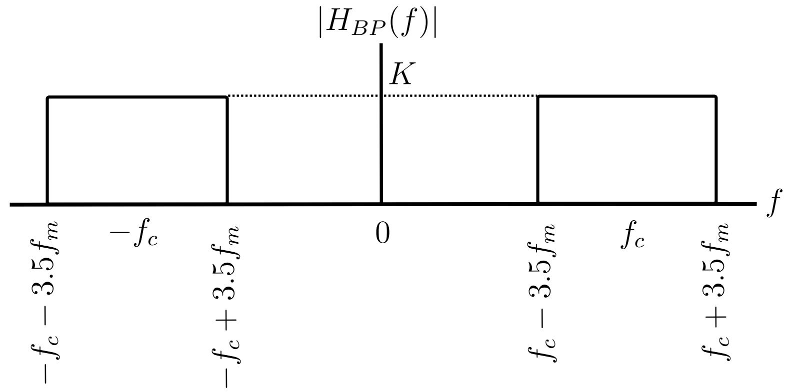

Figure 1: Frequency response of an ideal band-pas filter with a bandwidth $7f_m$

Question (03): Narrow-band Frequency Modulation [25 marks]

Consider a narrow-band frequency modulated (FM) signal approximately defined by

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \tag{2}$$

(a). The envelope of this FM signal is defined as

$$e(t) = \sqrt{s_I^2(t) + s_Q^2(t)},$$

where $s_I(t) = A_c$ and $s_Q(t) = \beta A_c \sin(2\pi f_m t)$. By using this definitions, determine the envelope of $s(t)$ as a function of β.

(b). Determine the ratio of the maximum to the minimum value of this envelope.

(c). Plot this ratio versus the modulation index (β) assuming β is restricted to the interval $0 \leq \beta \leq 0.3$.

(d). By using the Fourier transform of (2), determine the average power of the narrow-band FM signal, expressed as a percentage of the average power of the unmodulated carrier wave. Plot this result versus β, assuming that β is restricted to the interval $0 \leq \beta \leq 0.3$. What is the reason for the average power of the narrow-band FM signal not being equal to $A_c^2/2$.

(e). The instantaneous angle ($\theta_i(t)$) of the FM signal in (2) is given by

$$\theta_i(t) = 2\pi f_c t + \tan^{-1}\left[\frac{s_Q(t)}{s_I(t)}\right],$$

where $s_I(t) = A_c$ and $s_Q(t) = \beta A_c \sin(2\pi f_m t)$. By using this definition, determine the instantaneous angle of $s(t)$ as a function of β. By using the power series for $\tan^{-1}(\cdot)$, find the first three terms of the expansion of $\theta_i(t)$.

(f). The harmonic distortion is defined as the power ratio of the third harmonic and the first harmonic. Determine the harmonic distortion for $\beta = 0.37$.

Question (04): Phase modulation

(a) Consider the following sinusoidal modulating wave;

$$m(t) = A_m \cos(2\pi f_m t).$$

Assume that the above signal $m(t)$ is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier wave is also a sinusoidal signal with frequency f_C and amplitude A_c. This phase modulated signal is given by

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t)).$$

(a.1) Determine the spectrum of the resulting phase modulated signal, assuming that the maximum phase deviation $\beta_p = k_p A_m$ does not exceed 0.3 radians.

(a.2) Determine the average power of the above phase modulated signal

(b) Suppose that the phase modulated signal in part (a) has an arbitrary value for the maximum phase deviation β_p. That is the resulting phase modulated wave can be a wide-band signal.

(b.1) Determine the complex envelope $\tilde{s}(t)$ of the phase modulated signal.
[hint: $s(t) = \mathrm{Re}\left[\tilde{s}(t)\exp(j2\pi f_c t)\right]$]

(b.2) Show that this complex envelope $\tilde{s}(t)$ can be expressed in terms of a complex Fourier series as follows:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t),$$

where the complex Fourier co-efficient c_n is given by

$$c_n = A_c \exp\left(-\frac{jn\pi}{2}\right) J_{-n}\left(\beta_p\right).$$

Here $J_n(x)$ is the nth order Bessel function of the first kind defined as

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left[j\left(\beta \sin(x) - nx\right)\right] dx.$$

(b.3) Show that the original phase modulated signal $s(t)$ can be expressed in terms of the Fourier series representation of the complex envelop $\tilde{s}(t)$ as follows:

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_{-n}\left(\beta_p\right) \cos(2\pi(f_c + n f_m)t - \frac{n\pi}{2}).$$

(c) Assume that the phase modulated signal in part (b) is applied to an ideal band-pass filter shown in Fig. 2 with mid-band frequency f_c and a passband extending from $f_c - 1.5 f_m$ and $f_c + 1.5 f_m$.

(c.1) Determine the envelop, phase, and instantaneous frequency of the modulated signal at the filter output as functions of time.

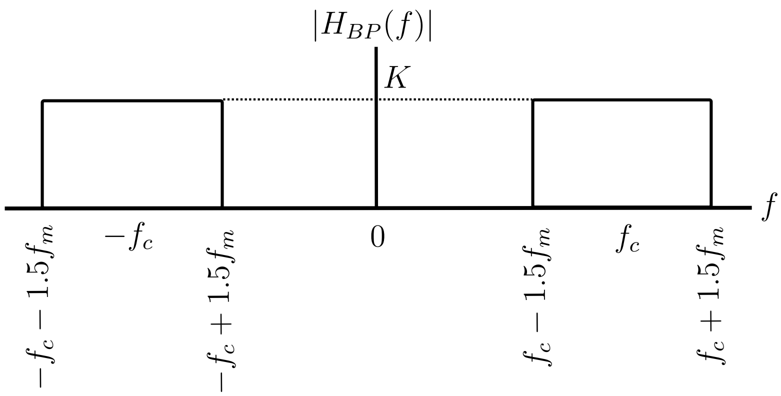

Figure 2: Frequency response of an ideal band-pas filter with a bandwidth $7f_m$