Homework 03

ECE 478/ECE 570 – Principles of Communication Systems

Posted date: 02/26/2025
Due by: 11.59 PM – 03/06/2025
Section: Amplitude modulation
Number of problems: 05
Policy: Late submissions will not be accepted.
Undergraduate students: (Q.3.c), (Q.4.b) and (Q.5) are optional
Undergraduate students can earn bonus marks by solving optional problems.

Question (01): Amplitude Modulation and Demodulation [25 marks]

Consider the triangular pulse $m_{T_0}(t)$ shown in Fig. 1.

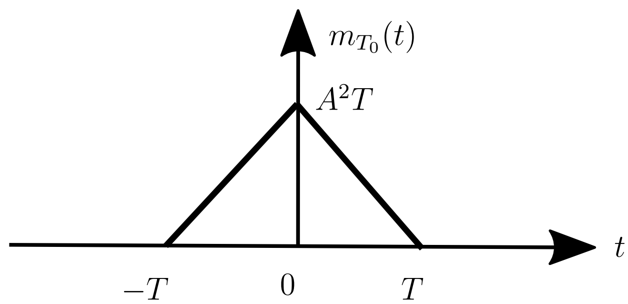

Figure 1: Triangular Pulse

(a). Show that the triangular pulse in Fig. 1 can be obtained by the convolution of two rectangular pulses given by $x(t) = A\text{rect}\left(\frac{t}{T}\right)$. In other words, show that $m_{T_0}(t) = x(t) \circledast x(t)$.

(b). Use the fact that $m_{T_0}(t) = x(t) \circledast x(t)$ to derive the Fourier transform of the triangular pulse $m_{T_0}(t)$.

(c). Consider the periodic triangular wave $m(t)$ shown in Fig. 2. Find the Fourier transform of $m(t)$ by using the results in part (b).

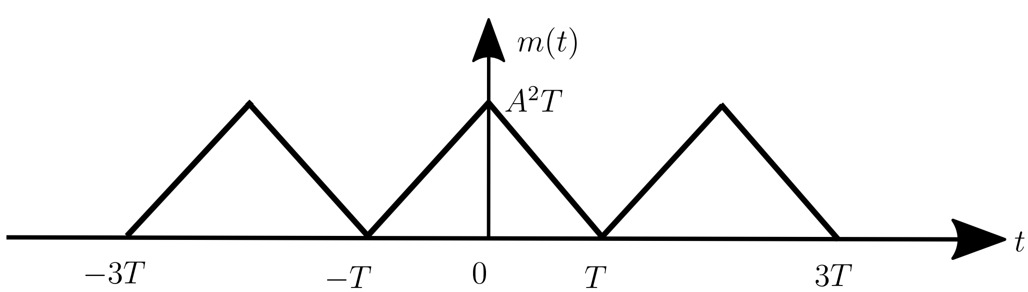

Figure 2: Periodic triangular wave

(d). Let the periodic triangular wave in Fig. 2 be an input to an amplitude modulator whose output signal is given by

$$s(t) = A_c\,[1 + k_a m(t)]\cos(2\pi f_c t),$$

where $c(t) = A_c \cos(2\pi f_c t)$ is the carrier signal, f_c is the carrier frequency, k_a is the modulator sensitivity and $m(t)$ is the input message signal.

(d.1) Sketch the amplitude modulated wave for (i) 75% modulation, (ii) 100% modulation, and (iii) 125% modulation by showing all relevant values.

(d.2) Sketch the output after passing $s(t)$ for three aforementioned cases (i.e., (i) 75% modulation, (ii) 100% modulation, and (iii) 125% modulation) through an ideal envelope detector. Does the envelop detector recover the input message without distortion? (Discuss your answers for all three cases.)

(d.3) Find the Fourier transform $S(f)$ of the amplitude modulated signal $s(t)$ for 75% modulation.

(d.4) Sketch the corresponding amplitude spectrum for 75% modulation.

(e). Let the periodic triangular wave in Fig. 2 be an input to a double side-band suppressed carrier (DSB-SC) modulator whose output signal is given by

$$s(t) = A_c m(t)\cos(2\pi f_c t),$$

where $c(t) = A_c \cos(2\pi f_c t)$ is the carrier signal, f_c is the carrier frequency, and $m(t)$ is the input message signal.

(e.1) Sketch the aforementioned DSB-SC modulated wave.

(e.2) Find the Fourier transform $S(f)$ of the DSB-SC modulated signal $s(t)$.

(e.3) Sketch the corresponding amplitude spectrum.

(f). Consider the coherent detector shown in Fig. 3. Let the DSB-SC modulated signal $s(t) = A_c m(t)\cos(2\pi f_c t)$ be an input to this coherent detector in Fig. 3.

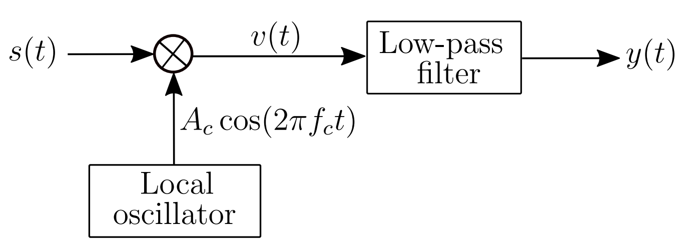

Figure 3: Coherent detector

(f.1) Write an expression for $v(t)$, and thereby, find its Fourier transform $V(f)$.

(f.2) Sketch the spectrum $V(f)$.

(f.3) Sketch the spectrum of the output of the coherent detector $|Y(f)|$ and thereby confirm that the original message signal can be decoded.

(f.4) Assume that the local oscillator has a constant phase error of ϕ. Therefore, the local oscillator output is given by $A_c \cos(2\pi f_c t + \phi)$. Derive an expression for the output signal $y(t)$.

Question (02): Multi-tone amplitude modulation and multi-tone DSB-SC modulation [15 marks]

(a) Consider the block diagram of an amplitude modulator shown in Fig. 4.

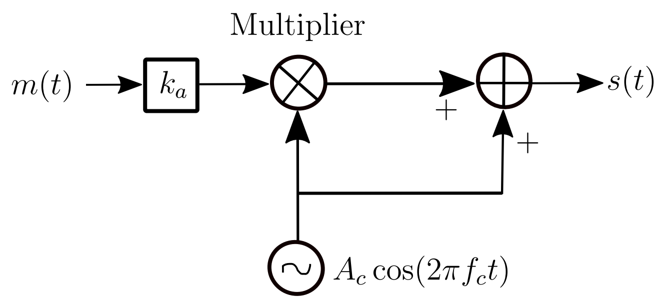

Figure 4: Block diagram of an amplitude modulator

Let the input message be a multi-tone signal given by

$$m(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t),$$

where $f_1 < f_2$ and $A_1 < A_2$.

(a.1) Write an expression for the amplitude modulated wave $s(t)$.

(a.2) Derive the corresponding frequency spectrum $S(f)$.

(a.3) Sketch the amplitude spectrum of the amplitude modulated signal $s(t)$.

(b) Consider the block diagram of a DSB-SC modulator shown in Fig. 5.

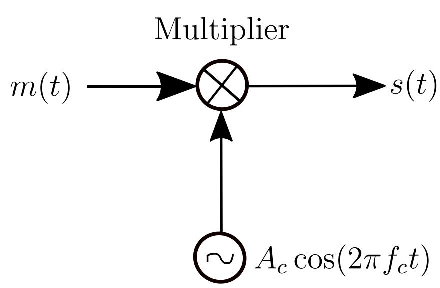

Figure 5: Block diagram of a DSB-SC modulator

Let the input message be a multi-tone signal given by

$$m(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t) + A_3 \cos(2\pi f_3 t),$$

where $f_1 < f_2 < f_3$ and $A_1 < A_2 < A_3$.

(b.1) Write an expression for the DSB-SC modulated wave $s(t)$.

(b.2) Derive the corresponding frequency spectrum $S(f)$.

(b.3) Sketch the amplitude spectrum of the DSB-SC modulated signal $s(t)$.

Question (03): Quadrature Amplitude Modulation and Demodulation [20 marks]

Quadrature amplitude multiplexing/modulation (QAM) provides the basis for the generation of amplitude modulation (AM) stereo signals. Stereo multiplexing is a form of frequency division multiplexing designed to transmit two separate signals via the same carrier. For instance, these two signals can be two different elements of the same program.

Let $m_l(t)$ and $m_r(t)$ denote the signal picked up by left-hand and right-hand microphones at the transmitter end of the system. They are applied to the simple matrixer in Fig. that generates the sum signal $(m_l(t) + m_r(t))$ and difference signal $(m_l(t) - m_r(l))$.

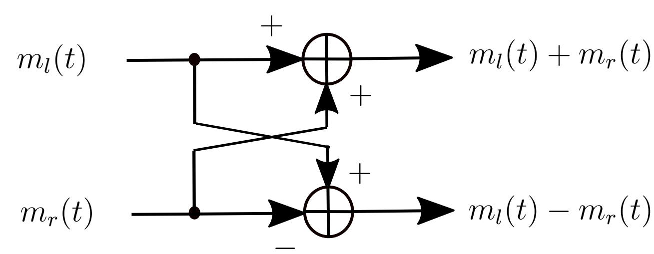

Figure 6: Matrixer

(a) Consider the QAM modulator shown in Fig. 7. Let the input signals to the in-phase and quadrature channels be $m_1(t) = A_0 + m_l(t) + m_r(t)$ and $m_2(t) = m_l(t) - m_r(t)$, respectively. Here, A_0 is a dc offset included for the purpose of transmitting the carrier component.

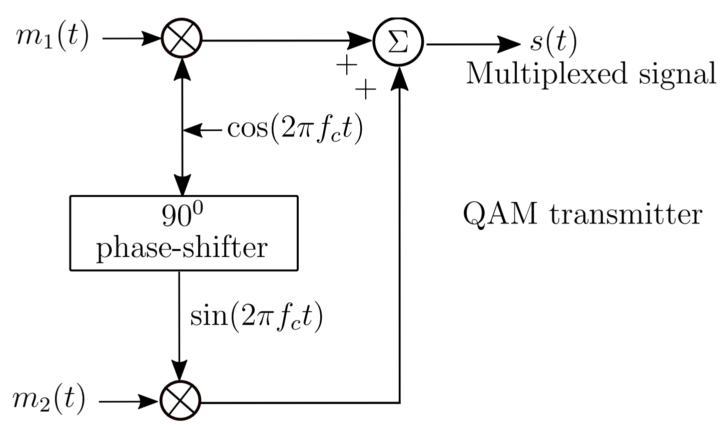

Figure 7: Quadrature amplitude multiplexer/modulator

(a.1) Write an expression for the multiplexed signal $s(t)$.

(a.2) Derive an expression for the envelope of $s(t)$.

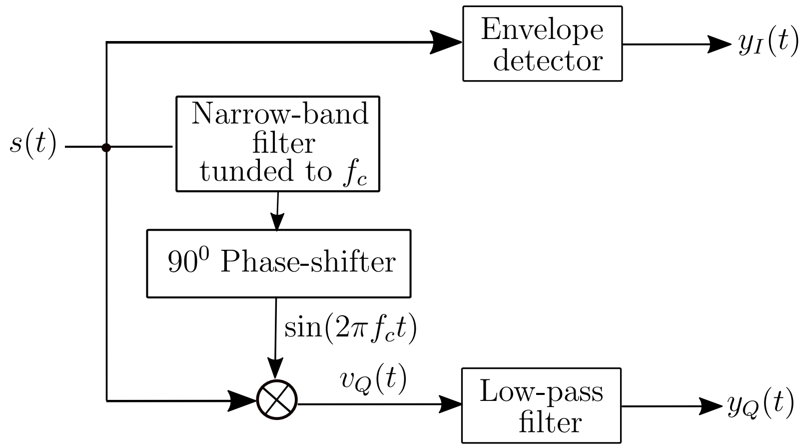

Figure 8: Demultiplexer/demodulator

(b) Consider the demultiplexer/demodulator shown in Fig. 8. Let the multiplexed signal $s(t)$ from part (a) be an input to this demultiplexer.

(b.1) Show that the output signal of the envelope detector $y_I(t)$ is equal to the sum signal $(m_l(t) + m_r(t))$ if A_0 is chosen to be very large.

(b.2) Show that the output signal of the coherent detector $y_Q(t)$ is equal to the the difference signal $(m_l(t) - m_r(t))$.

(b.3) How the desired $m_l(t)$ and $m_r(t)$ are recovered from the aforementioned sum and difference signals.

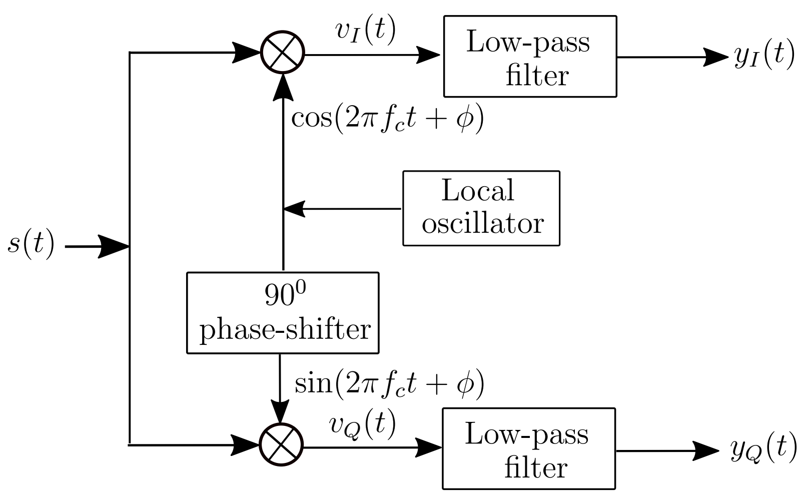

Figure 9: Quadrature amplitude demultiplexer/demodulator with phase errors

(c) Consider the quadrature amplitude demultiplexer/demodulator shown in Fig. 9. Here, the local carrier signal has not been perfectly synchronized to the carrier of the modulator. Assume that there exists a phase error of ϕ in the local oscillator.

Let the input this demodulator be $s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$.

(c.1) Write an expression for the intermediate signal $v_I(t)$ in the in-phase channel.

(c.2) Write an expression for the intermediate signal $v_Q(t)$ in the quadrature-phase channel.

(c.3) Derive the output signals $y_I(t)$ and $y_Q(t)$.

(c.4) Show that the phase error in the local oscillator causes cross-talks to arise between the two demodulated signals ($y_I(t)$ and $y_Q(t)$).

[hint: Here, cross-talk means that a portion of one message signal appears at the output belonging to the other message signal, and vice versa.]

Question (04): DSB-SC modulation and demodulation [20 marks]

(a) Consider the Balanced modulator shown in Fig. 10. Two amplitude modulators in Fig. 10 are identical and have the same modulation sensitivity k_a.

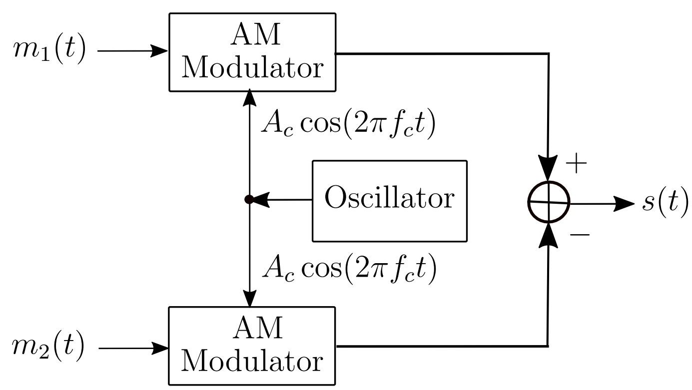

Figure 10: Balanced modulator

(a.1) Write an expression for the output signal $s(t)$ of this balanced modulator.

(a.2) Show that the output signal $s(t)$ becomes a DSB-SC modulated wave if $m_1(t) = m(t)$ and $m_2(t) = -m(t)$.

(b) Consider the coherent detector shown in Fig. 11. Note that the local oscillator's frequency and phase have bot been perfectly synchronized to the carrier signal of the modulator. Therefore, there exists a frequency error of Δf and a phase error of ϕ in this local oscillator.

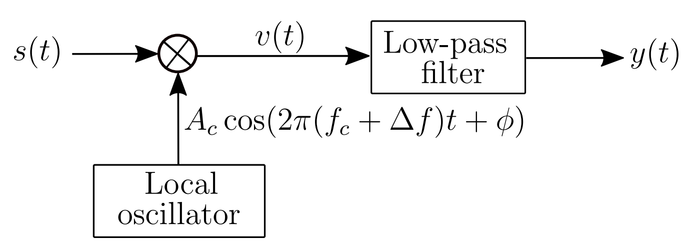

Figure 11: Coherence detector

Let the input signal to this coherence detector be the output signal of the balanced modulator $s(t)$ in Fig. 10.

(b.1) Write an expression for the intermediate signal denoted by $v(t)$ in Fig. 11.

(b.2) By assuming $\phi = 0$ and $\Delta f \neq 0$, derive the output signal $y(t)$.

(b.3) By assuming $\Delta f = 0$ and $\phi \neq 0$, derive the output signal $y(t)$.

(b.4) Drive the output signal $y(t)$ for the general case $\Delta f \neq 0$ and $\phi \neq 0$.

Question (05): SSB modulation and demodulation [20 marks]

(a) Consider the SSB modulator shown in Fig. 12. The transfer function $H(f)$ of the wide-band phase-shifter is given by

$$H(f) = -j\text{sgn}(f),$$

where $\text{sgn}(f)$ is the signum function. Typically, this wide-band phase-shifter is referred to as the Hilbert transformer.

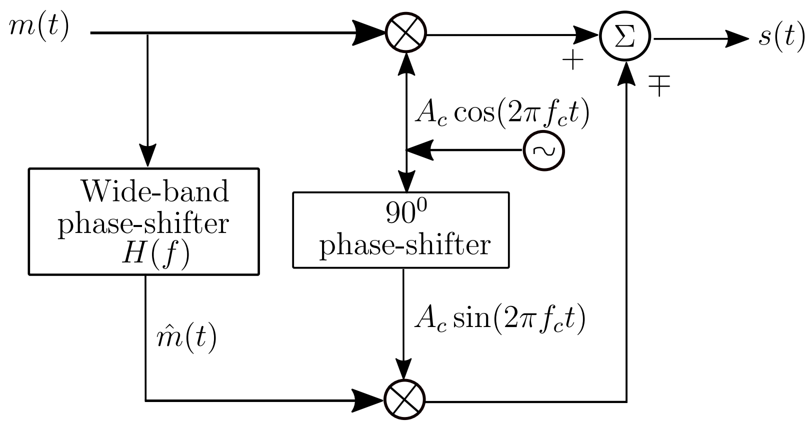

Figure 12: SSB transmitter

(a.1) The output of the wide-band phase-shifter can be written as $\hat{M}(f) = H(f)M(f)$, where $\hat{M}(f)$ and $M(f)$ are the Fourier transforms of $\hat{m}(t)$ and $m(t)$, respectively. Show that the amplitude spectrum of $\hat{m}(t)$ is same as that of $m(t)$. Further, show that the phase of $\hat{m}(t)$ differs by $\pm 90^0$.

(a.2) Write an expression for the output signal $s(t)$.

(a.3) By using the Fourier transform $S(f)$, prove the following two expressions.

$$\text{For the upper SSB: } S(f) = \begin{cases} A_c M(f - f_c), & f \geq f_c \\ 0, & 0 < f \leq f_c \end{cases}$$

$$\text{For the lower SSB: } S(f) = \begin{cases} 0, & f \geq f_c \\ A_c M(f - f_c), & 0 < f \leq f_c \end{cases}$$

(a.4) Discuss the advantages of SSB modulation scheme compared to both DSB-SC modulation and amplitude modulation.

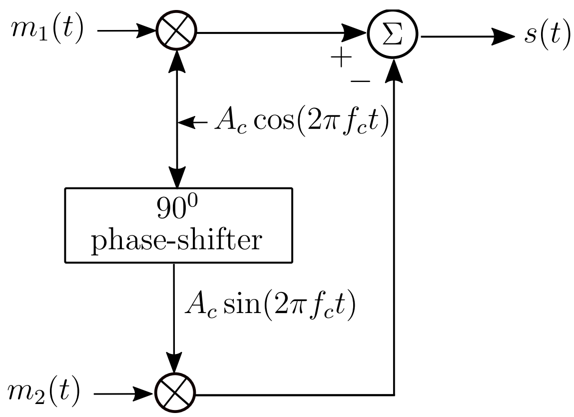

Figure 13: Coherence detector

(b) Consider the modulator shown in Fig. 13.

(b.1) Write an expression for the output signal $s(t)$ of this modulator.

(b.2) Let the input signals to this modulator be $m_1(t) = A_m \cos(2\pi f_c t)$ and $m_2(t) = A_m \sin(2\pi f_c t)$. Show that the output signal is given by $s(t) = A_m A_c \cos(2\pi(f_c + f_m)t)$.

(b.3) Find and sketch the spectrum $S(f)$ of the output signal.

(b.4) Find and sketch the spectrum of the output signal $s(t)$ if $m_1(t) = A_m \cos(2\pi f_c t)$ and $m_2(t) = -A_m \sin(2\pi f_c t)$.

(b.5) Name the corresponding modulation scheme.

(c) Consider the coherent detector shown in Fig. 13.

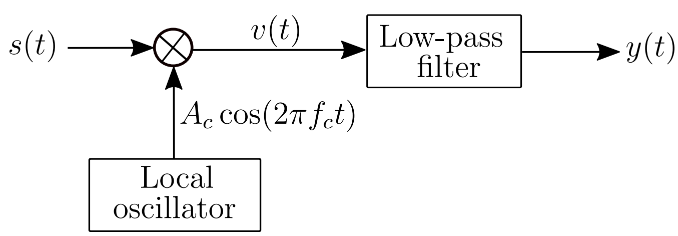

Figure 14: Coherent detector

Let the input to this coherent detector be $s(t)$ from part (b.1)

(c.1) Write an expression for $v(t)$.

(c.2) Find the spectrum $V(f)$ by taking the Fourier transform of $v(t)$.

(c.3) Show that message signal $m(t)$ can be recovered by passing the SSB modulated signal $s(t)$ through this coherent detector by first deriving the spectrum $Y(f)$ and then by obtaining an expression for $y(t)$.