ELE478

HW4

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## 1) FREQUENCY MODULATION

CONSIDER A SINUSOIDAL CARRIER WAVE w/ AMPLITUDE $A_c$ AND FREQUENCY $f_c$ FREQUENCY MODULATED (FM) WITH A SINUSOIDAL INFORMATION BEARING SIGNAL w/ AMPLITUDE $A_m$ AND FREQUENCY $f_m$; THE RESULT IS A WIDE-BAND FM SIGNAL.

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t]$$

$* \; \Delta f = k_f A_m$

$\beta = \Delta f / f_m$

### A) DETERMINE VALUES OF $\beta$ WHICH REDUCE THE CARRIER COMPONENT TO ZERO THE FIRST 4 TIMES.

THE CARRIER COMPONENT IS ASSOCIATED w/ $J_0(\beta)$ SINCE WHEN $n=0$

$\hookrightarrow \; s(t) = A_c J_0(\beta)\cos(2\pi f_c t)$

WHERE IF THE CARRIER IS $c(t)$, THEN IF $n=0$, $s(t) = J_0(\beta)\,c(t)$.

(IN REALITY FOR $n \in (-\infty, \infty)$

$s(t) = \cdots + J_0(\beta)c(t) + \cdots$)

FROM THE BESSEL TABLE

$k=1 \rightarrow J_0(2.4048) = 0$

$k=2 \rightarrow J_0(5.5201) = 0$

$k=3 \rightarrow J_0(8.6537) = 0$

$k=4 \rightarrow J_0(11.7915) = 0$

SO THE CARRIER COMPONENT IS REDUCED TO ZERO FOR $\boxed{\beta = 2.4048,\ 5.5201,\ 8.6537,\ 11.7915}$ FOR THE FIRST, SEC, THIRD, & FOURTH TIMES.

### B) IN AN EXPERIMENT, $f_m = 2\,kHz$ AND $A_m$ IS INCREASED TO 4V AND THE CARRIER COMPONENT IS REDUCED TO ZERO FOR THE FIRST TIME. FIND AN EXPRESSION/QTY FOR THE MODULATOR SENSITIVITY $k_f$.

FOR THE FIRST NULL, $\beta = 2.4048$

$$\Rightarrow \beta = \Delta f / f_m \Rightarrow \Delta f = \beta f_m = (2.4048)(2\,kHz) = 4.8096\,kHz$$

$$\Rightarrow \Delta f = k_f A_m \Rightarrow k_f = \frac{\Delta f}{A_m} = \frac{4.8096\,kHz}{4V} = 1.2024\,\frac{kHz}{V} \Rightarrow \boxed{k_f = 1.2024\,\frac{kHz}{V}}$$

### C) WRITE AN ANALYTICAL EXPRESSION TO FIND THE AMPLITUDE $A_m$ FOR WHICH THE CARRIER COMPONENT REDUCES TO ZERO; THEN, SOLVE FOR THE VALUES OF $A_m$ WHICH REDUCES THE CARRIER TO ZERO THE $1^{st}$, $2^{nd}$, $3^{rd}$, & $4^{th}$ TIMES.

$$\Rightarrow \beta = \frac{k_f A_m}{f_m} \rightarrow A_m = \frac{\beta f_m}{k_f} = \frac{2\,kHz}{1.2024\,\frac{kHz}{V}}\beta \Rightarrow \boxed{A_m = 1.663\beta \text{ Volts}}$$

| 1st | 2nd | 3rd | 4th |
|---|---|---|---|
| $A_m = 1.663(2.4048)\,V$ | $A_m = 1.663(5.5201)\,V$ | $A_m = 1.663(8.6537)\,V$ | $A_m = 1.663(11.7915)\,V$ |
| $= 3.999\,V$ | $= 9.180\,V$ | $= 14.391\,V$ | $= 19.609\,V$ |

$\boxed{A_m = 3.999V,\ 9.180V,\ 14.391V,\ 19.609V}$ REDUCES CARRIER COMP. TO ZERO THE FIRST 4 TIMES.

$\sim 4V \ldots$

## ② FREQUENCY MODULATION

CONSIDER THE WIDE-BAND FM SIGNAL:

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

### (B.1) DETERMINE $S(f)$.

$\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}(\delta(f-f_c) + \delta(f+f_c))$ AND $\mathcal{F}\{\cdot\}$ IS LINEAR $\therefore$

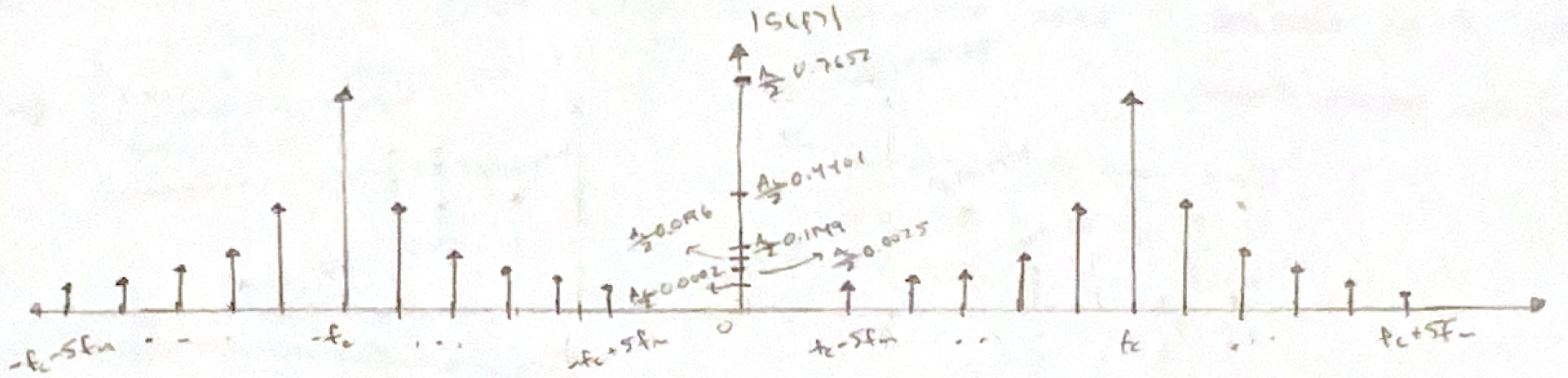
$$\rightarrow \boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m))]}$$

### (B.2) $\beta = 1$ & $n \in \{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$. THEN PLOT $|S(f)|$.

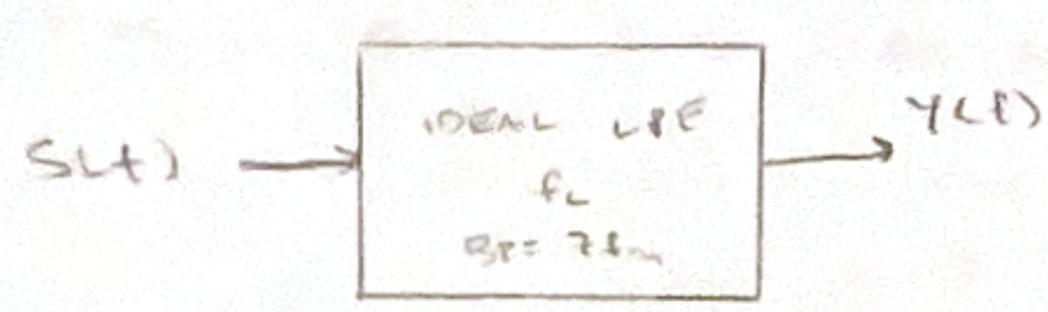
$* J_n(x) = (-1)^n J_{-n}(x)$

doesn't matter for $|S(f)|$ ...

FOR VALS OF $n$ & $\beta = 1$, $J_n(1) = \{-0.0002, 0.0025, -0.0196, 0.1149, -0.4401, 0.7652,$
$0.4401, 0.1149, 0.0196, 0.0025, 0.0002\}$




### (B.3) ASSUME THE FM SIGNAL IS PASSED THROUGH AN IDEAL BANDPASS FILTER, W/ CENTER FREQUENCY $f_c$ AND $B_T = 7f_m$. DETERMINE THE AMPLITUDE SPECTRUM OF THE FILTER OUTPUT.




THIS WILL RESECT THE COMPONENTS FOR $n = -5, -4, +4, +5$

$$\Rightarrow \boxed{|Y(f)| = \left| \frac{A_c}{2} \sum_{n=-3}^{3} J_n(1)[\delta(f - f_c + nf_m)] + \delta(f + (f_c + nf_m))] \right|}$$

THE PLOT WOULD BE THE SAME, BUT REMOVE ANY IMPULSES FOR $\pm 4f_m$, $\pm 5f_m$.

③ NARROW-BAND FM

$$\hookrightarrow s(t) \approx A_c\cos(2\pi f_c t) - \underbrace{\beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)}_{-\frac{\beta A_c}{2}\left[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c+f_m)t)\right]}$$

Ⓐ THE ENVELOPE $e(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$, WHERE $s_I(t) = A_c$ AND $s_Q(t) = \beta A_c \sin(2\pi f_m t)$.
DETERMINE $e(t)$ AS A FUNCTION OF $\beta$.

$$\Rightarrow e(t) = \sqrt{A_c^2 + \beta^2 A_c^2 \sin^2(2\pi f_m t)}$$

$$\rightarrow \boxed{e(t) = A_c\sqrt{1 + \beta^2\sin^2(2\pi f_m t)}}$$

Ⓑ DETERMINE RATIO OF MAX TO MIN VALUE OF ENVELOPE

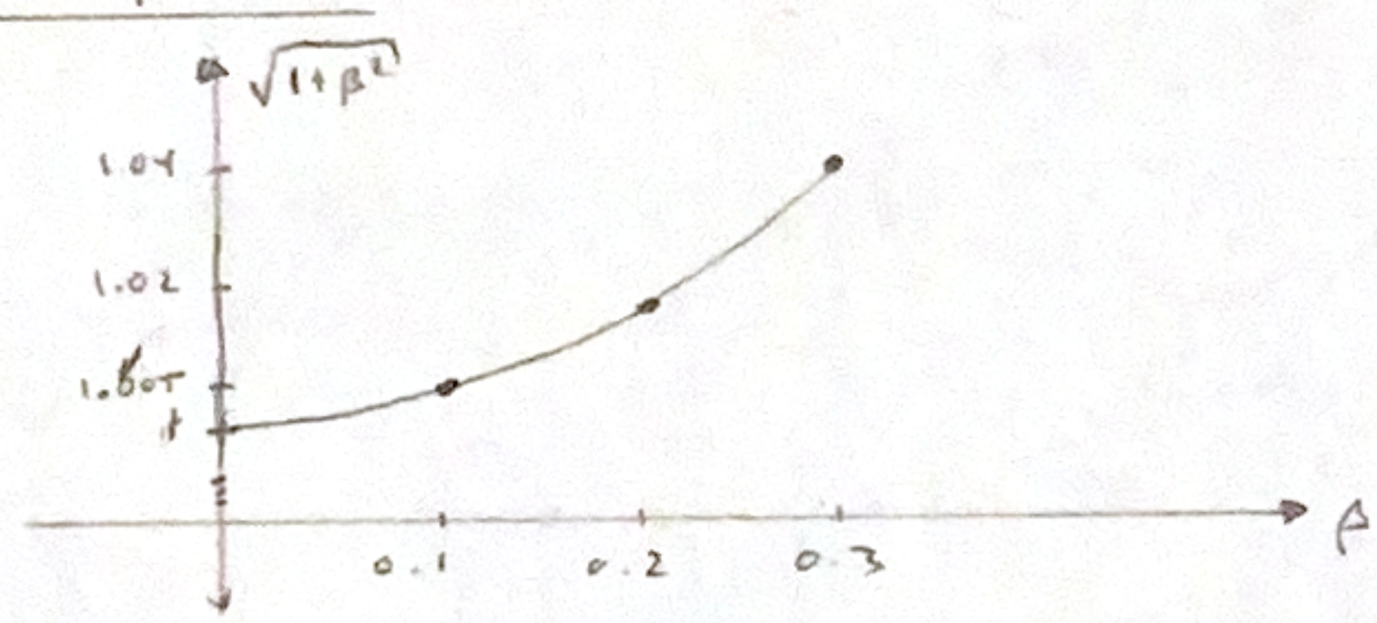
$e_{max}(t) = A_c\sqrt{1+\beta^2}$, $e_{min}(t) = A_c$ $\rightarrow \dfrac{A_c\sqrt{1+\beta^2}}{A_c} = \sqrt{1+\beta^2}$

$$\Rightarrow \boxed{\frac{e_{max}(t)}{e_{min}(t)} = \sqrt{1+\beta^2}}$$

Ⓒ PLOT RATIO FOR $0 \le \beta \le 0.3$

$f(\beta) = \sqrt{1+\beta^2}$

| $\beta$ | $f$ |
|---|---|
| 0 | 1 |
| 0.1 | $\sqrt{1.01} \cong 1.005$ |
| 0.2 | 1.02 |
| 0.3 | 1.04 |




Ⓓ USING FT OF $s(t)$, FIND AVG POWER AS A PERCENTAGE OF THE POWER OF THE UNMODULATED CARRIER WAVE. PLOT AGAINST $\beta$ FOR $0 \le \beta \le 0.3$. WHY NOT EQUAL TO $A_c^2/2$?

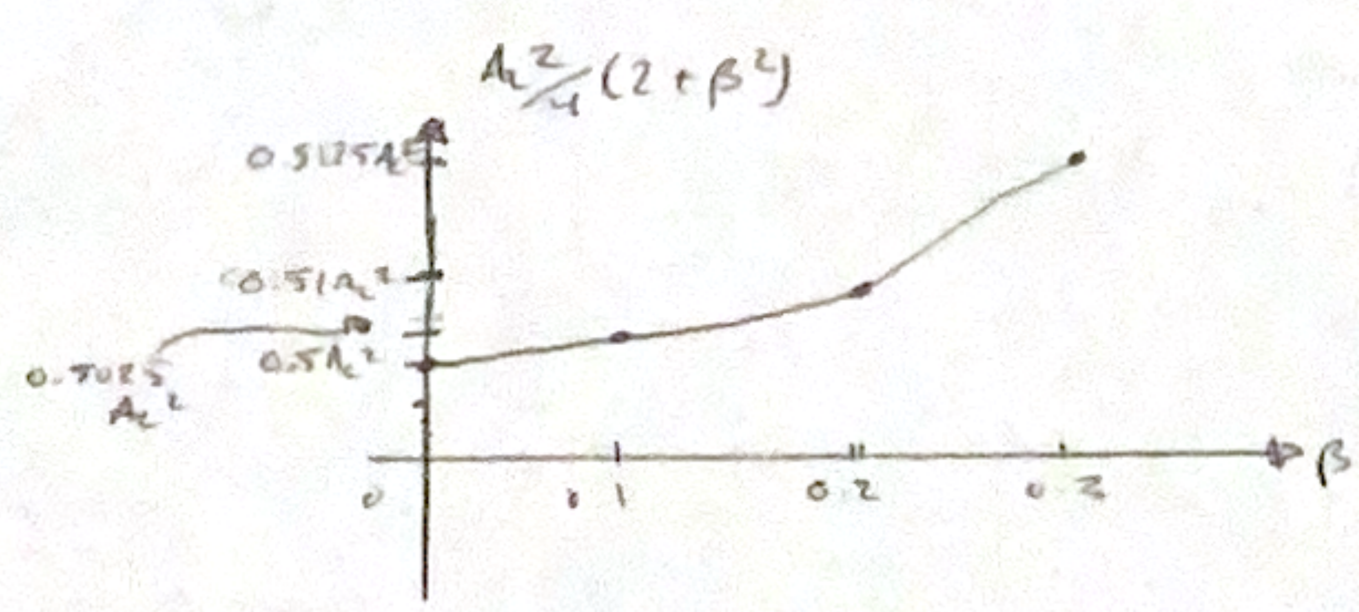
$$\rightarrow S(f) = \mathcal{F}\{s(t)\} = \frac{A_c}{2}\left(\delta(f-f_c) + \delta(f+f_c)\right) - \frac{\beta A_c}{4}\left(\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right)$$
$$+ \frac{\beta A_c}{4}\left(\delta(f-(f_c+f_m)) - \delta(f+(f_c+f_m))\right)$$

VIA RAYLEIGH'S $\Rightarrow$ $P_{avg,s} = \left(\frac{A_c}{2}\right)^2 + \left(\frac{A_c}{2}\right)^2 + \left(\frac{\beta A_c}{4}\right)^2 + \left(\frac{\beta A_c}{4}\right)^2 + \left(\frac{\beta A_c}{4}\right)^2 + \left(\frac{\beta A_c}{4}\right)^2$

$$= 2\frac{A_c^2}{4} + 4\frac{\beta^2 A_c^2}{16} = \frac{A_c^2}{2} + \frac{\beta^2 A_c^2}{4} = \frac{A_c^2}{2}\left(1 + \frac{\beta^2}{2}\right) \therefore \boxed{\left(1 + \frac{\beta^2}{2}\right)\%}$$

$g(\beta) = \frac{A_c^2}{4}(2+\beta^2)$

| $\beta$ | $g$ |
|---|---|
| 0 | $\frac{A_c^2}{4}(2) = \frac{A_c^2}{2}$ |
| 0.1 | $0.7025 A_c^2$ |
| 0.2 | $0.51 A_c^2$ |
| 0.3 | $0.7225 A_c^2$ |




THE AVERAGE POWER IS NOT $A_c^2/2$ SINCE THE EMBEDDED MESSAGE ADDS EXTRA SIDEBANDS EVEN THOUGH THERE IS CONSTANT AMPLITUDE.

(e) THE INSTANTANEOUS ANGLE $\theta_i(t) = 2\pi f_c t + \arctan\left(\frac{s_Q(t)}{s_I(t)}\right)$. DETERMINE $\theta_i(t)$ FOR $s(t)$, USING $\arctan(\cdot)$'s POWER SERIES FOR THE FIRST 3 TERMS OF $\theta_i(t)$.

$$\Rightarrow \theta_i(t) = 2\pi f_c t + \arctan\left(\frac{\beta A_c \sin(2\pi f_m t)}{A_c}\right)$$

$$= 2\pi f_c t + \arctan(\beta \sin(2\pi f_m t))$$

$$\approx \boxed{2\pi f_c t + \left[\beta \sin(2\pi f_m t) - \tfrac{1}{3}\beta^3 \sin^3(2\pi f_m t) + \tfrac{1}{5}\beta^5 \sin^5(2\pi f_m t)\right]}$$

$^*\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$

(f) HARMONIC DISTORTION IS THE POWER RATIO OF THE THIRD HARMONIC AND THE FIRST HARMONIC. DETERMINE FOR $\beta = 0.37$.

THE FIRST HARMONIC IS FOR $f_m$ AND THIRD FOR $3f_m$ TERMS...

$^*\ \sin^3(x) = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$ $\qquad$ $^*\ \sin^5(x) = \frac{10}{16}\sin x - \frac{5}{16}\sin 3x + \frac{1}{16}\sin 5x$

$$\Rightarrow \theta_i(t) = 2\pi f_c t + \beta\sin(2\pi f_m t) - \frac{\beta^3}{3}\left[\frac{3}{4}\sin(2\pi f_m t) - \frac{1}{4}\sin(2\pi 3 f_m t)\right] + \frac{\beta^5}{5}\left[\frac{10}{16}\sin(2\pi f_m t) - \frac{5}{16}\sin(2\pi 3 f_m t) + \frac{1}{16}\sin(2\pi 5 f_m t)\right]$$

$$\Rightarrow \quad = 2\pi f_c t + \left(\beta - \frac{\beta^3}{4} + \frac{\beta^5}{8}\right)\sin(2\pi f_m t) + \left(+\frac{\beta^3}{12} - \frac{\beta^5}{16}\right)\sin(2\pi 3 f_m t) + \ldots$$

$$P_{1st} = \left(\beta - \frac{\beta^3}{4} + \frac{\beta^5}{8}\right)^2$$

$$P_{3rd} = \left(\frac{\beta^3}{12} - \frac{\beta^5}{16}\right)^2$$

$$\Rightarrow \frac{P_{3rd}}{P_{1st}} = \left(\frac{\frac{\beta^3}{12} - \frac{\beta^5}{16}}{\beta - \frac{\beta^3}{4} + \frac{\beta^5}{8}}\right)^2 \xrightarrow{\beta = 0.37} = \boxed{0.010574}$$