Homework 05

ECE 478/ECE 570 – Principles of Communication Systems

Southern Illinois University Carbondale

Posted date: 03/30/2025
Due by: 5.00 PM – 04/09/2025
Section: Sampling
Number of problems: 04
Policy: Late submissions will not be accepted.

Question (01): Sampling Theorem [25 marks]

(a). Determine the Nyquist sampling rate and the Nyquist sampling interval for the following signals.

(a.1) $g(t) = \text{sinc}(200t)$

(a.2) $g(t) = \text{sinc}(200t) + \text{sinc}(300t)$

(a.3) $g(t) = \text{sinc}^2(200t)$

(a.4) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

(a.5) $g(t) = \text{sinc}(200t) + \text{sinc}(300t) + \text{sinc}^2(200t)$

(b). Twenty-four voice signals are sampled uniformly and then time-division multiplexed. The sampling operation uses flat-top samples with 1μs duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of sufficient amplitude and also 1 μs duration. The highest frequency component of each voice signal is 3.4 kHz.

(b.1) Assuming a sampling rate of 8 kHz, determine the spacing between successive pulses of the multiplexed signal.

(b.2) Repeat your calculation assuming the use of Nyquist rate sampling.

(c.) Thirty different message signals, each with a bandwidth of 10 kHz, are to be multiplexed and transmitted. Determine the minimum bandwidth required for each method if the multiplexing/modulation method used is

(c.1) Frequency division multiplexing (FDD) and single-side band (SSB) modulation

(c.2) Time division multiplexing (TDM) and pulse amplitude modulation (PAM).

Question (02):

An ideal sampled signal $m_\delta(t)$ can be obtained by multiplying the message signal $m(t)$ with an ideal sampling function as follows:

$$m_\delta(t) = m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s),$$

1. Suppose that the message is a sinusoidal signal given by $m(t) = A_m \cos(2\pi f_m t)$, where f_m is its frequency. Find and sketch the spectrum $M(f)$ of the message signal $m(t)$. What is the Nyquist sampling rate?

2. Find and sketch the spectrum $M_\delta(f)$ of the ideal sampled signal $m_\delta(t)$ by assuming that the message signal is sampled at Nyquist rate.

3. Suppose an ideal low-pass $H(f)$ is used to reconstruct the original message signal $m(t)$ from the sampled signal $m_\delta(t)$. The transfer function of this ideal reconstruction filter is given by
$$H(f) = \frac{1}{2W} \operatorname{rect}\left(\frac{f}{2W}\right).$$
Determine the bandwidth W of this reconstruction filter as a function of f_m.

4. Assume that the sampled signal $m_\delta(t)$ is applied to the above reconstruction filter and obtain a filter output $y(t)$. Find and sketch the spectrum $Y(f)$ of the filter output $y(t)$.

Question (03): Pulse code modulation (PCM) [25 marks]

(a). A PCM system utilizes a N-bit binary encoder. The bit rate of this PCM signal is equal to R_b bits/s.

(a.1) Determine the sampling duration.

(a.2) What is the maximum message bandwidth for which the system operates satisfactory?

(a.3) Suppose that L_1 such PCM signals are multiplexed to obtain the output carrier of the first multiplexing level. Assuming that M bits are added for frame synchronization, determine the final bit rate of this multiplexer output.

(a.4) The bit rate of the output of the second level multiplexer is equal to R_1 bits/s. Determine the number of maximum number of first level multiplexed carriers (L_2) at the input of the second level multiplexer.

(a.5) Determine the number of PCM signals in the second level multiplexed carrier.

(b). A voice signal is sampled at 8 kHz, and the resulting samples are encoded by using 8 bits per sample to obtain a PCM signal. Assume that the first-level multiplexer in the CCIT hierarchy is a synchronous voice-PCM channel bank with 30 input signals. The output bit rate of the first level multiplexer is 2.048 Mbps.

(b.1) Determine the bit rate of this resulting PCM signal

(b.2) Find the number of additional bits used for synchronization/signalling per frame.

Question (04): Pulse amplitude modulation (PAM) [25 marks]

(a). Suppose that the modulating signal $m(t) = A_m \cos(2\pi f_m t)$ is pulse amplitude modulated to obtain the following PAM signal

$$s(t) = \sum_{n=-\infty}^{\infty} [1 + k_a m(t)] h(t - nT_s)$$

where k_a is the modulation sensitivity and $h(t)$ is a rectangular pulse with unit amplitude and duration T.

(a.1) Show that the above PAM signal can be constructed by convolving the pulse shape $h(t)$ and an ideally sampled signal $m'_\delta(t)$, which is obtained by multiplying the message signal $m'(t) = 1 + k_a m(t)$ with an ideal sample function as follows:

$$m'_\delta(t) = m'(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s).$$

(a.2) Determine the modulating index k_a for 100% modulation.

(a.3) Determine the spectrum $M_\delta(f)$ of the ideal sampled message signal $m_\delta(t)$. Plot the amplitude spectrum $|M_\delta(f)|$.

(a.4) Determine the spectrum $S(f)$ of the PAM signal $s(t)$. Plot the corresponding amplitude spectrum $|S(f)|$ for $f_m = 0.25\,\mathrm{Hz}$, sampling period $T_s = 1\,\mathrm{s}$ and pulse duration $T = 0.45\,\mathrm{s}$.

(a.5) What should be the bandwidth of the ideal reconstruction filter? Plot the spectrum of the filter output.

(a.6) Is there any distortion at the output of this reconstruction filter? What should be the frequency response of the equalizer which can be cascaded with the reconstruction filter for mitigating a probable distortion.