

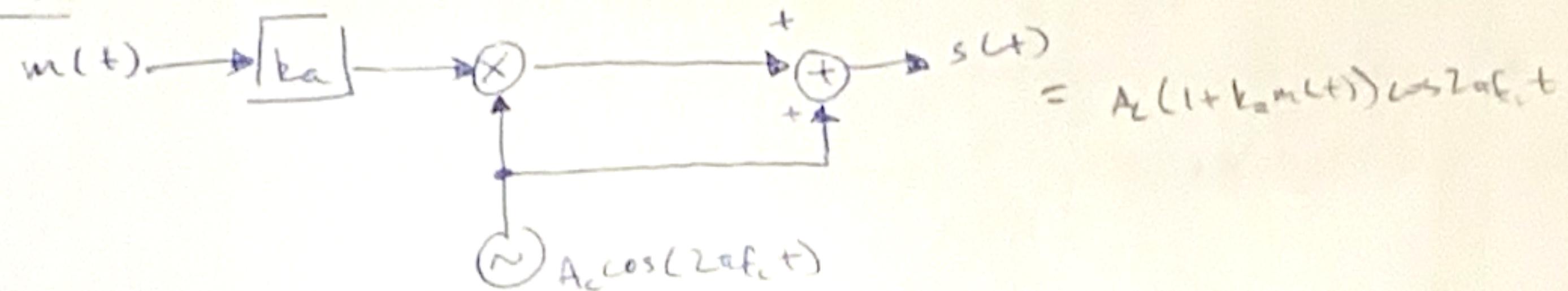
# Design and Analysis of Amplitude and Frequency Modulators

ECE478 Individual Project

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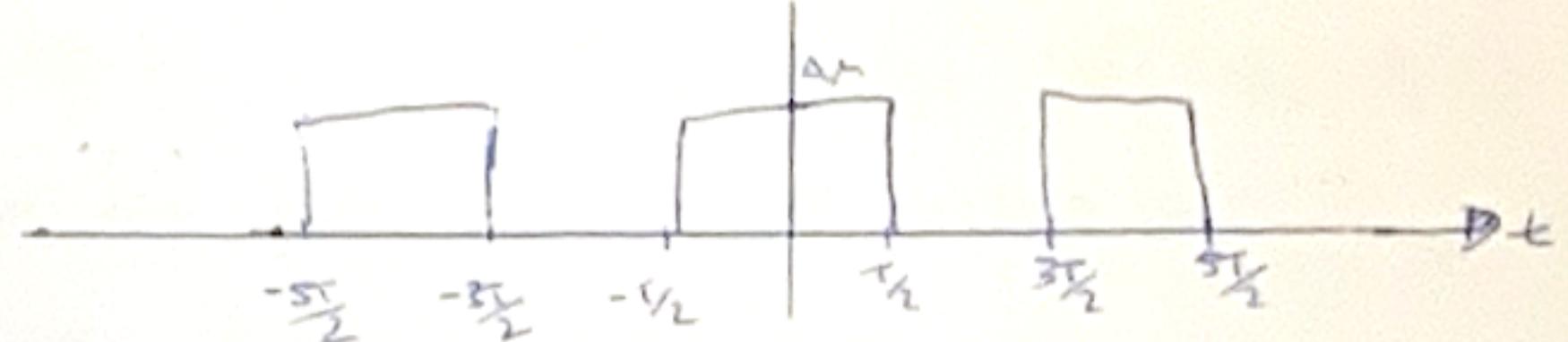
## ① AM MODULATOR DESIGN



YOU ARE ASKED TO DESIGN TWO AM MODULATORS;  $m(t)$  IS A PERIODIC SQUARE WAVE.

- 80% MOD.

- 120% MOD.

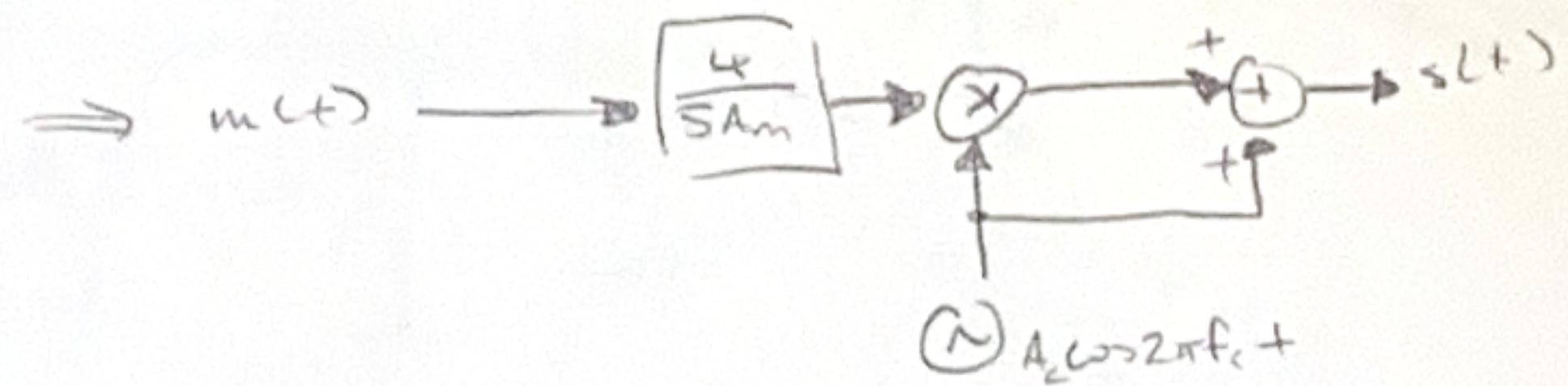


② PLOT THE INPUT SIGNALS OF YOUR AM MODULATORS IN THE TIME DOMAIN.  
THEN DISCUSS THE NECESSARY RELIEFERS FOR DECODING THE INFORMATION-BEARING SIGNALS.

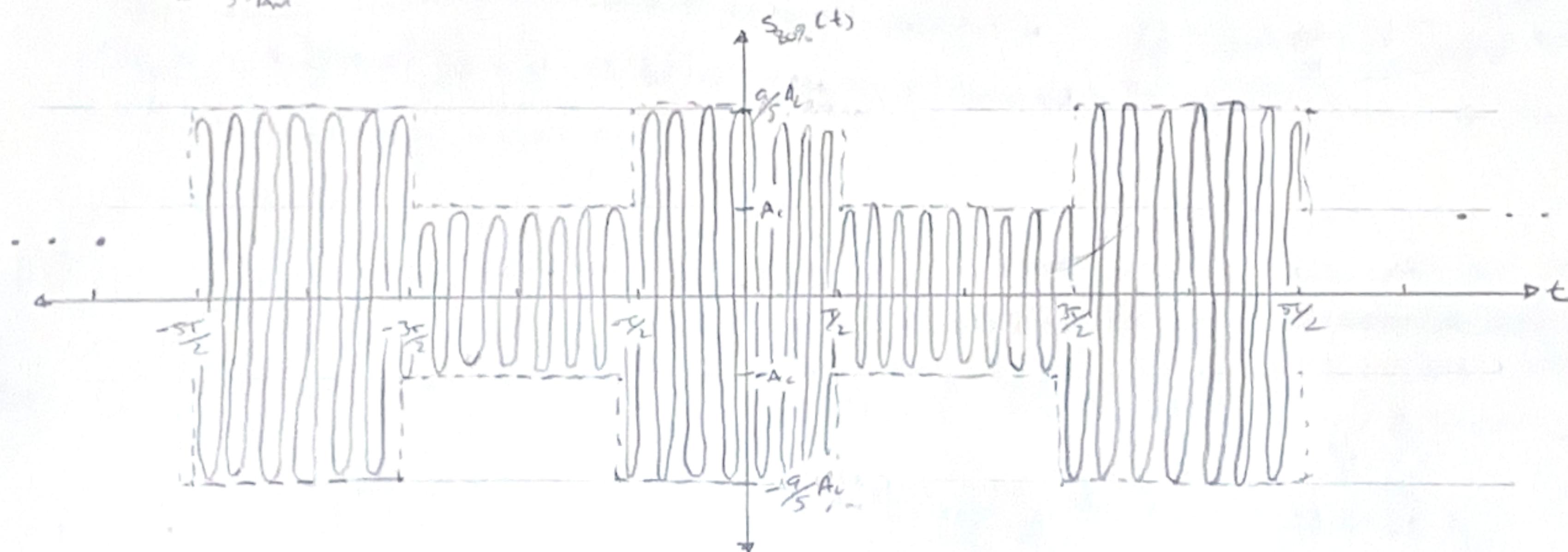
80%

$$\max |k_m m(t)| = 0.8$$

$$\Rightarrow k_m = \frac{0.8}{\max|m(t)|} = \frac{0.8}{A_m} = \frac{4}{5A_m}$$



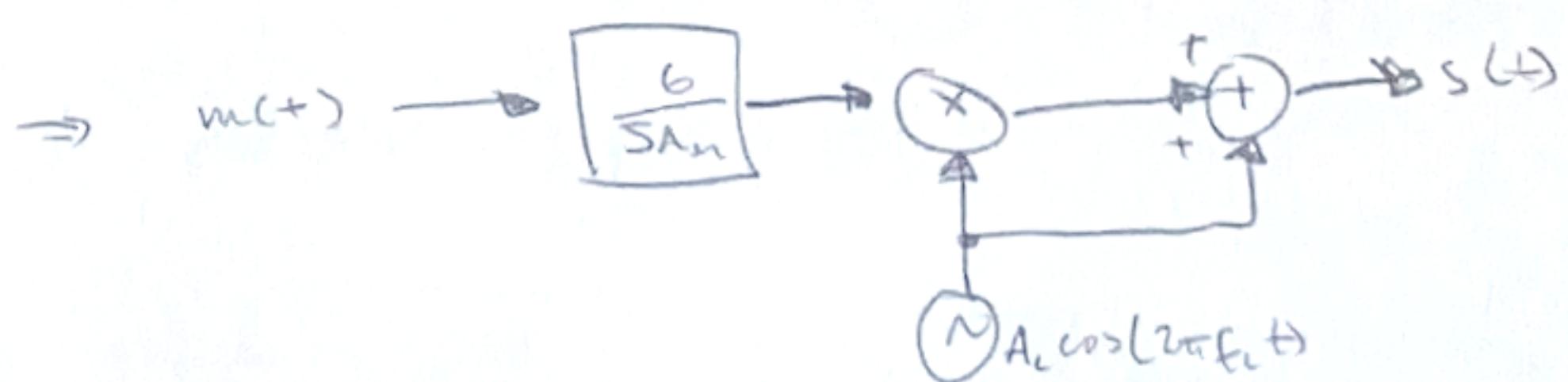
$$\Rightarrow s_{\text{max}} = +\frac{4}{5} A_c \quad ; \quad s_{\text{min}} = -\frac{4}{5} A_c$$



120%

$$\max |k_m m(t)| = 1.2$$

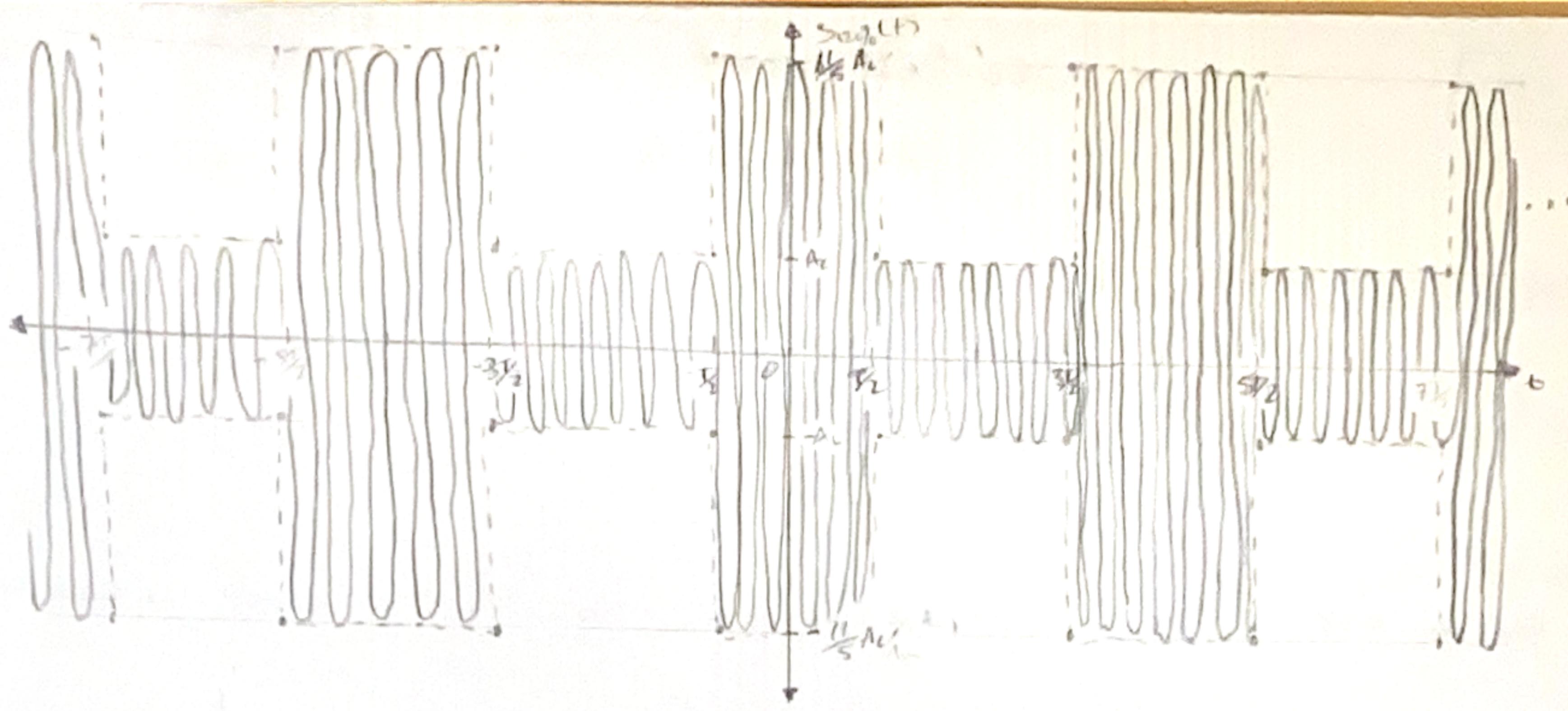
$$\Rightarrow k_m = \frac{1.2}{\max|m(t)|} = \frac{1.2}{A_m} = \frac{6}{5A_m}$$



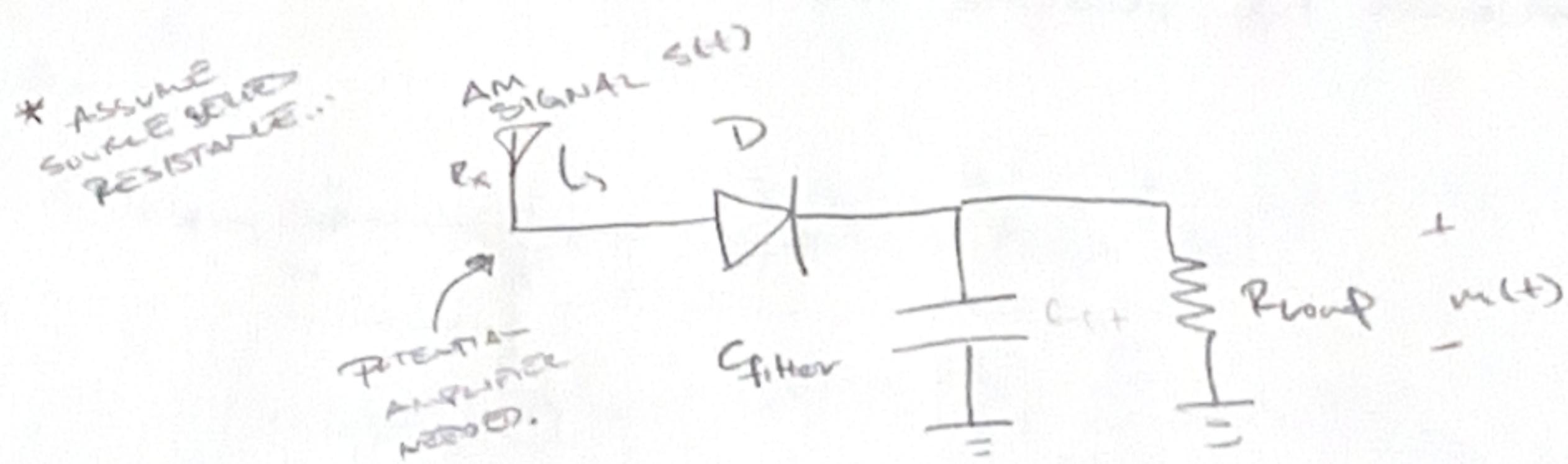
$$\Rightarrow s_{\text{max}} = +\frac{6}{5} A_c \quad ; \quad s_{\text{min}} = -\frac{6}{5} A_c$$

$$s_{\text{min}} = -\frac{6}{5} A_c$$

PLT  
ON  
NEW  
PAGE



AS BOTH THE 80% & 120% AM OUTPUTS HAVE NO ZERO-CROSSINGS, SO A SIMPLE RECTIFIER WITH A FILTER CAPACITOR AT THE DEMODULATOR WOULD RETRIEVE THE MESSAGE SIGNAL  $m(t)$ .



This design is simple and would be easy to build and maintain. However, the system is highly volatile to thermal noise, mostly due to D (as  $V_D = V_{TH} \ln(\frac{I_0}{I_D})$ ). A simple design, even if not robust, <sup>time dependent.</sup> brings communication to more people.

THE DIODE BLOCKS THE  
NEGATIVE HALF OF THE SYSTEM

→ MmMm - -

TEN Cycles (in consideration of  
CLT resistance)  
removes the  
obstruction/purple.

(B) FIND F.T. OF ALL SIGNALS; ANALYTICALLY DETERMINE THE AMPLITUDE SPECTRUMS.  
DISCUSS BANDWIDTH & POWER REQUIREMENTS.

$$s_{\text{Dop}}(t) = A_c \left\{ 1 + \frac{4}{5} \sum_{m=-\infty}^{\infty} \text{rect}\left(\frac{t - 2mT}{T}\right) \right\} \cos(2\pi f_c t)$$

$$+ \text{rect}(\gamma_f) \geq T_{\text{small}}(f)$$

$$= A_c \cos(2\pi f_c t) + \frac{A_c}{2} \sum_{m=-\infty}^{\infty} \text{rect}\left(\frac{t - 2mT}{T}\right) \cos\left(2\pi f_c t\right)$$

$$\Rightarrow \sum \{ S_{\text{soft}}^{(+)} \} = S_{\text{soft}}^{(f)} = \frac{A_c}{2} (\delta(f - f_c) + \delta(f + f_c)) + \frac{4A_c}{\pi S} \left( \frac{1}{2T} \sum_{n=-\infty}^{\infty} T \sin(\frac{n\pi}{T}) \delta(f - \frac{n}{2T}) \right) \otimes \frac{1}{2} (\delta(f - f_c) + \delta(f + f_c))$$

$$= \frac{A_c}{2} (\delta(f - f_c) + \delta(f + f_c)) + \frac{A_c}{\pi S} \left( \sum_{n=-\infty}^{\infty} \sin(\frac{n\pi}{T}) \delta(f - f_c - \frac{n}{2T}) + \sum_{n=-\infty}^{\infty} \sin(\frac{n\pi}{T}) \delta(f + f_c - \frac{n}{2T}) \right)$$

$$S_{\text{sig}}(f) = \frac{A_S}{2} \delta(f - f_c) + \frac{A_S}{2} \delta(f + f_c) + \sum_{n=-\infty}^{\infty} \left[ \sin\left(\frac{n\pi f}{2T}\right) \delta(f - f_c - \frac{n\pi}{2T}) + \sin\left(\frac{n\pi f}{2T}\right) \delta(f + f_c - \frac{n\pi}{2T}) \right]$$

[1209.]

$$S_{120\%}(t) = A_c \left[ 1 + \frac{6A_c}{5} \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-2nT}{T}\right) \right] \cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{6A_c}{5} \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-2nT}{T}\right) \cos(2\pi f_c t)$$

$$\rightarrow F\{S_{120\%}(t)\} = S_{120\%}(f) = \frac{A_c}{2} (\delta(f-f_c) + \delta(f+f_c)) + \frac{6A_c}{5} \left[ \left( \frac{1}{2T} \sum_{n=-\infty}^{\infty} T \sin\left(\frac{n\pi}{T}\right) \delta(f-\frac{n\pi}{T}) \right) \otimes \frac{1}{T} (\delta(f-f_c) * \delta(f+f_c)) \right]$$

$$= \frac{A_c}{2} (\delta(f-f_c) + \delta(f+f_c)) + \frac{3A_c}{10} \left[ \sin\left(\frac{\pi}{T}\right) \delta(f-f_c - \frac{\pi}{T}) + \sum_{n=-\infty}^{\infty} \sin\left(\frac{n\pi}{T}\right) \delta(f+f_c - \frac{n\pi}{T}) \right]$$

$$\Rightarrow S_{120\%}(f) = \frac{A_c}{2} \delta(f-f_c) + \frac{A_c}{2} \delta(f+f_c) + \frac{3A_c}{10} \sum_{n=-\infty}^{\infty} \sin\left(\frac{n\pi}{T}\right) \delta(f-f_c - \frac{n\pi}{T}) + \frac{3A_c}{10} \sum_{n=-\infty}^{\infty} \sin\left(\frac{n\pi}{T}\right) \delta(f+f_c - \frac{n\pi}{T})$$

**Power**

RAYLEIGH'S BASIS

$$P = \int_{-\infty}^{\infty} |S(f)|^2 df \Rightarrow P_{120\%} = \left(\frac{A_c}{2}\right)^2 + \left(\frac{A_c}{2}\right)^2 + \left(\frac{3A_c}{10}\right)^2 \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right) + \left(\frac{3A_c}{10}\right)^2 \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right)$$

$$= \frac{A_c^2}{4} + \frac{A_c^2}{4} + \frac{A_c^2}{25} \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right) + \frac{A_c^2}{25} \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right)$$

$$\Rightarrow P_{120\%} = \frac{A_c^2}{2} + \frac{2A_c^2}{25} \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right)$$

$$\Rightarrow P_{120\%} = \frac{A_c^2}{2} + 2 \left[ \left(\frac{3A_c}{10}\right)^2 \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right) \right]$$

$$\boxed{P_{120\%} = \frac{A_c^2}{2} + \frac{9A_c^2}{50} \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{n\pi}{T}\right)}$$

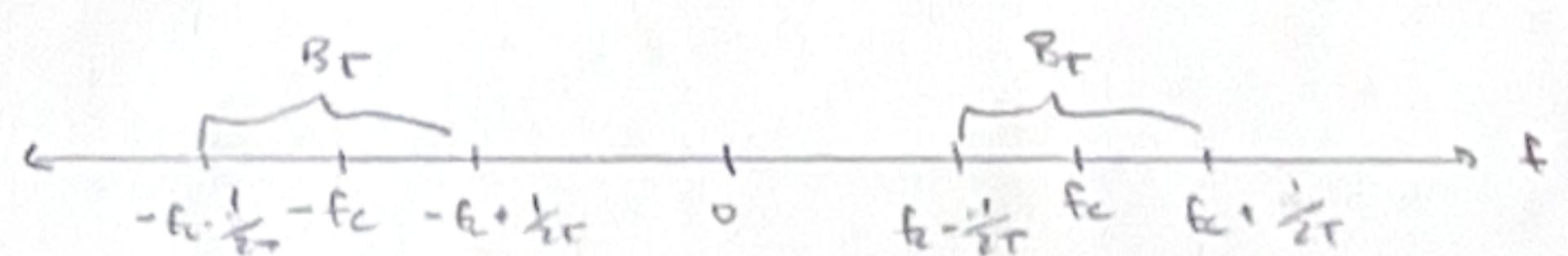
STILL FOLLOWS

PARRIER + PSB STRUCTURE,  
BUT SIDEBANDS ARE THE  
IMPLESES FROM THE  
PERIODIC RECTANGULAR TRSIN.

**BANDWIDTH**

WE CAN USE THE NULL-TO-NUL BANDWIDTH TO EVALUATE THE SPECTRAL REQUIREMENTS.  
THIS CAN BE DONE KNOWING THE SINUS PULSE ENVELOPE EXTENDS FROM  $n = \{-1, 1\}$  FOR THE  
FIRST LOBE.

$$\begin{aligned} n = -1: & \quad \delta(f-f_c + \frac{1}{2T}), \quad \delta(f+f_c + \frac{1}{2T}) \\ n = 1: & \quad \delta(f-f_c - \frac{1}{2T}), \quad \delta(f+f_c - \frac{1}{2T}) \end{aligned}$$



\* THIS IS  
A BASEBLED  
SIGNAL

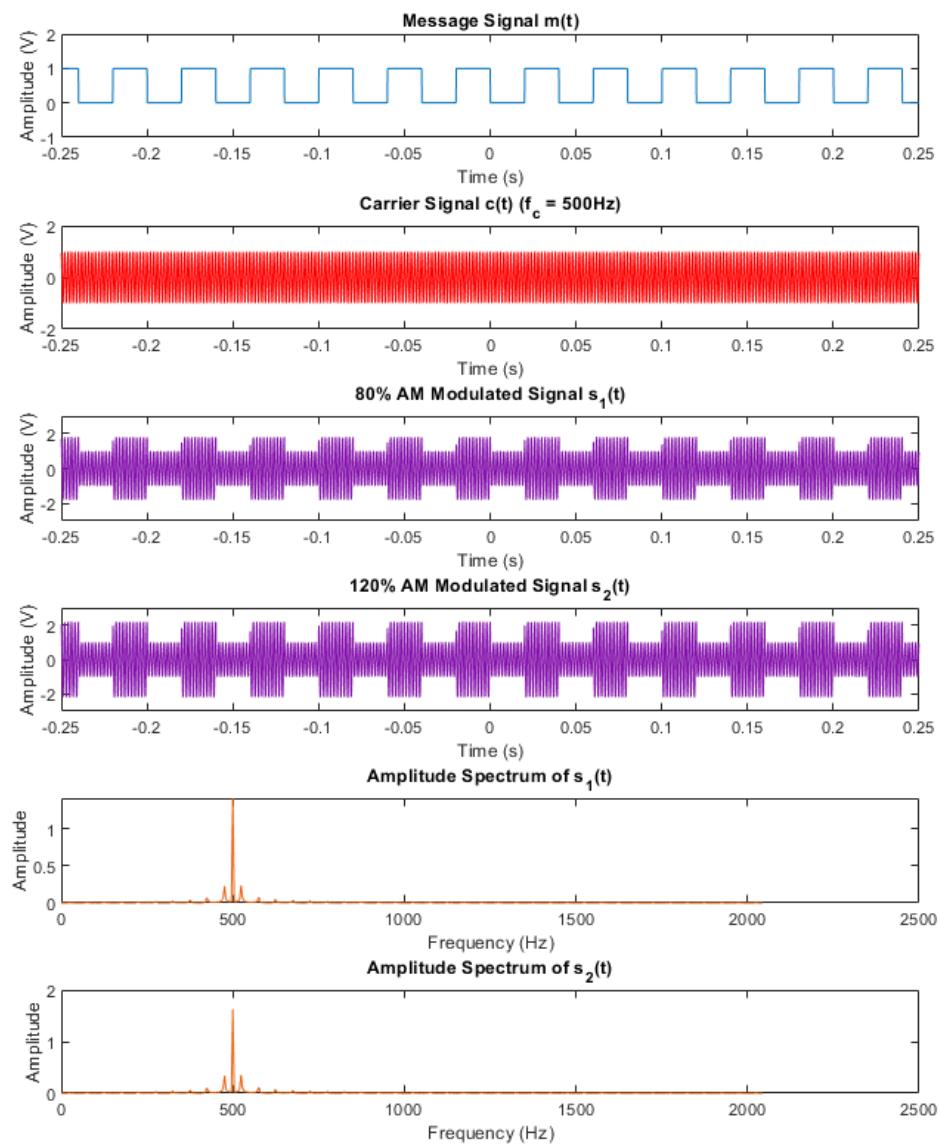
$$\Rightarrow B_T = (f_c + \frac{1}{2T}) - (f_c - \frac{1}{2T}) = \frac{1}{T} + \frac{1}{T} = \frac{1}{T} \Rightarrow B_T = \frac{1}{T}$$

THIS BANDWIDTH IS INDEPENDENT FOR PESENT MODULATION, SO  $S_{80\%} \triangleq S_{120\%}$   
BOTH HAVE NULL-TO-NUL BANDWIDTHS OF  $\frac{1}{T}$ .

SINCE  $B_T$  IS THE SAME FOR BOTH, BUT 80% MOD USES LESS POWER,  
THE 80% SCHEME IS MORE SPECTRALLY EFFICIENT.

## MATLAB Verification of AM Modulator Design

The code for the following discussion (*q1\_AM\_rect\_train.m*) can be found in Appendix A: MATLAB Code.



*Figure 1: Signal and Spectral Outputs of 80% and 120% AM Modulator Designs*

```
>> q1_AM_rect_train
Max of s1(t) = 1.800000
Max of s2(t) = 2.200000
Power of 80%: 0.002112
Power of 120%: 0.002923
120% Power by 80% Power: 1.384052
```

*Figure 2: Numerical Power and Power Ratio of 80% and 120% of AM Modulator Designs*

From Figures 1 and 2, we can see the behavior of the AM modulators, where  $s_1(t)$  and  $s_2(t)$  are the 80% and 120% designs, respectively. And by pure inspection, both perform the exact same task, but the 120% is obviously driven at a larger voltage.

From Figure 2, we can verify the accuracy of the analytical model for the modulators, where if we assume that  $A_c=1V$ :

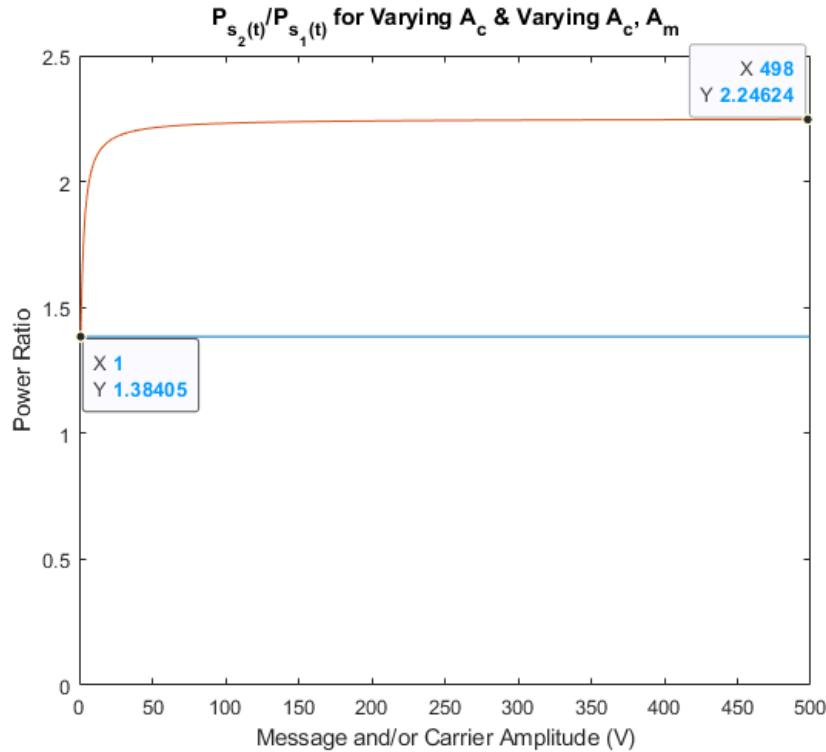
$$\max(s_1(t)) = \frac{9A_c}{5} = \frac{9}{5} = 1.8$$

$$\max(s_2(t)) = \frac{11A_c}{5} = \frac{11}{5} = 2.2$$

Also, we can verify that the power requirements for the 120% modulator are 1.384 times more than the 80% modulated signal when  $A_c = A_m = 1V$ . This value stays constant for any voltage we drive the carrier at, as long as  $A_m = 1V$ , but if  $A_m$  and  $A_c$  are both increased together, this ratio will saturate to 2.25 (shown in Figure 3). So, at worst, the 120% modulated signal will require 2.25 times more power to operate than the 80% modulated signal.

Analytically, we get 2.25 by taking the coefficients of the power derived using Rayleigh's Theorem (assuming  $A_c \rightarrow \infty$ ):

$$\frac{(9A_c^2/50)}{(2A_c^2/25)} = 9/4 = 2.25$$



*Figure 3: Power Ratio for Varying Amplitudes*

For the bandwidth of both signals, they are identical, which was expected based on the analytical model. This means that the spectral efficiency of the 80% modulator is better as it conveys the same information in the same channel with less power.

Below in Table 1, I discuss the applications for the two AM modulators. For critical applications, the 120% modulator is better since it is more robust to attenuation in a transmission channel, since we drive it with more power than the 80% modulator; critical applications include those for important national infrastructure, those which protect the safety of humans, and which need large outreach. The 80% modulator is better suited for less important, or more local applications, as it will attenuate faster, but uses much less power. The complexity of both are the same, but reliability is where they differ.

*Table 1: AM Modulator Design Applications*

Modulator Applications	
80% AM Modulator	120% AM Modulator
<b>Renewable Energy Radio</b>	<b>Tele-Medicine with High Reliability</b>
<b>Daily News for Rural Communities with Low Budget</b>	<b>National Traffic Information</b>
<b>Cultural Music with High Fidelity</b>	<b>Social Welfare Information</b>
<b>Social Information about Diversity and Demographics</b>	<b>Radio Service for Global Audience with High Fidelity</b>

## Frequency Modulation

The code for this section to analyze was provided by Dr. Baduge and his TA's.

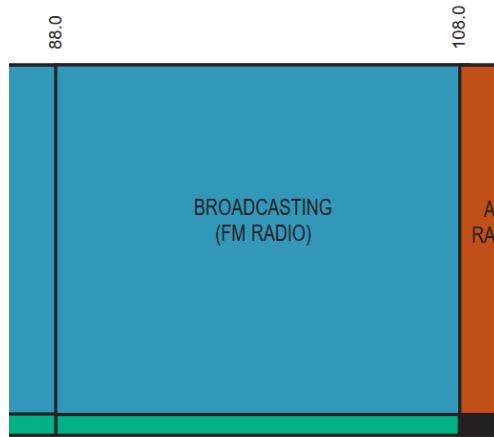
After running the code for the two FM modulator designs, independently designed by two communications engineers, are as follows:

```
=====
FM modulation: Design-1
Transmission bandwidth = 152000 Hz
Transmit power = 105000 W
Coverage = 116.95 km (with receiver sensitivity = -110.0 dBm and path-loss exponent = 3.0)
>> fm_design2
=====
FM modulation: Design -2
Transmission bandwidth = 44.00 Hz
Transmit power = 0.50 W
Coverage = 0.0000001516 km (with receiver sensitivity = 10.0 dBm and path-loss exponent = 4.0)
```

*Figure 4: FM Modulator Design Statistics*

(A)

The first design transmits at 100MHz, while the second transmits at 500Hz. Neither fit into the NIJ Standard-0201.01, as Design 1 falls in-between the Type 1 and Type 2 transmitter bands and Design 2 lives in the low voice range. However, according to the FCC United States Frequency Allocations, the first design falls into the 88-108MHz band.

*Figure 5: FCC Allocation for FM in the 88-108MHz band*

The NIJ standard is outdated, as the FCC allocates “Type-1”, “Type-2” and “Type-4” FM transmitters now for an assortment of land, maritime, and aeronautical mobile communications. “Type-3” transmitters for mobile, satellite, radio astronomy, and meteorological communications.

So again, in the late 1980s neither design follow the standard, but for today’s standards, Design 1 follows. So, the first design engineer followed their engineering ethics more than the second, who shouldn’t have been hired in the first place.

Additionally, whether the designs follow the 3dB output power variance in their sections of coverage, the first design accounts for 3dB of drop-off in their coverage calculation. The second design accounts for 4dB of output drop-off.

(B)

According to Title 47 Chapter 1C §73.211, the first design, which is the only valid design, just goes past the power of a Class C FM transmitter, which has a max ERP of 100kW, so a slight decrease in transmit power and the design will be ok. Note, that the standard would require the antenna for the transmitter to have a minimum height of 600m.

(B.1)

To be blunt, the second FM design is useless, as it can transmit data over 0.1516mm; a public broadcasting company would need at least 100km of coverage to provide services for a useful number of people. The first design, which uses significantly more power has coverage over 116.95km, which can adequately provide communications for a large region. It's transmit power of 0.5W also prevents it from being licensed by the FCC, as you need 100W minimum to have a licensed low-power FM transmitter.

The first design, when compared to the second, requires 3455 times more bandwidth and 210,000 times more power to cover 771,000,000 times more distance. Design 1 would be able to provide communications almost from Chicago to Milwaukee (Figure 5). In 2024, the average price of electricity was \$0.161 per kWh (according to U.S. Labor Statistics), which means that for an hour of

communications, the first design would burn through \$16,905. But, as it is the only usable design for public communications, Design 1 must be chosen.

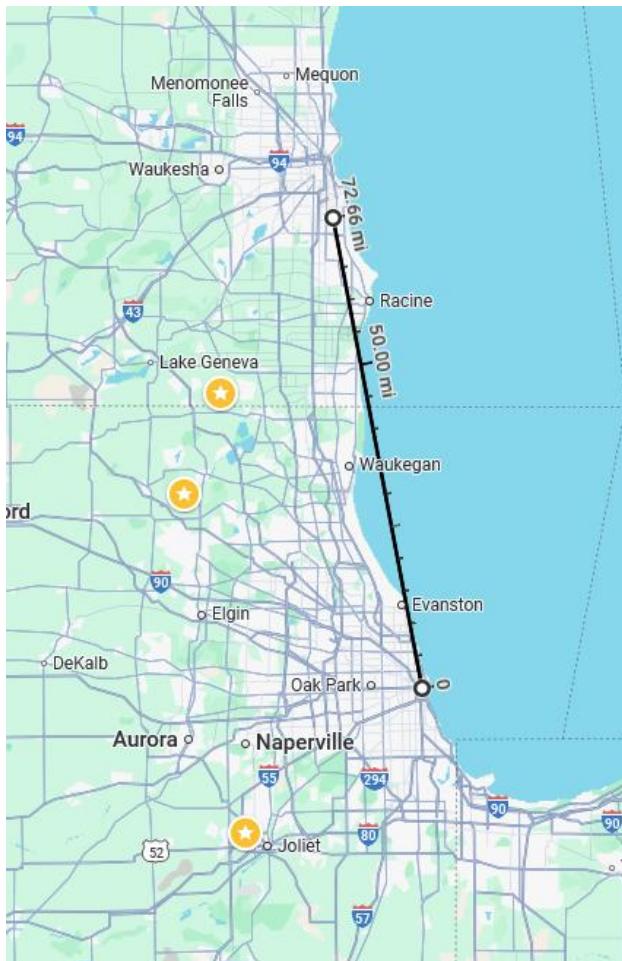


Figure 6: Coverage of Design 1 Between Chicago and Milwaukee

(B.2)

Again, Design 2 is unusable for anything, so Design 1 must be chosen for any application listed in the project description, regardless of optimization tradeoffs.

But in general, the more spectrally efficient you can make the FM transmitter, the better for society, both socially and economically, as using less energy to connect more people is better than connecting only a few while wasting energy. The reduction of indirect carbon emissions based on FM communications will always be the goal of an ethical engineer.

## Appendix A: MATLAB Code

q1\_AM\_rect\_train.m

```
% CHASE LOTITO - SIUC EE UNDEGRAD
% ECE428 INDIVIDUAL PROJECT
%
% 80% 120% BANDWIDTH AM MODULATOR

% NUMBER OF SUBPLOTS
nsub = 6;

% DEFINE MODULATION INDEX
mu1 = 0.8;
mu2 = 1.2;

% DEFINE FREQUENCIES & AMPLITUDES OF m(t), c(t)
Am = 1;           % MESSAGE AMPLITUDE
Ac = 1;           % CARRIER AMPLITUDE
fm = 25;          % MESSAGE FREQ.
fc = 500;          % CARRIER FREQ.

%-----%
% FROM fft.m:
%   For length N input vector x, the DFT is a length N vector X,
%   with elements
%           N
%   X(k) = sum  x(n)*exp(-j*2*pi*(k-1)*(n-1)/N), 1 <= k <= N.
%   n=1
%-----%
% DEFINE SAMPLING FREQUENCY (for N-point FFT)
N = 2048;          % N for the N-point FFT
fs = 8192;          % SAMPLING FREQ.
t = (-(N-1):N-1)/fs;

% GENERATE MESSAGE SIGNAL
T = 1/fm;           % PULSE TRAIN PERIOD
d = min(t):T:max(t);
mt = Am * pulstran(t, d, 'rectpuls', T/2);
subplot(nsub,1,1);
plot(t,mt);
title('Message Signal m(t)');
```

```
xlabel('Time (s)');
ylabel('Amplitude (V)');
ylim([-1 2]);

% GENERATE CARRIER SIGNAL
ct = Ac * cos(2*pi*fc*t);
subplot(nsub,1,2);
plot(t,ct, Color='r');
str = compose('Carrier Signal c(t) (f_c = %dHz)', fc);
title(str);
xlabel('Time (s)');
ylabel('Amplitude (V)');
ylim([-2 2]);

% GENERATE 80% AM SIGNAL
st1 = (1 + mu1 * mt) .* ct;
subplot(nsub,1,3);
plot(t,st1, Color="#8512ac");
title('80% AM Modulated Signal s_1(t)');
xlabel('Time (s)');
ylabel('Amplitude (V)');
ylim([-3 3]);

s1max = max(st1);
fprintf('Max of s1(t) = %f\n', s1max);

% GENERATE 120% AM SIGNAL
st2 = (1 + mu2 * mt) .* ct;
subplot(nsub,1,4);
plot(t,st2, Color="#8512ac");
title('120% AM Modulated Signal s_2(t)');
xlabel('Time (s)');
ylabel('Amplitude (V)');
ylim([-3 3]);

s2max = max(st2);
fprintf('Max of s2(t) = %f\n', s2max);

% GENERATE 80% AM AMPLITUDE SPRECTRUM
Sf1 = 2 / N * abs(fft(st1,N));
f = fs * (0: N/2) / N;
subplot(nsub, 1, 5);
plot(f(1:512), Sf1(1:512), Color="#e86412");
title('Amplitude Spectrum of s_1(t)');
xlabel('Frequency (Hz)');
ylabel('Amplitude');
```

```
% GENERATE 120% AM AMPLITUDE SPRECTRUM
Sf2 = 2 / N * abs(fft(st2,N));
f = fs * (0 : N/2) / N;
subplot(nsub, 1, 6);
plot(f(1:512), Sf2(1:512), Color="#e86412");
title('Amplitude Spectrum of s_2(t)');
xlabel('Frequency (Hz)');
ylabel('Amplitude');

% CALCULATE POWER REQUIREMENTS (THEN TAKE RATIO)
ps1 = sum(Sf1.^2) / N;
ps2 = sum(Sf2.^2) / N;
pratio = ps2/ps1;
fprintf("Power of 80%: %f\n", ps1);
fprintf("Power of 120%: %f\n", ps2);
fprintf("120% Power by 80% Power: %f\n", pratio);
```

power\_against\_voltage\_am.m

```
% CHASE LOTITO - SIUC EE UNDEGRAD
% ECE428 INDIVIDUAL PROJECT
%
% 80% 120% POWER ANALYSIS

% NUMBER OF SUBPLOTS
nsub = 6;

% DEFINE MODULATION INDEX
mu1 = 0.8;
mu2 = 1.2;

psratio = [];
psratio1 = [];

for Ac=1:1:500
    % DEFINE FREQUENCIES & AMPLITUDES OF m(t), c(t)
    Am = 1;          % MESSAGE AMPLITUDE
    fm = 25;         % MESSAGE FREQ.
    fc = 500;        % CARRIER FREQ.

    %-----
```

---

```
% FROM fft.m:
```

```

% For length N input vector x, the DFT is a length N vector X,
% with elements
%
%           N
%   X(k) = sum x(n)*exp(-j*2*pi*(k-1)*(n-1)/N), 1 <= k
<= N.
%           n=1
%-----
%-----  

% DEFINE SAMPLING FREQUENCY (for N-point FFT)
N = 2048;           % N for the N-point FFT
fs = 8192;          % SAMPLING FREQ.
t = (-(N-1):N-1)/fs;  

% GENERATE MESSAGE SIGNAL
T = 1/fm;           % PULSE TRAIN PERIOD
d = min(t):T:max(t);
mt = Am * pulstran(t, d, 'rectpuls', T/2);  

% GENERATE CARRIER SIGNAL
ct = Ac * cos(2*pi*fc*t);  

% GENERATE 80% AM SIGNAL
st1 = (1 + mu1 * mt) .* ct;  

% GENERATE 120% AM SIGNAL
st2 = (1 + mu2 * mt) .* ct;  

% GENERATE 80% AM AMPLITUDE SPRECTRUM
Sf1 = 2 / N * abs(fft(st1,N));  

% GENERATE 120% AM AMPLITUDE SPRECTRUM
Sf2 = 2 / N * abs(fft(st2,N));  

% CALCULATE POWER REQUIREMENTS (THEN TAKE RATIO)
ps1 = sum(Sf1.^2) / N;
ps2 = sum(Sf2.^2) / N;
pratio = ps2/ps1;
psratio = [psratio, pratio];
end  

for Ac=1:1:500
% DEFINE FREQUENCIES & AMPLITUDES OF m(t), c(t)
Am = 1;            % MESSAGE AMPLITUDE
fm = 25;           % MESSAGE FREQ.
fc = 500;          % CARRIER FREQ.

```

```

%-----%
% FROM fft.m:
% For length N input vector x, the DFT is a length N vector X,
% with elements
%           N
%     X(k) =   sum  x(n)*exp(-j*2*pi*(k-1)*(n-1)/N), 1 <= k
<= N.
%           n=1
%-----%

% DEFINE SAMPLING FREQUENCY (for N-point FFT)
N = 2048;           % N for the N-point FFT
fs = 8192;          % SAMPLING FREQ.
t = (-(N-1):N-1)/fs;

% GENERATE MESSAGE SIGNAL
T = 1/fm;           % PULSE TRAIN PERIOD
d = min(t):T:max(t);
mt = Ac * pulstran(t, d, 'rectpuls', T/2);

% GENERATE CARRIER SIGNAL
ct = Ac * cos(2*pi*fc*t);

% GENERATE 80% AM SIGNAL
st1 = (1 + mu1 * mt) .* ct;

% GENERATE 120% AM SIGNAL
st2 = (1 + mu2 * mt) .* ct;

% GENERATE 80% AM AMPLITUDE SPRECTRUM
Sf1 = 2 / N * abs(fft(st1,N));

% GENERATE 120% AM AMPLITUDE SPRECTRUM
Sf2 = 2 / N * abs(fft(st2,N));

% CALCULATE POWER REQUIREMENTS (THEN TAKE RATIO)
ps1 = sum(Sf1.^2) / N;
ps2 = sum(Sf2.^2) / N;
pratio = ps2/ps1;
psratio1 = [psratio1, pratio];
end

Ac = 1:1:500;
plot(Ac, psratio);
hold on;

```

```
plot(Ac, psratio1);
ylim([0,2.5]);
title('{P_{s_2(t)}}/{P_{s_1(t)}} for Varying A_c & Varying A_c, A_m');
xlabel('Message and/or Carrier Amplitude (V)');
ylabel('Power Ratio');
```