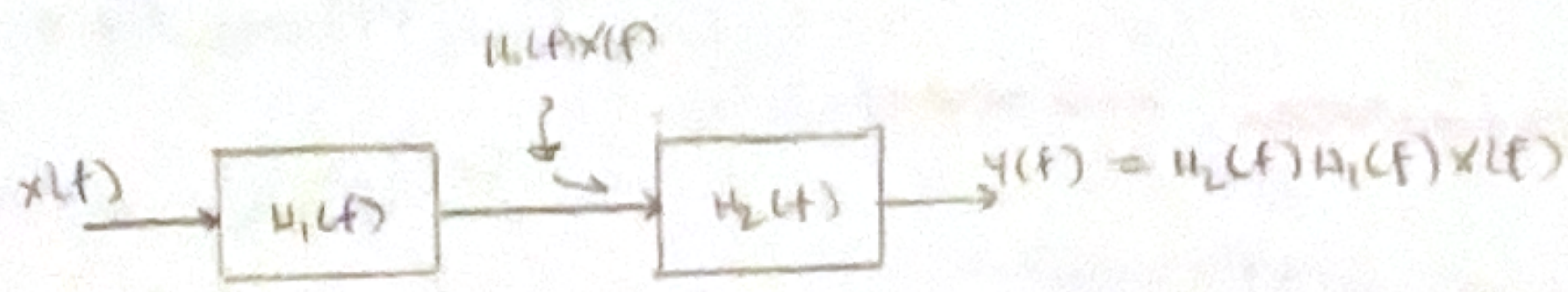
## ① BLOCK DIAGRAM ANALYSIS (LTI)

EACH COMMUNICATION SYSTEM IS MADE OF SUBSYSTEMS WITH TRANSFER FUNCTIONS $H_1(f)$ AND $H_2(f)$.

Ⓐ FIND THE OVERALL TRANSFER FUNCTIONS OF CASCADE, PARALLEL, AND FEEDBACK CONNECTIONS IN TERMS OF $H_1(f)$ AND $H_2(f)$.

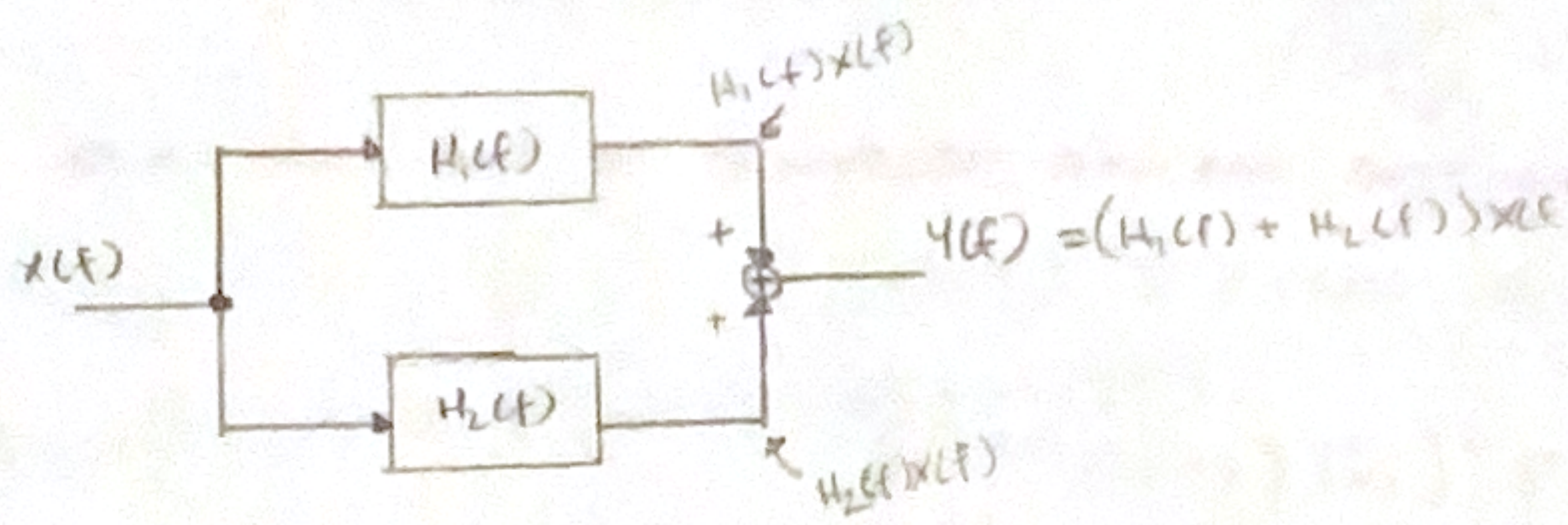
**CASCADE**




$$\Rightarrow Y(f) = H_2(f)H_1(f)X(f)$$

$$\Rightarrow \frac{Y(f)}{X(f)} = \boxed{H_{eq}(f) = H_2(f)H_1(f)}$$

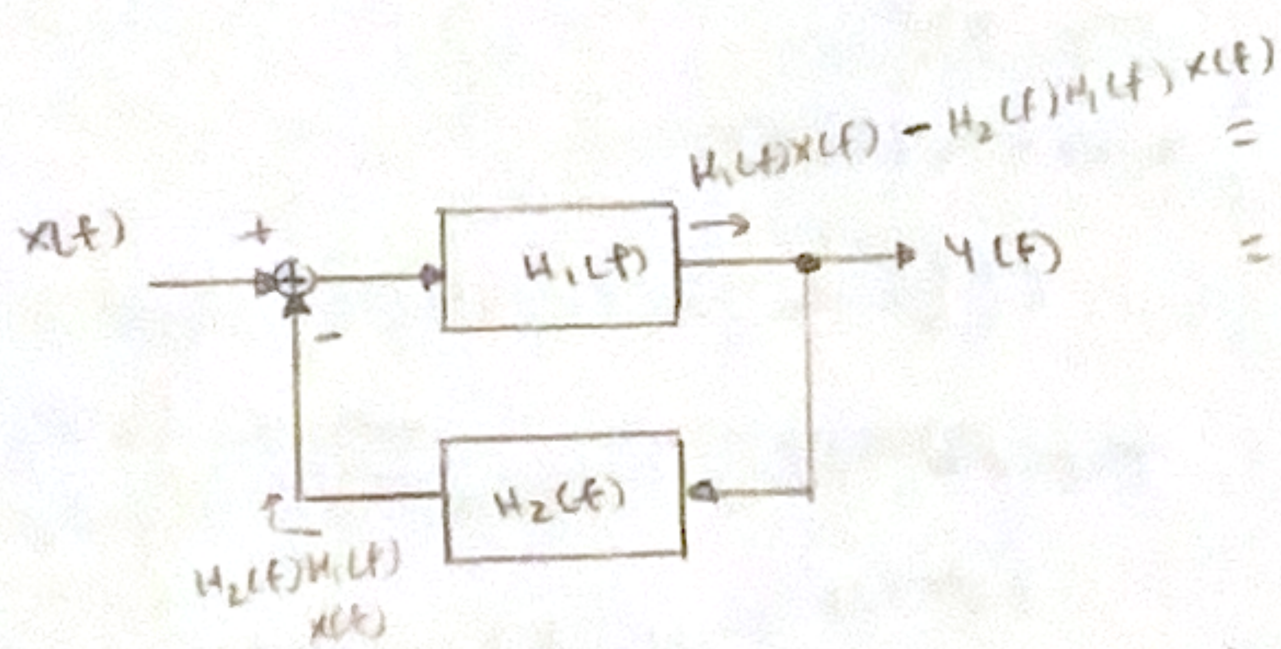
**PARALLEL**




$$\Rightarrow Y(f) = (H_1(f) + H_2(f))X(f)$$

$$\Rightarrow \frac{Y(f)}{X(f)} = \boxed{H_{eq}(f) = H_1(f) + H_2(f)}$$

**FEEDBACK**




$$= X(f)[H_1(f) - H_1(f)H_2(f)]$$

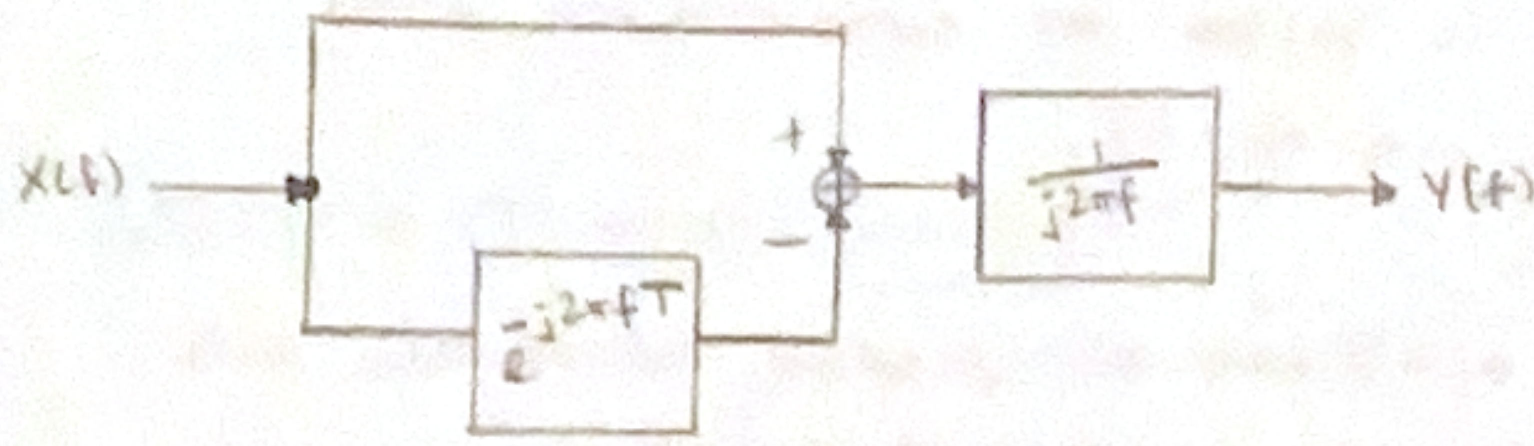
$$= X(f)[H_1(f)(1 - H_2(f))]$$

$$\Rightarrow Y(f) = [H_1(f)(1 - H_2(f))]X(f)$$

$$\Rightarrow \frac{Y(f)}{X(f)} = \boxed{H_{eq}(f) = H_1(f)(1 - H_2(f))}$$

(3) ANY LTI OPERATION HAS AN EQUIVALENT TRANSFER FUNCTION.

(3.1) DRAW BLOCK DIAGRAM FOR ZERO-HOLD SYSTEM IN FREQ. DOMAIN.




(3.2) FIND THE OVERALL TRANSFER FUNCTION.

$$Y(f) = \left(X(f) - X(f)e^{-j2\pi fT}\right)\frac{1}{j2\pi f}$$

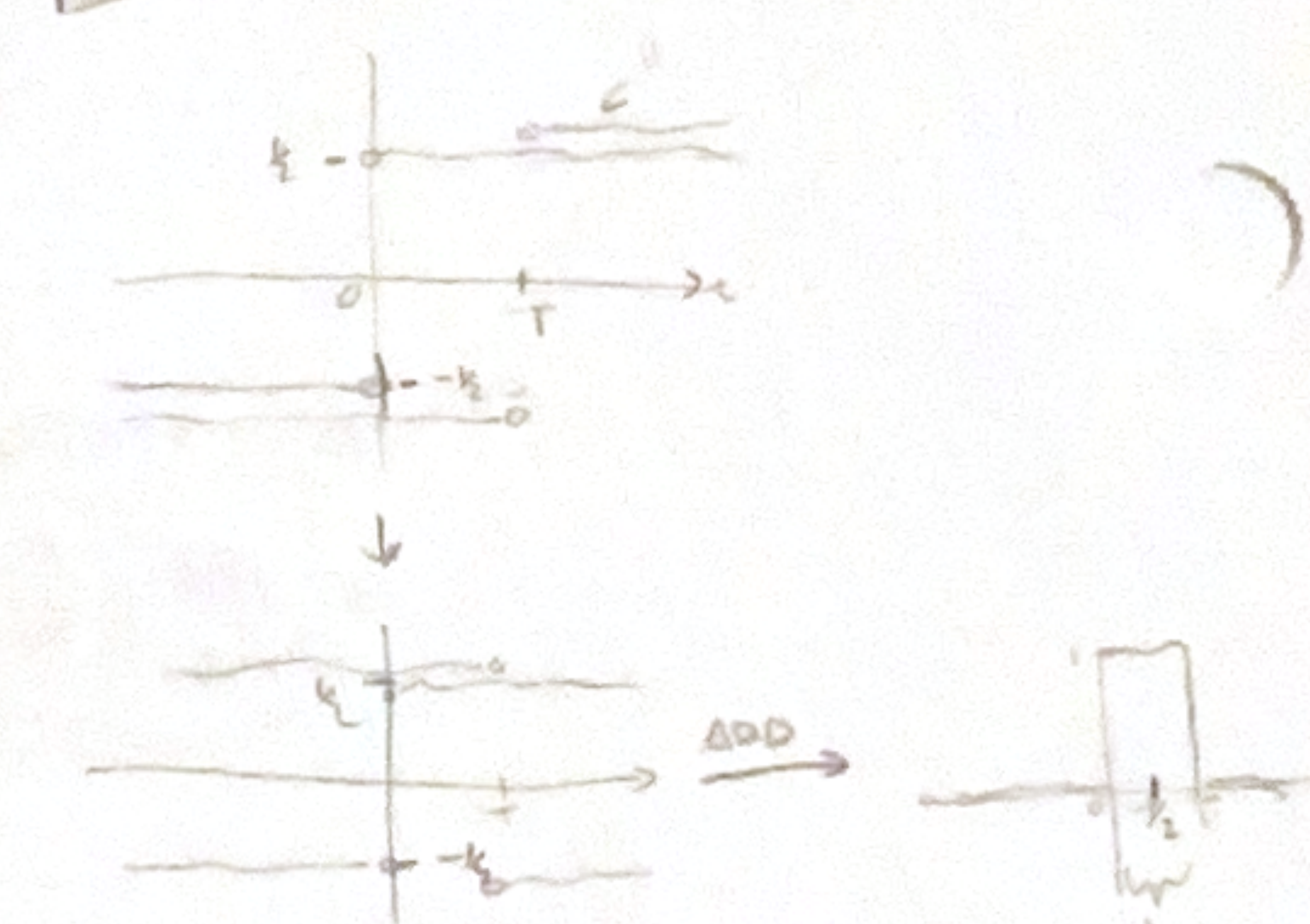
$$= X(f)\left[\frac{1}{j2\pi f}\left(1-e^{-j2\pi fT}\right)\right]$$

$$\Rightarrow \frac{Y(f)}{X(f)} = \boxed{H_{eq} = \frac{1}{j2\pi f}\left(1-e^{-j2\pi fT}\right)}$$

(3.3) FIND AND SKETCH THE IMPULSE RESPONSE OF THE ZERO HOLD.

NEED TO FIND $h(t) = \mathcal{F}^{-1}[H(f)]$.

VISUAL




$$\Rightarrow \mathcal{F}^{-1}[H(f)] = \mathcal{F}^{-1}\left[\frac{1}{j2\pi f}\left(1-e^{-j2\pi fT}\right)\right]$$

$$= \tfrac{1}{2}\operatorname{sgn}(t) \circledast \left(\delta(t) - \delta(t-T)\right)$$

$$= \tfrac{1}{2}\left(\operatorname{sgn}(t) \circledast \delta(t)\right) - \tfrac{1}{2}\left(\operatorname{sgn}(t) \circledast \delta(t-T)\right)$$

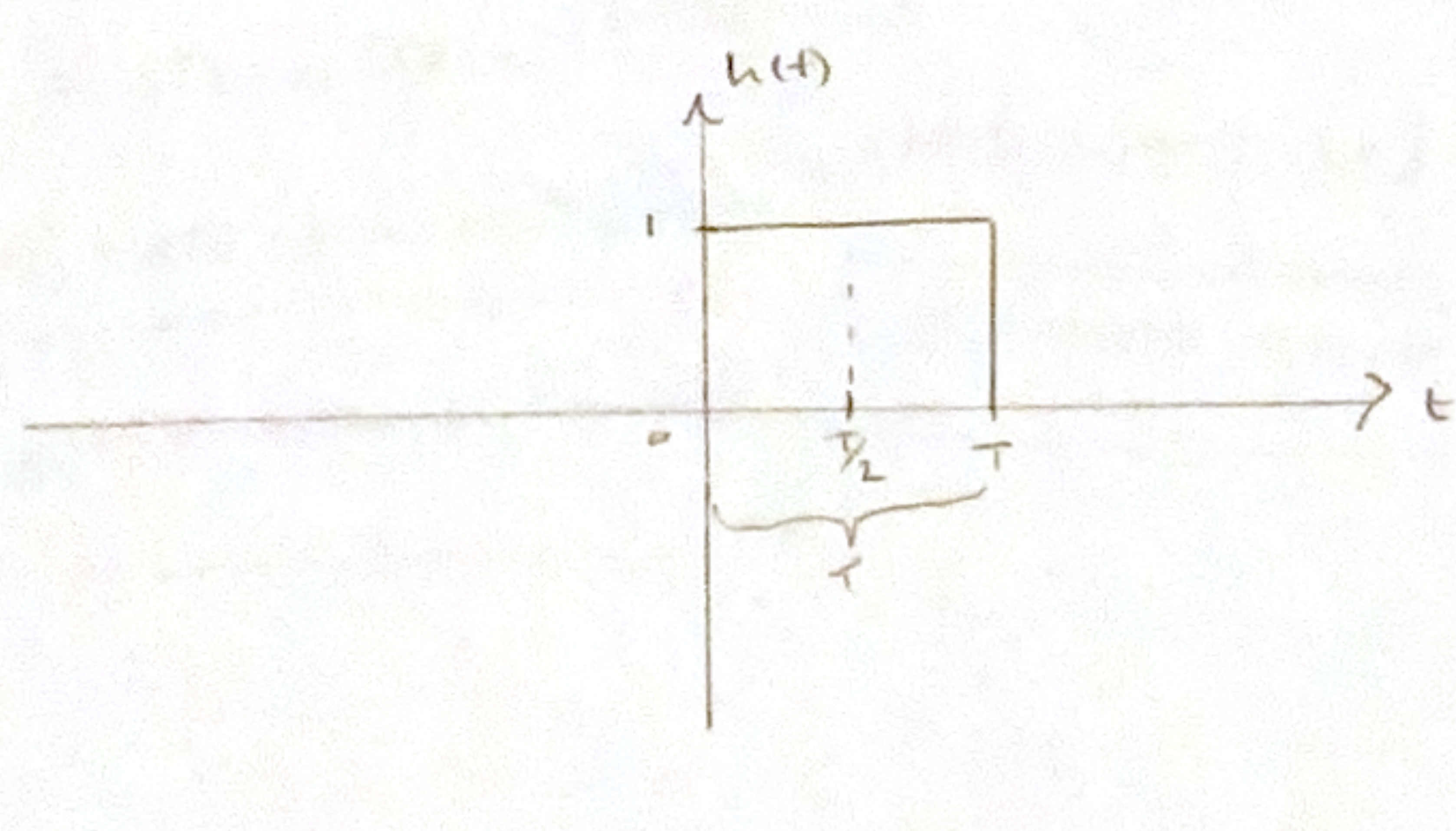
$$= \tfrac{1}{2}\left[\operatorname{sgn}(t) - \operatorname{sgn}(t-T)\right]$$

* $\operatorname{sgn}(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0 \end{cases}$

| FOR $t<0$ | FOR $t>0$ AND $t<T$ | FOR $t>T$ |
|---|---|---|
| $h(t) = \frac{1}{2}(-1+(+1))$ | $h(t) = \frac{1}{2}(1-(-1))$ | $h(t) = \frac{1}{2}(1-(1))$ |
| $= \frac{1}{2}(0)$ | $= \frac{1}{2}(+1)$ | $= \frac{1}{2}(0)$ |
| $= 0$ | $= \frac{1}{2}(2)$ | $= 0$ |
| $\Rightarrow$ FOR $t<0$, $h(t)=0$. | $= 1$ | FOR $t>T$, $h(t)=0$ |
| | FOR $0<t<T$, $h(t)=1$ | |

$\Rightarrow$ SO, $\boxed{h(t) = \operatorname{rect}\left(\frac{t-T/2}{T}\right)}$

SKETCH



3.4 LET $x(t) = A\,\text{rect}(t/\tau)$ BE APPLIED TO THE ZERO-~~[illegible]~~ ORDER HOLD. FIND $y(t)$ FOR THE FOLLOWING 3 CASES.

(i) $\tau \ll T$

$h(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$; $x(t) = A\,\text{rect}(t/\tau) \xrightarrow{\mathcal{F}} H(f) = T\,\text{sinc}(fT)e^{-j\pi fT}$; $X(f) = A\tau\,\text{sinc}(f\tau)$

$\Rightarrow Y(f) = X(f)H(f) = A T\tau\,\text{sinc}(fT)\,\text{sinc}(f\tau)\,e^{-j\pi fT}$

SMALL ANGLE APPROX.

$\text{sinc}(f\tau) = \frac{\sin(f\tau)}{f\tau}$

$\tau \ll T \rightarrow \approx \frac{f\tau}{f\tau} = 1$

(small angle) $= AT\tau\,\text{sinc}(fT)\,e^{-j\pi fT}$

so $Y(f) \approx AT\tau\,\text{sinc}(fT)\,e^{-j\pi fT}$

$\Rightarrow \mathcal{F}^{-1}\{Y(f)\} = AT\tau\,\mathcal{F}^{-1}\{\text{sinc}(fT)e^{-j\pi fT}\} = AT\tau\,\mathcal{F}^{-1}\{\text{sinc}(fT)e^{-j2\pi f(T/2)}\}$

$= \frac{AT\tau}{T}\,\text{rect}\left(\frac{t - T/2}{T}\right)$

$$\boxed{y(t) \approx A\tau\,\text{rect}\left(\frac{t - T/2}{T}\right)}$$

that assumes the small angle approx can be used of $\tau \ll T$. Otherwise it would look like a rect pulse w/ rise and fall times".

(ii) $\tau = T$

$x(t) = A\,\text{rect}(t/\tau) = A\,\text{rect}(t/T) \rightarrow \mathcal{F}\{x(t)\} = AT\,\text{sinc}(fT)$

$\Rightarrow Y(f) = X(f)H(f) = (AT\,\text{sinc}(fT))(T\,\text{sinc}(fT)e^{-j2\pi f(T/2)})$

$= AT^2\,\text{sinc}^2(fT)\,e^{-j2\pi f(T/2)}$

$\Rightarrow y(t) = \mathcal{F}^{-1}\{Y(f)\} = AT^2 \cdot \frac{1}{T}\Lambda\left(\frac{t - T/2}{T}\right) \rightarrow \boxed{y(t) = AT\Lambda\left(\frac{t - T/2}{T}\right)}$

(iii) $\tau \gg T$

$x(t) = A\,\text{rect}(t/\tau) \rightarrow X(f) = \mathcal{F}\{x(t)\} = A\tau\,\text{sinc}(f\tau)$

$\Rightarrow Y(f) = X(f)H(f) = (A\tau\,\text{sinc}(f\tau))(T\,\text{sinc}(fT)e^{-j2\pi f(T/2)})$

$= AT\tau\,\text{sinc}(f\tau)\,\text{sinc}(fT)\,e^{-j2\pi f(T/2)}$

$= AT\tau\,\text{sinc}(f\tau)\,e^{-j2\pi f(T/2)}$

* SMALL ANGLE APPROX

$\tau \gg T$

↑ "small"

$\text{sinc}(fT) = \frac{\sin(fT)}{fT} \approx \frac{fT}{fT} = 1$

$\Rightarrow y(t) = \mathcal{F}^{-1}\{Y(f)\} = AT\tau \cdot \frac{1}{\tau}\,\text{rect}\left(\frac{t - T/2}{\tau}\right)$

$$\Rightarrow \boxed{y(t) \approx AT\,\text{rect}\left(\frac{t - T/2}{\tau}\right)}$$

(c) SHOW THAT THE INTEGRATED VALUE OF THE SIGNAL $x(t)$ OVER THE INTERVAL & T $\left(g(t) = \int_{t-T}^{t} x(\lambda)\,d\lambda\right)$ CAN BE OBTAINED BY PASSING $x(t)$ THROUGH THE ZERO-ORDER HOLD SYSTEM.

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

$$\Rightarrow y(t) = X(f)H(f) = \left(\int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df\right)\left(T\operatorname{sinc}(fT)\, e^{-j2\pi f (T/2)}\right)$$

$$= T\int_{-\infty}^{\infty} \underbrace{X(f)\operatorname{sinc}(fT)\, e^{+j2\pi f(t - T/2)}}_{\text{NEEDS SOME WORK}}\, df$$

$$T\operatorname{sinc}(fT)\, e^{-j2\pi f(t - T/2)} = T\operatorname{sinc}(fT)\, e^{+j2\pi f t}\, e^{-j\pi f T}$$

$* \sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$

$$= T\,\frac{\sin(\pi f T)}{\pi f T}\, e^{-j\pi f T}\, e^{+j2\pi f t}$$

$$= \frac{1}{\pi f}\left[\frac{1}{j2}\left(e^{j\pi f T} - e^{-j\pi f T}\right)\right] e^{-j\pi f T}\, e^{+j2\pi f t}$$

$$= \frac{1}{j2\pi f}\left(e^{j\pi f T} e^{-j\pi f T} - e^{-j\pi f T} e^{-j\pi f T}\right) e^{+j2\pi f t}$$

$$= \frac{1}{j2\pi f}\left(e^{0} - e^{-j2\pi f T}\right) e^{+j2\pi f t}$$

$$= \frac{1 - e^{-j2\pi f T}}{j2\pi f}\, e^{+j2\pi f t}$$

$$= \int_{-\infty}^{\infty} \frac{1 - e^{-j2\pi f T}}{j2\pi f}\, X(f)\, e^{j2\pi f t}\, df$$

$* \frac{1}{j2\pi f} \approx \int dt$

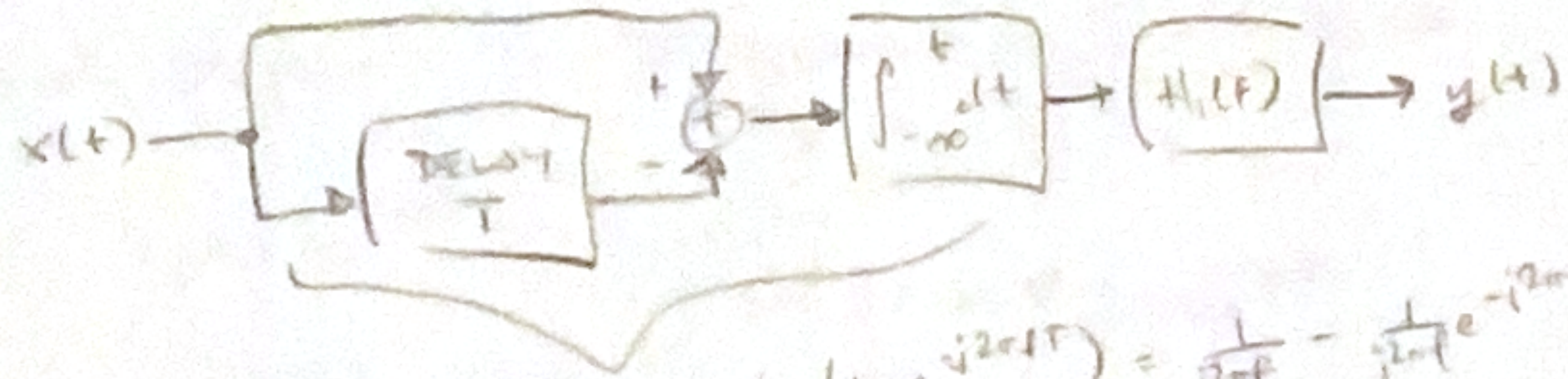
$$= \int_{-\infty}^{\infty} \frac{1}{j2\pi f}\left(X(f) - X(f)\, e^{-j2\pi f T}\right) e^{j2\pi f t}\, df$$

This shows the integral of the inverse Fourier transform of $X(f)$, i.e. $x(t)$, @ $t$ and $t-T$

$$= \int_{t-T}^{t} x(\lambda)\, d\lambda \quad \blacksquare$$

(2) THE ZERO-ORDER HOLD IS CASCADED W/ ANOTHER SUBSYSTEM W/ ~~[struck-through text]~~. TRANSFER FUNCTION $H_1(f)$.

(2.1) $h_1(t) = u(t) - u(t - T_0)$. FIND AND SKETCH THE OVERALL IMPULSE RESPONSE ~~[struck]~~ OF THE WHOLE SYSTEM FOR $T > T_0$.




$$H_2(f) = \frac{1}{j2\pi f}(1 - e^{-j2\pi f T}) = \frac{1}{j2\pi f} - \frac{1}{j2\pi f}e^{-j2\pi f T}$$

$\boxed{h_1(t) \rightleftharpoons H_1(f)}$

$h_1(t) = u(t) - u(t - T_0) \longrightarrow \mathcal{F}\{h_1(t)\} = H_1(f) = T_0 \,\mathrm{sinc}(fT_0)e^{-j2\pi f(T_0/2)}$

$= \mathrm{rect}\left(\frac{t - T_0/2}{T_0}\right)$

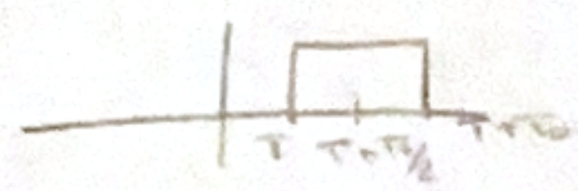
$$\Rightarrow H_{eq}(f) = H_1(f)H_2(f) = \left(T_0 \,\mathrm{sinc}(fT_0)e^{-j2\pi f(T_0/2)}\right)\left(\frac{1}{j2\pi f} - \frac{1}{j2\pi f}e^{-j2\pi f T}\right)$$

$$= \frac{1}{j2\pi f}\left(T_0 \,\mathrm{sinc}(fT_0)e^{-j2\pi f(T_0/2)}\right)(1 - e^{-j2\pi f T})$$

$$H_{eq}(f) = \frac{T_0}{j2\pi f}\,\mathrm{sinc}(fT_0)e^{-j2\pi f(T_0/2)} - \frac{T_0}{j2\pi f}\,\mathrm{sinc}(fT_0)e^{-j2\pi f(T + T_0/2)}$$

* integration & time-shift.

$$\Rightarrow h_{eq}(t) = \int_{-\infty}^{t} \mathrm{rect}\left(\frac{t - T_0/2}{T_0}\right)dt - \underbrace{\int_{-\infty}^{t} \mathrm{rect}\left(\frac{t - (T + T_0/2)}{T_0}\right)dt}$$




$$= \quad I_1 \quad - \quad I_2$$

CASES

for $t < 0$

$I_1 = 0,\ I_2 = 0 \;|\; 0$

for $0 \le t \le T_0$

$I_1 = \int_0^t dt = t\Big|_0^t = t$

$I_2 = 0$

for $T_0 < t < T$

$I_1 = 0$

$I_2 = 0$ $\quad / \; 0$

for $T \le t \le T + T_0$

$I_1 = 0$

$I_2 = \int_T^t dt = t\Big|_T^t = t - T$

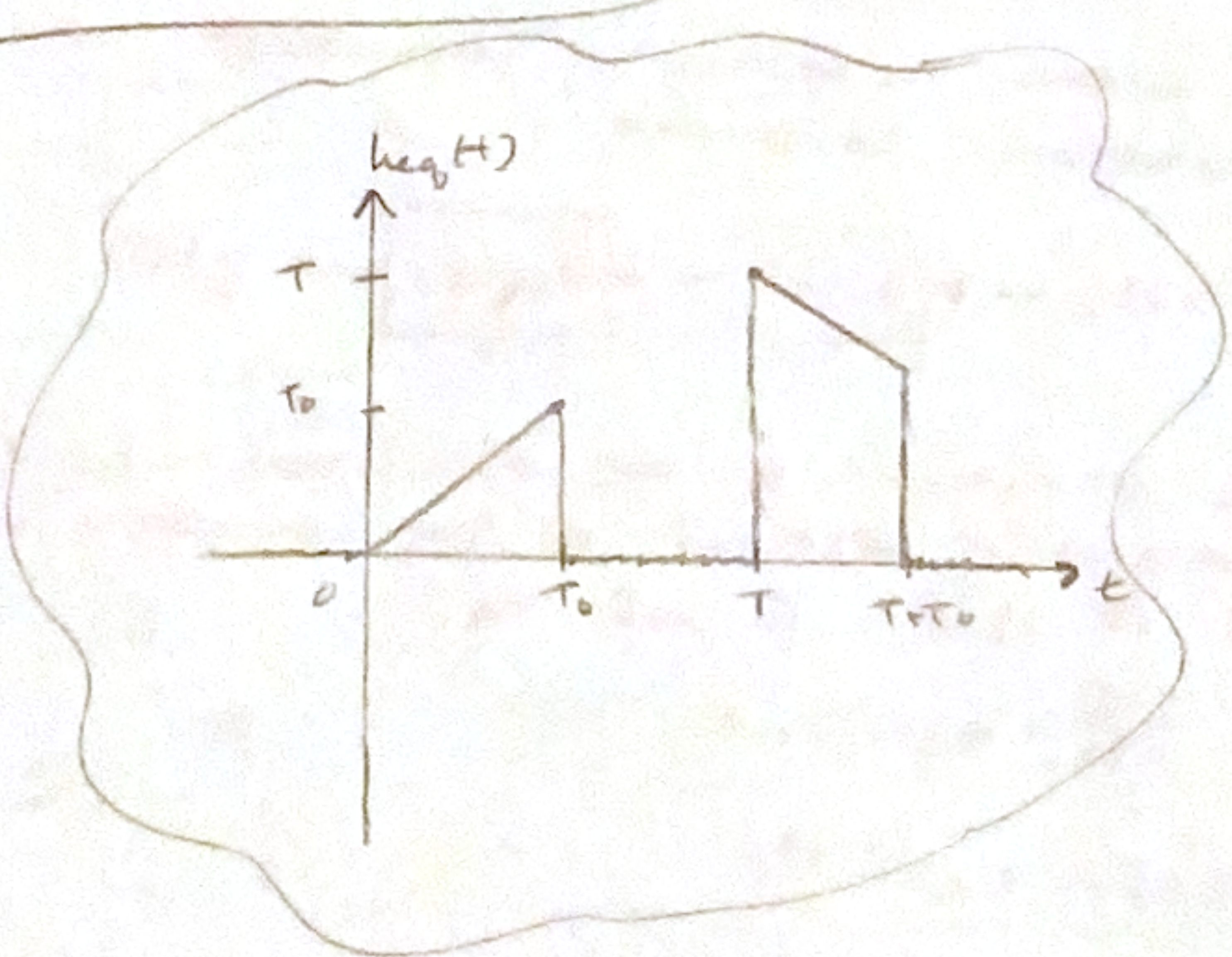
$\Rightarrow T - t$

for $t > T + T_0$

$I_1 = 0$

$I_2 = 0$ $\quad / \; 0$

$$h_{eq}(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \le t \le T_0 \\ 0, & T_0 < t < T \\ T - t, & T \le t \le T + T_0 \\ 0, & t > T_0 \end{cases}$$



D.2 LET $H_1(f) = \frac{1}{1+jf/B}$. FIND AND SKETCH OVERALL IMPULSE RESPONSE WHEN $T \gg 1/B$.

$$\Rightarrow H_{eq}(f) = H_1(f)\left(\frac{1}{j2\pi f}\left(1-e^{-j2\pi fT}\right)\right).$$

$$= \underbrace{\left(\frac{1}{1+jf/B}\right)}_{\mathcal{F}^{-1}}\underbrace{\left(\frac{1}{j2\pi f}\left(1-e^{-j2\pi fT}\right)\right)}_{\mathcal{F}^{-1}}$$

$^* \; e^{-at}u(t) \rightleftharpoons \frac{1}{a+j2\pi f}$

$a=1$

$1/B = 2\pi$

$$h_{eq}(t) = Be^{-Bt}u(t) \circledast \text{rect}\left(\frac{t-T/2}{T}\right)$$

$$= Be^{-Bt}u(t) \circledast \underbrace{(u(t)-u(t-T))}_{\text{only 1 from 0 to } T}$$

$$= \int_0^T Be^{-B(t-\tau)}\,d\tau$$

$$= \int_0^T Be^{-Bt}\cdot e^{B\tau}\,d\tau$$

$$= Be^{-Bt}\left(\frac{1}{B}e^{B\tau}\right)\Big|_0^T$$

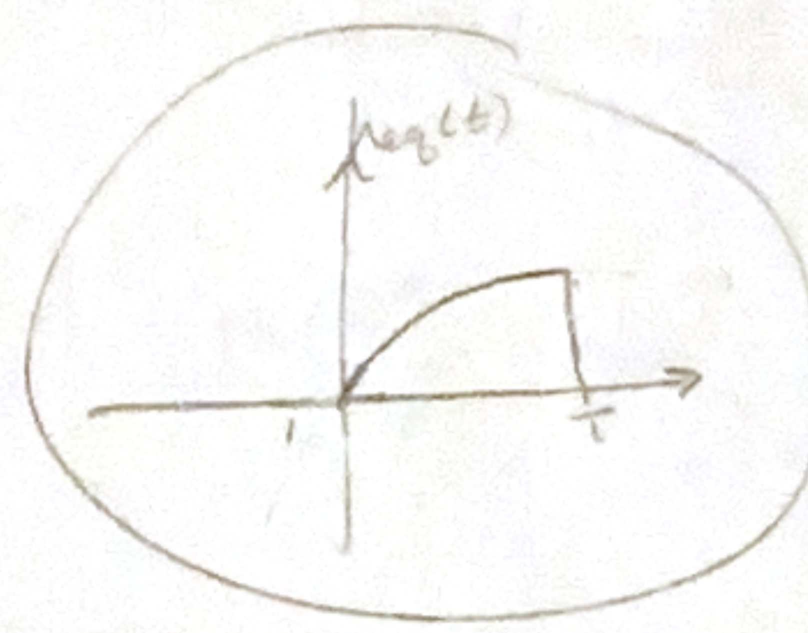
$$= e^{-Bt}\left(e^{BT}-e^{0}\right)$$

$$= e^{-BT}e^{Bt}-e^{-Bt}$$

$$= e^{0}-e^{-Bt}$$

$$= 1-e^{-Bt}$$

$$\boxed{h_{eq}(t) = 1-e^{Bt} \text{ for } 0 \le t \le T}$$




## 2 EQUILIZATION

RECALL THE TRANSFER FUNCTION OF A DISTORTION-LESS TRANSMISSION CHANNEL IS:

$$H(f) = Ke^{-j2\pi f t_d} \qquad (K, t_d \in \mathbb{R})$$

THEORETICALLY, LINEAR DISTORTION IN CHANNELS CAN BE MITIGATED BY USING EQUILIZATION. CONSIDER

$$x(t) \longrightarrow \boxed{H_c(f)} \longrightarrow \boxed{H_{eq}(f)} \longrightarrow y(t)$$

A FIND TRANSFER FUNCTION OF THE EQUILIZER $H_{eq}(f)$ WHICH TRANSFORMS THE OVERALL FREQUENCY RESPONSE OF THE CASCADED SYSTEM TO BE A DISTORTION-LESS ONE IN TERMS OF $H_c(f)$, $K$ and $t_d$.

OVERALL, $\frac{Y(f)}{X(f)} = H_{eq}(f)H_c(f) = H(f)$

$$\Rightarrow H_{eq}(f)H_c(f) = Ke^{-j2\pi f t_d}$$

$$\Rightarrow \boxed{H_{eq}(f) = \frac{Ke^{-j2\pi f t_d}}{H_c(f)}}$$

(3) WIRELESS TRANSMISSION SYSTEMS SUFFER FROM MULTIPATH DISTORTION DUE TO FREE PROPAGATION. THE CHANNEL OUTPUT IS:

$$y(t) = K_1 x(t-t_1) + K_2 x(t-t_2)$$

WHERE $t_2 > t_1$. THE SECOND TERM IS THE ECHO OF THE FIRST.

(3.1) FIND THE IMPULSE RESPONSE OF THE MULTIPATH CHANNEL $h_c(t)$.

FREQUENCY DOMAIN

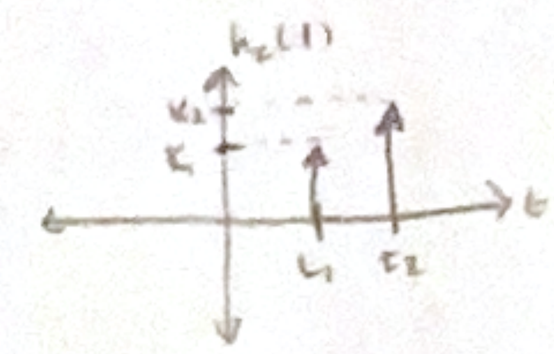
$$\mathcal{F}\{y(t)\} = K_1 X(f) e^{-j2\pi f t_1} + K_2 X(f) e^{-j2\pi f t_2}$$

$$\Rightarrow Y(f) = X(f)\left(K_1 e^{-j2\pi f t_1} + K_2 e^{-j2\pi f t_2}\right)$$

$$\Rightarrow H_c(f) = K_1 e^{-j2\pi f t_1} + K_2 e^{-j2\pi f t_2}$$

SUPERPOSITION OF TWO DISTORTIONLESS CHANNELS.

$$\mathcal{F}^{-1}\{H_c(f)\} = \boxed{h_c(t) = K_1 \delta(t-t_1) + K_2 \delta(t-t_2)}$$




(3.2) SHOW THAT THE TRANSFER FUNCTION OF THE MULTIPATH CHANNEL IS:

$$H_c(f) = K_1 e^{-j2\pi f t_1}\left[1 + k e^{-j2\pi f t_0}\right]$$

WHERE $k = K_2/K_1$, AND $t_0 = t_2 - t_1$.

FROM 3.1 $\rightarrow$ $H_c(f) = K_1 e^{-j2\pi f t_1} + K_2 e^{-j2\pi f t_2}$

$$= K_1 e^{-j2\pi f t_1}\left(1 + \overset{k}{\frac{K_2}{K_1}} e^{-j2\pi f t_2} e^{+j2\pi f t_1}\right)$$

$$= K_1 e^{-j2\pi f t_1}\left(1 + k e^{j2\pi f (\overset{-t_0}{t_1 - t_2})}\right) \rightarrow -t_0 = t_1 - t_2$$

$$= K_1 e^{-j2\pi f t_1}\left(1 + k e^{-j2\pi f t_0}\right) \boxtimes.$$

(3.3) IF $K = K_1$ AND $t_d = t_1$, SHOW THAT THE TRANSFER FUNCTION OF THE EQUALIZER $H_{eq}(f)$ WHICH MAKES THE OVERALL SYSTEM DISTORTIONLESS

$$H_{eq}(f) = \frac{1}{1 + k e^{-j2\pi f t_0}}$$

$$\text{RECALL} \rightarrow H_{eq}(f) = \frac{K e^{-j2\pi f t_d}}{H_c(f)}$$

$$= \frac{K_1 e^{-j2\pi f t_1}}{K_1 e^{-j2\pi f t_1}\left(1 + k e^{-j2\pi f t_0}\right)}$$

$$= \frac{1}{1 + k e^{-j2\pi f t_0}}$$

$\boxtimes$

(3.4) SHOW THAT $H_{eq}(f)$ CAN BE APPROXIMATED:

$$H_{eq}(f) \approx \left[e^{j2\pi f t_0} - k + k^2 e^{-j2\pi f t_0}\right] e^{-j2\pi f t_0}$$

[HINT: ASSUMING SMALL ECHO, I.E. $k^2 \ll 1$, THEN THE FIRST 3 TERMS
WHERE $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$ for $|x|<1$ ]

$$\Rightarrow \quad H_{eq}(f) = \frac{1}{1+ke^{-j2\pi f t_0}}$$

$$= \frac{1}{1-(-ke^{-j2\pi f t_0})}$$

$$= 1 + (-ke^{-j2\pi f t_0}) + (-ke^{-j2\pi f t_0})^2 + \dots$$

$k^2 \ll 1$ or $k_n \ll k_1$

$$\simeq 1 - ke^{-j2\pi f t_0} + k^2 e^{-j2\pi f t_0} e^{-j2\pi f t_0}$$

$$= e^{-j2\pi f t_0}\left(e^{+j2\pi f t_0} - ke^0 + k^2 e^{-j2\pi f t_0}\right)$$

$$= \left[e^{j2\pi f t_0} - k + k^2 e^{-j2\pi f t_0}\right] e^{-j2\pi f t_0}$$

(3.5) DRAW A BLOCK DIAGRAM FOR REPRESENTING THIS EQUILIZER IN TIME-DOMAIN BY USING DELAY AND MULTIPLICATION BLOCKS.

▷ : Gain/Multiplication