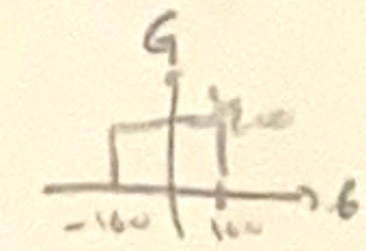
ELE478 | HW5 | C. LOTTER

SAMPLING THM

① NYQUIST RATE. FIND f_s & T_s

(A.1) $g(t) = \text{sinc}(200t)$

$$\Rightarrow \mathcal{F}\{g(t)\} = G(f) = \frac{1}{200}\text{rect}(f/200)$$

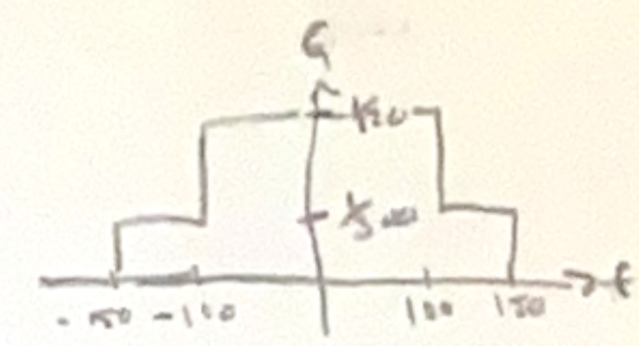


THE BANDWIDTH IS $W = 100\text{Hz}$ $\therefore$ $f_s \geq 2W = 2(100) = 200\text{Hz}$

$$\Rightarrow T_s = 1/f_s = 1/200 = 0.005\text{s} = 5\text{ms} \quad \therefore \quad \boxed{\begin{aligned} f_s &= 200\text{Hz} \\ T_s &= 5\text{ms} \end{aligned}}$$

(A.2) $g(t) = \text{sinc}(200t) + \text{sinc}(300t)$

$$\Rightarrow \mathcal{F}\{g(t)\} = G(f) = \frac{1}{200}\text{rect}(f/200) + \frac{1}{300}\text{rect}(f/300)$$

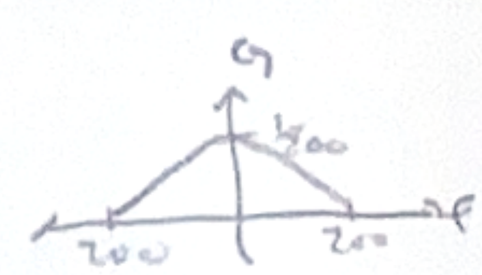


THE BANDWIDTH IS $B = 150\text{Hz}$ $\therefore$ $f_s \geq 2(150) = 300\text{Hz}$

$$\Rightarrow T_s = 1/f_s = \frac{1}{300\text{Hz}} = 3.33\text{ms} \Rightarrow \boxed{\begin{aligned} f_s &= 300\text{Hz} \\ T_s &= 3.33\text{ms} \end{aligned}}$$

(A.3) $g(t) = \text{sinc}^2(200t)$

$$\Rightarrow \mathcal{F}\{g(t)\} = G(f) = \frac{1}{200}\Delta(f/200)$$



THE BANDWIDTH IS $W = 200\text{Hz}$ $\rightarrow$ $f_s \geq 2W = 2(200\text{Hz}) = 400\text{Hz}$

$$\Rightarrow T_s = 1/f_s = \frac{1}{400\text{Hz}} = 2.5\text{ms} \Rightarrow \boxed{\begin{aligned} f_s &= 400\text{Hz} \\ T_s &= 2.5\text{ms} \end{aligned}}$$

(A.4) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

$$\Rightarrow \mathcal{F}\{g(t)\} = G(f) = \frac{1}{200}\text{rect}(f/200) + \frac{1}{200}\Delta(f/200)$$

$\Rightarrow W = 200\text{Hz} \rightarrow f_s \geq 2W = 2(200\text{Hz}) = 400\text{Hz}$

$\Rightarrow T_s = 1/f_s = \frac{1}{400\text{Hz}} = 2.5\text{ms}$ $\Rightarrow$ $\boxed{\begin{aligned} f_s &= 400\text{Hz} \\ T_s &= 2.5\text{ms} \end{aligned}}$

A.5

SAME AS A.3 & A.4

$$\Rightarrow \boxed{\begin{aligned} f_3 &= 400\text{Hz} \\ T_3 &= 2.5\text{ms} \end{aligned}}$$

B

③ PULSE CODE MODULATION

(A) PCM SYSTEM USES A N-BIT BINARY ENCODER. THE BIT RATE IS R_b.

(A.1) FIND T_s.

$$R_b = N_b f_s = N f_s = N/T_s \Rightarrow \boxed{T_s = N/R_b \ [s]}$$

(A.2) MAXIMUM MSG BW?

SAMPLING THEOREM $\Rightarrow f_s \geq 2W \quad \therefore \quad W \leq f_s/2 = \frac{1}{2T_s} = \frac{1}{2(N/R_b)} = \frac{R_b}{2N}$

$$\Rightarrow \boxed{W \leq R_b/2N \ [Hz]}$$

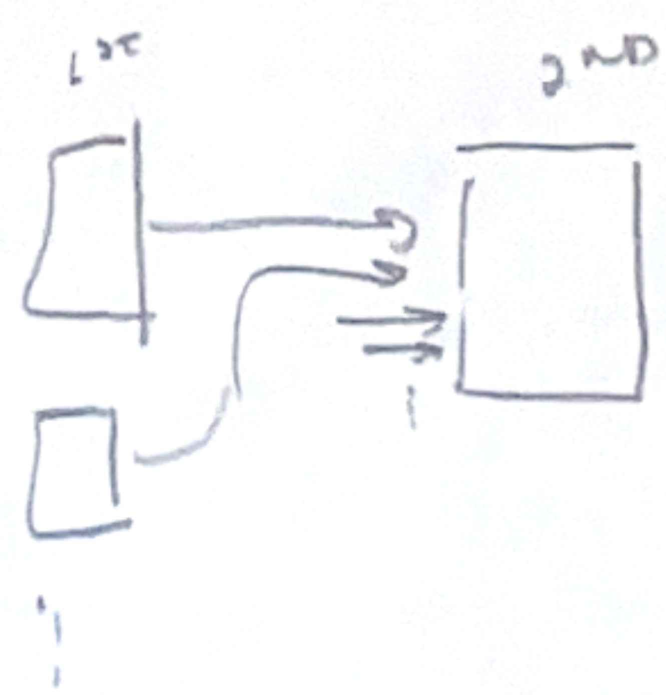
(A.3) L_1 SUCH SIGNALS ARE MULTIPLEXED W/ M-bits TO FRAME. DETERMINE BIT RATE OF MUX OUTPUT.

TOTAL BITS (IN FRAME) $= L_1 \cdot N + M$ [bit]

FRAME DURATION $= T_s$ [s]

DURATION PER BIT $= \frac{T_s}{L_1 N + M}$ [s/bit] $\longrightarrow \boxed{R_0 = \frac{L_1 N + M}{T_s} = \frac{L_1 N + M}{N/R_b} = \frac{R_b(L_1 N + M)}{N} = R_b(L_1 + M/N) \ [\text{bit/s}]}$

(A.4) # OF L_2 CARRIERS IN SECOND LEVEL $R_b = R_1$ bit/s.



$$\boxed{\#\text{ OF INPUTS TO 2ND LVL} = R_1/R_0 = \frac{R_1}{R_b(L_1 + M/N)} \text{ INPUTS}}$$

(A.5) # OF PCM SIGNALS AT 2ND LVL

$$\boxed{\#\text{ OF CARRIERS} = \frac{R_1 L_1}{R_b(L_1 + M/N)} \ [\text{PCM SIGNALS}]}$$

③ $f_s = 8\,kHz$, $N_b = 8$ bits/sample, PCM, 30 INPUTS TO FIRST LEVEL, $R_b^{(1)} = 2.048$ Mbps.

(3.1) DETERMINE BIT RATE OF PCM SIGNAL

$R_b^{(0)} = N_b f_s = (8 \text{ bits/sample})(8\,kHz) = \boxed{64\,kbps}$

(3.2) FIND # OF FRAMING BITS

bit duration $\Rightarrow$ $T_b^{(1)} = \frac{1}{R_b^{(1)}} = \frac{1}{2.048\,Mbps} = 0.488\,\mu s/\text{bit}$

frame duration $\Rightarrow$ $T_s = \frac{1}{f_s} = \frac{1}{8\,kHz} = 125\,\mu s$

$\frac{125\,\mu s}{0.488\,\mu s/\text{bit}} = \underline{\underline{256 \text{ bits}}}$

TOTAL BITS FOR ALL CHANNELS $\Rightarrow$ $N_b(30) = 30 \cdot 8 \text{ bits/sample} = 240$ bits

$N_{frame} = N_{tot} - N_b(30) = 256 - 240 = \boxed{16 \text{ bits for framing}}$

④ PULSE AMPLITUDE MODULATION.