Homework 01

ECE 478/ECE 570 Principles of Communication Systems

SIU Southern Illinois University CARBONDALE

Date: 02/04/2025
Due data: 02/10/2025
Section: Representations of Signals and Systems for Communication Systems
Submission instructions: Upload a single PDF file into D2L
(Adobe Scan mobile app is preferred)

---

(1) An exponentially decaying pulse is denoted as $g(t) = \exp(-at)u(t)$, where $u(t)$ is the unit step function. This pulse is depicted in Fig. 1.

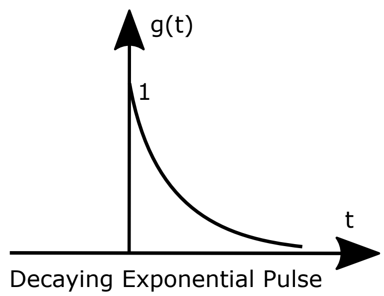

Figure 1: Exponential decaying pulse

a. Use the definition of Fourier transform to derive the frequency domain representation of the exponentially decaying pulse in Fig. 1.

b. Is its Fourier transform real-valued or complex-valued? Explain your answer by considering the real-valued and asymmetric nature of the time-domain pulse.

c. Let $G(f)$ denote the FT of the exponentially decaying pulse in Fig. 1. Determine the amplitude/magnitude spectrum $|G(f)|$ and phase spectrum $\theta(f)$ of $G(f)$. [Hint: A complex-valued function can be represented as $G(f) = X(f) + jY(f) = |G(f)| \exp(j\theta(f))$, where $|G(f)| = \sqrt{X^2(f) + Y^2(f)}$ and $\theta(f) = \tan^{-1}\left(\frac{Y(f)}{X(f)}\right)$.]

d. Plot the amplitude/magnitude spectrum and phase spectrum found in Part C using Matlab as a function of frequency in the x-axis.

e. Show that above FT $G(f)$ of the exponentially rising pulse exhibits conjugate symmetry. To this end, you need to show the amplitude spectrum is even-symmetric ($|G(-f)| = |G(f)|$) and the phase spectrum is odd-symmetric ($\theta(-f) = -\theta(f)$).

(2) Consider the time-domain representation of a low-pass filter given below:

$$g(t) = A\text{sinc}\,(2W(t-\tau))$$

a. Find the frequency-domain representation or Fourier Transform of this filter.

b. Plot its amplitude response as a function of frequency.

c. Plot its phase response as a function of frequency.

c. What is the effect of a time-shift in the frequency domain?

d. What is the significance of such a time-shift in designing filters?

(3) Consider the following time-domain function.

$$y(t) = \beta g(t)\cos(2\pi f_c t),$$

where β is a constants Moreover, the Fourier transform of $g(t)$ is given by $G(f)$.

(a.) Find the Fourier transform of $y(t)$ as a function of $G(f)$.

(b.) If $g(t) = \cos(2\pi f_0 t)$, then find the Fourier transform of $y(t)$. Plot its amplitude spectrum for the case $f_c \gg f_0$.

(c.) If $g(t) = \text{rect}(t/T_0)$, then find the Fourier transform of $y(t)$. Plot its amplitude spectrum for the case $f_c \gg f_0 = 1/T_0$.

(c.) If $g(t) = \Lambda(t/T_0)$, then find the Fourier transform of $y(t)$. Plot its amplitude spectrum for the case $f_c \gg f_0 = 1/T_0$.

(d.) What are the insights that you can obtain from your frequency domain analysis of a time-domain function of the form of $y(t) = \beta g(t)\cos(2\pi f_c t)$.

(4) Consider two rectangular pulses denoted by $g_1(t)$ and $g_2(t)$.

$$g_1(t) = \text{rect}(t) \qquad \text{and} \qquad g_2(t) = \text{rect}(t-1).$$

Let $g_{12}(t) = g_1(t) \circledast g_2(t)$, where $\circledast$ is the convolution operator. Hence, $g_{12}(t)$ has the following integral representation:

$$g_{12}(t) = g_1(t) \circledast g_2(t) = \int_{-\infty}^{\infty} g_1(\tau) g_2(t-\tau)\, d\tau.$$

(a.) Plot $g_1(t)$ and $g_2(t)$ in time-domain. Write down the Fourier transforms $G_1(f) = \mathcal{F}[g_1(t)]$ and $G_2(f) = \mathcal{F}[g_2(t)]$.

(b.) By using Property-11 (convolution in time-domain), find the Fourier transform of $g_{12}(t)$.

[Hint: Property 11: $\mathcal{F}[g_1(t) \circledast g_2(t)] = G_1(f)G_2(f)$, where $\mathcal{F}[g_1(t)] = G_1(f)$ and $\mathcal{F}[g_2(t)] = G_2(f)$.]

(c.) By evaluating the convolution integral, show that $g_{12}(t) = g_1(t) \circledast g_2(t)$ is a shifted triangular function given by

$$g_{12}(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t < 1 \\ -t+2, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases} = \Lambda(t-1).$$

Plot $g_{12}(t) = \Lambda(t-1)$ in time-domain.

(d.) By using that fact the Fourier transform of a standard triangular function is $\mathcal{F}[\Lambda(t)] = \text{sinc}^2(f)$, find the Fourier transform $G_{12}(f) = \mathcal{F}[\Lambda(t-1)]$ by using the time-shifting property.

[Hint: $\mathcal{F}[g(t-t_0)] = G(f)\exp(-j2\pi f t_0)$, where $\mathcal{F}[g(t)] = G(f)$.]

(e.) Verify that your answers for parts (b.) and (d.) are indeed equivalent.

(f.) Plot the amplitude-spectrum and phase-spectrum of $G_{12}(f) = \mathcal{F}[\Lambda(t-1)]$.

(5) Consider the periodic signal given below:

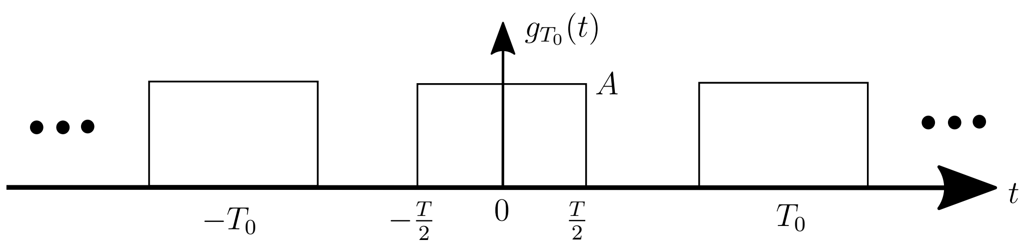

Figure 2: A periodic signal

(a.) Find the generating function $g(t)$ of the above periodic function $g_{T_0}(t)$.

(b.) Express the periodic function $g_{T_0}(t)$ in terms of its generating function $g(t)$.

(c.) Find the Fourier transform of the generating function $g(t)$ and plot its amplitude spectrum.

(d.) Find the Fourier transform of the periodic function $g_{T_0}(t)$ and plot its amplitude spectrum.

(e.) What are the insights that you can obtain from your frequency domain analysis of a periodic function in time-domain?