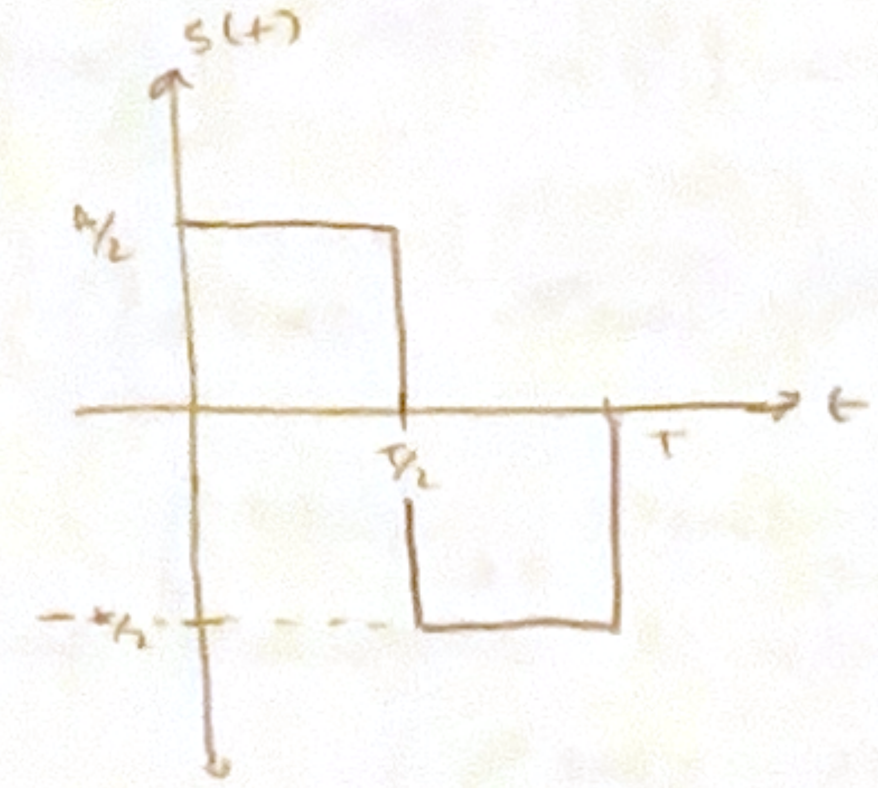
ELE478

HW8

C. LOTITO

① MATCHED FILTERING



Ⓐ DETERMINE THE IMPULSE RESPONSE OF A FILTER MATCHED TO THE SIGNAL s(t) AND PLOT.

$$h_{opt}(t) = kg^*(T-t)$$

$$\rightarrow s(t) = \tfrac{A}{2}\,\mathrm{rect}\left(\frac{t-T/4}{T/2}\right) - \tfrac{A}{2}\,\mathrm{rect}\left(\frac{t-3T/4}{T/2}\right)$$

$$\Rightarrow h(t) = ks^*(T-t)$$

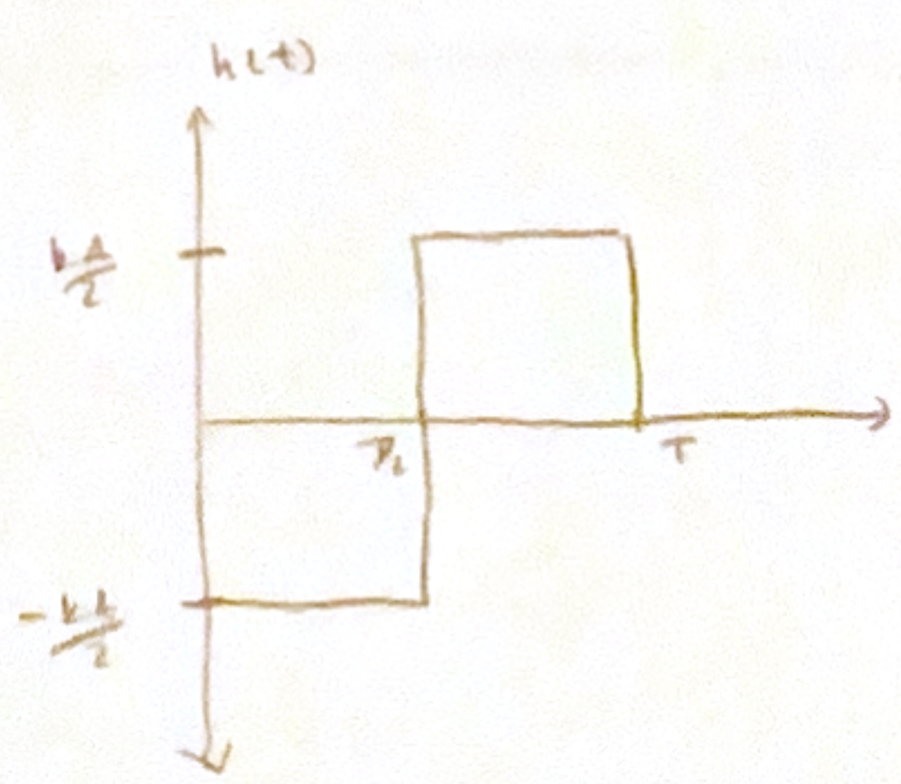
$$= k\left[\tfrac{A}{2}\,\mathrm{rect}\left(\frac{T-t-T/4}{T/2}\right) - \tfrac{A}{2}\,\mathrm{rect}\left(\frac{T-t-3T/4}{T/2}\right)\right]$$

$$= k\tfrac{A}{2}\left(\mathrm{rect}\left(\frac{3T/4-t}{T/2}\right) - \mathrm{rect}\left(\frac{T/4-t}{T/2}\right)\right)$$

$$= k\tfrac{A}{2}\left(\mathrm{rect}\left(\frac{-(t-3T/4)}{T/2}\right) - \mathrm{rect}\left(\frac{-(t-T/4)}{T/2}\right)\right)$$

* rect pulse is even, rect(t) = rect$(-t)$

$$\boxed{h(t) = k\tfrac{A}{2}\left(\mathrm{rect}\left(\frac{t-3T/4}{T/2}\right) - \mathrm{rect}\left(\frac{t-T/4}{T/2}\right)\right)}$$



(b) PASS THE SIGNAL THROUGH THE ~~[illegible]~~ FILTER. DETERMINE AND PLOT THE OUTPUT.

$$\Rightarrow y(t) = s(t) \circledast h(t)$$

$$= S(f)H(f)$$

$$Y(f) = \frac{kA^2}{4}\left[\frac{1}{T/2}\operatorname{sinc}\left(\frac{fT}{2}\right)e^{-j2\pi f T/4} - \frac{1}{T/2}\operatorname{sinc}\left(\frac{fT}{2}\right)e^{-j2\pi f 3T/4}\right]\left[\frac{1}{T/2}\operatorname{sinc}\left(\frac{fT}{2}\right)e^{-j2\pi f 3T/4} - \frac{1}{T/2}\operatorname{sinc}\left(\frac{fT}{2}\right)e^{-j2\pi f T/4}\right]$$

$$= \frac{kA^2}{4}\left(\frac{2}{T}\right)^2 \operatorname{sinc}^2\frac{fT}{2}\left[\left(e^{-j2\pi f T/4} - e^{-j2\pi f 3T/4}\right)\left(e^{-j2\pi f 3T/4} - e^{-j2\pi f T/4}\right)\right]$$

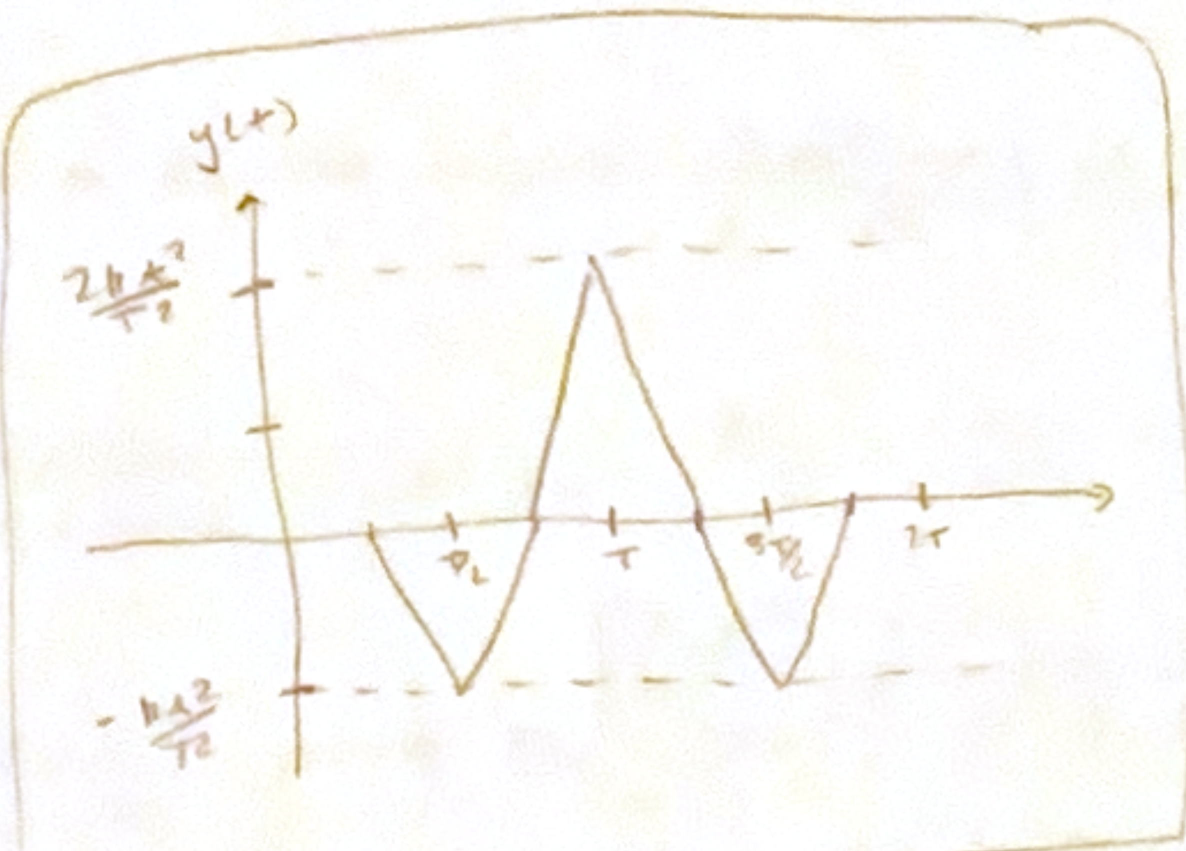
$$= \frac{kA^2}{T^2}\operatorname{sinc}^2\left(\frac{fT}{2}\right)\left[e^{-j2\pi f T} - e^{-j2\pi f T/2} - e^{-j2\pi f 3T/2} + e^{-j2\pi f T}\right]$$

$$= \frac{kA^2}{T^2}\operatorname{sinc}^2\left(\frac{fT}{2}\right)\left[-e^{-j2\pi f T/2} + 2e^{-j2\pi f T} - e^{-j2\pi f 3T/2}\right]$$

$$\Big\downarrow \mathcal{F}^{-1}$$

$$\boxed{y(t) = \frac{kA^2}{T^2}\left[-\Delta\left(\frac{t-T/2}{T/2}\right) + 2\Delta\left(\frac{t-T}{T/2}\right) - \Delta\left(\frac{t-3T/2}{T/2}\right)\right]}$$

$$s(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$



(c)

THE OUTPUT PEAKS @ $t = T$, $y(T) = \frac{2kA^2}{T^2}$.

SO THE IDEAL SAMPLING INSTANCE IS $\boxed{t = T}$.

② RAISED COSINE SPECTRUM

Ⓐ A RAISED COSINE SPECTRUM w/ A ROLLOFF FACTOR THAT CAN ELIMINATE INTER-SYMBOL INTERFERENCE (ISI) IS GIVEN BY:

$$P(f) = \begin{cases} \frac{1}{2W}, & 0 \le |f| < f_1 \\ \frac{1}{4W}\left[1 - \sin\left(\frac{\pi(|f| - W)}{2W - 2f_1}\right)\right], & f_1 \le |f| < 2W - f_1 \\ 0, & |f| \ge 2W - f_1 \end{cases}$$

WHERE THE FREQUENCY PARAMETER f_1 AND BANDWIDTH W ARE RELATED BY

$$\rightarrow \alpha = 1 - f_1/W$$

SUPPOSE A BINARY PAM WAVE IS TO BE TRANSMITTED OVER A BASEBAND CHANNEL w/ AN ABSOLUTE MAXIMUM BANDWIDTH OF 75kHz. THE BIT DURATION IS 10μs.

(A.1) FIND A RAISED COSINE SPECTRUM THAT SATISFIES THESE REQUIREMENTS.

OVERALL SYSTEM BANDWIDTH $\rightarrow W = R_b/2 = \frac{1}{2T_b} = \frac{1}{2(10\mu s)} = 50kHz \Rightarrow \underline{\underline{W = 50kHz}}$

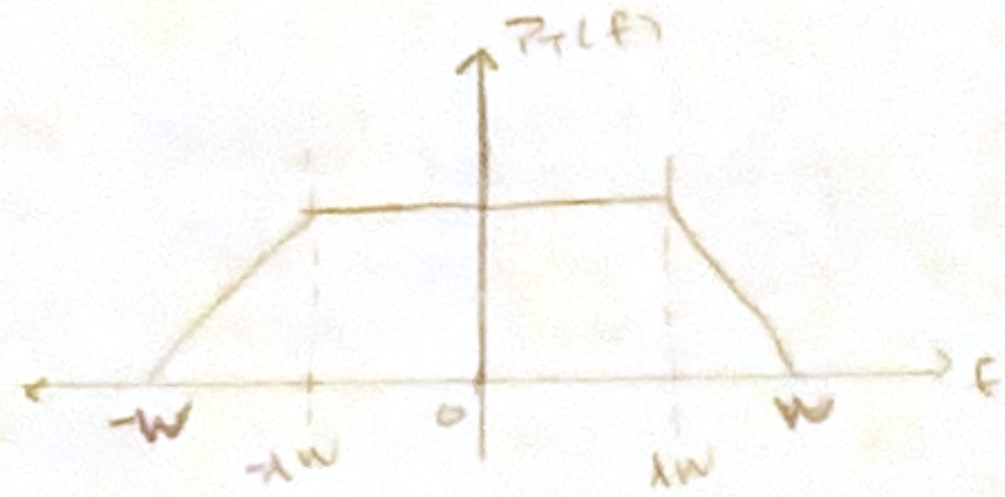
TRANSMISSION BANDWIDTH $\rightarrow B_T = 2W - f_1 \Rightarrow f_1 = 2W - B_T = 2(50kHz) - 75kHz = 25kHz$

$\Rightarrow \underline{\underline{f_1 = 25kHz}}$

$$\Rightarrow \alpha = 1 - f_1/W = 1 - \frac{25kHz}{50kHz} = 0.5$$

$$\boxed{\therefore P(f) = \begin{cases} \frac{1}{100kHz}, & 0 \le |f| < 25kHz \\ \frac{1}{200kHz}\left[1 - \sin\left(\frac{\pi(|f| - 50kHz)}{50kHz}\right)\right], & 25kHz \le |f| < 75kHz \\ 0, & |f| \ge 75kHz \end{cases}}$$

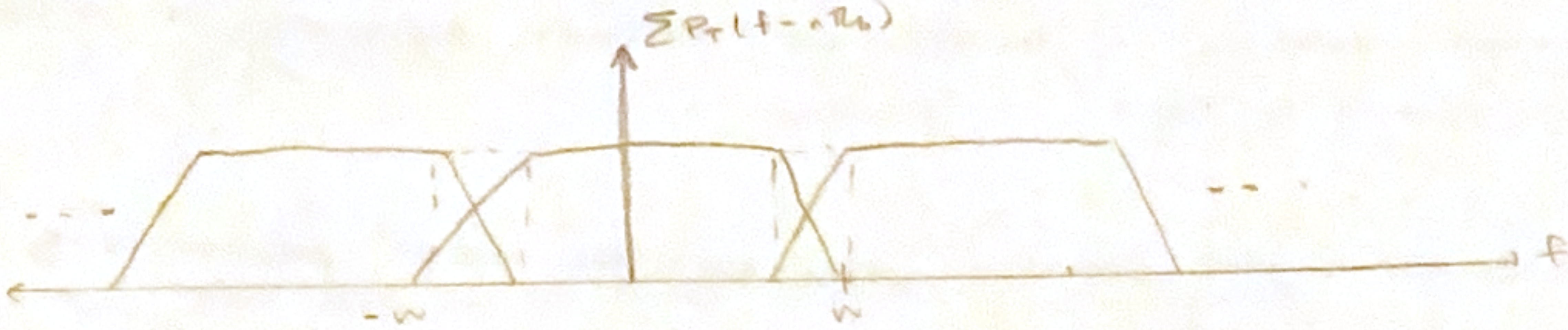
A.2 CONSIDER THE TRAPEZOIDAL PULSE SPECTRUM. JUSTIFY THAT THIS TWO SATISFIES THE ZERO ISI CONDITION.



TO HAVE ZERO ISI $\rightsquigarrow \sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b$

TO ENSURE THE PULSE $P_T(f)$ SATISFIES THE NYQUIST CRITERION, WE MUST SAMPLE AT A RATE S.T. THE LINEAR REGIONS ARE OVERLAPPING AND CONSTRUCTIVELY SUM TO EQUAL A CONSTANT, EQUIV. TO THE FLAT REGION OF $P_T(f)$.

ESSENTIALLY THE CENTERS OF THE LINEAR REGIONS MUST INTERSECT.



A.3 SUGGEST ANOTHER PULSE SPECTRUM THAT SATISFIES ZERO ISI

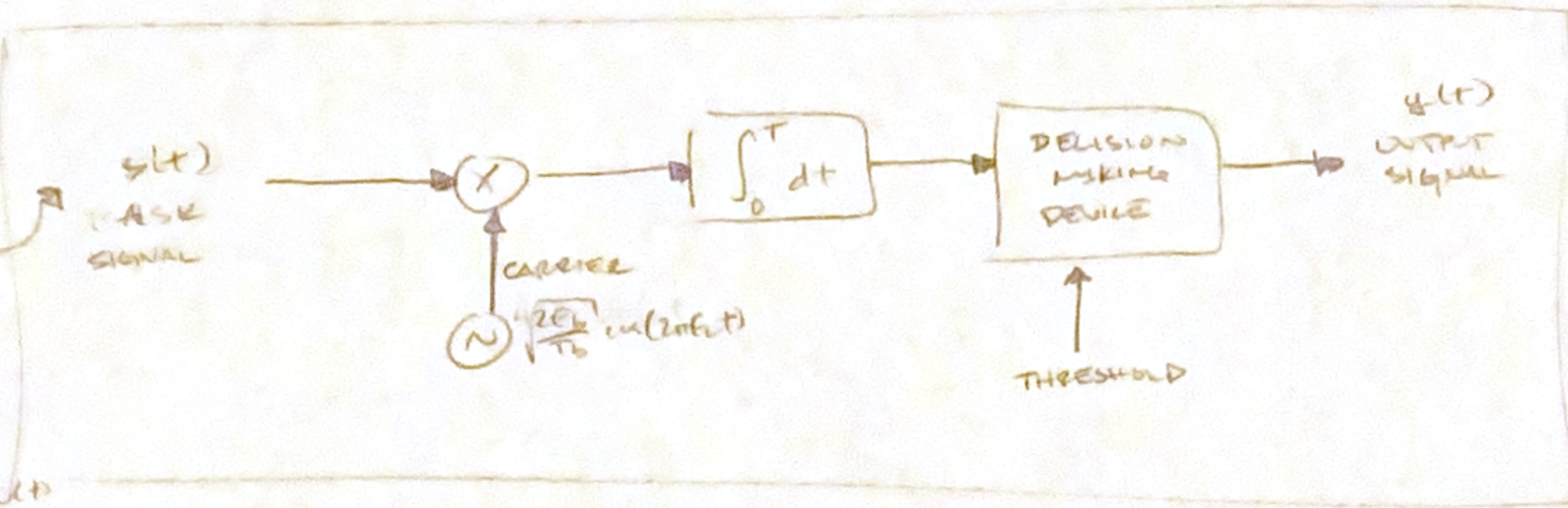
THE IDEAL PULSE SPECTRUM WOULD BE $\rightsquigarrow P(f) = \text{rect}(f/2W)$

WHICH SAMPLED AT A RATE OF $R_b = 2W$ WOULD PROVIDE THE PERFECT ZERO ISI CONDITION.

④ PROBABILITY OF ERROR FOR BINARY ASK & PSK

Ⓐ ON-OFF VERSION OF ASK SYSTEM, SYMBOL 1 IS REPRESENTED BY TRANSMITTING A SINUSOIDAL CARRIER OF AMPLITUDE $\sqrt{2E_b/T_b}$ WHERE E_b IS SIGNAL ENERGY PER BIT AND T_b IS BIT DURATION. SYMBOL 0 IS REPRESENTED BY SWITCHING OFF THE CARRIER. ASSUME 1 & 0 OCCUR w/ EQUAL PROBABILITY. THE NOISE IS GAUSSIAN WHITE NOISE w/ ZERO MEAN AND $P_w(f) = N/2$.

(A.1) PROVIDE BLOCK DIAGRAM FOR A ASK COHERENT RECIEVER.



contains $w(t)$

$s(t) = s_1(t) + w(t)$

w/ $s_0(t) + w(t) = w(t)$

* ONLY 1 STAGE w/ CARRIER AS IT IS THE ONLY BASIS FUNCTION $\phi_1(t)$.

$\phi(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$

WE COULD ALSO JUST USE AN ENVELOPE DETECTOR (o—▷|—┬—o)

(A.2) DETERMINE AVERAGE PROBABILITY OF ERROR FOR ASK SYSTEM w/ COHERENT RECEPTION.

$s(t)$ CAN EITHER BE $s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$ OR $s_0(t) = 0$.

FOR EQUIPROBABLE SIGNALING $\rightarrow$ $P_e = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right)$

$$\rho = \frac{1}{T_b}\int_0^{T_b} s_1(t)s_0(t)\,dt = \frac{1}{T_b}\int_0^{T_b} 0\,dt = 0$$

$$\Rightarrow \boxed{P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

(A.3) SUPPOSE 1 OCCURS w/ PROBABILITY 2/3 AND 0 w/ PROBABILITY 1/3. HOW DOES THE RECIEVER & P_e CHANGE TO REDUCE P_e?

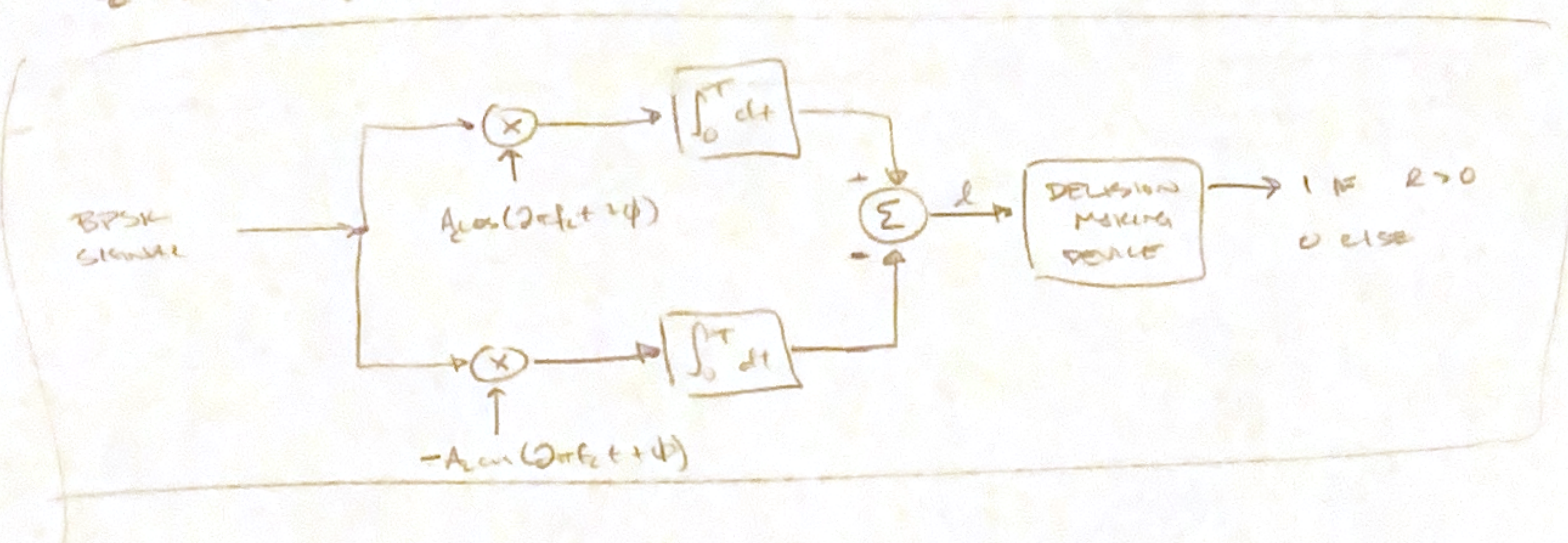
THE ONLY THING THAT CAN CHANGE IS THE THRESHOLD. ORIGINALLY THIS IS EQUIDISTANT BTWN THE SYMBOLS, BUT @ 2/3 PROBABILITY FOR "1", WE CAN LOWER THE THRESHOLD TO MAKE "1" DECISIONS MORE LIKELY, AS IT IS NOW TECHNICALLY THE SAFER BET.

(B) BPSK SIGNAL APPLIED TO COHERENT RECIEVER w/ PHASE REFERENCE w/in ϕ rad of CARRIER PHASE.

(B.1) BLOCK DIAGRAM OF COHERENT RECIEVER.

$s_1(t) = A_c\cos(2\pi f_c t)$

$s_0(t) = A_c\cos(2\pi f_c t + \pi) = -A_c\cos(2\pi f_c t)$



(B.2) FIND AVG PROB OF ERROR DETECTION

$\rho = -1$ AS $s_1(t) = -s_0(t)$ ...

$$\Rightarrow P_e = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right) = \boxed{Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$