Homework 02

ECE 478/ECE 570 – Principles of Communication Systems

SIU Southern Illinois University CARBONDALE

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Due by: 11.59 PM on 02/20/2025
Section 01: Representation communication systems
Number of problems: 03
Policy: Late submissions will not be accepted
(Q.3) is optimal for undergraduate students.
Bonus marks can be earned by solving (Q.3).
Submission instructions: Upload a single PDF file into D2L
(Adobe Scan mobile app is preferred)

Question (01): Block diagram analysis [35 marks]

Consider the block diagrams for cascade, parallel and feedback connections in Fig. 1. A communication system consists of many interconnected subsystems. Let us assume that these subsystems exhibit linear and lime-invariant (LTI) properties and the corresponding input-output relationship is described by their individual transfer functions $H_1(f)$ and $H_2(f)$. Whenever each subsystem is described by individual transfer functions, it is possible and desirable to obtain the equivalent transfer function of the overall system.

(a). Find the overall transfer functions of cascade, parallel and feedback connections in terms of $H_1(f)$ and $H_2(f)$.

(b). Any LTI operation has an equivalent transfer function. Table 1 lists four transfer functions obtained by applying transform properties of Fourier transform to four primitive time-domain operations.

Table 1: Transfer functions of four primitive time-domain operations

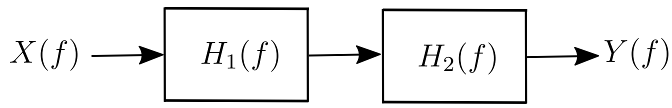
Time-domain operation	Description	Transfer function
Scalar multiplication	$y(t) = \pm K x(t)$	$H(f) = \pm K$
Time delay	$y(t) = x(t - t_d)$	$H(f) = \exp(-j2\pi f t_d)$
Differentiation	$y(t) = \frac{dx(t)}{dt}$	$H(f) = j2\pi f$
Integration	$y(t) = \int_{\infty}^{t} x(\lambda) d\lambda$	$H(f) = \frac{1}{j2\pi f}$

Consider the zero-order hold system in Fig. 1. It has several applications in communication systems. For instance, if a signal has discrete sample points, then a zero-order hold can be used to interpolate between the points.

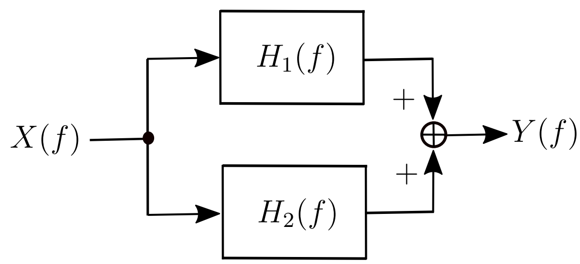
(b.1) Draw a block diagram to represent the zero-order hold in frequency domain.

(b.2) Find the overall transfer function of the zero-order hold.

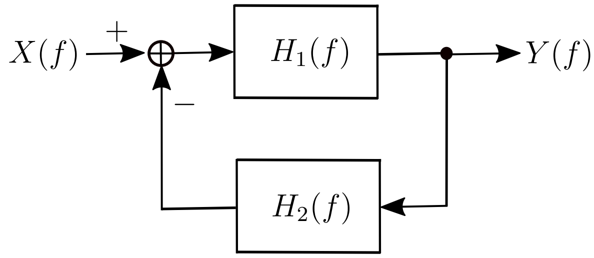
(b.3) Find and sketch the impulse response of the zero-order hold.



(a) Cascade connection



(b) Parallel connection



(c) Feedback connection

Figure 1: Basic block diagrams of communication systems

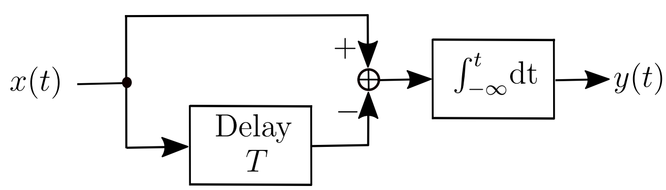


Figure 2: Zero-order hold system

(b.4) Let $x(t) = A \text{ rect}\left(\frac{t}{\tau}\right)$ be applied to the zero-order hold. Find $y(t)$ for the following three cases; (i) $\tau << T$, (ii) $\tau = T$ and (iii) $\tau >> T$.

(c) Show that the integrated value of the signal $x(t)$ over the interval T (i.e., $y(t) = \int_{t-T}^{t} x(\lambda)d\lambda$) can be obtained by passing $x(t)$ through the zero-order hold system. [hint: Express $x(t)$ in terms of its inverse Fourier transform and then use the transfer function of the zero-order hold system evaluated in part (b).]

(d) The zero-order hold is cascaded with another subsystem with a transfer function $H_1(f)$ as shown in Fig. 3.

(d.1) Let the impulse response of the second block in Fig. 3 be $h_1(t) = u(t) - u(t - T_0)$, where $u(t)$ is the unit step function. Find and sketch the overall impulse response of this cascaded system when $T > T_0$.

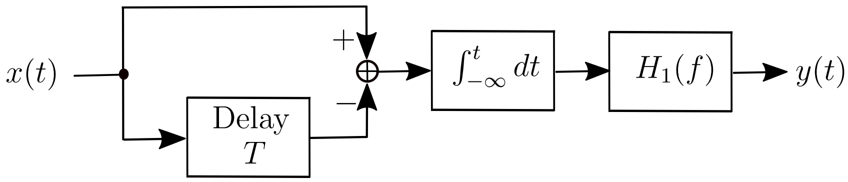


Figure 3: Cascaded zero-order hold system

(d.2) Let the transfer function of the second block in Fig. 3 be $H_1(f) = [1 + jf/B]^{-1}$. Find and sketch the overall impulse response of this cascaded system when $T >> 1/B$.

Question (02): Equalization [35 marks]

Recall that the transfer function of a distortion-less transmission channel can be written as

$$H(f) = K \exp(-j2\pi f t_d)$$

where K and t_d are constants (see lecture 5 for more information). Theoretically, linear distortion (in both amplitude and phase) in transmission channels can be mitigated by using equalization. Consider the transmission channel with the cascaded equalizer for linear distortion in Fig. 4.

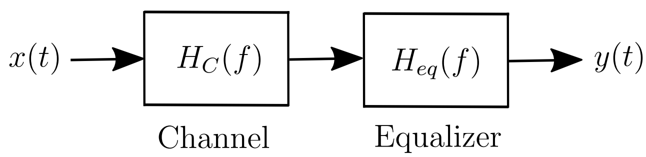


Figure 4: Channel with equalizer for linear distortion

(a) Find the transfer function of the equalizer $H_{eq}(f)$ which transforms the overall frequency response of the cascaded system to be a distortion-less one in terms $H_C(f)$, K and t_d.

(b) Typically, wireless transmission systems suffer from multipath distortion caused by multiple propagation paths between the transmitter and receiver. The channel output of such a system can be written as follows:

$$y(t) = K_1 x(t - t_1) + K_2 x(t - t_2),$$

where $t_2 > t_1$. This case ensures that the second term corresponds to an echo of the first term.

(b.1) Find the impulse response $h_c(t)$ of the aforementioned multipath channel.

(b.2) Show that the transfer function of the multipath channel is given by

$$H_C(f) = K_1 \exp(-j2\pi f t_1)[1 + k \exp(-j2\pi f t_0)],$$

where $k = K_2/K_1$ and $t_0 = t_2 - t_1$.

(b.3) If $K = K_1$ and $t_d = t_1$, show that the transfer function of the equalizer $H_{eq}(f)$ which makes the the overall frequency response of the system a distortion-less one can be written as

$$H_{eq}(f) = \frac{1}{1 + k\exp(-j2\pi f t_0)}.$$

(b.3) Show that $H_{eq}(f)$ can be approximated as follows:

$$H_{eq}(f) \approx \left[\exp(j2\pi f t_0) - k + k^2 \exp(-j2\pi f t_0)\right] \exp(-j2\pi f t_0)$$

[hint: Use binomial expansion assuming a small echo, so that $k^2 << 1$. Then take the first three terms by dropping all other higher order terms. Note that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$ for $|x| < 1$.]

(b.4) Draw a block diagram for representing this equalizer in time-domain by using delay and scaler multiplication blocks [hint: see Table 1].

Question (03): Low-pass, high-pass, band-pass and band-rejection filtering [30 marks]

Filters are important building-blocks of communication systems.

(a) Recall that the transfer function of an ideal low-pass filter can be written as

$$H_{LP}(f) = \begin{cases} K\exp(-j2\pi f t_d), & |f| \leq B, \\ 0, & |f| > B. \end{cases}$$

(a.1) Find and sketch the amplitude and phase response of this ideal low-pass filter. [hint: sketch the phase response for $|f| \leq B$ as it can be arbitrary for $|f| > B$.]

(a.2) Show that the impulse response of the low-pass filer is given by

$$h_{LP}(t) = 2KB\,\mathrm{sinc}(2B(t - t_d)) \quad \text{for} \quad -\infty < t < \infty.$$

(a.3) Sketch the impulse response $h_{LP}(t)$.

(b) Let the input to this ideal low-pass be a rectangular pulse given by $x(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$.

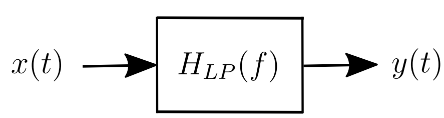


Figure 5: Low-pass filtering

(b.1) Let the time delay be zero (i.e., $t_d = 0$) in the aforementioned impulse response of the ideal low-pass filter. Show that the output signal of the low-pass filtering in Fig. 5 can be written as

$$y(t) = \frac{AK}{\pi}\left[\mathrm{Si}(2\pi B\,(t + T/2)) - \mathrm{Si}(2\pi B(t - T/2))\right],$$

where $\mathrm{Si}(x)$ is defined as the Sine Integral and given by

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(\lambda)}{\lambda} d\lambda.$$

(c) The impulse response of the ideal low-pass filter can be truncated to obtain a practically realizable causal function. The corresponding truncated impulse response can written as follows:

$$h(t) = \begin{cases} 2KB\ \mathrm{sinc}(2B(t-t_d)), & 0 < t < 2t_d \\ 0, & \text{elsewhere} \end{cases}$$

(c.1) Sketch this truncated impulse response $h(t)$.

(c.2) Show that the transfer function of this truncated impulse response is given by

$$H(f) = \frac{K}{\pi} \exp(-j2\pi f t_d) \left[\mathrm{Si}(2\pi\,(f+B)\,t_d) - \mathrm{Si}(2\pi(f-B)t_d)\right].$$

(d) The amplitude spectrum of an ideal high-pass filter is given by Fig. 6.

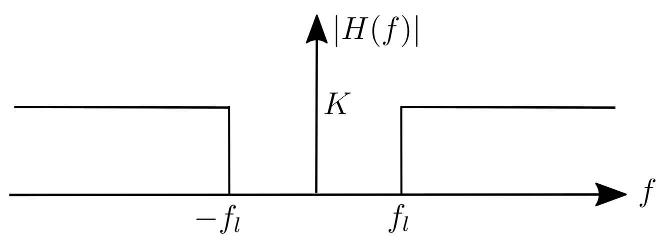


Figure 6: High-pass filtering

(d.1) Show that the transfer function of an ideal high-pass filter can be written as

$$H_{HP}(f) = K\exp(-j2\pi f t_d) - H_{LP}(f)$$

(d.2) Find and sketch the impulse response $h_{HP}(t)$ of this ideal high-pass filter.

(e) The amplitude spectrum of an ideal band-pass filter is given by Fig. 7.

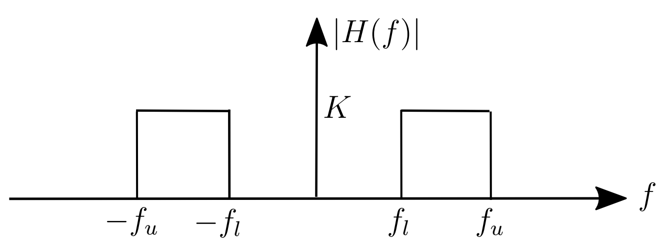


Figure 7: High-pass filtering

The transfer function of an ideal high-pass filter can be written as follows:

$$H_{BP}(f) = \begin{cases} K\exp(-j2\pi f t_d), & f_l \le |f| \le f_u \\ 0, & \text{elsewhere} \end{cases}$$

(e.1) Find and sketch the impulse response $h_{BP}(t)$ of this ideal band-pass filter.

(f) The amplitude spectrum of an ideal band-rejection filter is given by Fig. 8.

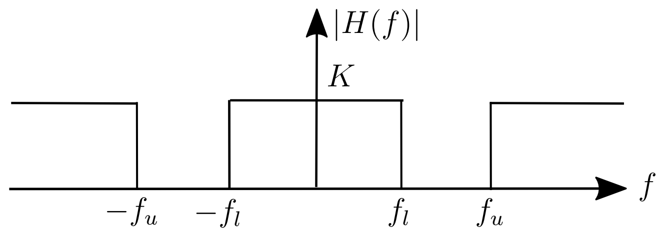


Figure 8: High-pass filtering

(f.1) Show that the transfer function of an ideal band-rejection filter can be written as

$$H_{BR}(f) = K \exp(-j2\pi f t_d) - H_{BP}(f)$$

(f.2) Find and sketch the impulse response $h_{BR}(t)$ of this ideal band-rejection filter.