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第四章 边界层
Chapter 4 Boundary Layers

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4.1 Introduction

Until the beginning of the twentieth century, analytical solutions of steady fluid flows were generally known for two typical situations.

One of these was that of parallel viscous flows and low Reynolds number flows, in which the nonlinear advective terms were zero and the balance of forces was that between pressure and viscous forces.

The second type of solution was that of inviscid flows around bodies of various shapes, in which the balance of forces was that between inertia and pressure forces.

4.1 Introduction

Although the equations of motion are nonlinear in this second situation, the velocity field can be determined by solving the linear Laplace equation.

These irrotational solutions predicted pressure forces on a streamlined body that agreed surprisingly well with experimental data for flow of fluids of small viscosity。

However, these solutions also predicted zero drag force and a nonzero tangential velocity at the body surface, features that did not agree with the experiments.

4.1 Introduction

In 1905 Ludwig Prandtl, an engineer by profession and therefore motivated to find realistic fields near bodies of various shapes, first hypothesized that,

for small viscosity, the viscous forces are negligible everywhere except close to solid boundaries where the noslip condition has to be satisfied.

The thickness of these boundary layers approaches zero as the viscosity goes to zero.

4.1 Introduction

Prandtl's hypothesis reconciled two rather contradictory facts.

It supported the intuitive idea that the effects of viscosity are indeed negligible in most of the flow field if ν is small, but it also accounted for drag by insisting that the no-slip condition must be satisfied at a solid surface, no matter how small the viscosity.

This reconciliation was Prandtl's aim, which he achieved brilliantly, and in such a simple way that it now seems strange that nobody before him thought of it.

4.1 Introduction

Since the time of Prandtl, the concept of the boundary layer has been generalized, and the mathematical techniques involved have been formalized, extended, and applied in other branches of physical science.

The concept of the boundary layer is considered a cornerstone in the intellectual foundation of fluid mechanics.

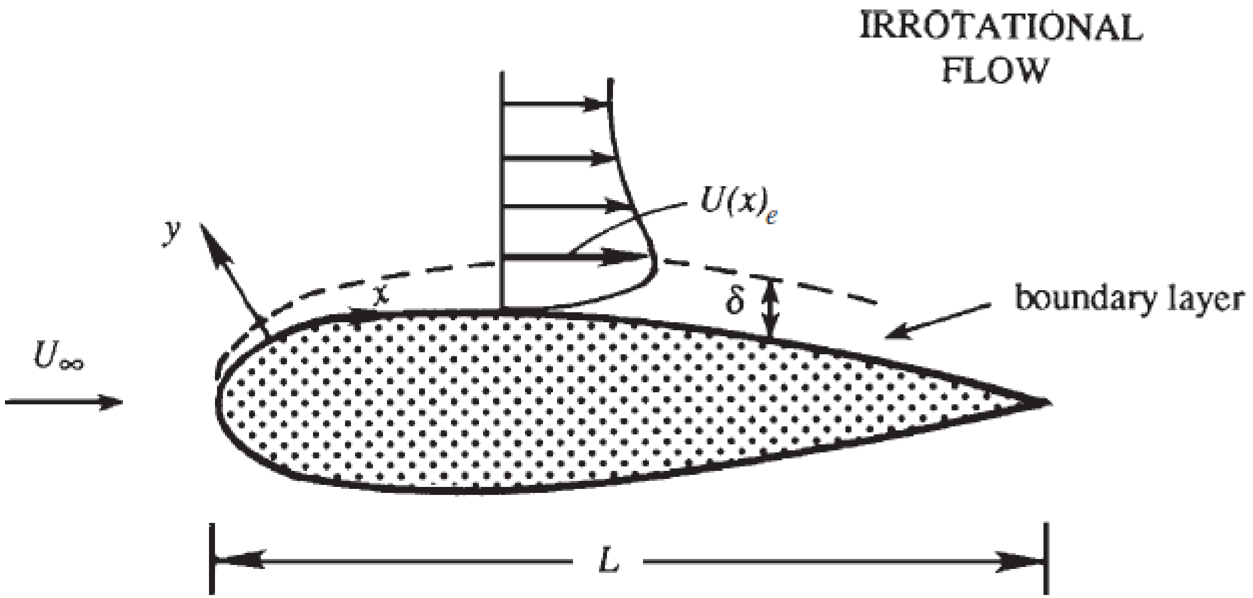
4.1 Introduction

The fundamental assumption of boundary-layer theory is that the layer is thin compared to other length scales such as the length of the surface or its local radius of curvature.

Across this thin layer, which can exist only in high Reynolds number flows, the velocity varies rapidly enough for viscous effects to be important.

4.1 Introduction

We shall refer to the solution of the irrotational flow outside the boundary layer as the ***outer*** problem and that of the boundary-layer flow as the ***inner*** problem.



4.1 Introduction

In summary, the simplifications of the boundary-layer assumption are as follows.

First, diffusion in the stream-wise direction is negligible compared to that in the wall normal direction.

Second, the pressure field can be found from the outer flow, so that it is regarded as a known quantity within the boundary layer.

Furthermore, a crude estimate of τ_0, the wall shear stress, can be made from the various scalings employed earlier:

$$\tau_0 \sim \mu U/\overline{\delta} \sim (\mu U/L)\mathrm{Re}^{1/2}$$

4.2 Boundary-layer Thickness

Since the fluid velocity in the boundary layer smoothly joins that of the outer flow, there is no obvious demarcation of the boundary layer's edge.

Thus, a variety of thickness definitions are used to define a boundary layer's character. The three most common thickness definitions are described here.

4.2 Boundary-layer Thickness

The first, δ_{99}, is an overall boundary-layer thickness that specifies the distance from the wall where the stream-wise velocity in the boundary layer is $0.99U_e$, where U_e is the local free-stream speed.

A second measure of the boundary-layer thickness, and one in which there is no arbitrariness, is the ***displacement thickness***, which is commonly denoted δ^*or δ_1. It is defined as the thickness of a layer of zero-velocity fluid that has the same velocity deficit as the actual boundary layer.

$$\int_{y=0}^{h} \underbrace{(U_e - u)}_{\text{velocity deficit}} dy = \int_{y=0}^{\delta^*} (U_e - 0) dy = U_e \delta^*, \text{ or } \delta^* = \int_{y=0}^{\infty} \left(1 - \frac{u}{U_e}\right) dy.$$

4.2 Boundary-layer Thickness

A third measure of the boundary-layer thickness is the ***momentum thickness*** θ or δ_2. It is defined such that $\rho U^2\theta$ is the momentum loss in the actual flow because of the presence of the boundary layer.

$$\theta = \int_{y=0}^{\infty} \frac{u}{U_e}\left(1 - \frac{u}{U_e}\right) dy$$

4.3 Transition

The process of changing from laminar to turbulent flow is called ***transition***, and it occurs in a wide variety of flows as the Reynolds number increases.

Interestingly for a high Reynolds number theory, the agreement of solutions to the laminar boundary equations with experimental data breaks down when the downstream-distance based Reynolds number Re_x is larger than some critical value, say Re_{cr} , that depends on fluctuations in the free stream above the boundary layer and on the surface shape, curvature, roughness, vibrations, and pressure gradient.

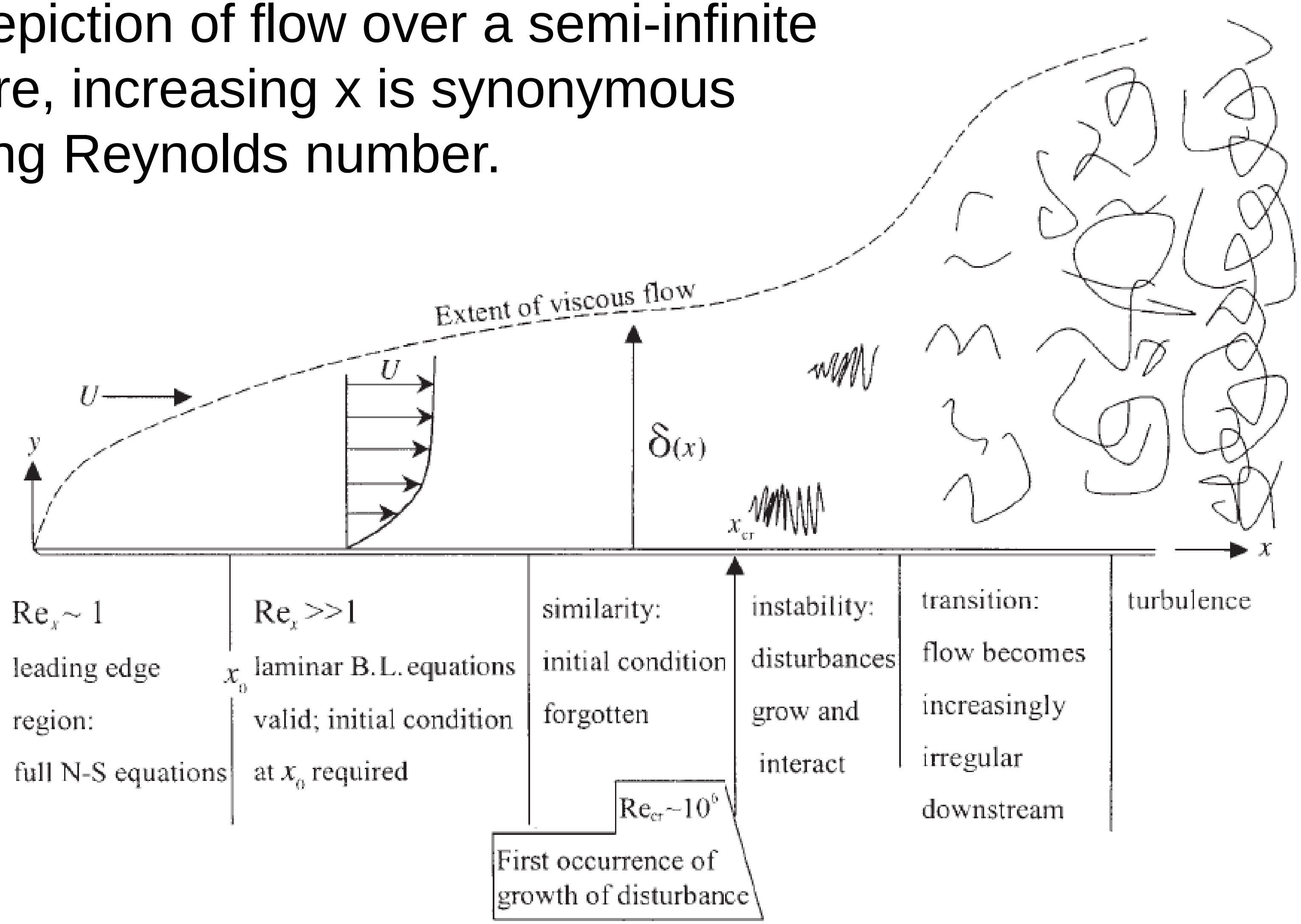
4.3 Transition

Above Re_{cr}, a laminar boundary-layer flow becomes unstable and transitions to turbulence.

Typically, the critical Reynolds number decreases when the surface roughness or free-stream fluctuation levels increase.

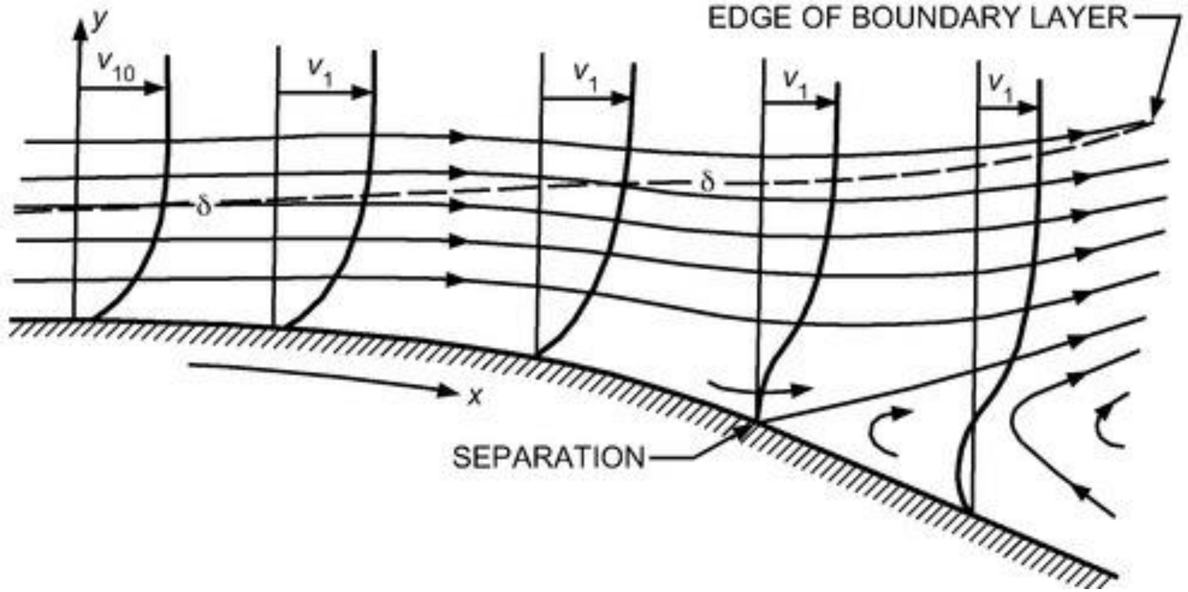
4.3 Transition

Schematic depiction of flow over a semi-infinite flat plate. Here, increasing x is synonymous with increasing Reynolds number.



4.4 Boundary-layer Separation

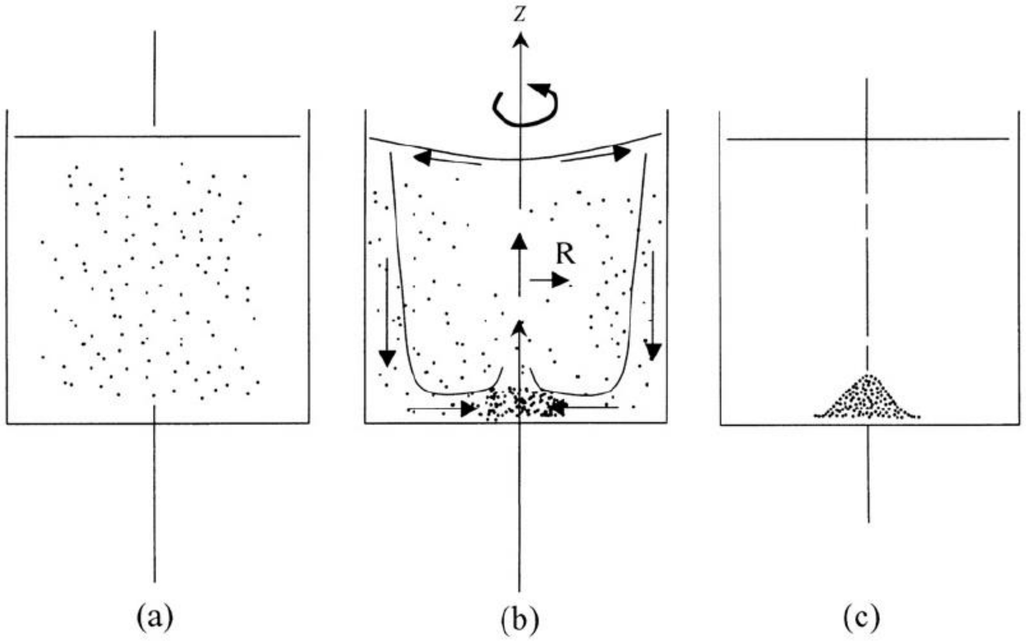
Boundary-layer separation occurs when the surface shear stress, τ_0, produced by the boundary layer vanishes and reverse (or upstream-directed) flow occurs near the surface.



4.5 Secondary Flows

Large Reynolds number flows with curved streamlines tend to generate additional velocity components because of the properties of boundary layers. These additional components are commonly called ***secondary flows***.

An example of such a flow is made dramatically visible by randomly dispersing finely crushed tea leaves into a cup of water, and then stirring vigorously in a circular motion.



4.5 Secondary Flows in Open Channel Flow