

Intermediate Microeconomics

EXCHANGE AND EFFICIENCY

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A pure exchange model economy

The only kind of agent in this model is the consumer—there are no firms that engage in production.

The consumer is described completely by his preferences for the goods and his endowment of the goods.

For simplicity we consider a case where there are:

- exactly 2 consumers (“Joey” and “Carrie”) and
- 2 goods (“Capri Sun” and “Sandwiches”).

Edgeworth box

A rectangular space with dimensions:

$$\begin{aligned} & [\text{Total endowment of Capri Sun}] \\ & \times [\text{Total endowment of sandwiches}]. \end{aligned}$$

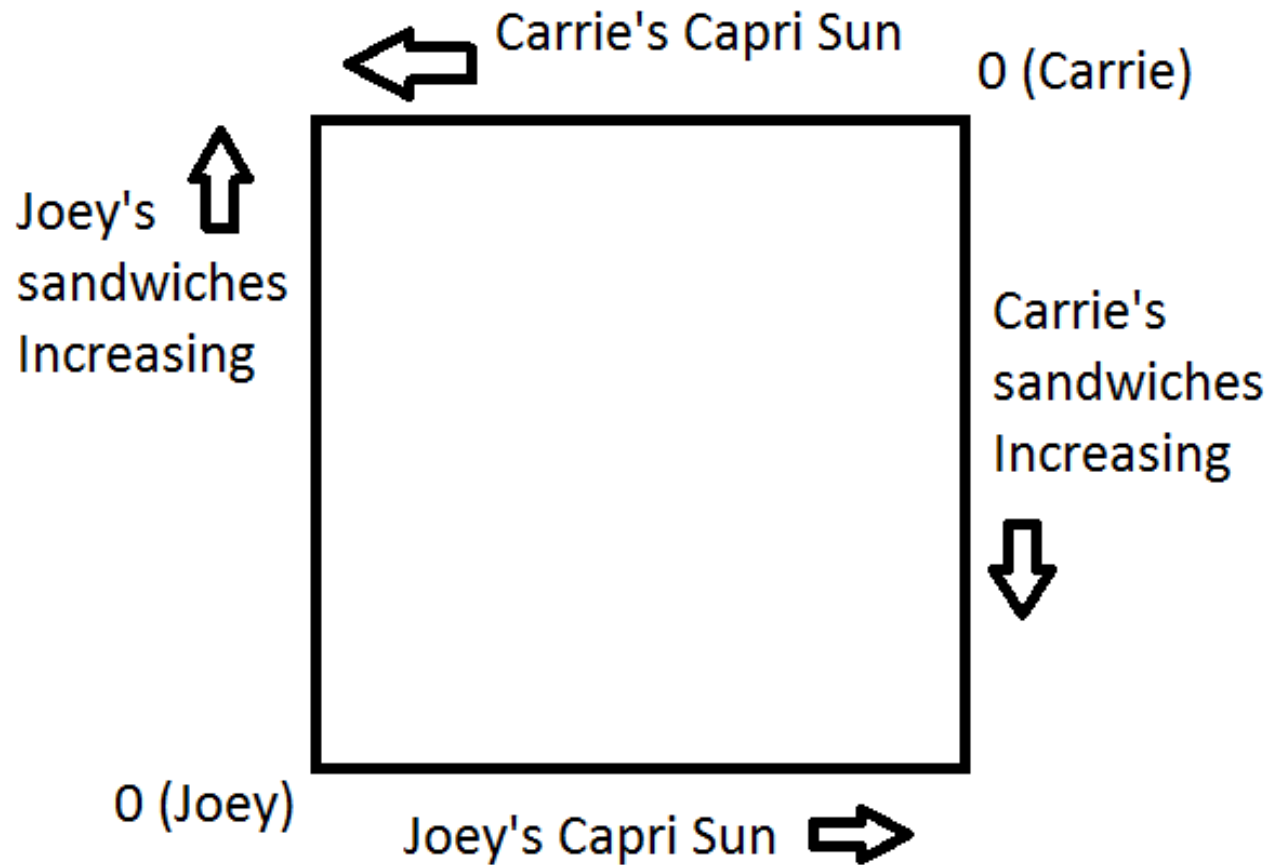
Each consumer has a different origin at which his consumption bundle is $[0, 0]$.

The origin for Joey is the lower left corner; the origin for Carrie is the upper right corner.

Any point in the Edgeworth Box denotes 4 things:

1. How much Capri Sun Joey consumes.
2. How much Capri Sun Carrie Consumes.
3. How many sandwiches Joey consumes.
4. How many sandwiches Carrie consumes.

Edgeworth box graphically



Allocation

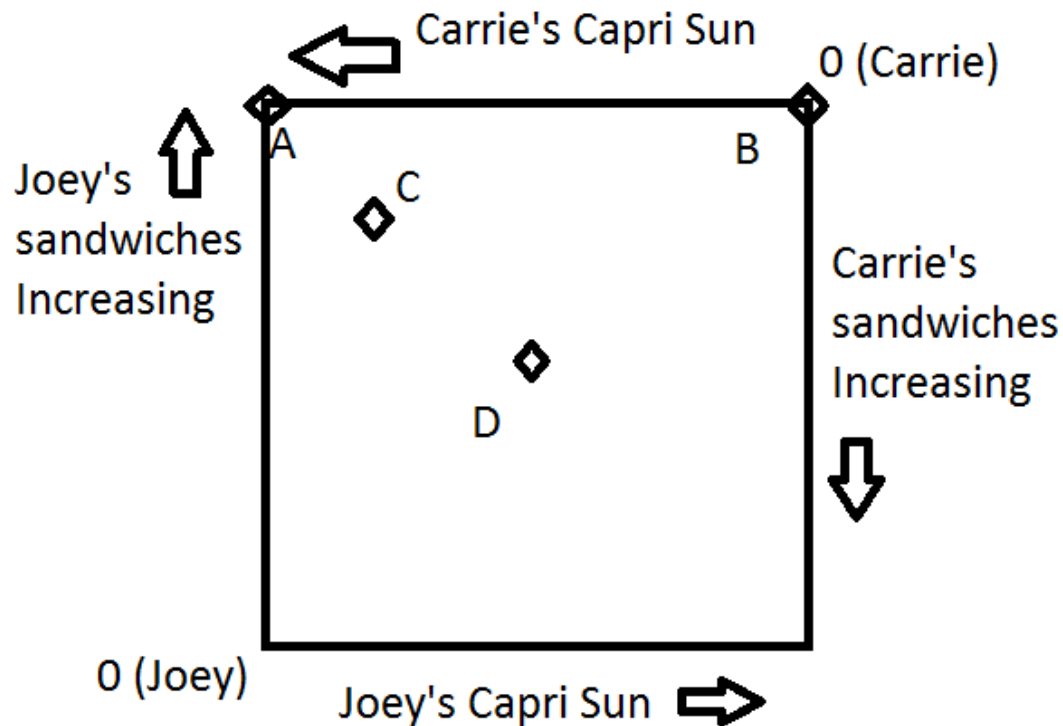
A point in the Edgeworth Box tells you how much of each good each consumer gets.

Any point in the Edgeworth Box is called an allocation.

Allocation: a collection of consumption bundles (one per consumer) describing what each agent holds.

- Example: “Joey gets 2 sandwiches and 2 Capri Suns; Carrie gets 1 sandwich and 3 Capri Suns” is one of many possible allocations.

Allocations graphically



Consider an economy where the total endowments of Capri Sun and sandwiches are each equal to 4.

Points A through D indicate four possible allocations of the endowment.

Allocations (interpretation)

Point A is an allocation in which Joey eats all the sandwiches and Carrie drinks all the Capri Sun.

Point B is an allocation in which Joey eats all the sandwiches *and* Capri Sun, leaving Carrie with nothing.

At point C, both consumers consume some of each good, but it is still skewed in favor of Joey eating sandwiches and Carrie drinking Capri Sun, e.g., Joey consumes $[3,1]$ and Carrie consumes $[1,3]$.

At point D, both consumers consume an equal quantity of both goods: Joey consumes $[2,2]$ and so does Carrie.

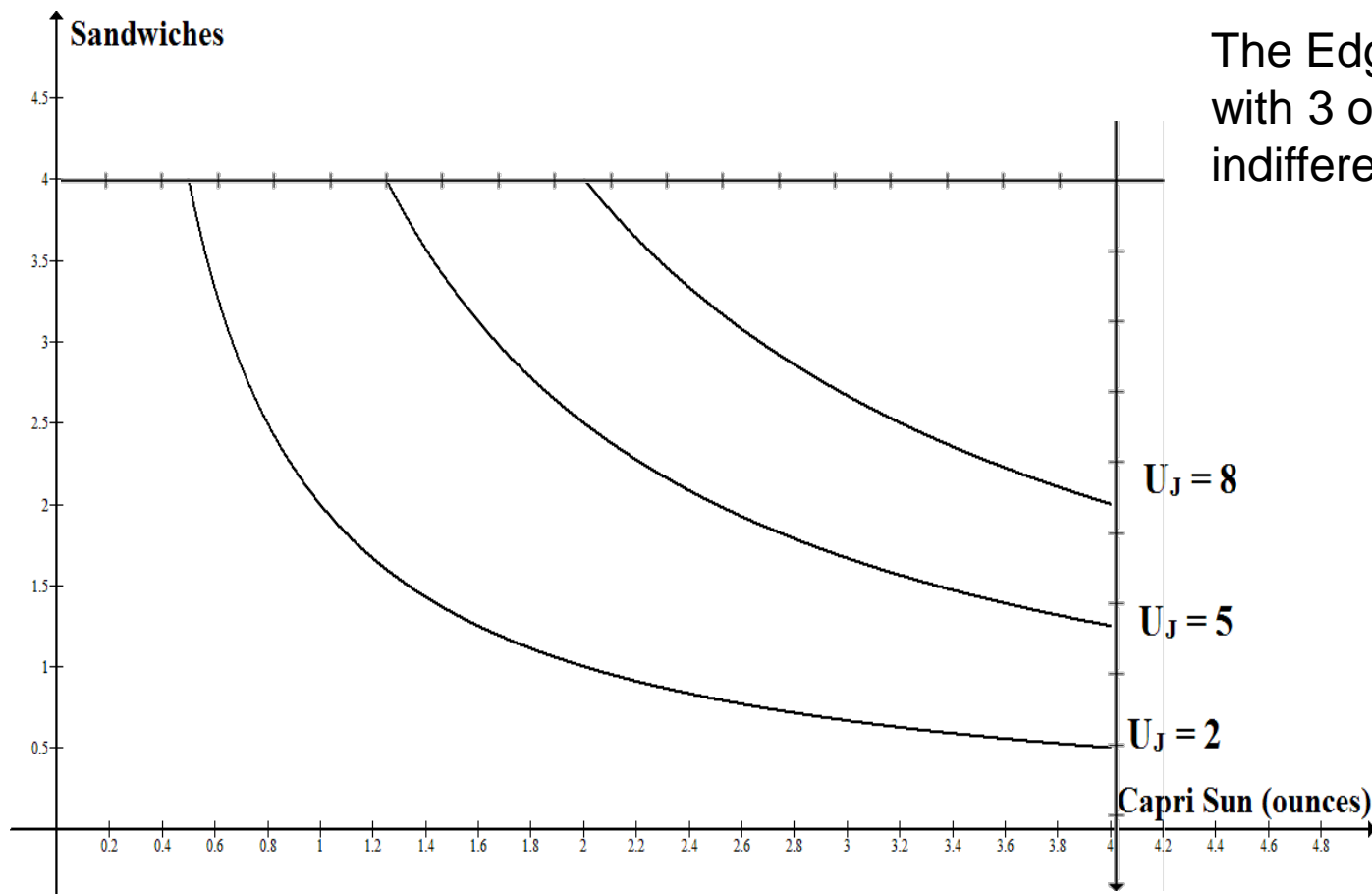
Edgeworth box and utility

Notice that the “space” created by an Edgeworth Box is the same as that created by a map of indifference curves (see previous notes on utility).

- When both axes on a graph represent quantities of different goods, we call the area goods space.
- An Edgeworth Box and an indifference curve map are both drawn in goods space.

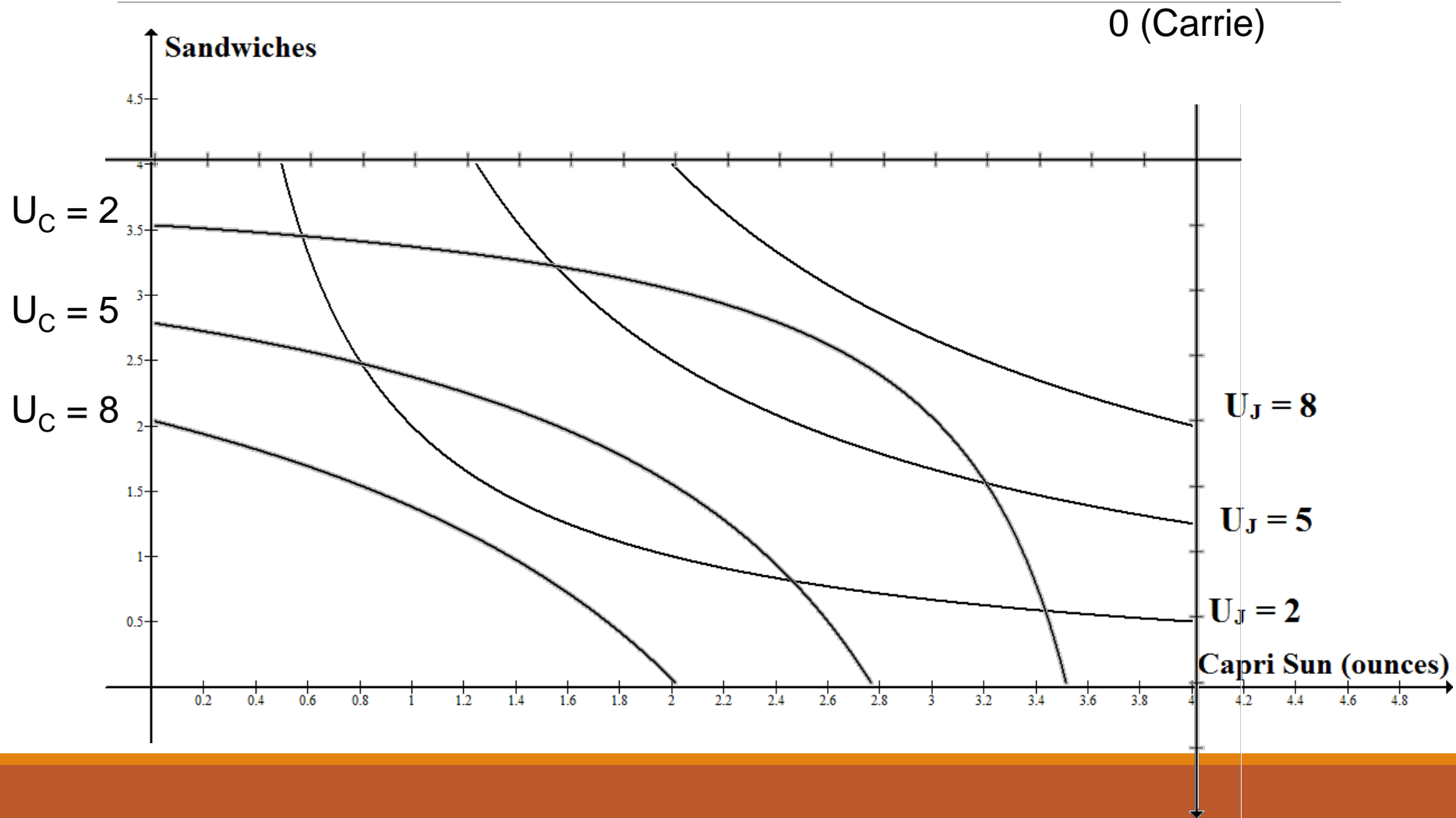
When this is the case, it is possible to graph indifference curves in the Edgeworth Box.

Preferences in the Edgeworth box



The Edgeworth Box
with 3 of Joey's
indifference curves.

Both consumers' preferences



The initial endowment and trade

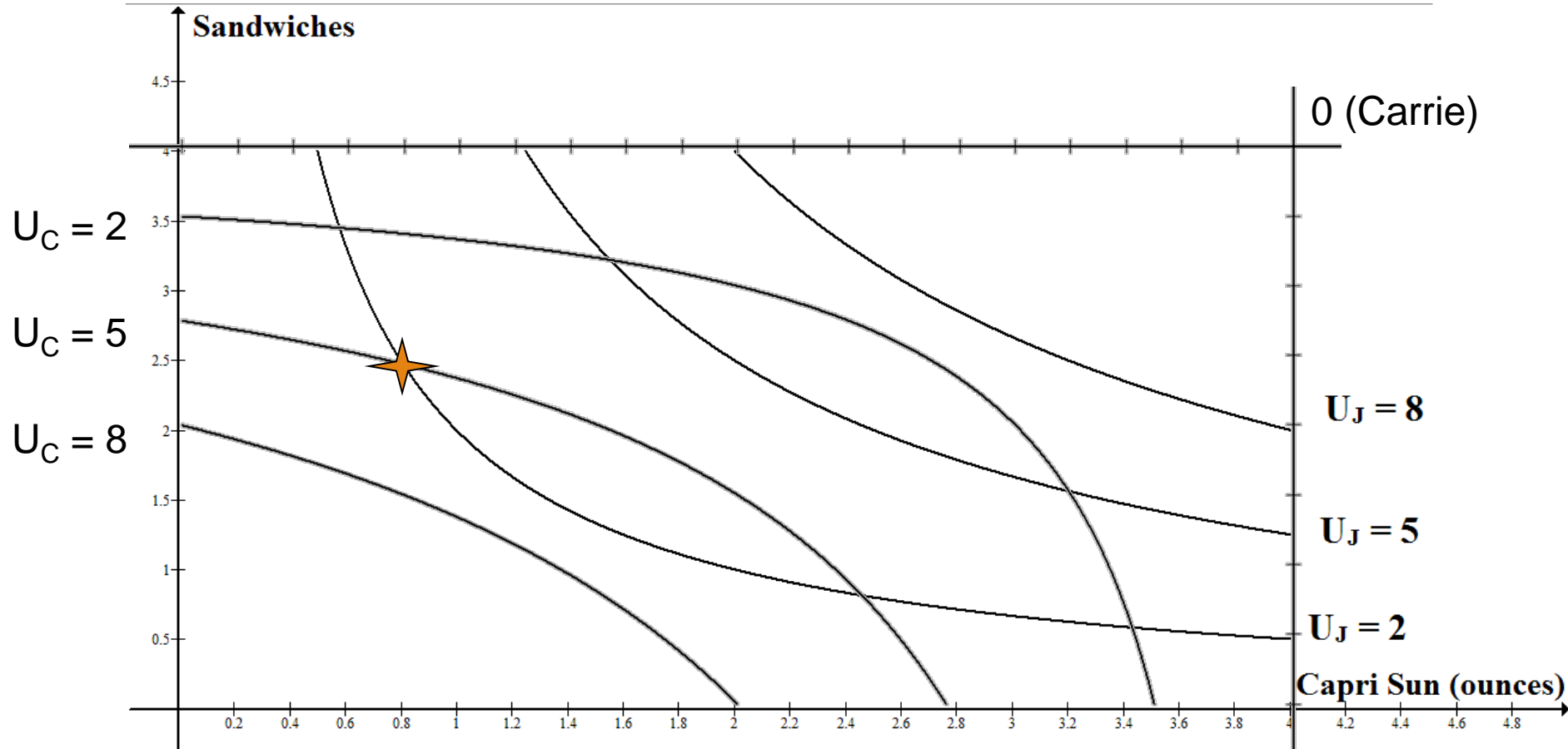
Any point in the Edgeworth Box can conceivably be an initial endowment:

- the quantity of each good each consumer has before trade.

When the consumers are allowed to trade, it is possible that both will attain a higher level of utility than when they are not allowed to trade.

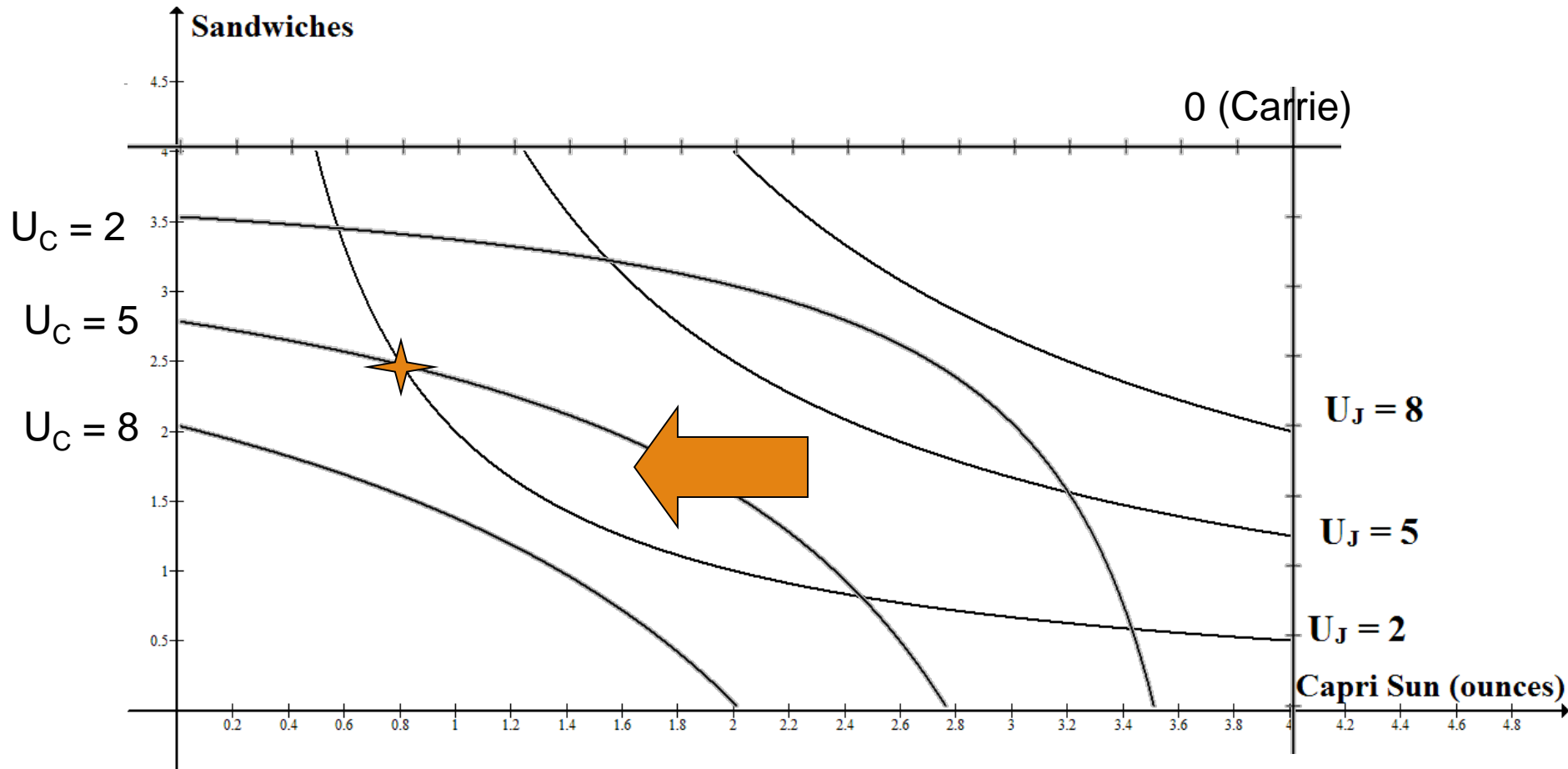
. . . so what will the allocation be when the consumers are allowed to trade?

Endowment graphically



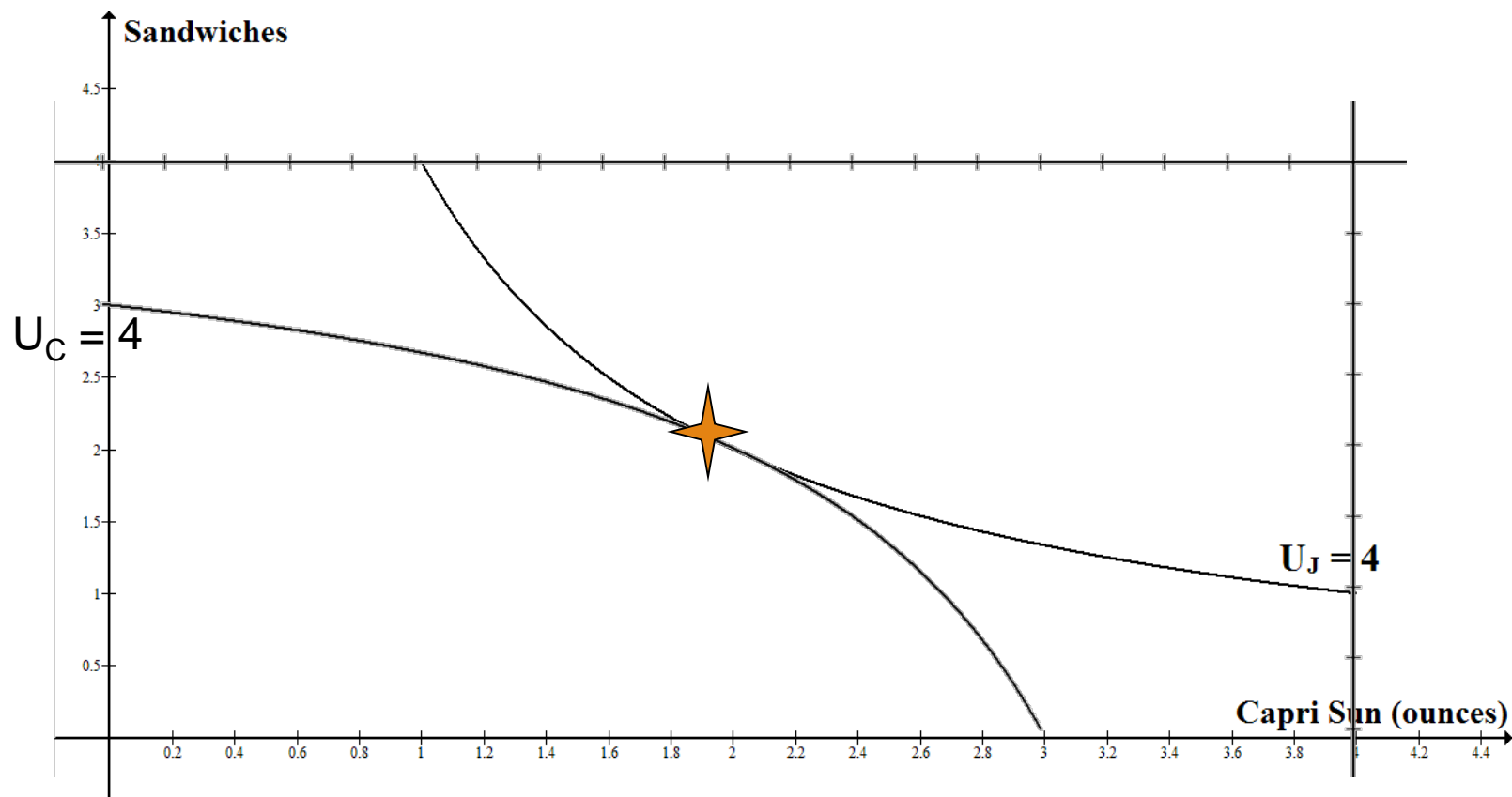
The star is the initial endowment; Joe has utility 2, and Carrie has utility 5.

Possibilities for voluntary trade



The area between the indifference curves $U_J = 2$ and $U_C = 5$ are allocations where both consumers have higher utility than they do initially.

Equilibrium in the exchange economy



The star is one allocation where both consumers' utilities are maximized.

Equilibrium

A condition in which all agents are making optimal consumption decisions at the same time.

In the exchange economy, it is sufficient to have an allocation in which it is impossible to increase one consumer's utility without decreasing the other consumer's utility.

- This occurs graphically wherever the consumers' indifference curves are tangent, as they are in the previous slide.
- The previous example is an equilibrium with both consumers getting 2 of each good.

Pareto efficiency

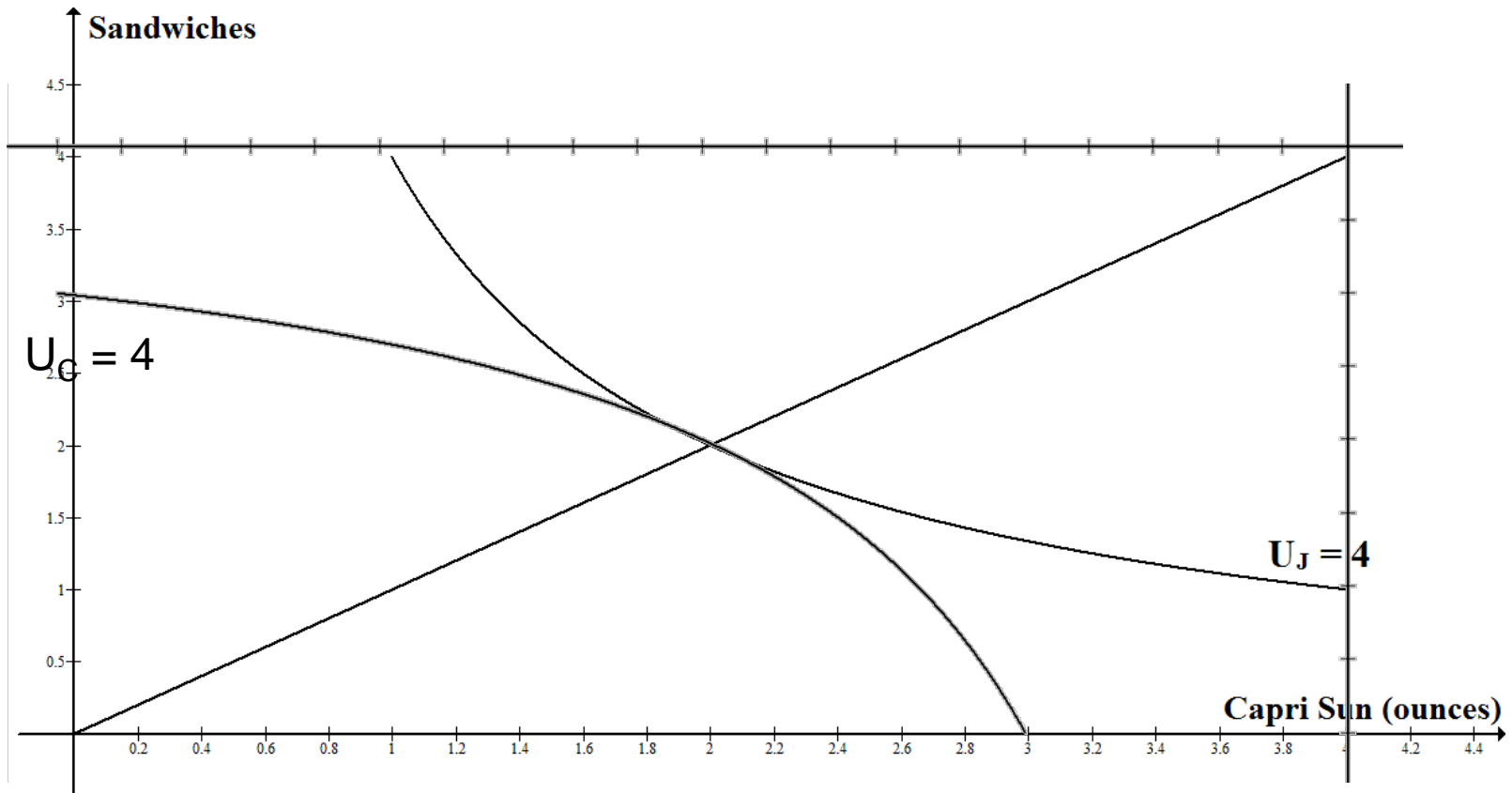
An allocation is Pareto efficient if there is no other allocation that would give all agents higher utility.

All points of tangency between the consumers' indifference curves in the Edgeworth Box (equilibria) are Pareto efficient.

The condition of equilibrium results from the exhaustion of all mutually beneficial trades.

- e.g., Joey and Carrie will continue trading with one another until it is impossible to reach a trade to which both parties agree.

The set of efficient allocations



The diagonal line is the set of efficient allocations for these consumers.

Efficiency and equity

Notice that there are highly unequal outcomes that are still Pareto efficient.

- e.g., a single consumer getting all the goods is still Pareto efficient.
- This notion of Pareto efficiency is not a restrictive criterion.

The initial endowment determines which of the Pareto efficient allocations will be the equilibrium.

The price ratio

What ratio of prices will move the exchange economy from its initial endowment point to equilibrium?

- i.e., what will be the equilibrium price of Capri Sun and what will be the equilibrium price of sandwiches?

To answer this, we will use a specific example of utility functions and endowments:

- Joe has utility function $U_J(X_J, Y_J) = X_J^{2/5}Y_J^{3/5}$
- Carrie has utility function $U_C(X_C, Y_C) = X_C^{1/2}Y_C^{1/2}$
- Joe is endowed with 1 Capri Sun and 3 sandwiches.
- Carrie is endowed with 3 Capri Suns and 1 sandwich.

The initial endowment

In this example, Joe's endowment is the set (1, 3) and Carrie's is the set (3,1).

Total endowment is 4 of each good:

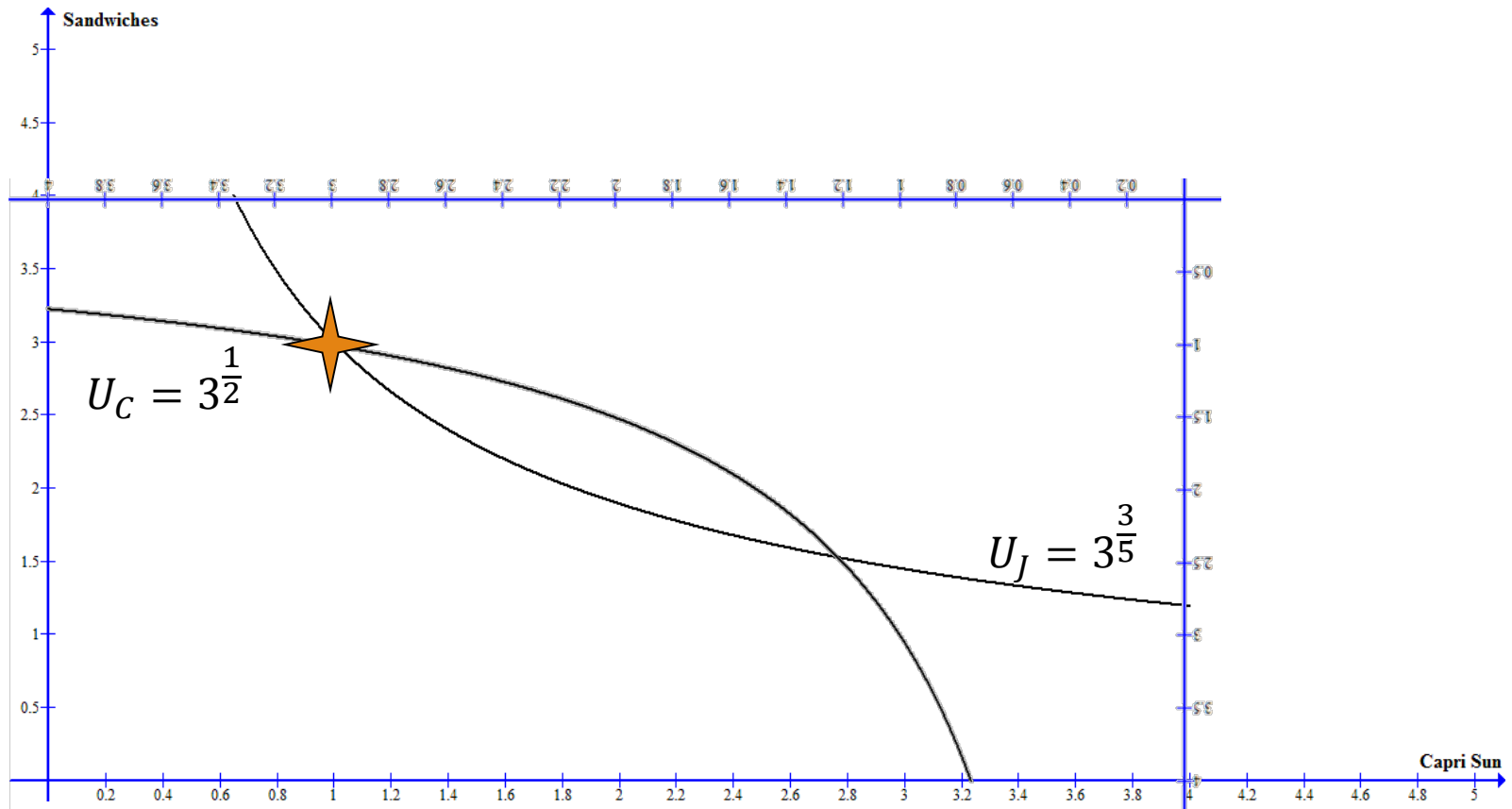
- Joe's endowment of Capri Sun plus Carrie's endowment equals 4.
- Joe's endowment of sandwiches plus Carrie's endowment equals 4.

This yields the utility levels,

$$U_J = 1^{\frac{2}{5}} 3^{\frac{3}{5}} = 3^{\frac{3}{5}}$$
$$U_C = 3^{\frac{1}{2}} 1^{\frac{1}{2}} = 3^{\frac{1}{2}}$$

The corresponding indifference curves are shown on the next slide.

The initial endowment



The initial endowment, and the initial indifference curves for each consumer.

What is the Pareto set?

All the allocations at which the two consumers have tangent indifference curves, i.e., identical MRS, are Pareto efficient.

Joe's marginal utilities:

$$\frac{\partial U_J}{\partial X_J} = \frac{2}{5} X_J^{-\frac{3}{5}} Y_J^{\frac{3}{5}}, \frac{\partial U_J}{\partial Y_J} = \frac{3}{5} X_J^{\frac{2}{5}} Y_J^{-\frac{2}{5}}$$

Carrie's marginal utilities:

$$\frac{\partial U_C}{\partial X_C} = \frac{1}{2} X_C^{-\frac{1}{2}} Y_C^{\frac{1}{2}}, \frac{\partial U_C}{\partial Y_C} = \frac{1}{2} X_C^{\frac{1}{2}} Y_C^{-\frac{1}{2}}$$

The ratio of each consumer's marginal utilities gives his/her MRS:

$$\frac{\partial Y_J}{\partial X_J} = \frac{2Y_J}{3X_J}, \text{ and } \frac{\partial Y_C}{\partial X_C} = \frac{Y_C}{X_C}.$$

Pareto set

Setting the MRS for both consumers equal yields:

$$\frac{Y_C}{X_C} = \frac{2Y_J}{3X_J}.$$

Unfortunately this equality has 4 variables in it . . . but we can use the endowments to make two substitutions:

$$X_C = 4 - X_J \text{ and } Y_C = 4 - Y_J.$$

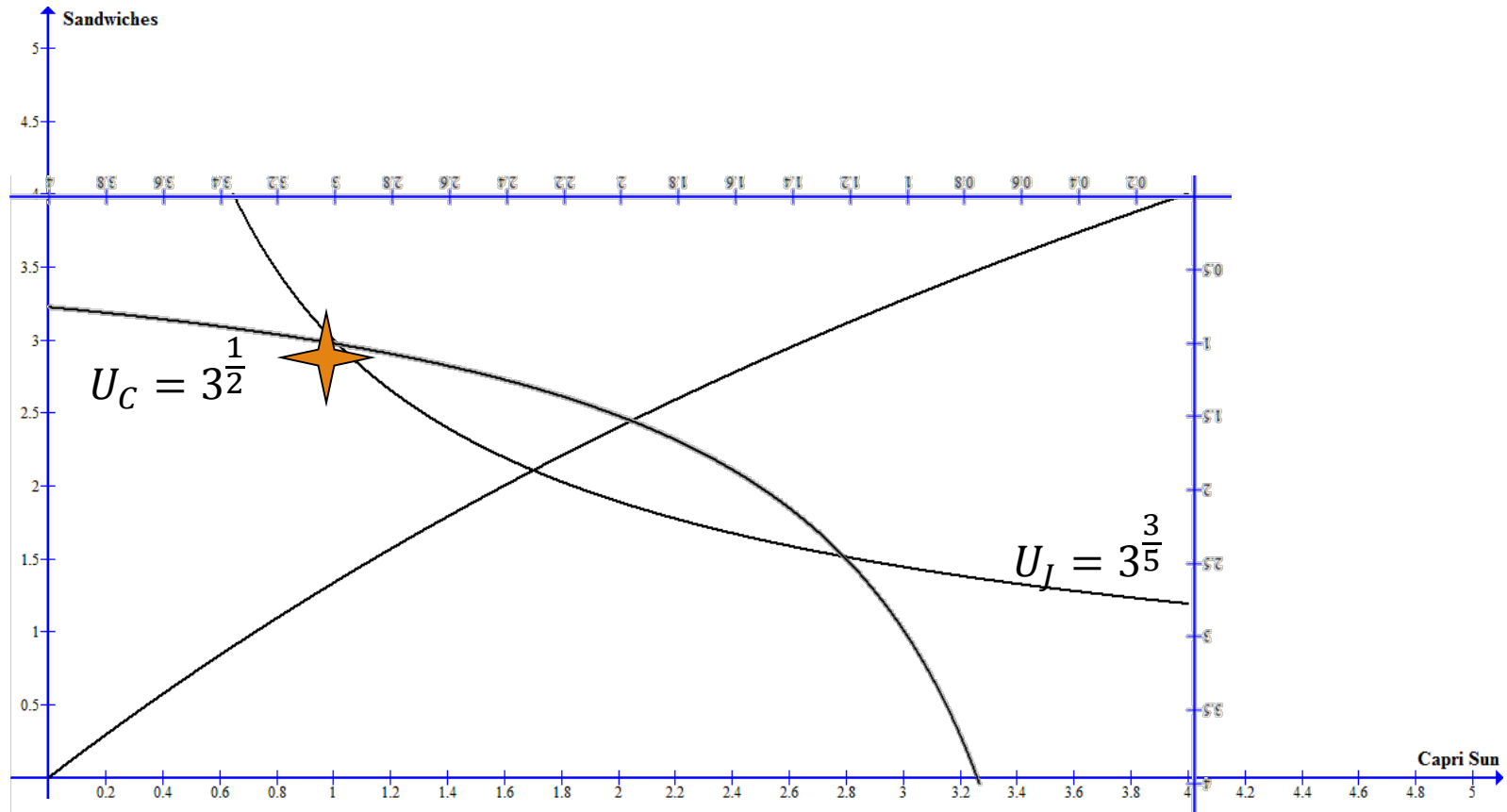
Now the Pareto set satisfies :

$$\frac{4 - Y_J}{4 - X_J} = \frac{2Y_J}{3X_J}$$

Rearranging algebraically enables you to write the Pareto condition in $Y_J = f(X_J)$ form:

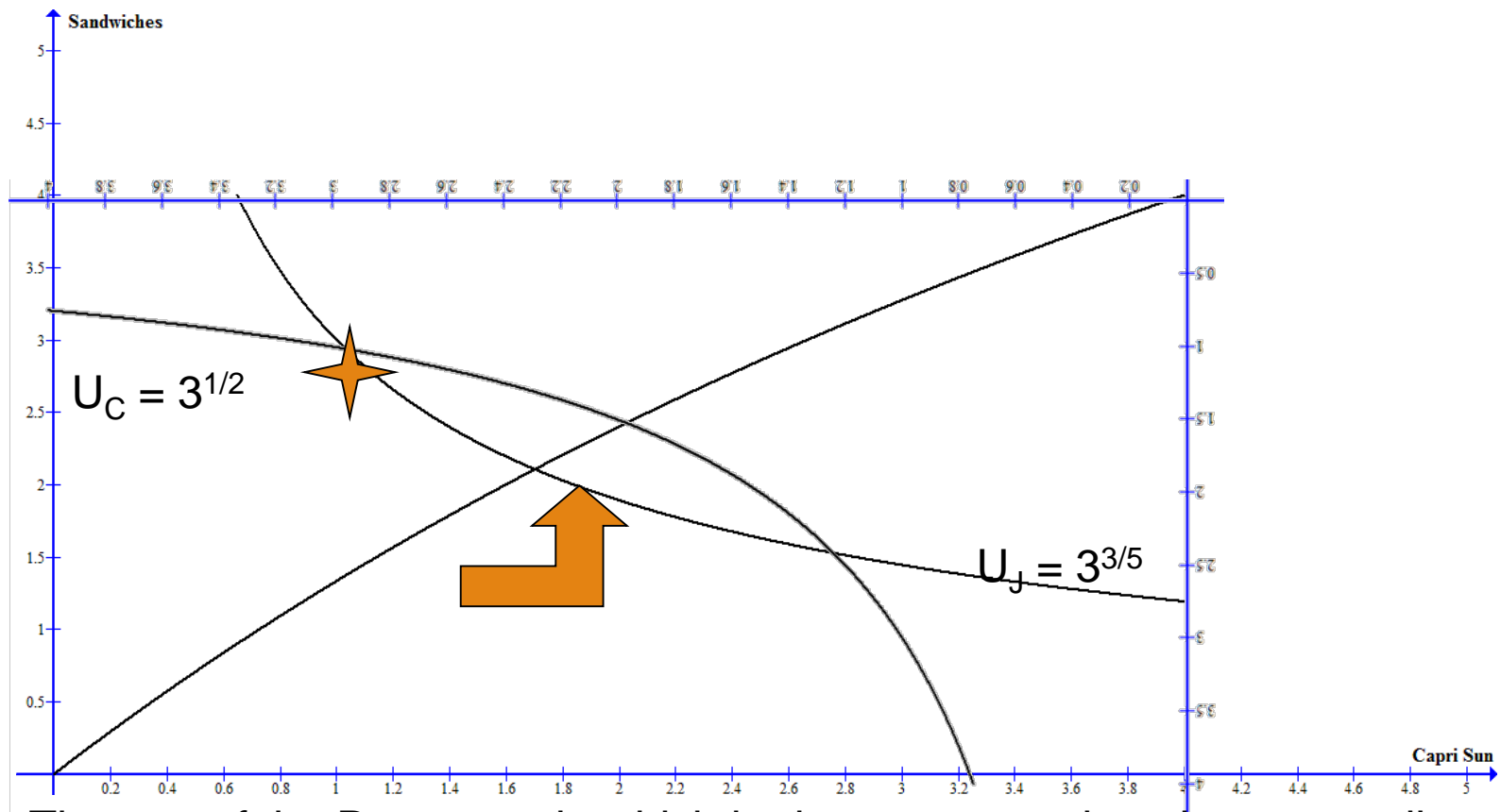
$$Y_J = \frac{12X_J}{8 + X_J}.$$

The Pareto set



The diagonal connecting both origins is the set of Pareto efficient allocations.

The contract curve



The part of the Pareto set in which both consumers do at least as well as their initial endowments—which will contain the equilibrium.

The budget constraints

Each consumer has a budget constraint of the form:

$$\text{Income} = \text{Spending on } X + \text{Spending on } Y.$$

Spending is quantity consumed times price:

$$\text{Joe's Income} = P_X X_J + P_Y Y_J \text{ and}$$

$$\text{Carrie's Income} = P_X X_C + P_Y Y_C.$$

But they get their income from their endowments:

$$P_X W_{XJ} + P_Y W_{YJ} = P_X X_J + P_Y Y_J \text{ and}$$

$$P_X W_{XC} + P_Y W_{YC} = P_X X_C + P_Y Y_C.$$

We have to find the prices that will maximize the consumers' utilities subject to these constraints.

We will only be able to define the price *ratio*, however, so we will designate one of the prices (P_Y) as the *numeraire* and set it equal to 1.

Maximizing utility

Each consumer's utility is maximized when his/her MRS equals the price ratio:

$$\begin{aligned} \text{(With numeraire } P_Y = 1) \quad \frac{P_X}{1} &= MRS_J = MRS_C \\ \frac{P_X}{1} &= \frac{2Y_J}{3X_J} \\ \frac{P_X}{1} &= \frac{Y_C}{X_C}. \end{aligned}$$

These conditions enable us to write the consumers' optimal choices of Y as a function of their choices of X:

$$Y_J = \frac{3X_J P_X}{2} \text{ and } Y_C = X_C P_X.$$

Optimal consumption

Substituting the two conditions from the previous slide into the budget constraints for each consumer:

$$P_X W_{XJ} + P_Y W_{YJ} = P_X X_J + P_Y \left(\frac{3X_J P_X}{2} \right) \text{ and}$$

$$P_X W_{XC} + P_Y W_{YC} = P_X X_C + P_Y (X_C P_X)$$

We can also substitute each consumer's endowments:

$$P_X 1 + P_Y 3 = P_X X_J + P_Y \left(\frac{3X_J P_X}{2} \right) \text{ and}$$

$$P_X 3 + P_Y 1 = P_X X_C + P_Y (X_C P_X).$$

Optimal consumption (cont'd)

Finally recalling that $P_Y = 1$ as numeraire:

$$P_X + 3 = P_X X_J + \left(\frac{3X_J P_X}{2} \right) \text{ and}$$
$$3P_X + 1 = P_X X_C + (X_C P_X).$$

Now we can write each consumer's consumption of X as a function of the price of X.

Equilibrium price of X

From the previous slide we have:

$$P_X + 3 = P_X X_J + \left(\frac{3X_J P_X}{2} \right) \text{ and} \\ 3P_X + 1 = P_X X_C + (X_C P_X).$$

Re-writing each expression for X yields:

$$X_J = \frac{P_X + 3}{\frac{5}{2}P_X} = \frac{2P_X + 6}{5P_X} \text{ and } X_C = \frac{3P_X + 1}{2P_X}$$

Finally the sum of the two consumers' X equals the endowment: $4 = X_J + X_C$.

$$\frac{2P_X + 6}{5P_X} + \frac{3P_X + 1}{2P_X} = 4$$

Equilibrium price of X (cont'd)

$$\frac{2P_X + 6}{5P_X} + \frac{3P_X + 1}{2P_X} = 4$$

Solving this expression for P_X gives us $P_X^* = \frac{51}{63} \approx 0.810$, the equilibrium price of X.

Equilibrium allocation

When we substitute the equilibrium price of X into each consumer's budget constraint, you get each consumer's optimal consumption:

$$X_J^* = \frac{2}{5} + \frac{6}{5} \left(\frac{63}{51} \right) \approx 1.882 \text{ and}$$

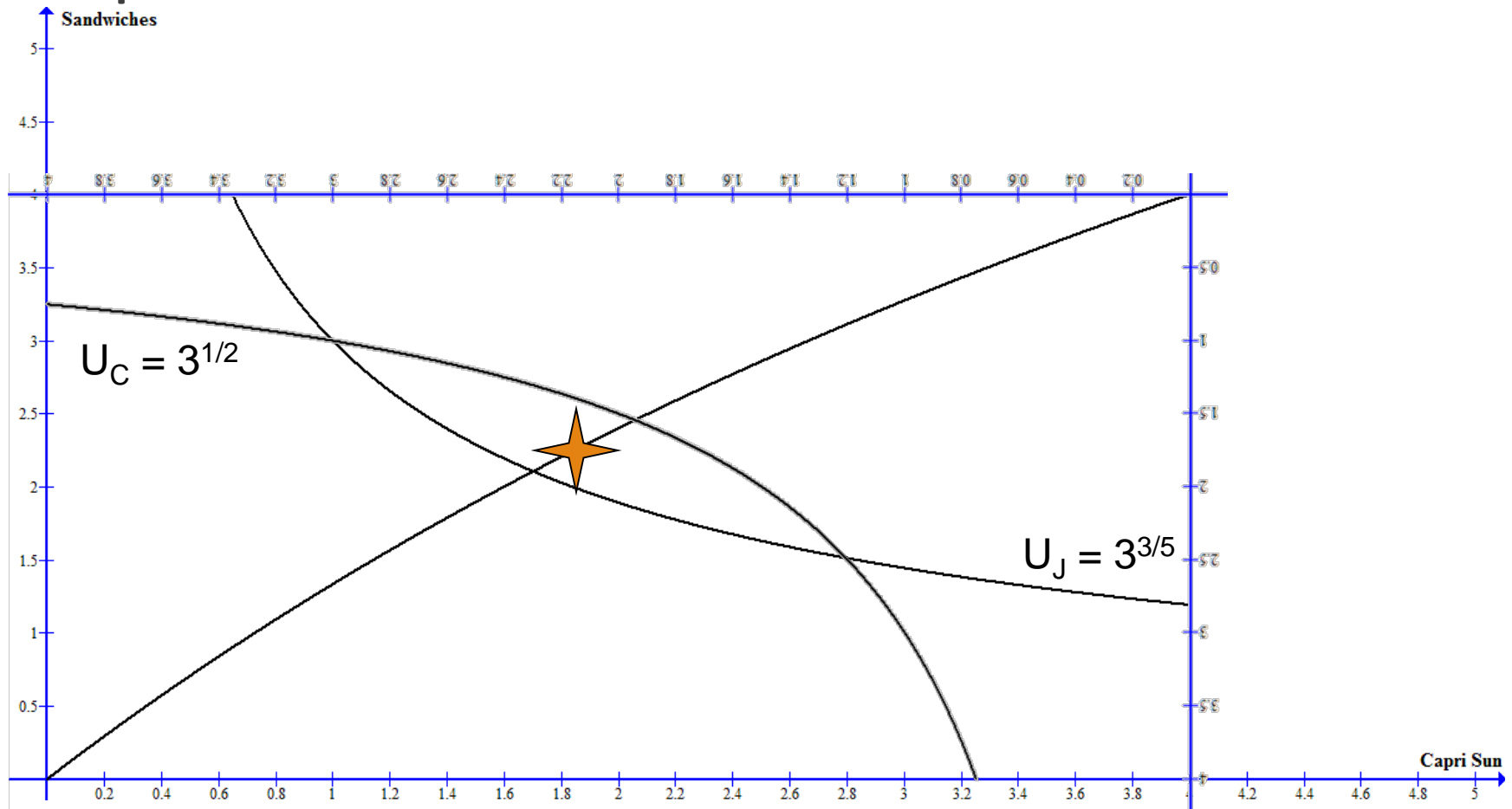
$$X_C^* = \frac{3}{2} + \frac{1}{2} \left(\frac{63}{51} \right) \approx 2.118.$$

Finally you can get each consumer's consumption of Y from the MRS:

$$Y_J^* = \frac{3}{2} * (1.882) \left(\frac{51}{63} \right) \approx 2.285 \text{ and}$$

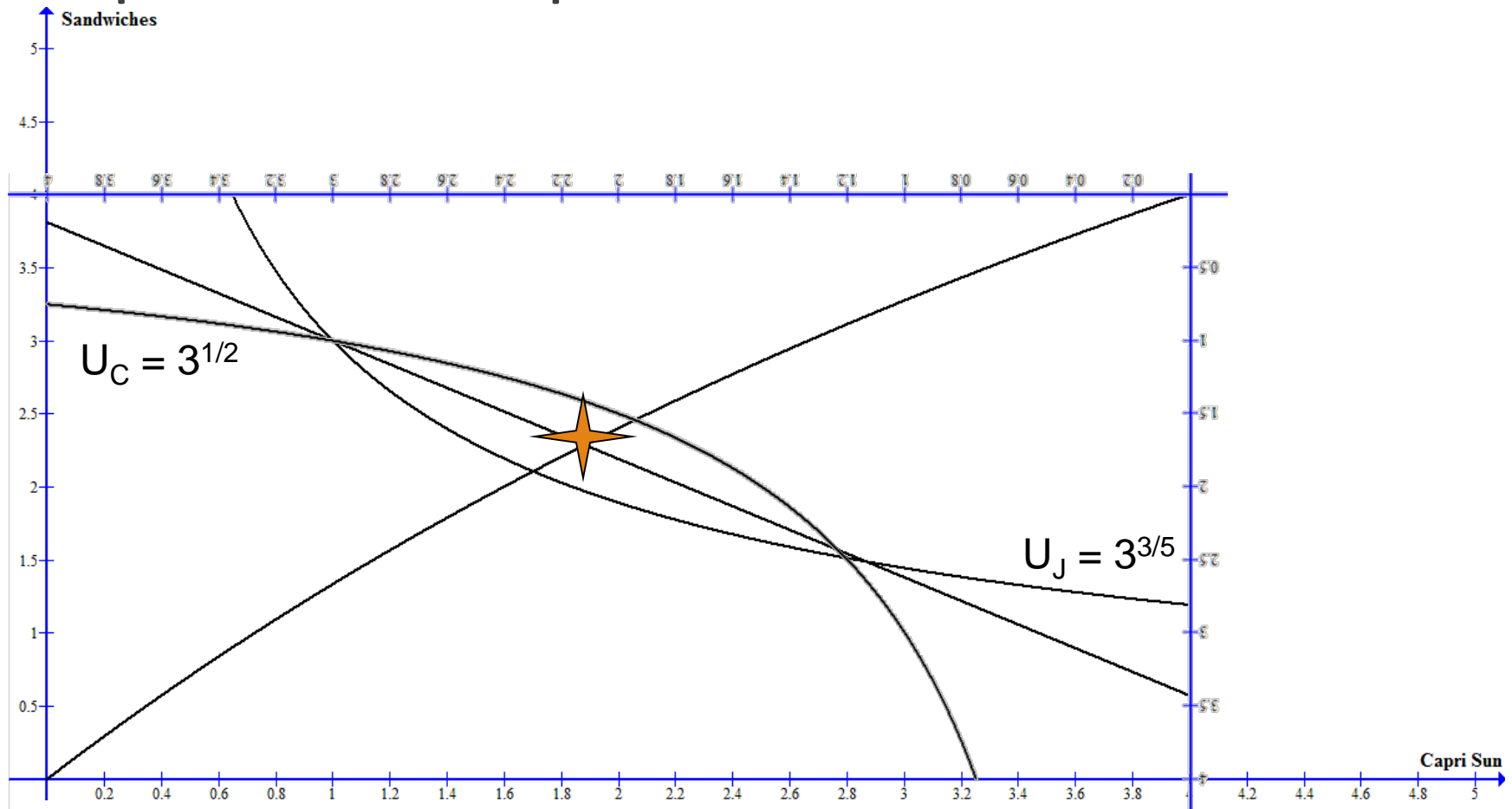
$$Y_C^* = 2.118 * \left(\frac{51}{63} \right) \approx 1.715.$$

Equilibrium allocation



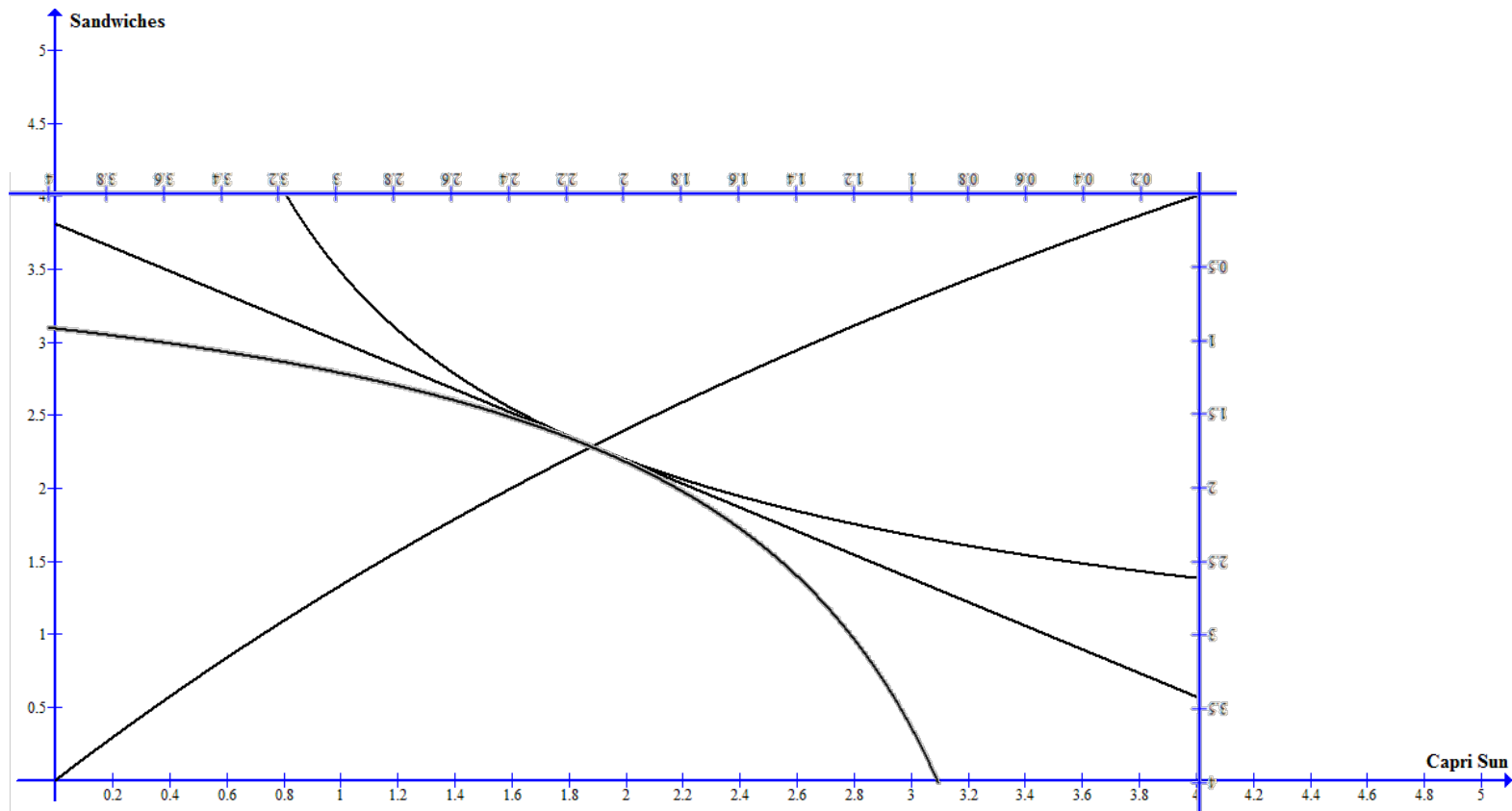
Equilibrium with $X_J^* = 1.882$, $X_C^* = 2.118$, $Y_J^* = 2.285$, $Y_C^* = 1.715$.

Equilibrium price



The equilibrium price ratio goes through the initial endowment and the equilibrium allocation.

Equilibrium



At equilibrium, the price ratio and both consumers' MRS are all equal.

“Supply and Demand”

In equilibrium, Joe is a “buyer” of Capri Sun and a “seller” of sandwiches:

- He consumes 1.882 Capri Sun but was only endowed with 1, so he “purchases” 0.882 units from Carrie.

Carrie is a “buyer” of sandwiches and a “seller” of Capri Sun:

- She consumes 1.715 sandwiches though she was only endowed with 1, so she purchases 0.715 units from Joe.

The price of Capri Sun in terms of sandwiches is $\frac{0.715}{0.882} \approx 0.81$ sandwiches.

- This is the same as the P_X (51/63) we found earlier.

Gains from trade

Finally note that in equilibrium with trade, both consumers are better off than initially.

- Joe has utility $U_J = 1.882^{\frac{2}{5}} 2.285^{3/5} \approx 2.114$, which is greater than his initial $3^{\frac{3}{5}}$ (≈ 1.993).
- Carrie has utility $U_C = 2.118^{\frac{1}{2}} 1.715^{\frac{1}{2}} \approx 1.906$, which is greater than her initial $3^{\frac{1}{2}}$ (≈ 1.732).

Summary

This example has demonstrated the features of general equilibrium using two consumers and two goods.

- It illustrates how consumers maximize utility under a budget constraint.
- It illustrates how consumers' preferences and endowments allocate goods through markets
- It illustrates that the market allocations are Pareto efficient (cannot improve one's welfare without making someone else worse off).

Notes on Hayek "The Use of Knowledge in Society" (AER 1945)

I. Maximizing Social Welfare (sum of individuals' utilities)
is easy if :

- A. Possess all information,
- B. Individuals preferences are given,
- C. Technology for all production is known.

Then allocate / produce goods according to :

$$MRS_{ij} = \frac{P_i}{P_j} \quad \forall \underbrace{i \text{ and } j}_{\text{goods}}$$

II. The "easy" allocation decision fails because, though all this information exists, it is not given to any one agent in totality, i.e., no "benevolent dictator." How to overcome this problem of communicating information to other agents who must act on it?

- A. Central planning?
- B. De-centralized planning?

Which is more efficient / better at using the relevant knowledge?

III. "Knowledge" is specific to time and place: everyone has some niche where ~~he~~ is more familiar with available ~~means~~ resources and potential uses than other agents.

A. How to go about "publicizing" this specific information?

1. Practical to invest in cataloging and accumulating it all in a centralized "encyclopedia"?

a. Then make all the decisions based on looking up info. in the encyclopedia ...

2. Depends on how often changes to the specifics occur.

a. } Frequently changing → a lot of wasted effort, time spent revising the encyclopedia; is it ever accurate and for how long?

b. } ~~Never~~ Never changing → a case for central planning

3. It is almost surely the former: dynamic, evolving, too volatile to accurately and thoroughly keep in one volume. Too dynamic, fast-changing to act intelligently upon, except by the "man on the spot" (p. 524).

(iii)

~~10~~ B. The "man on the spot" needs information from others, cannot make intelligent decisions purely on his own specific info.

1. How much info. does he need? Answer: he only needs to know ~~how much~~ ^{how much} more/less urgently needed are the resources at his disposal/needs for production.

~~20~~ a. "Why" is not important to get him to act.

II Prices communicate this to ~~him~~ the "man on the spot."

A. A price increase does not communicate to the buyer why he ought to economize on a good—merely that the good has become more scarce and some of its marginal uses must be foregone/substitutes found.

1. It could be a supply decrease or a demand increase, but the effect on an individual buyer is unimportant.

B. Prices convey information very economically (efficiently). They tell agents what they need to know—and only what they need to know to allocate resources to their most important uses (again, from the social welfare notion of "important").

C. "The whole acts as one market, not because any of its members surveys the whole field, but because their limited individual ~~interests~~ fields of vision sufficiently overlap so that through many intermediaries the relevant information is communicated to all." —p. 526.

D. The price system's real function is communicating information.

E. Crucially, observe how little each agent must know in order to act in the society's interest \approx "the Invisible Hand" of A. Smith.

1. "The marvel is that in a case like that of a scarcity of one raw material, without an order being issued, without more than perhaps a handful of people knowing the cause, tens of thousands of people whose identity could not be ascertained by months of investigation, are made to use ~~the~~ the material or its products more sparingly, i.e., they move in the right direction."

- p. 527

IV. Conclusion: the price system \leftarrow efficiently! communicates information, enabling the coordination of many agents' decisions about allocating resources. It overcomes the problem of ~~decent~~ decentralized, specific information.

A. The decisions it enables have produced division of labor and advances in the efficient use of resources which have made modern civilization possible.