

5. Explain how measurement affects the quantum state and probability outcomes. Also explain the measurement basis conversion.

* Measurement collapses the state into $|i\rangle$ with the probability $|\langle i|\psi\rangle|^2$.

* After measurement, state becomes $|i\rangle$.

* Measurement destroys the superposition.
 $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

Let $i=0$, $\Rightarrow |i\rangle = |0\rangle$ then the probability of getting $|0\rangle$ is

$$\Rightarrow |\langle 0|\psi\rangle|^2 = |\langle 0|\alpha|0\rangle + \beta|1\rangle|^2 = |\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle|^2$$

$$\langle 0|0\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \quad \Rightarrow \quad \langle 0|1\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow |\langle 0|\psi\rangle|^2 = |\alpha|1\rangle + \beta|0\rangle|^2 = |\alpha|^2$$

Let $i=1$, $|i\rangle = |1\rangle$ then the probability of getting $|1\rangle$ is $|\beta|^2$.

$$\Rightarrow |\langle 1|\psi\rangle|^2 = |\langle 1|\alpha|0\rangle + \beta|1\rangle|^2 = |\alpha\langle 1|0\rangle + \beta\langle 1|1\rangle|^2$$

$$\Rightarrow \langle 1|0\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 ; \quad \langle 1|1\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

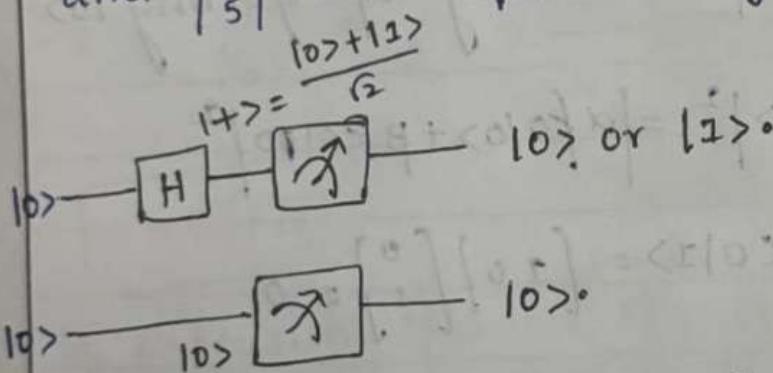
$$\Rightarrow |\alpha|0\rangle + \beta|1\rangle|^2 = |\beta|^2$$

Example:-

$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle - \boxed{\cancel{\chi}} - \left(\frac{3}{5}\right)^2$ is the probability of getting $|0\rangle$.

$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle - \boxed{\cancel{\chi}} - \left|\frac{4}{5}\right|^2$ is the probability of getting $|1\rangle$.

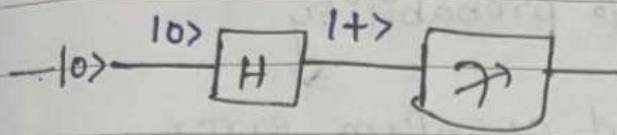
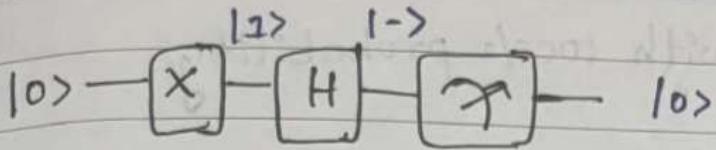
$\Rightarrow \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$ is destroyed to $|0\rangle$ or $|1\rangle$ after measurement with $\left(\frac{3}{5}\right)^2$ probability of getting $|0\rangle$ and $\left|\frac{4}{5}\right|^2$ is the probability of getting $|1\rangle$.



Possible states are $|0\rangle \otimes |0\rangle$ or $|1\rangle \otimes |0\rangle$.

$$\begin{aligned} &\Rightarrow \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \otimes |0\rangle &&\Rightarrow \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] \times |0\rangle \\ &= \left(\frac{|00\rangle + |10\rangle}{\sqrt{2}} \right). &&\Rightarrow \left[\frac{|00\rangle - |10\rangle}{\sqrt{2}} \right]. \end{aligned}$$

Anyway after measurement, post measurement state must be $|00\rangle$ or $|10\rangle$.



$$\Rightarrow 10 > \textcircled{X} 10 > \cdot$$

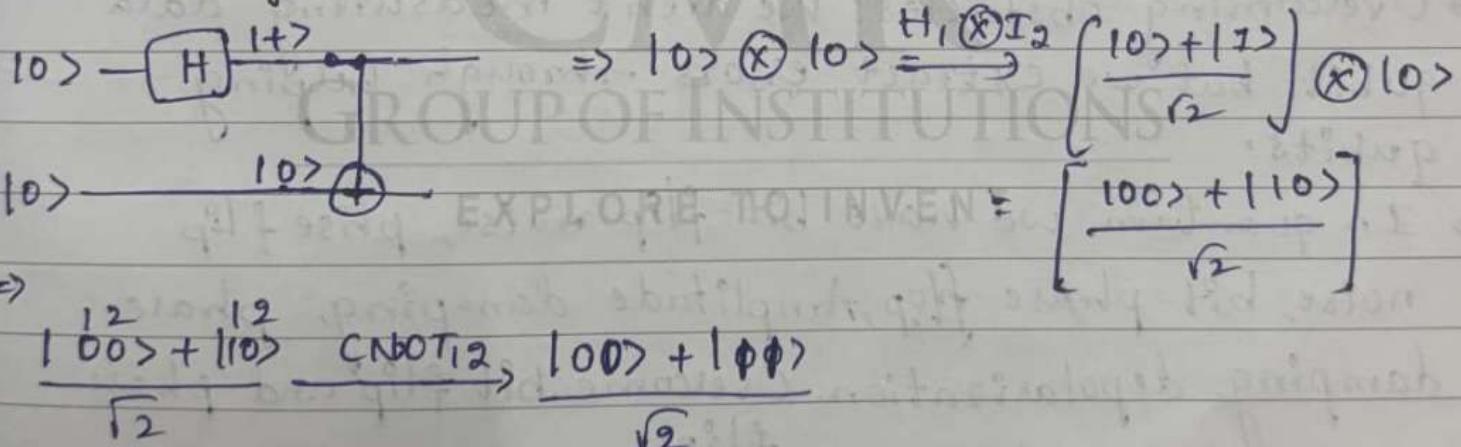
$\downarrow x_1 \textcircled{X} I_2$

$$\Rightarrow |1\rangle \otimes |0\rangle$$

$\downarrow H_1 \times H_2$

$$\Rightarrow \left[\frac{|10\rangle - |11\rangle}{\sqrt{2}} \right] \otimes \left[\frac{|10\rangle + |11\rangle}{\sqrt{2}} \right] = \left[\frac{|100\rangle - |111\rangle + |101\rangle - |110\rangle}{\sqrt{2}} \right].$$

After measurement, we can get $|00\rangle$ or $|01\rangle$ or $|10\rangle$ or $|11\rangle$.
Bell State generator circuit:



* This is called bell state/entangled pair.

$$\frac{100\rangle + 111\rangle}{\sqrt{2}} \rightarrow \boxed{\cancel{\chi}} \rightarrow \begin{matrix} 0 \\ 1 \end{matrix}$$

(or)

$$\rightarrow \boxed{\cancel{\chi}} \rightarrow \begin{matrix} 0 \\ 1 \end{matrix}$$