5. Explain how measurement affects the quantum state and probability outcomes. Also explain the measurement basis conversion.

* Measurement collapses the state into $|i\rangle$ with the probability $|\langle i|\psi\rangle|^2$.
* After measurement, state becomes $|i\rangle$.
* Measurement destorys the superposition.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Let $i=0$, $\Rightarrow |i\rangle = |0\rangle$ then the probability of getting $|0\rangle$ is

$$\Rightarrow |\langle 0|\psi\rangle|^2 = |\langle 0|\alpha|0\rangle + \beta|1\rangle|^2 = |\alpha\langle 0|0\rangle + \beta\langle 0|1\rangle|^2$$

$$\langle 0|0\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \Rightarrow \langle 0|1\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow |\langle 0|\psi\rangle|^2 = |\alpha|1\rangle + \beta|0\rangle|^2 = |\alpha|^2$$

Let $i=1$, $|i\rangle = |1\rangle$ then the probability of getting $|1\rangle$ is $|\beta|^2$

$$\Rightarrow |\langle 1|\psi\rangle|^2 = |\langle 1|\alpha|0\rangle + \beta|1\rangle|^2 = |\alpha\langle 1|0\rangle + \beta\langle 1|1\rangle|^2$$

$$\Rightarrow \langle 1|0\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 ; \quad [1\ 1] = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

$$\Rightarrow |\alpha|0\rangle + \beta|1\rangle|^2 = |\beta|^2$$

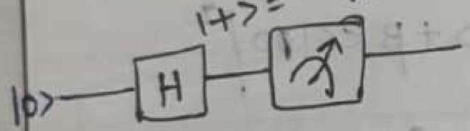
Example:-

$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$ — $\left(\frac{3}{5}\right)^2$ is the probability of getting $|0\rangle$.

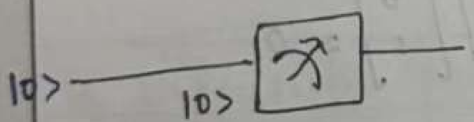
$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$ — $\left|\frac{4}{5}\right|^2$ is the probability of getting $|1\rangle$.

$\Rightarrow$ $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$ is destroyed to $|0\rangle$ or $|1\rangle$ after measurement with $\left(\frac{3}{5}\right)^2$ probability of getting $|0\rangle$ and $\left|\frac{4}{5}\right|^2$ is the probability of getting $|1\rangle$.

$|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$



$|0\rangle$ or $|1\rangle$.



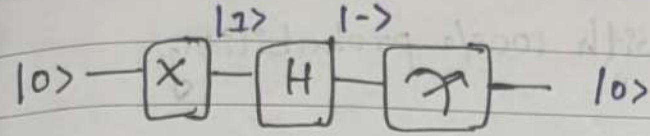
$|0\rangle$.

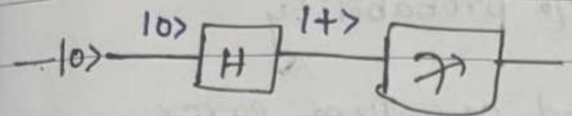
Possible states are $|0\rangle \otimes |0\rangle$ or $|1\rangle \otimes |0\rangle$.

$$\Rightarrow \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes |0\rangle \qquad \Rightarrow \left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right] \times |0\rangle$$

$$= \left(\frac{|00\rangle+|10\rangle}{\sqrt{2}}\right). \qquad \Rightarrow \left[\frac{|00\rangle-|10\rangle}{\sqrt{2}}\right].$$

Anyway after measurement, post measurement state must be $|00\rangle$ or $|10\rangle$.





$\Rightarrow$ $|0\rangle \otimes |0\rangle$

$\downarrow X_1 \otimes I_2$

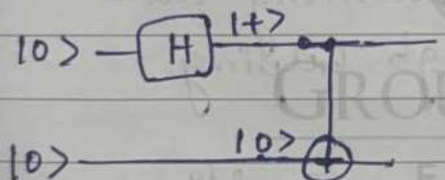
$\Rightarrow |1\rangle \otimes |0\rangle$

$\downarrow H_1 \otimes H_2$

$$\Rightarrow \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] \otimes \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right] = \left[\frac{|00\rangle - |11\rangle + |01\rangle - |10\rangle}{2}\right]$$

After measurement, we can get $|00\rangle$ or $|01\rangle$ or $|10\rangle$ or $|11\rangle$.

Bell State generator circuit:



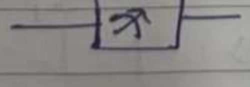
$$\Rightarrow |0\rangle \otimes |0\rangle \xrightarrow{H_1 \otimes I_2} \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right] \otimes |0\rangle = \left[\frac{|00\rangle + |10\rangle}{\sqrt{2}}\right]$$

$\Rightarrow$

$$\frac{|\overset{12}{00}\rangle + |\overset{12}{10}\rangle}{\sqrt{2}} \xrightarrow{CNOT_{12}} \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

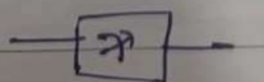
This is called bell state/entangled pair.

$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$



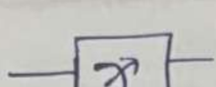
0 1

(or)

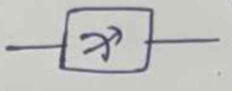


0 1.

$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$



then $|0\rangle$ with 100% probability.



if first, $|0\rangle$ with 50% probability