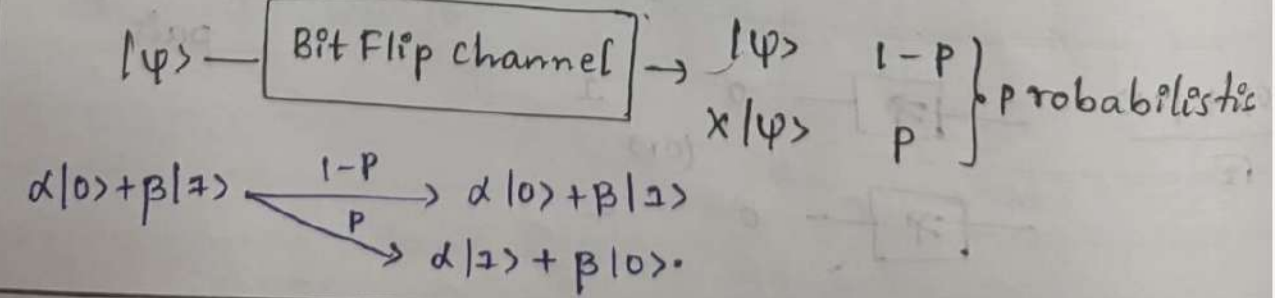
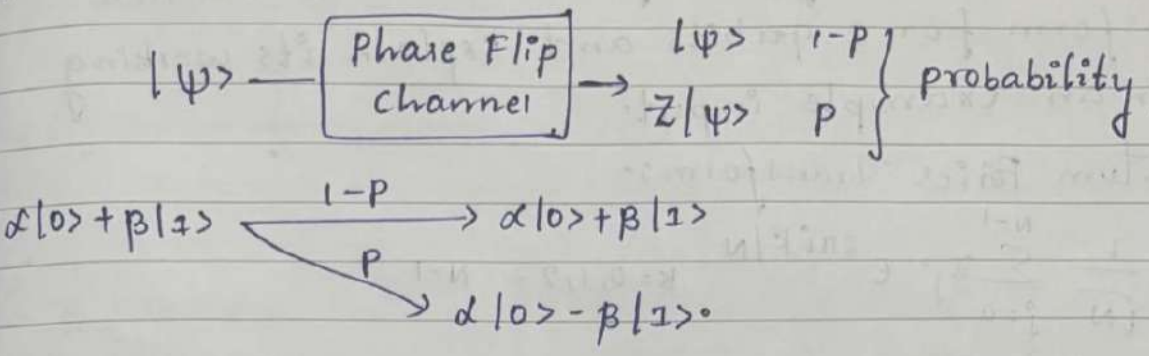
Bit-Flip Channel:



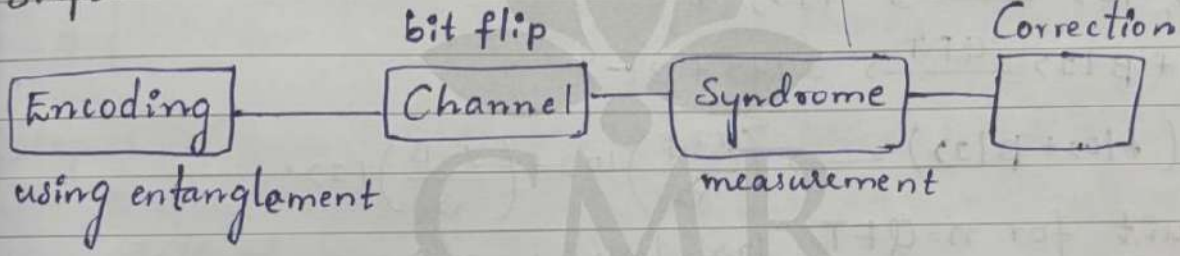
Ex. No. | Date | Page No.

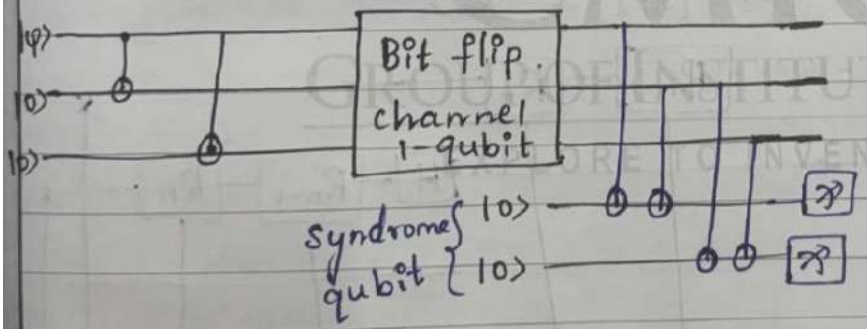
Phase Flip channel:



* So the goal of Quantum Error correction is to detect and correct the errors by using more no. of qubits.

Steps:





$\alpha|000\rangle_{123} + \beta|111\rangle_{123}$.

Case (i). Error on 1st qubit:

$\alpha|000\rangle+\beta|011\rangle$.

..after channel

$(\alpha|000\rangle+\beta|011\rangle)|00\rangle \Rightarrow \alpha|000000\rangle+\beta|011100\rangle$

$\xrightarrow{CX_{14}}$ $\Rightarrow \alpha|00000\rangle + \beta|01100\rangle$

$\xrightarrow{CX_{24}}$ $\Rightarrow \alpha|00010\rangle + \beta|01110\rangle$.

$\xrightarrow{CX_{25}}$ $\Rightarrow \alpha|00000\rangle + \beta|01111\rangle$.

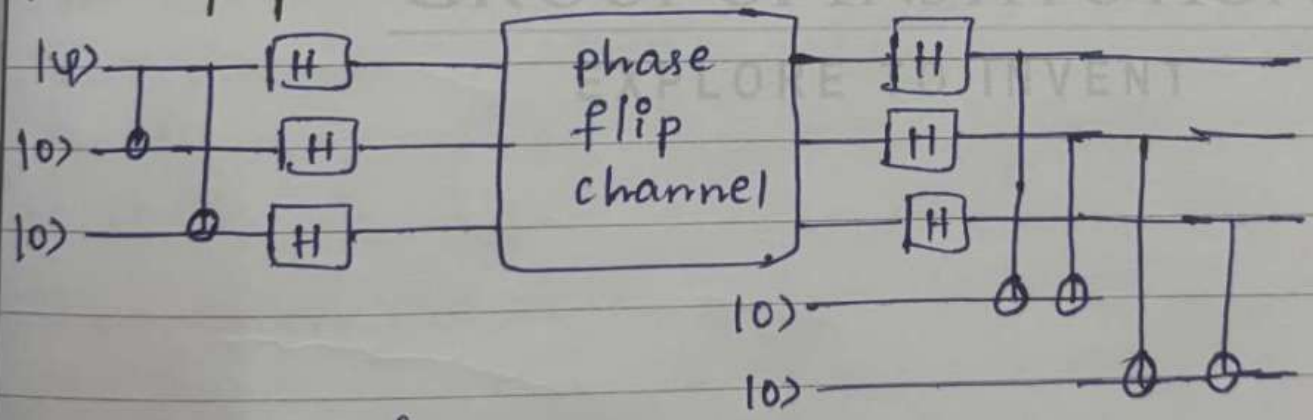
$\xrightarrow{CX_{35}}$ $\Rightarrow \alpha|10010\rangle + \beta|01110\rangle$.

$(\alpha|100\rangle + \beta|011)\,|10\rangle$.

encoded. * syndrome.

Syndrome	Error Qubit.
0 0	no error
0 1	3rd qubit
1 0	1st qubit
1 1	2nd qubit.

Phase flip Channel:



Error on 2nd qubit

$(\alpha|0\rangle + \beta|1\rangle)\,|00\rangle$.

$\Rightarrow \alpha|\overset{1}{0}\overset{2}{0}\overset{3}{0}\rangle + \beta|\overset{1}{1}\overset{2}{0}\overset{3}{0}\rangle \xrightarrow{CX_{12}} \alpha|000\rangle + \beta|110\rangle$

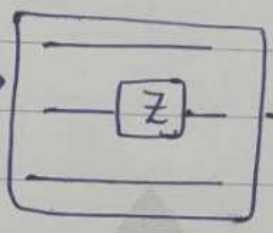
Date | Page No.

$\xrightarrow{CX_{13}} \alpha|\overset{1}{0}\overset{2}{0}\overset{3}{0}\rangle + \beta|\overset{1}{1}\overset{2}{1}\overset{3}{1}\rangle$.

$\xrightarrow{H \otimes H \otimes H} \alpha|+++\rangle + \beta|---\rangle$.

$\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha|++\rangle + \beta|--\rangle$.

$\alpha|+++\rangle+\beta|---\rangle$.

Error on 2nd qubit $\rightarrow$ 

$\alpha|+-+\rangle+\beta|-+-\rangle$.

Z-gate.

$|0\rangle \rightarrow |0\rangle$

$|1\rangle \rightarrow |-1\rangle$

$|+\rangle \rightarrow |+\rangle$

$|-\rangle \rightarrow -|-\rangle$.

$\xrightarrow{H_1 \otimes H_2 \otimes H_3}$

$\alpha|\overset{1}{0}\overset{2}{1}\overset{3}{0}\rangle + \beta|\overset{1}{1}\overset{2}{0}\overset{3}{1}\rangle$

Compare 12, 23. $CNOT_{12} \cdot |010\rangle + \beta|101\rangle$

$CNOT_{23}$ 11 11.

So the error is in 2nd qubit, then syndrome is 11.