Quantum Computing

Day 3 – Introduction To Quantum Computing

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#30DayQuantumComputingChallenge

Hilbert Space

Hilbert Space

- Complex vector space with inner product, complete uder induced indurd norm.
- Quantum states live in Hilbert spaces.
- Example: Single qubit: $\mathbb{C}^2$. n qubits: $\mathbb{C}^{2^n}$.

Dirac (Bra-Ket) Notation

- **Ket**: $|v\rangle = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \end{pmatrix}$.
- **Bra**: $\langle v| = (c_0^{**} \quad c_1^{**} \quad \dots)$.
- **Inner product**: $\langle u|v\rangle = u^\dagger v$.
- **Outer product**: $|u\rangle\langle v| = uv^\dagger$.
- Example: Qubit basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Probability Basis

- Probability: $0 \leq P(A) \leq 1$.
- Sum of probabilities: $\sum_i P(i) = 1$.
- Mutually exclusive events: $P(A \cup B) = P(A) + P(B)$.
- Independent events: $P(A \cap B) = P(A)P(B)$.

Classical vs Quantum Computing - Fundamentals

- **Unit of Information:**
  - Classical: Bit (0 or 1).
  - Quantum: Qubit ($\alpha|0\rangle + \beta|1\rangle$).
- **State:**
  - Classical: Definite state.
  - Quantum: Superposition of states.
- **Processing:**
  - Classical: Logic gates (AND, OR, NOT).
  - Quantum: Quantum gates (Hadamard, CNOT).

- **Phenomena:**
  - Classical: None.
  - Quantum: Superposition, entanglement, interference.
- **Computational Power:**
  - Classical: Struggles with factorization, molecular simulation.
  - Quantum: Speedups (e.g., Shor's, Grover's algorithms).

Take Away

- ✓ Importance Of Hilbert Space
- ✓ Representation Of Dirac Notations
- ✓ Probability Basis
- ✓ Fundamentals of Quantum