2. Describe the Quantum Teleportation protocol step by step.

Goal: Send $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ from Alice to Bob.

Requires: * 1 shared entangled pair
* 2 bits of classical communication.

Steps: 1. Alice entangles, measure her qubits.
2. Sends 2 bits to Bob
3. Bob applies correction based on bits.

⇒ Quantum information isn't about physically moving objects or people but it's about transferring the quantum information from one location to other without moving the practicle itself.

Why we need Quantum Teleportation?

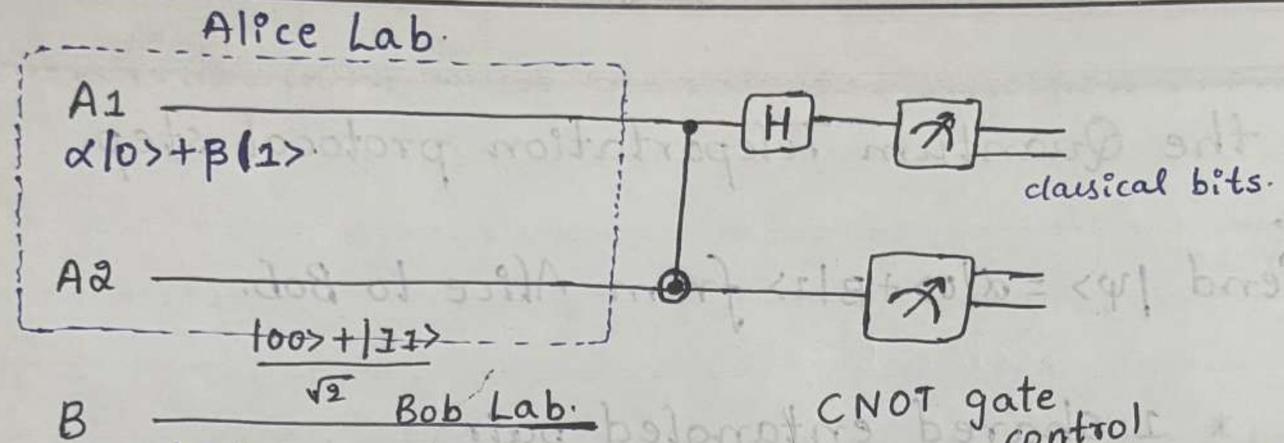
Quantum Teleportation allows the transfer of qubits between distant locations without directly sending the qubit through a channel. This is important because

- Qubits are fragile and disrupted by noise or loss over distance.

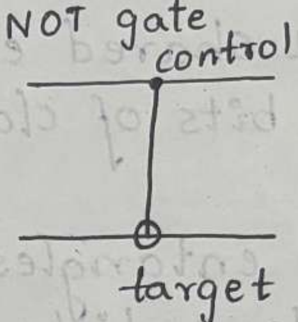
- Quantum teleportation requires the bell states before transfer of qubits between distant locations.

Example of bell state:

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



$\Rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$ (bell state).



control o/p same as i/p

Target o/p flips when control o/p is $|1\rangle$.

$$(\alpha|0\rangle + \beta|1\rangle)\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right).$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right].$$

Applying C-NOT$_{12}$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right].$$

Applying H-gate on 1st qubit.

$|0\rangle \leftrightarrow |+\rangle$

$|1\rangle \leftrightarrow |-\rangle$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\alpha|+00\rangle + \alpha|+11\rangle + \beta|-10\rangle + \beta|-01\rangle\right].$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\alpha|+\rangle|00\rangle + \alpha|+\rangle|11\rangle + \beta|-\rangle|10\rangle + \beta|-\rangle|01\rangle\right].$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|00\rangle + \alpha\left(\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|11\rangle\right)\right.$$

$$\left. + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|10\rangle + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|01\rangle\right].$$

Partial Measurement:

$$\frac{|\overset{1}{0}\overset{2}{0}\rangle + |\overset{1}{1}\rangle|\overset{2}{0}\rangle}{\sqrt{2}} \;—\boxed{M}—\; |0\rangle.$$ Measurement of 2^{nd} qubit

$$\frac{|\overset{1}{0}\overset{2}{0}\rangle + |\overset{1}{1}\overset{2}{0}\rangle}{\sqrt{2}} \;—\boxed{M}—\; |0\rangle \text{ or } |1\rangle.$$ Measurement of 1^{st} qubit.

Then,

$$= \frac{1}{2}\Big[\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle\Big].$$

$$= \frac{1}{2}\Big[\overset{A_1A_2}{|00\rangle}\overset{B}{[\alpha|0\rangle + \beta|1\rangle]} + \overset{A_1A_2}{|10\rangle}\overset{B}{[\alpha|0\rangle - \beta|1\rangle]} + \overset{A_1A_2}{|01\rangle}\overset{B}{[\alpha|1\rangle + \beta|0\rangle]} + \overset{A_1A_2}{11}\overset{B.}{[\alpha|1\rangle - \beta|0\rangle]}.\Big].$$

After measurement:

A_1 A_2	B.	Performs.
0 0	$\alpha|0\rangle+\beta|1\rangle$.	Do nothing
0 1	$\alpha|1\rangle+\beta|0\rangle$	X-gate.
1 0	$\alpha|0\rangle-\beta|1\rangle$	Z-gate.
1 1	$\alpha|1\rangle-\beta|0\rangle$.	Y-gate.