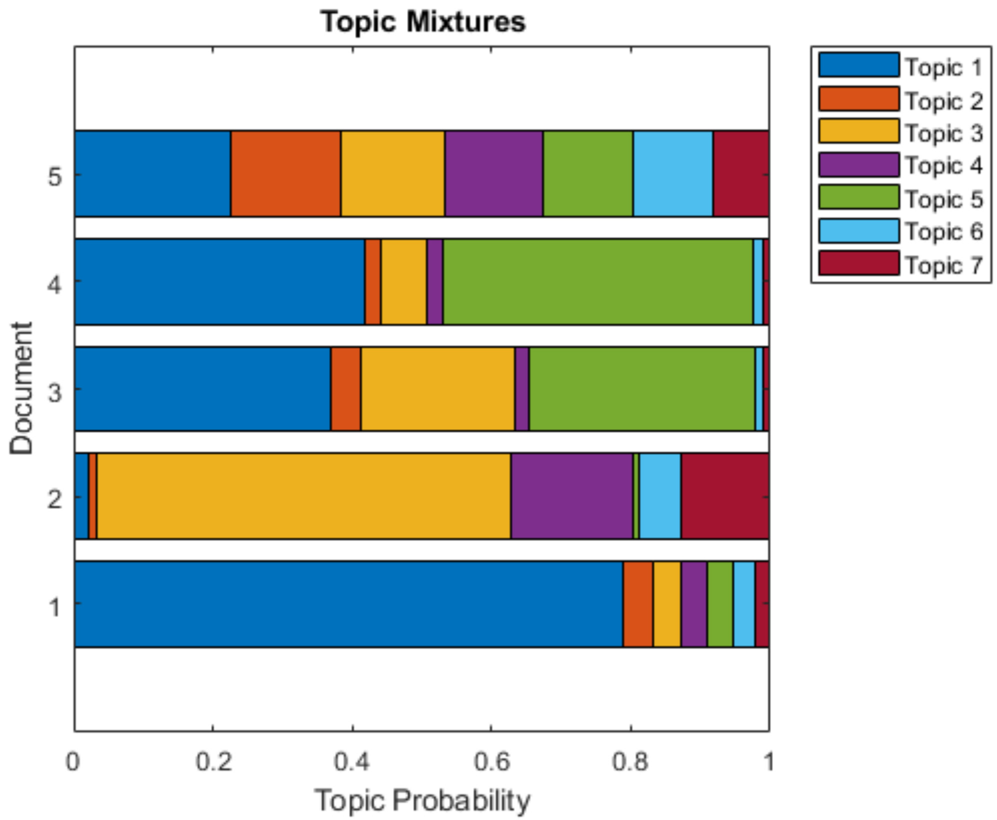
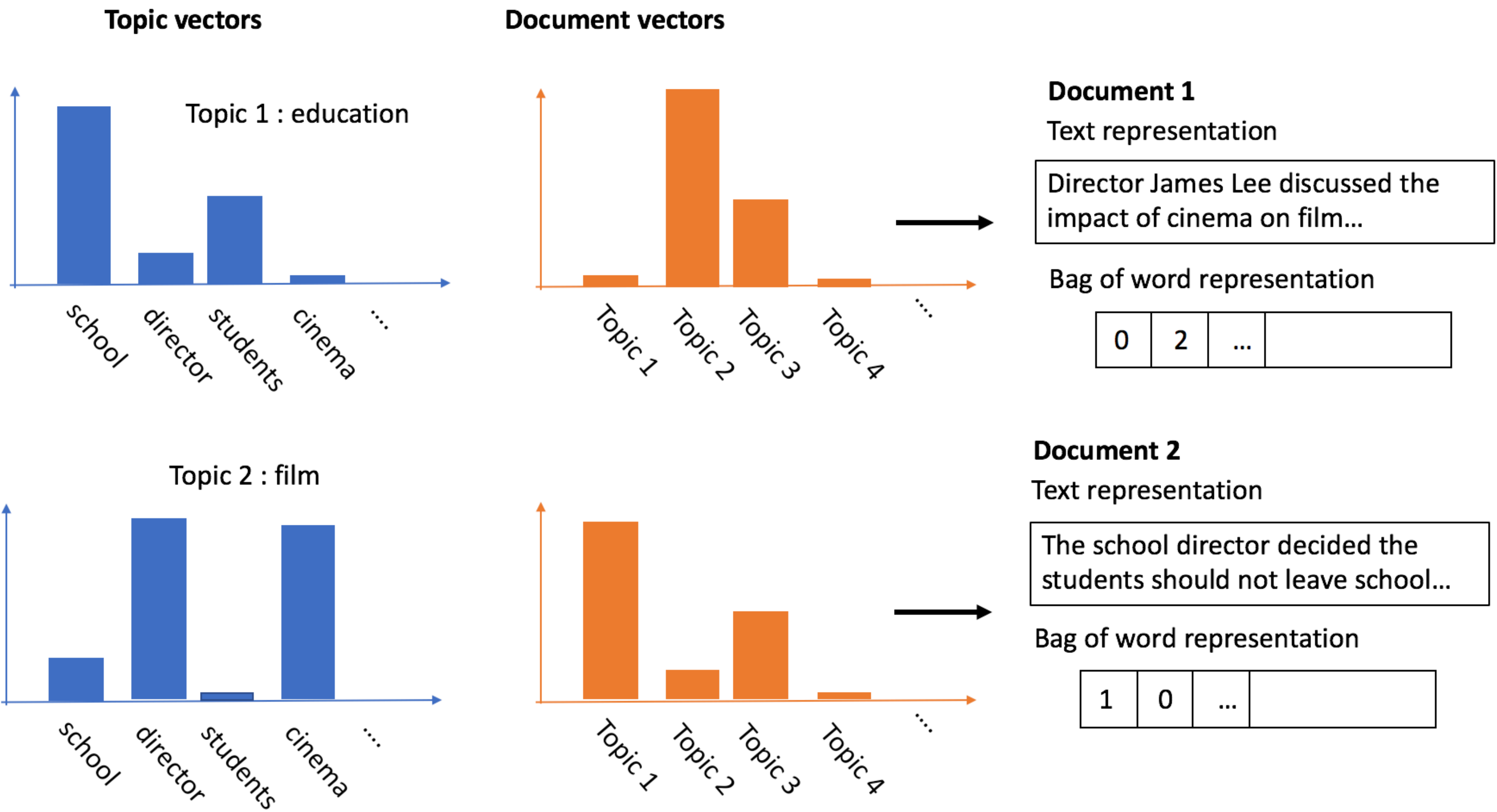
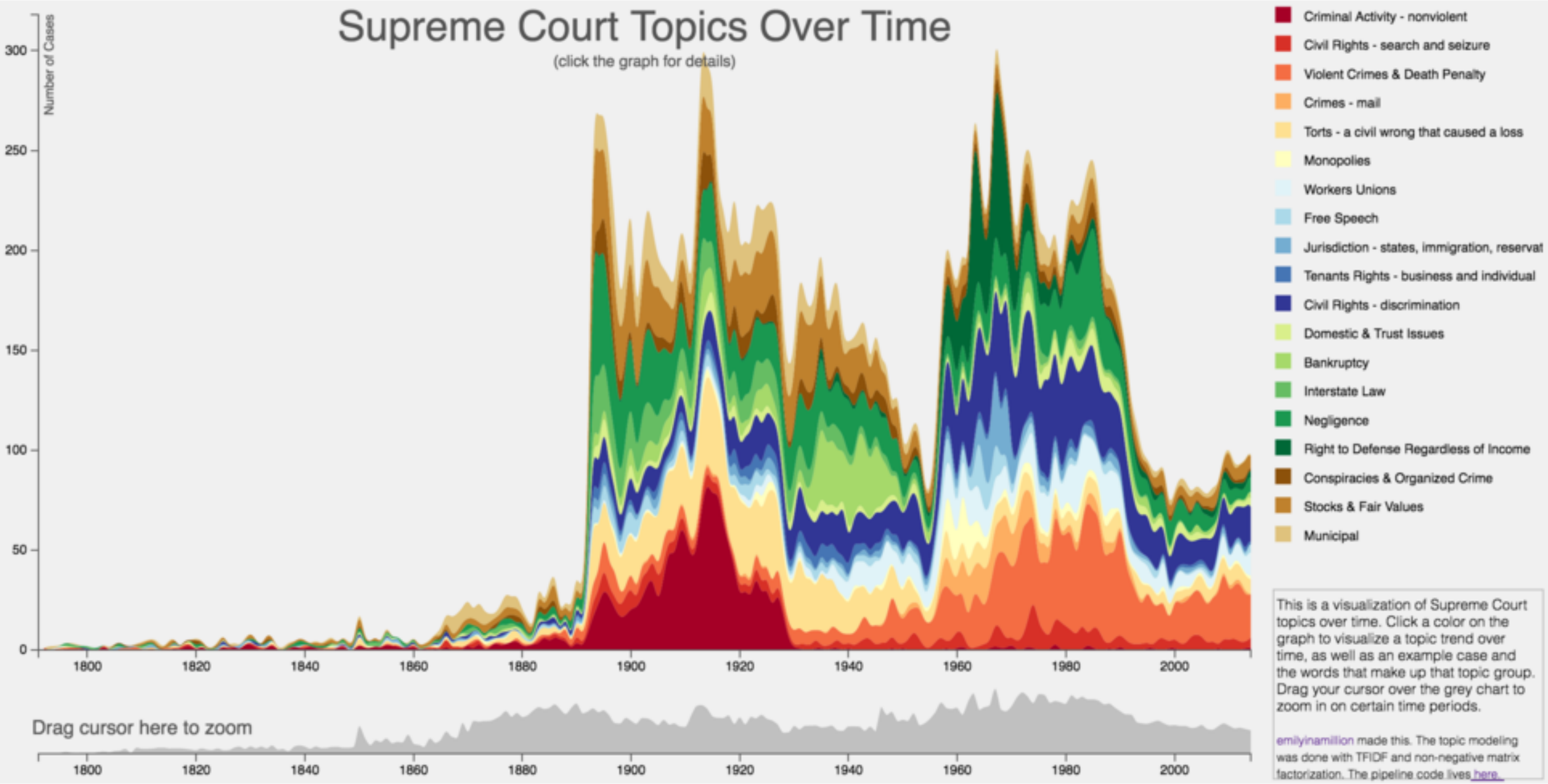
Latent Dirichlet Allocation


Topic Mixtures
Document
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
Topic Probability
Topic 1
Topic 2
Topic 3
Topic 4
Topic 5
Topic 6
Topic 7



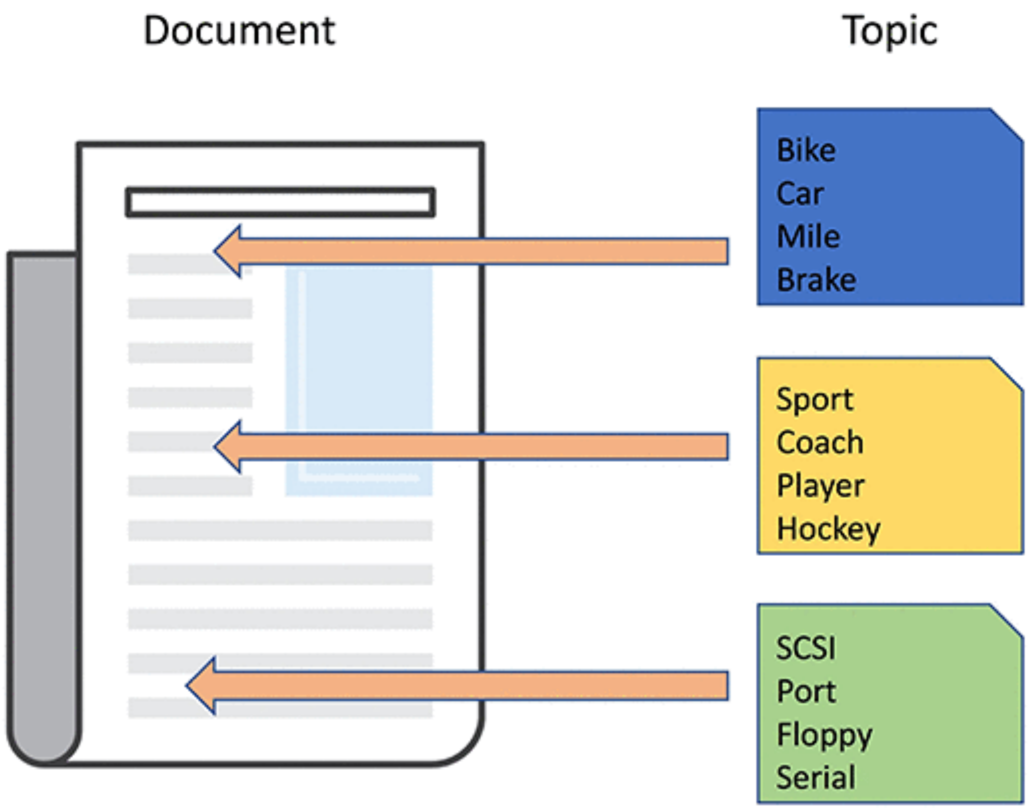
Topic vectors
Document vectors
Topic 1 : education
school
director
students
cinema
...
Topic 1
Topic 2
Topic 3
Topic 4
...
Document 1
Text representation
Director James Lee discussed the impact of cinema on film...
Bag of word representation
0
2
...
Topic 2 : film
school
director
students
cinema
...
Topic 1
Topic 2
Topic 3
Topic 4
...
Document 2
Text representation
The school director decided the students should not leave school...
Bag of word representation
1
0
...

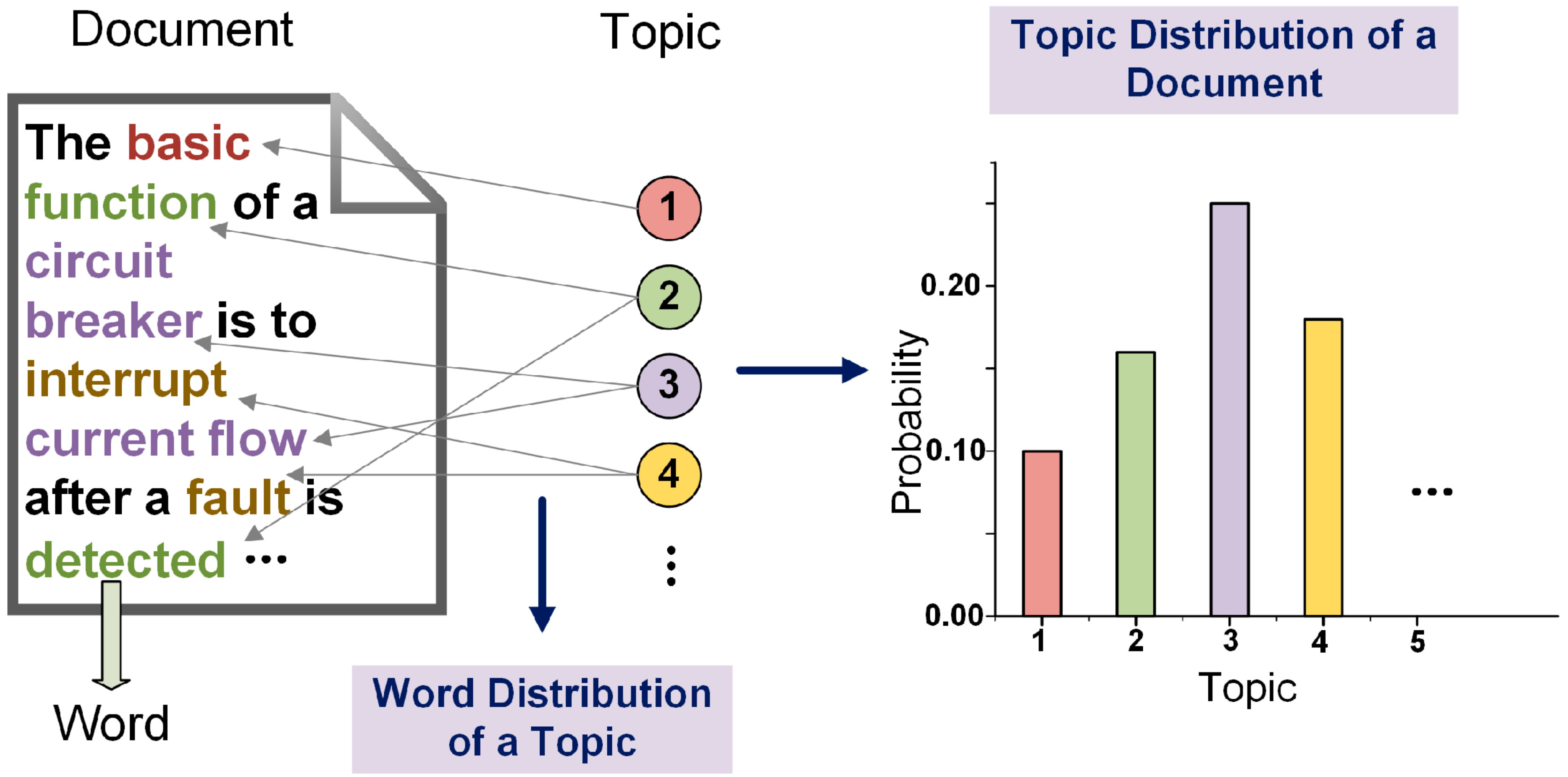

Supreme Court Topics Over Time
(click the graph for details)
Number of Cases
300
250
200
150
100
50
0
1800
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
Criminal Activity - nonviolent
Civil Rights - search and seizure
Violent Crimes & Death Penalty
Crimes - mail
Torts - a civil wrong that caused a loss
Monopolies
Workers Unions
Free Speech
Jurisdiction - states, immigration, reservat
Tenants Rights - business and individual
Civil Rights - discrimination
Domestic & Trust Issues
Bankruptcy
Interstate Law
Negligence
Right to Defense Regardless of Income
Conspiracies & Organized Crime
Stocks & Fair Values
Municipal
This is a visualization of Supreme Court topics over time. Click a color on the graph to visualize a topic trend over time, as well as an example case and the words that make up that topic group. Drag your cursor over the grey chart to zoom in on certain time periods.
emilyinamillion made this. The topic modeling was done with TFIDF and non-negative matrix factorization. The pipeline code lives here.
Drag cursor here to zoom

Topic Modeling

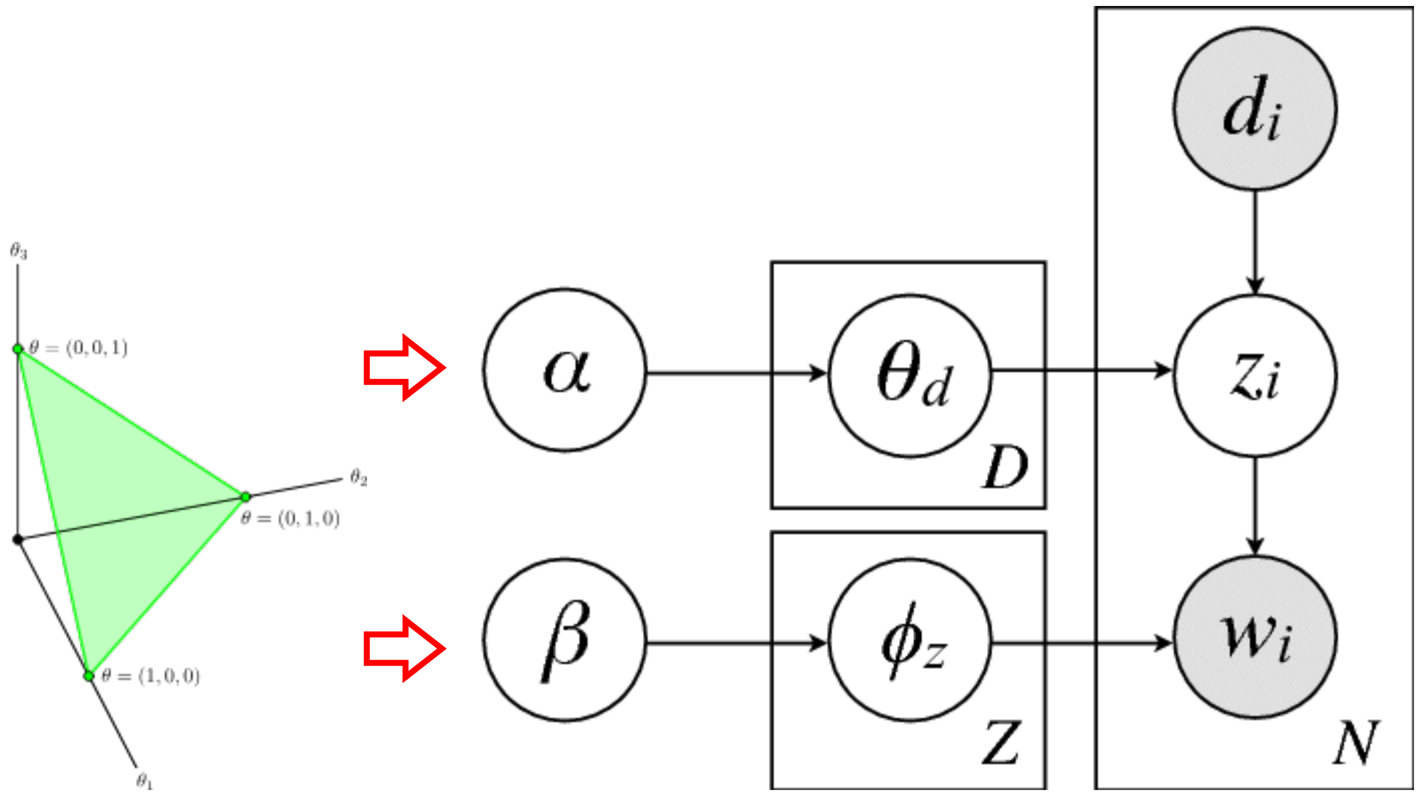
What we can identify are

- Topics
- The proportion of topics
- The most probable words in topics
- Text analysis without reading the whole corpus


Document
Topic
Bike
Car
Mile
Brake
Sport
Coach
Player
Hockey
SCSI
Port
Floppy
Serial



Document
Topic
Topic Distribution of a Document
The basic function of a circuit breaker is to interrupt current flow after a fault is detected ...
1
2
3
4
Word
Word Distribution of a Topic
Probability
0.20
0.10
0.00
1
2
3
4
5
Topic



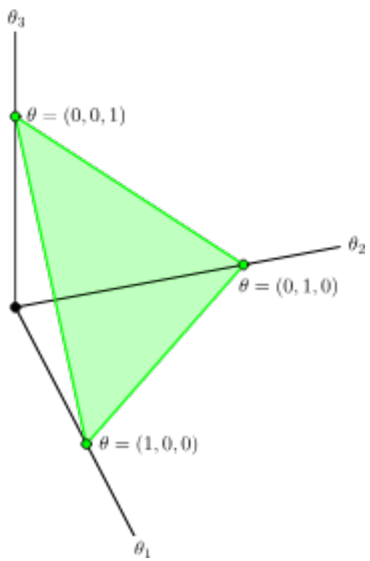
θ_3
$\theta = (0,0,1)$
θ_2
$\theta = (0,1,0)$
$\theta = (1,0,0)$
θ_1
α
θ_d
D
β
ϕ_z
Z
d_i
z_i
w_i
N

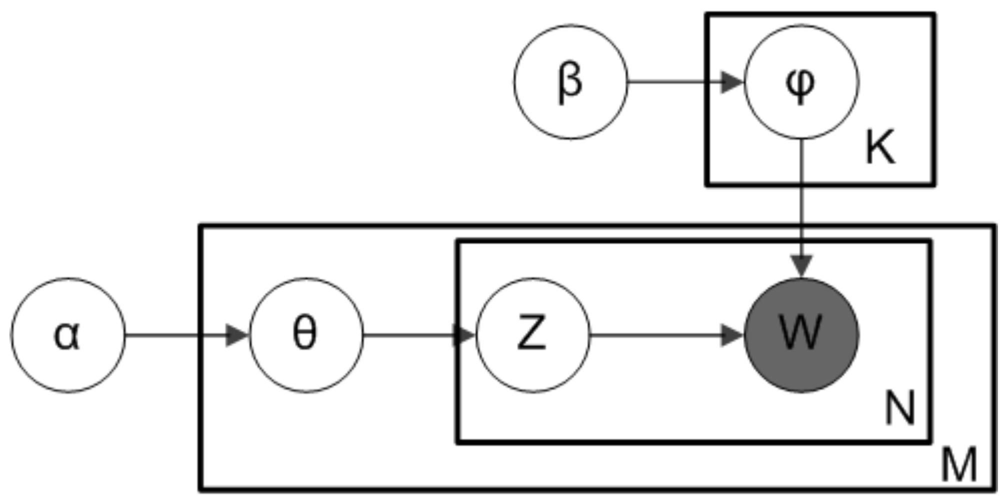
LDA

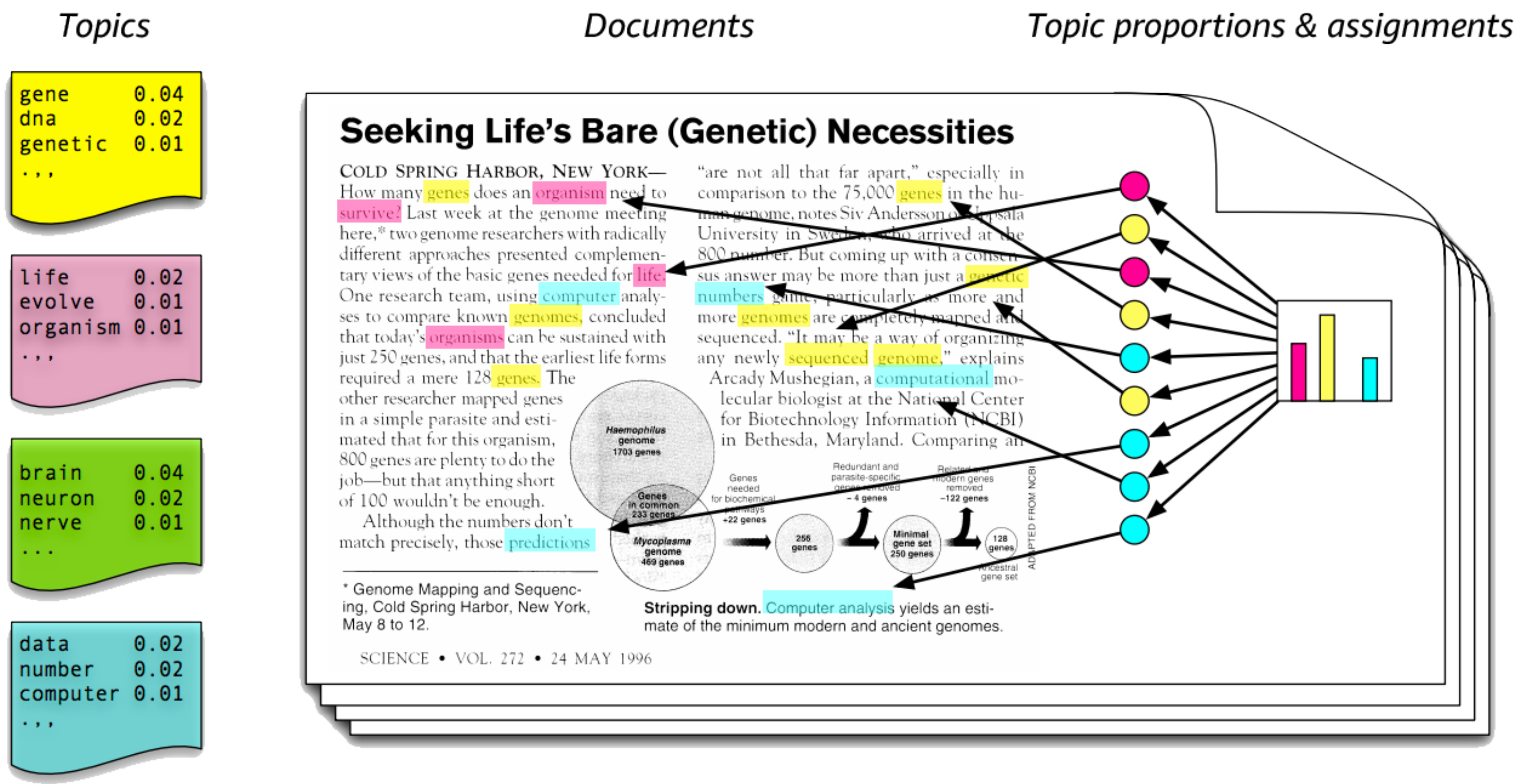
Latent Dirichlet Allocation

- Soft clustering in text data
- Has the structure of text corpus
- Is a Bayesian model with priors

For each word , sample topic assignment






Topics
Documents
Topic proportions & assignments
gene 0.04
dna 0.02
genetic 0.01
...
life 0.02
evolve 0.01
organism 0.01
...
brain 0.04
neuron 0.02
nerve 0.01
...
data 0.02
number 0.02
computer 0.01
...
Seeking Life's Bare (Genetic) Necessities
Haemophilus genome 1703 genes
Genes in common 233 genes
Mycoplasma genome 469 genes
256 genes
Minimal gene set 250 genes
128 genes
* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.
Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.
SCIENCE • VOL. 272 • 24 MAY 1996

Finding Topic Assignment/Word

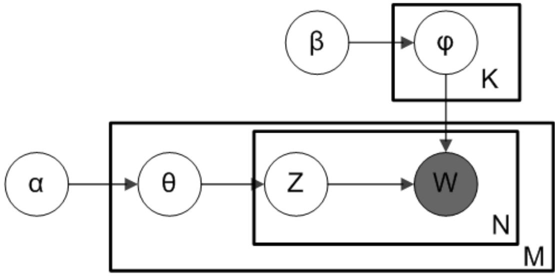
Generative process

$$\theta_i \sim Dir(\alpha),\ i \in \{1, \ldots, M\}$$

$$\varphi_k \sim Dir(\beta),\ k \in \{1, \ldots, K\}$$

$$z_{i,l} \sim Multi(\theta_i),\ i \in \{1, \ldots, M\},\ l \in \{1, \ldots, N\}$$

$$w_{i,l} \sim Multi\left(\varphi_{z_{i,l}}\right),\ i \in \{1, \ldots, M\},\ l \in \{1, \ldots, N\}$$

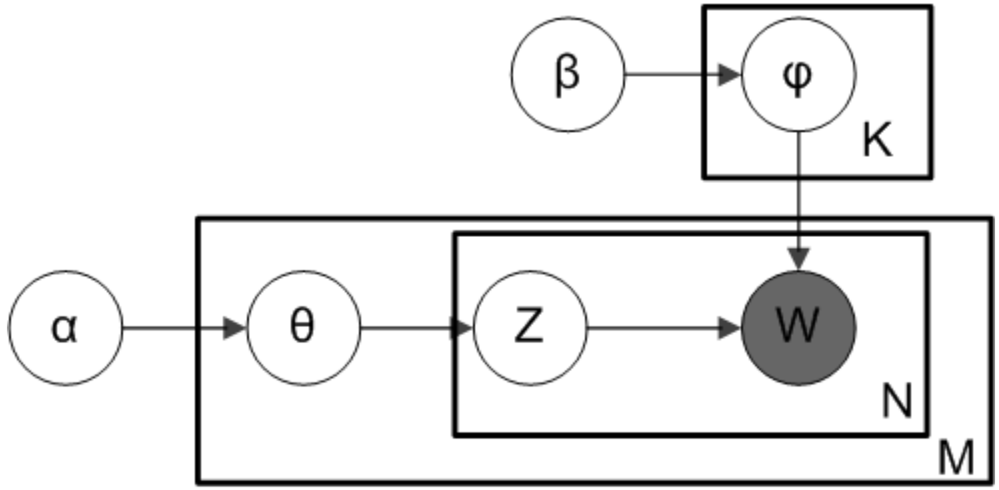


A word is generated from distribution of word-topic distribution

topic is generated from distribution of document-topic distribution

document topic distribution is generated from distribution of

word-topic distribution is generated from distribution of



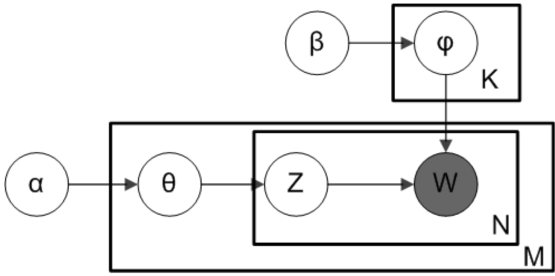
If we have distribution, we can find the most likely and

is topic distribution in a document

is word distribution in a topic

Finding the most likely allocation of is the key of inference on and

Gibbs Sampling



Finding the most likely assignment on → *Gibbs sampling*

Start with the factorization

$$P(\theta_j \mid \alpha) P(\alpha)$$

$$P(W,\ Z,\ \theta,\ \varphi;\ \alpha, \beta) = \prod_{i=1}^{K} P(\varphi_i; \beta) \prod_{j=1}^{M} P(\theta_j; \alpha) \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j) P(W_{j,l} \mid \varphi_{Z_{j,l}})$$

$$P(\varphi_i \mid \beta) P(\beta)$$

We are going to collapse and to leave only , , and

(Data point), (Sampling Target), and (Prior)

$$P\left(W,\ Z,\ \theta,\ \varphi;\ \alpha,\beta\right) = \prod_{i=1}^{K} P(\varphi_i;\beta) \prod_{j=1}^{M} P(\theta_j;\alpha) \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j) P(W_{j,l} \mid \varphi_{Z_{j,l}})$$

$$P\left(W,\ Z;\ \alpha,\beta\right) = \int_{\theta}\int_{\varphi} P(W,\ Z,\theta,\ \varphi;\ \alpha,\beta)\, d\varphi d\theta$$

→ *Marginalization, Summing out*

↓

Collapsed Gibbs sampling $= \int_{\varphi} \prod_{i=1}^{K} P(\varphi_i;\beta) \prod_{j=1}^{M} \prod_{l=1}^{N} P(W_{j,l} \mid \varphi_{z_{j,l}})\, d\varphi$

$$\times \int_{\theta} \prod_{j=1}^{M} P(\theta_j;\alpha) \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j)\, d\theta$$

$$= (1) \times (2)$$

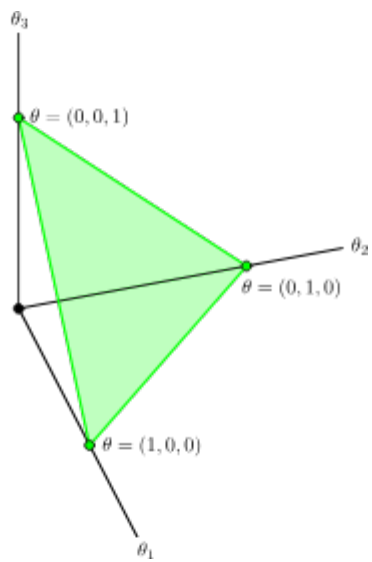
→ *Independence between two integrals*

→ *Need to remove the integrals*

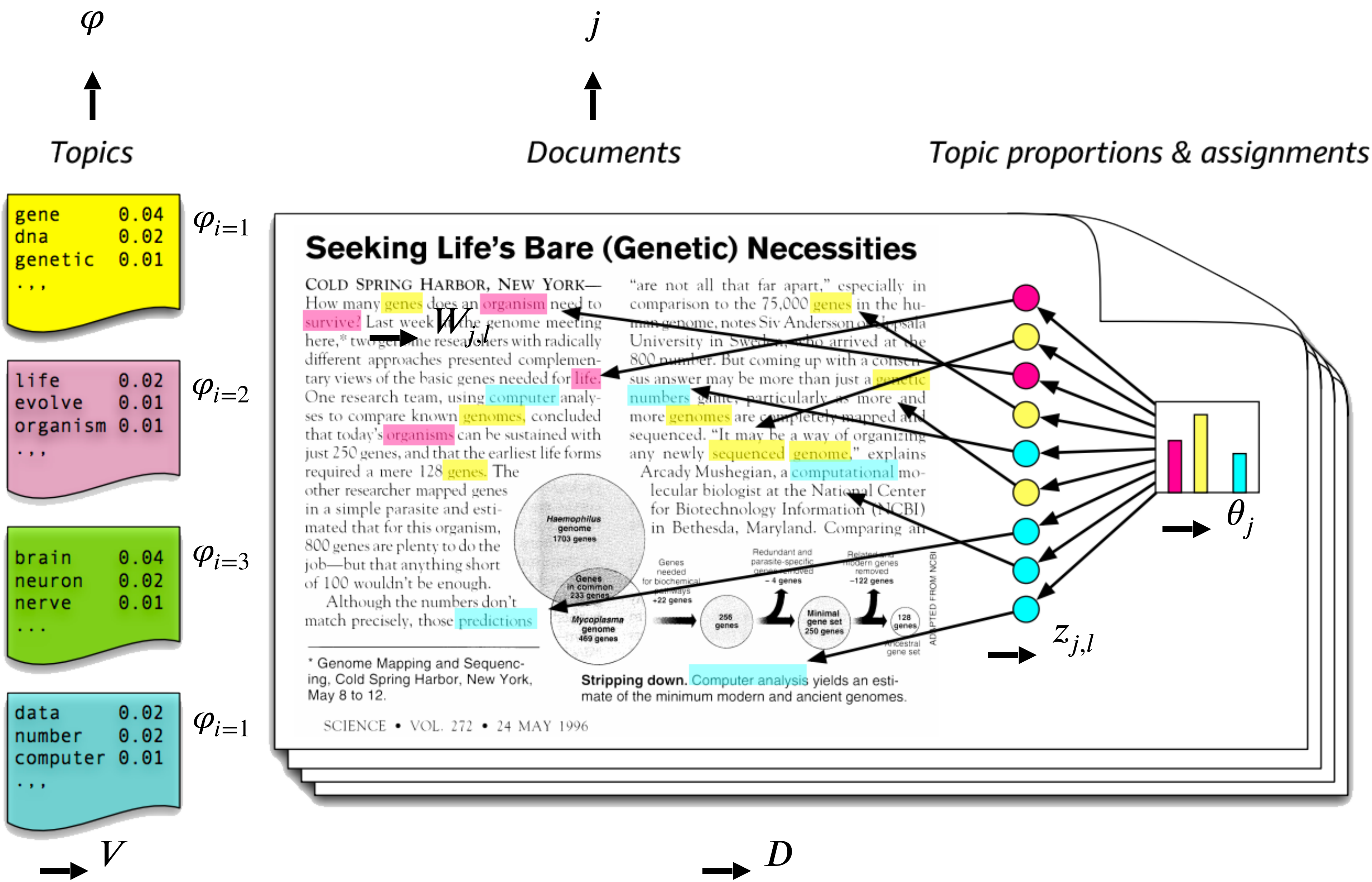
$$(1) = \int_{\varphi} \prod_{i=1}^{K} P(\varphi_i; \beta) \prod_{j=1}^{M} \prod_{l=1}^{N} P(W_{j,l} \mid \varphi_{z_{j,l}})\, d\varphi$$

$$= \prod_{i=1}^{K} \int_{\varphi_i} P(\varphi_i; \beta) \prod_{j=1}^{M} \prod_{l=1}^{N} P(W_{j,l} \mid \varphi_{z_{j,l}})\, d\varphi_i$$

$$= \prod_{i=1}^{K} \int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V} \beta_v)}{\prod_{v=1}^{V} \Gamma(\beta_v)} \prod_{v=1}^{V} {\varphi_{i,v}}^{\beta_v - 1} \prod_{j=1}^{M} \prod_{l=1}^{N} P(W_{j,l} \mid \varphi_{z_{j,l}})\, d\varphi_i$$



$$x \sim Dir(\alpha), \quad P(X \mid \alpha) = \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$


φ
j
Topics
Documents
Topic proportions & assignments
gene 0.04
dna 0.02
genetic 0.01
...
life 0.02
evolve 0.01
organism 0.01
...
brain 0.04
neuron 0.02
nerve 0.01
...
data 0.02
number 0.02
computer 0.01
...
$\varphi_{i=1}$
$\varphi_{i=2}$
$\varphi_{i=3}$
$\varphi_{i=1}$
Seeking Life's Bare (Genetic) Necessities
$W_{j,l}$
θ_j
$z_{j,l}$
V
D
Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.
SCIENCE • VOL. 272 • 24 MAY 1996

$$= \prod_{i=1}^{K} \int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V}\beta_v)}{\prod_{v=1}^{V}\Gamma(\beta_v)} \prod_{v=1}^{V} \varphi_{i,v}^{\beta_v-1} \prod_{j=1}^{M}\prod_{l=1}^{N} P(W_{j,l} \mid \varphi_{z_{j,l}})\, d\varphi_i$$

Dirichlet distribution

, number of words assigned to -th topic in -th document with -th unique word

$$= \prod_{i=1}^{K} \int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V}\beta_v)}{\prod_{v=1}^{V}\Gamma(\beta_v)} \prod_{v=1}^{V} \varphi_{i,v}^{\beta_v-1} \prod_{v=1}^{V} \varphi_{i,v}^{n^{i}_{(.),v}}\, d\varphi_i$$

$$= \prod_{i=1}^{K} \int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V}\beta_v)}{\prod_{v=1}^{V}\Gamma(\beta_v)} \prod_{v=1}^{V} \varphi_{i,v}^{n^{i}_{(.),v}+\beta_v-1}\, d\varphi_i$$

$$x \sim Dir(\alpha), \quad P(X \mid \alpha) = \frac{\Gamma(\sum_{i=1}^{K}\alpha_i)}{\prod_{i=1}^{K}\Gamma(\alpha_i)} \prod_{i=1}^{K} x_i^{\alpha_i-1}$$

$$= \prod_{i=1}^{K} \int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V} \beta_v)}{\prod_{v=1}^{V} \Gamma(\beta_v)} \prod_{v=1}^{V} {\varphi_{i,v}}^{n^i_{(.),v}+\beta_v-1} \, d\varphi_i$$

$$= \prod_{i=1}^{K} \frac{\prod_{v=1}^{V} \Gamma(n^i_{(.),v} + \beta_v) \Gamma(\sum_{v=1}^{V} \beta_v)}{\prod_{v=1}^{V} \Gamma(\beta_v) \Gamma(\sum_{v=1}^{V} n^i_{(.),v} + \beta_v)} \underbrace{\int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V} n^i_{(.),v} + \beta_v)}{\prod_{v=1}^{V} \Gamma(n^i_{(.),v} + \beta_v)} \prod_{v=1}^{V} {\varphi_{i,v}}^{n^i_{(.),v}+\beta_v-1} \, d\varphi_i}$$

$\longrightarrow$ *Dirichlet distribution*

$$= \prod_{i=1}^{K} \frac{\prod_{v=1}^{V} \Gamma(n^i_{(.),v} + \beta_v) \Gamma(\sum_{v=1}^{V} \beta_v)}{\prod_{v=1}^{V} \Gamma(\beta_v) \Gamma(\sum_{v=1}^{V} n^i_{(.),v} + \beta_v)}$$

$$
\begin{aligned}
P\left(W,\ Z;\ \alpha,\beta\right) &= \int_{\theta}\int_{\varphi} P(W,\ Z,\theta,\ \varphi;\ \alpha,\beta)\, d\varphi d\theta \\
&= \int_{\varphi} \prod_{i=1}^{K} P(\varphi_i;\beta) \prod_{j=1}^{M} \prod_{l=1}^{N} P(W_{j,l} \mid \varphi_{z_{j,l}})\, d\varphi \\
&\quad \times \int_{\theta} \prod_{j=1}^{M} P(\theta_j;\alpha) \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j)\, d\theta \\
&= (1) \times (2)
\end{aligned}
$$

$$
x \sim Dir(\alpha),\quad P(X \mid \alpha) = \frac{\Gamma\left(\sum_{i=1}^{K} \alpha_i\right)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} x_i^{\alpha_i - 1}
$$

$$(2) = \int_{\theta} \prod_{j=1}^{M} P(\theta_j; \alpha) \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j)\, d\theta$$

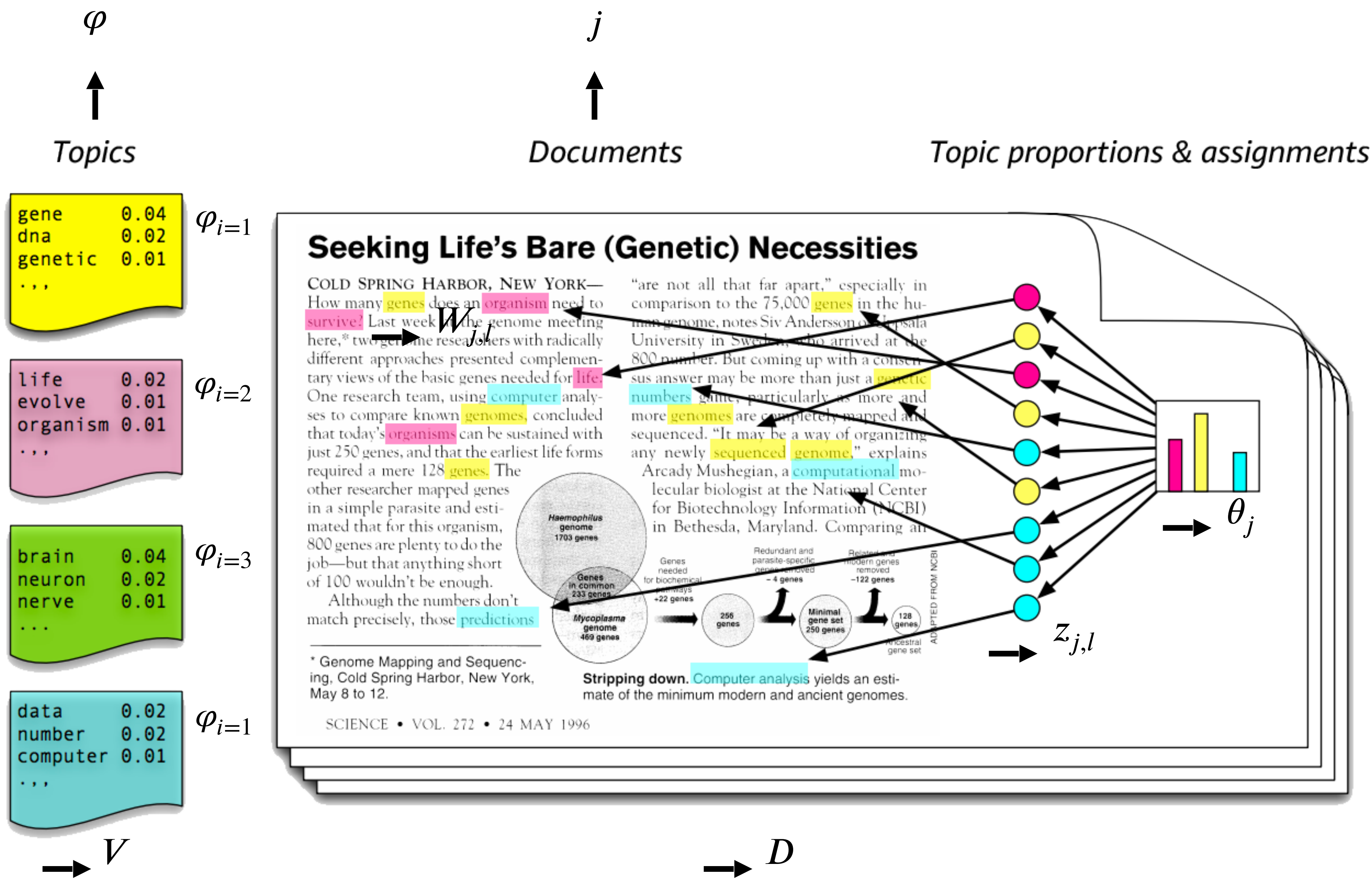
$$= \prod_{j=1}^{M} \int_{\theta_j} P(\theta_j; \alpha) \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j)\, d\theta_j$$

$$= \prod_{j=1}^{M} \int_{\theta_j} \frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \prod_{k=1}^{K} {\theta_{j,k}}^{\alpha_k - 1} \prod_{l=1}^{N} P(Z_{j,l} \mid \theta_j)\, d\theta_j$$

→ *Dirichlet distribution* → *, number of words assigned to -th topic in -th document with -th unique word*

$$= \prod_{j=1}^{M} \int_{\theta_j} \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} {\theta_{j,i}}^{\alpha_i - 1} \prod_{k=1}^{K} {\theta_{j,k}}^{n_{j,(.)}^{k}}\, d\theta_j$$

$$= \prod_{j=1}^{M} \int_{\theta_j} \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} {\theta_{j,i}}^{n_{j,(.)}^{i} + \alpha_i - 1}\, d\theta_j$$



φ
j
Topics
Documents
Topic proportions & assignments
gene 0.04
dna 0.02
genetic 0.01
.,,
φ i=1
life 0.02
evolve 0.01
organism 0.01
.,,
φ i=2
brain 0.04
neuron 0.02
nerve 0.01
...
φ i=3
data 0.02
number 0.02
computer 0.01
.,,
φ i=1
V
Seeking Life's Bare (Genetic) Necessities
W j,l
θ j
z j,l
D
Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.
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$$= \prod_{j=1}^{M} \int_{\theta_j} \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} {\theta_{j,i}}^{n_{j,(.)}^{i} + \alpha_i - 1} d\theta_j$$

$$= \prod_{j=1}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i} + \alpha_i)\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)} \int_{\theta_j} \frac{\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}{\prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i} + \alpha_i)} \prod_{i=1}^{K} {\theta_{j,i}}^{n_{j,(.)}^{i} + \alpha_i - 1} d\theta_j$$

$$= \prod_{j=1}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i} + \alpha_i)\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}$$

$$x \sim Dir(\alpha), \quad P(X \mid \alpha) = \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$$

Collapse from Conjugacy

Same mechanism to remove and

$$(1) = \prod_{i=1}^{K} \frac{\prod_{v=1}^{V} \Gamma(n_{(.),v}^{i} + \beta_v)\Gamma(\sum_{v=1}^{V} \beta_v)}{\prod_{v=1}^{V} \Gamma(\beta_v)\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)} \int_{\varphi_i} \frac{\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)}{\prod_{v=1}^{V} \Gamma(n_{(.),v}^{i} + \beta_v)} \prod_{v=1}^{V} {\varphi_{i,v}}^{n_{(.),v}^{i} + \beta_v - 1} \, d\varphi_i$$

$$(2) = \prod_{j=1}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i} + \alpha_i)\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)} \int_{\theta_j} \frac{\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}{\prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i} + \alpha_i)} \prod_{i=1}^{K} {\theta_{j,i}}^{n_{j,(.)}^{i} + \alpha_k - 1} \, d\theta_j$$

This is multiplication of the Dirichlet distribution and the multinomial distribution

After multiplication, another Dirichlet distribution emerges

In LDA,

In general,

Likelihood and prior multiplication results in the prior distribution

$\longrightarrow$ *Conjugate prior*

Gibbs Sampling Formula

$$P\left(W,\ Z;\ \alpha,\beta\right) = \prod_{i=1}^{K} \frac{\prod_{v=1}^{V} \Gamma(n^{i}_{(.),v} + \beta_v) \Gamma(\sum_{v=1}^{V} \beta_v)}{\prod_{v=1}^{V} \Gamma(\beta_v) \Gamma(\sum_{v=1}^{V} n^{i}_{(.),v} + \beta_v)} \prod_{j=1}^{M} \frac{\prod_{i=1}^{K} \Gamma(n^{i}_{j,(.)} + \alpha_i) \Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i) \Gamma(\sum_{i=1}^{K} n^{i}_{j,(.)} + \alpha_i)}$$

, and are assumed and data points, and is the target of sampling

Gibbs sampling iterates the element of , one by one

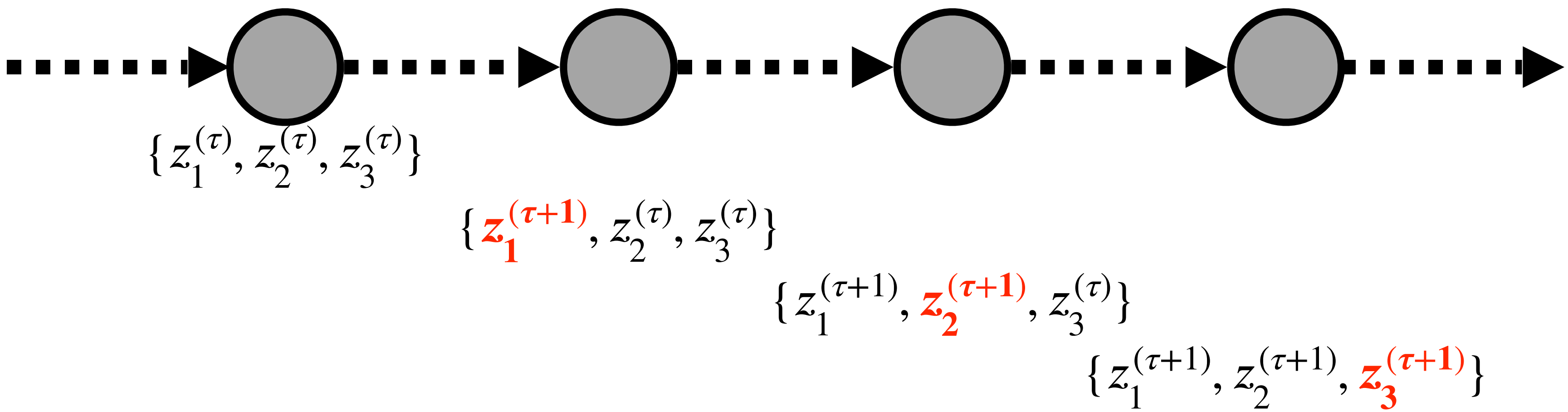
We need to derive a formula of a single element

when all other element of , , and are given

$$P\left(Z_{(m,l)} = k \mid Z_{-(m,l)}, W;\ \alpha,\beta\right) = \frac{P\left(Z_{(m,l)} = k, Z_{-(m,l)}, W;\ \alpha,\beta\right)}{P\left(Z_{-(m,l)}, W;\ \alpha,\beta\right)}$$

$$\propto P\left(Z_{(m,l)} = k, Z_{-(m,l)}, W;\ \alpha,\beta\right)$$

is the topic assignment on the -th word of -th document



$\{z_1^{(\tau)}, z_2^{(\tau)}, z_3^{(\tau)}\}$
$\{\mathbf{z_1^{(\tau+1)}}, z_2^{(\tau)}, z_3^{(\tau)}\}$
$\{z_1^{(\tau+1)}, \mathbf{z_2^{(\tau+1)}}, z_3^{(\tau)}\}$
$\{z_1^{(\tau+1)}, z_2^{(\tau+1)}, \mathbf{z_3^{(\tau+1)}}\}$

$$P\left(W,\ Z;\ \alpha,\beta\right) = \prod_{i=1}^{K} \frac{\prod_{v=1}^{V}\Gamma(n_{(.),v}^{i} + \beta_v)\Gamma(\sum_{v=1}^{V}\beta_v)}{\prod_{v=1}^{V}\Gamma(\beta_v)\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)} \prod_{j=1}^{M} \frac{\prod_{i=1}^{K}\Gamma(n_{j,(.)}^{i} + \alpha_i)\Gamma(\sum_{i=1}^{K}\alpha_i)}{\prod_{i=1}^{K}\Gamma(\alpha_i)\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}$$

$$= (\frac{\Gamma(\sum_{v=1}^{V}\beta_v)}{\prod_{v=1}^{V}\Gamma(\beta_v)})^{K} (\frac{\Gamma(\sum_{i=1}^{K}\alpha_i)}{\prod_{i=1}^{K}\Gamma(\alpha_i)})^{M} \prod_{i=1}^{K} \frac{\prod_{v=1}^{V}\Gamma(n_{(.),v}^{i} + \beta_v)}{\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)} \prod_{j=1}^{M} \frac{\prod_{i=1}^{K}\Gamma(n_{j,(.)}^{i} + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}$$

$$\propto \prod_{i=1}^{K} \frac{\prod_{v=1}^{V}\Gamma(n_{(.),v}^{i} + \beta_v)}{\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)} \prod_{j=1}^{M} \frac{\prod_{i=1}^{K}\Gamma(n_{j,(.)}^{i} + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}$$

Now, apply that

$$\propto \prod_{i=1}^{K} \frac{\prod_{v=1}^{V} \Gamma(n_{(.),v}^{i} + \beta_v)}{\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)} \prod_{j=1}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i} + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_{j,(.)}^{i} + \alpha_i)}$$

$$\propto \prod_{i=1}^{K} \frac{\prod_{v=1}^{V} \Gamma(n_{(.),v}^{i} + \beta_v)}{\Gamma(\sum_{v=1}^{V} n_{(.),v}^{i} + \beta_v)} \times \frac{\prod_{i=1}^{K} \Gamma(n_{m,(.)}^{i} + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_{m,(.)}^{i} + \alpha_i)}$$

→ *Fixing document by*

$$\propto \prod_{i=1}^{K} \frac{\Gamma(n_{(.),v}^{i} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{i} + \beta_r)} \times \frac{\prod_{i=1}^{K} \Gamma(n_{m,(.)}^{i} + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_{m,(.)}^{i} + \alpha_i)}$$

→ *Fixing word by*

$$\propto \prod_{i=1}^{K} \frac{\Gamma(n_{(.),v}^{i} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{i} + \beta_r)} \times \prod_{i=1}^{K} \Gamma(n_{m,(.)}^{i} + \alpha_i)$$

→ *Remove a constant*

$$P\Big(Z_{(m,l)} = k \mid Z_{-(m,l)}, W;\ \alpha, \beta\Big) \propto P\Big(Z_{(m,l)} = k, Z_{-(m,l)}, W;\ \alpha, \beta\Big)$$

$$\propto \prod_{i=1}^{K} \frac{\Gamma(n^{i}_{(.),v} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n^{i}_{(.),r} + \beta_r)} \times \prod_{i=1}^{K} \Gamma(n^{i}_{j,(.)} + \alpha_i)$$

Now, we set as excluding the count from the topic assignment of

$$\propto \prod_{i=1,\ i\neq k}^{K} \frac{\Gamma(n^{i,-(m,n)}_{(.),v} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n^{i,-(m,n)}_{(.),r} + \beta_r)} \times \prod_{i=1, i\neq k}^{K} \Gamma(n^{i,-(m,n)}_{j,(.)} + \alpha_i)$$

$$\times \frac{\Gamma(n^{k,-(m,n)}_{(.),v} + \beta_v + 1)}{\Gamma(\sum_{r=1}^{V} n^{k,-(m,n)}_{(.),r} + \beta_r + 1)} \times \Gamma(n^{k,-(m,n)}_{m,(.)} + \alpha_k + 1)$$

$$\propto \prod_{i=1,\, i\neq k}^{K} \frac{\Gamma(n_{(.),v}^{i,-(m,n)} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{i,-(m,n)} + \beta_r)} \times \prod_{i=1, i\neq k}^{K} \Gamma(n_{j,(.)}^{i,-(m,n)} + \alpha_i)$$

$$\times \frac{\Gamma(n_{(.),v}^{k,-(m,n)} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{k,-(m,n)} + \beta_r)} \times \Gamma(n_{m,(\,.\,)}^{k,-(m,n)} + \alpha_k)$$

$$\times \frac{n_{(.),v}^{k,-(m,n)} + \beta_v}{\sum_{r=1}^{V} n_{(.),r}^{k,-(m,n)} + \beta_r} \times (n_{m,(\,.\,)}^{k,-(m,n)} + \alpha_k)$$

Definition of

Therefor,

$$= (x-1)! \times x$$

$$= \Gamma(x) \times x$$

$$
\begin{aligned}
P\Big(Z_{(m,l)} = k \mid Z_{-(m,l)}, W;\ \alpha, \beta\Big) &\propto P\Big(Z_{(m,l)} = k, Z_{-(m,l)}, W;\ \alpha, \beta\Big) \\
&\propto \prod_{i=1,\ i\neq k}^{K} \frac{\Gamma(n_{(.),v}^{i,-(m,n)} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{i,-(m,n)} + \beta_r)} \times \prod_{i=1, i\neq k}^{K} \Gamma(n_{j,(.)}^{i,-(m,n)} + \alpha_i) \\
&\times \frac{\Gamma(n_{(.),v}^{k,-(m,n)} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{k,-(m,n)} + \beta_r)} \times \Gamma(n_{m,(.)}^{k,-(m,n)} + \alpha_k) \\
&\times \frac{n_{(.),v}^{k,-(m,n)} + \beta_v}{\sum_{r=1}^{V} n_{(.),r}^{k,-(m,n)} + \beta_r} \times (n_{m,(.)}^{k,-(m,n)} + \alpha_k) \\
&\propto \prod_{i=1}^{K} \frac{\Gamma(n_{(.),v}^{i,-(m,n)} + \beta_v)}{\Gamma(\sum_{r=1}^{V} n_{(.),r}^{i,-(m,n)} + \beta_r)} \times \prod_{i=1}^{K} \Gamma(n_{j,(.)}^{i,-(m,n)} + \alpha_i) \\
&\times \frac{n_{(.),v}^{k,-(m,n)} + \beta_v}{\sum_{r=1}^{V} n_{(.),r}^{k,-(m,n)} + \beta_r} \times (n_{m,(.)}^{k,-(m,n)} + \alpha_k)
\end{aligned}
$$

$$P\left(Z_{(m,l)} = k \mid Z_{-(m,l)}, W;\ \alpha, \beta\right) \propto P\left(Z_{(m,l)} = k, Z_{-(m,l)}, W;\ \alpha, \beta\right)$$

$$\propto \frac{n_{(.),v}^{k,-(m,n)} + \beta_v}{\sum_{r=1}^{V} n_{(.),r}^{k,-(m,n)} + \beta_r} \times (n_{m,(\,.\,)}^{k,-(m,n)} + \alpha_k)$$

Finally, simplified enough to calculate the likelihood of assigning to

To become a probability, we need to normalize the above formula

Parameter Inference

LDA(Documents , ,)

Randomly, initialize assignment on

Count with the initial assignment

While performance measure converges (***perplexity***)

For to ’s document number

For to ’s document word length

Sampling from

Adjust by assigning

Calculate the most likely estimation on and

Return and

Perplexity → *We just set the iteration number*

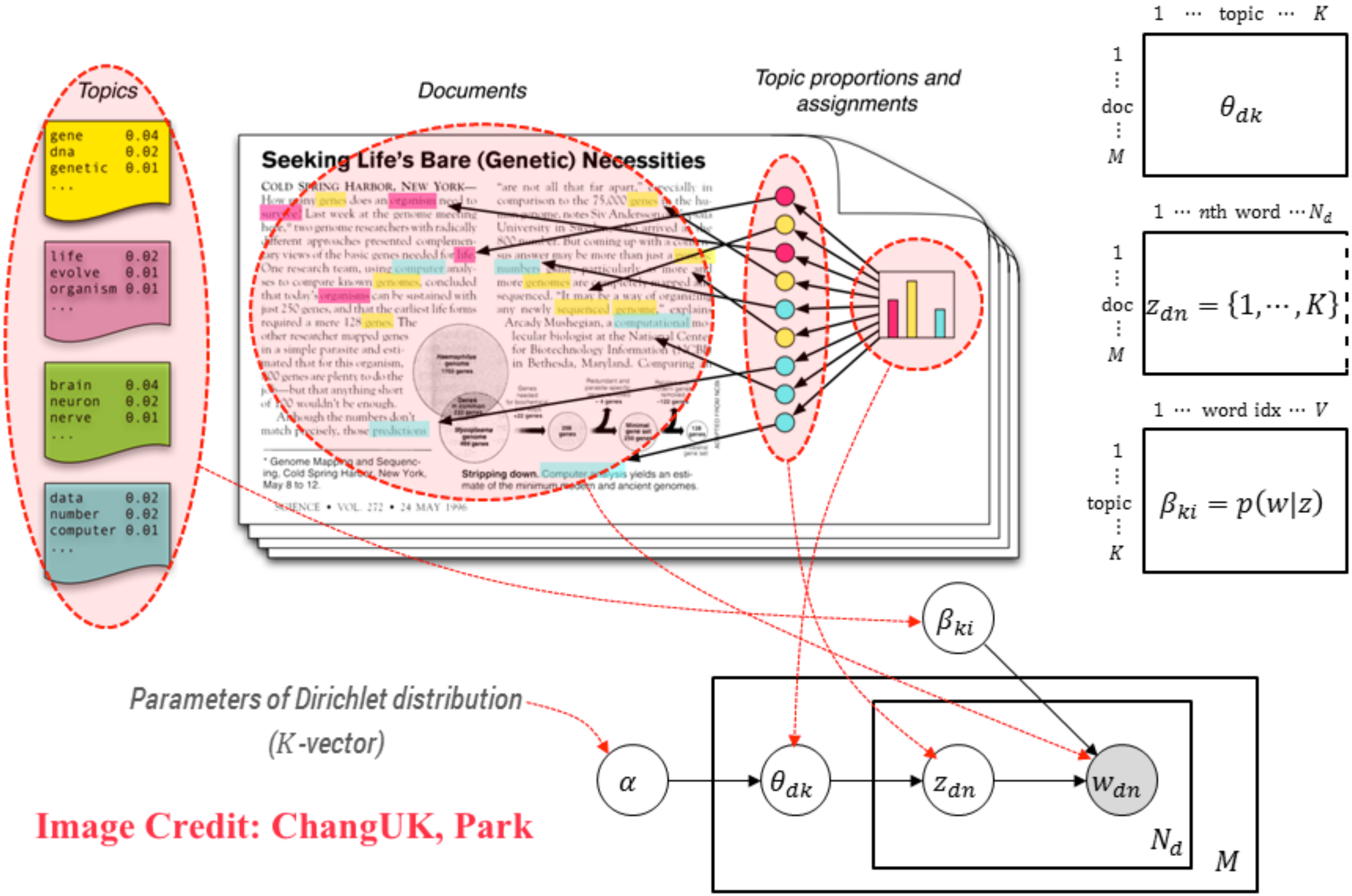


Image Credit: ChangUK, Park