Investigation of Error in Numerical Approximations

Table of Contents


```
In order to examine the rate at which numerical approximations of
$\int_0^pi \sin(x)\ dx $ converge to the actual value of the integral,
we are going to generate a scatter plot that shows the relationship
between error and the number of subintervals.
```

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end section

Inititialization

```
% The first thing we will do is choose the number experiements
 (trials) to
% perform.  Each trial will use a different number of subintervals.
numTrials = 15;


% Next we create arrays in which to record the outcomes of the trials.
% The arrays will be filled with zeros initially.  Those values will
 be
% replaced as we perform the experiments.  Each array will have 1 row
 with
% numTrials entries.

% In this array we will record the number of subintervals that we use
 to
% make an approximation.
numSubintervalsTrap = zeros(1, numTrials);  % an array filled with
 zeros

% In this array we will record the magnitude of the error in the
% approximation.
errorMagnitudeTrap = zeros(1, numTrials);   % an array filled with
 zeros
```

```
% end section
```

Error Experiment: Trapezoid Rule

```
% In this section we use the Trapezoid Rule to compute approximations
 of
% the integral with different numbers of subintervals.  We record
 outcomes
% in the arrays that we prepared above, and then display the results
 as a
% scatter plot.

for k = 1:1:numTrials
     % The counter k tells us which trial we're conducting.  The
 number of
     % subintervals increases by 5 with each new trial
     n = 5*k;

     % Compute the size of each subinterval
     Dx = pi/n;

     % Record the partition points at the endpoints of the
 subintervals
     x = 0:Dx:pi;

     % Calculate the function value at the partition points.  Note
 that f
     % is an array of numbers because x is an array of numbers.
     f = sin(x);

     % Assemble the array of coefficients for the Trapezoid Rule
     c = ones(1, length(f));
     c(2:end-1) = 2;

     % Calculate the Trapezoid Rule approximation
     T = sum(c.*f)*Dx/2;

     % Now we record the results of trial k in position k of our
 arrays
     % First, record the number of subintervals
     numSubintervalsTrap(k) = n;
     % Next, record the error in trial k
     errorMagnitudeTrap(k) = abs(2-T);
end

% Next we generate a scatter plot that shows the relationship between
 the
% number of subintervals, n, and the error in the approximation.
figure('Color','white')
hold on
    scatter( numSubintervalsTrap, errorMagnitudeTrap,
 75, 'filled', 'b')
```

```
    title('Number of subintervals and corresponding absolute error')
    xlabel('Number of subintervals')
    ylabel('Absolute error')

    grid on
    grid minor

hold off

% end section
```

A Power Law for Error in the Trapezoid Rule

Put MATLAB commands here

```
X = log( numSubintervalsTrap );
Y = log( errorMagnitudeTrap );

m = ( Y(end) - Y(1) )/(X(end) - X(1) );
s = sprintf('the slope of the line in the log-log plot is %f', m');

figure
hold on
scatter(X, Y, 75, 'filled', 'blue')
grid on %toggles the major grid lines
grid minor %toggles the minor grid lines
title(s)
xlabel( 'log(number of subintervals)' )
ylabel( 'log(absolute error)' )
hold off
% end section
```

Error Experiment: Simpson's method

Put MATLAB commands here

```
numTrials = 15;
numSubintervalsSimp = zeros(1, numTrials);
errorMagnitudeSimp = zeros(1, numTrials);

function integral = simpsonsrule(f,a,b,n)

h = (b-a)/n;
x = linspace(a,b,n);
x4=0;
x2=0;
for j=2:2:b
    x4 = x4 + f(x4);
end
for k=3:2:b
    x2= x2 + f(x2);

end
```

```
integral = (h/3)*(f(a)+ f(b) + 4*(x4)+ 2*(x2));
end

clear;
integral = simpsonsrule(@(x) sin(x), 0, pi, 15);
% end section
```

A Power Law for Error in Simpson's method

Put MATLAB commands here

```
X = log( numSubintervalsTrap );
Y = log( errorMagnitudeTrap );

m = ( Y(end) - Y(1) )/(X(end) - X(1) );
s = sprintf('the slope of the line in the log-log plot is %f', m');

figure
hold on
scatter(X, Y, 75, 'filled', 'blue')
grid on %toggles the major grid lines
grid minor %toggles the minor grid lines
title(s)
```

$e^{\pi i} + 1 = 0$

```
xlabel( 'log(number of subintervals)' )
ylabel( 'log(absolute error)' )
hold off
% end section
```

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