Bayesian Knowledge Tracing and Other Predictive Models

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Dr. Neil Heffernan, Advisor, WPI - Computer Science
Dr. Ryan S.J.d. Baker, WPI - Social Science and Policy Studies
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Outline of Talk

- Introduction to Knowledge Tracing
  - History
  - Intuition
  - Generative example
  - Influence of parameters
    - Demo?
  - Prior Per Student model
  - Variations (and other models)
  - MATLAB Code demo

Knowledge Tracing

History in the literature

- Introduced in 1995 (Corbett & Anderson)
- Four parameter simplification of ACT-R theory of skill acquisition (Anderson 1993)
- Computations based on a variation of Bayesian calculations proposed in 1972 (Atkinson)
- Formalized as equivalent to a Dynamic Bayesian Network (Rye, 2004) "Student modeling based on belief networks"

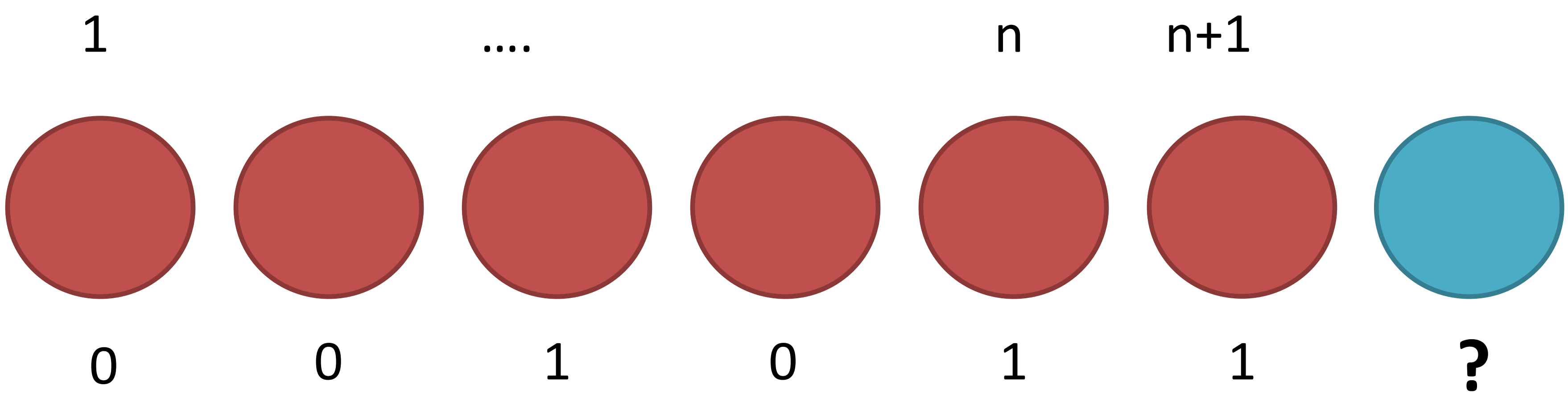
Knowledge Tracing

Real world deployment

- Used in the Cognitive Tutors (Carnegie Learning) to determine when a student has mastered a skill and can move on in the curriculum
- Replies on a skill model (tagging of skills to items)
- Parameters of the model can be learned with Expectation Maximization (EM) or grid search

For some Skill K:

Given a student’s response sequence 1 to n, predict n+1

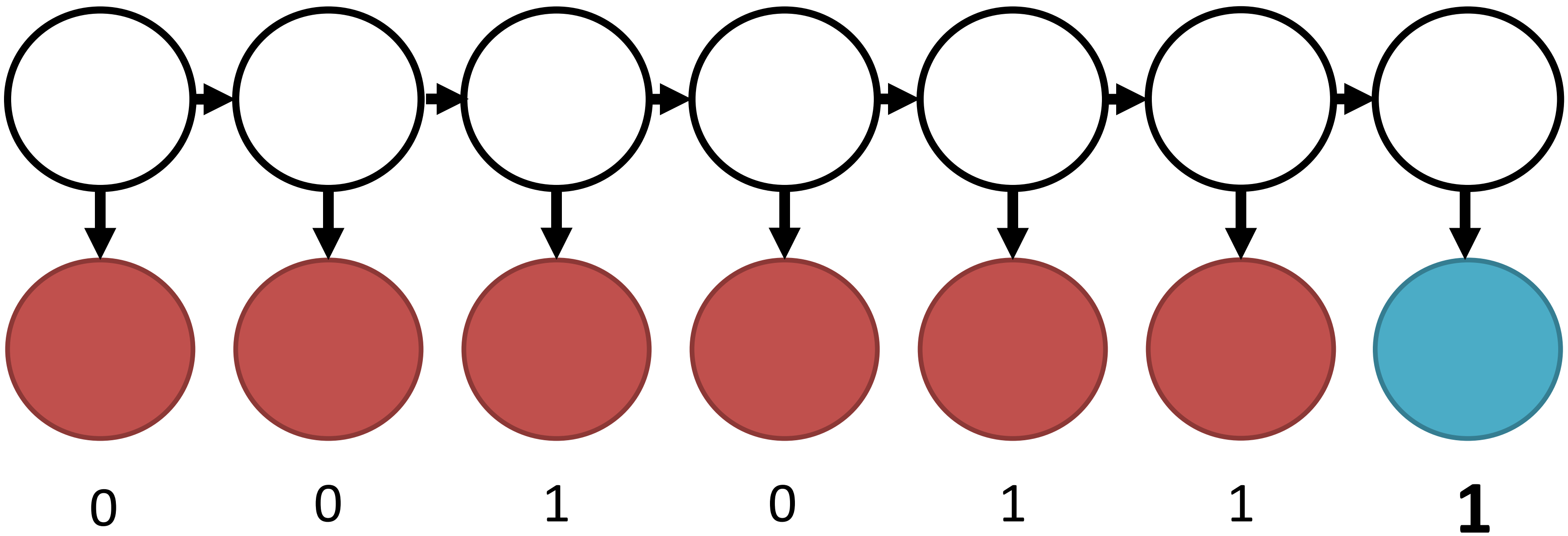


Chronological response sequence for student *Y*

[0 = Incorrect response   1 = Correct response]

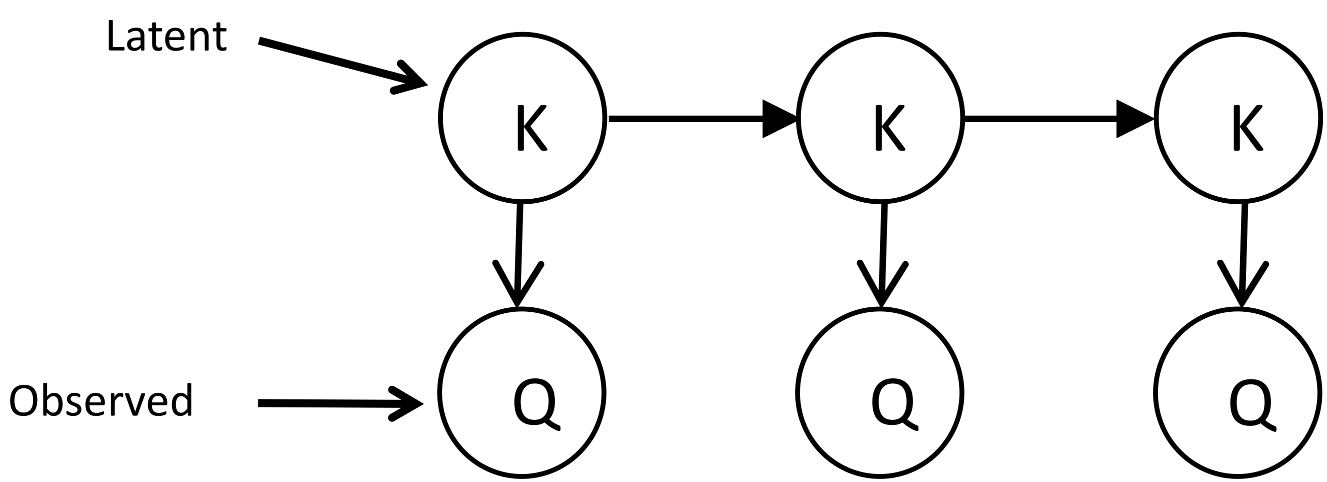
Intro to Knowledge Tracing

Track knowledge over time
(model of learning)



Intro to Knowledge Tracing

Knowledge Tracing (KT) can be represented as a simple HMM



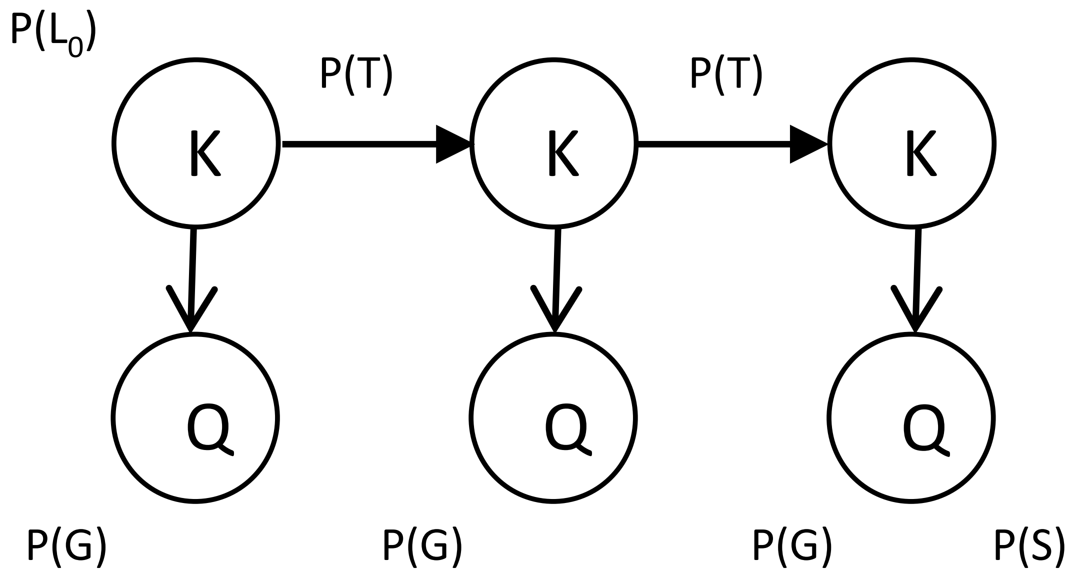
<u>Node representations</u>
K = Knowledge node
Q = Question node

<u>Node states</u>
K = Two state (0 or 1)
Q = Two state (0 or 1)

Intro to Knowledge Tracing

Four parameters of the KT model:

$P(L_0)$ = Probability of initial knowledge
$P(T)$ = Probability of learning
$P(G)$ = Probability of guess
$P(S)$ = Probability of slip



Probability of forgetting assumed to be zero (fixed)

Intro to Knowledge Tracing

Formulas for inference and prediction

$$If\ Correct_n$$

$$P(L_{n-1}) = \frac{P(L_{n-1})*(1-P(S))}{P(L_{n-1})*(1-P(S)) + (1-P(L_{n-1}))*(P(G))} \qquad (1)$$

$$Incorrect_n$$

$$P(L_{n-1}) = \frac{P(L_{n-1})*P(S)}{P(L_{n-1})*P(S) + (1-P(L_{n-1}))*(1-P(G))} \qquad (2)$$

$$P(L_n) = P((L_{n-1}) * (1 - P(F)) + ((1 - P(L_{n-1})) * P(T)) \qquad (3)$$

- Derivation (Reye, JAIED 2004):

$$p(L_{n-1} \mid C_n) = \frac{p(C_n \mid L_{n-1})p(L_{n-1})}{p(C_n \mid L_{n-1})p(L_{n-1}) + p(\neg L_{n-1})p(C_n \mid \neg L_{n-1})}$$

- Formulas use Bayes Theorem to make inferences about latent variable

Knowledge Tracing

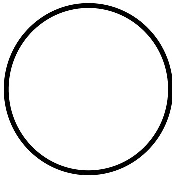
How a Bayesian Knowledge Tracing World Works

Generative - Example

How a Bayesian Knowledge Tracing World Works

Prior = 0.40

$P(L_0)$



Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40

$P(L_0)$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40

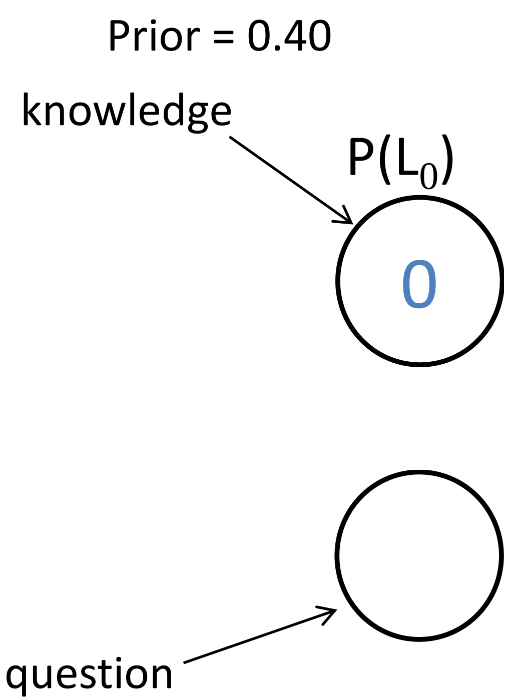
knowledge

$P(L_0)$

0

Knowledge Tracing

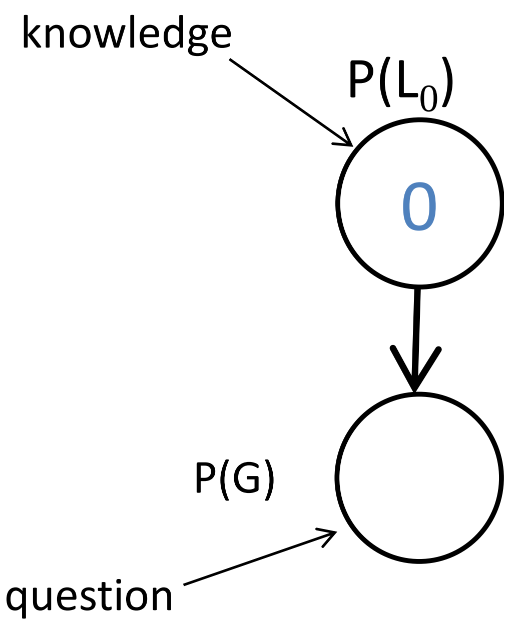
How a Bayesian Knowledge Tracing World Works



Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

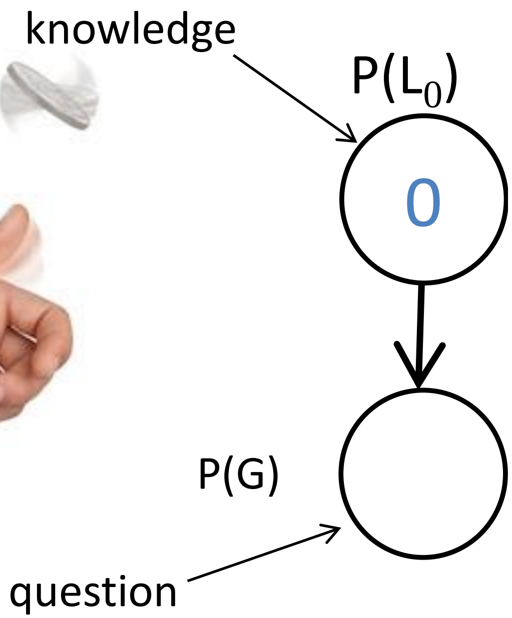
Prior = 0.40 Guess = 0.14



Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

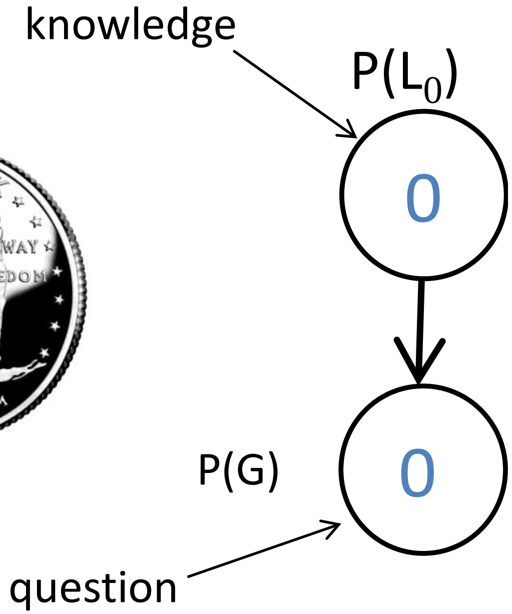
Prior = 0.40   Guess = 0.14



Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

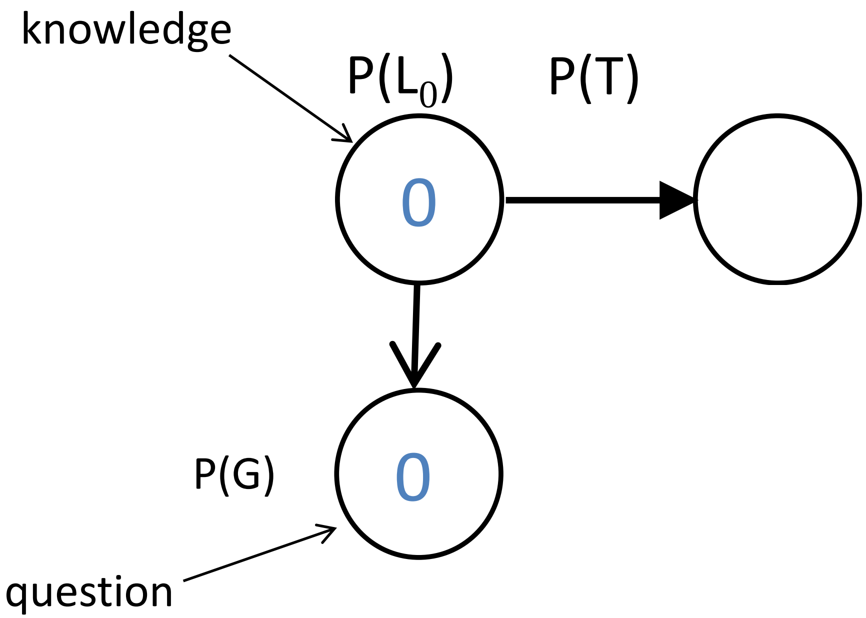
Prior = 0.40   Guess = 0.14



Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20



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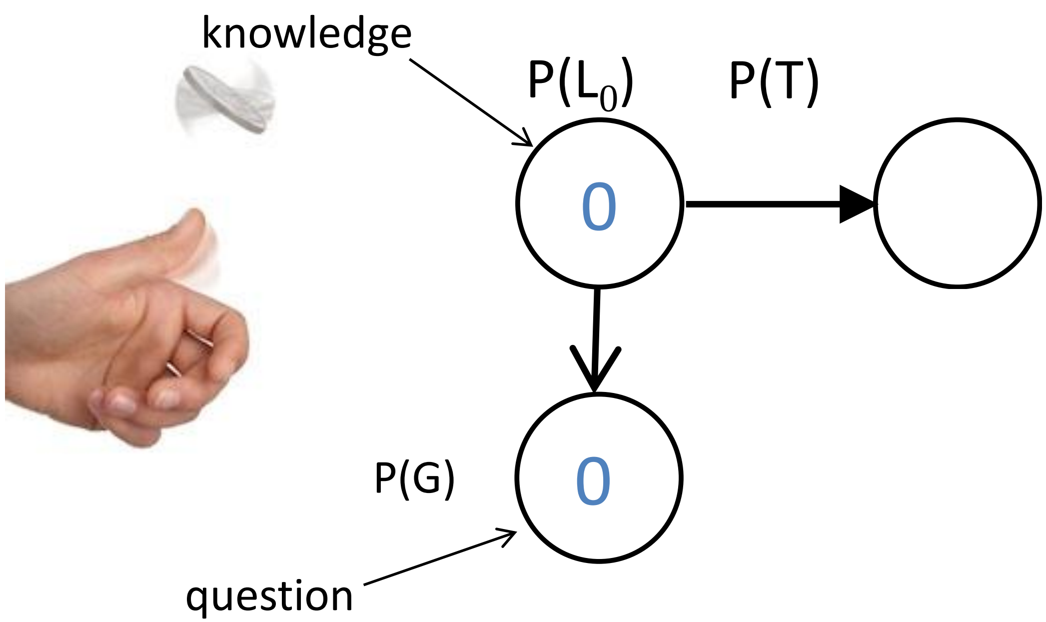

Knowledge Tracing

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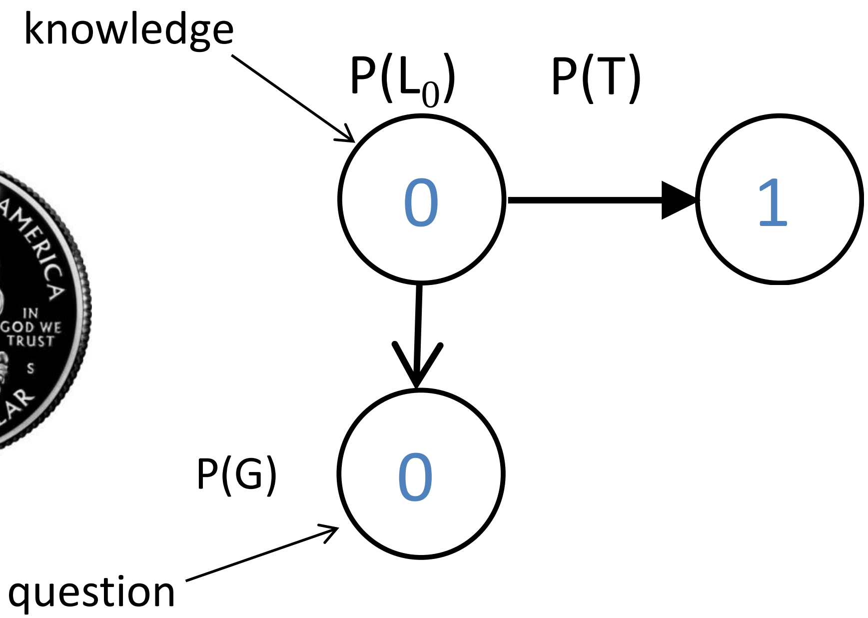
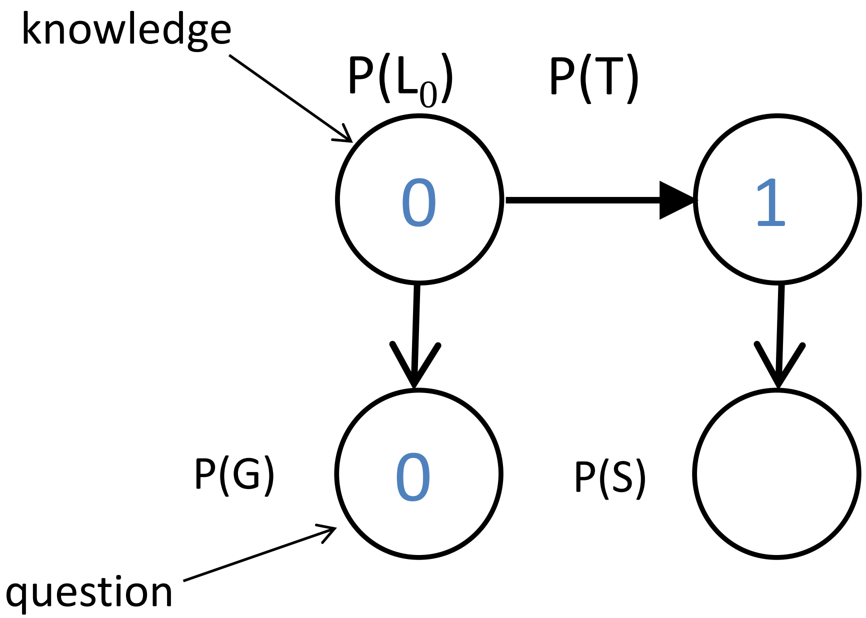


Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



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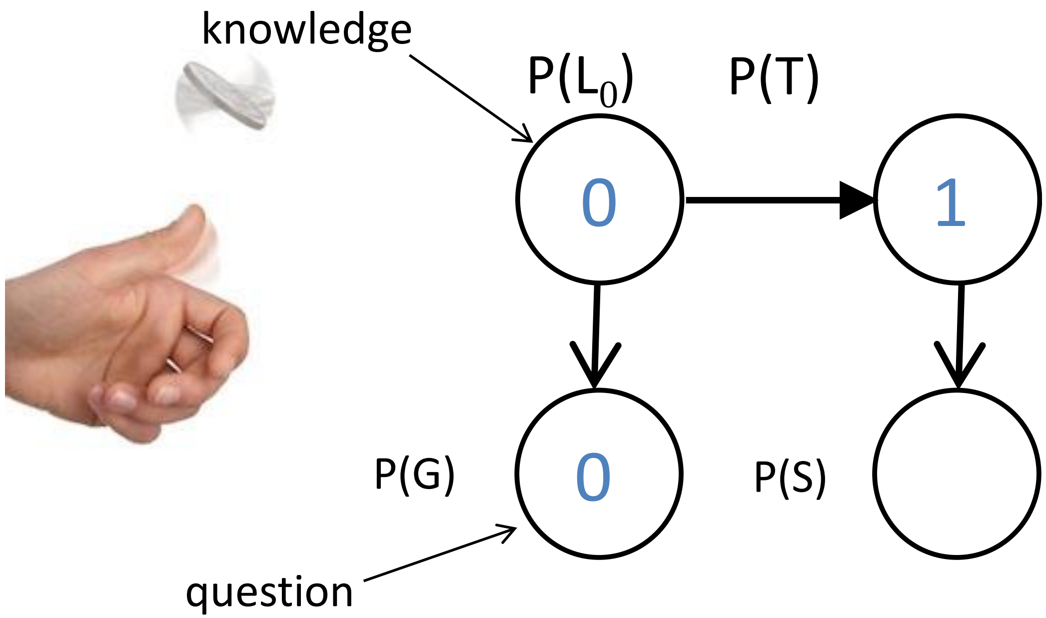

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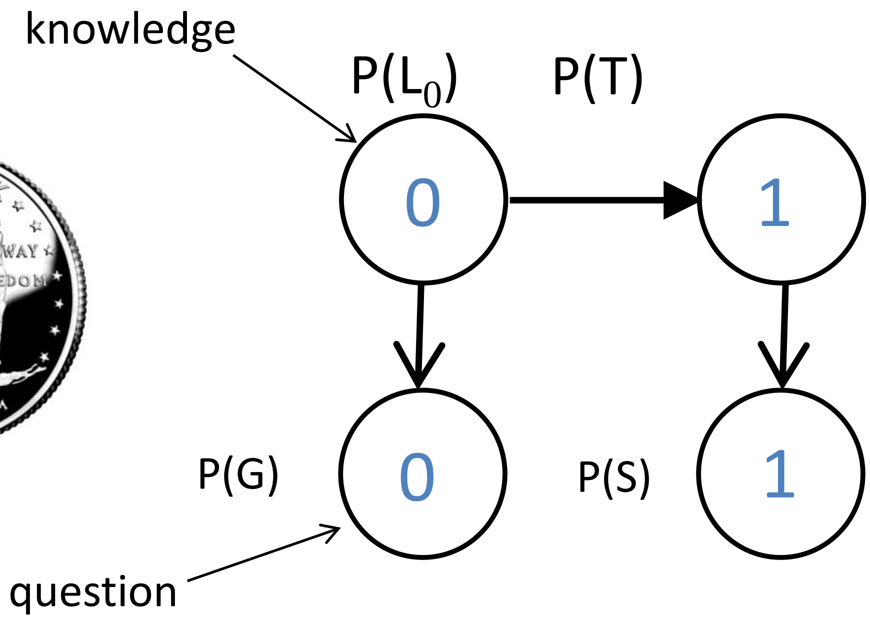

Knowledge Tracing

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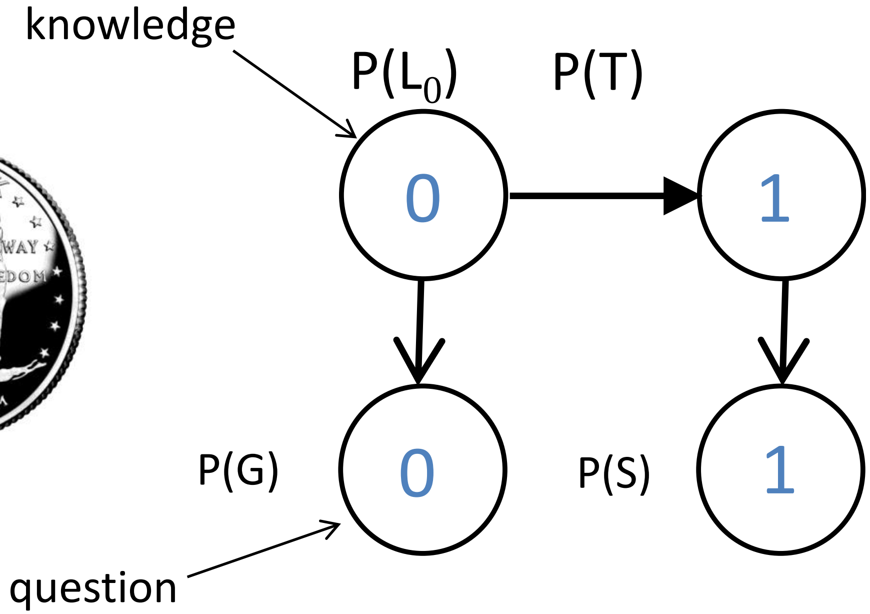


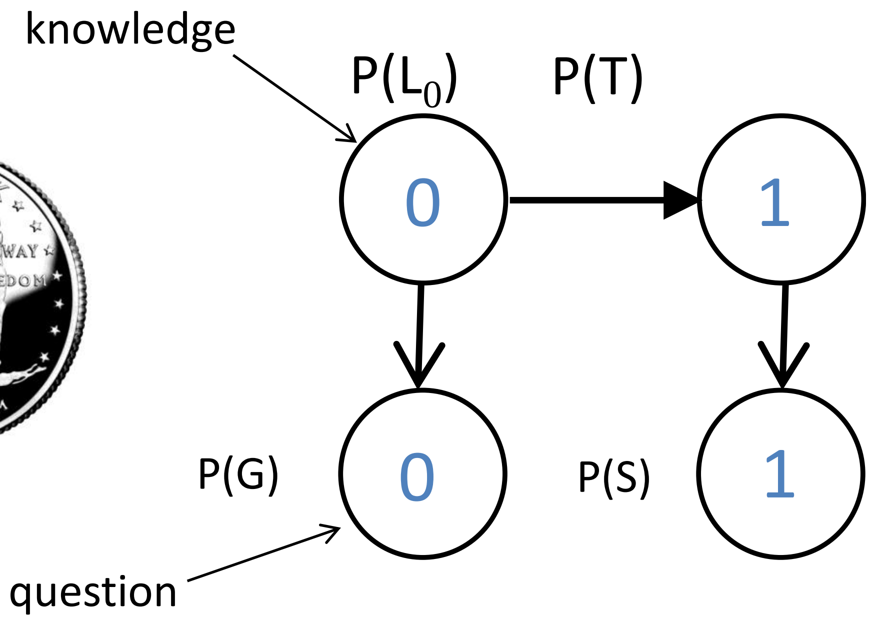
Generalization of the response prediction calculation:

$$P(Correct_n) = \mathrm{P}(L_n)\big(1 - P(S)\big) + (1 - \mathrm{P}(L_n))\mathrm{P(G)}$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



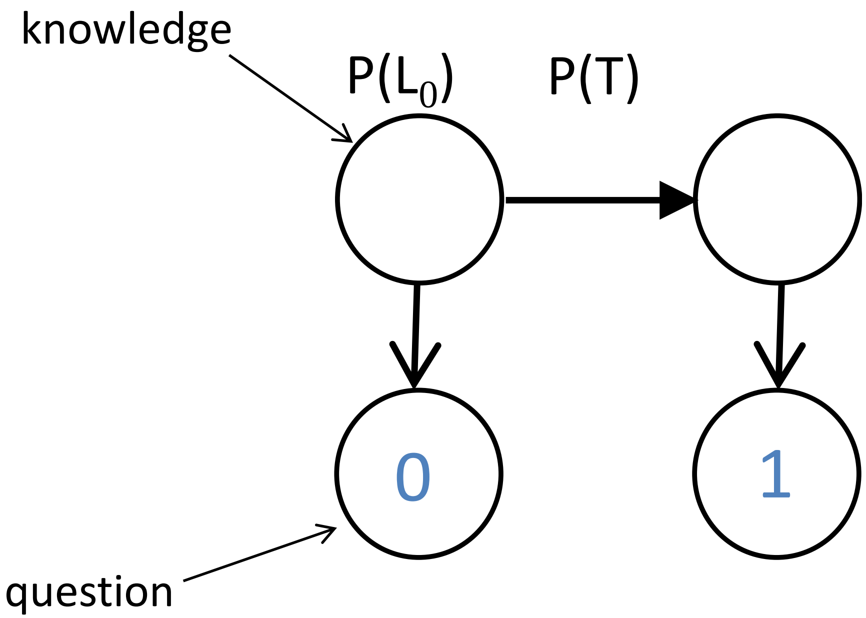
Generalization of the probability of learning calculation:

$$P(L_{n+1}) = \mathrm{P}(L_n) + (1 - \mathrm{P}(L_n))\mathrm{P(T)}$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

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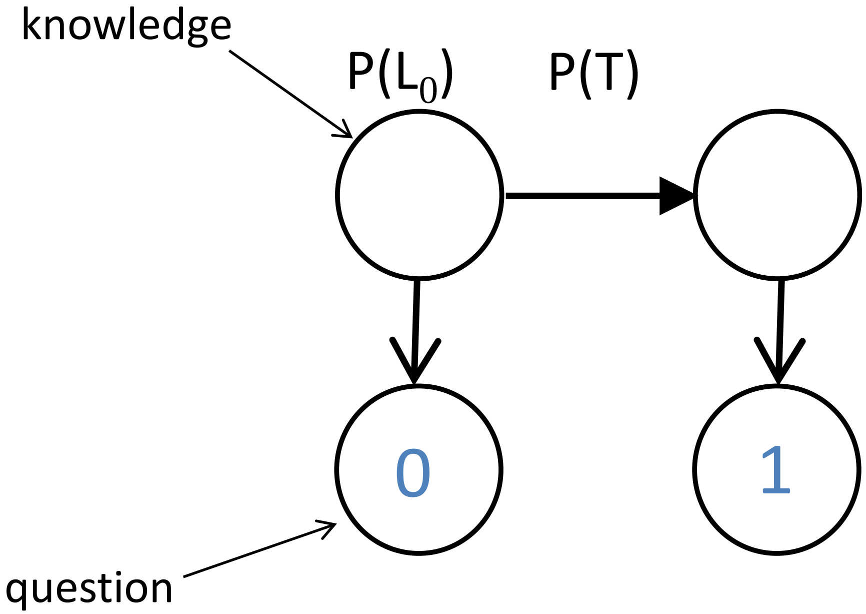


You want to infer $\mathrm{P}(L_n)$ from the student's responses

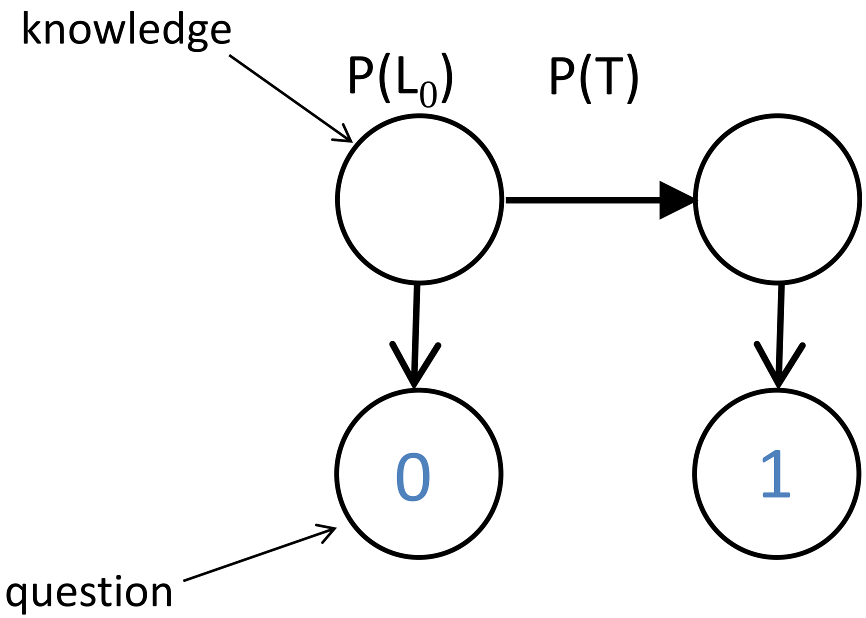
First, infer the knowledge at the first opportunity:

$$P(Knowledge|Response = 0) = \frac{\mathrm{P}(L_0)P(S)}{\mathrm{P}(L_0)P(S) + (1 - P(L_0))(1 - P(G))}$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

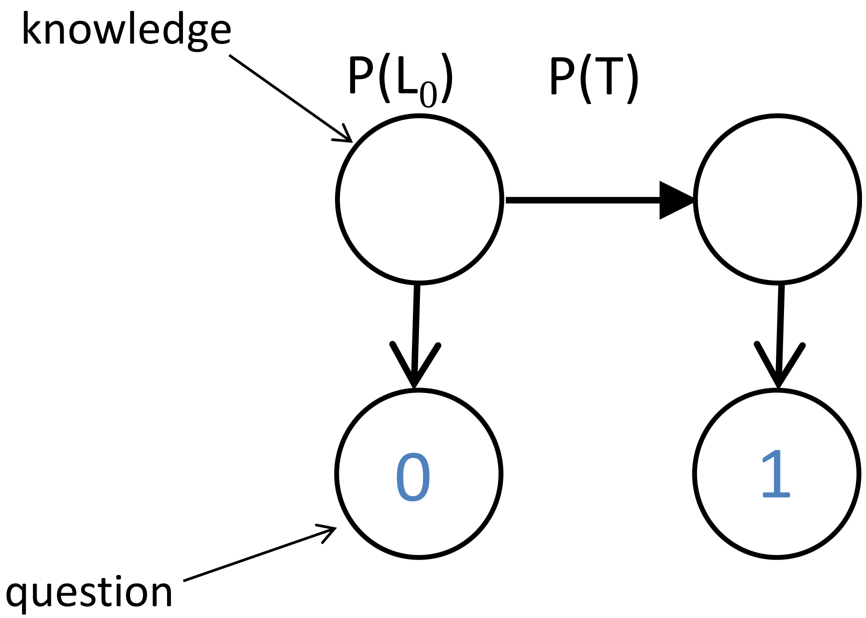
First, infer the knowledge at the first opportunity:

$$P(Knowledge|Response = 0) = \frac{0.40 \cdot P(S)}{0.40 \cdot P(S) + (1 - 0.40)(1 - P(G))}$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

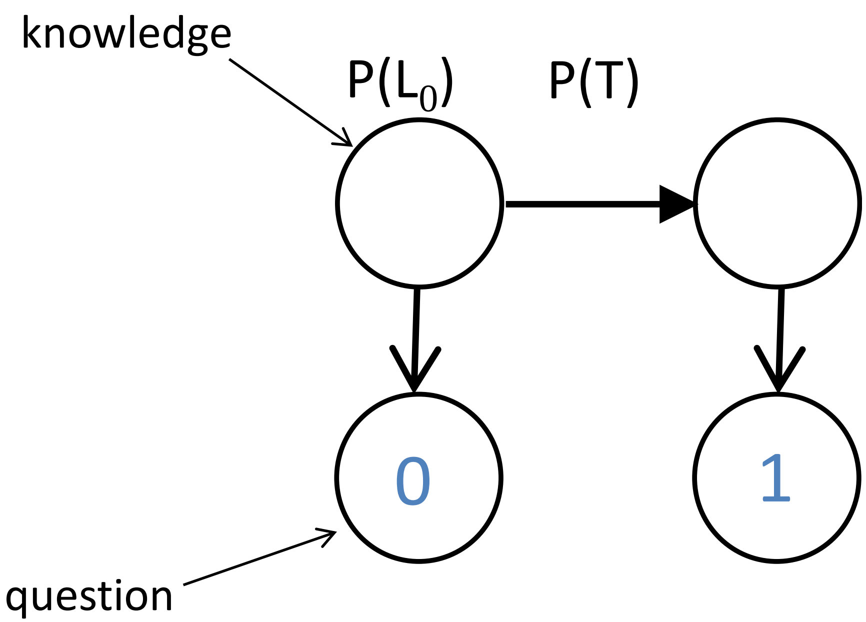
First, infer the knowledge at the first opportunity:

$$P(Knowledge|Response = 0) = \frac{0.40 \cdot 0.05}{0.40 \cdot 0.05 + (1 - 0.40)(1 - P(G))}$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

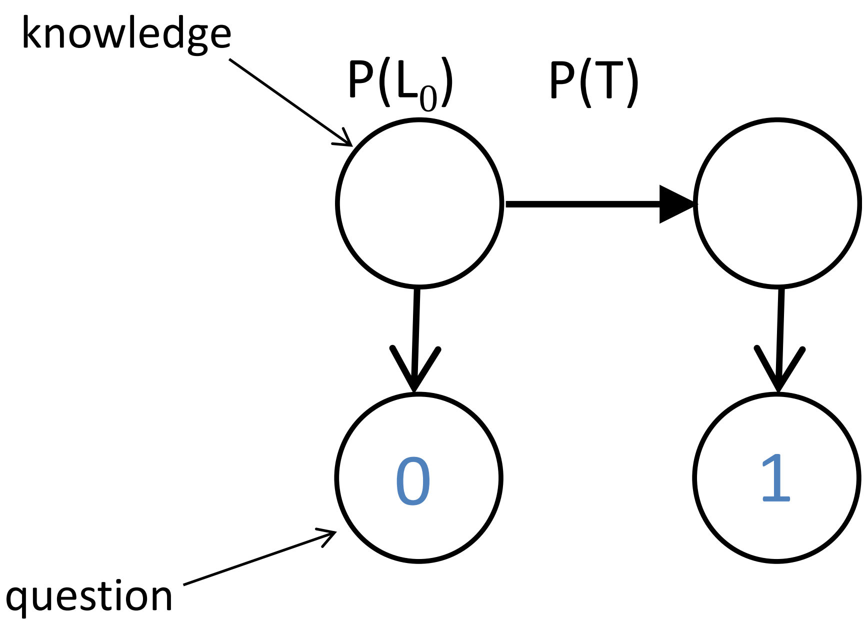
First, infer the knowledge at the first opportunity:

$$P(Knowledge|Response = 0) = \frac{0.40 \cdot 0.05}{0.40 \cdot 0.05 + (1 - 0.40)(1 - 0.14)}$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

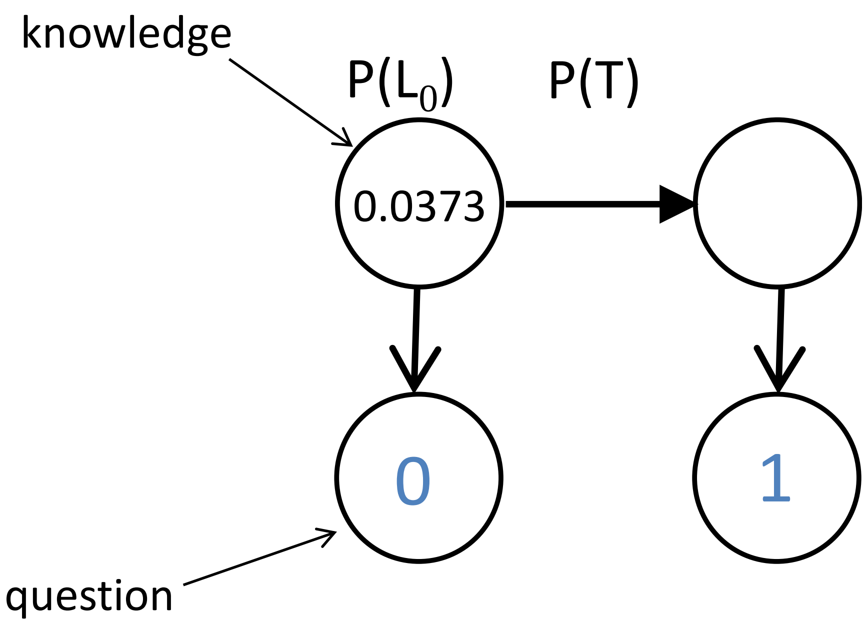
First, infer the knowledge at the first opportunity:

$$P(Knowledge|Response = 0) = \frac{0.40 \cdot 0.05}{0.40 \cdot 0.05 + (1 - 0.40)(1 - 0.14)} =$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

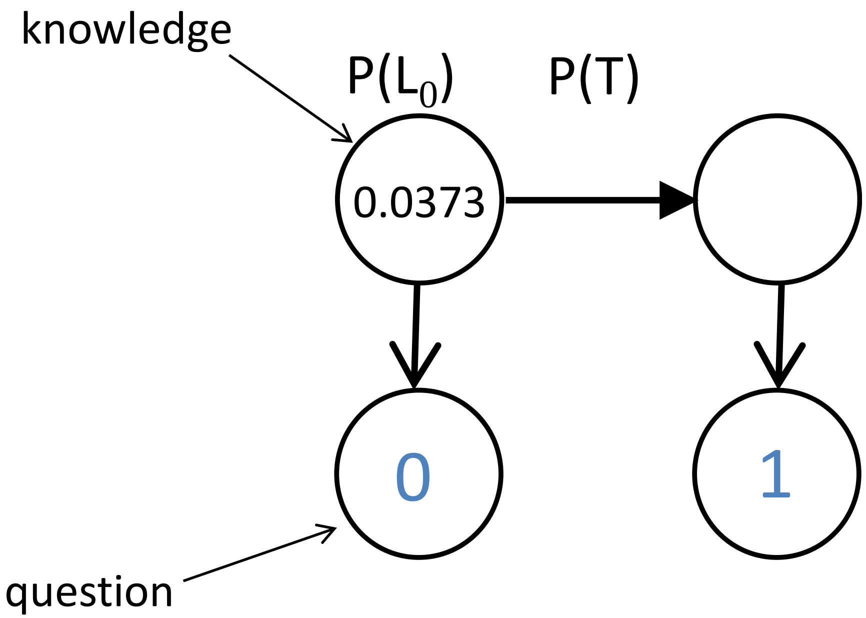
First, infer the knowledge at the first opportunity: Posterior probability of knowledge

$$P(Knowledge|Response = 0) = \frac{0.40 \cdot 0.05}{0.40 \cdot 0.05 + (1 - 0.40)(1 - 0.14)} = 0.0373$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

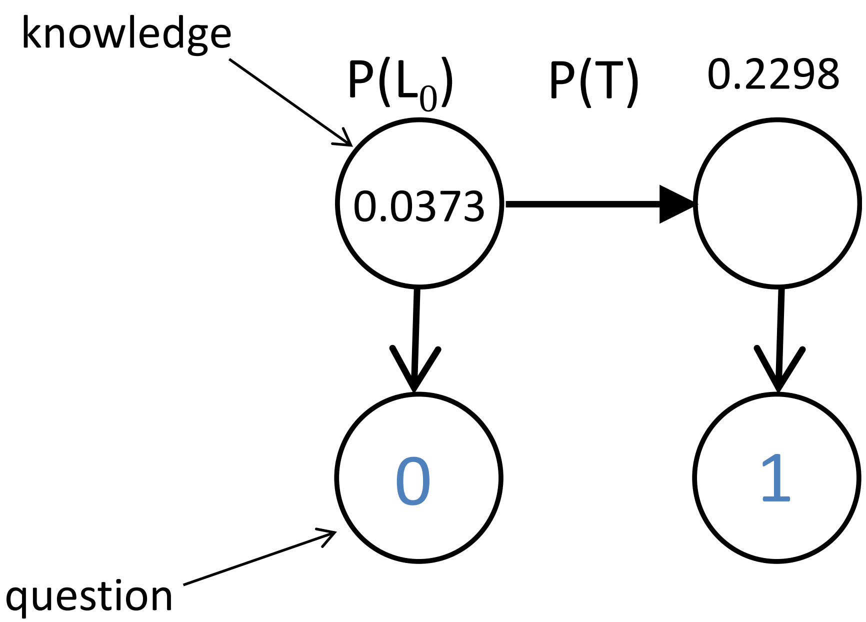
Next, apply the learning transition formula:

$$P(L_{n+1}) = 0.0373 + (1 - 0.0373)(0.20) =$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

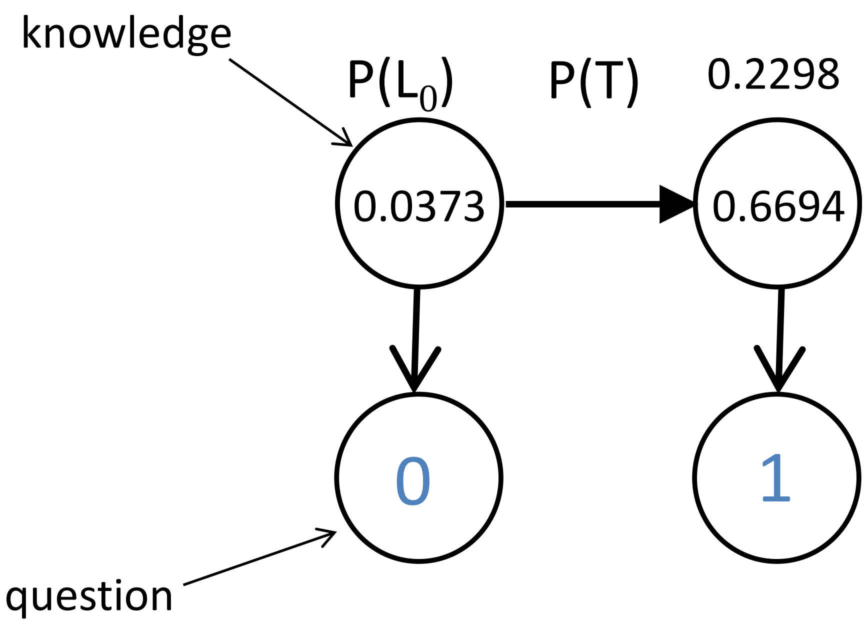
Next, apply the learning transition formula:

$P(L_{n+1}) = 0.0373 + (1 - 0.0373)(0.20)$ =0.2298 New prior for L_{n+1}

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

Done!

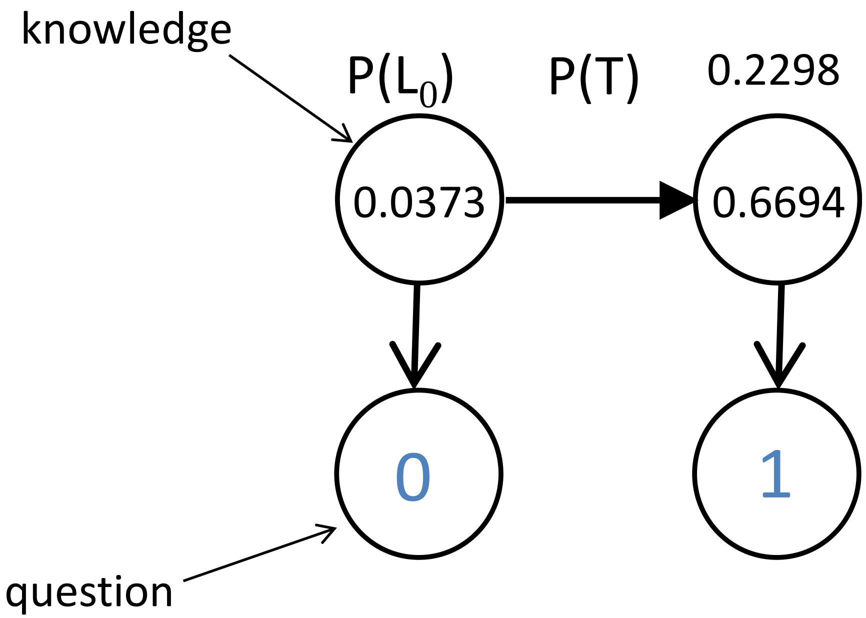
Lastly, infer the knowledge at the second opportunity:

$$P(Knowledge|Response = 1) = \frac{0.2298 \cdot (1 - 0.05)}{0.2298 \cdot (1 - 0.05) + (1 - 0.2298) \cdot 0.14} = 0.6694$$

Knowledge Tracing

How a Bayesian Knowledge Tracing World Works

Prior = 0.40 Guess = 0.14 Learn = 0.20 Slip = 0.05



You want to infer $\mathrm{P}(L_n)$ from the student's responses

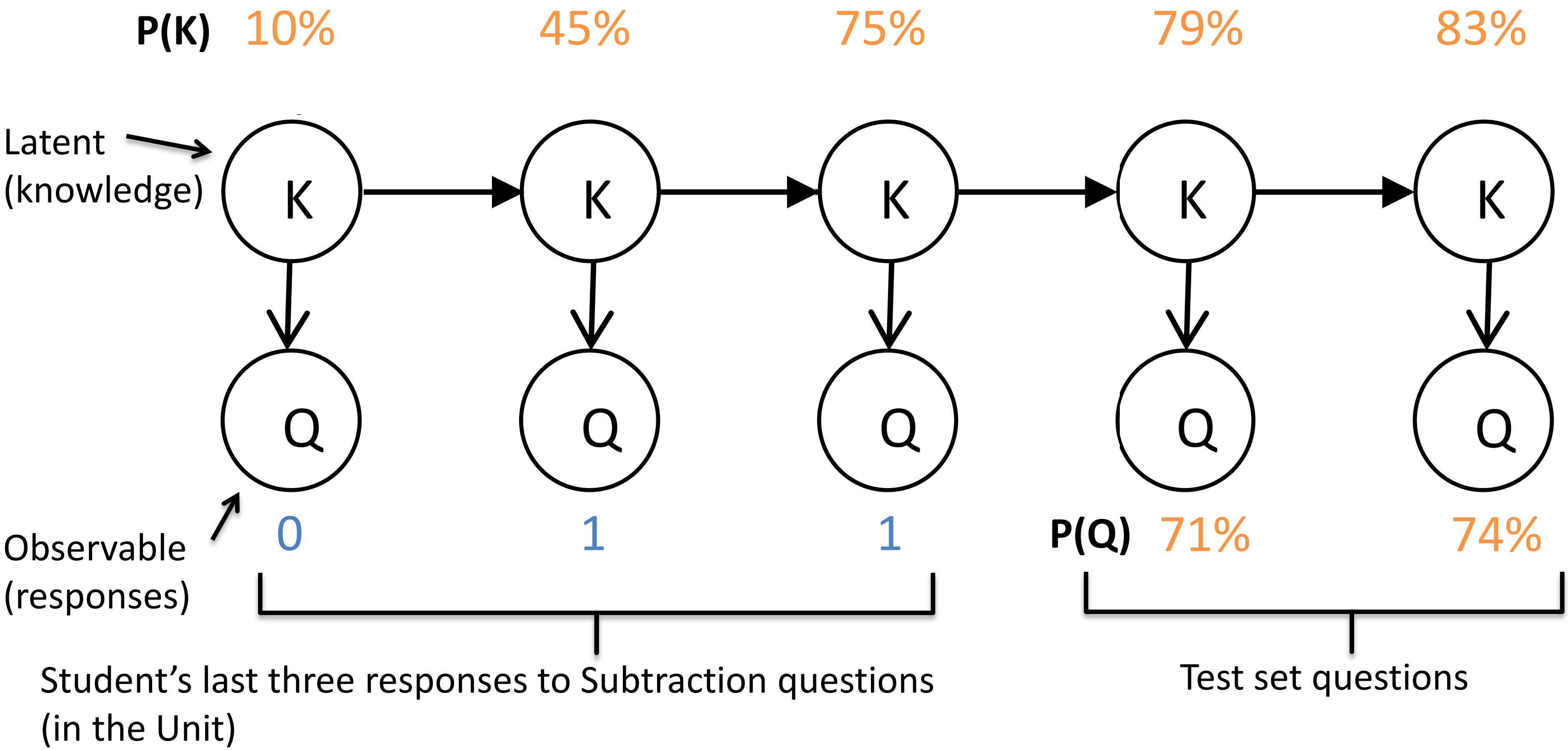
Done!

Inference calculations are applications of Bayes theorem: $P(K|\mathrm{Q}) = \dfrac{P(Q|K)P(K)}{P(Q)}$

Intro to Knowledge Tracing

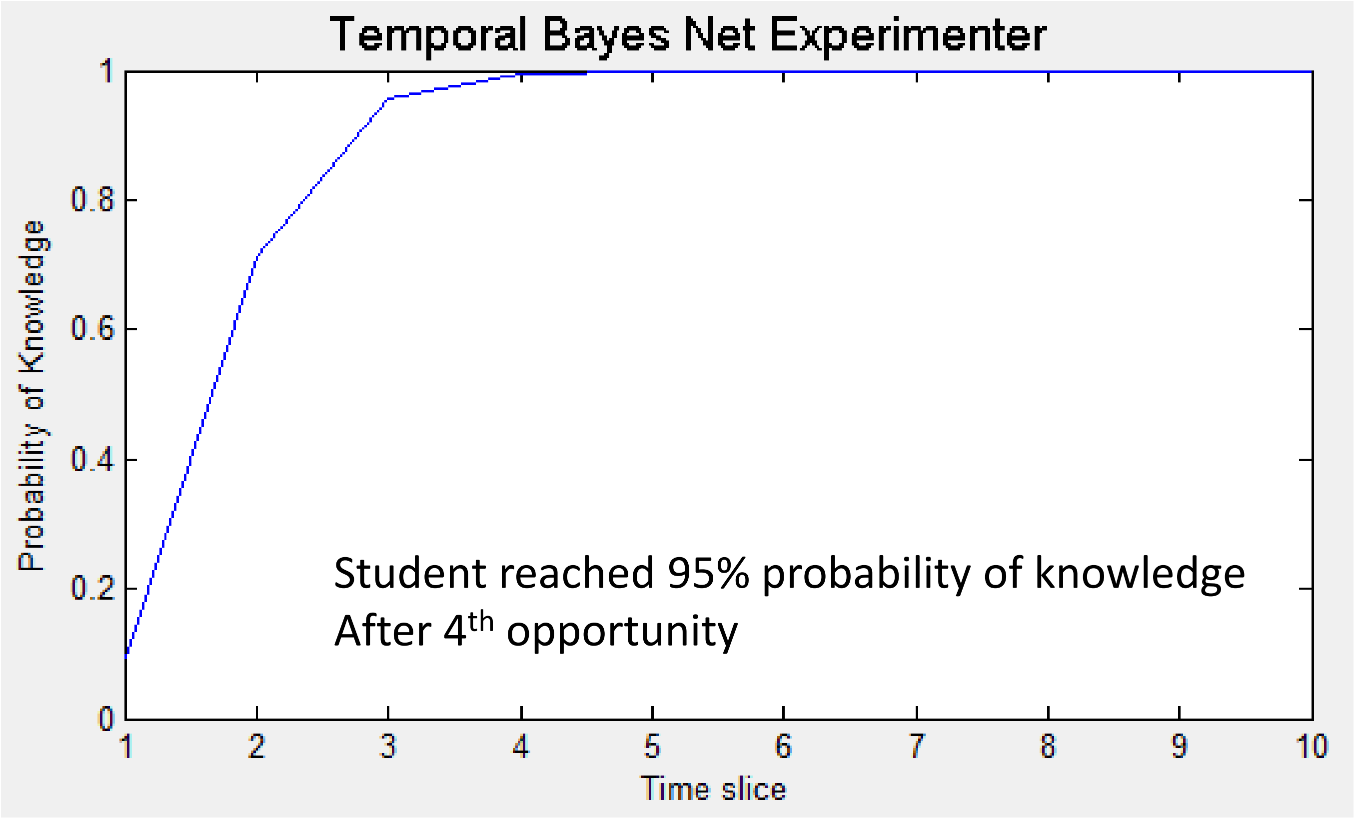
Model Prediction

Model Tracing Step – Skill: Subtraction



Influence of parameter values

Estimate of knowledge for student with response sequence: 0 1 1 1 1 1 1 1 1 1

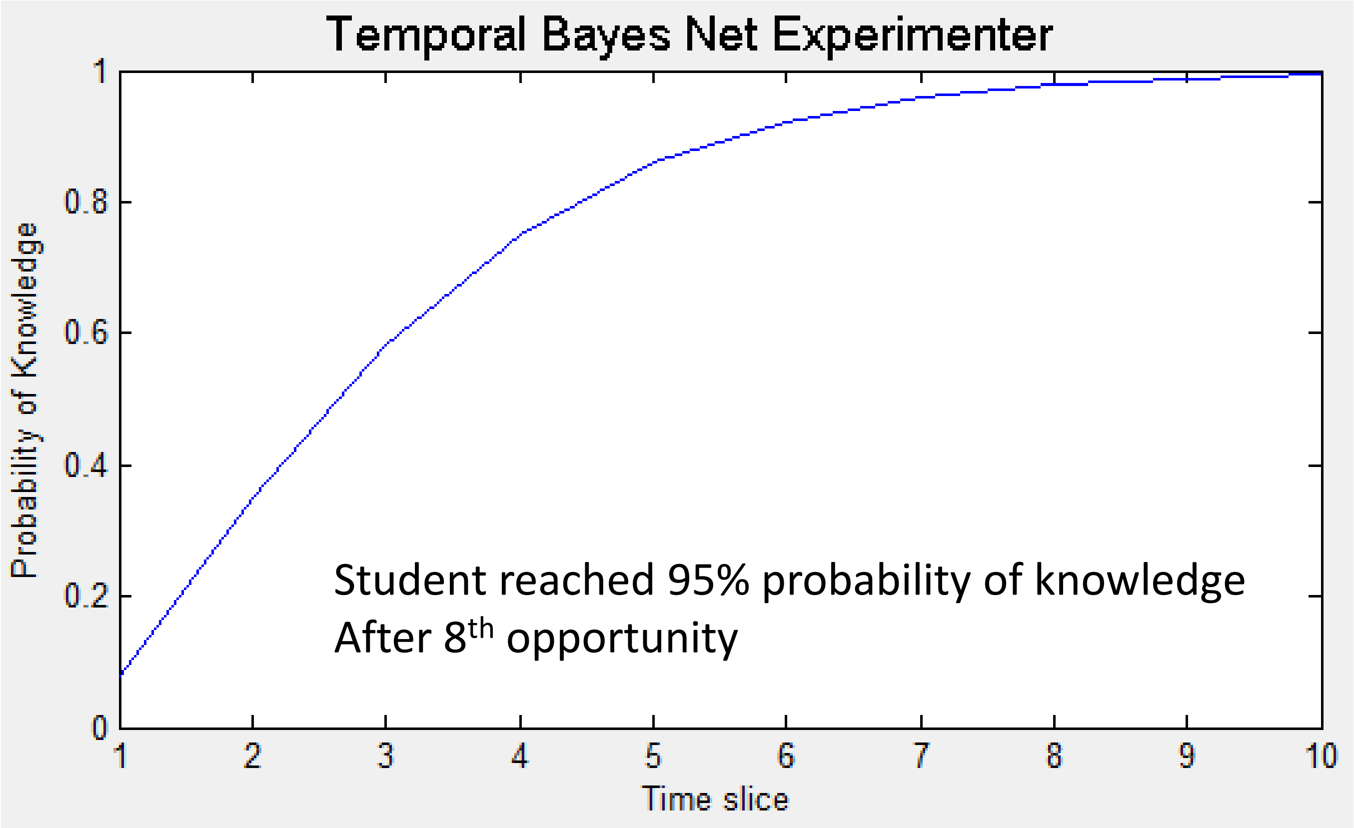


$P(L_0)$: 0.50 P(T): 0.20 P(G): 0.14 P(S): 0.09

Intro to Knowledge Tracing

Influence of parameter values

Estimate of knowledge for student with response sequence: 0 1 1 1 1 1 1 1 1 1



$P(L_0)$: 0.50 P(T): 0.20 P(G): 0.14 P(S): 0.09

$P(L_0)$: 0.50 P(T): 0.20 **P(G): 0.64 P(S): 0.03**

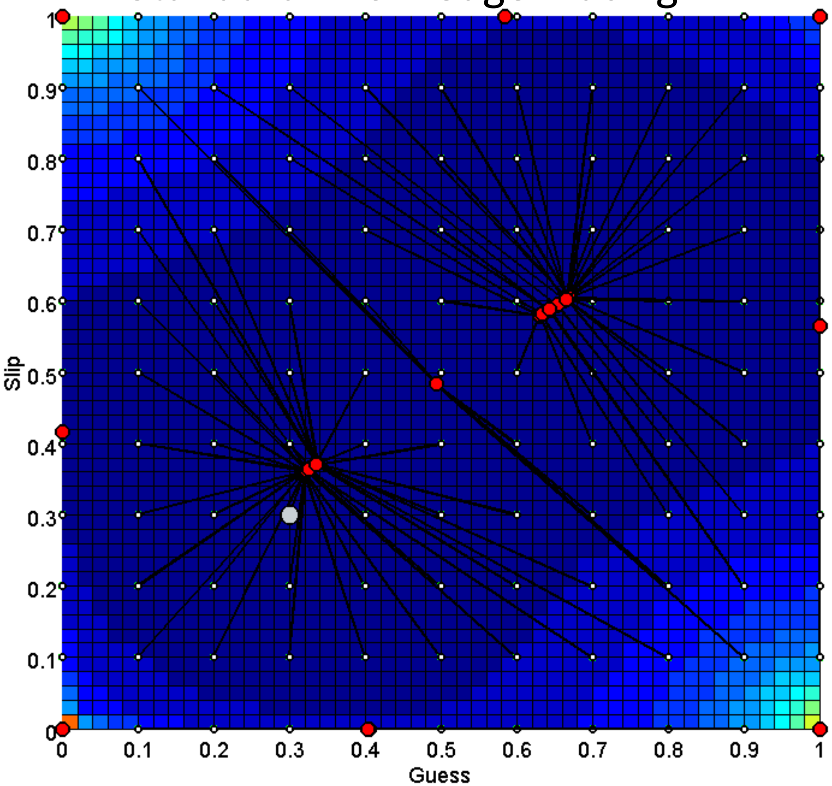
(Demo)

Parameter fitting

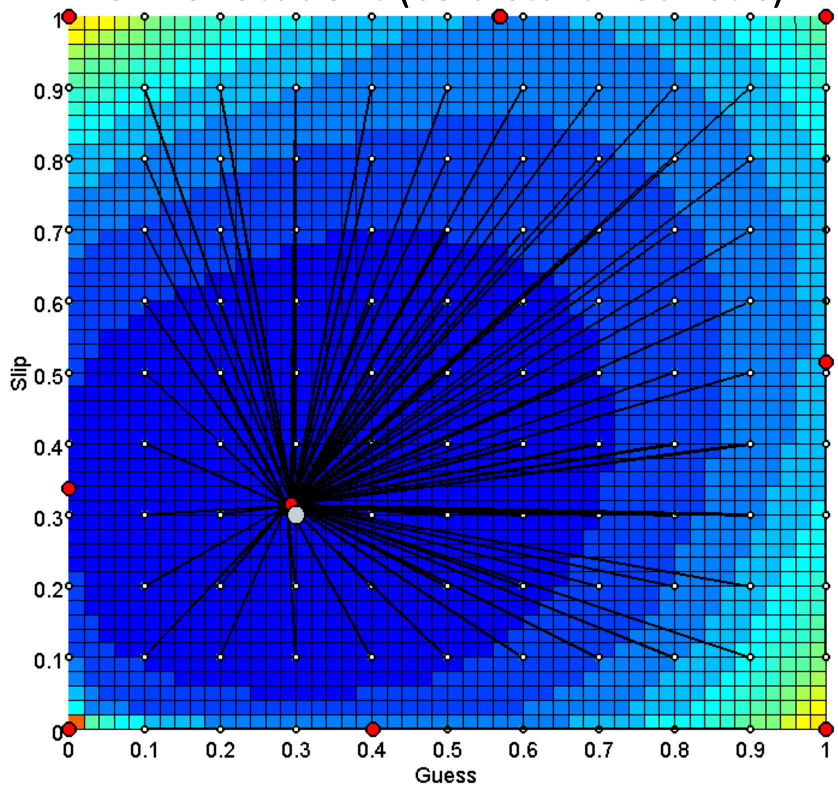
-EM, Grid-search, Spectral DS (Gordon)

-1st workshop on Parameter fitting (ITS 2012)

Standard Knowledge Tracing



Prior Per Student (cold start heuristic)



Pardos, Z. A., Heffernan, N. T. In Press (2010) **Navigating the parameter space of Bayesian Knowledge Tracing models: Visualizations of the convergence of the Expectation Maximization algorithm**. In *Proceedings of the 3rd International Conference on Educational Data Mining. Pittsburg*

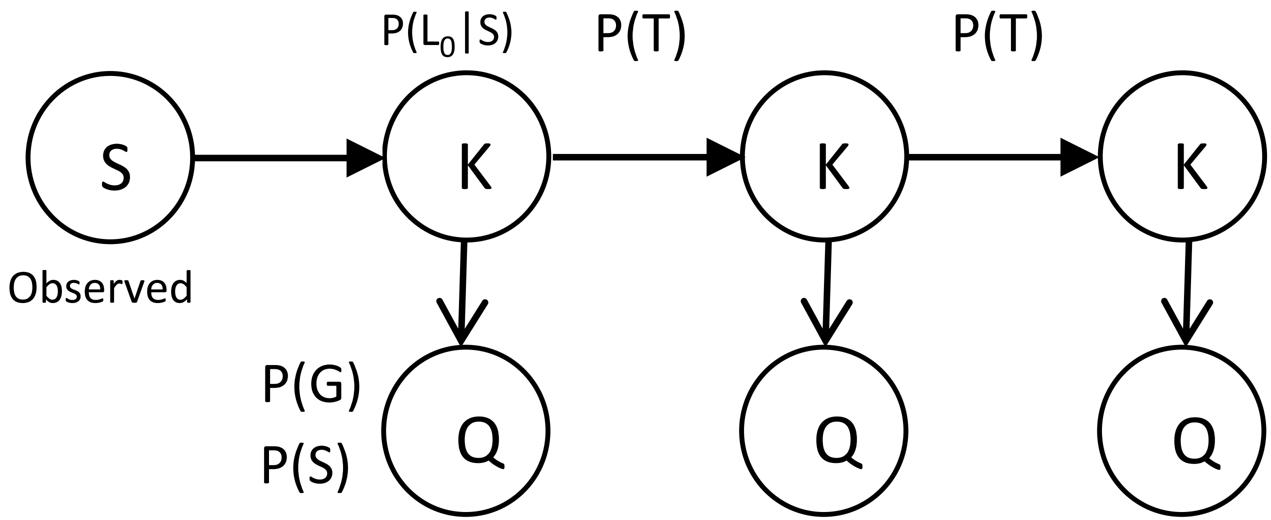
Prior Per Student Model

Student Individualization

- Knowledge Tracing, the current state of the art in knowledge assessment
  - Has no student specific parameters
    - Individual prior knowledge
    - Individual learn rates
  - Research objective is to add individualization to improve knowledge assessment and prediction accuracy.

Prior Individualization Approach

Do all students enter a lesson with the same background knowledge?



Node representations
K = Knowledge node
Q = Question node
S = Student node

Node states
K = Two state (0 or 1)
Q = Two state (0 or 1)
S = Multi state (1 to N)

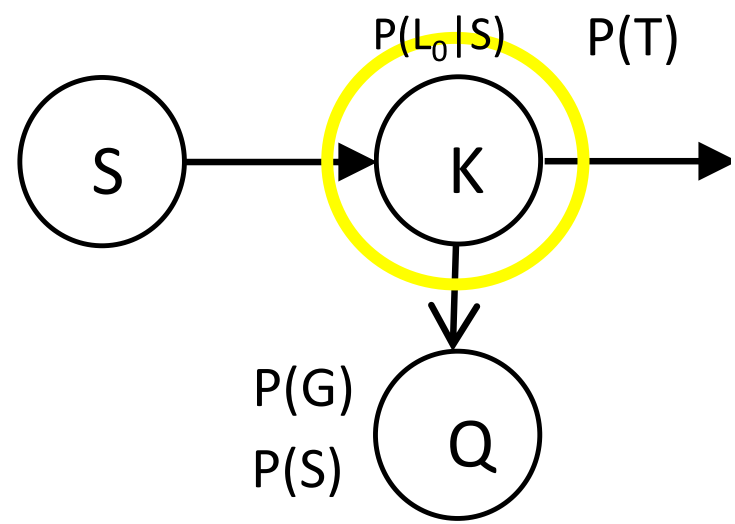
Prior Individualization Approach

Conditional Probability Table of Student node and Individualized Prior node

- Now that the model enables a prior parameter per student, how are these parameters going to be learned?

CPT of Individualized Prior node

S value	$P(L_0 \mid S)$
1	0.05
2	0.30
	.95
	:



Several strategies tried

	Most accurate predictor (of 42)		Avg. Correlation	
$P(L_0)$ Strategy	PPS	KT	PPS	KT
Percent correct heuristic	33	8	0.3515	0.1933
Cold start heuristic	30	12	0.3014	0.1726
Random parameter values	26	16	0.2518	0.1726

(Pardos & Heffernan, 2010a)

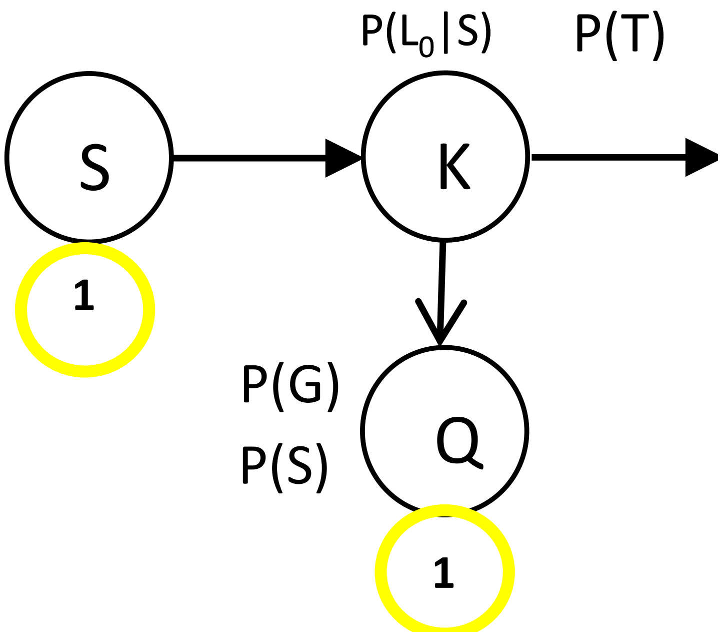
Prior Individualization Approach

Conditional Probability Table of Student node and Individualized Prior node

- Cold Start Heuristic

CPT of Individualized Prior node

S value	$P(L_0\|S)$
0	0.05
1	0.30



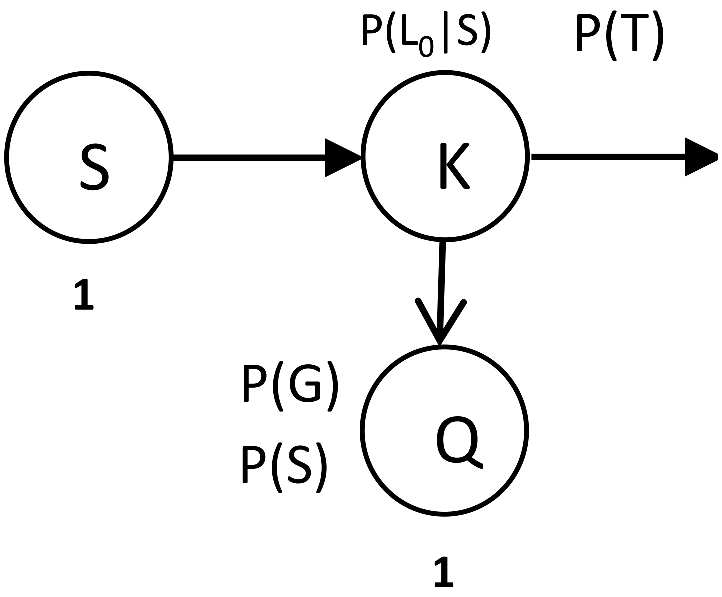
Prior Individualization Approach

What values to use for the two priors?

CPT of Individualized Prior node

S value	$P(L_0\|S)$
0	0.05
1	0.30

What values to use for the two priors?



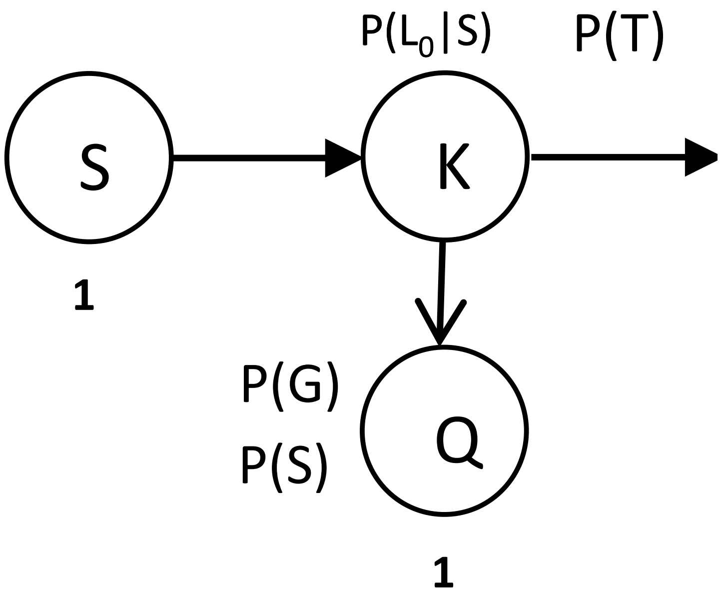
Prior Individualization Approach

What values to use for the two priors?

1. **Use ad-hoc values**

CPT of Individualized Prior node

S value	$P(L_0\|S)$
0	0.10
1	0.85



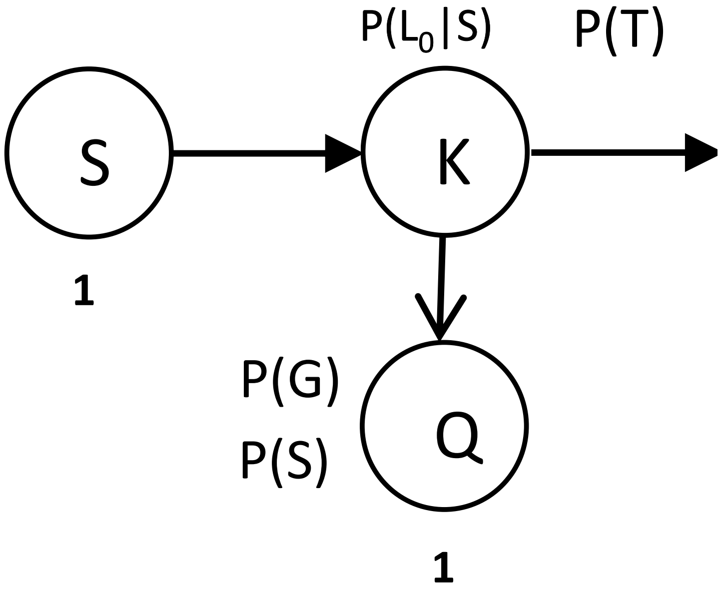
Prior Individualization Approach

What values to use for the two priors?

1. Use ad-hoc values
2. **Learn the values**

CPT of Individualized Prior node

S value	$P(L_0 \| S)$
0	EM
1	EM



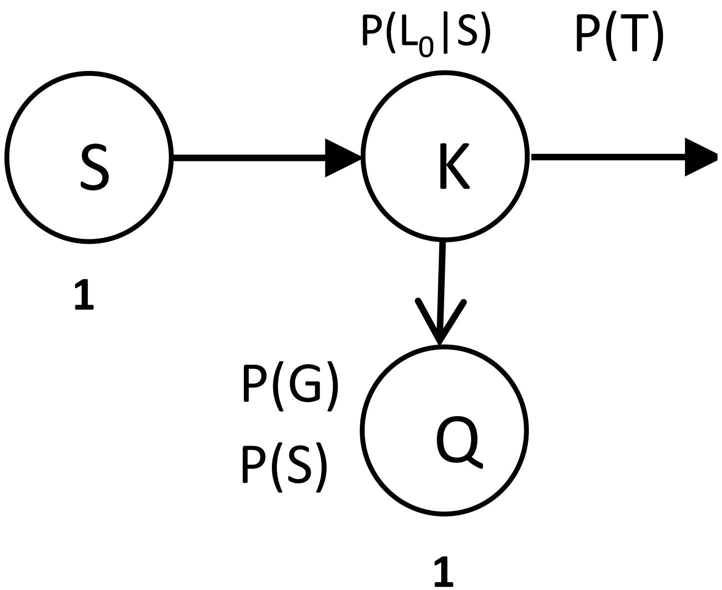
Prior Individualization Approach

What values to use for the two priors?

1. Use ad-hoc values
2. Learn the values
3. **Link with the guess/slip CPT**

CPT of Individualized Prior node

S value	$P(L_0\|S)$
0	*Slip*
1	1-*Guess*



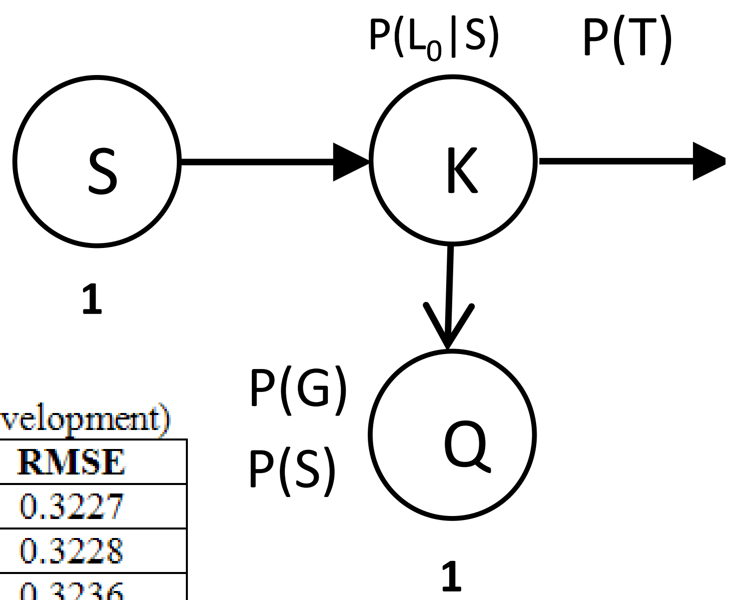
Prior Individualization Approach

What values to use for the two priors?

1. Use ad-hoc values
2. Learn the values
3. Link with the guess/slip CPT

CPT of Individualized Prior node

S value	$P(L_0 \mid S)$
0	*Slip*
1	1-*Guess*



Algebra (development)

	Strategy	RMSE
1	adjustable	0.3659
2	guess/slip	0.3660
3	*Ad-hoc*	0.3662

Bridge to Algebra (development)

	Strategy	RMSE
1	guess/slip	0.3227
2	adjustable	0.3228
3	*Ad-hoc*	0.3236

(Pardos & Heffernan, JMLR In Press)

With an ASSISTments Platform dataset, PPS (ad-hoc) achieved an R^2 of 0.301 (0.176 with KT)

(Pardos & Heffernan, UMAP 2010)

Prior Per Student Model

<u>Cold start heuristic was a success</u>

•Performed well, improvement in prediction over KT in 30/42 problem sets

•Requires no extra information outside of the responses in the problem set being predicted

•Reduces the free parameters to three instead of four
  - •Faster parameter training time with more accurate prediction

•The most simple individualization technique to add to existing KT models
  - •One binary node addition and one arc

•Parameters can be learned from one population of students to predict another

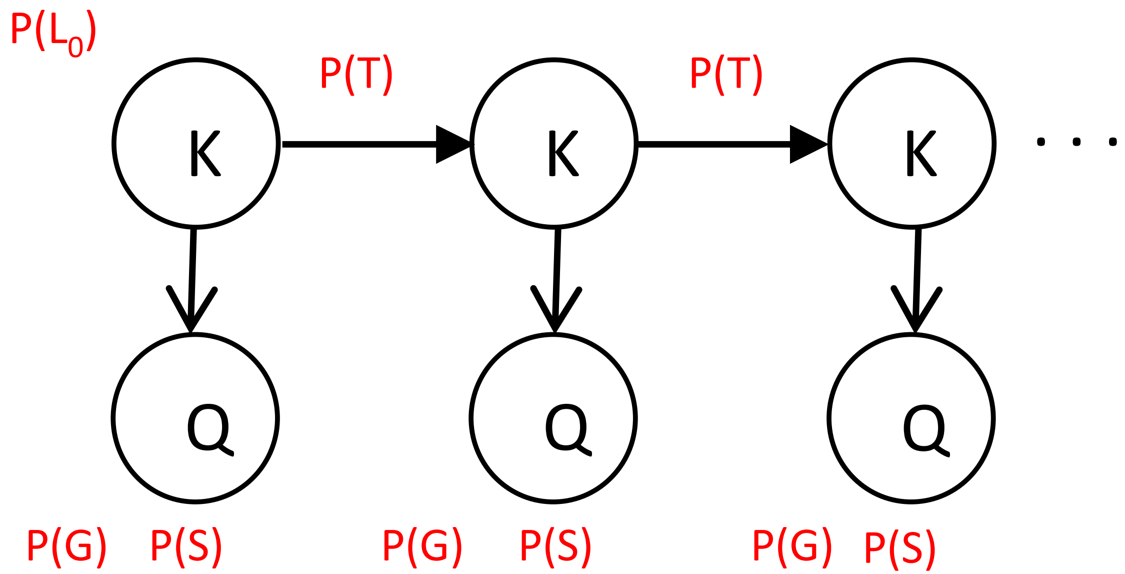
Variations on Knowledge Tracing
(and other models)

Intro to Knowledge Tracing

1. BKT-BF

Learns values for these parameters by performing a grid search (0.01 granularity) and chooses the set of parameters with the best squared error

$P(L_0)$ = Probability of initial knowledge
$P(T)$ = Probability of learning
$P(G)$ = Probability of guess
$P(S)$ = Probability of slip



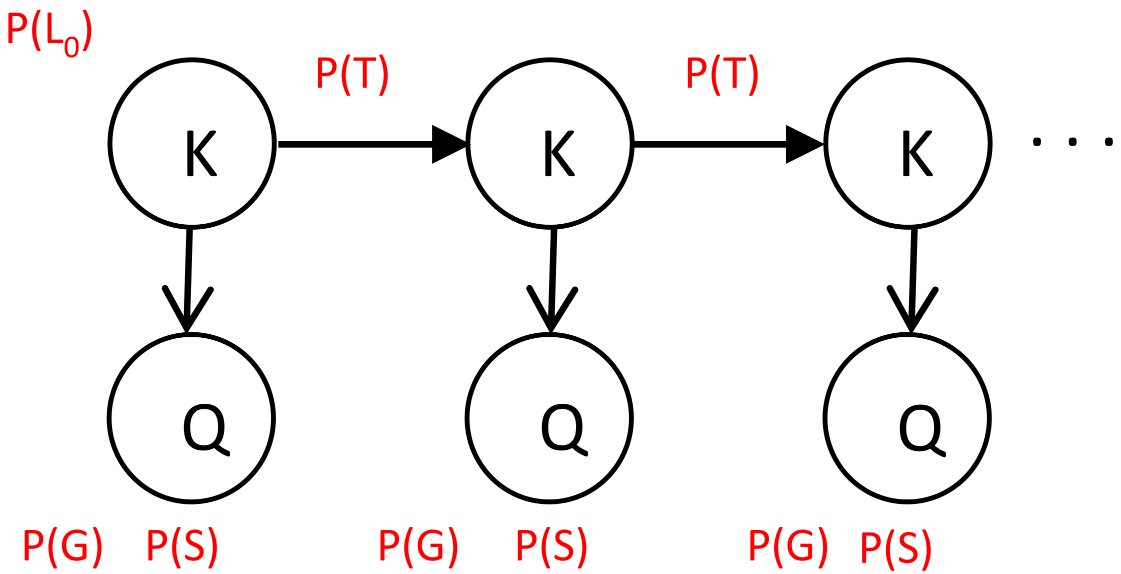
(Baker et al., 2010)

Intro to Knowledge Tracing

2. BKT-EM

Learns values for these parameters with Expectation Maximization (EM). Maximizes the log likelihood fit to the data

$P(L_0)$ = Probability of initial knowledge
$P(T)$ = Probability of learning
$P(G)$ = Probability of guess
$P(S)$ = Probability of slip

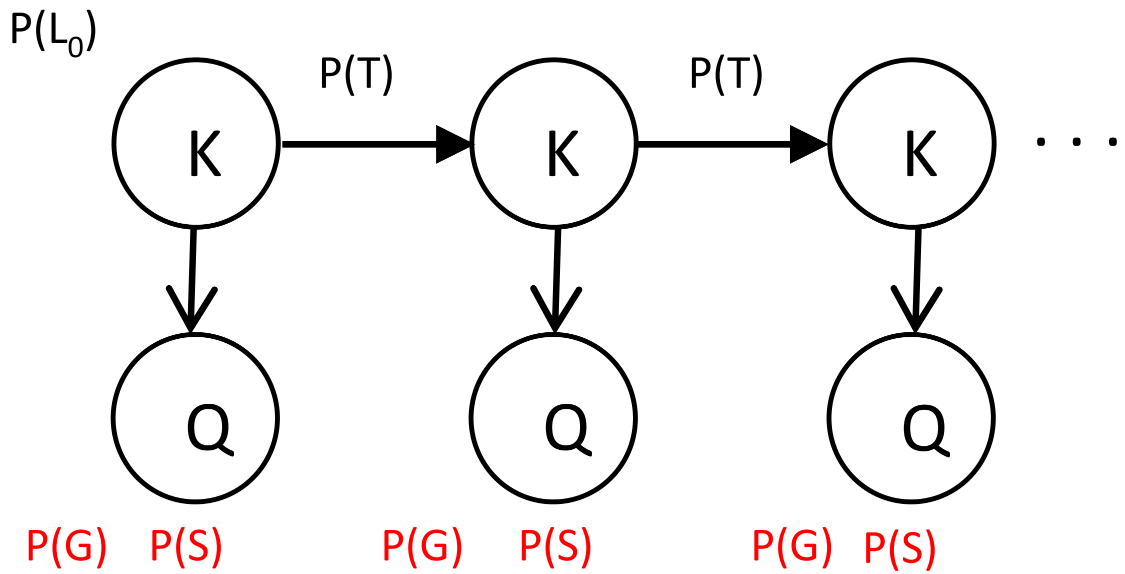


(Chang et al., 2006)

3. BKT-CGS

Guess and slip parameters are assessed contextually using a regression on features generated from student performance in the tutor

$P(L_0)$ = Probability of initial knowledge
$P(T)$ = Probability of learning
$P(G)$ = Probability of guess
$P(S)$ = Probability of slip

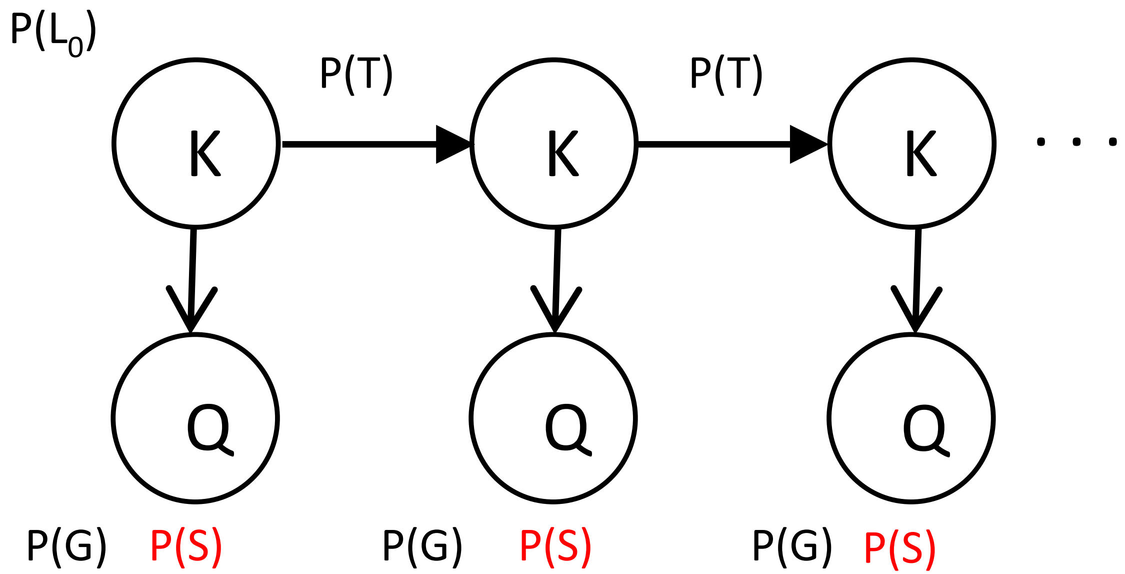


(Baker, Corbett, & Aleven, 2008)

4. BKT-CSlip

Uses the student's averaged <u>contextual</u> Slip parameter learned across all incorrect actions.

$P(L_0)$ = Probability of initial knowledge
$P(T)$ = Probability of learning
$P(G)$ = Probability of guess
$P(S)$ = Probability of slip



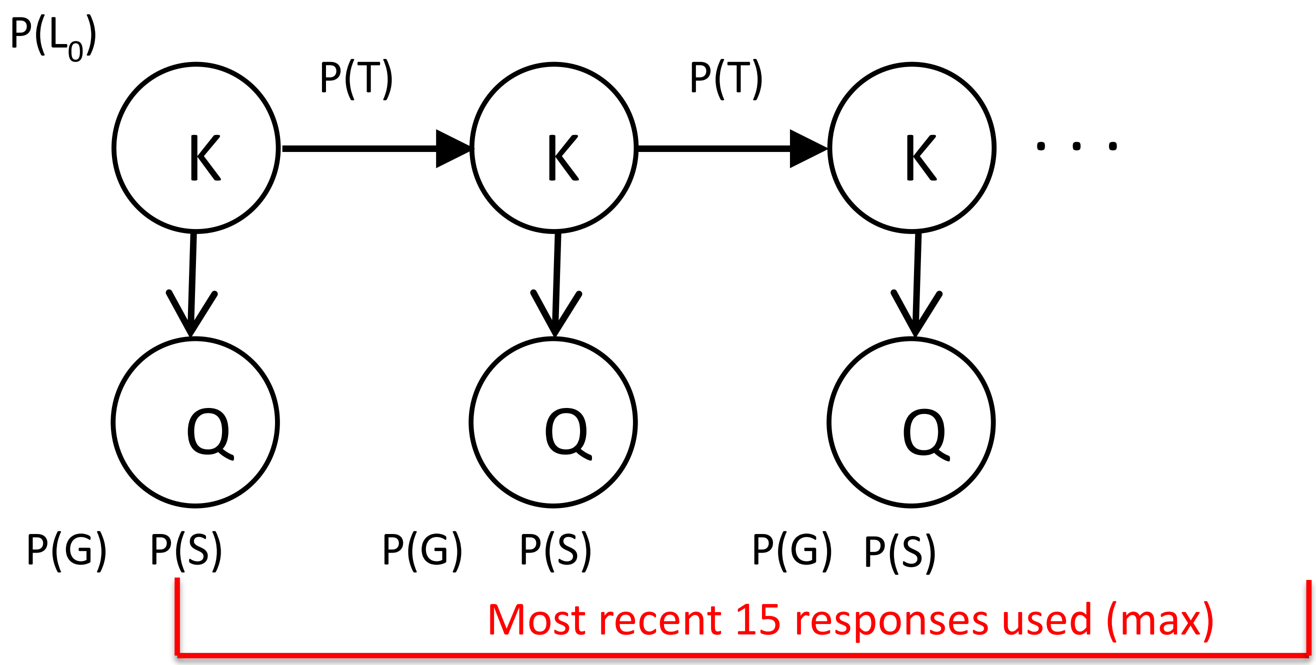
(Baker, Corbett, & Aleven, 2008)

Intro to Knowledge Tracing

5. BKT-LessData

Limits students response sequence length to the most recent 15 during EM training.

$P(L_0)$ = Probability of initial knowledge
$P(T)$ = Probability of learning
$P(G)$ = Probability of guess
$P(S)$ = Probability of slip

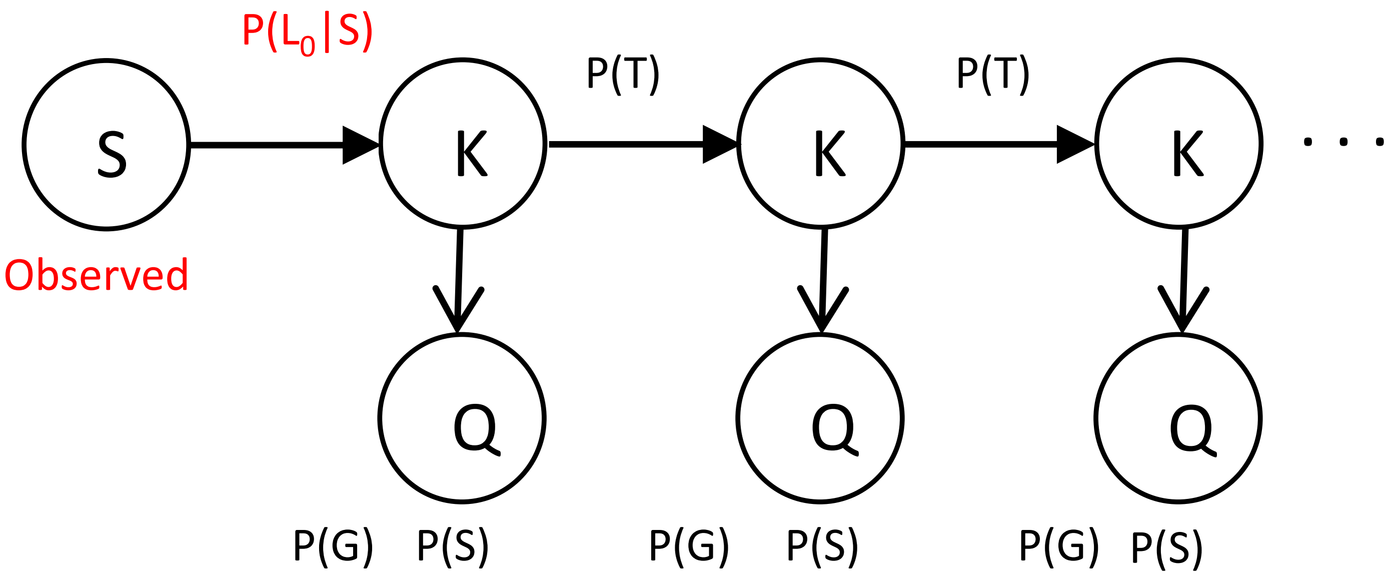


(Nooraiei et al, 2011)

6. BKT-PPS

Prior per student (PPS) model which individualizes the prior parameter. Students are assigned a prior based on their response to the first question.

$P(L_0)$ = Probability of initial knowledge
P(T) = Probability of learning
P(G) = Probability of guess
P(S) = Probability of slip



(Pardos & Heffernan, 2010)

7. CFAR

Correct on First Attempt Rate (CFAR) calculates the student's percent correct on the current skill up until the question being predicted.

Student responses for Skill X: 0 1 0 1 0 1 _

Predicted next response would be 0.50

(Yu et al., 2010)

8. Tabling

Uses the student's response sequence (max length 3) to predict the next response by looking up the average next response among student with the <u>same sequence</u> in the training set

Training set
Student A: 0 1 1 0
Student B: 0 1 1 1
Student C: 0 1 1 1

Max table length set to 3:
Table size was $2^0+2^1+2^2+2^3=15$

Test set student: 0 0 1 _

Predicted next response would be 0.66

(Wang et al., 2011)

9. IRT

Item response theory (IRT) the standard assessment tool used for GRE testing.

$$p(+|v,i) = \frac{\exp(\theta_v - \sigma_i)}{1+\exp(\theta_v - \sigma_i)} = \frac{1}{1+e^{-(\theta_v - \sigma_i)}}$$

Where $p(+|v,i)$ is the probability of positive performance of student v on test item i and σ_i is the difficulty of item i.

Extension which breaks down an item into cognitive operations (Scheiblechner, 1972)

$$\sigma_i = \sum_{j=1}^{m} q_{ij} n_j + c$$

Where *j* is a cognitive operation, q_{ij} is the number of times the operation occurs in item *i*, n_j is the difficulty of cognitive operation *j* and *c* is a scaling constant

Learning from the test addition:

$\sigma = \sum_{j=1}^{m} q_{ij} n_j - q_{uj} h^*{}_{ij} \beta_j) + c$

10. PFA

Performance Factors Analysis (PFA). Logistic regression model which elaborates on the Rasch IRT model. Predicts performance based on the count of student's prior failures and successes on the current skill.

An overall difficulty parameter β is also fit for each skill or each item in this formula the variant of PFA that fits β for each skill is shown. The PFA equation is:

$$m(i, j \in KCs, s, f) = \beta_j + \sum(\gamma_j S_{ij} + \rho_j F_{ij})$$

(Pavlik et al., 2009)

Conclusion

Time Left? If yes then KT-IDEM / IEM

Else

  next slide

Demo

Bayesian Knowledge Tracing
MATLAB Demo / code

Thank you

Questions?

Thesis

Pardos, Z.A., Heffernan, N.T. (2012) Tutor Modeling vs. Student Modeling. In *Proceedings of the 25th annual Florida Artificial Intelligence Research Society Conference *Invited paper*

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