CBSE Class–10 Mathematics

NCERT solution

Chapter - 1

Real Numbers - Exercise 1.1

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1. Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

Ans. (i) 135 and 225

We have 225 > 135,

So, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Here remainder 90 ≠ 0, we apply the division lemma again to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45 ≠ 0, and apply the division lemma to obtain

$$90 = 2 \times 45 + 0$$

Since at this time the remainder is zero, the process is stopped.

The divisor at this stage is 45

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

As the remainder is zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

We have 867 > 255,

So, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Here remainder 102 ≠ 0, we apply the division lemma again to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

Here remainder 51 ≠ 0, we apply the division lemma again to 102 and 51 to obtain

$$102 = 51 \times 2 + 0$$

As the remainder is zero, the process stops.

The divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

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2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.

Ans. Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$,

or $6q + 5$

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3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Ans. We have to find the HCF (616, 32) to find the maximum number of columns in which they can march.

To find the HCF, we can use Euclid's algorithm.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

Since, the last divisor is 8.

Therefore, the HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

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4. Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m. [Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Ans. Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 or (3q+1)^2 or (3q+2)^2$$
$$a^2 = (9q)^2 or 9q^2 + 6q + 1 or\ 9q^2 + 12q + 4$$
$$= 3 \times (3q^2)\ or\ 3(3q^2 + 2q) + 1\ or\ 3(3q^2 + 4q + 1) + 1$$
$$= 3k_1\ or\ 3k_2 + 1\ or\ 3k_3 + 1$$

Where $k_1, k_2,\ and\ k_3$ are some positive integers.

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

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5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Ans. Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $r = 0, 1, 2$ because $0 \leq r < 3$

a = 3q or 3q + 1 or 3q + 2

Therefore, every number can be represented as these three forms.

We have three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$$a^3 = (3q + 1)^3$$

$$a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$a^3 = 9\left(3q^3 + 3q^2 + q\right) + 1$$

$$a^3 = 9m + 1$$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$$a^3 = (3q + 2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9\left(3q^3 + 6q^2 + 4q\right) + 8$$

$$a^3 = 9m + 8$$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 1

Real Numbers -Exercise 1.2

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1. Express each number as product of its prime factors:

(i) 140
(ii) 156
(iii) 3825
(iv) 5005
(v) 7429

Ans. (i) 140 = $2\times2\times5\times7 = 2^2\times5\times7$
(ii) 156 = $2\times2\times3\times13 = 2^2\times3\times13$
(iii) 3825 = $3\times3\times5\times5\times17 = 3^2\times5^2\times17$
(iv) 5005 = $5\times7\times11\times13$
(v) 7429 = $17\times19\times23$

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2. Find the LCM and HCF of the following pairs of integers and verify that $\mathrm{LCM}\times\mathrm{HCF}$ = product of the two numbers.

(i) 26 and 91
(ii) 510 and 92
(iii) 336 and 54

Ans. (i) 26 and 91
$26 = 2 \times 13$
$91 = 7 \times 13$
HCF (26, 91) = 13
LCM (26, 91) = $2 \times 7 \times 13$ = 182
Product of two numbers 26 and 91 $= 26 \times 91$ = 2366
$HCF\times LCM = 13\times182$ = 2366
Hence, product of two numbers = $HCF \times LCM$

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF (510, 92) = 2

LCM (510, 92) = $2 \times 2 \times 3 \times 5 \times 17 \times 23$ = 23460

Product of two numbers 510 and 92 = 510×92 = 46920

$HCF \times LCM = 2 \times 23460$ = 46920

Hence, product of two numbers = $HCF \times LCM$

(iii) 336 and 54

336= $2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

54 = $2 \times 3 \times 3 \times 3 = 2 \times 3^3$

HCF (336, 54) = $2 \times 3 = 6$

LCM (336, 54) = $2^4 \times 3^3 \times 7$ = 3024

Product of two numbers 336 and 54 = 336×54 = 18144

$HCF \times LCM = 6 \times 3024$ = 18144

Hence, product of two numbers = $HCF \times LCM$

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3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

Ans. (i) 12, 15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF (12, 15, 21) = 3

LCM (12, 15, 21) = $2^2 \times 3 \times 5 \times 7$ = 420

(ii) 17, 23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

HCF (17, 23, 29) = 1

LCM (17, 23, 29) = $17 \times 23 \times 29$ = 11339

(iii) 8, 9 and 25

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5^2$

HCF (8, 9, 25) = 1

LCM (8, 9, 25) = $2^3 \times 3^2 \times 5^2$ = 1800

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4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Ans. HCF (306, 657) = 9

We know that, LCM × HCF = Product of two numbers

$LCM \times HCF = 306 \times 657$

$$LCM = \frac{306 \times 657}{HCF} = \frac{306 \times 657}{9}$$

LCM (306, 657) = 22338

---

5. Check whether 6^n can end with the digit 0 for any natural number *n*.

Ans. If any number ends with the digit 0, it should be divisible by 10.

In other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n .

Hence, for any value of *n*, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number *n*.

---

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Ans. Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78 = 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors other than 1 and number itself.

Therefore, it is a composite number.

$7\times6\times5\times4\times3\times2\times1+5$

$= 5\times(7\times6\times4\times3\times2\times1+1)$

$= 5 \times (1008 + 1) = 5 \times 1009$

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors other than 1 and number itself.

Hence, it is a composite number.

---

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Ans. It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2\times3\times3$ And, $12 = 2\times2\times3$

LCM of 12 and $18 = 2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

CBSE Class–10 Mathematics

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Chapter - 1

Real Numbers - Exercise 1.3

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1. Prove that $\sqrt{5}$ is irrational.

Ans. Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ ... **(1)**

It means that 5 is factor of a^2

Hence, **5 is also factor of *a*** by Theorem. ... **(2)**

If, **5 is factor of *a***, it means that we can write ***a*** = **5*c*** for some integer ***c***.

Substituting value of ***a*** in **(1)**,

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

It means that 5 is factor of b^2.

Hence, **5 is also factor of *b*** by Theorem. ... **(3)**

From **(2)** and **(3)**, we can say that **5** is factor of both ***a* and *b***.

But, ***a* and *b*** are **co-prime**.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

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2. Prove that (3 + 2 $\sqrt{5}$) is irrational.

Ans. We will prove this by contradiction.

Let us suppose that (3+2 $\sqrt{5}$) is rational.

It means that we have co-prime integers ***a* and *b*** (*b* ≠ 0) such that

$$\frac{a}{b} = 3 + 2\sqrt{5} \Rightarrow \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \; ... \; \textbf{(1)}$$

a* and *b are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational. It is not possible. Therefore, our supposition is wrong. (3+2 $\sqrt{5}$) cannot be rational.

Hence, (3+2 $\sqrt{5}$) is irrational.

---

3. Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6 + \sqrt{2}$

Ans. (i) We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers *a* and *b* (*b* ≠ 0) such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

⇒ $\sqrt{2} = \frac{b}{a}$ … **(1)**

R.H.S of **(1)** is rational but we know that $\sqrt{2}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

Hence, it is irrational.

(ii) We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers *a* and *b* (*b* ≠ 0) such that

$7\sqrt{5} = \frac{a}{b}$

⇒ $\sqrt{5} = \frac{a}{7b}$ … **(1)**

R.H.S of **(1)** is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

(iii) We will prove $6+\sqrt{2}$ irrational by contradiction.

Let us suppose that ($6+\sqrt{2}$) is rational.

It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{a}{b}-6$$

$$\Rightarrow \sqrt{2}=\frac{a-6b}{b} \quad \text{... } \mathbf{(1)}$$

a* and *b are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. ($6+\sqrt{2}$) cannot be rational.

Hence, ($6+\sqrt{2}$) is irrational.

CBSE Class–10 Mathematics

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Chapter - 1

Real Numbers - Exercise 1.4

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1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating decimal expansion.

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 \times 5^2}$

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

(viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$

(x) $\frac{77}{210}$

Ans. According to Theorem, any given rational number of the form $\frac{p}{q}$ **where *p* and *q* are co-prime**, has a terminating decimal expansion if q is of the form $2^n \times 5^m$, where m and n are non-negative integers.

(i) $\frac{13}{3125}$

$q = 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 2^0 \times 5^5$

Here, denominator is of the form $2^n \times 5^m$, where m = 5 and n = 0.

It means rational number $\frac{13}{3125}$ has a **terminating** decimal expansion.

(ii) $\frac{17}{8}$

$q = 8 = 2 \times 2 \times 2 = 2^3 = 2^3 \times 5^0$

Here, denominator is of the form $2^n \times 5^m$, where m = 0 and n = 3.

It means rational number $\frac{17}{8}$ has a **terminating** decimal expansion.

(iii) $\frac{64}{455}$

$q = 455 = 5 \times 91$

Here, denominator is not of the $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{64}{455}$ has a **non-terminating repeating** decimal expansion.

(iv) $\frac{15}{1600} = \frac{3}{320}$

$q = 320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 6.

It means rational number $\frac{15}{1600}$ has a **terminating** decimal expansion.

(v) $\frac{29}{343}$

$q = 343 = 7 \times 7 \times 7 = 7^3$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{29}{343}$ has **non-terminating repeating** decimal expansion.

(vi) $\frac{23}{2^3 \times 5^2}$

$q = 2^3 \times 5^2$

Here, denominator is of the form $2^n \times 5^m$, where m = 2 and n = 3 are non-negative integers.

It means rational number $\frac{23}{2^3 \times 5^2}$ has **terminating** decimal expansion.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

$q = 2^2 \times 5^7 \times 7^5$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\dfrac{129}{2^2 \times 5^7 \times 7^5}$ has **non-terminating repeating** decimal expansion.

(viii) $\dfrac{6}{15} = \dfrac{2}{5}$

$q = 5 = 5^1 = 2^0 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 0.

It means rational number $\dfrac{6}{15}$ has **terminating** decimal expansion.

(ix) $\dfrac{35}{50} = \dfrac{7}{10}$

$q = 10 = 2 \times 5 = 2^1 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 1.

It means rational number $\dfrac{35}{50}$ has **terminating decimal** expansion.

(x) $\dfrac{77}{210} = \dfrac{11}{30}$

$q = 30 = 5 \times 3 \times 2$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\dfrac{77}{210}$ has **non-terminating repeating** decimal expansion.

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2. Write down the decimal expansions of those rational numbers in Question 1 which

have terminating decimal expansions.

Ans. (i) $\frac{13}{3125}=\frac{13}{5^5}=\frac{13\times2^5}{5^5\times2^5}=\frac{416}{10^5}=0.00416$

(ii) $\frac{17}{8}=\frac{17}{2^3}=\frac{17\times5^3}{2^3\times5^3}=\frac{17\times5^3}{10^3}=\frac{2125}{10^3}=2.215$

(iv) $\frac{15}{1600}=\frac{15}{2^6\times5^2}=\frac{15\times5^4}{2^6\times5^2\times5^4}=\frac{15\times5^4}{10^6}=\frac{9375}{10^6}=0.009375$

(vi) $\frac{23}{2^3\times5^2}=\frac{23\times5^1}{2^3\times5^2\times5^1}=\frac{23\times5^1}{10^3}=\frac{115}{10^3}=0.115$

(viii) $\frac{6}{15}=\frac{2}{5}=\frac{2\times2}{5\times2}=\frac{4}{10}=0.4$

(ix) $\frac{35}{50}=\frac{7}{10}=0.7$

---

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If, they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000...

(iii) $43.\overline{123456789}$

Ans. (i) 43.123456789

It is rational because decimal expansion is terminating. Therefore, it can be expressed in $\frac{p}{q}$ form where q = 10^9 and factors of q are of the form $2^n \times 5^m$ where n and m are non-negative integers

(ii) 0.120112001200012000...

It is irrational because decimal expansion is neither terminating nor non-terminating repeating.

(iii) $43.\overline{123456789}$

It is rational because decimal expansion is non-terminating repeating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of q **are not** of the form $2^n \times 5^m$ where n and m are non-negative integers.

Thus, $43.123456789 = \frac{p}{q}$, where q = 999999999

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Chapter - 2

Polynomials - Exercise 2.1

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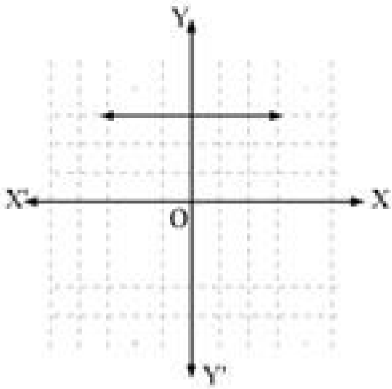
1. The graphs of y=p(x) are given to us, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

(i)



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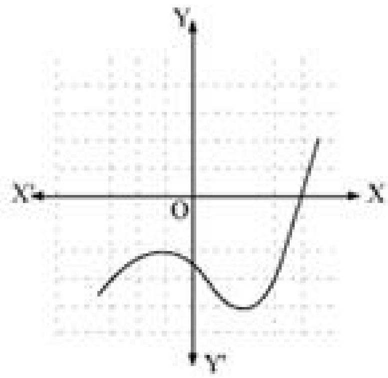

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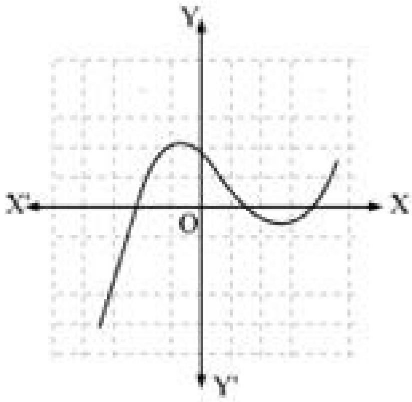

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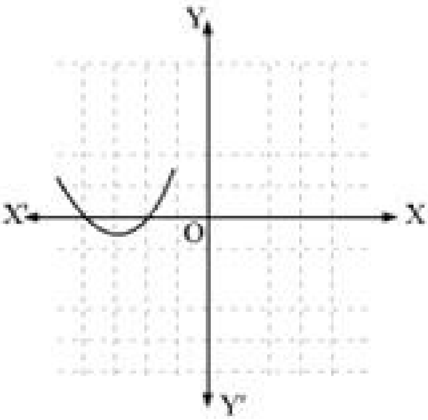
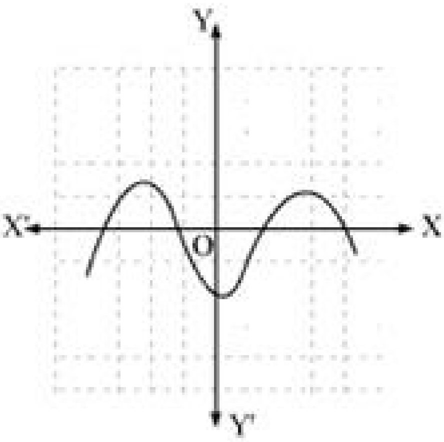
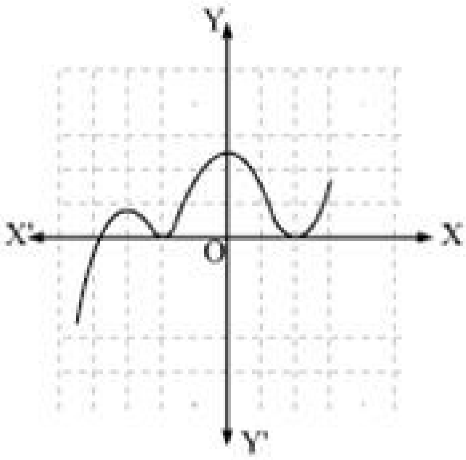


(v)



(vi)



Ans. (i) The given graph does not intersects x-axis at all. Hence, it does not have any zero.

(ii) Given graph intersects x-axis 1 time. It means this polynomial has 1 zero.

(iii) Given graph intersects x-axis 3 times. Therefore, it has 3 zeroes.

(iv) Given graph intersects x-axis 2 times. Therefore, it has 2 zeroes.

(v) Given graph intersects x-axis 4 times. It means it has 4 zeroes.

(vi) Given graph intersects x-axis 3 times. It means it has 3 zeroes.

CBSE Class–10 Mathematics

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Chapter - 2

Polynomials - Exercise 2.2

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1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Ans. (i) $x^2 - 2x - 8$

Comparing given polynomial with general form of quadratic polynomial $ax^2 + bx + c$,

We get a = 1, b = -2 and c = -8

We have, $x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x-4)+2(x-4) = (x-4)(x+2)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$(x-4)(x+2) = 0$

$\Rightarrow x = 4, -2$ are two zeroes.

Sum of zeroes = 4 + (– 2) = 2 =

=> $\frac{-(-2)}{1}$ = $\frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes = 4 × (−2) = −8

= $\frac{-8}{1} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2 - 4s + 1$

Here, a = 4, b = -4 and c = 1

We have, $4s^2 - 4s + 1$

= $4s^2 - 2s - 2s + 1$

=2s(2s−1)−1(2s−1)
= (2s−1)(2s−1)

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

⇒ (2s−1)(2s−1) = 0

⇒ s = $\frac{1}{2}, \frac{1}{2}$

Therefore, two zeroes of this polynomial are $\frac{1}{2}, \frac{1}{2}$

Sum of zeroes = $\frac{1}{2} + \frac{1}{2}$ = 1 = $\frac{-(-1)}{1} \times \frac{4}{4} = \frac{-(-4)}{4}$

= $\frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$= \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iii) $6x^2 - 3 - 7x \quad \Rightarrow \quad 6x^2 - 7x - 3$

Here, a = 6, b = -7 and c = -3

We have, $6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x-3)+1(2x-3) = (2x-3)(3x+1)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow (2x-3)(3x+1) = 0$

$\Rightarrow x = \frac{3}{2}, \frac{-1}{3}$

Therefore, two zeroes of this polynomial are $\frac{3}{2}, \frac{-1}{3}$

Sum of zeroes = $\frac{3}{2} + \frac{-1}{3} = \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = $\frac{3}{2} \times \frac{-1}{3} = \frac{-1}{2} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u$

Here, a = 4, b = 8 and c = 0

$4u^2 + 8u = 4u(u+2)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow 4u(u+2) = 0$

$\Rightarrow u = 0, -2$

Therefore, two zeroes of this polynomial are 0, −2

Sum of zeroes = 0−2 = −2

= $\frac{-2}{1}\times\frac{4}{4}=\frac{-8}{4}$ = $\frac{-b}{a}=\frac{-\text{Coefficient of }x}{\text{Coefficient of }x^2}$

Product of Zeroes $=$ 0×-2 = 0

= $\frac{0}{4}=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of }x^2}$

(v) t^2-15

Here, a = 1, b = 0 and c = -15

We have, $t^2-15 \Rightarrow t^2 = 15 \Rightarrow t = \pm\sqrt{15}$

Therefore, two zeroes of this polynomial are $\sqrt{15},-\sqrt{15}$

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{0}{1}=\frac{-b}{a}=\frac{-\text{Coefficient of }x}{\text{Coefficient of }x^2}$

Product of Zeroes = $\sqrt{15}\times\left(-\sqrt{15}\right)=-15$

$=\frac{-15}{1}=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of }x^2}$

(vi) $3x^2-x-4$

Here, a = 3, b = -1 and c = -4

We have, $3x^2-x-4 = 3x^2-4x+3x-4$

= $x(3x-4)+1(3x-4)$ = $(3x-4)(x+1)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

⇒ $(3x-4)(x+1) = 0$

$\Rightarrow x = \frac{4}{3}, -1$

Therefore, two zeroes of this polynomial are $\frac{4}{3}, -1$

Sum of zeroes = $\frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = $\frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

---

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}$ **, −1**

(ii) $\sqrt{2}$ **, 13**

(iii) 0, $\sqrt{5}$

(iv) 1, 1

(v) $\frac{-1}{4}, \frac{1}{4}$

(vi) 4, 1

Ans. (i) $\frac{1}{4}$, −1

Let quadratic polynomial be $ax^2 + bx + c$

Let α and β are two zeroes of above quadratic polynomial.

$\alpha+\beta = \frac{1}{4} = \frac{-b}{a}$

$\alpha \times \beta$ = -1 $= \frac{-1}{1} \times \frac{4}{4} = \frac{-4}{4}$ = $\frac{c}{a}$

On comparing, we get

$\therefore\ a = 4, b = -1, c = -4$

Putting the values of a, b and c in quadratic polynomial $ax^2 + bx + c$, we get

Quadratic polynomial which satisfies above conditions = $4x^2 - x - 4$

(ii) $\sqrt{2}, \frac{1}{3}$

Let quadratic polynomial be $ax^2 + bx + c$

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta$ = $\sqrt{2} \times \frac{3}{3} = \frac{3\sqrt{2}}{3}$ = $\frac{-b}{a}$

$\alpha \times \beta$ = $\frac{1}{3}$ which is equal to $\frac{c}{a}$

On comparing, we get

$\therefore\ a = 3, b = -3\sqrt{2}, c = 1$

Putting the values of a, b and c in quadratic polynomial $ax^2 + bx + c$, we get

Quadratic polynomial which satisfies above conditions = $3x^2 - 3\sqrt{2}x + 1$.

(iii) 0, $\sqrt{5}$

Let quadratic polynomial be $ax^2 + bx + c$

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta$ = 0 $= \frac{0}{1}$ = $\frac{-b}{a}$

$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$

On comparing, we get

$\therefore\ a = 1, b = 0, c = \sqrt{5}$

Putting the values of a, b and c in quadratic polynomial $ax^2 + bx + c$, we get

Quadratic polynomial which satisfies above conditions $= x^2 + \sqrt{5}$

(iv) 1, 1

Let quadratic polynomial be $ax^2 + bx + c$

Let α and β be two zeros of above quadratic polynomial.

$\alpha + \beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$

$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$

On comparing, we get

$\therefore\ a = 1, b = -1, c = 1$

Putting the values of a, b and c in quadratic polynomial $ax^2 + bx + c$, we get

Quadratic polynomial which satisfies above conditions = $x^2 - x + 1$

(v) $\frac{-1}{4}, \frac{1}{4}$

Let quadratic polynomial be $ax^2 + bx + c$

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = \frac{-1}{4} = \frac{-b}{a}$

$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$

On comparing, we get

$\therefore\ a = 4, b = 1, c = 1$

Putting the values of a, b and c in quadratic polynomial $ax^2 + bx + c$, we get

Quadratic polynomial which satisfies above conditions = $4x^2 + x + 1$

(vi) 4, 1

Let quadratic polynomial be $ax^2 + bx + c$

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = 4\ \frac{-(-4)}{1} = \frac{-b}{a}$

$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$

On comparing, we get

$\therefore\ a = 1, b = -4, c = 1$

Putting the values of a, b and c in quadratic polynomial $ax^2 + bx + c$, we get

Quadratic polynomial which satisfies above conditions $= x^2 - 4x + 1$

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Chapter - 2

Polynomials - Exercise 2.3

---

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following.

(i) $\mathbf{p}(\mathbf{x}) = x^3 - 3x^2 + 5x - 3,\ \mathbf{g}(\mathbf{x}) = x^2 - 2$

(ii) $\mathbf{p}(\mathbf{x}) = x^4 - 3x^2 + 4x + 5,\ \mathbf{g}(\mathbf{x}) = x^2 - x + 1$

(iii) $\mathbf{p}(\mathbf{x}) = x^4 - 5x + 6,\ \mathbf{g}(\mathbf{x}) = 2 - x^2$

Ans. (i)

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2-2 & x^3-3x^2+5x-3 \\
 & \pm x^3 \qquad \mp 2x \\
\cline{2-2}
 & -3x^2+7x-3 \\
 & \mp 3x^2 \qquad \pm 6 \\
\cline{2-2}
 & 7x-9
\end{array}
$$

Therefore, quotient = x – 3 and Remainder = 7x – 9

(ii)

$$
\begin{array}{r}
x^2 + x - 3 \\
\hline
x^2 - x + 1)\; x^4 - 3x^2 + 4x + 5 \\
\pm x^4 \pm x^2 \qquad \mp x^3 \\
\hline
-4x^2 + 4x + 5 + x^3 \\
\mp x^2 \pm x \qquad \pm x^3 \\
\hline
-3x^2 + 3x + 5 \\
\mp 3x^2 \pm 3x \mp 3 \\
\hline
8
\end{array}
$$

Therefore, quotient = $x^2 + x - 3$ and, Remainder = 8

(iii)

$$
\begin{array}{r}
-x^2 - 2 \\
\hline
-x^2 + 2)\; x^4 - 5x + 6 \\
\pm x^4 \qquad \mp 2x^2 \\
\hline
-5x + 6 + 2x^2 \\
\mp 4 \pm 2x^2 \\
\hline
-5x + 10
\end{array}
$$

Therefore, quotient = $-x^2 - 2$ and, Remainder = $-5x + 10$

---

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Ans. (i)

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2-3 & \cancel{2t^4}+3t^3-2t^2-9t-12 \\
 & \pm\cancel{2t^4} \qquad \mp 6t^2 \\
\hline
 & \quad +\cancel{3t^3}+4t^2-\cancel{9t}-12 \\
 & \quad \pm\cancel{3t^3} \qquad \mp\cancel{9t} \\
\hline
 & \qquad +\cancel{4t^2} \qquad -\cancel{12} \\
 & \qquad \pm\cancel{4t^2} \qquad \mp\cancel{12} \\
\hline
 & \qquad\quad 0
\end{array}
$$

∵ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(ii)

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & \pm 3x^4 \pm 9x^3 \pm 3x^2 \\
\hline
 & \quad -4x^3-10x^2+2x+2 \\
 & \quad \mp 4x^3 \mp 12x^2 \mp 4x \\
\hline
 & \qquad +2x^2+6x+2 \\
 & \qquad \pm 2x^2 \pm 6x \pm 2 \\
\hline
 & \qquad\quad 0
\end{array}
$$

∵ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(iii)

$$
\begin{array}{r|l}
 & x^2-1 \\
\cline{2-2}
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & \underline{\pm x^5 \mp 3x^3 \pm x^2} \\
 & \quad -x^3+3x+1 \\
 & \quad \underline{\mp x^3 \pm 3x \mp 1} \\
 & \qquad\qquad 2
\end{array}
$$

$\because$ Remainder $\neq 0$

Hence first polynomial is not factor of second polynomial.

---

3. Obtain all other zeroes of $\left(3x^4+6x^3-2x^2-10x-5\right)$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Ans. Two zeroes of $\left(3x^4+6x^3-2x^2-10x-5\right)$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ which means that $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=x^2-\frac{5}{3}$ = $3x^2-5$ is a factor of $\left(3x^4+6x^3-2x^2-10x-5\right)$.

Applying Division Algorithm to find more factors we get:

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad\quad 3x^2+6x+3 \\
\hline
x^2-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & \pm 3x^4 \quad\quad \mp 5x^2 \\
\hline
 & \quad +6x^3+3x^2-10x-5 \\
 & \quad \pm 6x^3 \quad\quad \mp 10x \\
\hline
 & \quad\quad +3x^2 \quad\quad -5 \\
 & \quad\quad \pm 3x^2 \quad\quad \mp 5 \\
\hline
 & \quad\quad\quad\quad 0
\end{array}
$$

We have $p(x) = g(x) \times q(x)$.

$\Rightarrow \left(3x^4+6x^3-2x^2-10x-5\right)$

$= \left(3x^2-5\right)\left(x^2+2x+1\right)$

$= \left(3x^2-5\right)(x+1)(x+1)$

Therefore, other two zeroes of $\left(3x^4+6x^3-2x^2-10x-5\right)$ are −1 and −1.

---

4. On dividing $\left(x^3-3x^2+x+2\right)$ by a polynomial g(x), the quotient and remainder were (x-2) and (-2x+4) respectively. Find g(x).

Ans. Let $p(x) = x^3-3x^2+x+2$, q(x) = (x – 2) and r(x) = (–2x+4)

According to Polynomial Division Algorithm, we have

p(x) = g(x).q(x) + r(x)

$\Rightarrow x^3-3x^2+x+2 = g(x).(x-2)-2x+4$

$\Rightarrow x^3-3x^2+x+2+2x-4 = g(x).(x-2)$

$\Rightarrow x^3-3x^2+3x-2 = g(x).(x-2)$

$\Rightarrow g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2}$

So, Dividing $\left(x^3 - 3x^2 + 3x - 2\right)$ by (x−2), we get

$$\begin{array}{r}
x^2 - x + 1 \\
\hline
x - 2)\ x^3 - 3x^2 + 3x - 2 \\
\underline{\pm x^3 \mp 2x^2 \qquad\qquad} \\
-x^2 + 3x - 2 \\
\underline{\mp x^2 \pm 2x \qquad} \\
x - 2 \\
\underline{\pm x \mp 2} \\
0
\end{array}$$

Therefore, we have $g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2} = x^2 - x + 1$

---

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Ans. (i) Let $p(x) = 3x^2 + 3x + 6$, g(x) = 3

$$
\begin{array}{r}
x^2+x+2 \\
3\overline{)\,3x^2+3x+6} \\
\underline{\pm 3x^2\qquad\quad} \\
+3x+6 \\
\underline{\pm 3x\quad\;} \\
+6 \\
\underline{\pm 6} \\
0
\end{array}
$$

So, we can see in this example that deg p(x) = deg q(x) = 2

(ii) Let $p(x) = x^3+5$ and $g(x) = x^2-1$

$$
\begin{array}{r}
x \\
x^2-1\overline{)\,x^3+5} \\
\underline{\pm x^3 \quad \mp x} \\
x+5
\end{array}
$$

We can see in this example that deg q(x) = deg r(x) = 1

(iii) Let $p(x) = x^2+5x-3$, g(x) = x+3

$$
\begin{array}{r}
x+2 \\
x+3\overline{)\,x^2+5x-3} \\
\underline{\pm x^2 \pm 3x\quad} \\
+2x-3 \\
\underline{\pm 2x \pm 6} \\
-9
\end{array}
$$

We can see in this example that deg r(x) = 0

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Chapter - 2

Polynomials - Exercise 2.4

---

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2; \frac{1}{2}, 1, -2$

(ii) $x^3-4x^2+5x-2; 2, 1, 1$

Ans. (i) Comparing the given polynomial with ax^3+bx^2+cx+d, we get

$a=2, b=1, c=-5$ and $d=2$.

$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$

$= \frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2 = \frac{1+1-10+8}{4} = \frac{0}{4} = 0$

$p(1)=2(1)^3+(1)^2-5(1)+2$

$= 2+1-5+2 = 0$

$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$

$= 2(-8)+4+10+2 = -16+16 = 0$

$\therefore \frac{1}{2}, 1$ and -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, $\alpha+\beta+\gamma$

$= \frac{1}{2}+1+(-2) = \frac{1+2-4}{2} = \frac{-1}{2} = \frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha$

$= \left(\frac{1}{2}\right)(1)+(1)(-2)+(-2)\left(\frac{1}{2}\right)$

$= \frac{1}{2}-2-1 = \frac{-5}{2} = \frac{c}{a}$

And $\alpha\beta\gamma = \frac{1}{2}\times 1\times(-2) = -1 = \frac{-2}{2} = \frac{-d}{a}$

(ii) Comparing the given polynomial with ax^3+bx^2+cx+d, we get

$a=1, b=-4, c=5$ and $d=-2.$

$p(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$= 8-16+10-2 = 0$

$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1-4+5-2 = 0$

$\therefore 2, 1$ and 1 are the zeroes of $x^3-4x^2+5x-2.$

Now, $\alpha+\beta+\gamma$

$= 2+1+1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha = (2)(1)+(1)(1)+(1)(2)$

$= 2+1+2 = \frac{5}{1} = \frac{c}{a}$

And $\alpha\beta\gamma$ = $2\times1\times1=2=\frac{-(-2)}{1}=\frac{-d}{a}$

---

2. Find a cubic polynomial with the sum of the product of its zeroes taken two at a time and the product of its zeroes are $2, -7, -14$ respectively.

Ans. Let the cubic polynomial be ax^3+bx^2+cx+d and its zeroes be α, β and γ.

Then $\alpha+\beta+\gamma$ = 2 = $\frac{-(-2)}{1}=\frac{-b}{a}$ and $\alpha\beta+\beta\gamma+\gamma\alpha$ = $-7=\frac{-7}{1}=\frac{c}{a}$

And $\alpha\beta\gamma$ = $-14=\frac{-14}{1}=\frac{d}{a}$

Here, $a=1, b=-2, c=-7$ and $d=14$

Hence, cubic polynomial will be $x^3-2x^2-7x+14$.

---

3. If the zeroes of the polynomial x^3-3x^2+x+1 are $a-b, a, a+b$, find a and b.

Ans. Since $(a-b), a, (a+b)$ are the zeroes of the polynomial x^3-3x^2+x+1.

$\therefore \alpha+\beta+\gamma$ = $a-b+b+a+b=\frac{-(-3)}{1}=3$

$\Rightarrow 3a=3$

$\Rightarrow a=1$

And $\alpha\beta+\beta\gamma+\gamma\alpha$

= $(a-b)a+a(a+b)+(a+b)(a-b)=\frac{1}{1}=1$

$\Rightarrow a^2-ab+a^2+ab+a^2-b^2=1$

$\Rightarrow 3a^2-b^2=1$

$\Rightarrow 3(1)^2 - b^2 = 1 \; [\because a = 1]$

$\Rightarrow 3 - b^2 = 1 \qquad \Rightarrow \qquad b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

Hence $a = 1$ and $b = \pm\sqrt{2}$.

---

4. If the two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Ans. Since $2 \pm \sqrt{3}$ are two zeroes of the polynomial $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$.

Let $x = 2 \pm \sqrt{3} \Rightarrow x - 2 = \pm\sqrt{3}$

Squaring both sides, $x^2 - 4x + 4 = 3$

$\Rightarrow x^2 - 4x + 1 = 0$

Now we divide $p(x)$ by $x^2 - 4x + 1$ to obtain other zeroes.

$$
\begin{array}{r l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1) & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & \pm x^4 \mp 4x^3 \pm x^2 \\
\hline
 & -2x^3 - 27x^2 + 138x \\
 & \mp 2x^3 \pm 8x^2 \mp 2x \\
\hline
 & -35x^2 + 140x - 35 \\
 & \mp 35x^2 \pm 140x \mp 35 \\
\hline
 & 0
\end{array}
$$

$\therefore p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2-4x+1)(x^2-7x+5x-35)$

$= (x^2-4x+1)[x(x-7)+5(x-7)]$

$= (x^2-4x+1)(x+5)(x-7)$

$\Rightarrow (x+5)$ and $(x-7)$ are the other factors of $p(x)$.

$\therefore -5$ and 7 are other zeroes of the given polynomial.

---

5. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$, find k and a.

Ans. Let us divide $x^4-6x^3+16x^2-25x+10$ by x^2-2x+k.

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\
 & \pm x^4 \mp 2x^3 \pm kx^2 \\
\hline
 & -4x^3+(16-k)x^2-25x+10 \\
 & \mp 4x^3 \pm 8x^2 \mp 4kx \\
\hline
 & (8-k)\ x^2+(4k-25)x+10 \\
 & \pm(8-k)x^2 \mp 2(8-k)x \pm (8-k)k \\
\hline
 & (2k-9)x-(8-k)k+10
\end{array}
$$

$\therefore$ Remainder = $(2k-9)x-(8-k)k+10$

On comparing this remainder with given remainder, i.e. $x+a$,

$2k-9=1 \Rightarrow 2k=10$

$\Rightarrow k=5$

And $-(8-k)k+10=a$

$\Rightarrow a=-(8-5)5+10=-5$

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Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.1

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Ans. Let the present age of Aftab and his daughter be x years and y years respectively.

Seven years ago, Age of Aftab = (x - 7) years and Age of his daughter = (y - 7) years.

According to the given condition,

$$(x-7)=7(y-7)$$

$$\Rightarrow x-7=7y-49$$

$$\Rightarrow x-7y=-42$$

Again, Three years hence, Age of Aftab = x + 3 and Age of his daughter = y + 3

According to the given condition,

$$(x+3)=3(y+3)$$

$$\Rightarrow x+3=3y+9$$

$$\Rightarrow x-3y=6$$

Thus, the given conditions can be algebraically represented as:

x - 7y = –42

$\Rightarrow$ x = –42 + 7y

Three solutions of this equation can be written in a table as follows:

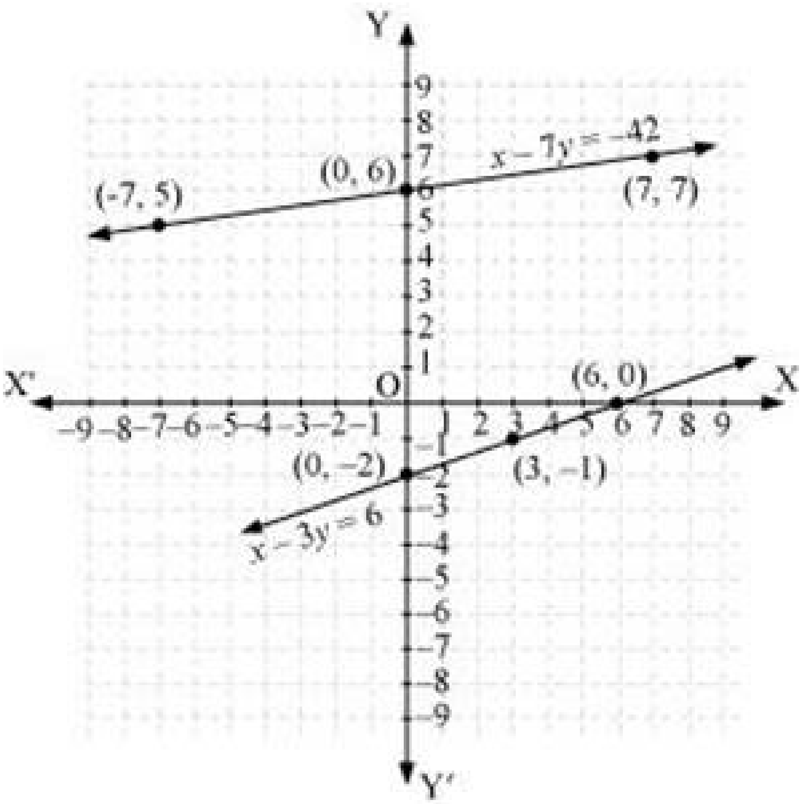
x	–7	0	7
y	5	6	7

And x – 3y = 6

$\Rightarrow x = 6 + 3y$

Three solutions of this equation can be written in a table as follows:

x	6	3	0
y	0	−1	−2



The graphical representation is as follows:

Concept insight: In order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

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2. The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 3 more balls of the same kind for Rs 1300. Represent this situation algebraically and graphically.

Ans. Let cost of 1 cricket bat = Rs x and let cost of 1 cricket ball= Rs y

According to given conditions, we have

$3x + 6y = 3900 \Rightarrow x + 2y = 1300$... **(1)**

And $x + 3y = 1300$... **(2)**

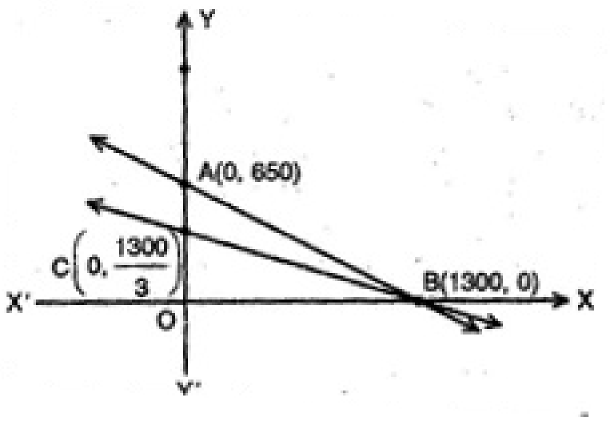
For equation $x + 2y = 1300$, we have following points which lie on the line

x	0	1300
y	650	0

For equation $x + 3y = 1300$, we have following points which lie on the line

x	0	1300
y	$\frac{1300}{3}$	0

We plot the points for both of the equations and it is the graphical representation of the given situation.



It is clear that these lines intersect at B (1300,0).

3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Ans. Let cost of 1 kg of apples = Rs x and let cost of 1 kg of grapes= Rs y

According to given conditions, we have

$2x + y = 160$... **(1)**

$4x + 2y = 300$

$\Rightarrow 2x + y = 150$... **(2)**

So, we have equations **(1)** and **(2),** $2x + y = 160$ and $2x + y = 150$ which represent given situation algebraically.

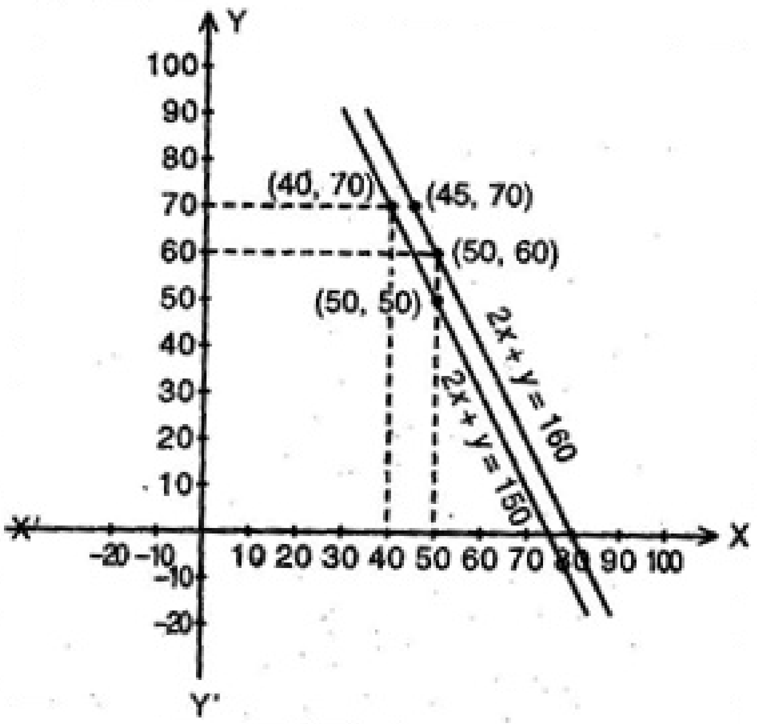
For equation 2x + y = 160, we have following points which lie on the line

x	50	45
y	60	70

For equation 2x + y = 150, we have following points which lie on the line

x	50	40
y	50	70

We plot the points for both of the equations and it is the graphical representation of the given situation.



CBSE Class–10 Mathematics

NCERT solution

Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of class X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Ans. (i) Let number of boys who took part in the quiz = x

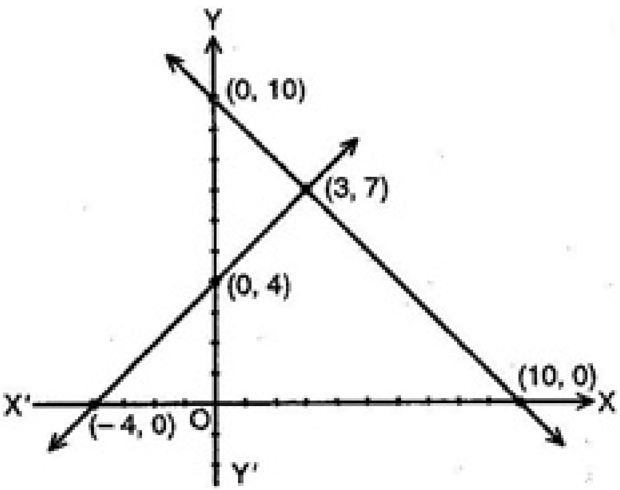
Let number of girls who took part in the quiz = y

According to given conditions, we have

$x + y = 10$... **(1)**

And, $y = x + 4$

$\Rightarrow x - y = -4$ ... **(2)**



For equation x + y = 10, we have following points which lie on the line

x	0	10
y	10	0

For equation x – y = –4, we have following points which lie on the line

x	0	- 4
y	4	0

We plot the points for both of the equations to find the solution.

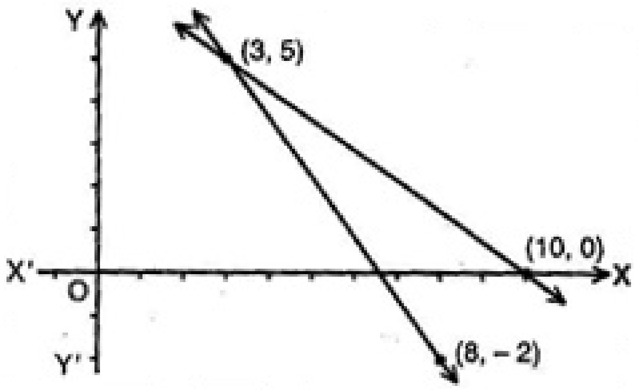
We can clearly see that the intersection point of two lines is **(3, 7).**

Therefore, number of boys who took park in the quiz = 3 and, number of girls who took part in the quiz = 7.

(ii) Let cost of one pencil = Rs x and Let cost of one pen = Rs y

According to given conditions, we have

$5x + 7y = 50$... **(1)**



$7x + 5y = 46$... **(2)**

For equation 5x + 7y = 50, we have following points which lie on the line

x	10	3
y	0	5

For equation 7x + 5y = 46, we have following points which lie on the line

x	8	3
y	−2	5

We can clearly see that the intersection point of two lines is **(3, 5).**

Therefore, cost of pencil = Rs 3 and, cost of pen = Rs 5

---

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2} and \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

(i) 5*x* − 4*y* + 8 = 0;   7x + 6y - 9 = 0

(ii)9*x* + 3*y* + 12 = 0; 18x + 6y + 24 = 0

(iii) 6*x* − 3*y* + 10 = 0; 2*x* – *y* + 9 = 0

Ans. (i) 5*x* − 4*y* + 8 = 0, 7*x* + 6*y* – 9 = 0

Comparing equation 5*x* − 4*y* + 8 = 0 with $a_1x+b_1y+c_1 = 0$ and 7*x* + 6*y* – 9 = 0 with $a_2x+b_2y+c_2 = 0$,

We get, $a_1 = 5, b_1 = -4, c_1 = 8, a_2 = 7, b_2 = 6, c_2 = -9$

We have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ because $\frac{5}{7} \neq \frac{-4}{6}$

Hence, lines have unique solution which means they intersect at one point.

(ii) 9*x* + 3*y* + 12 = 0, 18*x* + 6*y* + 24 = 0

Comparing equation 9*x* + 3*y* + 12 = 0 with $a_1x+b_1y+c_1 = 0$ and 18*x* + 6*y* + 24 = 0 with $a_2x+b_2y+c_2 = 0$,

We get, $a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18, b_2 = 6, c_2 = 24$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Hence, lines are coincide.

(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

Comparing equation $6x - 3y + 10 = 0$ with $a_1x + b_1y + c_1 = 0$ and $2x - y + 9 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 6$, $b_1 = -3$, $c_1 = 10$, $a_2 = 2$, $b_2 = -1$, $c_2 = 9$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9} \Rightarrow \frac{3}{1} = \frac{3}{1} \neq \frac{10}{9}$

Hence, lines are parallel to each other.

---

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$, $2x - 3y = 7$

(ii) $2x - 3y = 8$, $4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$, $9x - 10y = 14$

(iv) $5x - 3y = 11$, $-10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Ans. (i) $3x + 2y = 5$, $2x - 3y = 7$

Comparing equation $3x + 2y = 5$ with $a_1x + b_1y + c_1 = 0$ and $2x - 3y = 7$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 3, b_1 = 2, c_1 = -5,$ $a_2 = 2, b_2 = -3, c_2 = -7$

$\frac{a_1}{a_2} = \frac{3}{2}$ and $\frac{b_1}{b_2} = \frac{2}{-3}$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ which means equations have unique solution.

Hence they are consistent.

(ii) $2x - 3y = 8$, $4x - 6y = 9$

Comparing equation $2x - 3y = 8$ with $a_1x + b_1y + c_1 = 0$ and $4x - 6y = 9$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 2, b_1 = -3, c_1 = -8, a_2 = 4, b_2 = -6, c_2 = -9$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{2}{4} = \frac{-3}{-6} \neq \frac{-8}{-9} \Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{-8}{-9}$

Therefore, equations have no solution because they are parallel.

Hence, they are inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$, $9x - 10y = 14$

Comparing equation $\frac{3}{2}x + \frac{5}{3}y = 7$ with $a_1x + b_1y + c_1 = 0$ and $9x - 10y = 14$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7, a_2 = 9, b_2 = -10, c_2 = -14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}$ and $\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, equations have unique solution.

Hence, they are consistent.

(iv) $5x - 3y = 11$, $-10x + 6y = -22$

Comparing equation $5x - 3y = 11$ with $a_1x + b_1y + c_1 = 0$ and $-10x + 6y = -22$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 5, b_1 = -3, c_1 = -11, a_2 = -10, b_2 = 6, c_2 = 22$

$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinite many solutions.

Hence, they are consistent.

(v) $\frac{4}{3}x + 2y = 8;\ \ 2x + 3y = 12$

Comparing equation $\frac{4}{3}x + 2y = 8$ with $a_1x + b_1y + c_1 = 0$ and $2x + 3y = 12$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8, a_2 = 2, b_2 = 3, c_2 = -12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinite many solutions.

Hence, they are consistent.

---

4. Which of the following pairs of linear equations are consistent/inconsistent? If

consistent, obtain the solution graphically:

(i) $x + y = 5$**,** $2x + 2y = 10$

(ii) $x - y = 8$**,** $3x - 3y = 16$

(iii) $2x + y - 6 = 0$**,** $4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0$**,** $4x - 4y - 5 = 0$

Ans. (i) $x + y = 5$, $2x + 2y = 10$

For equation x + y = 5 we have following points which lie on the line

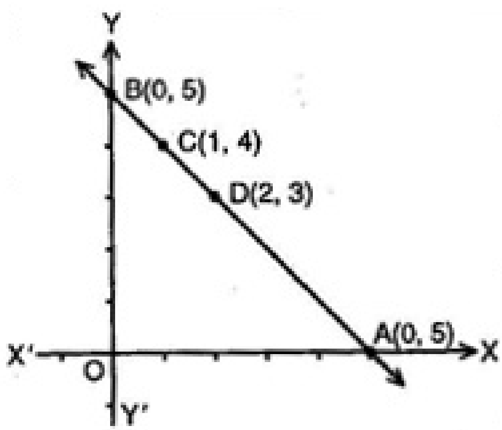
x	0	5
y	5	0

For equation 2x + 2y – 10 = 0, we have following points which lie on the line

x	1	2
y	4	3

We can see that both of the lines coincide. Hence, there are infinite many solutions. Any point which lies on one line also lies on the other. Hence, by using equation ($x + y - 5 = 0$), we can say that $x = 5 - y$

We can assume any random values for y and can find the corresponding value of x using the above equation. All such points will lie on both lines and there will be infinite number of such points.



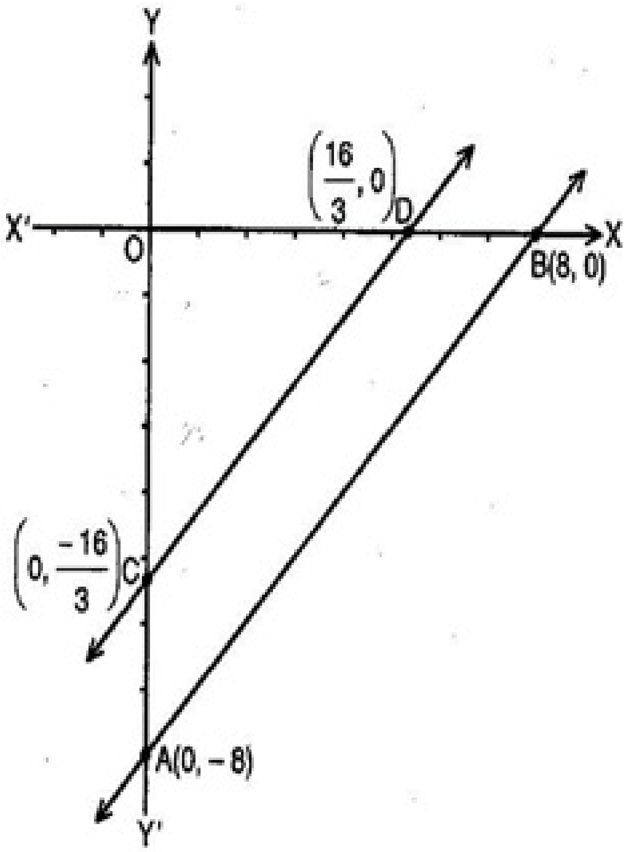
(ii) $x - y = 8$, $3x - 3y = 16$

For x – y = 8, the coordinates are:

x	0	8
y	– 8	0

And for 3x – 3y = 16, the coordinates

x	0	$\frac{16}{3}$
y	$\frac{-16}{3}$	0



Plotting these points on the graph, it is clear that both lines are parallel. So the two lines have no common point. Hence the given equations have no solution and lines are inconsistent.

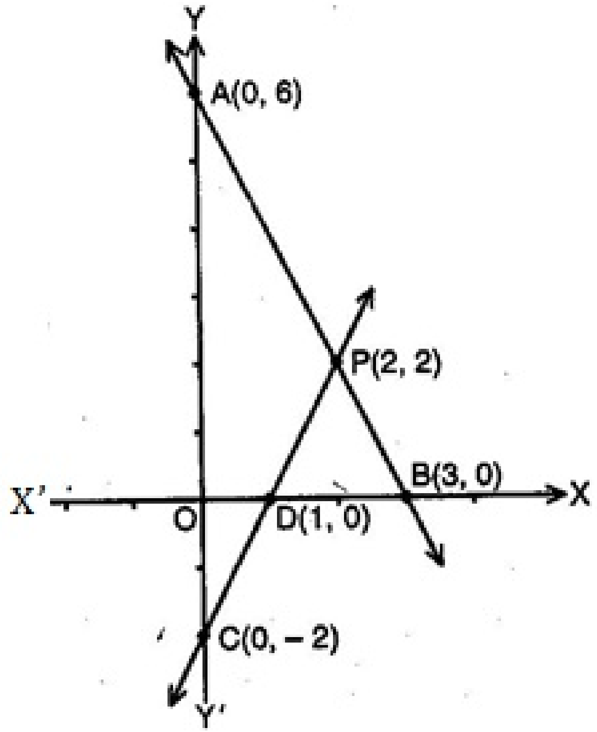
(iii) $2x + y - 6 = 0$, $4x - 2y - 4 = 0$

For equation 2x + y – 6 = 0, we have following points which lie on the line

x	0	3
y	6	0

For equation 4x – 2y – 4 = 0, we have following points which lie on the line

x	0	1
y	−2	0



We can clearly see that lines are intersecting at (2, 2) which is the solution.

Hence x = 2 and y = 2 and lines are consistent.

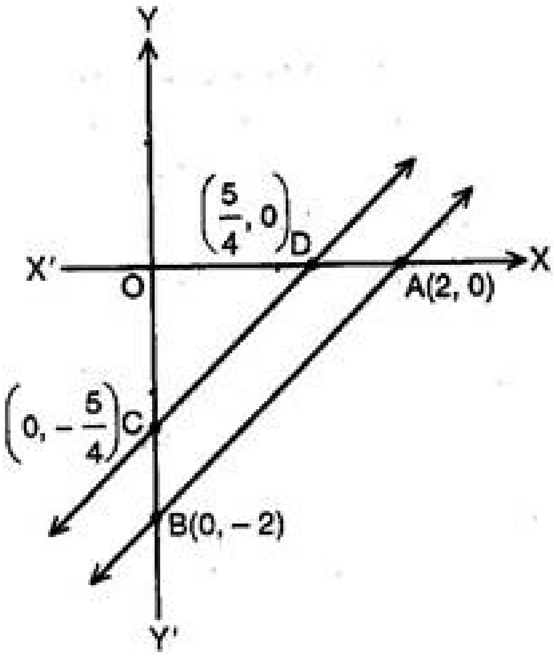
(iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

For 2x – 2y – 2 = 0, the coordinates are:

x	2	0
y	0	−2

And for 4x – 4y – 5 = 0, the coordinates

x	0	$\frac{5}{4}$
y	$\frac{-5}{4}$	0



Plotting these points on the graph, it is clear that both lines are parallel. So the two lines have no common point. Hence the given equations have no solution and lines are inconsistent.

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5. Half the perimeter of a rectangle garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Ans. Let length of rectangular garden = x metres

Let width of rectangular garden = y metres

According to given conditions, perimeter = 36 m

$\Rightarrow \frac{1}{2}\left[2\left(x+y\right)\right]=36$

$\Rightarrow$ x + y = 36 ……(i)

And x = y + 4

$\Rightarrow x - y = 4$........(ii)

Adding eq. (i) and (ii),

2x = 40

$\Rightarrow$ x = 20 m

Subtracting eq. (ii) from eq. (i),

2y = 32

$\Rightarrow$ y = 16 m

Hence, length = 20 m and width = 16 m

---

6. Given the linear equation (2x + 3y – 8 = 0), write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines

(ii) Parallel lines

(iii) Coincident lines

Ans. (i) Let the second line be equal to $a_2x + b_2y + c_2 = 0$

Comparing given line **2x + 3y – 8 = 0** with $a_1x + b_1y + c_1 = 0$,

We get $a_1 = 2, b_1 = 3 \, and \, c_1 = -8$

Two lines intersect with each other if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, second equation can be **x** + **2y** = **3** because $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Let the second line be equal to $a_2x + b_2y + c_2 = 0$

Comparing given line **2x** + **3y** – **8** = **0 with** $a_1x+b_1y+c_1 = 0$,

We get $a_1 = 2, b_1 = 3 \text{ and } c_1 = -8$

Two lines are parallel to each other if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, second equation can be **2x** + **3y** – **2** = **0** because $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) Let the second line be equal to $a_2x+b_2y+c_2 = 0$

Comparing given line **2x** + **3y** – **8** = **0 with** $a_1x+b_1y+c_1 = 0$,

We get $a_1 = 2, b_1 = 3 \text{ and } c_1 = -8$

Two lines are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, second equation can be **4x** + **6y** – **16** = **0** because $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

---

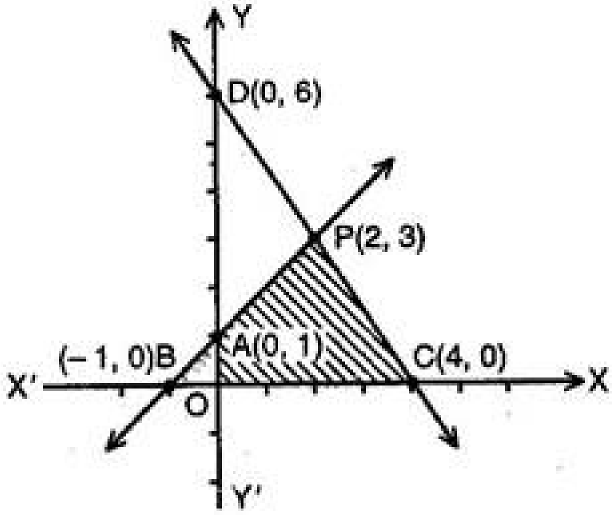
7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Ans. For equation x – y + 1 = 0, we have following points which lie on the line

x	0	–1
y	1	0

For equation 3x + 2y – 12 = 0, we have following points which lie on the line

x	4	0
y	0	6



We can see from the graphs that points of intersection of the lines with the x–axis are (–1, 0), (2, 3) and (4, 0).

CBSE Class–10 Mathematics

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Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.3

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(ii) $s - t = 3$

$$\frac{s}{3}+\frac{t}{2}=6$$

(iii) $3x - y = 3$

$9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$$

Ans. (i) $x + y = 14$ ...(1)

$x - y = 4$ ... (2)

$x = 4 + y$ from equation (2)

Putting this in equation (1), we get

$4 + y + y = 14$

$\Rightarrow 2y = 10 \Rightarrow y = 5$

Putting value of y in equation (1), we get

$x + 5 = 14$

$\Rightarrow x = 14 - 5 = 9$

Therefore, $x = 9$ and $y = 5$

(ii) $s - t = 3$ ... (1)

$\frac{s}{3} + \frac{t}{2} = 6$ ...(2)

Using equation (1), we can say that $s = 3 + t$

Putting this in equation (2), we get

$$\frac{3+t}{3} + \frac{t}{2} = 6$$

$\Rightarrow \frac{6+2t+3t}{6} = 6$

$\Rightarrow 5t + 6 = 36$

$\Rightarrow 5t = 30 \Rightarrow t = 6$

Putting value of t in equation (1), we get

$s - 6 = 3 \Rightarrow s = 3 + 6 = 9$

Therefore, $t = 6$ and $s = 9$

(iii) $3x - y = 3$ ... (1)

$9x - 3y = 9$ ... (2)

Comparing equation $3x - y = 3$ with $a_1x + b_1y + c_1 = 0$ and equation $9x - 3y = 9$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = -1, c_1 = -3, a_2 = 9, b_2 = -3 \; and \; c_2 = -9$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, we have infinite many solutions for x and y

(iv) $0.2x + 0.3y = 1.3$ ... (1)

$0.4x + 0.5y = 2.3$ ... (2)

Using equation (1), we can say that

$0.2x = 1.3 - 0.3y$

$\Rightarrow x = \frac{1.3 - 0.3y}{0.2}$

Putting this in equation (2), we get

$0.4\left(\frac{1.3 - 0.3y}{0.2}\right) + 0.5y = 2.3$

$\Rightarrow 2.6 - 0.6y + 0.5y = 2.3$

$\Rightarrow -0.1y = -0.3 \Rightarrow y = 3$

Putting value of y in (1), we get

$0.2x + 0.3(3) = 1.3$

$\Rightarrow 0.2x + 0.9 = 1.3$

$\Rightarrow 0.2x = 0.4 \Rightarrow x = 2$

Therefore, $x = 2$ and $y = 3$

(v) $\sqrt{2}x+\sqrt{3}y=0$ ....(1)

$\sqrt{3}x-\sqrt{8}y=0$ ....(2)

Using equation (1), we can say that

$x = \dfrac{-\sqrt{3}y}{\sqrt{2}}$

Putting this in equation (2), we get

$$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0 \Rightarrow \frac{-3y}{\sqrt{2}}-\sqrt{8}y=0$$

$$\Rightarrow y\left(\frac{-3}{\sqrt{2}}-\sqrt{8}\right)=0 \Rightarrow y = 0$$

Putting value of y in (1), we get $x = 0$

Therefore, $x = 0$ and $y = 0$

(vi) $\dfrac{3x}{2}-\dfrac{5y}{3}=-2$ ... (1)

$\dfrac{x}{3}+\dfrac{y}{2}=\dfrac{13}{6}$ ... (2)

Using equation (2), we can say that

$x = \left(\dfrac{13}{6}-\dfrac{y}{2}\right)\times 3$

$\Rightarrow x = \dfrac{13}{2}-\dfrac{3y}{2}$

Putting this in equation (1), we get

$$\frac{3}{2}\left(\frac{13}{2}-\frac{3y}{2}\right)-\frac{5y}{3}=\frac{-2}{1}$$

$$\Rightarrow \frac{39}{4}-\frac{9y}{4}-\frac{5y}{3}=-2$$

$$\Rightarrow \frac{-27y-20y}{12}=-2-\frac{39}{4}$$

$$\Rightarrow \frac{-47y}{12}=\frac{-8-39}{4}$$

$$\Rightarrow \frac{-47y}{12}=\frac{-47}{4} \Rightarrow y = 3$$

Putting value of y in equation (2), we get

$$\frac{x}{3}+\frac{3}{2}=\frac{13}{6}$$

$$\Rightarrow \frac{x}{3}=\frac{13}{6}-\frac{3}{2}=\frac{13-9}{6}=\frac{4}{6}=\frac{2}{3}$$

$$\Rightarrow \frac{x}{3}=\frac{2}{3}$$

$\Rightarrow$ x = 2

Therefore, $x = 2$ and $y = 3$

---

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which

$y = mx + 3$.

Ans. $2x + 3y = 11$ ... (1)

$2x - 4y = -24$ ... (2)

Using equation (2), we can say that

$2x = -24 + 4y$

$\Rightarrow x = -12 + 2y$

Putting this in equation (1), we get

$2(-12 + 2y) + 3y = 11$

$\Rightarrow -24 + 4y + 3y = 11$

$\Rightarrow 7y = 35 \Rightarrow y = 5$

Putting value of y in equation (1), we get

$2x + 3(5) = 11$

$\Rightarrow 2x + 15 = 11$

$\Rightarrow 2x = 11 - 15 = -4 \Rightarrow x = -2$

Therefore, $x = -2$ and $y = 5$

Putting values of x and y in $y = mx + 3$, we get

$5 = m(-2) + 3$

$\Rightarrow 5 = -2m + 3$

$\Rightarrow -2m = 2 \Rightarrow m = -1$

---

3. Form a pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii)The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats

and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Ans. (i) Let first number be x and second number be y.

According to given conditions, we have

$x - y = 26$ (assuming x > y) ... (1)

$x = 3y$(x > y)... (2)

Putting equation (2) in (1), we get

$3y - y = 26$

$\Rightarrow 2y = 26$

$\Rightarrow y = 13$

Putting value of y in equation (2), we get

$x = 3y = 3 \times 13 = 39$

Therefore, two numbers are 13 and 39.

(ii) Let smaller angle $= x$ and let larger angle $= y$

According to given conditions, we have

$y = x + 18$ … (1)

Also, $x + y = 180^0$ (Sum of supplementary angles) … (2)

Putting (1) in equation (2), we get

$x + x + 18 = 180$

$\Rightarrow 2x = 180 – 18 = 162$

$\Rightarrow x = 81^0$

Putting value of x in equation (1), we get

$y = x + 18 = 81 + 18 = 99^0$

Therefore, two angles are 81^0 *and* 99^0 .

(iii) Let cost of each bat = Rs x and let cost of each ball = Rs y

According to given conditions, we have

$7x + 6y = 3800$ … (1)

And, $3x + 5y = 1750$ … (2)

Using equation (1), we can say that

$7x = 3800 - 6y \Rightarrow x = \dfrac{3800-6y}{7}$

Putting this in equation (2), we get

$3\left(\dfrac{3800-6y}{7}\right) + 5y = 1750$

$\Rightarrow \left(\dfrac{11400-18y}{7}\right) + 5y = 1750$

$\Rightarrow \frac{5y}{1} - \frac{18y}{7} = \frac{1750}{1} - \frac{11400}{7}$

$\Rightarrow \frac{35y - 18y}{7} = \frac{12250 - 11400}{7}$

$\Rightarrow 17y = 850 \Rightarrow y = 50$

Putting value of y in (2), we get

$3x + 250 = 1750$

$\Rightarrow 3x = 1500 \Rightarrow x = 500$

Therefore, cost of each bat = Rs 500 and cost of each ball = Rs 50

(iv) Let fixed charge = Rs x and let charge for every km = Rs y

According to given conditions, we have

$x + 10y = 105$... (1)

$x + 15y = 155$... (2)

Using equation (1), we can say that

$x = 105 - 10y$

Putting this in equation (2), we get

$105 - 10y + 15y = 155$

$\Rightarrow 5y = 50 \Rightarrow y = 10$

Putting value of y in equation (1), we get

$x + 10\ (10) = 105$

$\Rightarrow x = 105 - 100 = 5$

Therefore, fixed charge = Rs 5 and charge per km = Rs 10

To travel distance of 25 Km, person will have to pay = Rs (x + 25y)

= Rs (5 + 25 × 10)

= Rs (5 + 250) = Rs 255

(v) Let numerator = x and let denominator = y

According to given conditions, we have

$\frac{x+2}{y+2} = \frac{9}{11}$ ... (1)

$\frac{x+3}{y+3} = \frac{5}{6}$ ... (2)

Using equation (1), we can say that

11 (x + 2) = 9y + 18

⇒ 11x + 22 = 9y + 18

⇒ 11x = 9y – 4

⇒ x = $\frac{9y-4}{11}$

Putting value of x in equation (2), we get

$6\left(\frac{9y-4}{11}+3\right)$ = 5 (y + 3)

⇒ $\frac{54y}{11} - \frac{24}{11} + 18 = 5y + 15$

⇒ $-\frac{24}{11} + \frac{3}{1} = \frac{5y}{1} - \frac{54y}{11}$

⇒ $\frac{-24+33}{11} = \frac{55y-54y}{11}$

$\Rightarrow y = 9$

Putting value of y in (1), we get

$$\frac{x+2}{9+2} = \frac{9}{11}$$

$\Rightarrow x + 2 = 9 \Rightarrow x = 7$

Therefore, fraction = $\frac{x}{y} = \frac{7}{9}$

(vi) Let present age of Jacob = x years

Let present age of Jacob’s son = y years

According to given conditions, we have

$(x + 5) = 3 (y + 5)$ … (1)

And, $(x - 5) = 7 (y - 5)$ … (2)

From equation (1), we can say that

$x + 5 = 3y + 15$

$\Rightarrow x = 10 + 3y$

Putting value of x in equation (2) we get

$10 + 3y - 5 = 7y - 35$

$\Rightarrow -4y = -40$

$\Rightarrow y = 10$ years

Putting value of y in equation (1), we get

$x + 5 = 3 (10 + 5) = 3 \times 15 = 45$

$\Rightarrow x = 45 - 5 = 40$ years

Therefore, present age of Jacob = 40 years and, present age of Jacob’s son = 10 years

CBSE Class–10 Mathematics

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Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x + y = 5$, $2x - 3y = 4$

(ii) $3x + 4y = 10$, $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$, $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1, x - \frac{y}{3} = 3$

Ans. (i) $x + y = 5$ … (1)

$2x - 3y = 4$ … (2)

Elimination method:

Multiplying equation (1) by 2, we get equation (3)

$2x + 2y = 10$ … (3)

$2x - 3y = 4$ … (2)

Subtracting equation (2) from (3), we get

$5y = 6 \Rightarrow y = \frac{6}{5}$

Putting value of y in (1), we get

$x + \frac{6}{5} = 5$

$\Rightarrow x = 5 - \frac{6}{5} = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

Substitution method:

$x + y = 5$ ... (1)

$2x - 3y = 4$ ... (2)

From equation (1), we get,

$x = 5 - y$

Putting this in equation (2), we get

$2(5 - y) - 3y = 4$

$\Rightarrow 10 - 2y - 3y = 4$

$\Rightarrow 5y = 6 \Rightarrow y = \frac{6}{5}$

Putting value of y in (1), we get

$x = 5 - \frac{6}{5} = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) $3x + 4y = 10$... (1)

$2x - 2y = 2$... (2)

Elimination method:

Multiplying equation (2) by 2, we get (3)

$4x - 4y = 4$ … (3)

$3x + 4y = 10$ … (1)

Adding (3) and (1), we get

$7x = 14 \Rightarrow x = 2$

Putting value of x in (1), we get

$3(2) + 4y = 10$

$\Rightarrow 4y = 10 - 6 = 4$

$\Rightarrow y = 1$

Therefore, $x = 2$ and $y = 1$

Substitution method:

$3x + 4y = 10$… (1)

$2x - 2y = 2$… (2)

From equation (2), we get

$2x = 2 + 2y$

$\Rightarrow x = 1 + y$ … (3)

Putting this in equation (1), we get

$3(1 + y) + 4y = 10$

$\Rightarrow 3 + 3y + 4y = 10$

$\Rightarrow 7y = 7 \Rightarrow y = 1$

Putting value of y in (3), we get $x = 1 + 1 = 2$

Therefore, $x = 2$ and $y = 1$

(iii) $3x - 5y - 4 = 0$ ... (1)

$9x = 2y + 7$... (2)

Elimination method:

Multiplying (1) by 3, we get (3)

$9x - 15y - 12 = 0$... (3)

$9x - 2y - 7 = 0$... (2)

Subtracting (2) from (3), we get

$-13y - 5 = 0$

$\Rightarrow -13y = 5$

$\Rightarrow y = \dfrac{-5}{13}$

Putting value of y in (1), we get

$3x - 5\left(\dfrac{-5}{13}\right) - 4 = 0$

$\Rightarrow 3x = 4 - \dfrac{25}{13} = \dfrac{52-25}{13} = \dfrac{27}{13}$

$\Rightarrow x = \dfrac{27}{13\times 3} = \dfrac{9}{13}$

Therefore, $x = \dfrac{9}{13}$ and $y = \dfrac{-5}{13}$

Substitution Method:

$3x - 5y - 4 = 0$ ... (1)

$9x = 2y + 7$... (2)

From equation (1), we can say that

$3x = 4 + 5y \Rightarrow x = \dfrac{4+5y}{3}$

Putting this in equation (2), we get

$9\left(\dfrac{4+5y}{3}\right) - 2y = 7$

$\Rightarrow 12 + 15y - 2y = 7$

$\Rightarrow 13y = -5 \Rightarrow y = \dfrac{-5}{13}$

Putting value of y in (1), we get

$3x - 5\left(\dfrac{-5}{13}\right) = 4$

$\Rightarrow 3x = 4 - \dfrac{25}{13} = \dfrac{52-25}{13} = \dfrac{27}{13}$

$\Rightarrow x = \dfrac{27}{13\times 3} = \dfrac{9}{13}$

Therefore, $x = \dfrac{9}{13}$ and $y = \dfrac{-5}{13}$

(iv) $\dfrac{x}{2} + \dfrac{2y}{3} = -1$ … (1)

$x - \dfrac{y}{3} = 3$ … (2)

Elimination method:

Multiplying equation (2) by 2, we get (3)

$2x - \frac{2}{3}y = 6$ … (3)

$\frac{x}{2} + \frac{2y}{3} = -1$ … (1)

Adding (3) and (1), we get

$\frac{5}{2}x = 5$ ⇒ $x = 2$

Putting value of x in (2), we get

$2 - \frac{y}{3} = 3$

⇒ $y = -3$

Therefore, $x = 2$ and $y = -3$

Substitution method:

$\frac{x}{2} + \frac{2y}{3} = -1$ … (1)

$x - \frac{y}{3} = 3$ … (2)

From equation (2), we can say that $x = 3 + \frac{y}{3} = \frac{9+y}{3}$

Putting this in equation (1), we get

$\frac{9+y}{6} + \frac{2}{3}y = -1$

⇒ $\frac{9+y+4y}{6} = -1$

⇒ $5y + 9 = -6$

$\Rightarrow 5y = -15 \Rightarrow y = -3$

Putting value of y in (1), we get

$$\frac{x}{2} + \frac{2}{3}(-3) = -1 \Rightarrow x = 2$$

Therefore, $x = 2$ and $y = -3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes ½ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as sonu. Ten years later, Nuri will be twice as old as sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two–digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Ans. (i) Let numerator $= x$ and let denominator $= y$

According to given condition, we have

$$\frac{x+1}{y-1} = 1 \text{ and } \frac{x}{y+1} = \frac{1}{2}$$

$\Rightarrow x + 1 = y - 1$ and $2x = y + 1$

$\Rightarrow x - y = -2$ ... **(1)** and $2x - y = 1$... **(2)**

So, we have equations **(1)** and **(2)**, multiplying equation **(1)** by 2 we get **(3)**

$2x - 2y = -4$... **(3)**

$2x - y = 1$... **(2)**

Subtracting equation **(2)** from **(3)**, we get

$-y = -5 \Rightarrow y = 5$

Putting value of y in **(1)**, we get

$x - 5 = -2 \Rightarrow x = -2 + 5 = 3$

Therefore, fraction = $\frac{x}{y} = \frac{3}{5}$

(ii) Let present age of Nuri = x years and let present age of Sonu = y years

5 years ago, age of Nuri = (x – 5) years

5 years ago, age of Sonu = (y – 5) years

According to given condition, we have

$(x - 5) = 3(y - 5)$

$\Rightarrow x - 5 = 3y - 15$

$\Rightarrow x - 3y = -10$... **(1)**

10 years later from present, age of Nuri = $(x + 10)$ years

10 years later from present, age of Sonu = $(y + 10)$ years

According to given condition, we have

$(x + 10) = 2(y + 10)$

$\Rightarrow x + 10 = 2y + 20$

$\Rightarrow x - 2y = 10$ ... **(2)**

Subtracting equation **(1)** from **(2)**, we get

$y = 10 - (-10) = 20$ years

Putting value of **y** in **(1)**, we get

$x - 3\,(20) = -10$

$\Rightarrow x - 60 = -10$

$\Rightarrow x = 50$ years

Therefore, present age of Nuri = 50 years and present age of Sonu = 20 years

(iii) Let digit at ten's place = x and Let digit at one's place = y

According to given condition, we have

$x + y = 9$ ... **(1)**

And $9\,(10x + y) = 2\,(10y + x)$

$\Rightarrow 90x + 9y = 20y + 2x$

$\Rightarrow 88x = 11y$

$\Rightarrow 8x = y$

$\Rightarrow 8x - y = 0$ ... **(2)**

Adding **(1)** and **(2)**, we get

$9x = 9 \Rightarrow x = 1$

Putting value of **x** in **(1)**, we get

$1 + y = 9$

$\Rightarrow y = 9 - 1 = 8$

Therefore, number = $10x + y = 10(1) + 8 = 10 + 8 = 18$

(iv) Let number of Rs 100 notes = x and let number of Rs 50 notes = y

According to given conditions, we have

$x + y = 25$ ... **(1)**

and $100x + 50y = 2000$

$\Rightarrow 2x + y = 40$ ... **(2)**

Subtracting **(2)** from **(1)**, we get

$-x = -15 \Rightarrow x = 15$

Putting value of **x** in **(1)**, we get

$15 + y = 25$

$\Rightarrow y = 25 - 15 = 10$

Therefore, number of Rs 100 notes = 15 and number of Rs 50 notes = 10

(v) Let fixed charge for 3 days = Rs x
Let additional charge for each day thereafter = Rs y
According to given condition, we have
$x + 4y = 27$ ... **(1)**
$x + 2y = 21$ ... **(2)**
Subtracting **(2)** from **(1)**, we get
$2y = 6 \Rightarrow y = 3$
Putting value of **y** in **(1)**, we get
$x + 4(3) = 27$
$\Rightarrow x = 27 - 12 = 15$
Therefore, fixed charge for 3 days = Rs 15 and additional charge for each day after 3 days = Rs 3

CBSE Class–10 Mathematics

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Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.5

1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0

3x – 9y – 2 = 0

(ii)2x + y = 5

3x + 2y = 8

(iii) 3x − 5y = 20

6x − 10y = 40

(iv) x − 3y – 7 = 0

3x − 3y – 15 = 0

Ans. (i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Comparing equation $x - 3y - 3 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 9y - 2 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -3, a_2 = 3, b_2 = -9, c_2 = -2$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ this means that the two lines are parallel.

Therefore, there is no solution for the given equations i.e. it is inconsistent.

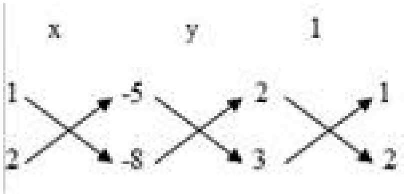
(ii) $2x + y = 5$

$3x + 2y = 8$

Comparing equation $2x + y = 5$ with $a_1x+b_1y+c_1 = 0$ and $3x + 2y = 8$ with $a_2x+b_2y+c_2 = 0$,

We get $a_1 = 2, b_1 = 1, c_1 = -5, a_2 = 3, b_2 = 2, c_2 = -8$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that there is unique solution for the given equations.



$$\frac{x}{(-8)(1)-(2)(-5)} = \frac{y}{(-5)(3)-(-8)(2)} = \frac{1}{(2)2-(3)1}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow x = 2$ and $y = 1$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

Comparing equation $3x - 5y = 20$ with $a_1x+b_1y+c_1 = 0$ and $6x - 10y = 40$ with $a_2x+b_2y+c_2 = 0$,

We get $a_1 = 3, b_1 = -5, c_1 = -20, a_2 = 6, b_2 = -10, c_2 = -40$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

It means lines coincide with each other.

Hence, there are infinitely many solutions.

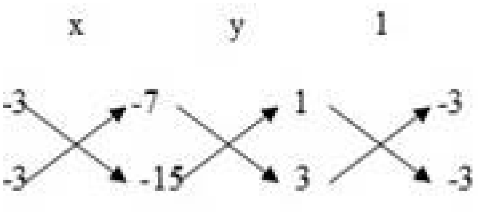
(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Comparing equation $x - 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 3y - 15 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -7, a_2 = 3, b_2 = -3, c_2 = -15$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that we have unique solution for these equations.



$$\frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{(-7)(3)-(-15)(1)} = \frac{1}{(-3)1-(-3)3}$$

$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$

$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

$\Rightarrow x = 4$ and $y = -1$

---

2. (i) For which values of a and b does the following pair of linear equations have an

infinite number of solutions?

$2x + 3y = 7$

$(a - b)\,x + (a + b)\,y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)\,x + (k - 1)\,y = 2k + 1$

Ans. (i) Comparing equation $2x + 3y - 7 = 0$ with $a_1x+b_1y+c_1 = 0$ and $(a - b)\,x + (a + b)\,y - 3a - b + 2 = 0$ with $a_2x+b_2y+c_2 = 0$

We get $a_1 = 2, b_1 = 3$ and $c_1 = -7, a_2 = (a-b), b_2 = (a+b)$ and $c_2 = 2 - b - 3a$

Linear equations have infinite many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{2-b-3a}$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b}$ and $\frac{3}{a+b} = \frac{-7}{2-b-3a}$

$\Rightarrow 2a + 2b = 3a - 3b$ and $6 - 3b - 9a = -7a - 7b$

$\Rightarrow a = 5b$... (1) and $-2a = -4b - 6$... (2)

Putting (1) in (2), we get

$-2\,(5b) = -4b - 6$

$\Rightarrow -10b + 4b = -6$

$\Rightarrow -6b = -6 \Rightarrow b = 1$

Putting value of b in (1), we get

$a = 5b = 5\ (1) = 5$

Therefore, $a = 5$ and $b = 1$

(ii) Comparing $(3x + y - 1 = 0)$ with $a_1x+b_1y+c_1 = 0$ and $(2k - 1)x + (k - 1)y - 2k - 1 = 0)$ with $a_2x+b_2y+c_2 = 0$,

We get $a_1 = 3, b_1 = 1$ and $c_1 = -1, a_2 = (2k-1), b_2 = (k-1)$ and $c_2 = -2k - 1$

Linear equations have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-2k-1}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1}$

$\Rightarrow 3\ (k - 1) = 2k - 1$

$\Rightarrow 3k - 3 = 2k - 1$

$\Rightarrow k = 2$

---

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

Ans. Substitution Method

$8x + 5y = 9$ … **(1)**

$3x + 2y = 4$ … **(2)**

From equation **(1)**,

$5y = 9 - 8x \Rightarrow y = \dfrac{9-8x}{5}$

Putting this in equation **(2)**, we get

$3x + 2\left(\dfrac{9-8x}{5}\right) = 4$

$\Rightarrow 3x + \dfrac{18-16x}{5} = 4$

$\Rightarrow 3x - \dfrac{16}{5}x = \dfrac{4}{1} - \dfrac{18}{5}$

$\Rightarrow 15x - 16x = 20 - 18$

$\Rightarrow x = -2$

Putting value of **x** in **(1)**, we get

$8(-2) + 5y = 9$

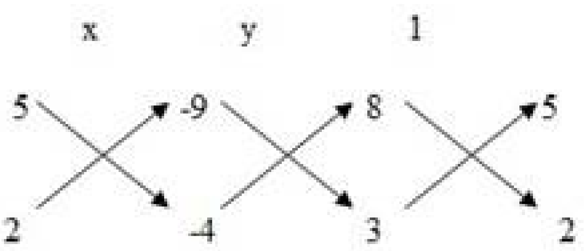
$\Rightarrow 5y = 9 + 16 = 25 \Rightarrow y = 5$

Therefore, $x = -2$ and $y = 5$

Cross multiplication method

$8x + 5y = 9$ … **(1)**

$3x + 2y = 4$ … **(2)**



$$\frac{x}{5(-4)-2(-9)}=\frac{y}{(-9)3-(-4)8}=\frac{1}{8\times2-5\times3}$$

$$\Rightarrow \frac{x}{-20+18}=\frac{y}{-27+32}=\frac{1}{16-15}$$

$$\Rightarrow \frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$

$\Rightarrow x = -2$ and $y = 5$

---

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the

rectangle.

Ans.(i) Let fixed monthly charge = Rs x and let charge of food for one day = Rs y

According to given conditions,

$x + 20y = 1000$ ... (1),

and $x + 26y = 1180$ ... (2)

Subtracting equation (1) from equation (2), we get

$6y = 180$

$\Rightarrow y = 30$

Putting value of y in (1), we get

$x + 20(30) = 1000$

$\Rightarrow x = 1000 - 600 = 400$

Therefore, fixed monthly charges = Rs 400 and, charges of food for one day = Rs 30

(ii) Let numerator = x and let denominator = y

According to given conditions,

$$\frac{x-1}{y} = \frac{1}{3} \quad ...(1) \quad \frac{x}{y+8} = \frac{1}{4} \quad ...(2)$$

$\Rightarrow 3x - 3 = y$ ... (1) $4x = y + 8$ ... (1)

$\Rightarrow 3x - y = 3$ ... (1) $4x - y = 8$ ... (2)

Subtracting equation (1) from (2), we get

$4x - y - (3x - y) = 8 - 3$

$\Rightarrow x = 5$

Putting value of x in (1), we get

$3\,(5) - y = 3$

$\Rightarrow 15 - y = 3$

$\Rightarrow y = 12$

Therefore, numerator = 5 and, denominator = 12

It means fraction = $\frac{x}{y} = \frac{5}{12}$

(iii) Let number of correct answers = x and let number of wrong answers = y

According to given conditions,

$3x - y = 40$ ... (1)

And, $4x - 2y = 50$ ... (2)

From equation (1), $y = 3x - 40$

Putting this in (2), we get

$4x - 2\,(3x - 40) = 50$

$\Rightarrow 4x - 6x + 80 = 50$

$\Rightarrow -2x = -30$

$\Rightarrow x = 15$

Putting value of x in (1), we get

$3\,(15) - y = 40$

$\Rightarrow 45 - y = 40$

$\Rightarrow y = 45 - 40 = 5$

Therefore, number of correct answers = x = 15and number of wrong answers = y = 5

Total questions = $x + y = 15 + 5 = 20$

(iv) Let speed of car which starts from part A = x km/hr

Let speed of car which starts from part B = y km/hr

According to given conditions,

$\frac{100}{x-y} = 5$ (Assuming x > y)

$\Rightarrow 5x - 5y = 100$

$\Rightarrow x - y = 20$ … (1)

And, $\frac{100}{x+y} = 1$

$\Rightarrow x + y = 100$ … (2)

Adding (1) and (2), we get

$2x = 120$

$\Rightarrow x = 60$ km/hr

Putting value of x in (1), we get

$60 - y = 20$

$\Rightarrow y = 60 - 20 = 40$ km/hr

Therefore, speed of car starting from point A = 60 km/hr

And, Speed of car starting from point B = 40 km/hr

(v) Let length of rectangle = x units and Let breadth of rectangle = y units

Area =xy square *units.* According to given conditions,

$xy - 9 = (x - 5)(y + 3)$

$\Rightarrow xy - 9 = xy + 3x - 5y - 15$

$\Rightarrow 3x - 5y = 6$ … (1)

And, $xy + 67 = (x + 3)(y + 2)$

$\Rightarrow xy + 67 = xy + 2x + 3y + 6$

$\Rightarrow 2x + 3y = 61$ … (2)

From equation (1),

$3x = 6 + 5y$

$\Rightarrow x = \dfrac{6+5y}{3}$

Putting this in (2), we get

$2\left(\dfrac{6+5y}{3}\right) + 3y = 61$

$\Rightarrow 12 + 10y + 9y = 183$

$\Rightarrow 19y = 171$

$\Rightarrow y = 9$ units

Putting value of y in (2), we get

$2x + 3(9) = 61$

$\Rightarrow 2x = 61 - 27 = 34$

$\Rightarrow x = 17$ units

Therefore, length = 17 units and, breadth = 9 units

CBSE Class–10 Mathematics

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Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.6

1. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}$ + 3y = 14

$\frac{3}{x}$ − 4y = 23

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$

(vi) $6x + 3y = 6xy$

$2x + 4y = 5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Ans. (i) $\frac{1}{2x}+\frac{1}{3y}=2$ ... (1)

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$ ... (2)

Let $\frac{1}{x}$ = p and $\frac{1}{y}$ = q

Putting this in equation (1) and (2), we get

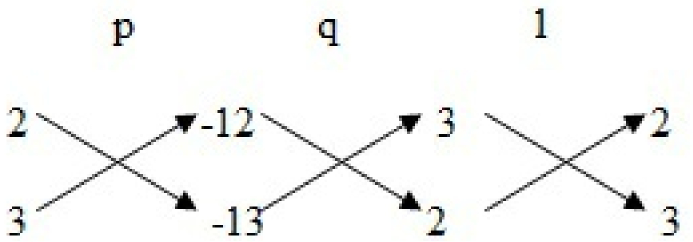
$\frac{p}{2}+\frac{q}{3}=2 \; and \; \frac{p}{3}+\frac{q}{2}=\frac{13}{6}$

Multiply both equation by 6, we get

⇒ $3p + 2q = 12$ and $2p + 3q = 13$

⇒ $3p + 2q – 12 = 0$ .... (3)

and $2p + 3q – 13 = 0$ .... (4)



$$\frac{p}{2(-13)-3(-12)} = \frac{q}{(-12)2-(-13)3} = \frac{1}{3\times 3-2\times 2}$$

$$\Rightarrow \frac{p}{-26+36} = \frac{q}{-24+39} = \frac{1}{9-4}$$

$$\Rightarrow \frac{p}{10} = \frac{q}{15} = \frac{1}{5}$$

$$\Rightarrow \frac{p}{10} = \frac{1}{5} \text{ and } \frac{q}{15} = \frac{1}{5}$$

$\Rightarrow p = 2$ and $q = 3$

But $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Putting value of p and q in this we get

$x = \frac{1}{2}$ and $y = \frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ … (1)

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$ … (2)

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

Putting this in (1) and (2), we get

$2p + 3q = 2$ … (3)

$4p - 9q = -1$ … (4)

Multiplying (3) by 2 and subtracting it from (4), we get

=> $4p - 9q - 2(2p + 3q) = -1 - 2(2)$

⇒ $4p - 9q - 4p - 6q = -1 - 4$

⇒ $-15q = -5$

⇒ $q = \frac{-5}{-15} = \frac{1}{3}$

Putting value of q in (3), we get

=> $2p + 1 = 2$

⇒ $2p = 1$

⇒ $p = ½$

Putting values of p and q in ($\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$), we get

$\frac{1}{\sqrt{x}} = \frac{1}{2}$ and $\frac{1}{\sqrt{y}} = \frac{1}{3}$

⇒ $\frac{1}{x} = \frac{1}{4} \text{ and } \frac{1}{y} = \frac{1}{9}$

⇒ $x = 4$ and $y = 9$

(iii) $\frac{4}{x} + 3y = 14$ … (1)

$\frac{3}{x}$ − 4y = 23 … (2)

Let $\frac{1}{x}$ = p

we get

4p + 3y = 14 … (3)

3p − 4y = 23 … (4)

Multiplying (3) by 3 and (4) by 4, we get

3 (4p + 3y - 14 =0) and, 4 (3p − 4y – 23 = 0)

⇒ 12p + 9y – 42 = 0 … (6) 12p − 16y – 92 = 0 … (7)

Subtracting (7) from (6), we get

9y − (−16y) – 42 − (−92) = 0

⇒ 25y + 50 = 0

⇒ y = $\frac{-50}{25}$ = −2

Putting value of y in (4), we get

4p + 3 (−2) = 14

⇒ 4p – 6 = 14

⇒ 4p = 20

⇒ p = 5

Putting value of p in (3), we get

$\frac{1}{x}$ = 5

$\Rightarrow x = \frac{1}{5}$

Therefore, $x = \frac{1}{5}$ and $y = -2$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$ ... (1)

$\frac{6}{x-1} - \frac{3}{y-2} = 1$ ... (2)

Let $\frac{1}{x-1} = p \;\; and \;\; \frac{1}{y-2} = q$

Putting this in (1) and (2), we get

$5p + q = 2$

$\Rightarrow 5p + q - 2 = 0$ ... (3)

And, $6p - 3q = 1$

$\Rightarrow 6p - 3q - 1 = 0$ ... (4)

Multiplying (3) by 3 and adding it to (4), we get

$3(5p + q - 2) + 6p - 3q - 1 = 0$

$\Rightarrow 15p + 3q - 6 + 6p - 3q - 1 = 0$

$\Rightarrow 21p - 7 = 0$

$\Rightarrow p = \frac{1}{3}$

Putting this in (3), we get

$5\left(\frac{1}{3}\right) + q - 2 = 0$

$\Rightarrow 5 + 3q = 6$

$\Rightarrow 3q = 6 - 5 = 1$

$\Rightarrow q = \frac{1}{3}$

Putting values of p and q in ($\frac{1}{x-1} = p \;\; and \;\; \frac{1}{y-2} = q$), we get

$$\frac{1}{x-1} = \frac{1}{3} and \frac{1}{y-2} = \frac{1}{3}$$

$\Rightarrow 3 = x - 1$ and $3 = y - 2$

$\Rightarrow x = 4$ and $y = 5$

(v) $7x - 2y = 5xy$ ... (1)

$8x + 7y = 15xy$ ... (2)

Dividing both the equations by xy, we get

$$\frac{7}{y} - \frac{2}{x} = 5 \qquad ...(3)$$

$$\frac{8}{y} + \frac{7}{x} = 15 \qquad ...(4)$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Putting these in (3) and (4), we get

$7q - 2p = 5$ ... (5)

$8q + 7p = 15$ ... (6)

From equation (5),

$2p = 7q - 5$

$\Rightarrow p = \dfrac{7q-5}{2}$

Putting value of p in (6), we get

$8q + 7\left(\dfrac{7q-5}{2}\right) = 15$

$\Rightarrow 16q + 49q - 35 = 30$

$\Rightarrow 65q = 30 + 35 = 65$

$\Rightarrow q = 1$

Putting value of q in (5), we get

$7(1) - 2p = 5$

$\Rightarrow 2p = 2$

$\Rightarrow p = 1$

Putting value of p and q in ($\dfrac{1}{x} = p$ and $\dfrac{1}{y} = q$), we get $x = 1$ and $y = 1$

(vi) $6x + 3y - 6xy = 0$ ... (1)

$2x + 4y - 5xy = 0$ ... (2)

Dividing both the equations by xy, we get

$$\frac{6}{y} + \frac{3}{x} - 6 = 0 \quad ...(3)$$

$$\frac{2}{y} + \frac{4}{x} - 5 = 0 \quad ...(4)$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Putting these in (3) and (4), we get

$6q + 3p – 6 = 0$ … (5)

$2q + 4p – 5 = 0$ … (6)

From (5),

$3p = 6 - 6q$

$\Rightarrow p = 2 - 2q$

Putting this in (6), we get

$2q + 4 (2 - 2q) – 5 = 0$

$\Rightarrow 2q + 8 - 8q – 5 = 0$

$\Rightarrow -6q = -3 \Rightarrow q = ½$

Putting value of q in (p = 2 – 2q), we get

$p = 2 – 2 (½) = 2 – 1 = 1$

Putting values of p and q in ($\frac{1}{x} = p$ and $\frac{1}{y} = q$), we get $x = 1$ and $y = 2$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$ … (1)

$\frac{15}{x+y} - \frac{5}{x-y} = -2$ …(2)

Let $\frac{1}{x+y} = p \text{ and } \frac{1}{x-y} = q$

Putting this in (1) and (2), we get

$10p + 2q = 4$ … (3)

$15p - 5q = -2$ … (4)

From equation (3),

$2q = 4 - 10p$

$\Rightarrow q = 2 - 5p$ … (5)

Putting this in (4), we get

$15p - 5(2 - 5p) = -2$

$\Rightarrow 15p - 10 + 25p = -2$

$\Rightarrow 40p = 8 \Rightarrow p = \frac{1}{5}$

Putting value of p in (5), we get

$q = 2 - 5\left(\frac{1}{5}\right) = 2 - 1 = 1$

Putting values of p and q in ($\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$), we get

$\frac{1}{x+y} = \frac{1}{5}$ and $\frac{1}{x-y} = \frac{1}{1}$

$\Rightarrow x + y = 5$ … (6) and $x - y = 1$ … (7)

Adding (6) and (7), we get

$2x = 6 \Rightarrow x = 3$

Putting $x = 3$ in (7), we get

$3 - y = 1$

$\Rightarrow y = 3 - 1 = 2$

Therefore, $x = 3$ and $y = 2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ … (1)

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$ … (2)

Let $\frac{1}{3x+y}=p \text{ and } \frac{1}{3x-y}=q$

Putting this in (1) and (2), we get

$p + q = \frac{3}{4}$ and $\frac{p}{2}-\frac{q}{2}=-\frac{1}{8}$

⇒ $4p + 4q = 3$ … (3) and $4p - 4q = -1$ … (4)

Adding (3) and (4), we get

$8p = 2 \Rightarrow p = \frac{1}{4}$

Putting value of p in (3), we get

$4\left(\frac{1}{4}\right) + 4q = 3$

⇒ $1 + 4q = 3$

⇒ $4q = 3 - 1 = 2$

⇒ $q = \frac{1}{2}$

Putting value of p and q in $\frac{1}{3x+y}=p \text{ and } \frac{1}{3x-y}=q$, we get

$\frac{1}{3x+y}=\frac{1}{4} \text{ and } \frac{1}{3x-y}=\frac{1}{2}$

⇒ $3x + y = 4$ … (5) and $3x - y = 2$ … (6)

Adding (5) and (6), we get

$6x = 6 \Rightarrow x = 1$

Putting $x = 1$ in (5) , we get

$3(1) + y = 4$

$\Rightarrow y = 4 - 3 = 1$

Therefore, $x = 1$ and $y = 1$

---

2. Formulate the following problems as a part of equations, and hence find their solutions.

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days.Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Ans. (i) Let speed of rowing in still water = x km/h

Let speed of current = y km/h

So, speed of rowing downstream = $(x + y)$ km/h

And, speed of rowing upstream = $(x - y)$ km/h

According to given conditions,

$$\frac{20}{x+y} = 2 \text{ and } \frac{4}{x-y} = 2$$

⇒ $2x + 2y = 20$ and $2x - 2y = 4$

⇒ $x + y = 10$ … (1) and $x - y = 2$ … (2)

Adding (1) and (2), we get

$2x = 12$

⇒ $x = 6$

Putting $x = 6$ in (1), we get

$6 + y = 10$

⇒ $y = 10 - 6 = 4$

Therefore, speed of rowing in still water = 6 km/h

Speed of current = 4 km/h

(ii) Let time taken by 1 woman alone to finish the work = x days

Let time taken by 1 man alone to finish the work = y days

So, 1 woman's 1-day work = $\left(\frac{1}{x}\right)$*th* part of the work

And, 1 man's 1-day work = $\left(\frac{1}{y}\right)$*th* part of the work

So, 2 women's 1-day work = $\left(\frac{2}{x}\right)$*th* part of the work

And, 5 men's 1-day work = $\left(\frac{5}{y}\right)$*th* part of the work

Therefore, 2 women and 5 men's 1-day work = ($\frac{2}{x}+\frac{5}{y}$)*th* part of the work… (1)

It is given that 2 women and 5 men complete work in = 4 days

It means that in 1 day, they will be completing $\frac{1}{4}$ *th* part of the work … (2)

Clearly, we can see that (1) = (2)

⇒ $\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$ … (3)

Similarly, $\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$ … (4)

Let $\frac{1}{x}=p \, and \, \frac{1}{y}=q$

Putting this in (3) and (4), we get

$2p + 5q = \frac{1}{4}$ and $3p + 6q = \frac{1}{3}$

⇒ $8p + 20q = 1$ … (5) and $9p + 18q = 1$ … (6)

Multiplying (5) by 9 and (6) by 8, we get

$72p + 180q = 9$ … (7)

$72p + 144q = 8$ … (8)

Subtracting (8) from (7), we get

$36q = 1$

⇒ $q = \frac{1}{36}$

Putting this in (6), we get

$9p + 18\left(\frac{1}{36}\right) = 1$

$\Rightarrow 9p = ½$

$\Rightarrow p = \frac{1}{18}$

Putting values of p and q in $\frac{1}{x} = p \text{ and } \frac{1}{y} = q$, we get $x = 18$ and $y = 36$

Therefore, 1 woman completes work in = 18 days

And, 1 man completes work in = 36 days

(iii) Let speed of train = x km/h and let speed of bus = y km/h

According to given conditions,

$$\frac{60}{x} + \frac{240}{y} = 4 \text{ and } \frac{100}{x} + \frac{200}{y} = 4 + \frac{10}{60}$$

Let $\frac{1}{x} = p \text{ and } \frac{1}{y} = q$

Putting this in the above equations, we get

$60p + 240q = 4$ … (1)

And $100p + 200q = \frac{25}{6}$ … (2)

Multiplying (1) by 5 and (2) by 3, we get

$300p + 1200q = 20$ … (3)

$300p + 600q = \frac{25}{2}$ … (4)

Subtracting (4) from (3), we get

$600q = 20 - \frac{25}{2} = 7.5$

$\Rightarrow q = \frac{7.5}{600}$

Putting value of q in (2), we get

$100p + 200\left(\frac{7.5}{600}\right) = \frac{25}{6}$

$\Rightarrow 100p + 2.5 = \frac{25}{6}$

$\Rightarrow 100p = \frac{25}{6} - 2.5$

$\Rightarrow p = \frac{10}{600}$

But $\frac{1}{x} = p \; and \; \frac{1}{y} = q$

Therefore, $x = \frac{600}{10} = 60$ km/h and $y = \frac{600}{7.5} = 80$ km/h

Therefore, speed of train = 60 km/h

And, speed of bus = 80 km/h

CBSE Class–10 Mathematics

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Chapter - 3

Pair of Linear Equations in Two Variables - Exercise 3.7

1. The age of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Ans. Let the age of Ani and Biju be x years and y years respectively.

Age of Dharam = 2x years and Age of Cathy = $\frac{y}{2}$ years

According to question, x – y = 3… (1)

And $2x - \frac{y}{2} = 30$

$\Rightarrow$ 4x – y = 60… (2)

Subtracting (1) from (2), we obtain:

3x = 60 – 3 = 57

$\Rightarrow$ x = Age of Ani = 19 years

Age of Biju = 19 – 3 = 16 years

Again, According to question, y – x = 3… (3)

And $2x - \frac{y}{2} = 30$

$\Rightarrow$ 4x – y = 60… (4)

Adding (3) and (4), we obtain:

3x = 63

$\Rightarrow$ x = 21

Age of Ani = 21 years

Age of Biju = 21 + 3 = 24 years

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]

Ans. Let the money with the first person and second person be Rs x and Rs y respectively. According to the question,

$x + 100 = 2(y - 100)$

$\Rightarrow x + 100 = 2y - 200$

$\Rightarrow x - 2y = -300$... (1)

Again, $6(x - 10) = (y + 10)$

$\Rightarrow 6x - 60 = y + 10$

$\Rightarrow 6x - y = 70$... (2)

Multiplying equation (2) by 2, we obtain:

$12x - 2y = 140$... (3)

Subtracting equation (1) from equation (3), we obtain:

$11x = 140 + 300$

$\Rightarrow 11x = 440$

$\Rightarrow x = 40$

Putting the value of x in equation (1), we obtain:

$40 - 2y = -300$

$\Rightarrow 40 + 300 = 2y$

$\Rightarrow$ 2y = 340

$\Rightarrow$ y = 170

Thus, the two friends had Rs 40 and Rs 170 with them.

---

3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Ans. Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Since Speed = $\dfrac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$

$\Rightarrow x = \dfrac{d}{t}$

$\Rightarrow d = xt$ ... (1)

According to the question

$x + 10 = \dfrac{d}{t-2}$

$\Rightarrow (x+10)(t-2) = d$

$\Rightarrow xt + 10t - 2x - 20 = d$ [Since, $xt = d$]

$\Rightarrow -2x + 10t = 20$ ......(2)[Using eq. (1)]

Again, $x - 10 = \dfrac{d}{t+3}$

$\Rightarrow (x-10)(t+3) = d$

$\Rightarrow xt - 10t + 3x - 30 = d$

$\Rightarrow 3x-10t=30$ ......(3)[Using eq. (1)] [Since, $xt = d$]

Adding equations (2) and (3), we obtain:
x = 50
Substituting the value of x in equation (2), we obtain:
$(-2)\times(50)+10t=20$

$\Rightarrow$ –100 + 10t = 20

$\Rightarrow$ 10t = 120

$\Rightarrow$ t = 12

From equation (1), we obtain:

d = xt = 50×12 = 600

Thus, the distance covered by the train is 600 km.

---

4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans. Let the number of rows be x and number of students in a row be y.
Total number of students in the class = Number of rows x Number of students in a row = xy

According to the question,

Total number of students = (x – 1) (y + 3)

$\Rightarrow$ xy = (x – 1) (y + 3)

$\Rightarrow$ xy = xy – y + 3x – 3

$\Rightarrow$ 3x – y – 3 = 0

$\Rightarrow$ 3x – y = 3... (1)

Total number of students = (x + 2) (y – 3)

$\Rightarrow xy = xy + 2y - 3x - 6$

$\Rightarrow 3x - 2y = -6$... (2)

Subtracting equation (2) from (1), we obtain:

$y = 9$

Substituting the value of y in equation (1), we obtain:

$3x - 9 = 3$

$\Rightarrow 3x = 9 + 3 = 12$

$\Rightarrow x = 4$

Number of rows = x = 4

Number of students in a row = y = 9

Hence, Total number of students in a class = xy = $4 \times 9 = 36$

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5. In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find three angles.

Ans. $\angle C = 3\angle B = 2(\angle A + \angle B)$

Taking $3\angle B = 2(\angle A + \angle B)$

$\Rightarrow \angle B = 2\angle A$

$\Rightarrow 2\angle A - \angle B = 0$ ....(1)

We know that the sum of the measures of all angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B + 3\angle B = 180°$

$\Rightarrow \angle A + 4\angle B = 180°$ ....(2)

Multiplying equation (1) by 4, we obtain:

$8\angle A - 4\angle B = 0$ ....(3)

Adding equations (2) and (3), we get

$9\angle A = 180°$

$\Rightarrow \angle A = 20°$

From eq. (2), we get,

$20° + 4\angle B = 180°$

$\Rightarrow \angle B = 40°$

And $\angle C = 3\times 40° = 120°$

Hence the measures of $\angle A$, $\angle B$ and $\angle C$ are $20°, 40°$ *and* $120°$ respectively.

---

6. Draw the graphs of the equations $5x-y=5$ and $3x-y=3$. Determine the co-ordinate of the vertices of the triangle formed by these lines and the $y-$axis.

Ans. $5x-y=5$

$\Rightarrow y=5x-5$

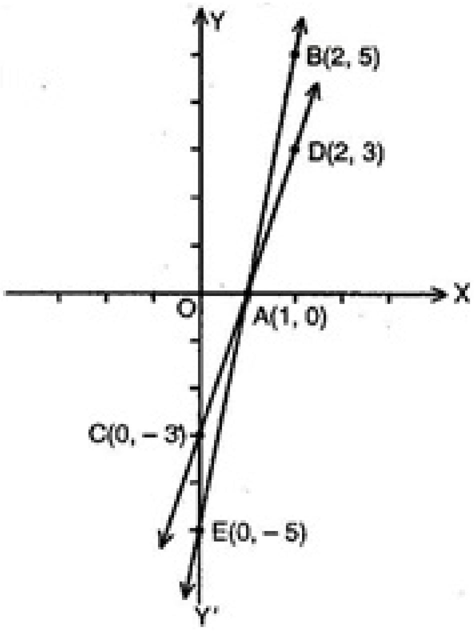
Three solutions of this equation can be written in a table as follows:

x	0	1	2
y	–5	0	5

$3x-y=3$

$\Rightarrow y=3x-3$

x	0	1	2
y	–3	0	3



It can be observed that the required triangle is ΔABC.

The coordinates of its vertices are A (1, 0), B (0, –3), C (0, –5).

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7. Solve the following pair of linear equations:

(i) $px + py = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Ans. (i) $px+qy=p-q$ ... (1)

$qx-py=p+q$ ... (2)

Multiplying equation (1) by p and equation (2) by q, we obtain:

$p^2x+pqy=p^2-pq$ ... (3)

$q^2x-pqy=pq+q^2$ ... (4)

Adding equations (3) and (4), we obtain:

$p^2x+q^2x=p^2+q^2$

$\Rightarrow \left(p^2+q^2\right)x=p^2+q^2$

$\Rightarrow x=\dfrac{p^2+q^2}{p^2+q^2}$ = 1

Substituting the value of x in equation (1), we obtain:

$p(1)+qy=p-q$

$\Rightarrow qy=-q \Rightarrow y=-1$

Hence the required solution is x = 1 and y = –1.

(ii) $ax+by=c$ ... (1)

$bx+ay=1+c$ ... (2)

Multiplying equation (1) by a and equation (2) by b, we obtain:

$a^2x+aby=ac$ ... (3)

$b^2x+aby=b+bc$ ... (4)

Subtracting equation (4) from equation (3),

$\left(a^2-b^2\right)x=ac-bc-b$

$\Rightarrow x=\dfrac{c(a-b)-b}{a^2-b^2}$

Substituting the value of x in equation (1), we obtain:

$a\left\{\dfrac{c(a-b)-b}{a^2-b^2}\right\}+by=c$

$$\Rightarrow \frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$\Rightarrow by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$\Rightarrow by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$\Rightarrow by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$\Rightarrow y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

$\Rightarrow bx-ay=0$ ....(1)

$ax+by=a^2+b^2$ ....(2)

Multiplying equation (1) and (2) by b and a respectively, we obtain:

$b^2x-aby=0$ ....(3)

$a^2x+aby=a^3+ab^2$ ....(4)

Adding equations (3) and (4), we obtain:

$b^2x+a^2x=a^3+ab^2$

$\Rightarrow x(b^2+a^2)=a(a^2+b^2)$

$\Rightarrow x=a$

Substituting the value of x in equation (1), we obtain:

$b(a)-ay=0$

$\Rightarrow ab-ay=0$

$\Rightarrow\ y=b$

(iv) $(a-b)x+(a+b)y=a^2-2ab-b^2$ ... (1)

$(a+b)(x+y)=a^2+b^2$

$\Rightarrow\ (a+b)x+(a+b)y=a^2+b^2$ ....(2)

Subtracting equation (2) from (1), we obtain:

$(a-b)x-(a+b)x=(a^2-2ab-b^2)-(a^2+b^2)$

$\Rightarrow\ (a-b-a-b)x=-2ab-2b^2$

$\Rightarrow\ -2bx=-2b(a+b)$

$\Rightarrow\ x=a+b$

Substituting the value of x in equation (1), we obtain:

$(a-b)(a+b)+(a+b)y=a^2-2ab-b^2$

$\Rightarrow\ a^2-b^2+(a+b)y=a^2-2ab-b^2$

$\Rightarrow\ (a+b)y=-2ab$

$\Rightarrow\ y=\dfrac{-2ab}{a+b}$

(v)152x – 378y = –74... (1)

–378x + 152y = –604 ... (2)

Adding the equations (1) and (2), we obtain:

–226x – 226y = –678

$\Rightarrow$x + y = 3 ....(3)

Subtracting the equation (2) from equation (1), we obtain:

530x – 530y = 530

$\Rightarrow$x – y = 1 ....(4)

Adding equations (3) and (4), we obtain:

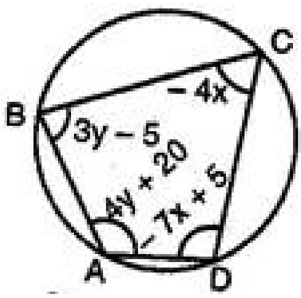
2x = 4

$\Rightarrow$x = 2

Substituting the value of x in equation (3), we obtain:

y = 1

8. ABCD is a cyclic quadrilateral (see figure). Find the angles of the cyclic quadrilateral.



Ans. We know that the sum of the measures of opposite angles in a cyclic quadrilateral is $180°$.

$\therefore \angle A + \angle C = 180°$

$\Rightarrow 4y + 20 - 4x = 180°$

$\Rightarrow -4x + 4y = 160°$

$\Rightarrow \ x - y = -40°$ ....(1)

Also $\angle B + \angle D = 180°$

$\Rightarrow 3y - 5 - 7x + 5 = 180°$

$\Rightarrow -7x + 3y = 180°$.........(2)

Multiplying equation (1) by 3, we obtain:

$3x - 3y = -120°$ ....(3)

Adding equations (2) and (3), we obtain:

$-4x = 60° \ \Rightarrow x = -15°$

Substituting the value of x in equation (1), we obtain:

$-15 - y = -40°$

$\Rightarrow \ y = -15 + 40 = 25$

$\therefore \angle A = 4y + 20 = 4 \times 25 + 20 = 120°$

$\angle B = 3y - 5 = 3 \times 25 - 5 = 70°$

$\angle C = -4x = -4 \times (-15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$

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Chapter - 4

Quadratic Equations - Exercise 4.1

1. Check whether the following are Quadratic Equations.

(i) $(x+1)^2 = 2(x-3)$

(ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$

(iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$

(vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Ans. (i) $(x+1)^2 = 2(x-3)$

$\left\{(a+b)^2 = a^2 + 2ab + b^2\right\}$

$\Rightarrow x^2 + 1 + 2x = 2x - 6$

$\Rightarrow x^2 + 7 = 0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

(ii) $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x - 2x + 6 = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3 = 0$

$\Rightarrow x^2 + x - 2x - 2 - x^2 - 3x + x + 3 = 0$

$\Rightarrow x - 2x - 2 - 3x + x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 + x - 6x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

Here, degree of equation is 2.

Therefore, it is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 + x - 5x + 5 = 0$

$\Rightarrow x^2 - 11x + 8 = 0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

(vi) $x^2 + 3x + 1 = (x-2)^2$

$\{(a-b)^2 = a^2 - 2ab + b^2\}$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow 7x - 3 = 0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

(vii) $(x+2)^3 = 2x(x^2 - 1)$

$\{(a+b)^3 = a^3 + b^3 + 3ab(a+b)\}$

$\Rightarrow x^3 + 2^3 + 3(x)(2)(x+2) = 2x(x^2 - 1)$

$\Rightarrow x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$\Rightarrow 2x^3 - 2x - x^3 - 8 - 6x^2 - 12x = 0$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

Here, degree of Equation is 3.

Therefore, it is not a quadratic Equation.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

$\{(a-b)^3 = a^3 - b^3 - 3ab(a-b)\}$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3(x)(2)(x-2)$

$\Rightarrow -4x^2 - x + 1 = -8 - 6x^2 + 12x$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

---

2. Represent the following situations in the form of Quadratic Equations:

(i) The area of rectangular plot is $528\,m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive numbers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) after 3 years will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at uniform speed. If, the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find speed of the train.

Ans. (i) We are given that area of a rectangular plot is $528\,m^2$.

Let breadth of rectangular plot be x metres

Length is one more than twice its breadth.

Therefore, length of rectangular plot is $(2x + 1)$ metres

Area of rectangle = length × breadth

$\Rightarrow 528 = x(2x + 1)$

$\Rightarrow 528 = 2x^2 + x$

$\Rightarrow 2x^2 + x - 528 = 0$

This is required Quadratic Equation.

(ii) Let two consecutive numbers be x and (x + 1).

It is given that $x(x + 1) = 306$

$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

This is the required Quadratic Equation.

(iii) Let present age of Rohan = x years

Let present age of Rohan's mother = (x + 26) years

Age of Rohan after 3 years = (x + 3) years

Age of Rohan's mother after 3 years = x + 26 + 3 = (x + 29) years

According to given condition:

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

This is the required Quadratic Equation.

(iv) Let speed of train be x km/h

Time taken by train to cover 480 km = $\frac{480}{x}$ hours

If, speed had been 8km/h less then time taken would be $\frac{480}{x-8}$ hours

According to given condition, if speed had been 8km/h less then time taken is 3 hours less.

Therefore, $\frac{480}{x-8} - \frac{480}{x} = 3$

⇒ $480(x - x + 8) = 3x(x - 8)$

⇒ $3840 = 3x^2 - 24x$

⇒ $3x^2 - 24x - 3840 = 0$

Dividing equation by 3, we get

⇒ $x^2 - 8x - 1280 = 0$

This is the required Quadratic Equation.

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Chapter - 4

Quadratic Equations - Exercise 4.2

1. Find the roots of the following Quadratic Equations by factorization.

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Ans. (i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x = 5, -2$

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (2x - 3)(x + 2) = 0$

$\Rightarrow x = \frac{3}{2}, -2$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2+2x+5x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x\left(x+\sqrt{2}\right)+5\left(x+\sqrt{2}\right)=0$

$\Rightarrow (\sqrt{2}x+5)(x+\sqrt{2})=0$

$\Rightarrow x=\frac{-5}{\sqrt{2}},-\sqrt{2}$

$\Rightarrow x=\frac{-5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}},-\sqrt{2}$

$\Rightarrow x=\frac{-5\sqrt{2}}{2},-\sqrt{2}$

(iv) $2x^2-x+\frac{1}{8}=0$

$\Rightarrow \frac{16x^2-8x+1}{8}=0$

$\Rightarrow 16x^2-8x+1=0$

$\Rightarrow 16x^2-4x-4x+1=0$

$\Rightarrow 4x(4x-1)-1(4x-1)=0$

$\Rightarrow (4x-1)(4x-1)=0$

$\Rightarrow x=\frac{1}{4},\frac{1}{4}$

(v) $100x^2-20x+1=0$

$\Rightarrow 100x^2-10x-10x+1=0$

$\Rightarrow 10x(10x-1)-1(10x-1)=0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow x = \frac{1}{10}, \frac{1}{10}$

2. Solve the following problems given:

(i) $x^2 - 45x + 324 = 0$

(ii) $x^2 - 55x + 750 = 0$

Ans. (i) $x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 9)(x - 36) = 0$

$\Rightarrow x = 9, 36$

(ii) $x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 30)(x - 25) = 0$

$\Rightarrow x = 30, 25$

3.Find two numbers whose sum is 27 and product is 182.

Ans. Let first number be x and let second number be $(27 - x)$

According to given condition, the product of two numbers is 182.

Therefore,

$x(27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 14x - 13x + 182 = 0$

$\Rightarrow x(x-14) - 13(x-14) = 0$

$\Rightarrow (x-14)(x-13) = 0$

$\Rightarrow x = 14, 13$

Therefore, the first number is equal to 14 or 13

And, second number is = 27 – x = 27 – 14 = 13 or Second number = 27 – 13 = 14

Therefore, two numbers are 13 and 14.

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4. Find two consecutive positive integers, sum of whose squares is 365.

Ans. Let first number be x and let second number be $(x + 1)$

According to given condition,

$x^2 + (x+1)^2 = 365$

$\{(a+b)^2 = a^2 + b^2 + 2ab\}$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

Dividing equation by 2

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x+14) - 13(x+14) = 0$

$\Rightarrow (x + 14)(x - 13) = 0$

$\Rightarrow x = 13, -14$

Therefore, first number = 13 {We discard -14 because it is negative number)

Second number = $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers are 13 and 14 whose sum of squares is equal to 365.

5.The altitude of right triangle is 7 cm less than its base. If, hypotenuse is 13 cm. Find the other two sides.

Ans. Let base of triangle be x cm and let altitude of triangle be $(x - 7)$ cm

It is given that hypotenuse of triangle is 13 cm

According to Pythagoras Theorem,

$(13)^2 = x^2 + (x - 7)^2$ [Since, $(a + b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow 169 = x^2 + x^2 + 49 - 14x$

$\Rightarrow 169 = 2x^2 - 14x + 49$

$\Rightarrow 2x^2 - 14x - 120 = 0$

Dividing equation by 2

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow (x - 12)(x + 5) = 0$

$\Rightarrow x = -5, 12$

We discard $x = -5$ because length of side of triangle cannot be negative.

Therefore, base of triangle = 12 cm

Altitude of triangle = $(x - 7) = 12 - 7 = 5$ cm

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6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Ans. Let cost of production of each article be Rs x

We are given total cost of production on that particular day = Rs 90

Therefore, total number of articles produced that day = 90/x

According to the given conditions,

$$x = 2\left(\frac{90}{x}\right) + 3$$

$$\Rightarrow x = \frac{180}{x} + 3$$

$$\Rightarrow x = \frac{180 + 3x}{x}$$

$$\Rightarrow x^2 = 180 + 3x$$

$$\Rightarrow x^2 - 3x - 180 = 0$$

$$\Rightarrow x^2 - 15x + 12x - 180 = 0$$

$\Rightarrow x\ (x - 15) + 12\ (x - 15) = 0$

$\Rightarrow (x - 15)\ (x + 12) = 0 \Rightarrow x = 15, -12$

Cost cannot be in negative, therefore, we discard $x = -12$

Therefore, $x = Rs\ 15$ which is the cost of production of each article.

Number of articles produced on that particular day = $\frac{90}{15} = 6$

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Chapter - 4

Quadratic Equations -Exercise 4.3

1. Find the roots of the following quadratic equations if they exist by the method of completing square.

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Ans. (i) $2x^2 - 7x + 3 = 0$

First we divide equation by 2 to make coefficient of x^2 equal to 1,

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

We divide middle term of the equation by 2x, we get $\frac{7}{2}x \times \frac{1}{2x} = \frac{7}{4}$

We add and subtract square of $\frac{7}{4}$ from the equation $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$,

=> $x^2 - \frac{7}{2}x + \frac{3}{2} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = 0$

$\Rightarrow x^2 + \left(\frac{7}{4}\right)^2 - \frac{7}{2}x + \frac{3}{2} - \left(\frac{7}{4}\right)^2 = 0$

$\left\{(a-b)^2 = a^2 + b^2 - 2ab\right\}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2+\frac{24-49}{16}=0$

$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{49-24}{16}$

=> $\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$

Taking Square root on both sides,

$\Rightarrow x-\frac{7}{4}=\pm\frac{5}{4}$

$\Rightarrow x=\frac{5}{4}+\frac{7}{4}=\frac{12}{4}=3 \text{ and } x=-\frac{5}{4}+\frac{7}{4}=\frac{2}{4}=\frac{1}{2}$

Therefore, $x=\frac{1}{2},3$

(ii) $2x^2+x-4=0$

Dividing equation by 2,

$x^2+\frac{x}{2}-2=0$

Following procedure of completing square,

=> $x^2+\frac{x}{2}-2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-2-\frac{1}{16}=0$

$\left\{(a+b)^2=a^2+b^2+2ab\right\}$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2 - \frac{33}{16} = 0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2 = \frac{33}{16}$$

Taking square root on both sides,

$$\Rightarrow x+\frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{\sqrt{33}}{4} - \frac{1}{4} = \frac{\sqrt{33}-1}{4} \text{ and } x = -\frac{\sqrt{33}}{4} - \frac{1}{4} = \frac{-\sqrt{33}-1}{4}$$

Therefore, $x = \frac{\sqrt{33}-1}{4}, \frac{-\sqrt{33}-1}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Dividing equation by 4,

$$x^2 + \sqrt{3}x + \frac{3}{4} = 0$$

Following the procedure of completing square,

$$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 0$$

$$\Rightarrow x^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \sqrt{3}x + \frac{3}{4} - \frac{3}{4} = 0$$

$$\left\{(a+b)^2 = a^2 + b^2 + 2ab\right\}$$

$$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2=0$$

$$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0$$

$$\Rightarrow x+\frac{\sqrt{3}}{2}=0, x+\frac{\sqrt{3}}{2}=0$$

$$\Rightarrow x=-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

(iv) $2x^2+x+4=0$

Dividing equation by 2,

$$x^2+\frac{x}{2}+2=0$$

Following the procedure of completing square,

$$\Rightarrow x^2+\frac{x}{2}+2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$$

$$\Rightarrow x^2+\left(\frac{1}{4}\right)^2+\frac{x}{2}+2-\left(\frac{1}{4}\right)^2=0$$

$$\left\{(a+b)^2=a^2+b^2+2ab\right\}$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2+2-\frac{1}{16}=0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}-2=\frac{1-32}{16}=\frac{-31}{16}$$

Right hand side does not exist because square root of negative number does not exist.

Therefore, there is no solution for quadratic equation $2x^2+x+4=0$

2. Find the roots of the following Quadratic Equations by applying quadratic formula.

(i) $2x^2-7x+3=0$

(ii) $2x^2+x-4=0$

(iii) $4x^2+4\sqrt{3}x+3=0$

(iv) $2x^2+x+4=0$

Ans. (i) $2x^2-7x+3=0$

Comparing quadratic equation $2x^2-7x+3=0$ with general form $ax^2+bx+c=0$, we get a = 2, b = -7 and c = 3

Putting these values in quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{7\pm\sqrt{(-7)^2-4(2)(3)}}{2\times2}$$

$$\Rightarrow x=\frac{7\pm\sqrt{49-24}}{4}$$

$$\Rightarrow x=\frac{7\pm5}{4}$$

$$\Rightarrow x=\frac{7+5}{4},\frac{7-5}{4}$$

$\Rightarrow x = 3, \frac{1}{2}$

(ii) $2x^2+x-4=0$

Comparing quadratic equation $2x^2 + x - 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = -4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\Rightarrow x = \frac{-1 + \sqrt{33}}{4}, \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2 \times 4}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2}$$

A quadratic equation has two roots. Here, both the roots are equal.

Therefore, $x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

Comparing quadratic equation $2x^2 + x + 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = 4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(4)}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

But, square root of negative number is not defined.

Therefore, Quadratic Equation $2x^2 + x + 4 = 0$ has no solution.

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3. Find the roots of the following equations:

(i) $\frac{x-1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Ans. (i) $x - \frac{1}{x} = 3 \ where \ x \neq 0$

$$\Rightarrow \frac{x^2 - 1}{x} = 3$$

$$\Rightarrow x^2 - 1 = 3x$$

$$\Rightarrow x^2 - 3x - 1 = 0$$

Comparing equation $x^2 - 3x - 1 = 0$ with general form $ax^2 + bx + c = 0$, We get $a = 1$, $b = -3$ and $c = -1$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

$$\Rightarrow x = \frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}$$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ *where* $x \neq -4, 7$

$$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow -30 = x^2 - 7x + 4x - 28$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

Comparing equation $x^2 - 3x + 2 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -3$ and $c = 2$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(2)}}{2 \times 1}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{1}}{2}$$

$$\Rightarrow x = \frac{3+\sqrt{1}}{2}, \frac{3-\sqrt{1}}{2}$$

$\Rightarrow x = 2, 1$

4. The sum of reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Ans. Let present age of Rehman = x years

Age of Rehman 3 years ago = $(x - 3)$ years.

Age of Rehman after 5 years = $(x + 5)$ years

According to the given condition:

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$\Rightarrow 3\,(2x + 2) = (x - 3)\,(x + 5)$

$\Rightarrow 6x + 6 = x^2 - 3x + 5x - 15$

$\Rightarrow x^2 - 4x - 15 - 6 = 0$

$\Rightarrow x^2 - 4x - 21 = 0$

Comparing quadratic equation $x^2 - 4x - 21 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -4$ and $c = -21$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-21)}}{2 \times 1}$$

$\Rightarrow x = \frac{4 \pm \sqrt{16+84}}{2}$

$\Rightarrow x = \frac{4 \pm \sqrt{100}}{2} = \frac{4 \pm 10}{2}$

$\Rightarrow x = \frac{4+10}{2}, \frac{4-10}{2}$

$\Rightarrow x = 7, -3$

We discard $x=-3$. Since age cannot be in negative.

Therefore, present age of Rehman is 7 years.

---

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Let Shefali's marks in Mathematics = x

Let Shefali's marks in English = $30 - x$

If, she had got 2 marks more in Mathematics, her marks would be = $x + 2$

If, she had got 3 marks less in English, her marks in English would be = $30 - x - 3 = 27 - x$

According to given condition:

=> $(x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -25$ and $c = 156$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(156)}}{2 \times 1}$$

$$\Rightarrow x = \frac{25 \pm \sqrt{625 - 624}}{2}$$

$$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$$

$$\Rightarrow x = \frac{25+1}{2}, \frac{25-1}{2}$$

$\Rightarrow x = 13, 12$

Therefore, Shefali's marks in Mathematics = 13 or 12

Shefali's marks in English = 30 – x = 30 – 13 = 17

Or Shefali's marks in English = 30 – x = 30 – 12 = 18

Therefore, her marks in Mathematics and English are (13, 17) or (12, 18).

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6. The diagonal of a rectangular field is 60 metres more than the shorter side. If, the longer side is 30 metres more than the shorter side, find the sides of the field.

Ans. Let shorter side of rectangle = $x$metres

Let diagonal of rectangle = (x + 60) metres

Let longer side of rectangle = (x + 30) metres

According to pythagoras theorem,

=> $(x + 60)^2 = (x + 30)^2 + x^2$

$\Rightarrow x^2 + 3600 + 120x = x^2 + 900 + 60x + x^2$

$\Rightarrow x^2 - 60x - 2700 = 0$

Comparing equation $x^2 - 60x - 2700 = 0$ with standard form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -60$ and $c = -2700$

Applying quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{60 \pm \sqrt{(-60)^2 - 4(1)(-2700)}}{2 \times 1}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{3600 + 10800}}{2}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{14400}}{2} = \frac{60 \pm 120}{2}$$

$$\Rightarrow x = \frac{60 + 120}{2}, \frac{60 - 120}{2}$$

$\Rightarrow x = 90, -30$

We ignore –30. Since length cannot be in negative.

Therefore, $x = 90$ which means length of shorter side = 90 metres

And length of longer side = $x + 30 = 90 + 30 = 120$ metres

Therefore, length of sides are 90 and 120 in metres.

---

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans. Let the larger number be x, then square of smaller number be 8x and square of larger number be x^2.

According to condition:

$x^2 - 8x = 180$

$\Rightarrow \quad x^2 - 8x - 180 = 0$

$\Rightarrow \quad x^2 - 18x + 10x - 180 = 0$

$\Rightarrow \quad x(x - 18) + 10(x - 18) = 0$

$\Rightarrow \quad (x - 18)(x + 10) = 0$

$\Rightarrow \quad x - 18 = 0 \quad \text{or} \quad x + 10 = 0$

$\Rightarrow \quad x = 18 \quad x = -10$

When $x = 18$, then square of smaller number = 144

Then smaller number = ± 12

Therefore, two numbers are (12, 18) or (−12, 18)

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8. A train travels 360 km at a uniform speed. If, the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans. Let the speed of the train = x km/hr

If, speed had been 5 km/hr more, train would have taken 1 hour less.

So, according to this condition

=> $\frac{360}{x} = \frac{360}{x+5} + 1$

$\Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$

$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right) = 1$

$\Rightarrow 360 \times 5 = x^2 + 5x$

$\Rightarrow x^2 + 5x - 1800 = 0$

Comparing equation $x^2 + 5x - 1800 = 0$ with general equation $ax^2 + bx + c = 0$,

We get $a = 1$, $b = 5$ and $c = -1800$

Applying quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-1800)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25 + 7200}}{2}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{7225}}{2} = \frac{-5 \pm 85}{2}$$

$$\Rightarrow x = \frac{-5 + 85}{2}, \frac{-5 - 85}{2}$$

$\Rightarrow x = 40, -45$

Since speed of train cannot be in negative. Therefore, we discard $x = -45$

Therefore, speed of train = 40 km/hr

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9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans. Let time taken by tap of smaller diameter to fill the tank = x hours

Let time taken by tap of larger diameter to fill the tank = $(x - 10)$ hours

It means that tap of smaller diameter fills $\frac{1}{x}^{th}$ part of tank in 1 hour.... (1)

And, tap of larger diameter fills $\frac{1}{x-10}^{th}$ part of tank in 1 hour. ... (2)

When two taps are used together, they fill tank in $\frac{75}{8}$ hours.

In 1 hour, they fill $\frac{8}{75}^{th}$ part of tank $\left(\frac{1}{\frac{75}{8}} = \frac{8}{75}\right)$ ... (3)

From (1), (2) and (3),

=> $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

⇒ $\frac{x-10+x}{x(x-10)} = \frac{8}{75}$

⇒ 75 (2x − 10) = $8\left(x^2 - 10x\right)$

⇒ 150x – 750 = $8x^2 - 80x$

⇒ $8x^2 - 80x - 150x + 750 = 0$

⇒ $4x^2 - 115x + 375 = 0$

Comparing equation $4x^2 - 115x + 375 = 0$ with general equation $ax^2 + bx + c = 0$,

We get a = 4, b = −115 and c = 375

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{115 \pm \sqrt{(-115)^2 - 4(4)(375)}}{2 \times 4}$$

$$\Rightarrow x = \frac{115 \pm \sqrt{13225 - 6000}}{8}$$

$$\Rightarrow x = \frac{115 \pm \sqrt{7225}}{8}$$

$$\Rightarrow x = \frac{115 \pm 85}{8}$$

$$\Rightarrow x = \frac{115 + 85}{8}, \frac{115 - 85}{8}$$

$\Rightarrow x = 25, 3.75$

Time taken by larger tap = $x - 10 = 3.75 - 10 = -6.25$ hours

Time cannot be in negative. Therefore, we ignore this value.

Time taken by larger tap = $x - 10 = 25 - 10 = 15$ hours

Therefore, time taken by larger tap is 15 hours and time taken by smaller tap is 25 hours.

---

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If, the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of two trains.

Ans. Let average speed of passenger train = x km/h

Let average speed of express train = $(x + 11)$ km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours

According to the given condition,

$$\Rightarrow \frac{132}{x} = \frac{132}{x+11} + 1$$

$$\Rightarrow 132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$$

$$\Rightarrow 132\left(\frac{x+11-x}{x(x+11)}\right) = 1$$

$\Rightarrow 132\ (11) = x\ (x + 11)$

$\Rightarrow 1452 = x^2 + 11x$

$\Rightarrow x^2 + 11x - 1452 = 0$

Comparing equation $x^2 + 11x - 1452 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = 11$ and $c = -1452$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(-1452)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-11 \pm \sqrt{121 + 5808}}{2}$$

$$\Rightarrow x = \frac{-11 \pm \sqrt{5929}}{2}$$

$$\Rightarrow x = \frac{-11 \pm 77}{2}$$

$$\Rightarrow x = \frac{-11+77}{2}, \frac{-11-77}{2}$$

$\Rightarrow x = 33, -44$

As speed cannot be in negative. Therefore, speed of passenger train = 33 km/h

And, speed of express train = x + 11 = 33 + 11 = 44 km/h

11. Sum of areas of two squares is 468 m^2. If, the difference of their perimeters is 24 metres, find the sides of the two squares.

Ans. Let perimeter of first square = x metres

Let perimeter of second square = (x + 24) metres

Length of side of first square = $\frac{x}{4}$ metres {Perimeter of square = $4 \times$ *length of side*}

Length of side of second square = $\left(\frac{x+24}{4}\right)$ metres

Area of first square = side × side = $\frac{x}{4} \times \frac{x}{4} = \frac{x^2}{16} m^2$

Area of second square = $\left(\frac{x+24}{4}\right)^2 m^2$

According to given condition:

=> $\frac{x^2}{16} + \left(\frac{x+24}{4}\right)^2 = 468$

⇒ $\frac{x^2}{16} + \frac{x^2 + 576 + 48x}{16} = 468$

⇒ $\frac{x^2 + x^2 + 576 + 48x}{16} = 468$

⇒ $2x^2 + 576 + 48x = 468 \times 16$

⇒ $2x^2 + 48x + 576 = 7488$

$\Rightarrow 2x^2 + 48x - 6912 = 0$

$\Rightarrow x^2 + 24x - 3456 = 0$

Comparing equation $x^2 + 24x - 3456 = 0$ with standard form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = 24$ and $c = -3456$

Applying Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-3456)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-24 \pm \sqrt{576 + 13824}}{2}$$

$$\Rightarrow x = \frac{-24 \pm \sqrt{14400}}{2} = \frac{-24 \pm 120}{2}$$

$$\Rightarrow x = \frac{-24 + 120}{2}, \frac{-24 - 120}{2}$$

$\Rightarrow x = 48, -72$

Perimeter of square cannot be in negative. Therefore, we discard $x = -72$.

Therefore, perimeter of first square = 48 metres

And, Perimeter of second square = $x + 24 = 48 + 24 = 72$ metres

$\Rightarrow$ Side of First square = $\dfrac{Perimeter}{4} = \dfrac{48}{4} = 12\ m$

And, Side of second Square = $\dfrac{Perimeter}{4} = \dfrac{72}{4} = 18\ m$

CBSE Class–10 Mathematics

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Chapter - 4

Quadratic Equations - Exercise 4.4

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Ans. (i) $2x^2 - 3x + 5 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,

We get *a* = 2, *b* = −3 and *c* = 5

Discriminant = $b^2 - 4ac = (-3)^2 - 4$ (2) (5)

= 9 – 40 = −31

Discriminant is less than 0 which means equation has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,

We get *a* = 3, *b* = $-4\sqrt{3}$ and c = 4

Discriminant = $b^2 - 4ac$ = $\left(-4\sqrt{3}\right)^2$ − 4 (3) (4)

= 48 – 48 = 0

Discriminant is equal to zero which means equations has equal real roots.

Applying quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find roots,

$$x=\frac{4\sqrt{3}\pm\sqrt{0}}{6}=\frac{2\sqrt{3}}{3}$$

Because, equation has two equal roots, it means $x=\dfrac{2\sqrt{3}}{3},\dfrac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$

Comparing equation with general equation $ax^2+bx+c= 0$,

We get a = 2, b = −6, and c = 3

Discriminant = $b^2 - 4ac = (-6)^2 - 4$ (2) (3)

= 36 – 24 = 12

Value of discriminant is greater than zero.

Therefore, equation has distinct and real roots.

Applying quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find roots,

$$x=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm2\sqrt{3}}{4}$$

$$\Rightarrow x=\frac{3\pm\sqrt{3}}{2}$$

$$\Rightarrow x=\frac{3+\sqrt{3}}{2},\frac{3-\sqrt{3}}{2}$$

2. Find the value of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) *kx* (*x* − 2) + 6 = 0

Ans. (i) $2x^2 + kx + 3 = 0$

We know that quadratic equation has two equal roots only when the value of discriminant is equal to zero.

Comparing equation $2x^2 + kx + 3 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$ and $c = 3$

Discriminant = $b^2 - 4ac = k^2 - 4$ (2) (3) = $k^2 - 24$

Putting discriminant equal to zero

$k^2 - 24 = 0 \Rightarrow k^2 = 24$

⇒ $k = \pm\sqrt{24} = \pm 2\sqrt{6}$

⇒ $k = 2\sqrt{6}, -2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

⇒ $kx^2 - 2kx + 6 = 0$

Comparing quadratic equation $kx^2 - 2kx + 6 = 0$ with general form $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$

Discriminant = $b^2 - 4ac = (-2k)^2 - 4$ (k) (6)

= $4k^2 - 24k$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.

Putting discriminant equal to zero

$4k^2 - 24k = 0$

$\Rightarrow 4k\,(k-6) = 0 \Rightarrow k = 0, 6$

The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2+bx+c=0$, where $a \neq 0$.

Therefore, in equation $kx^2-2kx+6=0$, we cannot have k = 0.

Therefore, we discard k = 0.

Hence the answer is k = 6.

---

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800\,m^2$? If so, find its length and breadth.

Ans. Let breadth of rectangular mango grove = x metres

Let length of rectangular mango grove = $2x$ metres

Area of rectangle = $length \times breadth = x \times 2x = 2x^2\,m^2$

According to given condition:

$\Rightarrow 2x^2 = 800$

$\Rightarrow 2x^2 - 800 = 0$

$\Rightarrow x^2 - 400 = 0$

Comparing equation $x^2-400=0$ with general form of quadratic equation $ax^2+bx+c=0$, we get $a = 1$, $b = 0$ and $c = -400$

Discriminant $= b^2 - 4ac = (0)^2 - 4\,(1)\,(-400) = 1600$

Discriminant is greater than 0 means that equation has two disctinct real roots.

Therefore, it is possible to design a rectangular grove.

Applying quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{0 \pm \sqrt{1600}}{2 \times 1} = \frac{\pm 40}{2} = \pm 20$$

⇒ $x = 20, -20$

We discard negative value of x because breadth of rectangle cannot be in negative.

Therefore, x = breadth of rectangle = 20 metres

Length of rectangle = $2x$ = 2×20 = 40 metres

---

4. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans. Let age of first friend = x years

then age of second friend = $(20 - x)$ years

Four years ago, age of first friend = $(x - 4)$ years

Four years ago, age of second friend = $(20 - x) - 4 = (16 - x)$ years

According to given condition,

=> $(x - 4)(16 - x) = 48$

⇒ $16x - x^2 - 64 + 4x = 48$

⇒ $20x - x^2 - 112 = 0$

⇒ $x^2 - 20x + 112 = 0$

Comparing equation, $x^2 - 20x + 112 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$ and $c = 112$

Discriminant = $b^2 - 4ac = (-20)^2 - 4$ (1) (112) = 400 – 448 = −48 < 0

Discriminant is less than zero which means we have no real roots for this equation.

Therefore, the give situation is not possible.

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5. Is it possible to design a rectangular park of perimeter 80 metres and area 400 m^2. If so, find its length and breadth.

Ans. Let length of park = *x* metres

We are given area of rectangular park = $400 m^2$

Therefore, breadth of park = $\frac{400}{x}$ metres{Area of rectangle = length × *breadth*}

Perimeter of rectangular park = 2 (*length* + *breath*) = $2\left(x+\frac{400}{x}\right)$ metres

We are given perimeter of rectangle = 80 metres

According to condition:

=> $2\left(x+\frac{400}{x}\right)=80$

⇒ $2\left(\frac{x^2+400}{x}\right)=80$

⇒ $2x^2 + 800 = 80x$

⇒ $2x^2 - 80x + 800 = 0$

⇒ $x^2 - 40x + 400 = 0$

Comparing equation, $x^2 - 40x + 400 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get *a* = 1, *b* = −40 and *c* = 400

Discriminant = $b^2 - 4ac = (-40)^2 - 4$ (1) (400) = 1600 – 1600 = 0

Discriminant is equal to 0.

Therefore, two roots of equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area $400m^2$.

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{40 \pm \sqrt{0}}{2} = \frac{40}{2} = 20$$

Here, both the roots are equal to 20.

Therefore, length of rectangular park = 20 metres

Breadth of rectangular park = $\frac{400}{x} = \frac{400}{20} = 20\ m$

CBSE Class–10 Mathematics

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Chapter - 5

Arithmetic Progressions - Exercise 5.1

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1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes one fourth of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

(iv) The amount of money in the account every year, when Rs 10,000 is deposited at compound Interest at 8% per annum.

Ans. (i) Taxi fare for 1st km = Rs 15, Taxi fare after 2 km = 15 + 8 = Rs 23

Taxi fare after 3 km = 23 + 8 = Rs 31

Taxi fare after 4 km = 31 + 8 = Rs 39

Therefore, the sequence is 15, 23, 31, 39…

It is an arithmetic progression because difference between any two consecutive terms is equal which is 8. (23 – 15 = 8, 31 – 23 = 8, 39 – 31 = 8, …)

(ii) Let amount of air initially present in a cylinder = V

Amount of air left after pumping out air by vacuum pump = $V-\frac{V}{4}=\frac{4V-V}{4}=\frac{3V}{4}$

Amount of air left when vacuum pump again pumps out air

$= \frac{3}{4}V - \left(\frac{1}{4} \times \frac{3}{4}V\right) = \frac{3}{4}V - \frac{3}{16}V = \frac{12V - 3V}{16} = \frac{9}{16}V$

So, the sequence we get is like $V, \frac{3}{4}V, \frac{9}{16}V \ldots$

Checking for difference between consecutive terms …

$\frac{3}{4}V - V = -\frac{V}{4}, \frac{9}{16}V - \frac{3}{4}V = \frac{9V - 12V}{16} = \frac{-3V}{16}$

Difference between consecutive terms is not equal.

Therefore, it is not an arithmetic progression.

(iii) Cost of digging 1 meter of well = Rs 150

Cost of digging 2 meters of well = 150 + 50 = Rs 200

Cost of digging 3 meters of well = 200 + 50 = Rs 250

Therefore, we get a sequence of the form 150, 200, 250…

It is an arithmetic progression because difference between any two consecutive terms is equal. (200 – 150 = 250 – 200 = 50…)

Here, difference between any two consecutive terms which is also called common difference is equal to 50.

(iv) Amount in bank after Ist year = $10000\left(1 + \frac{8}{100}\right)$ … (1)

Amount in bank after two years = $10000\left(1 + \frac{8}{100}\right)^2$ … (2)

Amount in bank after three years = $10000\left(1 + \frac{8}{100}\right)^3$ … (3)

Amount in bank after four years = $10000\left(1+\frac{8}{100}\right)^4$ ... (4)

It is not an arithmetic progression because (2) − (1) ≠ (3) − (2)

(Difference between consecutive terms is not equal)

Therefore, it is not an Arithmetic Progression.

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2. Write first four terms of the AP, when the first term a and common difference d are given as follows:

(i)a = 10, d = 10

(ii) a = -2, d = 0

(iii) a = 4, d = -3

(iv) a = -1, d = $\frac{1}{2}$

(v) a = -1.25, d = -0.25

Ans. (i) First term = a = 10, d = 10

Second term = a + d = 10 + 10 = 20

Third term = second term + d = 20 + 10 = 30

Fourth term = third term + d = 30 + 10 = 40

Therefore, first four terms are: 10, 20, 30, 40

(ii) First term = a = –2 , d = 0

Second term = a + d = –2 + 0 = –2

Third term = second term + d = –2 + 0 = –2

Fourth term = third term + d = –2 + 0 = –2

Therefore, first four terms are: –2, –2, –2, –2

(iii) First term = a = 4, d =–3

Second term = a + d = 4 – 3 = 1

Third term = second term + d = 1 – 3 = –2

Fourth term = third term + d = –2 – 3 = –5

Therefore, first four terms are: 4, 1, –2, –5

(iv) First term = a = –1, d = $\frac{1}{2}$

Second term = a + d = –1 + $\frac{1}{2}$ = $-\frac{1}{2}$

Third term = second term + d = $-\frac{1}{2} + \frac{1}{2} = 0$

Fourth term = third term + d = 0 + $\frac{1}{2} = \frac{1}{2}$

Therefore, first four terms are: –1, $-\frac{1}{2}$, 0, $\frac{1}{2}$

(v) First term = a = –1.25, d = –0.25

Second term = a + d = –1.25 – 0.25 = –1.50

Third term = second term + d = –1.50 – 0.25 = –1.75

Fourth term = third term + d

= –1.75 – 0.25 = –2.00

Therefore, first four terms are: –1.25, –1.50, –1.75, –2.00

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3. For the following APs, write the first term and the common difference.

(i) 3, 1, –1, –3 ...

(ii) –5, –1, 3, 7...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

(iv) 0.6, 1.7, 2.8, 3.9 ...

Ans. (i) 3, 1, –1, –3...

First term = a = 3,

Common difference (d) = Second term – first term = Third term – second term and so on

Therefore, Common difference (d) = 1 – 3 = –2

(ii) –5, –1, 3, 7...

First term = a = –5

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = –1 – (–5) = –1 + 5 = 4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

First term = a = $\frac{1}{3}$

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9...

First term = a = 0.6

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = 1.7 − 0.6 = 1.1

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4. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16...

(ii) 2, $\frac{5}{2}$ **, 3,** $\frac{7}{2}$ **...**

(iii) −1.2, −3.2, −5.2, −7.2...

(iv) −10, −6, −2, 2...

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}...$

(vi) 0.2, 0.22, 0.222, 0.2222...

(vii) 0, −4, −8, −12...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}...$

(ix) 1, 3, 9, 27...

(x) ***a*****, 2*****a*****, 3*****a*****, 4*****a*****...**

(xi) $a, a^2, a^3, a^4...$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}...$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}...$

(xiv) $1^2, 3^2, 5^2, 7^2...$

(xv) $1^2, 5^2, 7^2, 73...$

Ans. (i) 2, 4, 8, 16...

It is not an AP because difference between consecutive terms is not equal.

As4 – 2 ≠ 8 – 4

(ii)2, $\frac{5}{2}$, 3, $\frac{7}{2}$ ...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \frac{5}{2}-2=3-\frac{5}{2}=\frac{1}{2}$

Common difference (d) = $\frac{1}{2}$

Fifth term = $\frac{7}{2}+\frac{1}{2}=4$ Sixth term = 4 + ½ = $\frac{9}{2}$

Seventh term = $\frac{9}{2}+\frac{1}{2}=5$

Therefore, next three terms are 4, $\frac{9}{2}$ and 5.

(iii)−1.2, −3.2, −5.2, −7.2...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow$ −3.2 − (−1.2)

= −5.2 − (−3.2)

= −7.2 − (−5.2) = −2

Common difference (d) = −2

Fifth term = −7.2 – 2 = −9.2Sixth term = −9.2 – 2 = −11.2

Seventh term = −11.2 – 2 = −13.2

Therefore, next three terms are −9.2, −11.2 and −13.2

(iv) −10, −6, −2, 2…

It isan AP because difference between consecutive terms is equal.

$\Rightarrow$ −6 − (−10) = −2 − (−6)

= 2 − (−2) = 4

Common difference (d) = 4

Fifth term = 2 + 4 = 6 Sixth term = 6 + 4 = 10

Seventh term = 10 + 4 = 14

Therefore, next three terms are 6, 10 and 14

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}\ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 3+\sqrt{2}-3$

$=\sqrt{2}, 3+2\sqrt{2}-(3+\sqrt{2})$

$=3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2}$

Sixth term = $3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}$

Seventh term = $3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}$

Therefore, next three terms are $\left(3+4\sqrt{2}\right), \left(3+5\sqrt{2}\right), \left(3+6\sqrt{2}\right)$

(vi) 0.2, 0.22, 0.222, 0.2222…

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 0.22 - 0.2 \neq 0.222 - 0.22$

(vii) 0, −4, −8, −12…

It is an AP because difference between consecutive terms is equal.

$\Rightarrow -4 - 0 = -8 - (-4)$

$= -12 - (-8) = -4$

Common difference (d) = −4

Fifth term = −12 – 4 =−16 Sixth term = −16 – 4 = −20

Seventh term = −20 – 4 = −24

Therefore, next three terms are −16, −20 and −24

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -\frac{1}{2}-\left(-\frac{1}{2}\right)=-\frac{1}{2}-\left(-\frac{1}{2}\right)=0$$

Common difference (d) = 0

Fifth term = $-\frac{1}{2}+0=-\frac{1}{2}$ Sixth term = $-\frac{1}{2}+0=-\frac{1}{2}$

Seventh term = $-\frac{1}{2}+0=-\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2}, -\frac{1}{2}$ *and* $-\frac{1}{2}$

(ix) 1, 3, 9, 27...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3 - 1 \neq 9 - 3$

(x) a, $2a$, $3a$, $4a$...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2a - a = 3a - 2a = 4a - 3a = a$

Common difference (d) = a

Fifth term = $4a + a = 5a$ Sixth term = $5a + a = 6a$

Seventh term = $6a + a = 7a$

Therefore, next three terms are $5a$, $6a$ and $7a$

(xi) a, a^2, a^3, a^4...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow a^2 - a \neq a^3 - a^2$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$...

$\Rightarrow \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$ Sixth term = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

Seventh term = $6\sqrt{2}+\sqrt{2}=7\sqrt{2}$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \sqrt{6}-\sqrt{3} \neq \sqrt{9}-\sqrt{6}$

(xiv) $1^2, 3^2, 5^2, 7^2\ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3^2 - 1^2 \neq 5^2 - 3^2$

(xv) $1^2, 5^2, 7^2, 73\ldots$

$\Rightarrow$ 1, 25, 49, 73...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 5^2 - 1^2$

$= 7^2 - 5^2 = 73 - 7^2 = 24$

Common difference (d) = 24

Fifth term = 73 + 24 = 97 Sixth term = 97 + 24 = 121

Seventh term = 121 + 24 = 145

Therefore, next three terms are 97, 121 and 145

CBSE Class–10 Mathematics

NCERT solution

Chapter - 5

Arithmetic Progressions - Exercise 5.2

1. Find the missing variable from a, d, n and a_n, where a is the first term, d is the common difference and a_n is the nth term of AP.

(i) a = 7, d = 3, n = 8

(ii) a = –18, n = 10, $a_n = 0$

(iii) d = –3, n = 18, $a_n = -5$

(iv) a = –18.9, d = 2.5, $a_n = 3.6$

(v) a = 3.5, d = 0, n = 105

Ans. (i) a = 7, d = 3, n = 8

We need to find a_n here.

Using formula $a_n = a + (n-1)d$

Putting values of a, d and n,

a_n = 7 + (8 − 1) 3

= 7 + (7) 3 = 7 + 21 = 28

(ii) a = –18, n= 10, $a_n = 0$

We need to find d here.

Using formula $a_n = a + (n-1)d$

Putting values of a, a_n and n,

$0 = -18 + (10 - 1)\, d$

$\Rightarrow 0 = -18 + 9d$

$\Rightarrow 18 = 9d \Rightarrow d = 2$

(iii) d = –3, n = 18, $a_n = -5$

We need to find a here.

Using formula $a_n = a + (n - 1)d$

Putting values of d, a_n and n,

$-5 = a + (18 - 1)(-3)$

$\Rightarrow -5 = a + (17)(-3)$

$\Rightarrow -5 = a - 51 \Rightarrow a = 46$

(iv) a = –18.9, d = 2.5, $a_n = 3.6$

We need to find n here.

Using formula $a_n = a + (n - 1)d$

Putting values of d, a_n and n,

$3.6 = -18.9 + (n - 1)(2.5)$

$\Rightarrow 3.6 = -18.9 + 2.5n - 2.5$

$\Rightarrow 2.5n = 25 \Rightarrow n = 10$

(v) a = 3.5, d = 0, n = 105

We need to find a_n here.

Using formula $a_n = a + (n - 1)d$

Putting values of d, n and a,

$a_n = 3.5 + (105 - 1)(0)$

$\Rightarrow a_n = 3.5 + 104 \times 0$

$\Rightarrow a_n = 3.5 + 0 \Rightarrow a_n = 3.5$

2.Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10, 7, 4... is

(A) 97

(B) 77

(C) –77

(D) –87

(ii) 11th term of the AP: −3, −12, 2... is

(A) 28

(B) 22

(C) –38

(D) $-48\frac{1}{2}$

Ans.(i) 10, 7, 4...

First term = a = 10, Common difference = d = 7 – 10 = 4 – 7 = –3

And n = 30{Because, we need to find 30th term}

$a_n = a + (n-1)d$

$\Rightarrow a_{30} = 10 + (30 - 1)(-3) = 10 - 87 = -77$

Therefore, the answer is (C).

(ii) −3, −½, 2…

First term = a = –3, Common difference = d = $-\frac{1}{2} - (-3) = 2 - (-\frac{1}{2}) = \frac{5}{2}$

And n = 11 (Because, we need to find 11th term)

$a_n = -3 + (11 - 1)\frac{5}{2} = -3 + 25 = 22$

Therefore 11th term is 22 which means answer is (B).

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3. In the following AP's find the missing terms:

(i) 2, __ , 26

(ii) __, 13, __, 3

(iii) 5, __, __, $9\frac{1}{2}$

(iv) –4. __, __, __, __, 6

(v) __, 38, __, __, __, –22

Ans. (i) 2, __ , 26

We know that difference between consecutive terms is equal in any A.P.

Let the missing term be x.

$x - 2 = 26 - x$

$\Rightarrow 2x = 28 \Rightarrow x = 14$

Therefore, missing term is 14.

(ii)__, 13, __, 3

Let missing terms be x and y.

The sequence becomes x, 13, y, 3

We know that difference between consecutive terms is constant in any A.P.

$y - 13 = 3 - y$

$\Rightarrow 2y = 16 \Rightarrow y = 8$

And $13 - x = y - 13$

$\Rightarrow x + y = 26$

But, we have $y = 8$,

$\Rightarrow x + 8 = 26 \Rightarrow x = 18$

Therefore, missing terms are 18 and 8.

(iii) 5, __, __, $9\frac{1}{2}$

Here, first term = a = 5 And, 4th term = $a_4 = 9\frac{1}{2}$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$a_4 = 5 + (4 - 1)\,d$

$\Rightarrow \frac{19}{2} = 5 + 3d$

$\Rightarrow 19 = 2\,(5 + 3d)$

$\Rightarrow 19 = 10 + 6d$

$\Rightarrow 6d = 19 - 10$

$\Rightarrow 6d = 9 \Rightarrow d = \frac{3}{2}$

Therefore, we get common difference = d = $\frac{3}{2}$

Second term = a + d = $5 + \frac{3}{2} = \frac{13}{2}$

Third term = second term + d = $\frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8$

Therefore, missing terms are $\frac{13}{2}$ and 8

(iv)–4. __, __, __, __, 6

Here, First term = a = –4 and 6^{th} term = $a_6 = 6$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_6 = -4 + (6-1)d$

$\Rightarrow 6 = -4 + 5d$

$\Rightarrow 5d = 10 \Rightarrow d = 2$

Therefore, common difference = d = 2

Second term = first term + d = a + d = –4 + 2 = –2

Third term = second term + d = –2 + 2 = 0

Fourth term = third term + d = 0 + 2 = 2

Fifth term = fourth term + d = 2 + 2 = 4

Therefore, missing terms are –2, 0, 2 and 4.

(v) __, 38, __, __, __, –22

We are given 2^{nd} and 6^{th} term.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_2 = a + (2 - 1)\,d$ and $a_6 = a + (6 - 1)\,d$

$\Rightarrow 38 = a + d$ and $-22 = a + 5d$

These are equations in two variables, we can solve them using any method.

Using equation ($38 = a + d$), we can say that $a = 38 - d$.

Putting value of a in equation ($-22 = a + 5d$),

$-22 = 38 - d + 5d$

$\Rightarrow 4d = -60$

$\Rightarrow d = -15$

Using this value of d and putting this in equation $38 = a + d$,

$38 = a - 15 \Rightarrow a = 53$

Therefore, we get $a = 53$ and $d = -15$

First term = a = 53

Third term = second term + d = 38 – 15 = 23

Fourth term = third term + d = 23 – 15 = 8

Fifth term = fourth term + d = 8 – 15 = –7

Therefore, missing terms are 53, 23, 8 and –7.

---

4. Which term of the AP: 3, 8, 13, 18 ... is 78?

Ans. First term = a = 3, Common difference = d = 8 – 3 = 13 – 8 = 5 and $a_n = 78$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_n = 3 + (n - 1)\, 5$,

$\Rightarrow 78 = 3 + (n - 1)\, 5$

$\Rightarrow 75 = 5n - 5$

$\Rightarrow 80 = 5n \Rightarrow n = 16$

It means 16^{th} term of the given AP is equal to 78.

---

5. Find the number of terms in each of the following APs:

(i) 7, 13, 19…., 205

(ii) 18, $15\frac{1}{2}$, 13…, −47

Ans. (i) 7, 13, 19 …, 205

First term = a = 7, Common difference = d = 13 – 7 = 19 – 13 = 6

And $a_n = 205$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$205 = 7 + (n - 1)\, 6 = 7 + 6n - 6$

$\Rightarrow 205 = 6n + 1$

$\Rightarrow 204 = 6n \Rightarrow n = 34$

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$, 13 …, −47

First term = a =18, Common difference = d = $15\frac{1}{2}-18=\frac{31}{2}-18=\frac{31-36}{2}=\frac{-5}{2}$

And $a_n = -47$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$-47 = 18 + (n - 1)\left(-\frac{5}{2}\right)$

$= 36 - \frac{5}{2}n + \frac{5}{2}$

$\Rightarrow -94 = 36 - 5n + 5$

$\Rightarrow 5n = 135 \Rightarrow n = 27$

Therefore, there are 27 terms in the given arithmetic progression.

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6. Check whether –150 is a term of the AP: 11, 8, 5, 2...

Ans. Let −150 is the nth of AP 11, 8, 5, 2... which means that $a_n = -150$

Here, First term = a = 11, Common difference = d = 8 – 11 = –3

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$-150 = 11 + (n - 1)(-3)$

$\Rightarrow -150 = 11 - 3n + 3$

$\Rightarrow 3n = 164 \Rightarrow n = \frac{164}{3}$

But, n cannot be in fraction.

Therefore, our supposition is wrong. −150 cannot be term in AP.

---

7. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

Ans. Here $a_{11} = 38 \, and \, a_{16} = 73$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

38 = a + (11 − 1) (d)And73 = a + (16 − 1) (d)

⇒ 38 = a + 10d And 73 = a + 15d

These are equations consisting of two variables.

We have, 38 = a + 10d

⇒ a = 38 − 10d

Let us put value of a in equation (73 = a + 15d),

73 = 38 − 10d + 15d

⇒ 35 = 5d

Therefore, Common difference = d = 7

Putting value of d in equation 38 = a + 10d,

38 = a + 70

⇒ a = −32

Therefore, common difference = d = 7 and First term = a = –32

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

a_{31} = −32 + (31 − 1) (7)

= −32 + 210 = 178

Therefore, 31st term of AP is 178.

8. An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

Ans. An AP consists of 50 terms and the 50^{th} term is equal to 106 and $a_3 = 12$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{50} = a + (50 - 1)\,d$ And $a_3 = a + (3 - 1)\,d$

$\Rightarrow 106 = a + 49d$ And$12 = a + 2d$

These are equations consisting of two variables.

Using equation $106 = a + 49d$, we get $a = 106 - 49d$

Putting value of a in the equation $12 = a + 2d$,

$12 = 106 - 49d + 2d$

$\Rightarrow 47d = 94 \Rightarrow d = 2$

Putting value of d in the equation, $a = 106 - 49d$,

$a = 106 - 49\,(2) = 106 - 98 = 8$

Therefore, First term $= a = 8$ and Common difference $= d = 2$

To find 29^{th} term, we use formula $a_n = a + (n-1)d$ which is used to find n^{th} term of arithmetic progression,

$a_{29} = 8 + (29 - 1)\,2 = 8 + 56 = 64$

Therefore, 29th term of AP is equal to 64.

9. If the third and the ninth terms of an AP are 4 and –8 respectively, which term of this AP is zero?

Ans. It is given that 3^{rd} and 9^{th} term of AP are 4 and –8 respectively.

It means $a_3 = 4 and\ a_9 = -8$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

4 = a + (3 – 1) d And, –8 = a + (9 – 1) d

⇒ $4 = a + 2d$ and $-8 = a + 8d$

These are equations in two variables.

Using equation $4 = a + 2d$, we can say that $a = 4 - 2d$

Putting value of a in other equation $-8 = a + 8d$,

$-8 = 4 - 2d + 8d$

⇒ $-12 = 6d$ ⇒ $d = -2$

Putting value of d in equation $-8 = a + 8d$,

$-8 = a + 8(-2)$

⇒ $-8 = a - 16$ ⇒ $a = 8$

Therefore, first term = a = 8 and Common Difference = d = −2

We want to know which term is equal to zero.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$0 = 8 + (n - 1)(-2)$

⇒ $0 = 8 - 2n + 2$

⇒ $0 = 10 - 2n$

⇒ $2n = 10$ ⇒ $n = 5$

Therefore, 5th term is equal to 0.

10. The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Ans. $a_{17} = a_{10} + 7 \ldots (1)$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_{17} = a + 16d \ldots (2)$

$a_{10} = a + 9d \ldots (3)$

Putting (2) and (3) in equation (1),

$a + 16d = a + 9d + 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

---

11. Which term of the AP: 3, 15, 27, 39... will be 132 more than its 54^{th} term?

Ans. Lets first calculate 54^{th} of the given AP.

First term = a = 3, Common difference = d = 15 – 3 = 12

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_{54} = a + (54 - 1)\, d = 3 + 53\,(12) = 3 + 636 = 639$

We want to find which term is 132 more than its 54^{th} term.

Let us suppose it is n^{th} term which is 132 more than 54^{th} term.

$a_n = a_{54} + 132$

$\Rightarrow 3 + (n - 1)\, 12 = 639 + 132$

$\Rightarrow 3 + 12n – 12 = 771$

$\Rightarrow 12n - 9 = 771$

$\Rightarrow 12n = 780$

$\Rightarrow n = 65$

Therefore, 65^{th} term is 132 more than its 54^{th} term.

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12. Two AP's have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms.

Ans. Let first term of 1^{st} AP = a

Let first term of 2^{nd} AP = a'

It is given that their common difference is same.

Let their common difference be d.

It is given that difference between their 100^{th} terms is 100.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a + (100 - 1)d - [a' + (100 - 1)d]$

$= a + 99d - a' - 99d = 100$

$\Rightarrow a - a' = 100$... (1)

We want to find difference between their 1000^{th} terms which means we want to calculate:

$a + (1000 - 1)d - [a' + (1000 - 1)d]$

$= a + 999d - a' - 999d = a - a'$

Putting equation (1) in the above equation,

$a + (1000 - 1)d - [a' + (1000 - 1)d]$

$= a + 999d - a' + 999d = a - a' = 100$

Therefore, difference between their 1000^{th} terms would be equal to 100.

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13. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105, 112, 119..., 994

Let 994 is the n^{th} term of AP.

We need to find n here.

First term = a = 105, Common difference = d = 112 – 105 = 7

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$994 = 105 + (n - 1)(7)$

$\Rightarrow 994 = 105 + 7n - 7$

$\Rightarrow 896 = 7n \Rightarrow n = 128$

It means 994 is the 128^{th} term of AP.

Therefore, there are 128 terms in AP.

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14. How many multiples of 4 lie between 10 and 250?

Ans. First multiple of 4 which lie between 10 and 250 is 12.

The last multiple of 4 which lie between 10 and 250 is 248.

Therefore, AP is of the form 12, 16, 20... ,248

First term = a = 12, Common difference = d = 4

Using formula $a_n = a + (n – 1)\, d$, to find n^{th} term of arithmetic progression,

$248 = 12 + (n - 1)(4)$

$\Rightarrow 248 = 12 + 4n - 4$

$\Rightarrow 240 = 4n$

$\Rightarrow n = 60$

It means that 248 is the 60^{th} term of AP.

So, we can say that there are 60 multiples of 4 which lie between 10 and 250.

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15. For what value of n, are the nth terms of two AP's: 63, 65, 67... and 3, 10, 17... equal?

Ans. Lets first consider AP 63, 65, 67...

First term = a = 63, Common difference = d = 65 – 63 = 2

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_n = 63 + (n - 1)(2)$... (1)

Now, consider second AP 3, 10, 17...

First term = a = 3, Common difference = d = 10 – 3 = 7

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_n = 3 + (n - 1)(7)$... (2)

According to the given condition:

(1) = (2)

$\Rightarrow 63 + (n - 1)(2) = 3 + (n - 1)(7)$

$\Rightarrow 63 + 2n – 2 = 3 + 7n - 7$

$\Rightarrow 65 = 5n \Rightarrow n = 13$

Therefore, 13^{th} terms of both the AP's are equal.

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16. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Ans. Let first term of AP = a

Let common difference of AP = d

It is given that its 3^{rd} term is equal to 16.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$16 = a + (3 - 1)(d)$

$\Rightarrow 16 = a + 2d$... (1)

It is also given that 7^{th} term exceeds 5^{th} term by12.

According to the given condition:

$a_7 = a_5 + 12$

$\Rightarrow a + (7 - 1)d = a + (5 - 1)d + 12$

$\Rightarrow 2d = 12 \Rightarrow d = 6$

Putting value of d in equation $16 = a + 2d$,

$16 = a + 2(6) \Rightarrow a = 4$

Therefore, first term = a = 4

And, common difference = d = 6

Therefore, AP is 4, 10, 16, 22...

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17. Find the 20th term from the last term of the AP: 3, 8, 13... , 253.

Ans. We want to find 20th term from the last term of given AP.

So, let us write given AP in this way:253 ... 13, 8, 3

Here First term = a = 253, Common Difference = d = 8 – 13 = –5

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

a_{20} = 253 + (20 − 1) (−5)

⇒ a_{20} = 253 + 19 (−5) = 253 – 95 = 158

Therefore, the 20th term from the last term of given AP is 158.

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18. The sum of the 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. Find the three terms of the AP.

Ans. The sum of 4th and 8th terms of an AP is 24 and sum of 6th and 10th terms is 44.

$a_4 + a_8 = 24$

and $a_6 + a_{10} = 44$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

=> $a + (4 - 1)d + [a + (8 - 1)d] = 24$

And, $a + (6 - 1)d + [a + (10 - 1)d] = 44$

⇒ $a + 3d + a + 7d = 24$

And $a + 5d + a + 9d = 44$

⇒ $2a + 10d = 24$ And $2a + 14d = 44$

⇒ $a + 5d = 12$ And $a + 7d = 22$

These are equations in two variables.

Using equation, $a + 5d = 12$, we can say that $a = 12 - 5d$... (1)

Putting (1) in equation $a + 7d = 22$,

$12 - 5d + 7d = 22$

$\Rightarrow 12 + 2d = 22$

$\Rightarrow 2d = 10$

$\Rightarrow d = 5$

Putting value of d in equation $a = 12 - 5d$,

$a = 12 - 5\ (5) = 12 - 25 = -13$

Therefore, first term = $a = -13$ and, Common difference = $d = 5$

Therefore, AP is –13, –8, –3, 2...

Its first three terms are –13, –8 and –3.

---

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Ans. Subba Rao's starting salary = Rs 5000

It means, first term = a = 5000

He gets an increment of Rs 200 after every year.

Therefore, common difference = d = 200

His salary after 1 year = 5000 + 200 = Rs 5200

His salary after two years = 5200 + 200 = Rs 5400

Therefore, it is an AP of the form 5000, 5200, 5400, 5600... , 7000

We want to know in which year his income reaches Rs 7000.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

7000 = 5000 + (n − 1) (200)

⇒ 7000 = 5000 + 200n − 200

⇒ 7000 – 5000 + 200 = 200n

⇒ 2200 = 200n

⇒ n = 11

It means after 11 years, Subba Rao's income would be Rs 7000.

---

20. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Ans. Ramkali saved Rs. 5 in the first week of year. It means first term = a = 5
Ramkali increased her weekly savings by Rs 1.75.
Therefore, common difference = d = Rs 1.75
Money saved by Ramkali in the second week = a + d = 5 + 1.75 = Rs 6.75
Money saved by Ramkali in the third week = 6.75 + 1.75 = Rs 8.5
Therefore, it is an AP of the form:5, 6.75, 8.5 … , 20.75
We want to know in which week her savings become 20.75.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

20.75 = 5 + (n − 1) (1.75)

⇒ 20.75 = 5 + 1.75n − 1.75

⇒ 17.5 = 1.75n

⇒ n = 10

It means in the 10th week her savings become Rs 20.75.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 5

Arithmetic Progressions - Exercise 5.3

1. Find the sum of the following AP's.

(i) 2, 7, 12… to 10 terms

(ii) –37, –33, –29… to 12 terms

(iii) 0.6, 1.7, 2.8… to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ **to 11 terms**

Ans. (i) 2, 7, 12… to 10 terms

Here First term = a = 2, Common difference = d = 7 – 2 = 5 and n = 10

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{10}{2}\left[4 + (10-1)5\right] = 5(4+45) = 5 \times 49 = 245$$

(ii) –37, –33, –29… to 12 terms

Here First term = a = –37, Common difference = d = –33 – (–37) = 4

And n = 12

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{12}{2}\left[-74 + (12-1)4\right] = 6(-74+44) = 6 \times (-30) = -180$$

(iii) 0.6, 1.7, 2.8... to 100 terms

Here First term = a = 0.6, Common difference = d = 1.7 – 0.6 = 1.1

And n = 100

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$Sn = \frac{100}{2}[1.2 + (100 - 1)\,1.1] = 50\,(1.2 + 108.9) = 50 \times 110.1 = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ to 11 terms

Here First term =a = $\frac{1}{15}$ Common difference = d = $\frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_n = \frac{11}{2}\left[\frac{2}{15} + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right) = \frac{11}{2}\left(\frac{4+5}{30}\right) = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$$

2. Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) 34 + 32 + 30 + ... + 10

(iii) –5 + (–8) + (–11) + ... + (–230)

Ans. (i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

Here First term = a = 7, Common difference = d = $\frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2} = 3.5$

And Last term = l = 84

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$[7 + (n - 1)(3.5)] = 84$

$\Rightarrow 7 + (3.5)n - 3.5 = 84$

$\Rightarrow 3.5n = 84 + 3.5 - 7$

$\Rightarrow 3.5n = 80.5$

$\Rightarrow n = 23$

Therefore, there are 23 terms in the given AP.

It means n = 23.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{23} = \frac{23}{2}(7 + 84)$$

$$\Rightarrow S_{23} = \frac{23}{2} \times 91 = 1046.5$$

(ii) 34 + 32 + 30 + … + 10

Here First term = a = 34, Common difference = d = 32 – 34 = –2

And Last term = l = 10

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$[34 + (n - 1)(-2)] = 10$

$\Rightarrow 34 - 2n + 2 = 10$

$\Rightarrow -2n = -26 \Rightarrow n = 13$

Therefore, there are 13 terms in the given AP.

It means n = 13.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 286$$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Here First term = a = –5, Common difference = d = –8 – (–5) = –8 + 5 = –3

And Last term = l = −230

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$[-5 + (n - 1)(-3)] = -230$

$\Rightarrow -5 - 3n + 3 = -230$

$\Rightarrow -3n = -228 \Rightarrow n = 76$

Therefore, there are 76 terms in the given AP.

It means n = 76.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{76} = \frac{76}{2}(-5-230) = 38 \times (-235) = -8930$$

3. In an AP

(i) given a = 5, d = 3, $a_n = 50$, find n and S_n.

(ii) given a = 7, $a_{13} = 35$, find d and S_{13}.

(iii) given $a_{12} = 37$, d = 3, find a and S_{12}.

(iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) given d = 5, $S_9 = 75$, find a and a_9.

(vi) given a = 2, d = 8, $S_n = 90$, find n and a_n.

(vii) given a = 8, $a_n = 62$, $S_n = 210$, find n and d.

(viii) given $a_n = 4$, d = 2, $S_n = -14$, find n and a.

(ix) given a = 3, n = 8, S = 192, find d.

(x) given l = 28, S = 144, and there are total of 9 terms. Find a.

Ans. (i) Given a = 5, d = 3, $a_n = 50$, find n and S_n.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_n = 5 + (n − 1) (3)

⇒ 50 = 5 + 3n − 3

⇒ 48 = 3n⇒ n = 16

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{16} = \frac{16}{2}[10 + (16-1)3] = 8(10+45) = 8 \times 55 = 440$$

Therefore, n = 16 and $S_n = 440$

(ii) Given a = 7, $a_{13} = 35$, find d and S_{13}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{13} = 7 + (13 − 1) (d)

⇒ 35 = 7 + 12d

⇒ 28 = 12d⇒ $d = \frac{7}{3}$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}\left[14 + (13-1)\frac{7}{3}\right] = \frac{13}{2}(14+28) = \frac{13}{2} \times 42 = 273$$

Therefore, $d = \frac{7}{3}$ and $S_{13} = 273$

(iii) Given $a_{12} = 37$, d = 3, find a and S_{12}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{12} = a + (12 − 1) 3

⇒ 37 = a + 33 ⇒ a = 4

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{12} = \frac{12}{2}[8 + (12-1)3] = 6(8+33) = 6 \times 41 = 246$$

Therefore, $a = 4$ and $S_{12} = 246$

(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$a_3 = a + (3 - 1)(d)$

⇒ $15 = a + 2d$

⇒ $a = 15 - 2d$... (1)

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$

⇒ $125 = 5(2a + 9d) = 10a + 45d$

Putting (1) in the above equation,

$125 = 5[2(15 - 2d) + 9d] = 5(30 - 4d + 9d)$

⇒ $125 = 150 + 25d$

⇒ $125 – 150 = 25d$

⇒ $-25 = 25d$ ⇒ $d = -1$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$a_{10} = a + (10 - 1)d$

Putting value of d and equation (1) in the above equation,

$a_{10} = 15 - 2d + 9d = 15 + 7d$

$= 15 + 7(-1) = 15 – 7 = 8$

Therefore, $d = -1$ and $a_{10} = 8$

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_9 = \frac{9}{2}[2a + (9-1)5]$$

$$\Rightarrow 75 = \frac{9}{2}[2a + 40]$$

$\Rightarrow 150 = 18a + 360$

$\Rightarrow -210 = 18a$

$$\Rightarrow a = \frac{-35}{3}$$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$$a_9 = \frac{-35}{3} + (9-1)(5)$$

$$= \frac{-35}{3} + 40 = \frac{-35+120}{3} = \frac{85}{3}$$

Therefore, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$90 = \frac{n}{2}[4 + (n-1)8]$$

$\Rightarrow 90 = \frac{n}{2}[4 + 8n - 8]$

$\Rightarrow 90 = \frac{n}{2}[8n - 4]$

$\Rightarrow 8n^2 - 4n - 180 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$

$\Rightarrow (n - 5)(2n + 9) = 0$

$\Rightarrow n = 5, -9/2$

We discard negative value of n because here n cannot be in negative or fraction.

The value of n must be a positive integer.

Therefore, $n = 5$

Using formula $a_n = a + (n-1)d$,to find nth term of arithmetic progression,

$a_5 = 2 + (5 - 1)(8) = 2 + 32 = 34$

Therefore, $n = 5$ and $a_n = 34$

(vii) Given $a = 8$, $a_n = 62, S_n = 210$, find n and d.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$62 = 8 + (n - 1)(d) = 8 + nd - d$

$\Rightarrow 62 = 8 + nd - d$

$\Rightarrow nd - d = 54$

$\Rightarrow nd = 54 + d$... (1)

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$210 = \frac{n}{2}[16+(n-1)d] = \frac{n}{2}(16+nd-d)$$

Putting equation (1) in the above equation,

$$210 = \frac{n}{2}[16+54+d-d] = \frac{n}{2}\times 70$$

$$\Rightarrow n = \frac{210\times 2}{70} = 6 \Rightarrow n = 6$$

Putting value of n in equation (1),

$6d = 54 + d \Rightarrow d = \frac{54}{5}$

Therefore, $n = 6$ and $d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$4 = a + (n - 1)(2) = a + 2n - 2$

$\Rightarrow 4 = a + 2n - 2$

$\Rightarrow 6 = a + 2n$

$\Rightarrow a = 6 - 2n$... (1)

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$-14 = \frac{n}{2}[2a+(n-1)2] = \frac{n}{2}(2a+2n-2)$$

$$\Rightarrow -14 = \frac{n}{2}(2a+2n-2)$$

Putting equation (1) in the above equation, we get

$-28 = n\,[2\,(6 - 2n) + 2n - 2]$

$\Rightarrow -28 = n\,(12 - 4n + 2n - 2)$

$\Rightarrow -28 = n\,(10 - 2n)$

$\Rightarrow 2n^2 - 10n - 28 = 0$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow n\,(n - 7) + 2\,(n - 7) = 0$

$\Rightarrow (n + 2)\,(n - 7) = 0$

$\Rightarrow n = -2, 7$

Here, we cannot have negative value of n.

Therefore, we discard negative value of n which means $n = 7$.

Putting value of n in equation (1), we get

$a = 6 - 2n = 6 - 2\,(7) = 6 - 14 = -8$

Therefore, $n = 7$ and $a = -8$

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

Using formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$192 = \frac{8}{2}$ [6 + (8 − 1) d] = 4 (6 + 7d)

⇒ 192 = 24 + 28d

⇒ 168 = 28d ⇒ d = 6

(x) Given l = 28, S = 144, and there are total of 9 terms. Find a.

Applying formula, $S_n = \frac{n}{2}[a+l]$, to find sum of n terms, we get

$144 = \frac{9}{2}$ [a + 28]

⇒ 288 = 9 [a + 28]

⇒ 32 = a + 28⇒ a = 4

4. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

Ans. First term = a = 9, Common difference = d = 17 – 9 = 8, S_n = 636

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$636 = \frac{n}{2}$ [18 + (n − 1) (8)]

⇒ 1272 = n (18 + 8n − 8)

⇒ $1272 = 18n + 8n^2 - 8n$

⇒ $8n^2 + 10n - 1272 = 0$

⇒ $4n^2 + 5n - 636 = 0$

Comparing equation $4n^2 + 5n - 636 = 0$ with general form $an^2 + bn + c = 0$, we get

$a = 4$, $b = 5$ and $c = -636$

Applying quadratic formula, $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and putting values of a, b and c, we get

$$n = \frac{-5 \pm \sqrt{5^2 - 4(4)(-636)}}{8}$$

$$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8}$$

$$\Rightarrow n = \frac{-5 \pm 101}{8}$$

$$\Rightarrow n = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{106}{8}$$

We discard negative value of n here because n cannot be in negative, n can only be a positive integer.

Therefore, $n = 12$

Therefore, 12 terms of the given sequence make sum equal to 636.

---

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. First term = a = 5, Last term = l = 45, $S_n = 400$

Applying formula, $S_n = \frac{n}{2}[a + l]$ to find sum of n terms of AP, we get

$$400 = \frac{n}{2}[5 + 45]$$

$$\Rightarrow \frac{400}{50} = \frac{n}{2} \Rightarrow n = 16$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$$400 = \frac{16}{2}[10 + (16 - 1)d]$$

⇒ $400 = 8(10 + 15d)$

⇒ $400 = 80 + 120d$

⇒ $320 = 120d$

⇒ $d = \frac{320}{120} = \frac{8}{3}$

---

6. The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9, how many terms are there and what is their sum?

Ans. First term = a = 17, Last term = l = 350 and Common difference = d = 9

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression, we get

$350 = 17 + (n - 1)(9)$

⇒ $350 = 17 + 9n - 9$

⇒ $342 = 9n$ ⇒ $n = 38$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP and putting value of n, we get

$$S_{38} = \frac{38}{2}[34 + (38 - 1)9]$$

⇒ $S_{38} = 19(34 + 333) = 6973$

Therefore, there are 38 terms and their sum is equal to 6973.

---

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Ans. It is given that 22nd term is equal to 149 $\Rightarrow a_{22} = 149$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

149 = *a* + (22 − 1) (7)

⇒ 149 = *a* + 147⇒ *a* = 2

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of a, we get

$$S_{22} = \frac{22}{2}[4 + (22-1)7]$$

⇒ S_{22} = 11 (4 + 147)

⇒ S_{22} = 1661

Therefore, sum of first 22 terms of AP is equal to 1661.

---

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans. It is given that second and third term of AP are 14 and 18 respectively.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

14 = *a* + (2 − 1) *d*

⇒ 14 = *a* + *d* ... (1)

And, 18 = *a* + (3 − 1) *d*

⇒ 18 = *a* + 2*d* ... (2)

These are equations consisting of two variables.

Using equation (1), we get, $a = 14 - d$

Putting value of a in equation (2), we get

$18 = 14 - d + 2d$

$\Rightarrow d = 4$

Therefore, common difference $d = 4$

Putting value of d in equation (1), we get

$18 = a + 2\ (4)$

$\Rightarrow a = 10$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{51} = \frac{51}{2}[20 + (51-1)d] = \frac{51}{2}(20 + 200) = \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Therefore, sum of first 51 terms of an AP is equal to 5610.

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Ans. It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$49 = \frac{7}{2}[2a + (7-1)d]49$$

$\Rightarrow 98 = 7\ (2a + 6d)$

$\Rightarrow 7 = a + 3d \Rightarrow a = 7 - 3d$ ... (1)

And, $289 = \frac{17}{2}[2a+(17-1)d]$

$\Rightarrow 578 = 17\ (2a + 16d)$

$\Rightarrow 34 = 2a + 16d$

$\Rightarrow 17 = a + 8d$

Putting equation (1) in the above equation, we get

$17 = 7 - 3d + 8d$

$\Rightarrow 10 = 5d \Rightarrow d = 2$

Putting value of d in equation (1), we get

$a = 7 - 3d = 7 - 3\ (2) = 7 - 6 = 1$

Again applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{n}{2}[2(1)+(n-1)2]$$

$$\Rightarrow S_n = \frac{n}{2}[2+2n-2]$$

$$\Rightarrow S_n = \frac{n}{2} \times 2n \Rightarrow S_n = n^2$$

Therefore, sum of n terms of AP is equal to n^2.

---

10. Show that $a_1, a_2 \ldots a_n$ form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Ans. (i) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 3 + 4n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 3+4=7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3+12=15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

So, the sequence is of the form 7, 11, 15, 19 ...

Let us check difference between consecutive terms of this sequence.

11 – 7 = 4, 15 – 11 = 4, 19 – 15 = 4

Therefore, the difference between consecutive terms is constant which means terms a_1, a_2 ... a_n form an AP.

We have sequence 7, 11, 15, 19 ...

First term = a = 7 and Common difference = d = 4

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}\left[14+(15-1)4\right] = \frac{15}{2}(14+56) = \frac{15}{2}\times 70 = 15\times 35 = 525$$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 9 - 5n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 9 - 5n$

$a_1 = 9 - 5(1)$ = 9-5 = 4 $\quad a_2$= 9 – 5 (2) = 9 – 10 = −1

$a_3 = 9 - 5(3)$ = 9-15=-6 $\quad a_4$= 9 – 5 (4) = 9 – 20 = −11

So, the sequence is of the form 4, −1, −6, −11 ...

Let us check difference between consecutive terms of this sequence.

–1 – (4) = –5,–6 – (–1)

= –6 + 1 = –5,–11 – (–6)

= –11 + 6 = –5

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 ... a_n$ form an AP.

We have sequence 4, −1, −6, −11 ...

First term = a = 4 and Common difference = d = –5

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP , we get

$$S_{15} = \frac{15}{2}[8 + (15-1)(-5)] = \frac{15}{2}(8-70) = \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

Therefore, sum of first 15 terms of AP is equal to –465.

---

11. If the sum of the first n terms of an AP is $(4n - n^2)$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Ans. It is given that the sum of n terms of an AP is equal to $(4n - n^2)$

It means $S_n = 4n - n^2$

Let us calculate $S_1 and\ S_2$ using $S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

First term = a = S_1 = 3 … (1)

Let us find common difference now.

We can write any AP in the form of general terms like a , a + d, a + 2d …

We have calculated that sum of first two terms is equal to 4 i.e. $S_2 = 4$

Therefore, we can say that a + (a + d) = 4

Putting value of a from equation (1), we get

2a + d = 4

⇒ 2 (3) + d = 4

⇒ 6 + d = 4

⇒ d = –2

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

Second term of AP = a_2 = a + (2 – 1) d = 3 + (2 – 1) (–2) = 3 – 2 = 1

Third term of AP = a_3 = a + (3 – 1) d = 3 + (3 – 1) (–2) = 3 – 4 = –1

Tenth term of AP = a_{10} = a + (10 – 1) d = 3 + (10 – 1) (–2) = 3 – 18 = –15

n^{th} term of AP = a_n = a + (n – 1) d = 3 + (n – 1) (–2) = 3 – 2n + 2 = 5 – 2n

---

12. Find the sum of the first 40 positive integers divisible by 6.

Ans. The first 40 positive integers divisible by 6 are 6, 12, 18, 24 … 40 terms.

Therefore, we want to find sum of 40 terms of sequence of the form:

6, 12, 18, 24 ... 40 terms

Here, first term = a = 6 and Common difference = d = 12 – 6 = 6, n = 40

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{40} = \frac{40}{2}\left[12+(40-1)6\right]$$

$= 20\ (12\ +\ 39\ \times\ 6)$

= 20 (12 + 234)

$= 20\ \times\ 246\ =\ 4920$

---

13. Find the sum of the first 15 multiples of 8.

Ans. The first 15 multiples of 8 are 8,16, 24, 32 ... 15 terms

First term = a = 8 and Common difference = d = 16 – 8 = 8, n = 15

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}\left[16+(15-1)8\right] = \frac{15}{2}(16+14\times 8) = \frac{15}{2}(16+112) = \frac{15}{2}\times 128 = 15\times 64 = 960$$

---

14. Find the sum of the odd numbers between 0 and 50.

Ans. The odd numbers between 0 and 50 are 1, 3, 5, 7 ... 49

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a = 1, Common difference = 3 – 1 = 2, Last term = l = 49

We do not know how many odd numbers are present between 0 and 50.

Therefore, we need to find n first.

Using formula $a_n = a + (n - 1) d$, to find nth term of arithmetic progression, we get

$49 = 1 + (n - 1) 2$

$\Rightarrow 49 = 1 + 2n - 2$

$\Rightarrow 50 = 2n \Rightarrow n = 25$

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$$S_{25} = \frac{25}{2}(1 + 49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$$

---

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250

Penalty for third day = Rs 300

It is given that penalty for each succeeding day is Rs 50 more than the preceding day.

It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form200, 250, 300, 350 … 30 terms

First term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}[400 + (30-1)50]$$

$\Rightarrow S_n = 15\ (400\ +\ 29 \times 50)$

$\Rightarrow S_n$ = 15 (400 + 1450) = 27750

Therefore, penalty for 30 days is Rs. 27750.

---

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is R.s 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize =Rs (a − 20)

Let value of third prize = Rs (a − 40)

So, we have sequence of the form:

a, a − 20, a − 40, a – 60 ...

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a, Common difference = d = (a – 20) – a = –20

n = 7 (Because there are total of seven prizes)

$S_7 = Rs\ 700$ {given}

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_7 = \frac{7}{2}[2a + (7-1)(-20)]$$

$$\Rightarrow 700 = \frac{7}{2}[2a - 120]$$

⇒ 200 = 2a – 120

⇒ 320 = 2a ⇒ a = 160

Therefore, value of first prize = Rs 160

Value of second prize = 160 – 20 = Rs 140

Value of third prize = 140 – 20 = Rs 120

Value of fourth prize = 120 – 20 = Rs 100

Value of fifth prize = 100 – 20 = Rs 80

Value of sixth prize = 80 – 20 = Rs 60

Value of seventh prize = 60 – 20 = Rs 40

---

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections $\times 1 = 3 \times 1 = 3$

The number of trees planted by class II = number of sections $\times 2 = 3 \times 2 = 6$

The number of trees planted by class III = number of sections $\times 3 = 3 \times 3 = 9$

Therefore, we have sequence of the form 3, 6, 9 … 12 terms

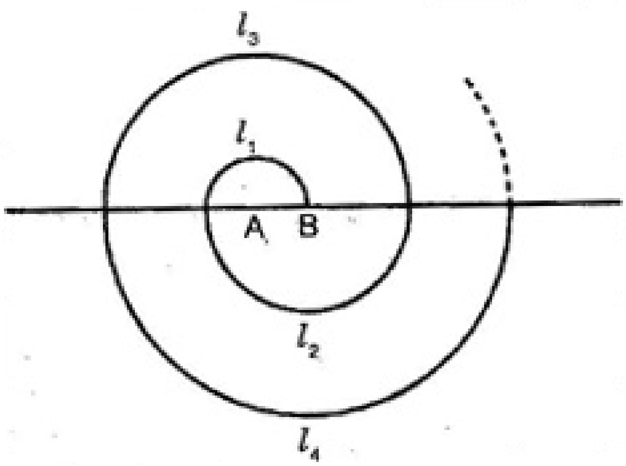
To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 … 12 terms.

First term = a = 3, Common difference = d= 6 – 3 = 3 and n = 12

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP , we get

$$S_{12} = \frac{12}{2}\left[6+(12-1)3\right] = 6(6+33) = 6\times 39 = 234$$

18. A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, … What is the total length of such a spiral made up of thirteen consecutive semicircles.



Ans. Length of semi–circle = $\dfrac{\text{Circumference of circle}}{2} = \dfrac{2\pi r}{2} = \pi r$

Length of semi-circle of radii 0.5 cm = π (0.5) cm

Length of semi-circle of radii 1.0 cm = π (1.0) cm

Length of semi-circle of radii 1.5 cm = π (1.5) cm

Therefore, we have sequence of the form:

π (0.5), π (1.0), π (1.5) … 13 terms {There are total of thirteen semi–circles}.

To find total length of the spiral, we need to find sum of the sequence π (0.5), π (1.0), π (1.5) … 13 terms

Total length of spiral = π (0.5) + π (1.0) + π (1.5) … 13 terms

⇒ Total length of spiral = π (0.5 + 1.0 + 1.5) … 13 terms … (1)

Sequence 0.5, 1.0, 1.5 … 13 terms is an arithmetic progression.

Let us find the sum of this sequence.

First term = a = 0.5, Common difference = 1.0 – 0.5 = 0.5 and n = 13

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{13} = \frac{13}{2}\left[1+(13-1)0.5\right] = 6.5(1+6) = 6.5\times 7 = 45.5$$

Therefore, 0.5 + 1.0 + 1.5 + 2.0 … 13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral= π (0.5 + 1.5 + 2.0 + … 13 terms) = π (45.5) = 143 cm

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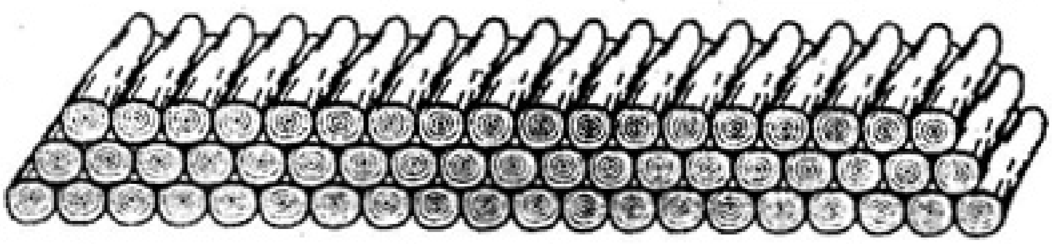
19. 200 logs are stacked in the following manner:20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = a = 20, Common difference = d = 19 – 20 = –1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$$200 = \frac{n}{2}[40+(n-1)(-1)]$$

⇒ 400 = n (40 – n + 1)

⇒ 400 = $40n - n^2 + n$

⇒ $n^2 - 41n + 400 = 0$

It is a quadratic equation, we can factorize to solve the equation.

⇒ n^2 − 25n − 16n + 400 = 0

⇒ n (n − 25) – 16 (n − 25) = 0

⇒ (n − 25) (n − 16)

⇒ n = 25, 16

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a+l)$ to find sum of n terms of AP, we get

200 = 8 (20 + l)

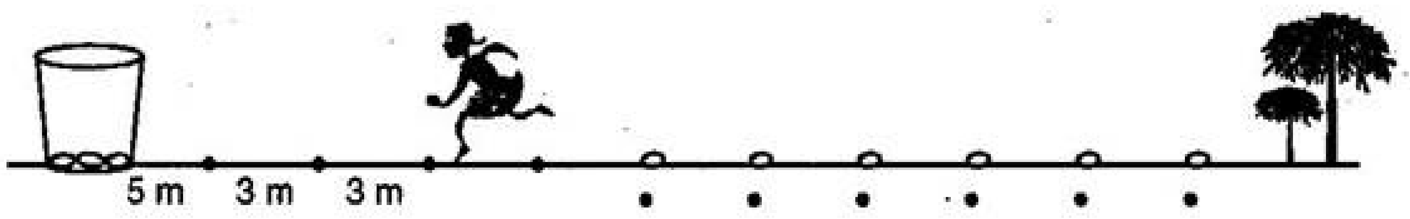
⇒ 200 = 160 + 8l

⇒ 40 = 8l ⇒ l = 5

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

---

20. In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pick up first potato and put it in bucket = 5×2 = 10 meters

The distance of Second potato from the starting point = 5 + 3 = 8 meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pick up 2nd potato and put it in bucket = 8×2 = 16 meters

The distance of third potato from the starting point = 8 + 3 = 11 meters

Therefore, the distance covered by competitor to pick up 3rd potato and put it in bucket = 11×2 = 22 meters

Therefore, we have a sequence of the form 10, 16, 22 … 10 terms

(There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find:

10 + 16 + 22 + … 10 terms

First term = a = 10, Common difference = d = 16 – 10 = 6

n = 10{There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{n10} = \frac{10}{2}\left[20+(10-1)6\right] = 5(20+54) = 5\times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 5

Arithmetic Progressions - Exercise 5.4

1. Which term of the AP: 121, 117, 113, ... is its first negative term?

Ans. Given: 121, 117, 113, ...

Here a=121, d=117-121=-4

Now, $a_n = a + (n-1)d$

= $121 + (n-1)(-4)$

= $121 - 4n + 4$ = $125 - 4n$

For the first negative term, $a_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow \frac{125}{4} < n$

$\Rightarrow 31\frac{1}{4} < n$

n is an integer and $n > 31\frac{1}{4}$.

Hence, the first negative term is 32^{nd} term.

2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of sixteen terms of the AP.

Ans. Let the AP be $a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, \ldots$

Then, $a_3 = a-2d$, $a_7 = a+2d$

$\Rightarrow a_3 + a_7 = a-2d+a+2d = 6$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$ ....(i)

Also $(a-2d)(a+2d) = 8$

$\Rightarrow a^2 - 4d^2 = 8$

$\Rightarrow 4d^2 = a^2 - 8$

$\Rightarrow 4d^2 = 3^2 - 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$

Taking $d = \frac{1}{2}$,

$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$

$= 8\left[2\times\left(3-4\times\frac{1}{2}\right)+15\times\frac{1}{2}\right]$

$= 8\left[2+\frac{15}{2}\right] = 8\times\frac{19}{2} = 76$

Taking $d = \frac{-1}{2}$,

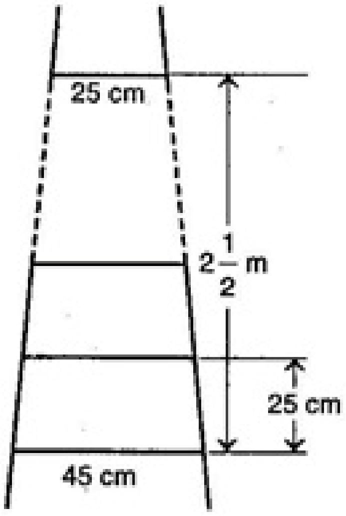
$$S_{16}=\frac{16}{2}\left[2\times(a-4d)+(16-1)\,d\right]$$

$$= 8\left[2\times\left(3-4\times\frac{-1}{2}\right)+15\times\frac{-1}{2}\right]$$

$$= 8\left[\frac{20-15}{2}\right] = 8\times\frac{5}{2} = 20$$

$\therefore S_{16} = 20$ and 76

3. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



Ans. Number of rungs $(n)=\dfrac{2\frac{1}{2}\times 100}{25}$ = 10

The length of the wood required for rungs = sum of 10 rungs

$$= \frac{10}{2}[25+45] = 5\times 70 = 350 \text{ cm}$$

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Ans. Here $a=1$ and $d=1$

$$\therefore S_{x-1} = \frac{x-1}{2}\left[2\times1+(x-1-1)\times1\right]$$

$$= \frac{x-1}{2}(2+x-2)$$

$$\frac{(x-1)x}{2} = \frac{x^2-x}{2}$$

$$S_x = \frac{x}{2}\left[2\times1+(x-1)\times1\right]$$

$$= \frac{x}{2}(x+1) = \frac{x^2+x}{2}$$

$$S_{49} = \frac{49}{2}\left[2\times1+(49-1)\times1\right]$$

$$= \frac{49}{2}(2+48) = 49 \text{ x } 25$$

According to question,

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \frac{x^2-x}{2} = 49\times25-\frac{x^2+x}{2}$$

$$\Rightarrow \frac{x^2-x}{2} + \frac{x^2+x}{2} = 49\times25$$

$$\Rightarrow \frac{x^2-x+x^2+x}{2} = 49\times25$$

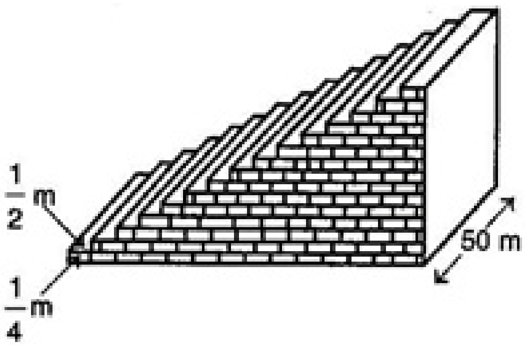
$$\Rightarrow x^2 = 49\times25$$

$$\Rightarrow x = \pm35$$

Since, x is a counting number, so negative value will be neglected.

$$\therefore x = 35$$

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.



Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, ... (in m^2) are

$$\frac{1}{4}\times\frac{1}{2}\times 50, \left(2\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \left(3\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \ldots$$

$$\Rightarrow \frac{50}{8}, 2\times\frac{50}{8}, 3\times\frac{50}{8}, \ldots\ldots$$

$\therefore$ Total volume of concrete required = $\frac{50}{8}+2\times\frac{50}{8}+3\times\frac{50}{8}+\ldots\ldots$

$= \frac{50}{8}[1+2+3+\ldots\ldots]$

$= \frac{50}{8}\times\frac{15}{2}\left[2\times 1+(15-1)\times 1\right]\left[\because n=15\right]$

$= \frac{50}{8}\times\frac{15}{2}\times 16$

$= 750\ m^3$

CBSE Class–10 Mathematics

NCERT solution

Chapter - 6

Triangles - Exercise 6.1

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1. Fill in the blanks using the correct word given in brackets:

(i) All circles are ______________. (congruent, similar)

(ii) All squares are ______________. (similar, congruent)

(iii) All ______________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ______________ and (b) their corresponding sides are ______________. (equal, proportional)

Ans. (i) similar

(ii) similar

(iii) equilateral

(iv) equal, proportional

---

2. Give two different examples of pair of:

(i) similar figures

(ii) non-similar figures

Ans. (i) Two different examples of a pair of similar figures are:

(a) Any two rectangles

(b) Any two squares

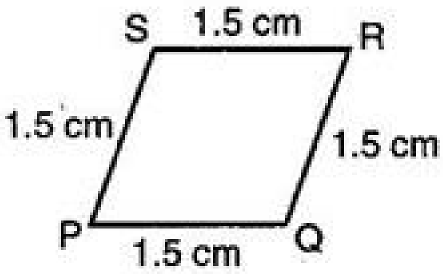
(ii) Two different examples of a pair of non-similar figures are:

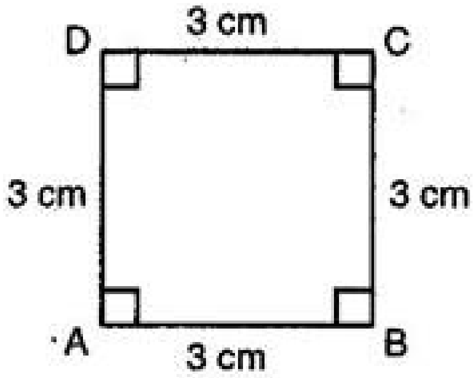
(a) A scalene and an equilateral triangle

(b) An equilateral triangle and a right angled triangle

---

3. State whether the following quadrilaterals are similar or not:





Ans. On looking at the given figures of the quadrilaterals, we can say that they are not similar because their angles are not equal.

CBSE Class–10 Mathematics

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Chapter - 6

Triangles - Exercise 6.2

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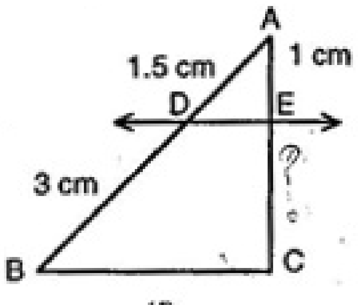
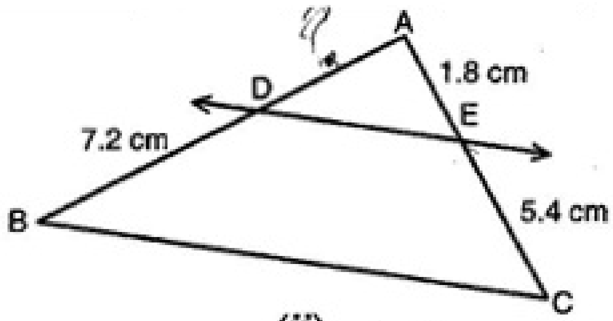
1. In figure (i) and (ii), DE $\parallel$ BC. Find EC in (i) and AD in (ii).



(i)



(ii)

Ans. (i) Since DE $\parallel$ BC,

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow EC = \frac{3}{1.5}$$

$\Rightarrow$ EC = 2 cm

(ii) Since DE $\parallel$ BC,

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$\Rightarrow$ AD = $\dfrac{1.8 \times 7.2}{5.4}$

$\Rightarrow$ EC = 2.4 cm

---

2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF $\|$ QR:

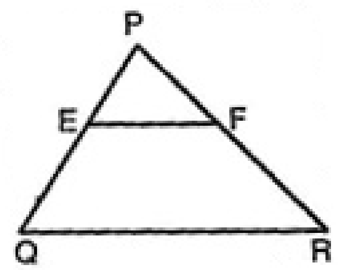
(i) PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Ans. (i) Given: PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm

Now, $\dfrac{PE}{EQ} = \dfrac{3.9}{4}$ = 0.97 cm



And $\dfrac{PF}{FR} = \dfrac{3.6}{2.4}$ = 1.2 cm

$\because \dfrac{PE}{EQ} \neq \dfrac{PF}{FR}$

Therefore, EF does not divide the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is not parallel to QR.

(ii) Given: PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Now, $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ cm

And $\frac{PF}{FR} = \frac{8}{9}$ cm

$\because \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is parallel to QR.

(iii) Given: PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

$\Rightarrow$ EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm

And ER = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$ cm

And $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ cm

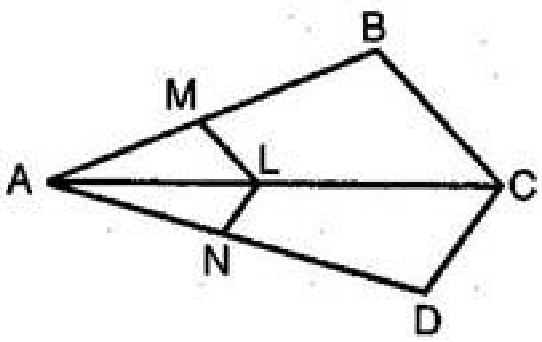
$\because \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is parallel to QR.

---

3. In figure, if LM $\parallel$ CB and LN $\parallel$ CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Ans. In $\triangle ABC$, $LM \parallel CB$

$\therefore \dfrac{AM}{AB} = \dfrac{AL}{AC}$ [Basic Proportionality theorem] ....(i)

And in $\triangle ACD$, $LN \parallel CD$

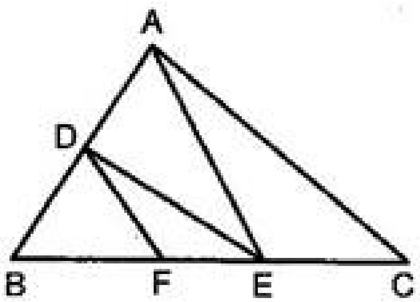
$\therefore \dfrac{AL}{AC} = \dfrac{AN}{AD}$ [Basic Proportionality theorem] ....(ii)

From eq. (i) and (ii), we have

$$\frac{AM}{AB} = \frac{AN}{AD}$$

---

4. In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\dfrac{BF}{FE} = \dfrac{BE}{EC}$.



Ans. In $\triangle BCA$, $DE \parallel AC$

$\therefore \dfrac{BE}{EC} = \dfrac{BD}{DA}$ [Basic Proportionality theorem] ....(i)

And in $\triangle BEA$, $DF \parallel AE$

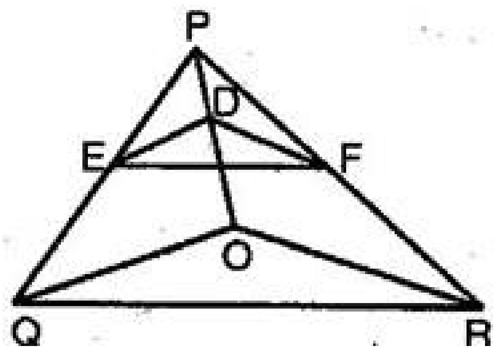
$\therefore \frac{BE}{FE} = \frac{BD}{DA}$ [Basic Proportionality theorem] ....(ii)

From eq. (i) and (ii), we have

$$\frac{BF}{FE} = \frac{BE}{EC}$$

---

5. In the given figure, DE $\parallel$ OQ and DF $\parallel$ OR. Show that EF $\parallel$ QR.



Ans. In ΔPQO, DE $\parallel$ OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ [Basic Proportionality theorem] ....(i)

And in ΔPOR, DF $\parallel$ OR

$\therefore \frac{PD}{DO} = \frac{PF}{FR}$ [Basic Proportionality theorem] ....(ii)

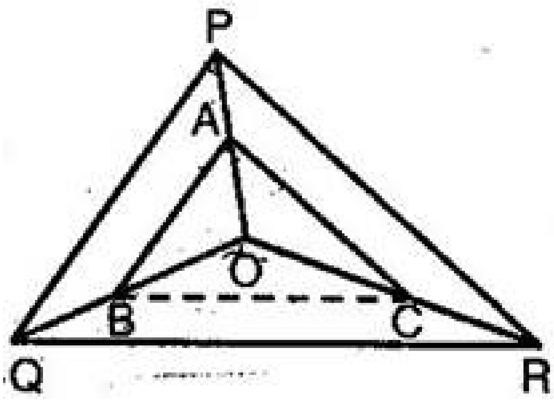
From eq. (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore$ EF $\parallel$ QR [By the converse of BPT]

---

6. In the given figure, A, B, and C are points on OP, OQ and OR respectively such that AB $\parallel$ PQ and AC $\parallel$ PR. Show that BC $\parallel$ QR.



Ans. Given: O is any point in $\triangle$PQR, in which AB $\parallel$ PQ and AC $\parallel$ PR.

To prove: BC $\parallel$ QR

Construction: Join BC.

Proof: In $\triangle$OPQ, AB $\parallel$ PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ [Basic Proportionality theorem] ....(i)

And in $\triangle$OPR, AC $\parallel$ PR

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ [Basic Proportionality theorem] ....(ii)

From eq. (i) and (ii), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$\because$ In $\triangle$OQR, B and C are points dividing the sides OQ and OR in the same ratio.

$\therefore$ By the converse of Basic Proportionality theorem,

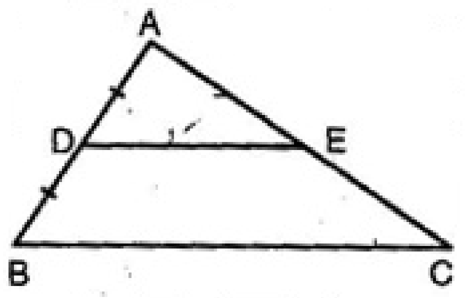
$\Rightarrow$ BC $\parallel$ QR

---

7. Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Ans. Given: A triangle ABC, in which D is the midpoint of side AB

and the line DE is drawn parallel to BC, meeting AC at E.



To prove: AE = EC

Proof: Since DE $\parallel$ BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Basic Proportionality theorem] ....(i)

But AD = DB [Given]

$\Rightarrow \frac{AD}{DB} = 1$

$\Rightarrow \frac{AE}{EC} = 1$ [From eq. (i)]

$\Rightarrow$ AE = EC

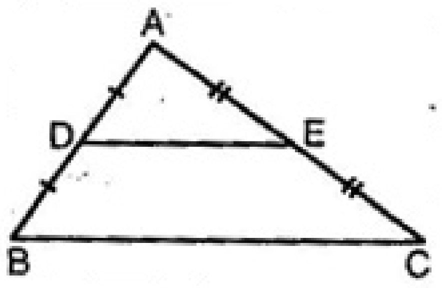
Hence, E is the midpoint of the third side AC.

---

8. Using Theorem 6.2, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Ans. Given: A triangle ABC, in which D and E are the midpoints of

sides AB and AC respectively.



To Prove: DE || BC

Proof: Since D and E are the midpoints of AB and AC

respectively.

∴ AD = DB and AE = EC

Now, AD = DB

$\Rightarrow \dfrac{AD}{DB} = 1$ and AE = EC

$\Rightarrow \dfrac{AE}{EC} = 1$

$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC} = 1$

$\Rightarrow \dfrac{AD}{DB} = \dfrac{AE}{EC}$

Thus, in triangle ABC, D and E are points dividing the sides AB and AC in the same ratio.

Therefore, by the converse of Basic Proportionality theorem, we have

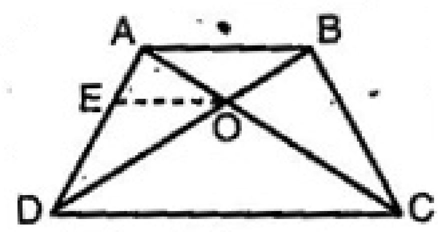
DE || BC

---

9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\dfrac{AO}{BO} = \dfrac{CO}{DO}$.

Ans. Given: A trapezium ABCD, in which AB $\parallel$ DC and its diagonals

AC and BD intersect each other at O.



To Prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE $\parallel$ AB, i.e. OE $\parallel$ DC.

Proof: In ΔADC, we have OE $\parallel$ DC

$\therefore \frac{AE}{ED} = \frac{AO}{CO}$ [By Basic Proportionality theorem]..........(i)

Again, in ΔABD, we have OE $\parallel$ AB[Construction]

$\therefore \frac{ED}{AE} = \frac{DO}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{AE}{ED} = \frac{BO}{DO}$ ....(ii)

From eq. (i) and (ii), we get

$$\frac{AO}{CO} = \frac{BO}{DO}$$

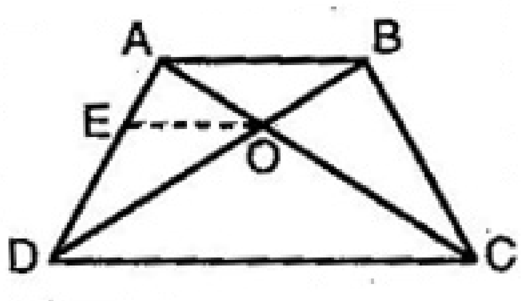
$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

---

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. **Show that ABCD is a trapezium.**

Ans. Given: A quadrilateral ABCD, in which its diagonals AC and

BD intersect each other at O such that $\frac{AO}{BO}=\frac{CO}{DO}$, i.e.



$\frac{AO}{CO}=\frac{BO}{DO}$.

To Prove: Quadrilateral ABCD is a trapezium.

Construction: Through O, draw OE $\parallel$ AB meeting AD at E.

Proof: In $\triangle$ADB, we have OE $\parallel$ AB [By construction]

$\therefore \frac{DE}{EA}=\frac{OD}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{EA}{DE}=\frac{BO}{DO}$

$\Rightarrow \frac{EA}{DE}=\frac{BO}{DO}=\frac{AO}{CO}$

$\left[\because \frac{AO}{CO}=\frac{BO}{DO}\right]$

$\Rightarrow \frac{EA}{DE}=\frac{AO}{CO}$

Thus in $\triangle$ADC, E and O are points dividing the sides AD and AC in the same ratio. Therefore by the converse of Basic Proportionality theorem, we have

EO $\parallel$ DC

But EO $\parallel$ AB[By construction]

$\therefore$ AB $\parallel$ D

$\therefore$ Quadrilateral ABCD is a trapezium

CBSE Class–10 Mathematics

NCERT solution

Chapter - 6

Triangles - Exercise 6.3

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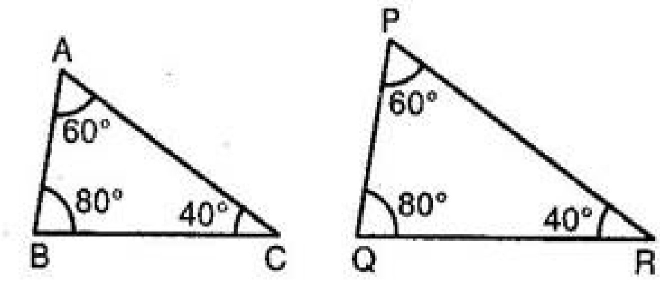
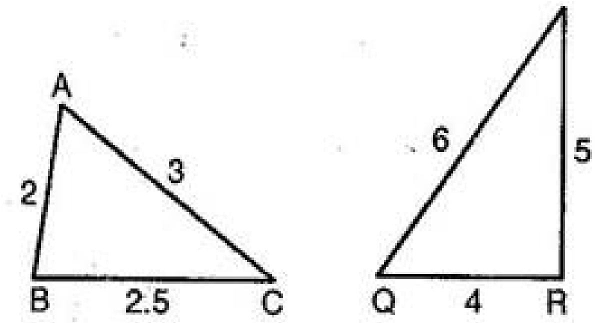
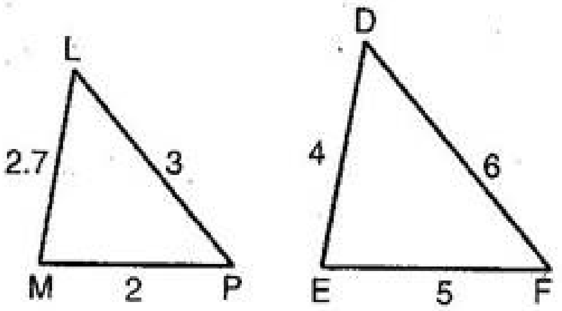
1. State which pairs of triangles in the given figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



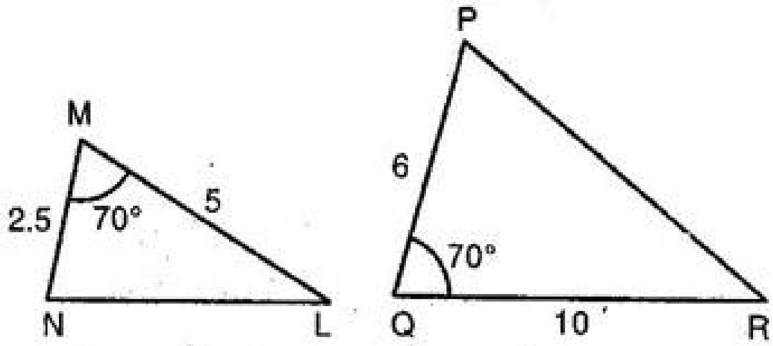
(i)



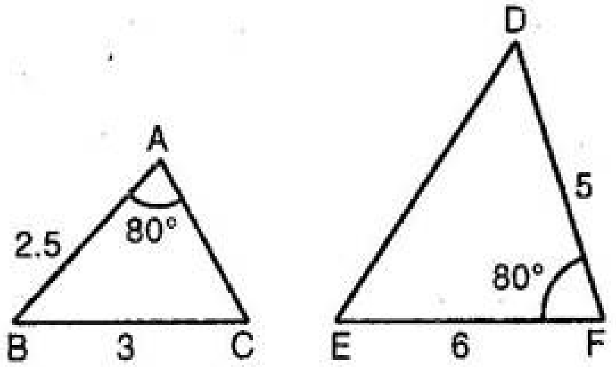
(ii)



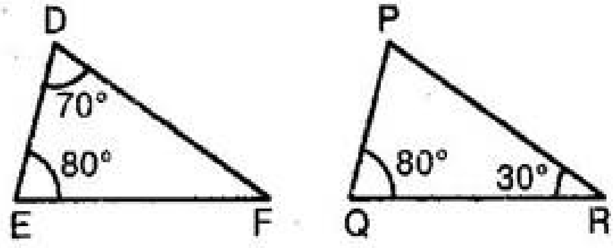
(iii)



(iv)



(v)



(vi)

Ans. (i) In Δs ABC and PQR, we observe that,

$\angle A = \angle P = 60^\circ$, $\angle B = \angle Q = 80^\circ$ and $\angle C = \angle R = 40^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta ABC \sim \Delta PQR$

(ii) In Δs ABC and PQR, we observe that,

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ} = \frac{1}{2}$$

$\therefore$ By SSS criterion of similarity, $\Delta ABC \sim \Delta PQR$

(iii) In Δs LMP and DEF, we observe that the ratio of the sides of these triangles is not equal.

Therefore, these two triangles are not similar.

(iv) In Δs MNL and QPR, we observe that, $\angle M = \angle Q = 70^\circ$

But, $\frac{MN}{PQ} \neq \frac{ML}{QR}$

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

(v) In Δs ABC and FDE, we have, $\angle A = \angle F = 80^\circ$

But, $\frac{AB}{DE} \neq \frac{AC}{DF}$ [$\because$ AC is not given]

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

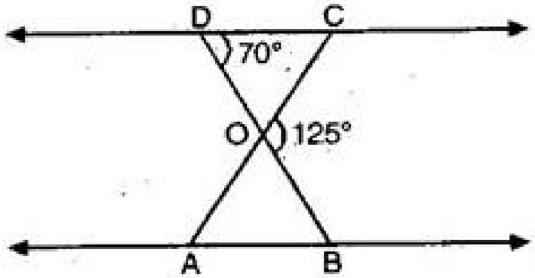
(vi) In Δs DEF and PQR, we have, $\angle D = \angle P = 70^\circ$

[$\because$ $\angle P = 180^\circ - 80^\circ - 30^\circ = 70^\circ$]

And $\angle E = \angle Q = 80^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta DEF \sim \Delta PQR$

2. In figure, Δ ODC $\sim$ Δ OBA, $\angle$ BOC = $125°$ and $\angle$ CDO = $70°$. Find $\angle$ DOC, $\angle$ DCO and $\angle$ OAB.



Ans. Since BD is a line and OC is a ray on it.

$\therefore \ \angle DOC + \angle BOC = 180°$

$\Rightarrow \ \angle DOC + 125° = 180°$

$\Rightarrow \ \angle DOC = 55°$

In Δ CDO, we have $\angle CDO + \angle DOC + \angle DCO = 180°$

$\Rightarrow \ 70° + 55° + \angle DCO = 180°$

$\Rightarrow \ \angle DCO = 55°$

It is given that Δ ODC $\sim$ Δ OBA

$\therefore \ \angle OBA = \angle ODC, \angle OAB = \angle OCD$

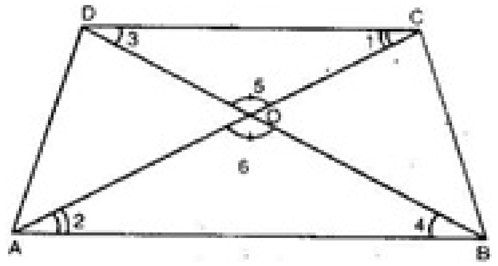
$\Rightarrow \ \angle OBA = 70°, \angle OAB = 55°$

Hence $\angle$ DOC = $55°$, $\angle$ DCO = $55°$ and $\angle$ OAB = $55°$

---

3. Diagonals AC and BD of a trapezium ABCD with AB || CD intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Ans. Given: ABCD is a trapezium in which AB $\|$ DC.



To Prove: $\frac{OA}{OC}=\frac{OB}{OD}$

Proof: In Δs OAB and OCD, we have,

$\angle 5 = \angle 6$ [Vertically opposite angles]

$\angle 1 = \angle 2$ [Alternate angles]

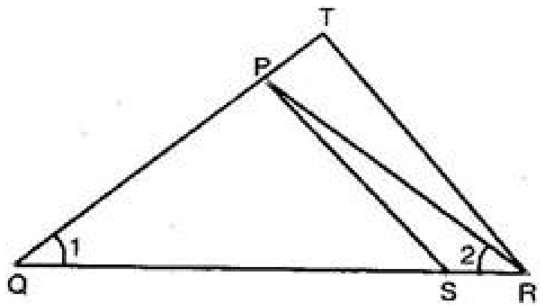
And $\angle 3 = \angle 4$ [Alternate angles]

$\therefore$ By AAA criterion of similarity, $\Delta OAB \sim \Delta ODC$

Hence, $\frac{OA}{OC}=\frac{OB}{OD}$

---

4. In figure, $\frac{QR}{QS}=\frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Ans. We have, $\frac{QR}{QS}=\frac{QT}{PR}$

$\Rightarrow \frac{QT}{QR}=\frac{PR}{QS}$ ....(1)

Also, $\angle 1 = \angle 2$ [Given]

$\therefore$ PR = PQ ....(2) [$\because$ Sides opposite to equal $\angle$s are equal]

From eq.(1) and (2), we get

$$\frac{QT}{QR} = \frac{PR}{QS} \Rightarrow \frac{PQ}{QT} = \frac{QS}{QR}$$

In Δs PQS and TQR, we have,

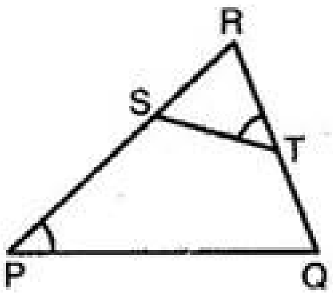
$\frac{PQ}{QT} = \frac{QS}{QR}$ and $\angle$PQS = $\angle$TQR = $\angle$Q

$\therefore$ By SAS criterion of similarity, ΔPQS $\sim$ ΔTQR

---

5. S and T are points on sides PR and QR of a ΔPQR such that $\angle$P = $\angle$RTS. Show that ΔRPQ $\sim$ ΔRTS.

Ans. In Δs RPQ and RTS, we have



$\angle$RPQ = $\angle$RTS [Given]

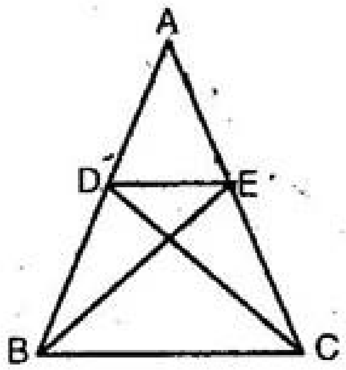
$\angle$PRQ = $\angle$TRS [Common]

$\therefore$ By AA-criterion of similarity,

ΔRPQ $\sim$ ΔRTS

---

6. In the given figure, if ΔABE $\cong$ ΔACD, show that ΔADE $\sim$ ΔABC.



Ans. It is given that $\triangle ABE \cong \triangle ACD$

$\therefore$ AB = AC and AE = AD

$\Rightarrow \dfrac{AB}{AD} = \dfrac{AC}{AE}$

$\Rightarrow \dfrac{AB}{AC} = \dfrac{AD}{AE}$ ....(1)

$\therefore$ In $\triangle$s ADE and ABC, we have,

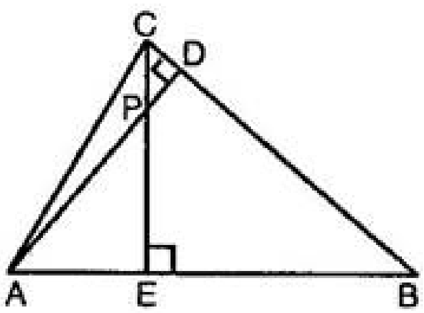
$\dfrac{AB}{AC} = \dfrac{AD}{AE}$ [from eq.(1)]

And $\angle$BAC = $\angle$DAE [Common]

Thus, by SAS criterion of similarity, $\triangle ADE \sim \triangle ABC$

---

7. In figure, altitude AD and CE of a $\triangle$ABC intersect each other at the point P. Show that:



(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle$ **ABD** $\sim$ $\triangle$ **CBE**

(iii) $\triangle$ **AEP** $\sim$ $\triangle$ **ADB**

(iv) $\triangle$ **PDC** $\sim$ $\triangle$ **BEC**

Ans. (i) In $\triangle$s AEP and CDP, we have,

$\angle$ AEP = $\angle$ CDP = 90° [$\because$ CE $\perp$ AB, AD $\perp$ BC]

And $\angle$ APE = $\angle$ CPD[Vertically opposite]

$\therefore$ By AA-criterion of similarity, $\triangle$AEP $\sim$ $\triangle$ CDP

(ii) In $\triangle$s ABD and CBE, we have,

$\angle$ ADB = $\angle$ CEB = 90°

And $\angle$ ABD = $\angle$ CBE[Common]

$\therefore$ By AA-criterion of similarity, $\triangle$ABD $\sim$ $\triangle$ CBE

(iii) In $\triangle$s AEP and ADB, we have,

$\angle$ AEP = $\angle$ ADB = 90° [$\because$ AD $\perp$ BC, CE $\perp$ AB]

And $\angle$ PAE = $\angle$ DAB[Common]

$\therefore$ By AA-criterion of similarity, $\triangle$AEP $\sim$ $\triangle$ ADB

(iv) In $\triangle$s PDC and BEC, we have,

$\angle$ PDC = $\angle$ BEC = 90° [$\because$ CE $\perp$ AB, AD $\perp$ BC]

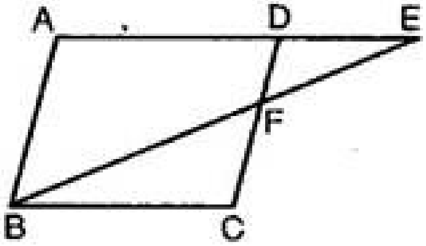
And $\angle$ PCD = $\angle$ BEC[Common]

$\therefore$ By AA-criterion of similarity, $\triangle$PDC $\sim$ $\triangle$ BEC

---

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle$ ABE $\sim$ $\triangle$ CFB.

Ans. In $\triangle$s ABE and CFB, we have,



$\angle$ AEB = $\angle$ CBF[Alt. $\angle$ s]

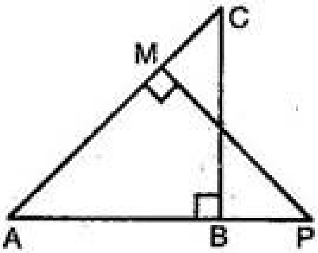
$\angle$ A = $\angle$ C [opp. $\angle$ s of a $\parallel$ gm]

$\therefore$ By AA-criterion of similarity, we have

$\triangle$ABE $\sim$ $\triangle$CFB

---

9. In the given figure, ABC and AMP are two right triangles, right angles at B and M respectively. Prove that:



(i) $\triangle$**ABC** $\sim$ $\triangle$ **AMP**

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Ans. (i) In $\triangle$s ABC and AMP, we have,

$\angle$ ABC = $\angle$ AMP = 90° [Given]

$\angle$ BAC = $\angle$ MAP [Common angles]

$\therefore$ By AA-criterion of similarity, we have

$\triangle$ABC $\sim$ $\triangle$AMP

(ii) We have $\Delta ABC \sim \Delta AMP$ [As prove above]

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$$

---

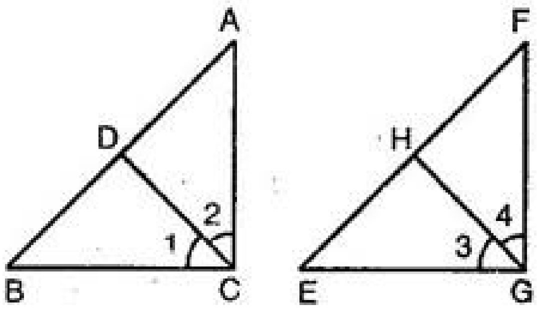
10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE at ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Δ **DCB** $\sim \Delta$ **HE**

(iii) Δ **DCA** $\sim \Delta$ **HGF**

Ans. We have, $\Delta ABC \sim \Delta FEG$



$\Rightarrow \angle A = \angle F$.........(1)

And $\angle C = \angle G$

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \angle 1 = \angle 3$ and $\angle 2 = \angle 4$ ....(2)

[$\because$ CD and GH are bisectors of $\angle C$ and $\angle G$

respectively]

$\therefore$ In Δs DCA and HGF, we have

$\angle A = \angle F$[From eq.(1)]

$\angle 2 = \angle 4$[From eq.(2)]

$\therefore$ By AA-criterion of similarity, we have

$\triangle DCA \sim \triangle HGF$

Which proves the (iii) part

We have, $\triangle DCA \sim \triangle HGF$

$$\Rightarrow \frac{AG}{FG} = \frac{CD}{GH}$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

Which proves the (i) part

In $\triangle$s DCA and HGF, we have

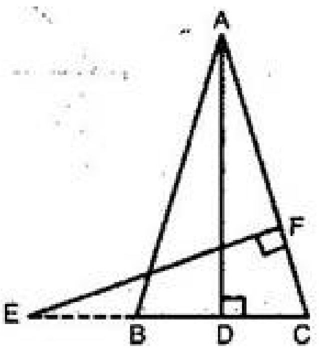
$\angle 1 = \angle 3$[From eq.(2)]

$\angle B = \angle E$[$\because \triangle DCB \sim \triangle HE$]

Which proves the (ii) part

---

11. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that $\triangle ABD \sim \triangle ECF$.



Ans. Here $\triangle ABC$ is isosceles with AB = AC

$\therefore \angle B = \angle C$

In $\triangle$s ABD and ECF, we have

$\angle$ ABD = $\angle$ ECF[$\because$ $\angle$ B = $\angle$ C]

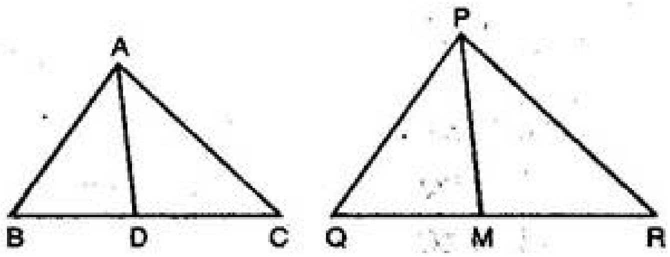
$\angle$ ABD = $\angle$ ECF = 90° [$\because$ AD $\perp$ BC and EF $\perp$ AC]

$\therefore$ By AA-criterion of similarity, we have

$\triangle$ABD $\sim$ $\triangle$ECF

---

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of a $\triangle$PQR (see figure). Show that $\triangle$ABC $\sim$ $\triangle$PQR.



Ans. Given: AD is the median of $\triangle$ ABC and PM is the median of $\triangle$ PQR such that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To prove: $\triangle$ ABC $\sim$ $\triangle$ PQR

Proof: BD = $\frac{1}{2}$ BC [Given]

And QM = $\frac{1}{2}$ QR [Given]

Also $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\therefore \triangle ABD \sim \triangle PQM$[By SSS-criterion of similarity]

$\Rightarrow \angle B = \angle Q$[Similar triangles have corresponding angles equal]

And $\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

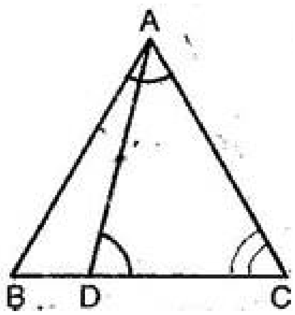
$\therefore$ By SAS-criterion of similarity, we have

$\triangle ABC \sim \triangle PQR$

---

13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

ANS. In triangles ABC and DAC,



$\angle ADC = \angle BAC$ [Given]

and $\angle C = \angle C$[Common]

$\therefore$ By AA-similarity criterion,

$\triangle ABC \sim \triangle DAC$

$$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$

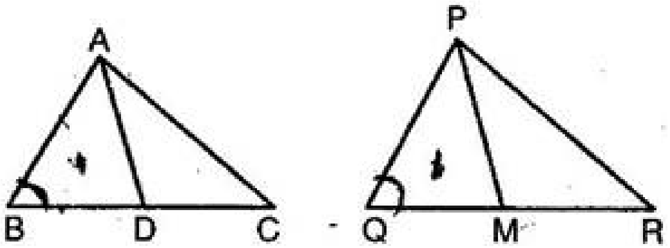
$$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$$

$\Rightarrow CA^2$ = CB.CD

---

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle$ABC $\sim$ $\triangle$PQR.

ANS. Given: AD is the median of $\triangle$ ABC and PM is the median of $\triangle$ PQR such that

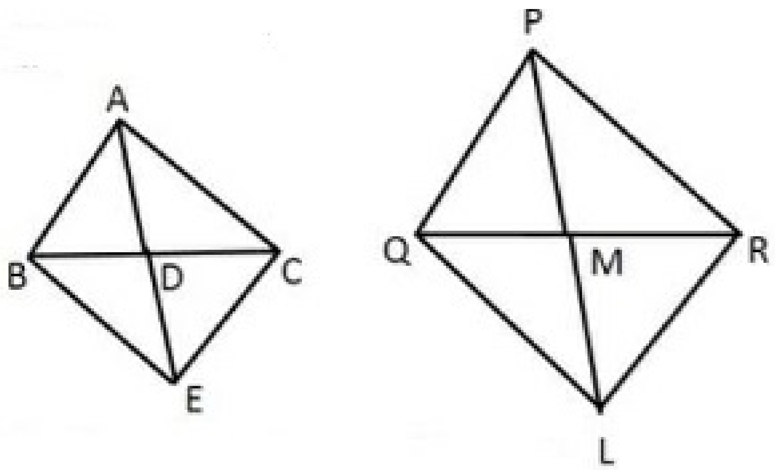


$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \ldots\ldots\ldots(1)$$

To prove: $\triangle$ ABC $\sim$ $\triangle$ PQR

Proof:

Let us extend AD to point D such that AD = DE and PM up to point L such that PM = ML



Join B to E. C to E,and Q to L, and R to L
We know that medians is the bisector of opposite side
Hence
BD = DC
Also, AD = DE (by construction)

Hence in quadrilateral ABEC, diagonals AE and BC bisect each other at point D.
Therefore, quadrilateral ABEC is a parallelogram.

AC = BE

AB = EC (opposite sides of | |gm are equal) .... (2)

Similarly, we can prove that PQLR is a parallelogram

PR = QL

PQ = LR opposite sides of | |gm are equal) .... (3)

Given that

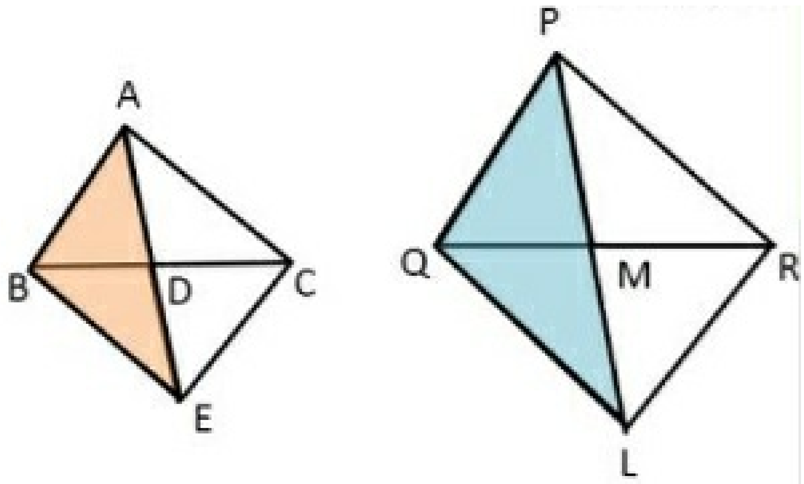
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AD}{PM}$ [from (2) (3)]

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$ $[as\ AD = DE, AE = AD + DE = 2AD$
$PM = ML. PL = PM + ML = 2PM$

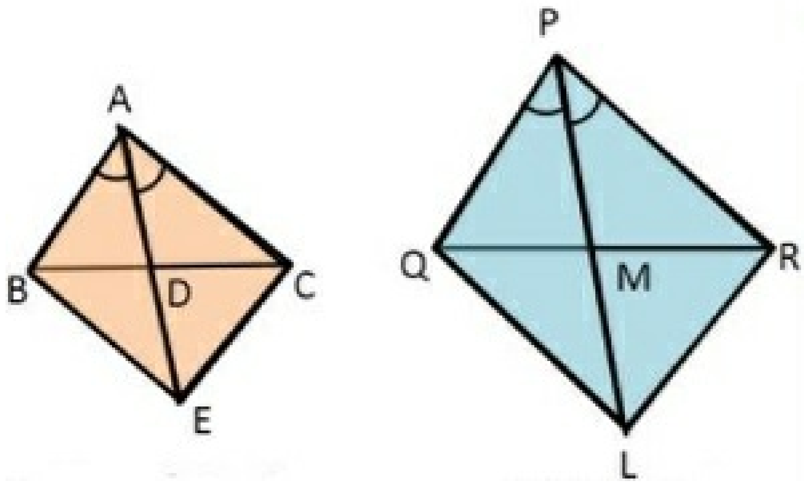
$\triangle ABE \sim \triangle PQL$ (By SSS Similiarity Criteria)



We know that corresponding angles of similar triangles are equal.

$\angle BAE = \angle QPL$ (4)

Similarly, we can prove that $\triangle AEC \sim \triangle PLR$.



We know that corresponding angles of similar triangles are equal.

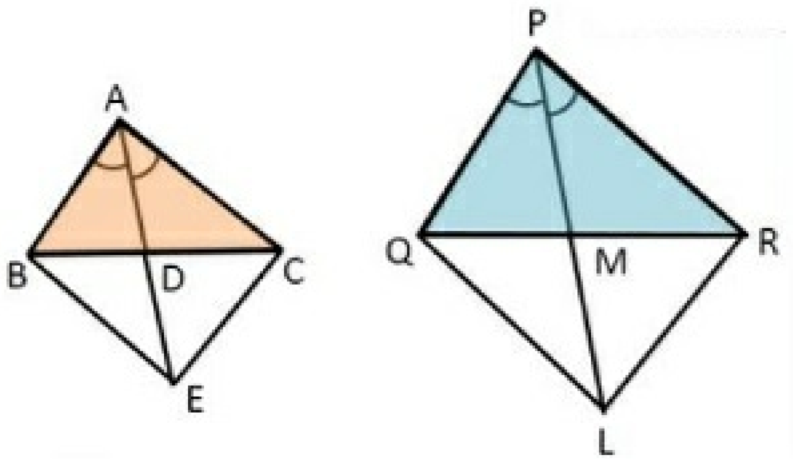
$\angle CAE = \angle RPL$ (5)

Adding (4) and (5),

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\angle CAB = \angle RPQ$

$In\ \triangle ABC\ and\ \triangle PQR,$



$\frac{AB}{PQ} = \frac{AC}{PR}$

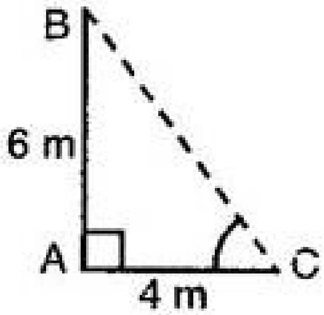
$\angle CAB = \angle RPQ$

$\triangle ABC \sim \triangle PQR$

Hence proved

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Ans. Let AB the vertical pole and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Joined BC and EF.



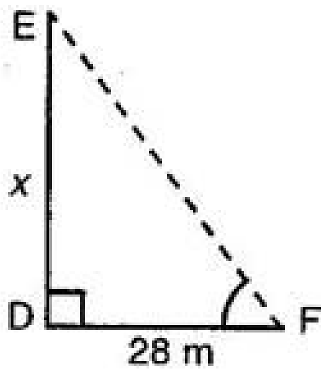
Let DE = x meters

Here, AB = 6 m, AC = 4 m and DF = 28 m

In the triangles ABC and DEF,

$\angle A = \angle D = 90^\circ$

And $\angle C = \angle F$[Each is the angular elevation of the sun]



$\therefore$ By AA-similarity criterion,

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{6}{x} = \frac{4}{28}$

$\Rightarrow \frac{6}{x} = \frac{1}{7}$

$\Rightarrow x = 42$ m

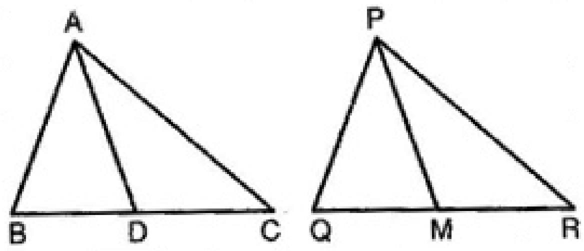
Hence, the height of the tower is 42 meters.

---

16. If AD and PM are medians of triangles ABC and PQR respectively, where $\Delta ABC \sim \Delta$ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Ans. Given: AD and PM are the medians of triangles

ABC and PQR respectively, where



$\Delta ABC \sim \Delta PQR$

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof: In triangles ABD and PQM,

$\angle B = \angle Q$ [Given]

And $\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$ [$\because$ AD and PM are the medians of BC and QR respectively]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

$\therefore$ By SAS-criterion of similarity,

$\Delta ABD \sim \Delta PQM$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

1. Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Ans. We have, $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC^2}{EF^2}$

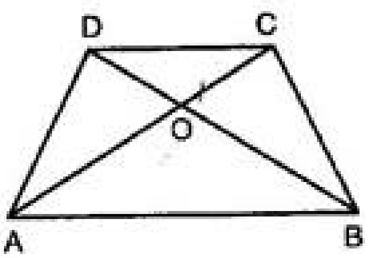
$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$\Rightarrow$ BC = $\left(\frac{8}{11} \times 15.4\right)$ cm = 11.2 cm

2. Diagonals of a trapezium ABCD with AB $\parallel$ DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Ans. In Δs AOB and COD, we have,



$\angle$ AOB = $\angle$ COD[Vertically opposite angles]

$\angle$ OAB = $\angle$ OCD[Alternate angles]

By AA-criterion of similarity,

$\therefore\ \Delta AOB \sim \Delta COD$

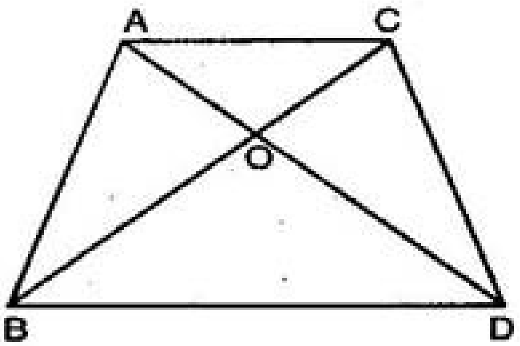
$$\therefore\ \frac{\text{Area}\ (\Delta AOB)}{\text{Area}\ (\Delta COD)} = \frac{AB^2}{DC^2}$$

$$\Rightarrow \frac{\text{Area}\ (\Delta AOB)}{\text{Area}\ (\Delta COD)} = \frac{(2DC)^2}{DC^2} = \frac{4}{1}$$

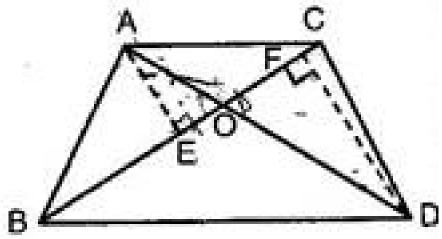
Hence, Area (Δ AOB) : Area (Δ COD) = 4 : 1

---

3. In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Ans. Given: Two Δ s ABC and DBC which stand on the same base but on the opposite sides of BC.



To Prove: $\frac{\text{Area}\ (\Delta ABC)}{\text{Area}\ (\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw AE $\perp$ BC and DF $\perp$ BC.

Proof: In Δs AOE and DOF, we have, $\angle$AEO = $\angle$DFO = 90°

and $\angle$AOE = $\angle$DOF[Vertically opposite)

$\therefore$ ΔAOE ~ ΔDOF[By AA-criterion]

$\therefore$ $\frac{AE}{DF} = \frac{AO}{OD}$ ....(i)

Now, $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$

$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \frac{AE}{DF}$

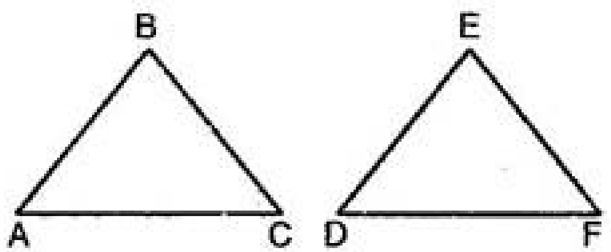
$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \frac{AO}{OD}$ [using eq. (i)]

---

4. If the areas of two similar triangles are equal, prove that they are congruent.

Ans. Given: Two Δs ABC and DEF such that ΔABC ~ ΔDEF

And Area(ΔABC) = Area (ΔDEF)



To Prove: ΔABC $\cong$ ΔDEF

Proof: ΔABC ~ ΔDEF

$\therefore$ $\angle$A = $\angle$D, $\angle$B = $\angle$E, $\angle$C = $\angle$F

And $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

To establish $\Delta ABC \cong \Delta DEF$, it is sufficient to prove that, AB = DE, BC = EF and AC = DF

Now, Area(Δ ABC) = Area (Δ DEF)

$\therefore \frac{\text{Area}\ (\Delta ABC)}{\text{Area}\ (\Delta DEF)} = 1$

$\Rightarrow \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$

$\Rightarrow$ AB = DE, BC = EF, AC = DF

Hence, $\Delta ABC \cong \Delta DEF$

---

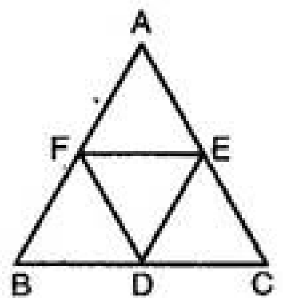
5. D, E and F are respectively the midpoints of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Ans. Since D and E are the midpoints of the sides BC and CA of ΔABC respectively.

$\therefore$ DE $\|$ BA $\Rightarrow$ DE $\|$ FA ....(i)

Since D and F are the midpoints of the sides BC and AB of ΔABC respectively.

$\therefore$ DF $\|$ CA $\Rightarrow$ DE $\|$ AE ....(ii)



From (i) and (ii), we can say that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in Δs DEF and ABC, we have

$\angle$FDE = $\angle$A[opposite angles of $\|$gm AFDE]

And $\angle$DEF = $\angle$B[opposite angles of $\|$gm BDEF]

$\therefore$ By AA-criterion of similarity, we have $\Delta DEF \sim \Delta ABC$

$$\Rightarrow \frac{\text{Area }(\Delta DEF)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{AB^2} = \frac{\left(\frac{1}{2}AB\right)^2}{AB^2}\frac{1}{4}$$

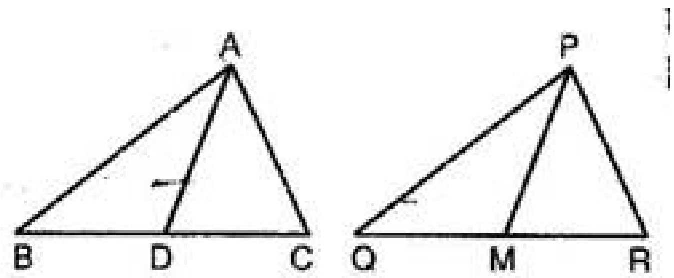
[$\because$ DE = $\frac{1}{2}$AB]

Hence, Area (ΔDEF): Area (ΔABC) = 1 : 4

---

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Ans. Given: $\Delta ABC \sim \Delta PQR$, AD and PM are the medians of Δs ABC and PQR respectively.



To Prove: $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AB^2}{PQ^2} \text{ ....(1)}$$

But, $\frac{AB}{PQ} = \frac{AD}{PM}$ ....(2)

$\therefore$ From eq. (1) and (2), we have,

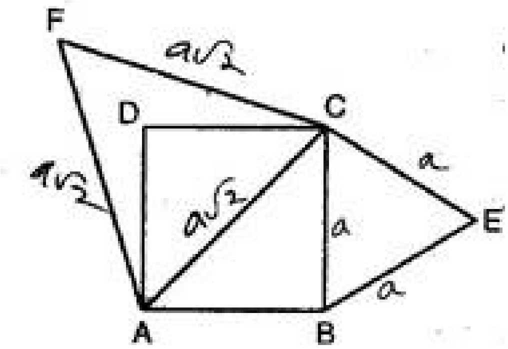
$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AD^2}{PM^2}$$

---

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of the diagonals.

Ans. Given: A square ABCD,

Equilateral Δs BCE and ACF have been drawn on side BC and the diagonal AC respectively.



To Prove: Area (Δ BCE) = $\frac{1}{2}$ Area (Δ ACF)

Proof: $\Delta BCE \sim \Delta ACF$

[Being equilateral so similar by AAA criterion of

similarity]

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{\left(\sqrt{2}BC\right)^2}$$

[$\because$ Diagonal = $\sqrt{2}$ side $\Rightarrow$ AC = $\sqrt{2}$ BC]

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{1}{2}$$

Tick the correct answer and justify:

8. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. The ratio of the areas of triangles ABC and BDE is:

(A) 2: 1

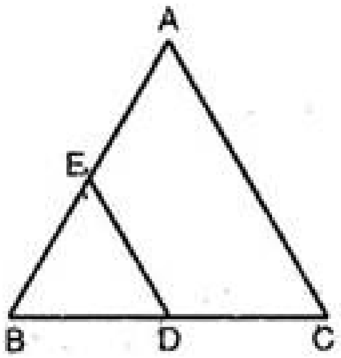
(B) 1: 2

(C) 4: 1

(D) 1: 4

Ans. (C) Since ΔABC and ΔBDE are equilateral, they are equiangular and hence,

$\Delta ABC \sim \Delta BDE$



$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{BC^2}{BD^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{(2BD)^2}{BD^2}$$

[$\because$ D is the midpoint of BC]

$$\Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta BDE)} = \frac{4}{1}$$

$\therefore$ (C) is the correct answer.

---

9. Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio:

(A) 2: 3

(B) 4: 9

(C) 81: 16

(D) 16: 81

Ans. (D) Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides. Therefore,

Ratio of areas = $\frac{(4)^2}{(9)^2} = \frac{16}{81}$

$\therefore$ (D) is the correct answer.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 6

Triangles - Exercise 6.5

---

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Ans. (i) Let a = 7 cm, b = 24 cm and c = 25 cm

Here the larger side is c = 25 cm.

We have, $a^2+b^2=7^2+24^2=49+576=625=c^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 25 cm

(ii) Let a = 3 cm, b = 8 cm and c = 6 cm

Here the larger side is b = 8 cm.

We have, $a^2+c^2=3^2+6^2=9+36=45\neq b^2$

So, the triangle with the given sides is not a right triangle.

(iii) Let a = 50 cm, b = 80 cm and c = 100 cm

Here the larger side is c = 100 cm.

We have, $a^2+b^2=50^2+80^2=2500+6400=8900\neq c^2$

So, the triangle with the given sides is not a right triangle.

(iv) Let a = 13 cm, b = 12 cm and c = 5 cm

Here the larger side is a = 13 cm.

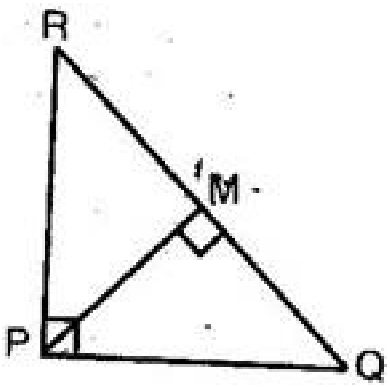
We have, $b^2 + c^2 = 12^2 + 5^2 = 144 + 25 = 169 = a^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 13 cm

---

2. PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that PM^2 = QM x MR.

Ans. Given: PQR is a triangle right angles at P and PM $\perp$ QR



To Prove: PM^2 = QM.MR

Proof: Since PM $\perp$ QR

$\therefore \Delta QMP \sim \Delta PMR$

$\Rightarrow \frac{QM}{PM} = \frac{PM}{RM}$

$\Rightarrow PM^2 = QM.MR$

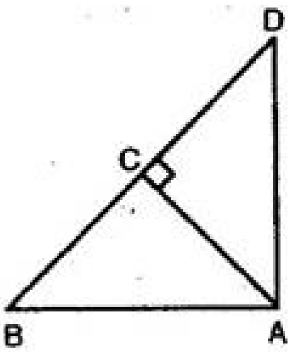
---

3. In the given figure, ABD is a triangle right angled at A and AC $\perp$ BD. Show that:

(i) AB^2 = BC.BD

(ii) AC^2 = BC.DC

(iii) AD^2 = BD.CD



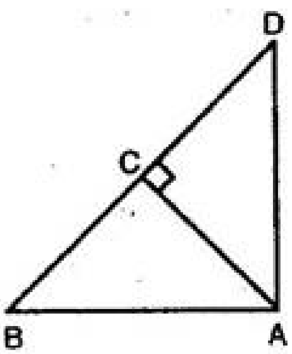
Ans. Given: ABD is a triangle right angled at A and AC $\perp$ BD.

To Prove: **(i)** $AB^2 = BC.BD$, **(ii)** $AC^2 = BC.DC$, **(iii)** $AD^2 = BD.CD$

Proof:**(i)** Since AC $\perp$ BD

$\therefore$ $\triangle CBA \sim \triangle CAD$ and each triangle is similar to $\triangle ABD$

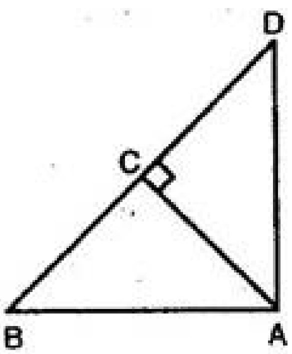
$\therefore$ $\triangle ABD \sim \triangle CBA$



$$\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$$

$$\Rightarrow AB^2 = BC.BD$$

(ii) Since $\triangle ABC \sim \triangle DAC$



$$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC}$$

$\Rightarrow AC^2 = BC.DC$

(iii) Since $\Delta CAD \sim \Delta ABD$

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$$

$\Rightarrow AD^2 = BD.CD$

---

4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Ans. Since ABC is an isosceles right triangle, right angled at C.

$\Rightarrow AB^2 = AC^2 + BC^2$
$\Rightarrow AB^2 = AC^2 + AC^2$ [BC = AC, given]
$\Rightarrow AB^2 = 2AC^2$

---

5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

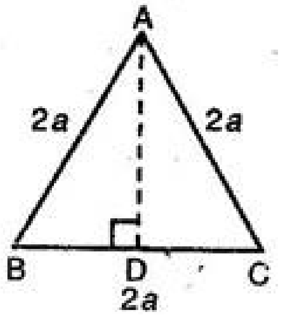
Ans. Since ABC is an isosceles right triangle with AC = BC and $AB^2 = 2AC^2$
$\Rightarrow AB^2 = AC^2 + AC^2$
$\Rightarrow AB^2 = AC^2 + BC^2$ [BC = AC, given]

$\therefore \Delta ABC$ is right angled at C.

---

6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Ans. Let ABC be an equilateral triangle of side $2a$ units.



Draw AD $\perp$ BC. Then, D is the midpoint of BC.

$$\Rightarrow BD = \frac{1}{2}BC = \frac{1}{2}\times 2a = a$$

Since, ABD is a right triangle, right triangle at D.

$$\therefore AB^2 = AD^2 + BD^2$$

$$\Rightarrow (2a)^2 = AD^2 + (a)^2$$

$$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$$

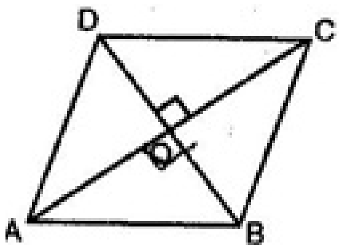
$\therefore$ Each of its altitude = $\sqrt{3}a$

---

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of squares of its diagonals.

Ans. Let the diagonals AC and BD of rhombus ABCD intersect each other at O. Since the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$ and AO = CO, BO = OD



Since AOB is a right triangle, right angled at O.

$$\therefore AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$$

[$\because$ OA = OC and OB = OD]

$$\Rightarrow 4AB^2 = AC^2 + BD^2 \text{ ....(1)}$$

Similarly, we have $4BC^2 = AC^2 + BD^2$..........(2)

$4CD^2 = AC^2 + BD^2$..........(3)

$4AD^2 = AC^2 + BD^2$..........(4)

Adding all these results, we get

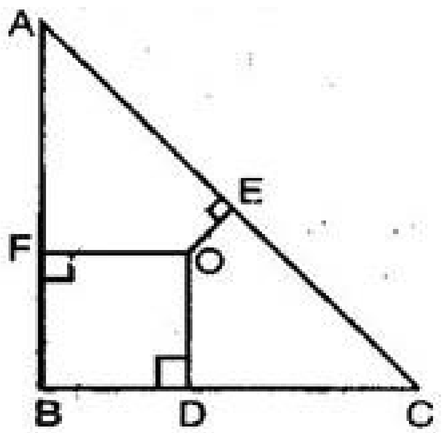
$$4\left(AB^2 + BC^2 + CD^2 + DA^2\right) = 4\left(AC^2 + BD^2\right)$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

---

8. In the given figure, O is a point in the interior of a triangle ABC, OD $\perp$ BC,

OE $\perp$ AC and OF $\perp$ AB. Show that:



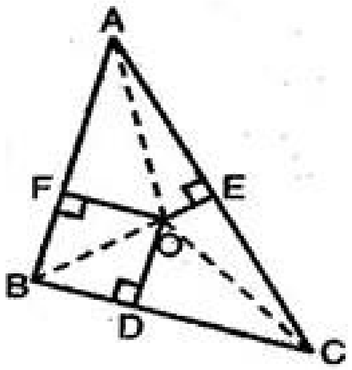
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Ans. Join AO, BO and CO.

(i) In right Δs OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2, OB^2 = BD^2 + OD^2$ and $OC^2 = CE^2 + OE^2$



Adding all these, we get

$$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$$

$$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$$

(ii) In right Δs ODB and ODC, we have

$OB^2 = BD^2 + OD^2$ and $OC^2 = OD^2 + CD^2$

$\Rightarrow OB^2 - OC^2 = BD^2 - CD^2$ ....(1)

Similarly, we have $OB^2 - OC^2 = BD^2 - CD^2$ ....(2)

and $OB^2 - OC^2 = BD^2 - CD^2$ ....(3)

Adding equations (1), (2) and (3), we get

$$= \left(OB^2 - OC^2\right) + \left(OC^2 - OA^2\right) + \left(OA^2 - OB^2\right)$$

$$= \left(BD^2 - CD^2\right) + \left(CE^2 - AE^2\right) + \left(AF^2 - BF^2\right)$$

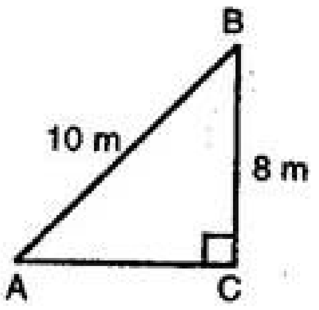
$$\Rightarrow \left(BD^2 + CE^2 + AF^2\right) - \left(AE^2 + CD^2 + BF^2\right) = 0$$

$$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$$

---

9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Ans. Let AB be the ladder, B be the window and CB be the wall. Then, ABC is a right triangle, right angled at C.



$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow 10^2 = AC^2 + 8^2$

$\Rightarrow AC^2 = 100 - 64$

$\Rightarrow AC^2 = 36$

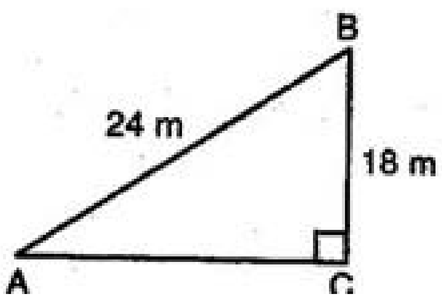
$\Rightarrow$ AC = 6

Hence, the foot of the ladder is at a distance 6 m from the base of the wall.

---

10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other hand. How far from the base of the pole should the stake be driven so that the wire will be taut?

Ans. Let AB (= 24m) be a guy wire attached to a vertical pole. BC of height 18 m. To keep the wire taut, let it be fixed to a stake at A. Then, ABC is a right triangle, right angled at C.



$\therefore AB^2 = AC^2 + BC^2$

$\Rightarrow 24^2 = AC^2 + 18^2$

$\Rightarrow$ $AC^2 = 576 - 324$

$\Rightarrow$ $AC^2 = 252$

$\Rightarrow$ AC = $6\sqrt{7}$

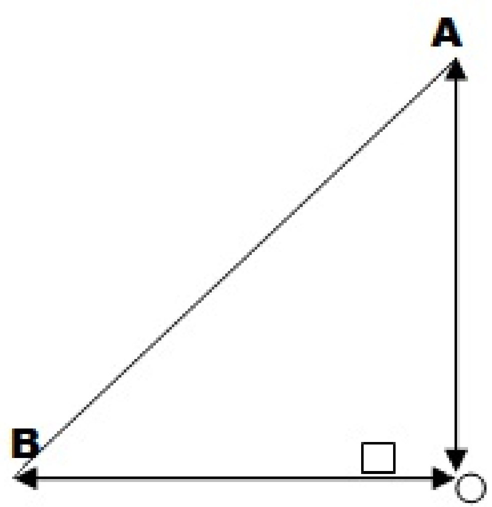
Hence, the stake may be placed at a distance of $6\sqrt{7}$ m from the base of the pole.

---

11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Ans. Let the first aeroplane starts from O and goes upto A towards north where

OA = $\left(1000 \times \frac{3}{2}\right)$ km = 1500 km



Let the second aeroplane starts from O at the same time and goes upto

B towards west where

OB = $\left(1200 \times \frac{3}{2}\right)$ km = 1800 km

According to the question the required distance = BA

In right angled triangle ABC, by Pythagoras theorem, we have,

$AB^2 = OA^2 + OB^2$

$= (1500)^2 + (1800)^2$

= 2250000 + 3240000

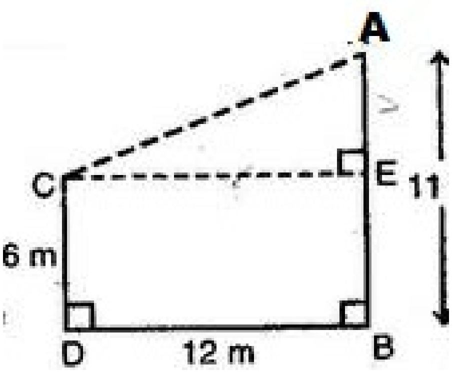
= 5490000 = $9 \times 61 \times 100 \times 100$

$\Rightarrow$ AB = $300\sqrt{61}$ km

---

12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Ans. Let AB = 11 m and CD = 6 m be the two poles such that BD = 12 m



Draw CE $\perp$ AB and join AC.

$\therefore$ CE = DB = 12 m

AE = AB – BE = AB – CD = (11 – 6)m = 5 m

In right angled triangle ACE, by Pythagoras theorem, we have

$AC^2 = CE^2 + AE^2 = 12^2 + 5^2$

= 144 + 25 = 169

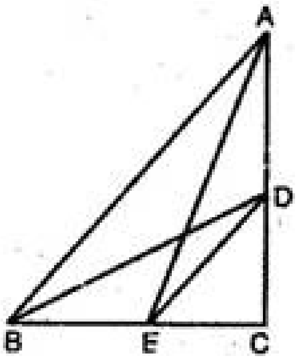
$\Rightarrow$ AC = 13 m

Hence, the distance between the tops of the two poles is 13 m.

---

13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Ans. In right angled Δs ACE and DCB, we have



$AE^2 = AC^2 + CE^2$ and $BD^2 = DC^2 + BC^2$

$\Rightarrow AE^2 + BD^2 = \left(AC^2 + CE^2\right) + \left(DC^2 + BC^2\right)$

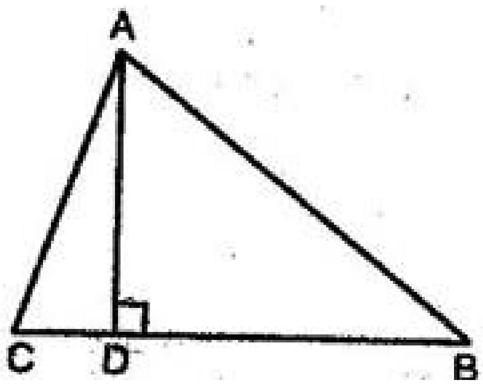
$\Rightarrow AE^2 + BD^2 = \left(AC^2 + BC^2\right) + \left(DC^2 + CE^2\right)$

$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$

[By Pythagoras theorem, $AC^2 + BC^2 = AB^2$ and $DC^2 + CE^2 = DE^2$]

---

14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Ans. We have, DB = 3CD

Now, BC = DB + CD

$\Rightarrow$ BC = 3CD + CD

$\Rightarrow$ BC = 4CD

$\therefore$ CD = $\frac{1}{4}$ BC and DB = 3CD = $\frac{3}{4}$ BC ....(1)

Since, Δ ABD is a right triangle, right angled at D. Therefore by Pythagoras theorem, we have,

$AB^2 = AD^2 + DB^2$ ....(2)

Similarly, from ΔACD, we have, $AC^2 = AD^2 + CD^2$ ....(3)

From eq. (2) and (3) $AB^2 - AC^2 = DB^2 - CD^2$

$\Rightarrow AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$ [Using eq.(1)]

$\Rightarrow AB^2 - AC^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2$

$\Rightarrow AB^2 - AC^2 = \frac{1}{2}BC^2$

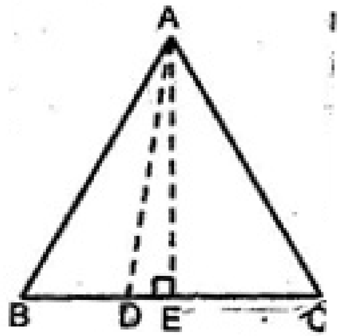
$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

---

15. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$.

Ans. Let ABC be an equilateral triangle and let D be a point on BC such that BD = $\frac{1}{3}$ BC



Draw AE $\perp$ BC, Join AD.

In Δs AEB and AEC, we have,

AB = AC[$\because$ ΔABC is equilateral]

$\angle$AEB = $\angle$AEC [$\because$ each 90°]

And AE = AE

$\therefore$ By SAS-criterion of similarity, we have

ΔAEB ~ ΔAEC

$\Rightarrow$ BE = EC

Thus, we have, BD = $\frac{1}{3}$ BC, DC = $\frac{2}{3}$ BC and BE = EC = $\frac{1}{3}$ BC ....(1)

Since, $\angle$C = 60°

$\therefore$ ΔADC is an acute angle triangle.

$\therefore$ $AD^2 = AC^2 + DC^2 - 2DC \times EC$

= $AC^2 + \left(\frac{2}{3}\mathrm{BC}\right)^2 - 2 \times \frac{2}{3}\mathrm{BC} \times \frac{1}{2}\mathrm{BC}$ [using eq.(1)]

$\Rightarrow AD^2 = AC^2 + \frac{4}{9}\mathrm{BC}^2 - \frac{2}{3}\mathrm{BC}^2$

$= AB^2 + \frac{4}{9}AB^2 - \frac{2}{3}AB^2$ [$\because$ AB = BC = AC]

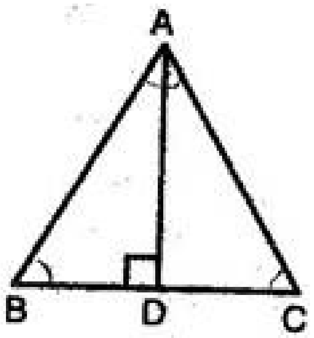
$\Rightarrow AD^2 = \frac{(9+4-6)AB^2}{9} = \frac{7}{9}AB^2$

$\Rightarrow 9AD^2 = 7AB^2$

---

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Ans. Let ABC be an equilateral triangle and let AD $\perp$ BC. In Δs ADB and ADC, we have,



AB = AC [Given]

$\angle$B = $\angle$C = 60° [Given]

And $\angle$ADB = $\angle$ADC[Each = 90°]

$\therefore$ ΔADB $\cong$ ΔADC[By RHS criterion of congruence]

$\Rightarrow$ BD = DC

$\Rightarrow$ BD = DC = $\frac{1}{2}$ BC

Since ΔADB is a right triangle, right angled at D, by Pythagoras theorem, we have,

$AB^2 = AD^2 + BD^2$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$$

$$\Rightarrow AB^2 = AD^2 + \frac{1}{4}BC^2$$

$$\Rightarrow AB^2 = AD^2 + \frac{AB^2}{4} \text{ [}\because \text{ BC = AB]}$$

$$\Rightarrow \frac{3}{4}AB^2 = AD^2$$

$$\Rightarrow 3AB^2 = 4AD^2$$

---

17. Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. the angles A and B are respectively:

(A) 90° **and** 30°

(B) 90° **and** 60°

(C) 30° **and** 90°

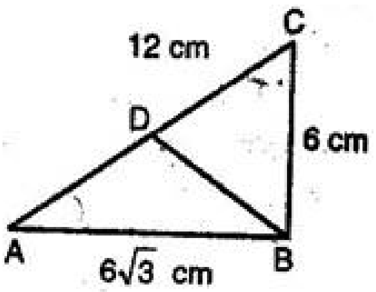
(D) 60° **and** 90°

Ans. (C) In ΔABC, we have, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

Now, $AB^2 + BC^2 = \left(6\sqrt{3}\right)^2 + (6)^2 = 36\times3+36 = 108+36 = 144 = (AC)^2$

Thus, ΔABC is a right triangle, right angled at B.

$\therefore \angle$B = 90°



Let D be the midpoint of AC. We know that the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

AD = BD = CD

$\Rightarrow$ CD = BD = 6 cm [$\because$ CD = $\frac{1}{2}$ AC]

Also, BC = 6 cm

$\therefore$ In ΔBDC, we have, BD = CD = BC

$\Rightarrow$ ΔBDC is equilateral

$\Rightarrow$ $\angle$ACB = 60°

$\therefore$ $\angle$A = $180^\circ - (\angle B + \angle C) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

Thus, $\angle$A = 30° and $\angle$B = 90°

CBSE Class–10 Mathematics

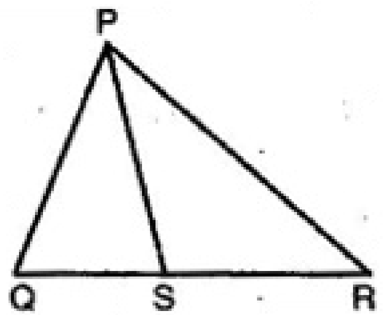
NCERT solution

Chapter - 6

Triangles - Exercise 6.6 (Optional)*

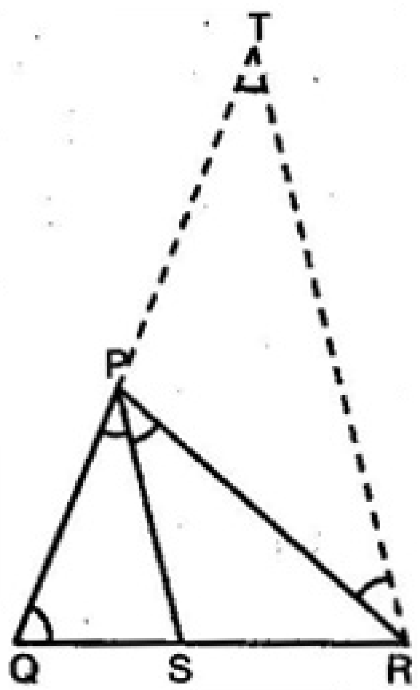
1. In the given figure, PS is the bisector of $\angle$ QPR of $\triangle$ PQR. Prove that

$$\frac{QS}{SR} = \frac{PQ}{PR}.$$



Ans. Given: PQR is a triangle and PS is the internal bisector of $\angle$ QPR

meeting QR at S.



$\therefore$ $\angle$ QPS = $\angle$ SPR

To prove: $\frac{QS}{SR} = \frac{PQ}{PR}$

Construction: Draw RT $\parallel$ SP to cut QP produced at T.

Proof: Since PS $\parallel$ TR and PR cuts them, hence,

$\angle$SPR = $\angle$PRT ....(i) [Alternate $\angle$s]

And $\angle$QPS = $\angle$PTR ....(ii)[Corresponding $\angle$s]

But $\angle$QPS = $\angle$SPR [Given]

$\therefore$ $\angle$PRT = $\angle$PTR[From eq. (i) & (ii)]

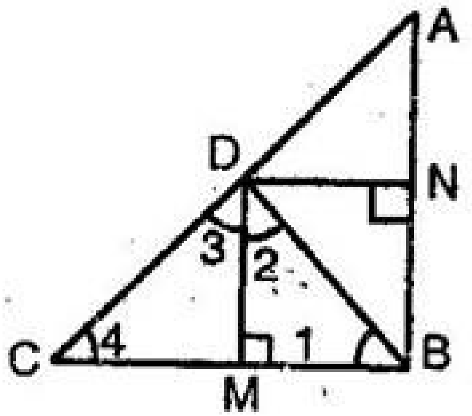
$\Rightarrow$ PT = PR..........(iii)

[Sides opposite to equal angles are equal]

Now, in ΔQRT,

RT $\parallel$ SP[By construction]

$\therefore \frac{QS}{SR} = \frac{PQ}{PT}$ [Thales theorem]



$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$ [From eq. (iii)]

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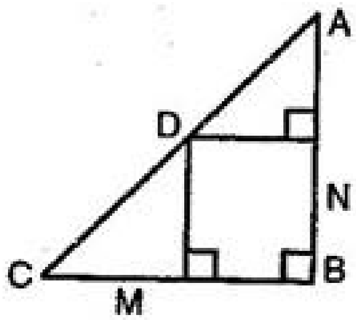
2. In the given figure, D is a point on hypotenuse AC of ΔABC, BD $\perp$ AC, DM $\perp$ BC and

DN $\perp$ AB. Prove that:



(i) DM^2 = DN.MC

(ii) DN^2 = DM.AN

Ans. Since AB $\perp$ BC and DM $\perp$ BC

$\Rightarrow$ AB $\parallel$ DM

Similarly, BC $\perp$ AB and DN $\perp$ AB

$\Rightarrow$ CB $\parallel$ DN

$\therefore$ quadrilateral BMDN is a rectangle.

$\therefore$ BM = ND

(i) In ΔBMD, $\angle 1 + \angle BMD + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + 90^\circ + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ$

Similarly, in ΔDMC, $\angle 3 + \angle 4 = 90^\circ$

Since BD $\perp$ AC,

$\therefore \angle 2 + \angle 3 = 90^\circ$

Now, $\angle 1 + \angle 2 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 3$

Also, $\angle 3 + \angle 4 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 3 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 4 = \angle 2$

Thus, in ΔBMD and ΔDMC,

$\angle 1 = \angle 3$ and $\angle 4 = \angle 2$

$\therefore \Delta BMD \sim \Delta DMC$

$$\Rightarrow \frac{BM}{DM} = \frac{MD}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \text{ [BM = ND]}$$

$\Rightarrow DM^2$ = DN.MC

(ii) Processing as in (i), we can prove that

$\Delta BND \sim \Delta DNA$

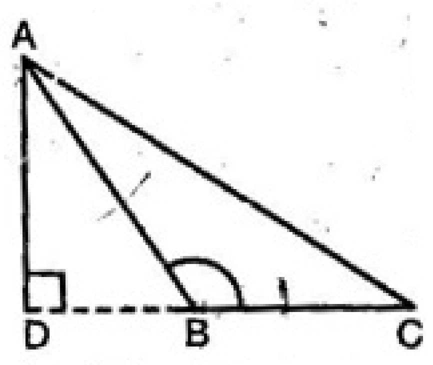
$$\Rightarrow \frac{BN}{DN} = \frac{ND}{NA}$$

$$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN} \text{ [BN = DM]}$$

$\Rightarrow DN^2$ = DM.AN

---

3. **In the given figure, ABC is a triangle in which $\angle$ABC > 90° and AD $\perp$ CB produced. Prove that:**



$AC^2 = AB^2 + BC^2 + 2BC$ .BD

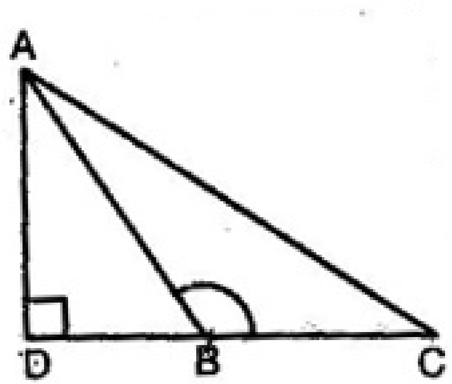
Ans. Given: ABC is a triangle in which $\angle ABC > 90^\circ$ and AD $\perp$ CB produced.

To prove: $AC^2 = AB^2 + BC^2 + 2BC$ .BD

Proof: Since $\triangle$ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + DB^2$ ....(i)

Again, $\triangle$ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,



$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB$ .BC

$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 + 2DB$ .BC

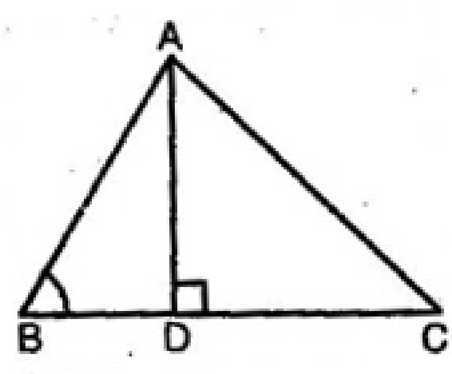
$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB$ .BC

[Using eq. (i)]

---

4. **In the given figure, ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC produced. Prove that:**

$$AC^2 = AB^2 + BC^2 - 2BC.BD$$



Ans. Given: ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC produced.

To prove: $AC^2 = AB^2 + BC^2 - 2BC.BD$

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ....(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

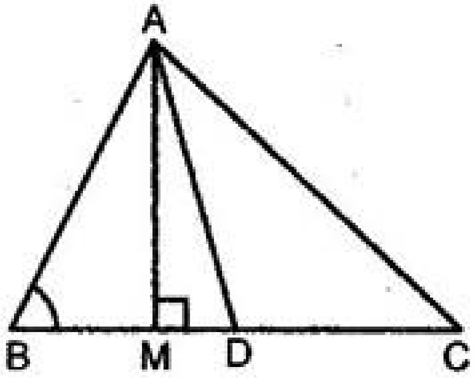
$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (BC - BD)^2$

$\Rightarrow AC^2 = AD^2 + BC^2 + BD^2 - 2BC.BD$

$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 - 2DB.BC$

$\Rightarrow AC^2 = AB^2 + BC^2 - 2DB.BC$



[Using eq. (i)]

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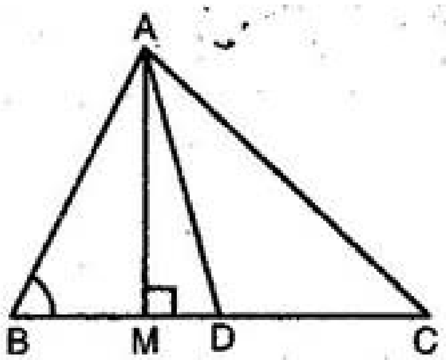
5. In the given figure, AD is a median of a triangle ABC and AM $\perp$ BC. Prove that:



(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC$

Ans. Since $\angle$ AMD = 90°, therefore $\angle$ ADM < 90° and $\angle$ ADC > 90°

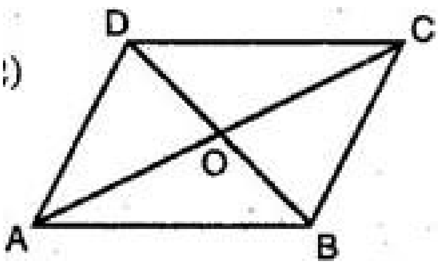
Thus, $\angle$ ADC is the acute angle and $\angle$ ADC is an obtuse angle.

(i) In ΔADC, $\angle$ ADC is an obtuse angle.

$\therefore AC^2 = AD^2 + DC^2 + 2DC.DM$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + 2.\frac{BC}{2}.DM$$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC.DM$$



$$\Rightarrow AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 \text{..........(i)}$$

(ii) In ΔABD, $\angle$ADM is an acute angle.

$$AB^2 = AD^2 + BD^2 - 2BD.DM$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - 2.\frac{BC}{2}.DM$$

$$\Rightarrow AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2 \text{..........(ii)}$$

(iii) From eq. (i) and eq. (ii),

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

---

6. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Ans. If AD is a median of ΔABC, then

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2 \text{ [See Q.5 (iii)]}$$

Since the diagonals of a parallelogram bisect each other, therefore, BO and DO are medians of triangles ABC and ADC respectively.

$\therefore AB^2 + BC^2 = 2BO^2 + \frac{1}{2}AC^2$ ....(i)

And $AD^2 + CD^2 = 2DO^2 + \frac{1}{2}AC^2$ ....(ii)

Adding eq. (i) and (ii),

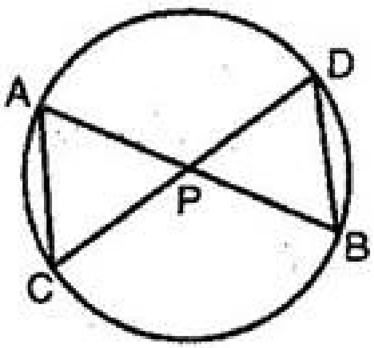
$AB^2 + BC^2 + AD^2 + CD^2 = 2(BO^2 + DO^2) + AC^2$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = 2\left(\frac{1}{4}BD^2 + \frac{1}{4}BD^2\right) + AC^2 \left[DO = \frac{1}{2}BD\right]$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = AC^2 + BD^2$

---

7. In the given figure, two chords AB and CD intersect each other at the point P. Prove that:



(i) $\Delta APC \sim \Delta DPB$

(ii) AP.PB = CP.DP

Ans. (i) In the triangles APC and DPB,

$\angle APC = \angle DPB$ [Vertically opposite angles]

$\angle CAP = \angle BDP$ [Angles in same segment of a circle are equal]

$\therefore$ By AA-criterion of similarity,

$\Delta APC \sim \Delta DPB$

(ii) Since $\Delta APC \sim \Delta DPB$

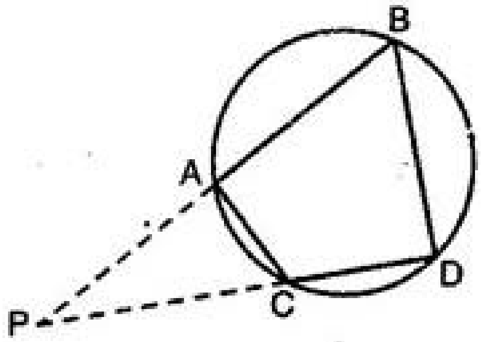
$\therefore \frac{AP}{DP} = \frac{CP}{PB} \Rightarrow$ AP x PB = CP x DP

---

8. In the give figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

(i) $\Delta PAC \sim \Delta PDB$

(ii) PA.PB = PC.PD



Ans. (i) In the triangles PAC and PDB,

$\angle APC = \angle DPB$ [Common]

$\angle CAP = \angle BDP$ [$\because \angle BAC = 180^\circ - \angle PAC$ and $\angle PDB = \angle CDB$]

$= 180^\circ - \angle BAC = 180^\circ - (180^\circ - \angle PAC) = \angle PAC$]

$\therefore$ By AA-criterion of similarity,

$\Delta APC \sim \Delta DPB$

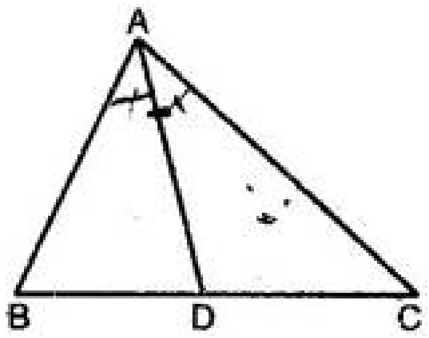
(ii) Since $\Delta APC \sim \Delta DPB$

$\therefore \frac{AP}{DP} = \frac{CP}{PB}$

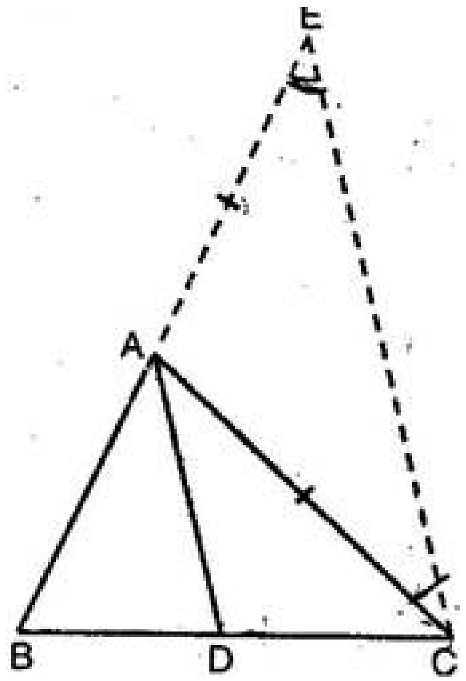
$\Rightarrow$ PA.PB = PC.PD

---

9. In the given figure, D is appointing on side BC of $\triangle$ABC such that $\frac{BD}{CD}=\frac{AB}{AC}$. Prove that AD is the bisector of $\angle$ BAC.



Ans. Given: ABC is a triangle and D is a point on BC such that $\frac{BD}{CD}=\frac{AB}{AC}$



To prove: AD is the internal bisector of $\angle$ BAC.

Construction: Produce BA to E such that AE = AC. Join CE.

Proof: In $\triangle$AEC, since AE = AC

$\therefore$ $\angle$ AEC = $\angle$ ACE ....(i)

[Angles opposite to equal side of a triangle are equal]

Now, $\frac{BD}{CD} = \frac{AB}{AC}$ [Given]

$\Rightarrow \frac{BD}{CD} = \frac{AB}{AE}$ [$\because$ AE = AC, by construction]

$\therefore$ By converse of Basic Proportionality Theorem,

DA $\parallel$ CE

Now, since CA is a transversal,

$\therefore$ $\angle$BAD = $\angle$AEC ....(ii) [Corresponding $\angle$s]

And $\angle$DAC = $\angle$ACE ....(iii) [Alternate $\angle$s]

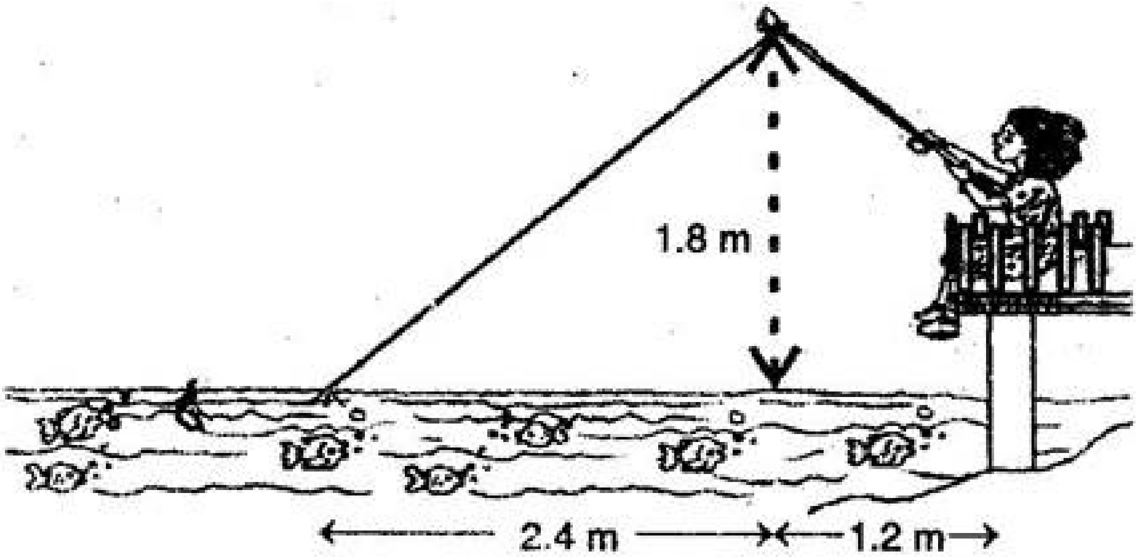
Also $\angle$AEC = $\angle$ACE [From eq. (i)]

Hence, $\angle$BAD = $\angle$DAC [From eq. (ii) and (iii)]

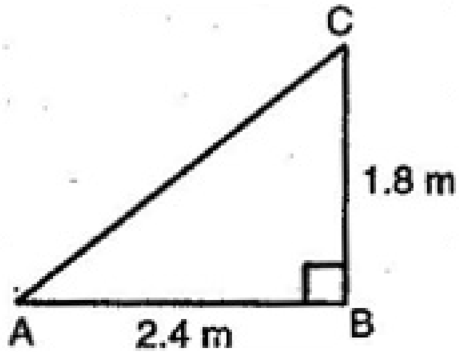
Thus, AD bisects $\angle$BAC internally.

---

10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig.)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Ans. I. To find The length of AC.



By Pythagoras theorem,

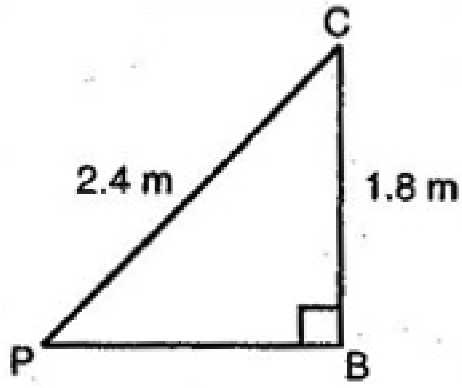
$AC^2 = (2.4)^2 + (1.8)^2$

$\Rightarrow AC^2 = 5.76 + 3.24 = 9.00$

$\Rightarrow$ AC = 3 m

$\therefore$ Length of string she has out= 3 m

Length of the string pulled at the rate of 5 cm/sec in 12 seconds



= (5 x 12) cm = 60 cm = 0.60 m

∴ Remaining string left out = 3 – 0.6 = 2.4 m

II. To find: The length of PB

$PB^2 = PC^2 - BC^2$

$= (2.4)^2 - (1.8)^2$

= 5.76 – 3.24 = 2.52

$\Rightarrow PB = \sqrt{2.52} = 1.59$ (approx.)

Hence, the horizontal distance of the fly from Nazima after 12 seconds

= 1.59 + 1.2 = 2.79 m (approx.)

CBSE Class–10 Mathematics

NCERT solution

Chapter - 7

Coordinate Geometry - Exercise 7.1

1. Find the distance between the following pairs of points:

(i) (2, 3), (4,1)

(ii) (–5, 7), (–1, 3)

(iii) (a, b), (–a, –b)

Ans. (i) Applying Distance Formula to find distance between points (2, 3) and (4,1), we get

d = $\sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(ii) Applying Distance Formula to find distance between points (–5, 7) and (–1, 3), we get

d = $\sqrt{[-1-(-5)]^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

(iii) Applying Distance Formula to find distance between points (a, b) and (–a, –b), we get

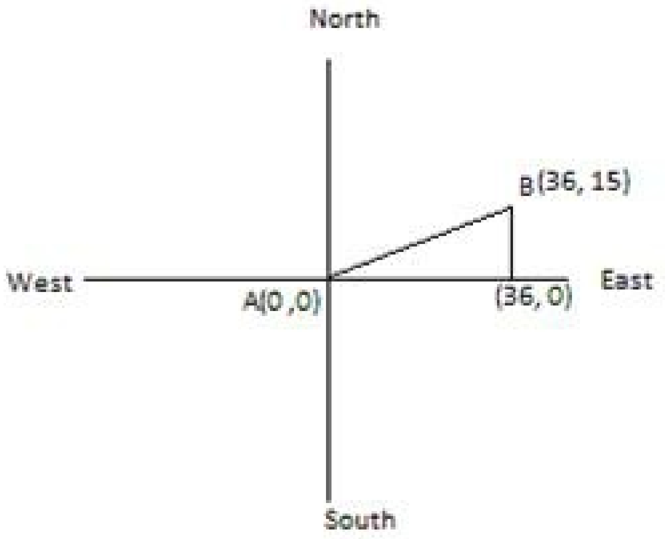
d = $\sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2} = \sqrt{4a^2+4b^2} = \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$

2. Find the distance between the points (0, 0) and (36, 15). Also, find the distance between towns A and B if town B is located at 36 km east and15 km north of town A.

Ans. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

d = $\sqrt{(36-0)^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ units

Town B is located at 36 km east and15 km north of town A. So, the location of town A and B can be shown as:



Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

d = $\sqrt{[36-0]^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ Km

3. Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Ans. Let A = (1, 5), B = (2, 3) and C = (–2, –11)

Using Distance Formula to find distance AB, BC and CA.

AB = $\sqrt{[2-1]^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{[-2-2]^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

CA = $\sqrt{[-2-1]^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+256} = \sqrt{265}$

Since AB + AC ≠ BC, BC + AC $\neq$ AB and AC $\neq$ BC.

Therefore, the points A, B and C are not collinear.

4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

Ans. Let A = (5, –2), B = (6, 4) and C = (7, –2)

Using Distance Formula to find distances AB, BC and CA.

AB = $\sqrt{[6-5]^2 + [4-(-2)]^2} = \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$

BC = $\sqrt{[7-6]^2 + (-2-4)^2} = \sqrt{(1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$

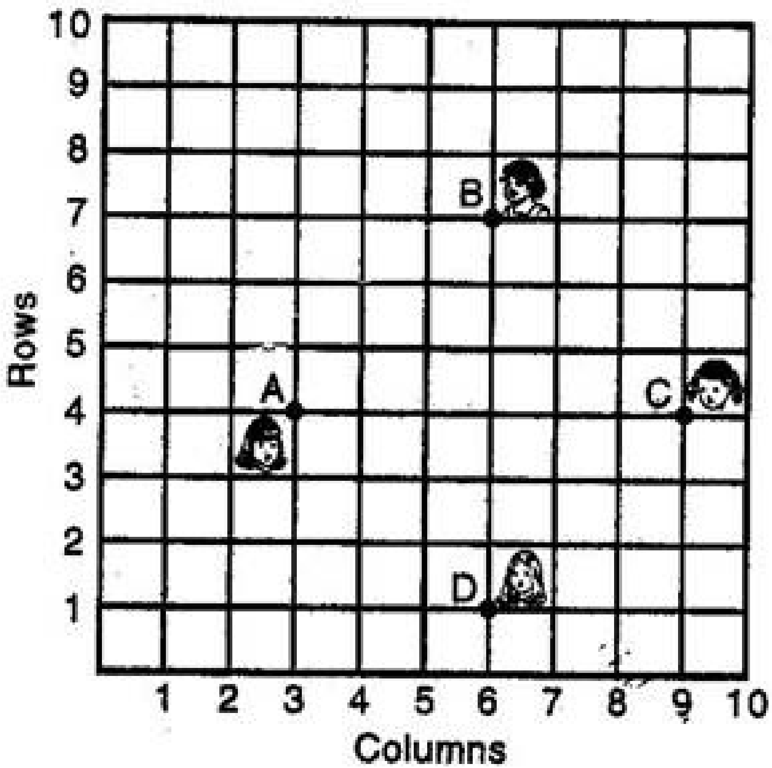
CA = $\sqrt{[7-5]^2 + [-2-(-2)]^2} = \sqrt{(2)^2 + (0)^2} = \sqrt{4+0} = \sqrt{4} = 2$

Since AB = BC.

Therefore, A, B and C are vertices of an isosceles triangle.

---

5. In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. "Don't you think ABCD is a square?"Chameli disagrees. Using distance formula, find which of them is correct.



Ans. We have A = (3, 4), B = (6, 7), C = (9, 4) and D = (6, 1)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[6-3]^2+[7-4]^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

BC = $\sqrt{[9-6]^2+[4-7]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[6-9]^2+[1-4]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

DA = $\sqrt{[6-3]^2+[1-4]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Therefore, All the sides of ABCD are equal here. … (1)

Now, we will check the length of its diagonals.

AC = $\sqrt{[9-3]^2+[4-4]^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36+0} = 6$

BD = $\sqrt{[6-6]^2+[1-7]^2} = \sqrt{(0)^2+(-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$

So, Diagonals of ABCD are also equal. … (2)

From (1) and (2), we can definitely say that ABCD is a square.

Therefore, Champa is correct.

---

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (–3, 0)

(ii) (–3, 5), (3, 1), (0, 3), (–1, –4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Ans. (i) Let A = (–1, –2), B = (1, 0), C= (–1, 2) and D = (–3, 0)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[1-(-1)]^2+[0-(-2)]^2} = \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{[-1-1]^2+[2-0]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $\sqrt{[-3-(-1)]^2+[0-2]^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{[-3-(-1)]^2+[0-(-2)]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Therefore, all four sides of quadrilateral are equal. … (1)

Now, we will check the length of diagonals.

AC = $\sqrt{[-1-(-1)]^2+[2-(-2)]^2} = \sqrt{(0)^2+(4)^2} = \sqrt{0+16} = \sqrt{16} = 4$

BD = $\sqrt{[-3-1]^2+[0-0]^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16+0} = \sqrt{16} = 4$

Therefore, diagonals of quadrilateral ABCD are also equal. … (2)

From (1) and (2), we can say that ABCD is a square.

(ii) Let A = (–3, 5), B= (3, 1), C= (0, 3) and D= (–1, –4)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[3-(-3)]^2+[1-5]^2} = \sqrt{(6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

BC = $\sqrt{[0-3]^2+[3-1]^2} = \sqrt{(-3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

CD = $\sqrt{[-1-0]^2+[-4-3]^2} = \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

DA = $\sqrt{[-1-(-3)]^2+[-4-5]^2} = \sqrt{(2)^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD.

(iii) Let A = (4, 5), B= (7, 6), C= (4, 3) and D= (1, 2)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[7-4]^2+[6-5]^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{[4-7]^2+[3-6]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[1-4]^2+[2-3]^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

DA = $\sqrt{[1-4]^2+[2-5]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Here opposite sides of quadrilateral ABCD are equal. … (1)

We can now find out the lengths of diagonals.

AC = $\sqrt{[4-4]^2+[3-5]^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

BD = $\sqrt{[1-7]^2+[2-6]^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Here diagonals of ABCD are not equal. … (2)

From (1) and (2), we can say that ABCD is not a rectangle therefore it is a parallelogram.

---

7. Find the point on the x–axis which is equidistant from (2, –5) and (–2, 9).

Ans. Let the point be (x, 0) on x–axis which is equidistant from (2, –5) and (–2, 9).

Using Distance Formula and according to given conditions we have:

$$\sqrt{[x-2]^2+[0-(-5)]^2} = \sqrt{[x-(-2)]^2+[(0-9)]^2}$$

$\Rightarrow \sqrt{x^2+4-4x+25} = \sqrt{x^2+4+4x+81}$

Squaring both sides, we get

$\Rightarrow x^2+4-4x+25 = x^2+4+4x+81$

$\Rightarrow -4x + 29 = 4x + 85$

$\Rightarrow 8x = -56$

$\Rightarrow x = -7$

Therefore, point on the x–axis which is equidistant from (2, –5) and (–2, 9) is (–7, 0)

---

8. Find the values of y for which the distance between the points P (2, –3) and Q (10, y) is 10 units.

Ans. Using Distance formula, we have

$$10 = \sqrt{(2-10)^2 + (-3-y)^2}$$

$$\Rightarrow 10 = \sqrt{(-8)^2 + 9 + y^2 + 6y}$$

$$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$$

Squaring both sides, we get

$$100 = 73 + y^2 + 6y$$

$$\Rightarrow y^2 + 6y - 27 = 0$$

Solving this Quadratic equation by factorization, we can write

$$\Rightarrow y^2 + 9y - 3y - 27 = 0$$

$\Rightarrow y (y + 9) – 3 (y + 9) = 0$

$\Rightarrow (y + 9) (y - 3) = 0$

$\Rightarrow y = 3, -9$

---

9. If, Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Ans. It is given that Q is equidistant from P and R. Using Distance Formula, we get

PQ = RQ

$\Rightarrow \sqrt{(0-5)^2+[1-(-3)]^2}=\sqrt{(0-x)^2+(1-6)^2}$

$\Rightarrow \sqrt{(-5)^2+[4]^2}=\sqrt{(-x)^2+(-5)^2}$

$\Rightarrow \sqrt{25+16}=\sqrt{x^2+25}$

Squaring both sides, we get

$\Rightarrow 25+16=x^2+25$

$\Rightarrow x^2=16$

$\Rightarrow x = 4, -4$

Thus, Q is (4, 6) or (–4, 6).

Using Distance Formula to find QR, we get

Using value of x = 4 QR = $\sqrt{(4-0)^2+(6-1)^2}=\sqrt{16+25}=\sqrt{41}$

Using value of x = –4 QR = $\sqrt{(-4-0)^2+(6-1)^2}=\sqrt{16+25}=\sqrt{41}$

Therefore, QR = $\sqrt{41}$

Using Distance Formula to find PR, we get

Using value of x = 4 PR = $\sqrt{(4-5)^2+[6-(-3)]^2}=\sqrt{1+81}=\sqrt{82}$

Using value of x =–4 PR = $\sqrt{(-4-5)^2+[6-(-3)]^2}=\sqrt{81+81}=\sqrt{162}$ = $9\sqrt{2}$

Therefore, x = 4, –4

QR = $\sqrt{41}$, PR = $\sqrt{82}, 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (–3, 4).

Ans. It is given that (x, y) is equidistant from (3, 6) and (–3, 4).

Using Distance formula, we can write

$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{[x-(-3)]^2+(y-4)^2}$$

$$\Rightarrow \sqrt{x^2+9-6x+y^2+36-12y}=\sqrt{x^2+9+6x+y^2+16-8y}$$

Squaring both sides, we get

$$\Rightarrow x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$$

$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$

$\Rightarrow 12x + 4y = 20$

$\Rightarrow 3x + y = 5$

CBSE Class–10 Mathematics

NCERT solution

Chapter - 7

Coordinate Geometry - Exercise 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2:3.

Ans. Let $x_1 = -1,\ x_2 = 4,\ y_1 = 7$ and $y_2 = -3,\ m_1 = 2\ and\ m_2 = 3$

Using Section Formula to find coordinates of point which divides join of (–1, 7) and (4, –3) in the ratio 2:3, we get

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2\times4+3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times(-3)+3\times7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Therefore, the coordinates of point are (1, 3) which divides join of (–1, 7) and (4, –3) in the ratio 2:3.

2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Ans. 

We want to find coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

We are given AC = CD = DB

We want to find coordinates of point C and D.

Let coordinates of point C be (x_1, y_1) and let coordinates of point D be (x_2, y_2).

Clearly, point C divides line segment AB in 1:2 and point D divides line segment AB in 2:1.

Using Section Formula to find coordinates of point C which divides join of (4, –1) and (–2, –3) in the ratio 1:2, we get

$$x_1 = \frac{1\times(-2)+2\times4}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$y_1 = \frac{1\times(-3)+2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

Using Section Formula to find coordinates of point D which divides join of (4, –1) and (–2, –3) in the ratio 2:1, we get

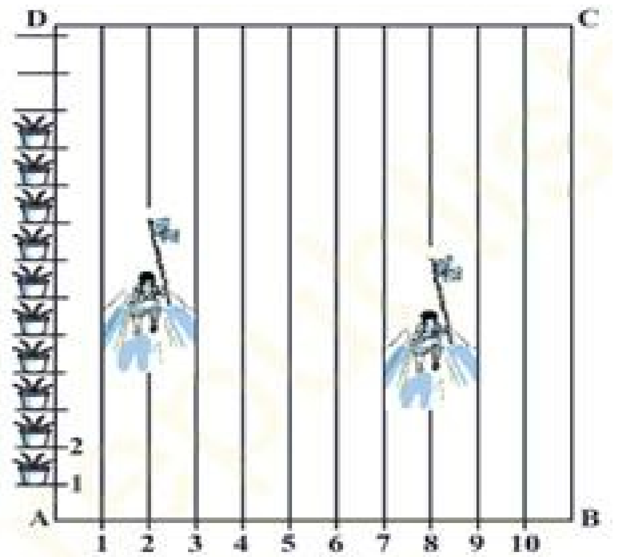
$$x_2 = \frac{2\times(-2)+1\times4}{1+2} = \frac{-4+4}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2\times(-3)+1\times(-1)}{1+2} = \frac{-6-1}{3} = \frac{-7}{3}$$

Therefore, coordinates of point C are $(2, -\frac{5}{3})$ and coordinates of point D are $(0, -\frac{7}{3})$

---

3. To conduct sports day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag. Preet runs 15th of the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Ans. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag.

There are 100 flower pots. It means, she stops at 25th flower pot.

Therefore, the coordinates of point where she stops are (2 m, 25 m).

Preet runs 15th of the distance AD on the eighth line and posts a red flag. There are 100 flower pots. It means, she stops at 20th flower pot.

Therefore, the coordinates of point where she stops are (8, 20).

Using Distance Formula to find distance between points (2 m, 25 m) and (8 m, 20 m), we get

$$d = \sqrt{(2-8)^2 + (25-20)^2} = \sqrt{(-6)^2 + 5^2} = \sqrt{36+25} = \sqrt{61}m$$

Rashmi posts a blue flag exactly halfway the line segment joining the two flags.

Using section formula to find the coordinates of this point, we get

$$x = \frac{2+8}{2} = \frac{10}{2} = 5$$

$$y = \frac{25+20}{2} = \frac{45}{2}$$

Therefore, coordinates of point, where Rashmi posts her flag are $(5, \frac{45}{2})$ (5, $\frac{45}{2}$).

It means she posts her flag in 5th line after covering $\frac{45}{2}$ = 22.5 m of distance.

4. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6).

Ans. Let (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in k:1.

Using Section formula, we get

$$-1 = \frac{(-3)\times 1 + 6\times k}{k+1}$$

$\Rightarrow -k - 1 = (-3 + 6k)$

$\Rightarrow -7k = -2$

$\Rightarrow k = \frac{2}{7}$

Therefore, the ratio is $\frac{2}{7}$:1 which is equivalent to 2:7.

Therefore, (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in 2:7.

---

5. Find the ratio in which the line segment joining A (1, –5) and B (–4, 5) is divided by the x–axis. Also find the coordinates of the point of division.

Ans. Let the coordinates of point of division be (x, 0) and suppose it divides line segment joining A (1, –5) and B (–4, 5) in k:1.

According to Section formula, we get

$$x = \frac{1\times 1+(-4)\times k}{k+1} = \frac{1-4k}{k+1} \text{ and } 0 = \frac{(-5)\times 1+5k}{k+1} \quad \dots (1)$$

$$0 = \frac{(-5)\times 1+5k}{k+1}$$

$\Rightarrow 5 = 5k$

$\Rightarrow k = 1$

Putting value of k in (1), we get

$$x = \frac{1\times 1+(-4)\times 1}{1+1} = \frac{1-4}{2} = \frac{-3}{2}$$

Therefore, point ($\frac{-3}{2}$, 0) on x–axis divides line segment joining A (1, –5) and B (–4, 5) in 1:1.

---

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Ans. Let A = (1, 2), B = (4, y), C = (x, 6) and D = (3, 5)

We know that diagonals of parallelogram bisect each other. It means that coordinates of midpoint of diagonal AC would be same as coordinates of midpoint of diagonal BD. ... (1)

Using Section formula, the coordinates of midpoint of AC are:

$$\frac{1+x}{2}, \frac{2+6}{2} = \frac{1+x}{2}, 4$$

Using Section formula, the coordinates of midpoint of BD are:

$$\frac{4+3}{2}, \frac{5+y}{2} = \frac{7}{2}, \frac{5+y}{2}$$

According to condition (1), we have

$$\frac{1+x}{2} = \frac{7}{2}$$

$\Rightarrow (1 + x) = 7$

$\Rightarrow x = 6$

Again, according to condition (1), we also have

$$4 = \frac{5+y}{2}$$

$\Rightarrow 8 = 5 + y$

$\Rightarrow y = 3$

Therefore, x = 6 and y = 3

---

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Ans. We want to find coordinates of point A. AB is the diameter and coordinates of center are (2, –3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$$2 = \frac{x+1}{2}$$

$\Rightarrow 4 = x + 1$

$\Rightarrow x = 3$

Using section formula, we get

$$-3 = \frac{4+y}{2}$$

$\Rightarrow -6 = 4 + y$

$\Rightarrow y = -10$

Therefore, Coordinates of point A are (3, –10).

---

8. If A and B are (–2, –2) and (2, –4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

Ans. A = (–2, –2) and B = (2, –4)

It is given that AP= $\frac{3}{7}$ AB

PB = AB – AP = AB − $\frac{3}{7}$ AB = $\frac{4}{7}$ AB

So, we have AP: PB = 3: 4

Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

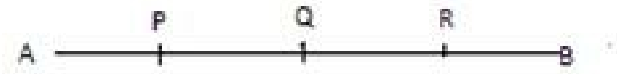
$$x = \frac{(-2)\times 4 + 2\times 3}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{(-2)\times 4 + (-4)\times 3}{3+4} = \frac{-8-12}{7} = \frac{-20}{7}$$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

9. Find the coordinates of the points which divides the line segment joining A (–2, 2) and B (2, 8) into four equal parts.

Ans. A = (–2, 2) and B = (2, 8)



Let P, Q and R are the points which divide line segment AB into 4 equal parts.

Let coordinates of point $P = (x_1, y_1)$, $Q = (x_2, y_2)$ *and* $R = (x_3, y_3)$

We know AP = PQ = QR = RS.

It means, point P divides line segment AB in 1:3.

Using Section formula to find coordinates of point P, we get

$$x_1 = \frac{(-2)\times 3 + 2\times 1}{1+3} = \frac{-6+2}{4} = \frac{-4}{4} = -1$$

$$y_1 = \frac{2\times 3 + 8\times 1}{1+3} = \frac{6+8}{4} = \frac{14}{4} = \frac{7}{2}$$

Since, AP = PQ = QR = RS.

It means, point Q is the mid-point of AB.

Using Section formula to find coordinates of point Q, we get

$$x_2 = \frac{(-2)\times 1 + 2\times 1}{1+1} = \frac{-2+2}{2} = \frac{0}{2} = 0$$

$$y_2 = \frac{2\times 1 + 8\times 1}{1+1} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Because, AP = PQ = QR = RS.

It means, point R divides line segment AB in 3:1

Using Section formula to find coordinates of point P, we get

$$x_3 = \frac{(-2)\times 1 + 2\times 3}{1+3} = \frac{-2+6}{4} = \frac{4}{4} = 1$$

$$y_3 = \frac{2\times 1 + 8\times 3}{1+3} = \frac{2+24}{4} = \frac{26}{4} = \frac{13}{2}$$

Therefore, P = (–1, $\frac{7}{2}$), Q= (0, 5)and R = (1, $\frac{13}{2}$)

---

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (–2, –1) taken in order. {Hint: Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)}

Ans. Let A = (3, 0), B = (4, 5), C = (–1, 4) and D = (–2, –1)

Using Distance Formula to find length of diagonal AC, we get

$$AC = \sqrt{[3-(-1)]^2 + (0-4)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Using Distance Formula to find length of diagonal BD, we get

$$BD = \sqrt{[4-(-2)]^2 + [5-(-1)]^2} = \sqrt{6^2 + 6^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

∵ Area of rhombus = ½ (product of its diagonals)

= $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ = 24 sq. units

CBSE Class–10 Mathematics

NCERT solution

Chapter - 7

Coordinate Geometry - Exercise 7.3

1. Find the area of the triangle whose vertices are:

(i) (2, 3), (–1, 0), (2, –4)

(ii) (–5, –1), (3, –5), (5, 2)

Ans. (i) (2, 3), (–1, 0), (2, –4)

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [2 {0 − (−4)} – 1 (−4 − 3) + 2 (3 − 0)]

= $\frac{1}{2}$ [2 (0 + 4) – 1 (−7) + 2 (3)]

= $\frac{1}{2}$ (8 + 7 + 6) = $\frac{21}{2}$ sq. units

(ii) (–5, –1), (3, –5), (5, 2)

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [−5 (−5 − 2) + 3 {2 − (−1)} + 5 {−1 − (−5)}]

= $\frac{1}{2}$ [−5 (−7) + 3 (3) + 5 (4)]

= $\frac{1}{2}$ (35 + 9 + 20)

$= \frac{1}{2}$ (64) = 32 sq. units

2. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, –2), (5, 1), (3, k)

(ii) (8, 1), (k, –4), (2, –5)

Ans. (i) (7, –2), (5, 1), (3, k)

Since, the given points are collinear, it means the area of triangle formed by them is equal to zero.

Area of Triangle = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

$\Rightarrow \frac{1}{2}$ [7 (1 − k) + 5 {k − (−2)} + 3 (−2 − 1)] = 0

$= \frac{1}{2}$ (7 − 7k + 5k + 10 − 9) = 0

$\Rightarrow \frac{1}{2}$ (7 − 7k + 5k + 1) = 0

$\Rightarrow \frac{1}{2}$ (8 − 2k) = 0

⇒ 8 − 2k = 0

⇒ 2k = 8

⇒ k = 4

(ii) (8, 1), (k, –4), (2, –5)

Since, the given points are collinear, it means the area of triangle formed by them is equal to zero.

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}$ [8 {−4 − (−5)} + k (−5 − 1) + 2 {1 − (−4)}] = 0

$\Rightarrow \frac{1}{2}$ (8 − 6k + 10) = 0

$\Rightarrow \frac{1}{2}$ (18 − 6k) = 0

⇒18 − 6k = 0

⇒ 18 = 6k

⇒ k = 3

---

3. Find the area of the triangle formed by joining the mid–points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Ans. Let A = (0, –1) = (x_1, y_1), B = (2, 1) = (x_2, y_2) and

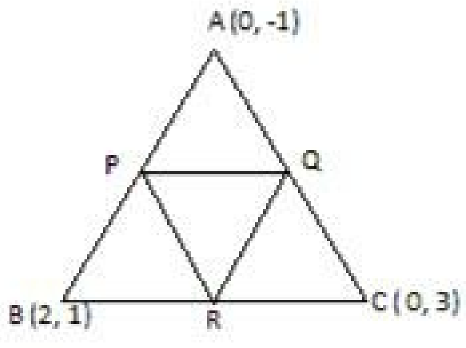
C = (0, 3) = (x_3, y_3)

Area of △ABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

⇒ Area of △ABC

= $\frac{1}{2}$ [0 (1 − 3) + 2 {3 − (−1)} + 0 (−1 − 1)] = $\frac{1}{2} \times 8$

= 4 sq. units



P, Q and R are the mid–points of sides AB, AC and BC respectively.

Applying Section Formula to find the vertices of P, Q and R, we get

$$P = \frac{0+2}{2}, \frac{1-1}{2} = (1,0)$$

$$Q = \frac{0+0}{2}, \frac{-1+3}{2} = (0,1)$$

$$R = \frac{2+0}{2}, \frac{1+3}{2} = (1,2)$$

Applying same formula, Area of $\triangle$PQR = $\frac{1}{2}$ [1 (1 − 2) + 0 (2 − 0) + 1 (0 − 1)] = $\frac{1}{2}\ |-2|$

= 1 sq. units (numerically)

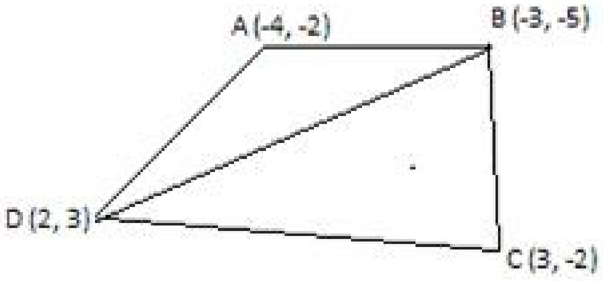
Now, $\dfrac{Area\ of\ \Delta PQR}{Area\ of\ \Delta ABC} = \dfrac{1}{4} = 1:4$

4. Find the area of the quadrilateral whose vertices taken in order are (–4, –2), (–3, –5), (3, –2) and (2, 3).

Ans. Area of Quadrilateral ABCD

= Area of Triangle ABD +

Area of Triangle BCD … (1)



Using formula to find area of triangle:

Area of ΔABD

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [-4 (-5 - 3) - 3 \{3 - (-2)\} + 2 \{-2 - (-5)\}]$$

$$= \frac{1}{2} (32 - 15 + 6)$$

$$= \frac{1}{2} (23) = 11.5 \text{ sq units} \ldots (2)$$

Again using formula to find area of triangle:

Area of $\triangle$BCD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2} [-3 (-2 - 3) + 3 \{3 - (-5)\} + 2 \{-5 - (-2)\}]$$

$$= \frac{1}{2} (15 + 24 - 6)$$

$$= \frac{1}{2} (33) = 16.5 \text{ sq units} \ldots (3)$$

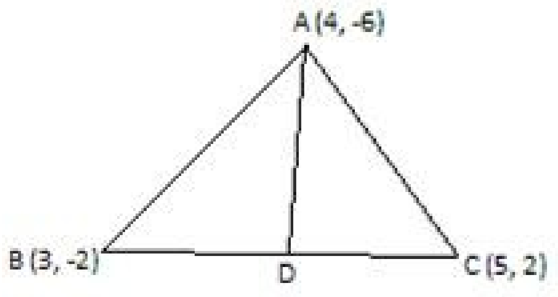
Putting (2) and (3) in (1), we get

Area of Quadrilateral ABCD = 11.5 + 16.5 = 28 sq units

5. We know that median of a triangle divides it into two triangles of equal areas. Verify this result for △ABC whose vertices are A (4, –6), B (3, –2) and C (5, 2).

Ans. We have △ABC whose vertices are given.

We need to show that ar(△ABD) = ar(△ACD).



Let coordinates of point D are (x, y)

Using section formula to find coordinates of D, we get

$$x = \frac{3+5}{2} = \frac{8}{2} = 4$$

$$y = \frac{-2+2}{2} = \frac{0}{2} = 0$$

Therefore, coordinates of point D are (4, 0)

Using formula to find area of triangle:

Area of △ABD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [4 (−2 − 0) + 3 {0 − (−6)} + 4 {−6 − (−2)}]

$= \frac{1}{2}$ (−8 + 18 −16)

$= \frac{1}{2}$ (−6) = −3 sq units

Area cannot be in negative.

Therefore, we just consider its numerical value.

Therefore, area of △ABD = 3 sq units … (1)

Again using formula to find area of triangle:

Area of △ACD = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$= \frac{1}{2}$ [4 (2 − 0) + 5 {0 − (−6)} + 4 {−6 −2)}]

$= \frac{1}{2}$ (8 + 30 − 32) = ½ (6) = 3 sq units … (2)

From (1) and (2), we get ar(△ABD) = ar(△ACD)

Hence Proved.

CBSE Class–10 Mathematics

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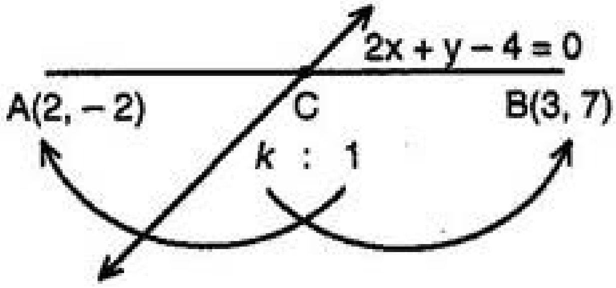
Chapter - 7

Coordinate Geometry - Exercise 7.4

1. Determine the ratio in which the line $2x+y-4=0$ divides the line segment joining the points A$(2,-2)$ and B$(3,7)$.

Ans. Let the line $2x+y-4=0$ divides the line segment joining A$(2,-2)$ and B (3, 7) in the ratio $k:1$ at point C. Then, the coordinates of C are $\left(\frac{3k+2}{k+1},\frac{7k-2}{k+1}\right)$.



But C lies on $2x+y-4=0$, therefore

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow 6k+4+7k-2-4k-4=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Hence, the required ratio if 2: 9 internally.

2. Find a relation between x and y if the points (x,y), $(1,2)$ and $(7,0)$ are collinear.

Ans. The points A(x,y), B (1, 2) and C (7, 0) will be collinear if

Area of triangle = 0

$$\Rightarrow \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]=0$$

$$\Rightarrow 2x-y+7y-14=0$$

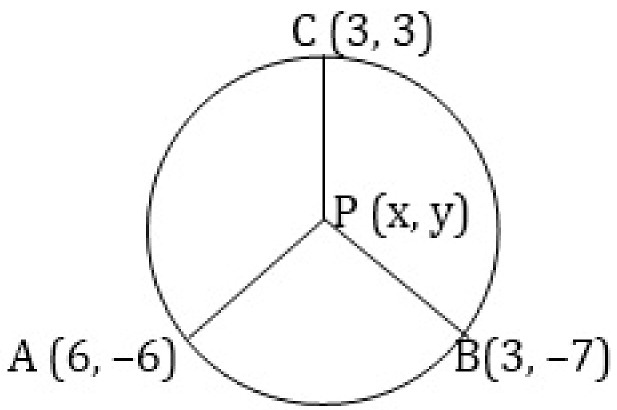
$$\Rightarrow 2x+6y-14=0$$

$$\Rightarrow x+3y-7=0$$

3. Find the centre of a circle passing through the points $(6,-6),(3,-7)$ and $(3,3)$.

Ans. Let P(x,y), be the centre of the circle passing through the points A$(6,-6)$, B$(3,-7)$ and C (3, 3). Then AP = BP = CP.

Taking AP = BP



$$\Rightarrow AP^2 = BP^2$$

$$\Rightarrow (x-6)^2+(y+6)^2=(x-3)^2+(y+7)^2$$

$$\Rightarrow x^2-12x+36+y^2+12y+36=x^2-6x+9+y^2+14y+49$$

$$\Rightarrow -12x+6x+12y-14y+72-58=0$$

$$\Rightarrow -6x-2y+14=0$$

$\Rightarrow 3x+y-7=0$ ....(i)

Again, taking BP = CP

$\Rightarrow BP^2 = CP^2$

$\Rightarrow (x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow x^2-6x+9+y^2+14y+49=x^2-6x+9+y^2-6y+9$

$\Rightarrow -6x+6x+14y+6y+58-18=0$

$\Rightarrow 20y+40=0$

$\Rightarrow y=-2$

Putting the value of y in eq. (i),

$3x+y-7=0$

$\Rightarrow 3x=9$

$\Rightarrow x=3$

Hence, the centre of the circle is $(3,-2)$.

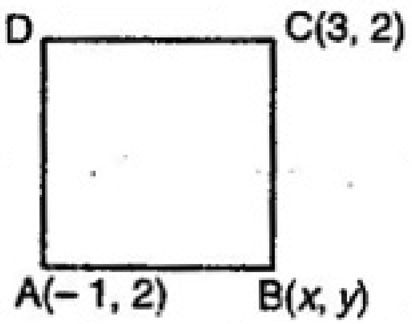
---

4. The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.

Ans. Let ABCD be a square and B(x,y) be the unknown vertex.

AB = BC

$\Rightarrow AB^2 = BC^2$



$\Rightarrow (x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow x^2+1+2x+y^2+4-4y=x^2+9-6x+y^2+4-4y$

$\Rightarrow 2x+1=-6x+9$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ....(i)

In ΔABC, $AB^2+BC^2=AC^2$

$\Rightarrow (x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2=(3+1)^2+(2-2)^2$

$\Rightarrow 2(y-2)^2+\left[(x+1)^2+(x-3)^2\right]=16+0$

$\Rightarrow 2(y-2)^2+\left[(1+1)^2+(1-3)^2\right]=16$

$\Rightarrow 2\left(y^2-4y+4\right)+8=16$

$\Rightarrow y^2-4y+4=4$

$\Rightarrow y(y-4)=0$

$\Rightarrow y$ = 0 or 4

Hence, the required vertices of the square are (1, 0) and (1, 4).

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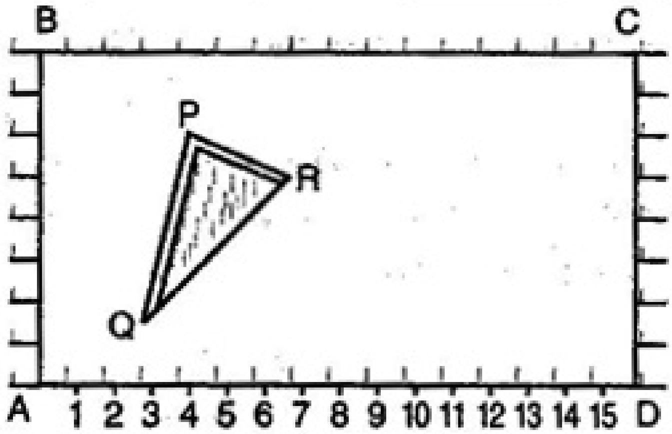
5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted

on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.**



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of $\triangle$PQR if C is the origin? Also calculate the area of the triangle in these cases. What do you observe?

Ans. (i) Taking A as the origin, AD and AB as the coordinate axes. Clearly, the points P, Q and R are (4, 6), (3, 2) and (6, 5) respectively.

(ii) Taking C as the origin, CB and CD as the coordinate axes. Clearly, the points P, Q and R are given by (12, 2), (13, 6) and (10, 3) respectively.

We know that the area of the triangle = $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$\therefore$ Area of $\triangle$PQR (First case) = $\frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$

= $\frac{1}{2}\left[4(-3)+3(-1)+6(4)\right]$

= $\frac{1}{2}\left[-12-3+24\right]$ = $\frac{9}{2}$ sq. units

And Area of $\triangle$ PQR (Second case) = $\frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$

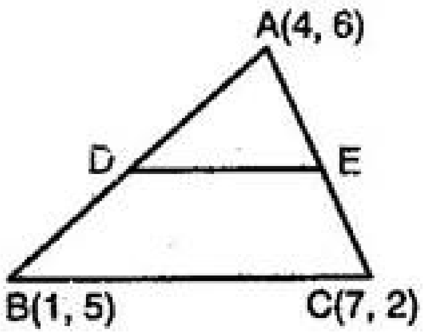
$= \frac{1}{2}[12(3)+13(1)+10(-4)]$

$= \frac{1}{2}[36+13-40] = \frac{9}{2}$ sq. units

Hence, the areas are same in both the cases.

6. The vertices of a $\triangle$ABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the $\triangle$ADE and compare it with the area of $\triangle$ABC.

Ans. Since, $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$



$\therefore$ DE $\parallel$ BC[By Thales theorem]

$\therefore$ $\triangle$ADE ~ $\triangle$ABC

$\therefore \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ABC)}=\frac{AD^2}{AB^2}$

$=\left(\frac{AD}{AB}\right)^2=\left(\frac{1}{4}\right)^2=\frac{1}{16}$ ....(i)

Now, Area ($\triangle$ABC) = $\frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$

$= \frac{1}{2}[12 - 4 + 7] = \frac{15}{2}$ sq. units..........(ii)

From eq. (i) and (ii),

Area (ΔADE) = $\frac{1}{16} \times$ Area (ΔABC) = $\frac{1}{16} \times \frac{15}{2} = \frac{15}{32}$ sq. units

$\therefore$ Area (ΔADE): Area (ΔABC) = 1: 16

---

7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP: PD = 2: 1.

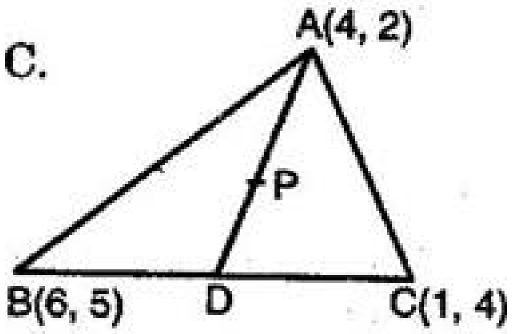
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR : RF = 2 : 1.

(iv) What do you observe?

(Note: The point which is common to all the three medians is called *centroid* and this point divides each median in the ratio 2: 1)

(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Ans. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.



(i) Since AD is the median of ΔABC.

$\therefore$ D is the mid-point of BC.

$\therefore$ Its coordinates are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Since P divides AD in the ratio 2: 1

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Since BE is the median of ΔABC.

$\therefore$ E is the mid-point of AD.

$\therefore$ Its coordinates are $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Since Q divides BE in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Since CF is the median of ΔABC.

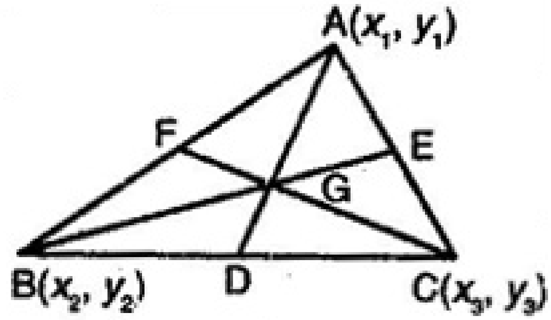
$\therefore$ F is the mid-point of AB.

$\therefore$ Its coordinates are $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Since R divides CF in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) We observe that the points P, Q and R coincide, i.e., the medians AD, BE and CF are concurrent at the point $\left(\frac{11}{3}, \frac{11}{3}\right)$. This point is known as the centroid of the triangle.



(v) According to the question, D, E, and F are the mid-points of BC, CA and AB respectively.

∴ Coordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Coordinates of a point dividing AD in the ratio 2: 1 are

$$\left(\frac{1.x_1+2\left(\frac{x_2+x_2}{2}\right)}{1+2}, \frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$.

∴ The coordinates of a point dividing BE in the ratio 2: 1 are

$$\left(\frac{1.x_2+2\left(\frac{x_1+x_3}{2}\right)}{1+2}, \frac{1.y_2+2\left(\frac{y_1+y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Similarly, the coordinates of a point dividing CF in the ratio 2: 1 are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Thus, the point $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 2: 1.

$\therefore$ The median of a triangle are concurrent and the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

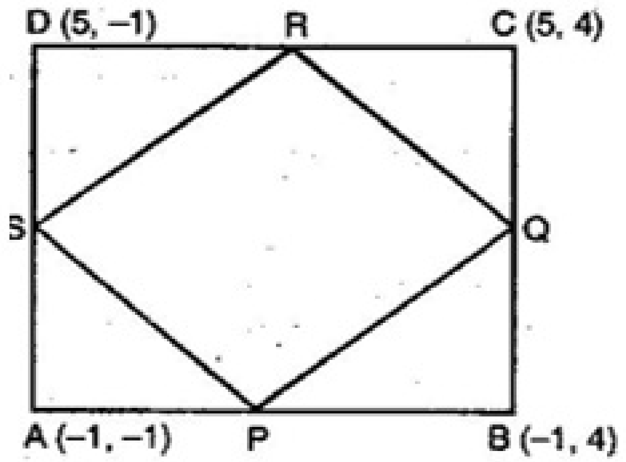
---

8. ABCD is a rectangle formed by joining points A$(-1,-1)$, B$(-1,4)$, C$(5,4)$ and D $(5,-1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? Or a rhombus? Justify your answer.

Ans. Since P is mid-point of AB, therefore, the coordinates of P are $\left(-1, \frac{3}{2}\right)$.

Similarly, the coordinates of Q are (2, 4)., the coordinates of R are $\left(5, \frac{3}{2}\right)$ and the coordinates of S are $(2, -1)$.

Using distance formula, PQ = $\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}$



$= \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

QR = $\sqrt{(5-2)^2+\left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

RS = $\sqrt{(2-5)^2+\left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

SP = $\sqrt{(-1-2)^2+\left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

$\Rightarrow$ PQ = QR = RS = SP

Now, PR = $\sqrt{(5+1)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36} = 6$

And SQ = $\sqrt{(2-2)^2+(4+1)^2} = \sqrt{25} = 5$

$\Rightarrow$ PR $\neq$ SQ

Since all the sides are equal but the diagonals are not equal.

$\therefore$ PQRS is a rhombus.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.1

---

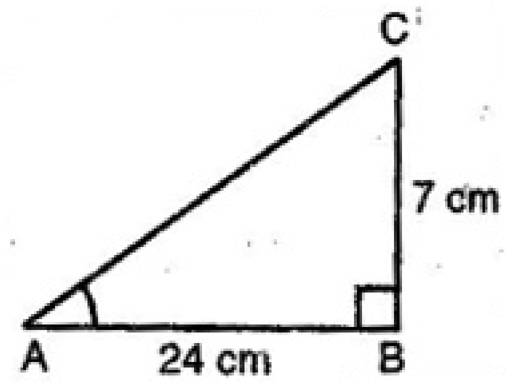
1. In $\triangle$ ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$

Ans. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,



Let AC = 24k and BC = 7k

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

= $(24)^2 + (7)^2$ = 576 + 49 = 625

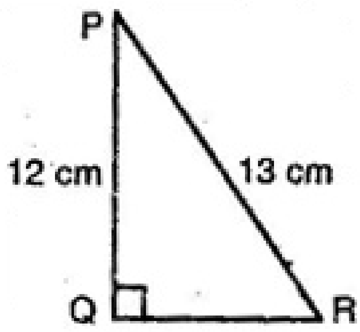
$\Rightarrow$ AC = 25 cm

(i) $\sin \mathrm{A} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{7}{25}$, $\cos \mathrm{A} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{24}{25}$

(ii) $\sin \mathrm{C} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{24}{25}$, $\cos \mathrm{C} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{7}{25}$

---

2. In adjoining figure, find $\tan P - \cot R$ **:**



Ans. In triangle PQR, Using Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (13)^2 = (12)^2 + QR^2$$

$$\Rightarrow QR^2 = 169 - 144 = 25$$

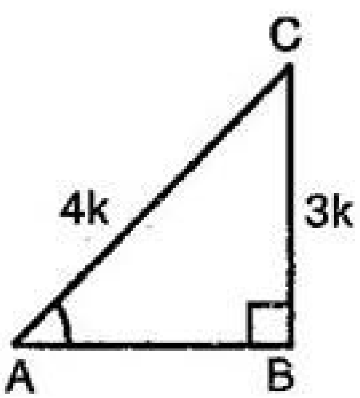
$\Rightarrow$ QR = 5 cm

$$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{12} - \frac{5}{12} = 0$$

---

3. If $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.

Ans. Given: A triangle ABC in which $\angle$ B = 90°



Let BC = $3k$ and AC = $4k$

Then, Using Pythagoras theorem,

AB = $\sqrt{(AC)^2-(BC)^2}$ = $\sqrt{(4k)^2-(3k)^2}$

= $\sqrt{16k^2-9k^2}$ = $k\sqrt{7}$

$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

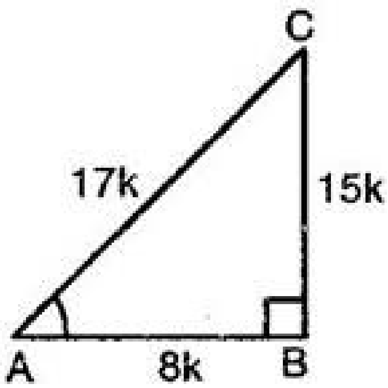
---

4. Given $15\cot A = 8$, find $\sin A$ and $\sec A$

Ans. Given: A triangle ABC in which $\angle$ B = 90°

$15\cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$



Let AB = $8k$ and BC = $15k$

Then using Pythagoras theorem,

AC = $\sqrt{(AB)^2+(BC)^2}$

= $\sqrt{(8k)^2+(15k)^2}$

= $\sqrt{64k^2+225k^2}$

= $\sqrt{289k^2}$ = $17k$

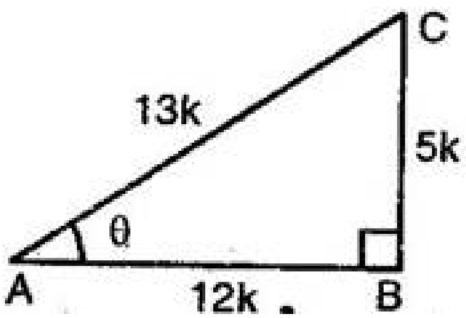
$\therefore \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{H}{B} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

---

5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Ans. Consider a triangle ABC in which $\angle$A = θ and $\angle$B = 90°



Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2 - (AB)^2}$

= $\sqrt{(13k)^2 - (12k)^2}$

= $\sqrt{169k^2 - 144k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin\theta = \frac{P}{H} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{B}{H} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{P}{B} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

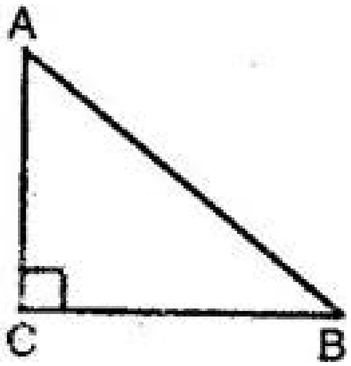
$\cot\theta = \frac{B}{P} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$\cos ec\theta = \frac{\text{H}}{\text{P}} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$

---

6. If $\angle$ And $\angle$ B are acute angles such that $\cos A = \cos B$, then show that $\angle$ A = $\angle$ B.

Ans. In right triangle ABC,



$\cos A = \frac{\text{AC}}{\text{AB}}$ and $\cos B = \frac{\text{BC}}{\text{AB}}$

But $\cos A = \cos B$ [Given]

$\Rightarrow \frac{\text{AC}}{\text{AB}} = \frac{\text{BC}}{\text{AB}}$

$\Rightarrow$ AC = BC

$\Rightarrow$ $\angle$ A = $\angle$ B

[Angles opposite to equal sides are equal]

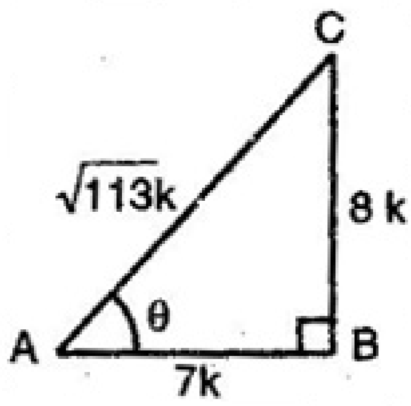
---

7. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°



Let AB = $7k$ and BC = $8k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2+(AB)^2}$

= $\sqrt{(8k)^2+(7k)^2}$

= $\sqrt{64k^2+49k^2}$

= $\sqrt{113k^2}$ = $\sqrt{113}k$

$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ = $\dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

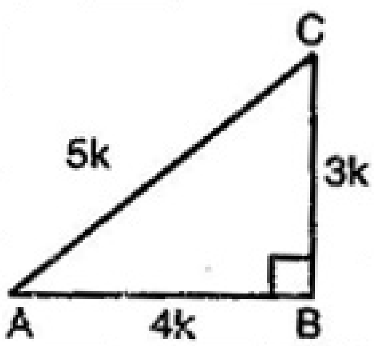
= $\dfrac{1-\dfrac{64}{113}}{1-\dfrac{49}{113}}$ = $\dfrac{113-64}{113-49}$ = $\dfrac{49}{64}$

(ii) $\cot^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{49/113}{64/113} = \dfrac{49}{64}$

---

8. If $3\cot A = 4$, **check whether** $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Ans. Consider a triangle ABC in which $\angle$ B = 90°.



And $3\cot A = 4$

$\Rightarrow \cot A = \dfrac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC = $\sqrt{(\text{BC})^2 + (\text{AB})^2}$

= $\sqrt{(3k)^2 + (4k)^2}$

= $\sqrt{16k^2 + 9k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin A = \dfrac{\text{BC}}{\text{AC}} = \dfrac{3k}{5k} = \dfrac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

And $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

= $\frac{16-9}{16+9}$ = $\frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A$ = $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

= $\frac{16}{25} - \frac{9}{25}$ = $\frac{7}{25}$

$\because$ L.H.S. = R.H.S.

$\therefore$ $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\cos^2 A - \sin^2 A$

---

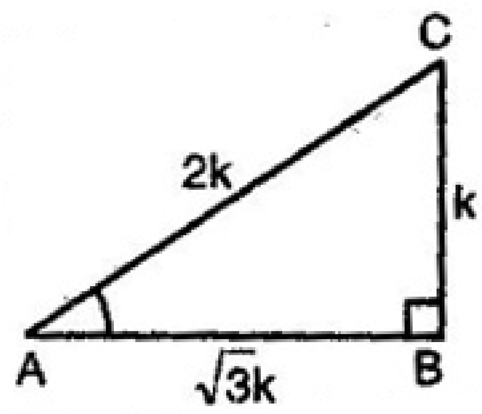
9. In ΔABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, find value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Ans. Consider a triangle ABC in which $\angle$B = 90°.

Let BC = k and AB = $\sqrt{3}k$



Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$

= $\sqrt{k^2 + 3k^2}$ = $\sqrt{4k^2}$ = $2k$

$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle$C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$ = $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

= $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$ = 1

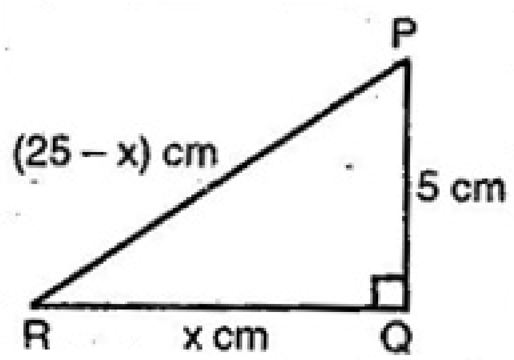
(ii) $\cos A \cos C - \sin A \sin C$ = $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

= $\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

---

10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P$, $\cos P$ **and** $\tan P$.

Ans. In ΔPQR, right angled at Q.



PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm, then PR = $(25 - x)$ cm

Using Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25 - x)^2 = (x)^2 + (5)^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = 12$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$\therefore \sin P = \frac{RQ}{RP} = \frac{12}{13}$

$\cos P = \frac{PQ}{RP} = \frac{5}{13}$

And $\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

---

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin \theta = \frac{4}{3}$ for some angle θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of ‘cot’ and A. ‘cot’ is separated from A has no meaning.

(v) False as $\sin \theta$ cannot be > 1.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.2

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1. Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Ans. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}$

$= \dfrac{3}{4} + \dfrac{1}{4} = \dfrac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= 2 + \dfrac{3}{4} - \dfrac{3}{4} = 2$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2+2\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)}$

$= \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\times 2(3-1)}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{4\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$= \dfrac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \dfrac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4}$

$= \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \dfrac{3\sqrt{3}-4}{3\sqrt{3}-4}$

$= \dfrac{27+16-24\sqrt{3}}{27-16}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \dfrac{43-24\sqrt{3}}{11}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$= \dfrac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \dfrac{5\times\frac{1}{4} + 4\times\frac{4}{3} - 1}{\frac{1}{4}+\frac{3}{4}} = \dfrac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}}$

$= \dfrac{15+64-12}{12} = \dfrac{67}{12}$

---

2. Choose the correct option and justify:

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ =

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

(iii) $\sin 2A = 2\sin A$ **is true when A** =

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ =

(A) $\cos 60^\circ$

(B) $\sin 60^\circ$

(C) $\tan 60^\circ$

(D) None of these

Ans. (i) (A) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$$= \frac{2 \times 1/\sqrt{3}}{1+\left(1/\sqrt{3}\right)^2}$$

$= \frac{2}{\sqrt{3}} \times \frac{3}{3+1}$ = $\frac{\sqrt{3}}{2}$ = $\sin 60^\circ$

(ii) (D) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ = $\frac{1-1}{1+1}$ = $\frac{0}{2}$ = 0

(iii) (A) Since A = 0, then

$\sin 2A = \sin 0^\circ = 0$ and $2\sin A = 2\sin 0^\circ$

= $2 \times 0 = 0$

$\therefore$ $\sin 2A = \sin A$ when A = 0

(iv) (C) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$= \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ = $\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{3-1}$ = $\sqrt{3} = \tan 60^\circ$

---

3. If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^\circ < A+B \le 90^\circ$; $A > B$, **find A and B.**

Ans. $\tan(A+B) = \sqrt{3}$

$\Rightarrow$ $\tan(A+B) = \tan 60^\circ$

$\Rightarrow$ A + B = 60° ....(i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^\circ$

$\Rightarrow$ A – B = 30°..........(ii)

On adding eq. (i) and (ii), we get,

2A = $90^\circ \Rightarrow$ A = 45°

On Subtracting eq. (i) and eq. (ii), we get

2B = $30^\circ \Rightarrow$ B = 15°

---

4. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ **increases as** θ **increases.**

(iii) The value of $\cos\theta$ **increases as** θ **increases.**

(iv) $\sin\theta = \cos\theta$ **for all values of** θ.

(v) $\cot A$ **is not defined for** $A = 0^\circ$.

Ans. (i) False, because, let A = 60° and B = 30°

Then, $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ$ = 1

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$ = $\frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

(ii) True, because it is clear from the table below:

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Therefore, it is clear, the value of $\sin\theta$ increases as θ increases.

(iii) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

(v) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

= $\frac{1}{0}$ i.e. undefined.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.3

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1. Evaluate:

(i) $\dfrac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\dfrac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\cos ec 31^\circ - \sec 59^\circ$

Ans. (i) $\dfrac{\sin 18^\circ}{\cos 72^\circ}$ = $\dfrac{\sin\left(90^\circ - 72^\circ\right)}{\cos 72^\circ}$

= $\dfrac{\cos 72^\circ}{\cos 72^\circ}$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

= 1

(ii) $\dfrac{\tan 26^\circ}{\cot 64^\circ}$ = $\dfrac{\tan\left(90^\circ - 64^\circ\right)}{\cot 64^\circ}$

= $\dfrac{\cot 64^\circ}{\cot 64^\circ}$ [Since $\tan(90^\circ - \theta) = \cot\theta$]

= 1

(iii) $\cos 48^\circ - \sin 42^\circ$

= $\cos\left(90^\circ - 42^\circ\right) - \sin 42^\circ$

= $\sin 42° - \sin 42°$ [Since $\cos(90° - \theta) = \sin\theta$]

= 0

(iv) $\operatorname{cosec} 31° - \sec 59°$

= $\operatorname{cosec}(90° - 59°) - \sec 59°$

= $\sec 59° - \sec 59°$ [Since $\operatorname{cosec}(90° - \theta) = \sec\theta$]

=0

---

2. Show that:

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$

Ans. (i) L.H.S. $\tan 48° \tan 23° \tan 42° \tan 67°$

= $\tan(90° - 42°) \tan(90° - 67°) \tan 42° \tan 67°$

= $\cot 42° \cot 67° \tan 42° \tan 67°$

= $\frac{1}{\tan 42°} \cdot \frac{1}{\tan 67°} \cdot \tan 42° \cdot \tan 67°$ = 1 = R.H.S.

(ii) R.H.S. $\cos 38° \cos 52° - \sin 38° \sin 52°$

= $\cos(90° - 52°) \cdot \cos(90° - 38°) - \sin 38° \cdot \sin 52°$

= $\sin 52° \sin 38° - \sin 38° \sin 52°$ = 0 = R.H.S.

---

3. If $\tan 2A = \cot(A - 18°)$, where 2A is an acute angle, find the value of A.

Ans. Given: $\tan 2A = \cot(A - 18°)$

$\Rightarrow \cot(90° - 2A) = \cot(A - 18°)$ [Since $\tan(90° - \theta) = \cot\theta$]

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow -2A - A = -18^\circ - 90^\circ$

$\Rightarrow -3A = -108^\circ$

$\Rightarrow$ A = 36°

---

4. If $\tan A = \cot B$, **prove that** $A + B = 90^\circ$.

Ans. Given: $\tan A = \cot B$

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow \quad 90^\circ = \mathrm{A} + \mathrm{B}$

$\Rightarrow$ A + B = 90°

---

5. If $\sec 4A = \cos ec(A - 20^\circ)$, **where 4A is an acute angle, find the value of A.**

Ans. Given: $\sec 4A = \cos ec(A - 20^\circ)$

$\Rightarrow \cos ec(90^\circ - 4A) = \cos ec(A - 20^\circ)$ [Since $\sec(90^\circ - \theta) = \cos ec\theta$]

$\Rightarrow 90^\circ - 4A = A - 20^\circ$

$\Rightarrow -4A - A = -20^\circ - 90^\circ$

$\Rightarrow -5A = -110^\circ$

$\Rightarrow$ A = 22°

---

6. If A, B and C are interior angles of a Δ **ABC, then show that** $\sin\left(\frac{\mathrm{B}+\mathrm{C}}{2}\right) = \cos\frac{\mathrm{A}}{2}$.

Ans. Given: A, B and C are interior angles of a ΔABC.

$\therefore$ A + B + C = 180° [Triangle sum property]

Dividing both sides by 2, we get

$$\Rightarrow \frac{A+B+C}{2} = 90^\circ$$

$$\Rightarrow \frac{A}{2} + \frac{B+C}{2} = 90^\circ$$

$$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

---

7. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Ans. $\sin 67^\circ + \cos 75^\circ$

= $\sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$ [Since $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$]

= $\cos 23^\circ + \sin 15^\circ$

CBSE Class–10 Mathematics
NCERT solution
Chapter - 8
Introduction to Trigonometry - Exercise 8.4

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1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans. For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

---

2. Write the other trigonometric ratios of A in terms of $\sec A$

Ans. For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,

$$\cos A = \frac{1}{\sec A}$$

For $\tan A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\cos ecA$,

$$\cos ecA = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

For $\cot A$,

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

---

3. Evaluate:

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$$

$$= \frac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$$

$$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$$

$$= \frac{1}{1} = 1 \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$$= \sin 25°.\cos(90° - 25°) + \cos 25°.\sin(90° - 25°)$$

$$= \sin 25°.\sin 25° + \cos 25°.\cos 25°$$

$\left[\because \sin\left(90^\circ - \theta\right) = \cos\theta, \cos\left(90^\circ - \theta\right) = \sin\theta\right]$

= $\sin^2 25^\circ + \cos^2 25^\circ$ = 1

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

---

4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $\left(1 + \tan\theta + \sec\theta\right)\left(1 + \cot\theta - \cos ec\theta\right)$ =

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $\left(\sec A + \tan A\right)\left(1 - \sin A\right)$ =

(A) $\sec A$

(B) $\sin A$

(C) $\cos ecA$

(D) $\cos A$

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$ =

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$

= $9\left(\sec^2 A - \tan^2 A\right)$

= $9 \times 1 = 9$ [Since $\sec^2\theta - \tan^2\theta = 1$]

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

= $\left(1+\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\cos\theta}\right)\left(1+\dfrac{\cos\theta}{\sin\theta}-\dfrac{1}{\sin\theta}\right)$

= $\left(\dfrac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\dfrac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\dfrac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\dfrac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\dfrac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

= $\dfrac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$ = 2

(iii)(D) $(\sec A+\tan A)(1-\sin A)$

= $\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

= $\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

= $\frac{1-\sin^2 A}{\cos A}$ [Since $(a+b)(a-b)=a^2-b^2$]

= $\frac{\cos^2 A}{\cos A}$

= $\cos A$ $\left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv)(D) $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\frac{\sec^2 A-\tan^2 A+\tan^2 A}{\operatorname{cosec}^2 A-\cot^2 A+\cot^2 A}$

= $\frac{\sec^2 A}{\operatorname{cosec}^2 A}$ = $\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

= $\frac{\sin^2 A}{\cos^2 A}$ = $\tan^2 A$

---

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A$, **using the identity** $\operatorname{cosec}^2 A=1+\cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$

$=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)$

$=\frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Ans. (i) L.H.S. $(\operatorname{cosec}\theta-\cot\theta)^2$

$=\operatorname{cosec}^2\theta+\cot^2\theta-2\operatorname{cosec}\theta\cot\theta$ [Since $(a-b)^2=a^2+b^2-2ab$]

$=\frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}-2\times\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}$

$= \dfrac{1+\cos^2\theta}{\sin^2\theta} - \dfrac{2\cos\theta}{\sin^2\theta}$

$= \dfrac{1+\cos^2\theta-2\cos\theta}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$ $\left[\because a^2+b^2-2ab=(a-b)^2\right]$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$ = R.H.S.

(ii) L.H.S. $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}$

$= \dfrac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$

$= \dfrac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$

$= \dfrac{1+1+2\sin A}{(1+\sin A)\cos A}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \dfrac{2+2\sin A}{(1+\sin A)\cos A}$ = $\dfrac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \dfrac{2}{\cos A}$ = $2\sec A$ = R.H.S

(iii) L.H.S. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}+\dfrac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}+\dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta}\times\dfrac{\sin\theta}{\sin\theta-\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\times\dfrac{\cos\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\dfrac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\dfrac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$\left[\because a^3-b^3=(a-b)(a^2+b^2+ab)\right]$

$= \dfrac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \dfrac{1}{\sin\theta\cos\theta}+\dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}+1 = 1+\dfrac{1}{\sin\theta\cos\theta}$

$= 1+\sec\theta\,\mathrm{cosec}\,\theta$

(iv) L.H.S. $\frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \times \frac{\cos A}{1} = 1+\cos A$

$= 1+\cos A \times \frac{1-\cos A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos A}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \frac{\sin^2 A}{1-\cos A}$ = R.H.S.

(v) L.H.S. $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$= \frac{\cot A-1+\cos ecA}{\cot A+1-\cos ecA} = \frac{\cot A+\cos ecA-1}{\cot A-\cos ecA+1}$

$= \frac{(\cot A+\cos ecA)-(\cos ec^2 A-\cot^2 A)}{(1+\cot A-\cos ecA)}$ [Since $\cos ec^2\theta - \cot^2\theta = 1$]

$= \frac{(\cot A+\cos ecA)+(\cot^2 A-\cos ec^2 A)}{(1+\cot A-\cos ecA)}$

$= \frac{(\cot A+\cos ecA)+(\cot A+\cos ecA)(\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \frac{(\cot A+\cos ecA)(1+\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \cot A+\cos ecA$ = R.H.S.

(vi) L.H.S. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$ $[\because 1-\sin^2\theta = \cos^2\theta]$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \sec A + \tan A$ = R.H.S.

(vii) L.H.S. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta[2(1-\sin^2\theta)-1]}$

$[\because 1-\sin^2\theta = \cos^2\theta]$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)} = \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$ = R.H.S

(viii) L.H.S. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \left(\sin A + \dfrac{1}{\sin A}\right)^2 + \left(\cos A + \dfrac{1}{\cos A}\right)^2$

$= \sin^2 A + \dfrac{1}{\sin^2 A} + 2.\sin A.\dfrac{1}{\sin A} + \cos^2 A + \dfrac{1}{\cos^2 A} + 2.\cos A.\dfrac{1}{\cos A}$

$= 2 + 2 + \sin^2 A + \cos^2 A + \dfrac{1}{\sin^2 A} + \dfrac{1}{\cos^2 A}$

$= 4 + 1 + \dfrac{1}{\sin^2 A} + \dfrac{1}{\cos^2 A}$

$= 5 + \operatorname{cosec}^2 A + \sec^2 A$

$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$

$\left[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta, \sec^2\theta = 1 + \tan^2\theta\right]$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.

(ix) L.H.S. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \left(\dfrac{1 - \sin^2 A}{\sin A}\right)\left(\dfrac{1 - \cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A}{\sin A} \times \dfrac{\sin^2 A}{\cos A} = \sin A.\cos A$

$= \dfrac{\sin A.\cos A}{\sin^2 A + \cos^2 A}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

Dividing all the terms by $\sin A.\cos A$,

$$= \dfrac{\dfrac{\sin A.\cos A}{\sin A.\cos A}}{\dfrac{\sin^2 A}{\sin A.\cos A} + \dfrac{\cos^2 A}{\sin A.\cos A}}$$

$$= \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$= \dfrac{1}{\tan A + \cot A}$ = R.H.S.

(x) L.H.S. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\cos ec^2 A}$

$[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta]$

$= \dfrac{1}{\cos^2 A} \times \dfrac{\sin^2 A}{1}$ = $\tan^2 A$ = R.H.S.

Now, Middle side = $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2$ = $\left(\dfrac{1-\tan A}{1-\dfrac{1}{\tan A}}\right)^2$

$$= \left(\dfrac{1-\tan A}{\dfrac{\tan A - 1}{\tan A}}\right)^2$$

$= \left(\dfrac{1-\tan A}{\dfrac{-(1-\tan A)}{\tan A}}\right)^2$ = $(-\tan A)^2$

$= \tan^2 A$ = R.H.S.

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Chapter - 9

Some Applications of Trigonometry - Exercise 9.1

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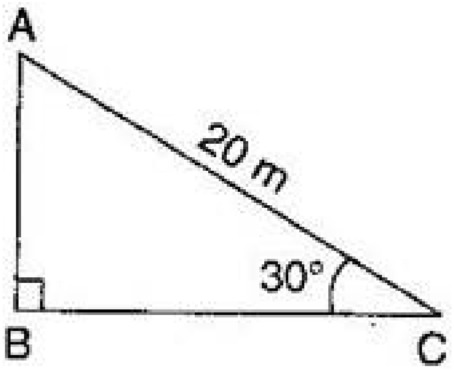
1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).



Ans. In right triangle ABC,

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{AB}{20}$$

AB= 20/2

$\Rightarrow$ AB = 10 m

Hence, the height of the pole is 10 m.

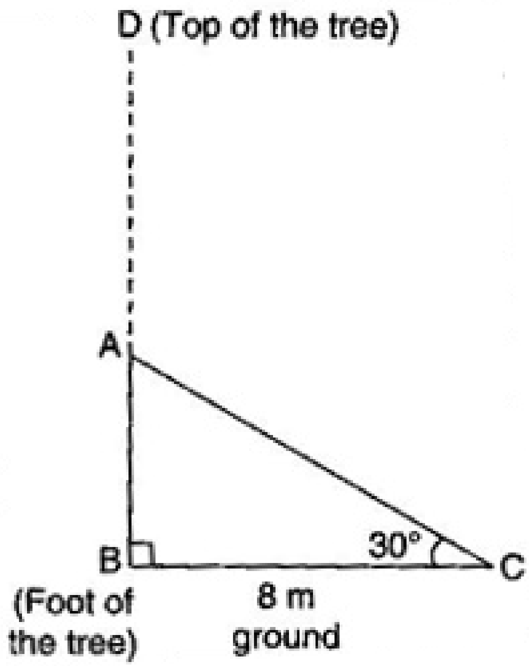
---

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Ans. Let AC be the broken part of tree

In right triangle ABC,

$$\cos 30^\circ = \frac{BC}{AC}$$



$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$$

$$\Rightarrow AC = \frac{16}{\sqrt{3}} \text{ m}$$

Again, $\tan 30^\circ = \frac{AB}{BC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$\Rightarrow AB = \frac{8}{\sqrt{3}} \text{ m}$$

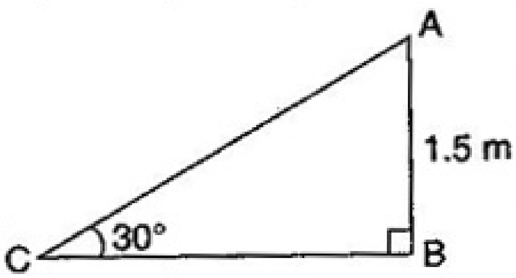
∴ Height of the tree = AB + AD=AB+AC

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$$

$= \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3}$ m

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Ans. In right triangle ABC,

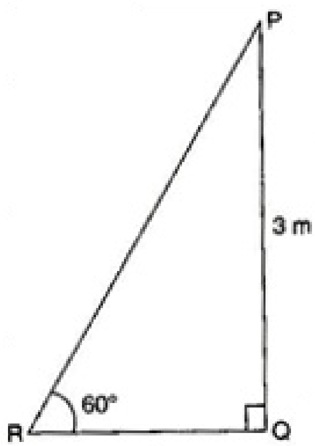


$\sin 30^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{1.5}{AC}$

$\Rightarrow$ AC = 3 m

In right triangle PQR,



$\sin 60^\circ = \frac{PQ}{PR}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{PR}$

$\Rightarrow$ PR = $2\sqrt{3}$ m

Hence, the lengths of the slides are 3 m and $2\sqrt{3}$ m respectively.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 30°. Find the height of the tower.

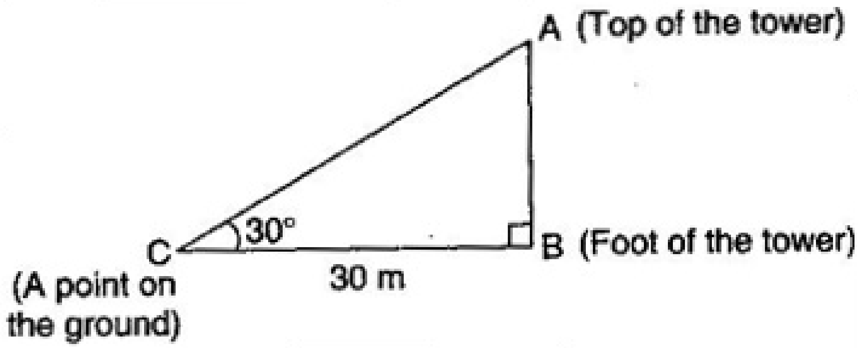
Ans. In right triangle ABC, AB be the height of the tower.

$\tan 30^\circ = \frac{AB}{BC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

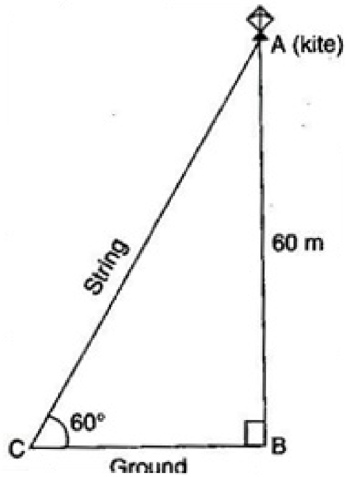
$$\Rightarrow AB = \frac{30}{\sqrt{3}} \text{ m}$$

$$\Rightarrow \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$



5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Ans. In right triangle ABC, AC is the length of the string



$$\sin 60^\circ = \frac{AB}{AC}$$

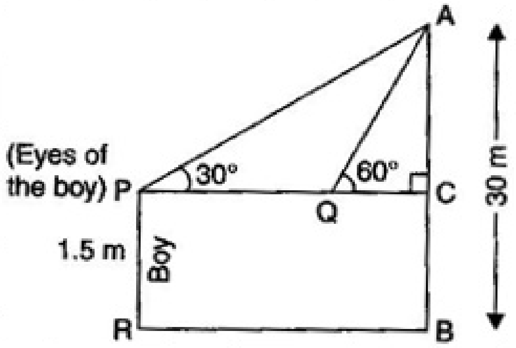
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$\Rightarrow AC = 40\sqrt{3} \text{ m}$$

Hence the length of the string is $40\sqrt{3}$ m.

6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Ans. AB = 30 m and PR = 1.5 m



AC = AB – BC

= AB – PR (As, BC = PR)

= 30 – 1.5

= 28.5 m

In right triangle ACQ,

$$\tan 60^\circ = \frac{AC}{QC}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{QC} \Rightarrow QC = \frac{28.5}{\sqrt{3}} \text{ m}$$

In right triangle ACP,

$$\tan 30^\circ = \frac{AC}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ + QC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ + \frac{28.5}{\sqrt{3}}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5 \times \sqrt{3}}{PQ\sqrt{3} + 28.5}$$

$$\Rightarrow PQ\sqrt{3} + 28.5 = 85.5$$

$$\Rightarrow PQ\sqrt{3} = 57$$

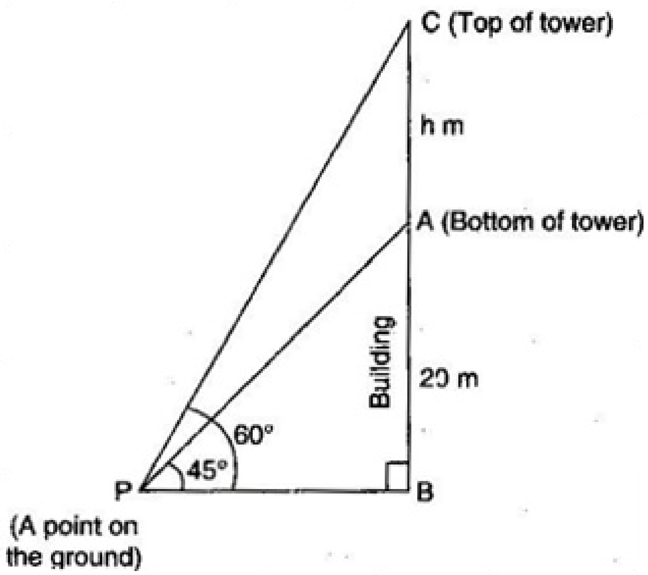
$$\Rightarrow PQ = \frac{57}{\sqrt{3}}$$

$$\Rightarrow PQ = \frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 19\sqrt{3} \text{ m}$$

Hence, the distance the boy walked towards the building is $19\sqrt{3}$ m.

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans. Let the height of the tower be h m. Then, in right triangle CBP,



$$\tan 60^\circ = \frac{BC}{BP}$$

$$\Rightarrow \sqrt{3} = \frac{AB + AC}{BP}$$

$$\Rightarrow \sqrt{3} = \frac{20 + h}{BP} \text{ ....(i)}$$

In right triangle ABP,

$$\tan 45^\circ = \frac{AB}{BP}$$

$$\Rightarrow 1 = \frac{20}{BP} \Rightarrow BP = 20 \text{ m}$$

Putting this value in eq. (i), we get,

$\sqrt{3} = \dfrac{20+h}{20}$

$\Rightarrow 20\sqrt{3} = 20 + h$

$\Rightarrow h = 20\sqrt{3} - 20$

$\Rightarrow h = 20\left(\sqrt{3} - 1\right)$ m

$\therefore$ The height of the tower is $20\left(\sqrt{3}-1\right)$ m.

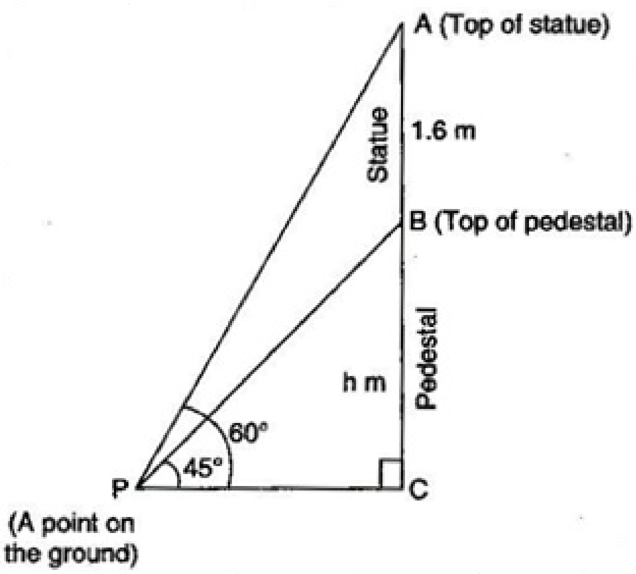
---

8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Ans. Let the height of the pedestal be h m.

$\therefore$ BC = h m

In right triangle ACP,



$\tan 60^\circ = \dfrac{AC}{PC}$

$$\Rightarrow \sqrt{3} = \frac{AB + BC}{PC}$$

$$\Rightarrow \sqrt{3} = \frac{1.6 + h}{PC} \quad ....(i)$$

In right triangle BCP,

$$\tan 45^\circ = \frac{BC}{PC}$$

$$\Rightarrow 1 = \frac{h}{PC} \Rightarrow PC = h$$

$$\therefore \sqrt{3} = \frac{1.6 + h}{h} \quad \text{[From eq. (i)]}$$

$$\Rightarrow \sqrt{3}h = 1.6 + h$$

$$\Rightarrow h\left(\sqrt{3} - 1\right) = 1.6$$

$$\Rightarrow h = \frac{1.6}{\sqrt{3} - 1}$$

$$\Rightarrow h = \frac{1.6\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)}$$

$$\Rightarrow h = \frac{1.6\sqrt{3} + 1}{3 - 1}$$

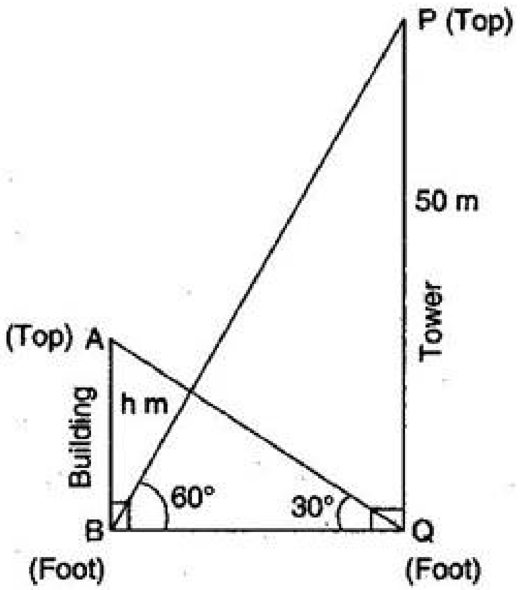
$$\Rightarrow h = \frac{1.6\left(\sqrt{3} + 1\right)}{2}$$

$$\Rightarrow h = 0.8\left(\sqrt{3} + 1\right) \text{ m}$$

Hence, the height of the pedestal is $0.8\left(\sqrt{3} + 1\right)$ m.

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Ans. Let the height of the building be h m.



In right triangle PQB,

$$\tan 60^\circ = \frac{PQ}{BQ} \Rightarrow \sqrt{3} = \frac{50}{BQ}$$

$\Rightarrow$ BQ = $\frac{50}{\sqrt{3}}$ m..........(i)

In right triangle ABQ,

$$\tan 30^\circ = \frac{AB}{BQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BQ}$$

$\Rightarrow$ BQ = $h\sqrt{3}$ m..........(ii)

From eq. (i) and (ii),

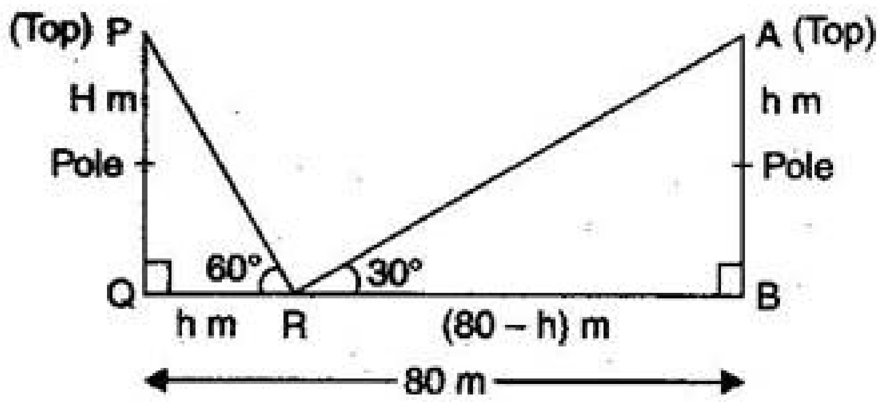
$$h\sqrt{3} = \frac{50}{\sqrt{3}} \Rightarrow h = \frac{50}{3} = 16\frac{2}{3} \text{ m}$$

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Ans. Let the height of each poles be H m

AB = PQ = H

In right triangle PRQ,



(in figure change AB= H m rather than h m)

$$\tan 60^\circ = \frac{PQ}{QR} \Rightarrow \sqrt{3} = \frac{H}{h}$$

$\Rightarrow$ H = $h\sqrt{3}$ m..........(i)

In right triangle ABR,

$$\tan 30^\circ = \frac{AB}{BR}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H}{80-h}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h\sqrt{3}}{80-h} \text{ [From eq. (i)]}$$

$$\Rightarrow 80 - h = 3h$$

$\Rightarrow 4h = 80$

$\Rightarrow h = 20$ m

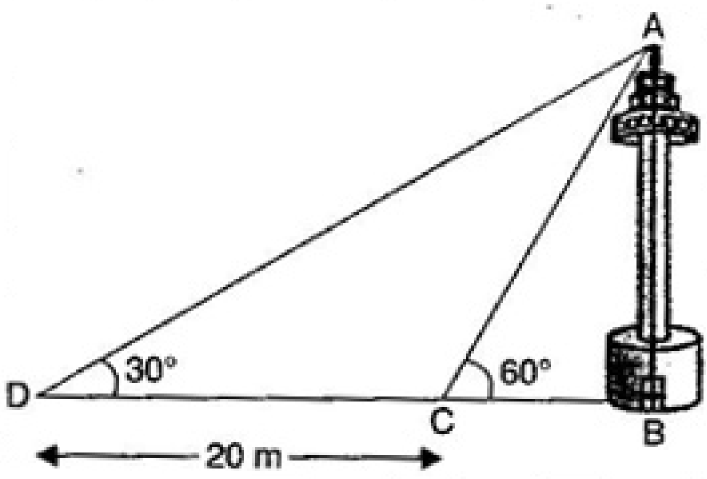
$\therefore$ H = $h\sqrt{3}$ = $20\sqrt{3}$ m

Also, BR = $80 - h$ = 80 – 20 = 60 m

Hence the heights of the poles are $20\sqrt{3}$ m each and the distances of the point from poles are 20 m and 60 m respectively.

---

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the canal.



Ans. Let AB be the TV tower .

In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{BC}$$

$\Rightarrow$ AB = $BC\sqrt{3}$ m..........(i)

In right triangle ABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$

$\Rightarrow$ AB = $\frac{BC + 20}{\sqrt{3}}$ m……….(ii)

From eq. (i) and (ii),

$BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$

$\Rightarrow$ 3BC = BC + 20

$\Rightarrow$ BC = 10 m

From eq. (i), AB = $10\sqrt{3}$ m

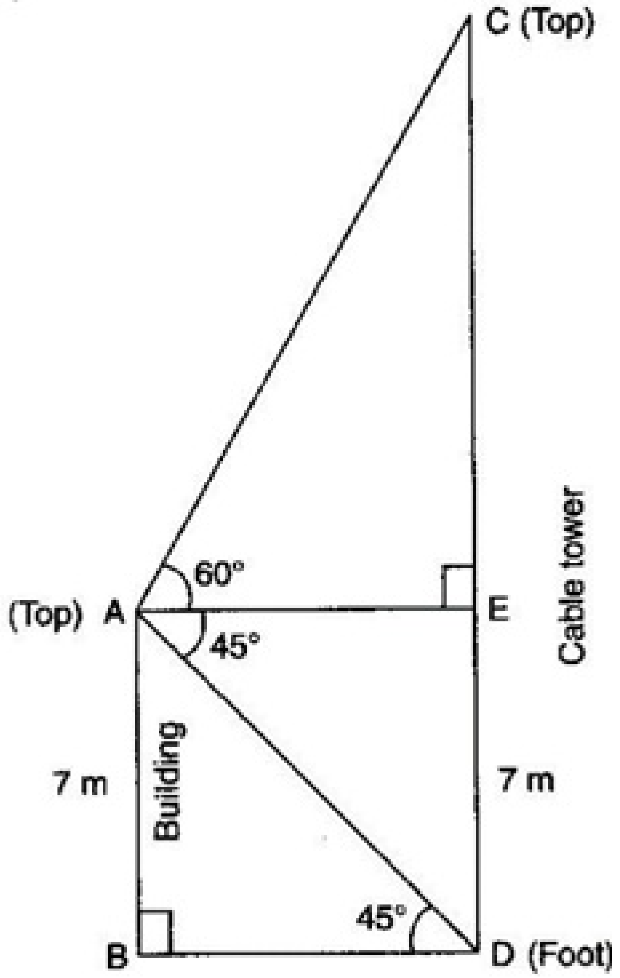
Hence height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

---

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Ans. In right triangle ABD,

$\tan 45^\circ = \frac{AB}{BD}$



$\Rightarrow 1 = \frac{7}{BD}$

$\Rightarrow$ BD = 7 m

$\Rightarrow$ AE = 7 m

In right triangle AEC,

$\tan 60^\circ = \frac{CE}{AE}$

$\Rightarrow \sqrt{3} = \frac{CE}{7}$

$\Rightarrow$ CE = $7\sqrt{3}$ m

$\therefore$ CD = CE + ED

$= CE + AB$ (As $AB = ED$)

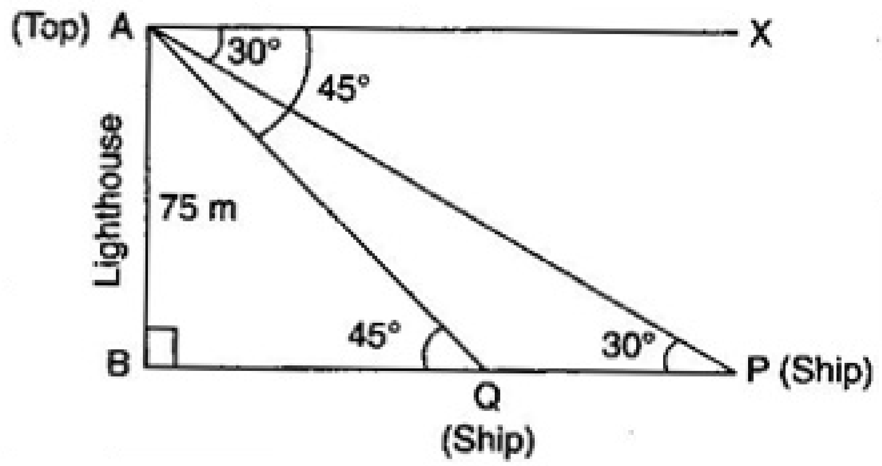
$= 7\sqrt{3} + 7 = 7\left(\sqrt{3}+1\right)$ m

Hence height of the tower is $7\left(\sqrt{3}+1\right)$ m.

---

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.

Ans. In right triangle ABQ,



$$\tan 45^\circ = \frac{AB}{BQ}$$

$$\Rightarrow 1 = \frac{75}{BQ}$$

$\Rightarrow$ BQ = 75 m..........(i)

In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BQ + QP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{75 + QP} \quad \text{[From eq. (i)]}$$

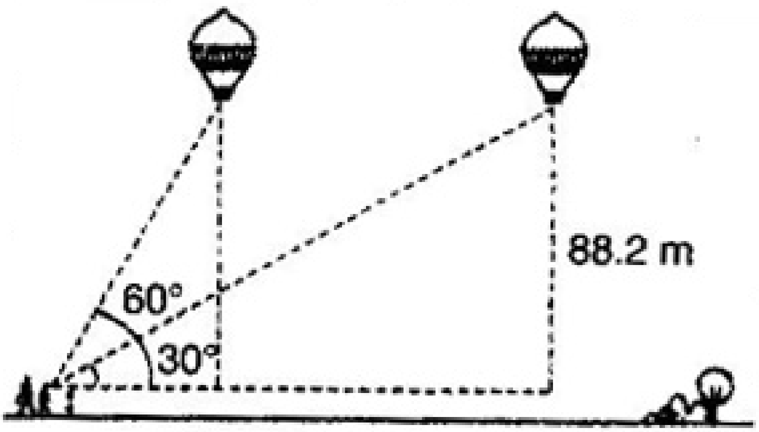
$$\Rightarrow 75 + QP = 75\sqrt{3}$$

$$\Rightarrow QP = 75\left(\sqrt{3} - 1\right) \text{ m}$$

Hence the distance between the two ships is $75\left(\sqrt{3} - 1\right)$ m.

---

14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any distant is 60^0 After some time, the angle of elevation reduces to 30^0 (see figure). Find the distance travelled by the balloon during the interval.

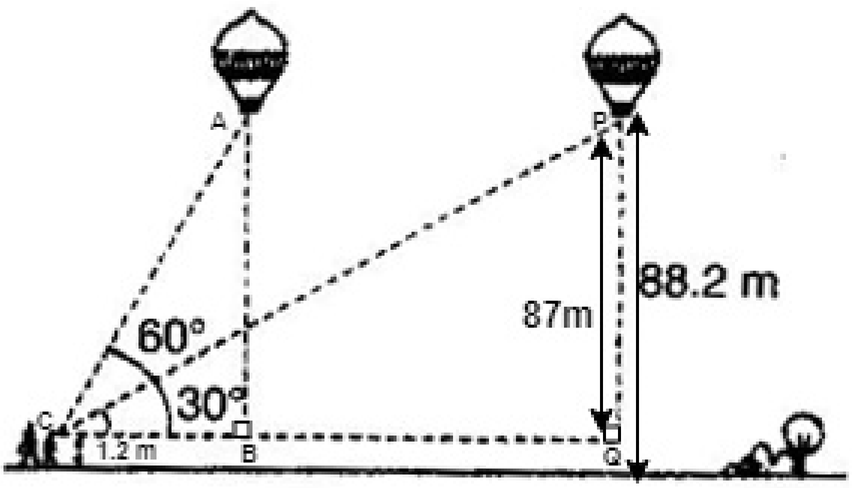


Ans. As, per question;

AB = PQ = 88.2 - 1.2 = 87 m

In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC}$$



$\Rightarrow \sqrt{3} = \frac{87}{BC}$

$\Rightarrow BC = \frac{87}{\sqrt{3}} = 29\sqrt{3}$ m

In right triangle PQC,

$$\tan 30^\circ = \frac{PQ}{CQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{29\sqrt{3}+BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{88.2}{\frac{88.2}{\sqrt{3}}+BQ}$$

$$\Rightarrow 29\sqrt{3} + BQ = 87\sqrt{3}$$

$$\Rightarrow BQ = 58\sqrt{3} \text{ m}$$

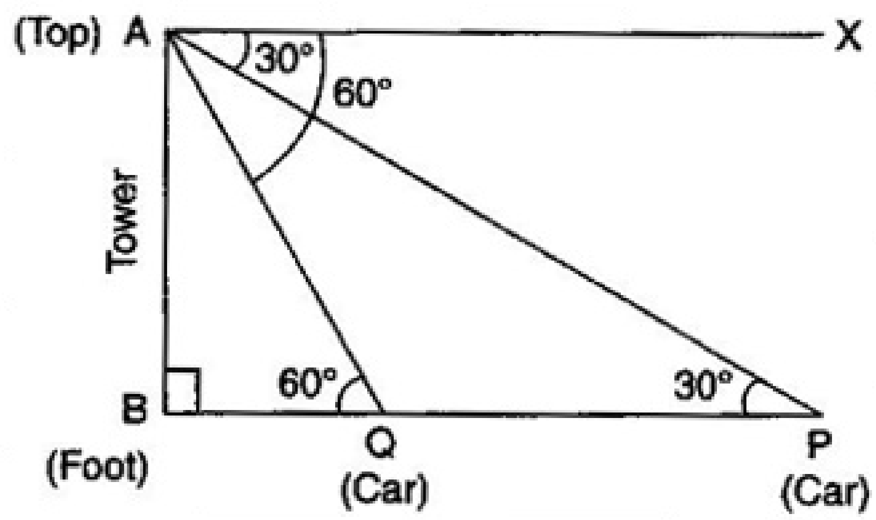
Hence the distance travelled by the balloon during the interval is $58\sqrt{3}$ m.

---

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Ans. In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BP}$$

$\Rightarrow$ BP = AB $\sqrt{3}$ ....(i)

In right triangle ABQ,

$$\tan 60^\circ = \frac{AB}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$$

$\Rightarrow$ BQ = $\frac{AB}{\sqrt{3}}$ ....(ii)

$\because$ PQ = BP – BQ

$\therefore$ PQ = AB $\sqrt{3}$ $-$ $\frac{AB}{\sqrt{3}}$

$= \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}}$ = 2BQ [From eq. (ii)]

$\Rightarrow$ BQ = $\frac{1}{2}$ PQ

$\because$ Time taken by the car to travel a distance PQ = 6 seconds.

$\therefore$ Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}$ PQ = $\frac{1}{2}$ x 6 = 3 seconds.

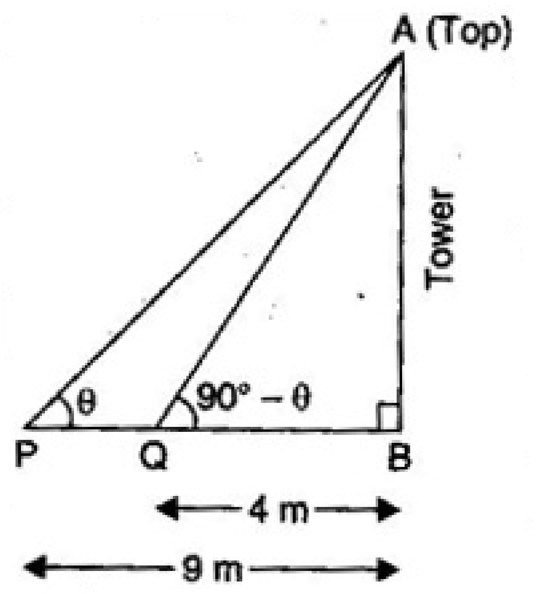
Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

---

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Ans. Let $\angle$ APB = θ

Then, $\angle$ AQB = $\left(90^\circ - \theta\right)$



[$\angle$ APB and $\angle$ AQB are complementary]

In right triangle ABP,

$$\tan\theta = \frac{AB}{PB}$$

$$\Rightarrow \tan\theta = \frac{AB}{9} \text{ ....(i)}$$

In right triangle ABQ,

$$\tan(90^\circ - \theta) = \frac{AB}{QB}$$

$$\Rightarrow \cot\theta = \frac{AB}{4} \text{ ....(ii)}$$

Multiplying eq. (i) and eq. (ii),

$$\frac{AB}{9}.\frac{AB}{4} = \tan\theta.\cot\theta$$

$$\Rightarrow \frac{AB^2}{36} = 1 \Rightarrow AB^2 = 36$$

$$\Rightarrow AB = 6 \text{ m}$$

Hence, the height of the tower is 6 m.

Proved.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 10

Circles - Exercise 10.1

1. How many tangents can a circle have?

Ans. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

2. Fill in the blanks:

(i) A tangent to a circle intersects it in _______________ point(s).
(ii) A line intersecting a circle in two points is called a _______________.

(iii) A circle can have _______________ parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called _________.

Ans. (i) A tangent to a circle intersects it in <u>exactly one</u> point.
(ii) A line intersecting a circle in two points is called a <u>secant</u>.
(iii) A circle can have <u>two</u> parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called <u>point of contact</u>.

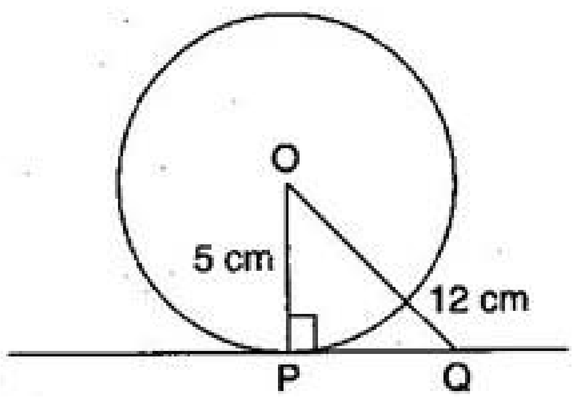
3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Ans. (D) $\because$ PQ is the tangent and OP is the radius through the point of contact.

$\therefore \angle OPQ = 90^\circ$ [The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ In right triangle OPQ,



$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$\Rightarrow (12)^2 = (5)^2 + PQ^2$

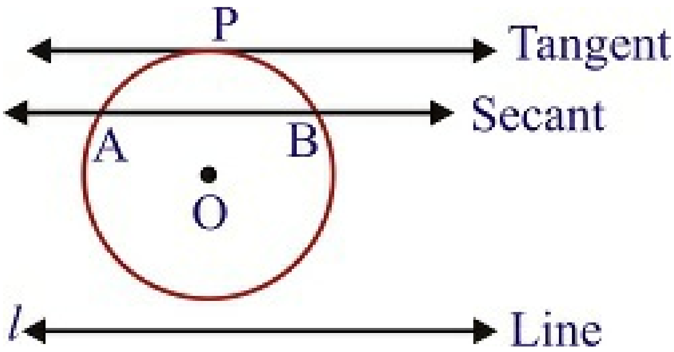
$\Rightarrow 144 = 25 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans.



CBSE Class–10 Mathematics

NCERT solution

Chapter - 10

Circles - Exercise 10.2

In Q 1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

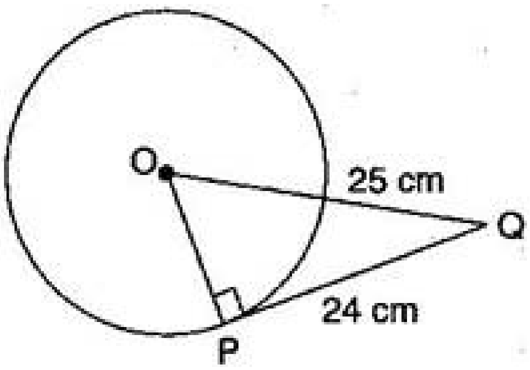
(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Ans. (A)

$\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]



$\therefore$ In right triangle OPQ,

$OQ^2 = OP^2 + PQ^2$

[By Pythagoras theorem]

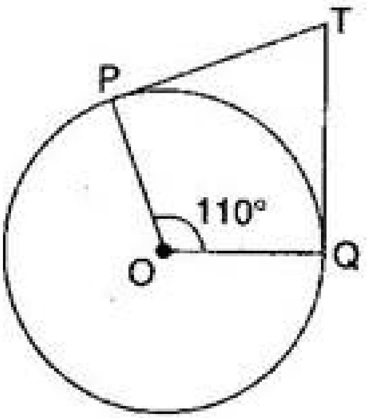
$\Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow 625 = OP^2 + 576$

$\Rightarrow$ $OP^2 = 625 - 576 = 49$

$\Rightarrow$ OP = 7 cm

2. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$POQ = 110°, then $\angle$PTQ is equal to:



(A) 60° **(B)** 70° **(C)** 80° **(D)** 90°

Ans. (B)

$\angle$POQ = 110°, $\angle$OPT = 90° and $\angle$OQT = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In quadrilateral OPTQ,

$\angle$POQ + $\angle$OPT + $\angle$OQT + $\angle$PTQ = 360°

[Angle sum property of quadrilateral]

$\Rightarrow$ $110^\circ + 90^\circ + 90^\circ$ + $\angle$PTQ = 360°

$\Rightarrow$ 290° + $\angle$PTQ = 360°

$\Rightarrow$ $\angle$PTQ = $360^\circ - 290^\circ$

$\Rightarrow$ $\angle$PTQ = 70°

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to:

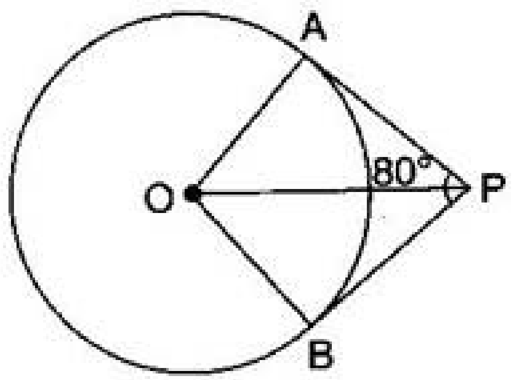
(A) 50° **(B)** 60° **(C)** 70° **(D)** 80°

Ans. (A)

$\because$ $\angle$ OAP = 90°

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]



$\angle$ OPA $= \frac{1}{2} \angle$ BPA $= \frac{1}{2} \times 80^\circ = 40^\circ$

[Centre lies on the bisector of the

angle between the two tangents]

In $\triangle$ OPA,

$\angle$ OAP + $\angle$ OPA + $\angle$ POA = 180°

[Angle sum property of a triangle]

$\Rightarrow 90^\circ + 40^\circ$ + $\angle$ POA = 180°

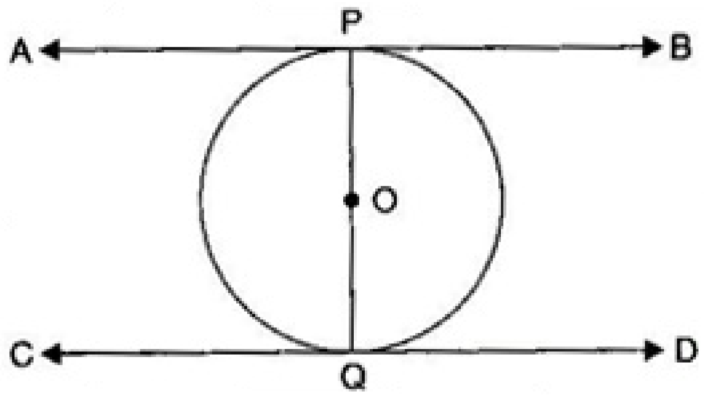
$\Rightarrow$ 130° + $\angle$ POA = 180°

$\Rightarrow$ $\angle$ POA = 50°

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans. Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.



To Prove: AB $\parallel$ CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle OPA = 90^\circ$ ....(i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle OQD = 90^\circ$ ....(ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

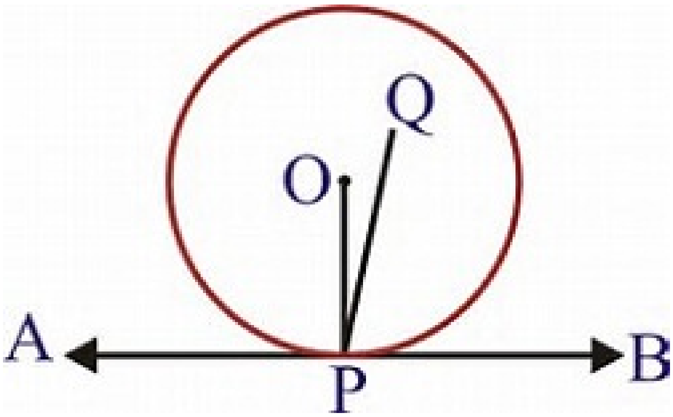
From eq. (i) and (ii), $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

$\therefore$ AB || CD

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans. Let AB be the tangent drawn at the point P on the circle with O.



If possible, let PQ be perpendicular to AB, not passing through O.

Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, $AB \perp OP$ $\Rightarrow$ $\angle OPB = 90^{o}$

Also, $\angle QPB = 90^{O}$ [By construction]

Therefore, $\angle QPB = \angle OPB$, which is not possible as a part cannot be equal to whole.

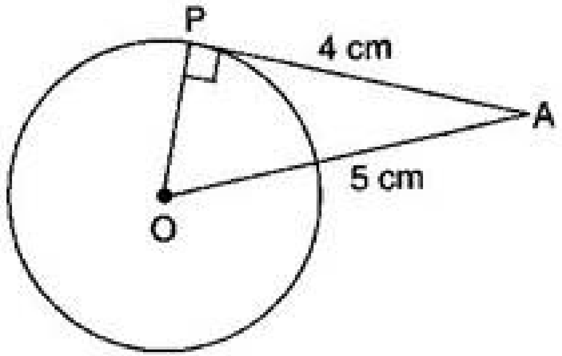
Thus, it contradicts our supposition.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

---

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Ans. We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.



$\therefore \angle$ OPA = 90°

$\therefore OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (\text{OP})^2 + (4)^2$

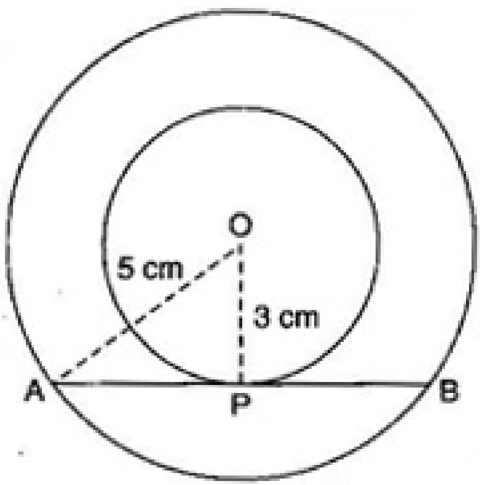
$\Rightarrow 25 = (\text{OP})^2 + 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow$ OP = 3 cm

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = 4 cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

$\Rightarrow$ AB = AP + BP

= AP + AP = 2AP

= 2×4 = 8 cm

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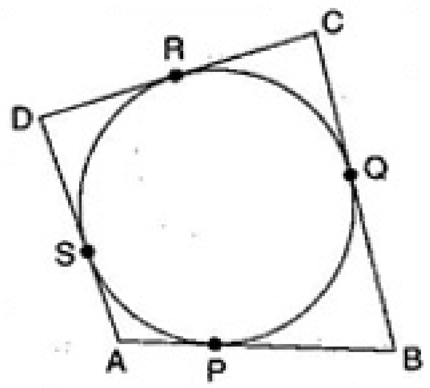
8. A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

AB + CD = AD + BC



Ans. We know that the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

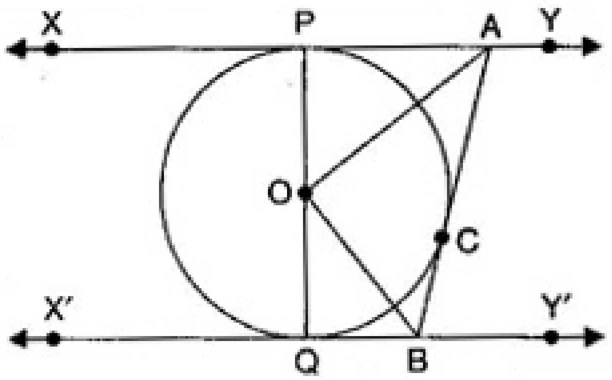
= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

---

9. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle$ AOB = 90°.

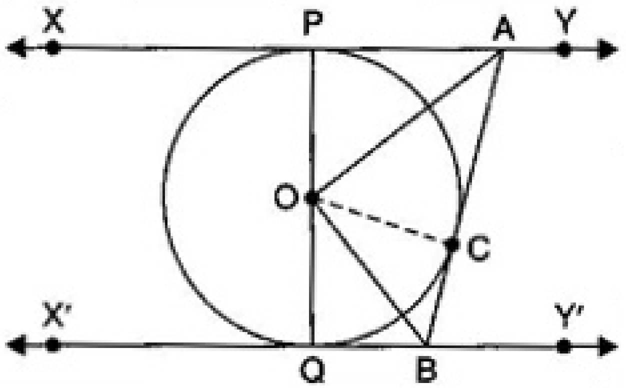


Ans. Given: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove: $\angle$ AOB = 90°

Construction: Join OC

Proof: $\angle$ OPA = 90°..........(i)



$\angle$ OCA = 90°..........(ii)

[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

In right angled triangles OPA and OCA,

$\angle$ OPA = $\angle$ OCA = 90^0

OA = OA [Common]

AP = AC [Tangents from an external

point to a circle are equal]

$\therefore \Delta OPA \cong \Delta OCA$

[RHS congruence criterion]

$\therefore \angle OAP = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \angle OAC = \frac{1}{2}\angle PAB$ ....(iii)

Similarly, $\angle OBQ = \angle OBC$

$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$ ....(iv)

$\because$ XY $\parallel$ X'Y' and a transversal AB intersects them.

$\therefore \angle PAB + \angle QBA = 180^\circ$

[Sum of the consecutive interior angles on the same side of the transversal is 180°]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA$

$= \frac{1}{2}\times 180^\circ$..........(v)

$\Rightarrow \angle OAC + \angle OBC = 90^\circ$

[From eq. (iii) & (iv)]

In ΔAOB,

$\angle OAC + \angle OBC + \angle AOB = 180^\circ$

[Angel sum property of a triangle]

$\Rightarrow 90^\circ + \angle AOB = 180^\circ$ [From eq. (v)]

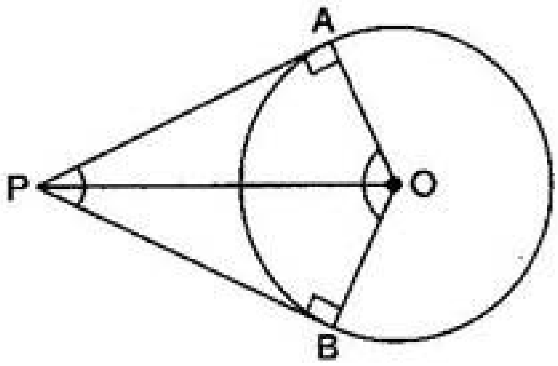
$\Rightarrow \angle AOB = 90^\circ$

Hence proved.

10. Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Ans. $\angle$ OAP = 90° ....(i)

$\angle$ OBP = 90° ....(ii)



[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore$ $\angle$ APB + $\angle$ AOB + $\angle$ OAP + $\angle$ OBP = 360°

[Angle sum property of a quadrilateral]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB + 90° + 90° = 360°

[From eq. (i) & (ii)]

$\Rightarrow$ $\angle$ APB + $\angle$ AOB = 180°

$\therefore$ $\angle$ APB and $\angle$ AOB are supplementary.

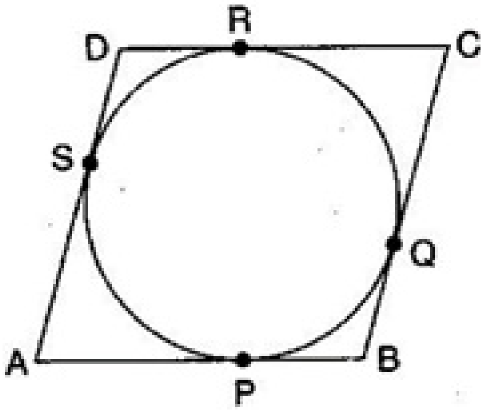
11. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans. Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ....(i)



BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD

[Opposite sides of $\|$ gm are equal]

$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB = AD

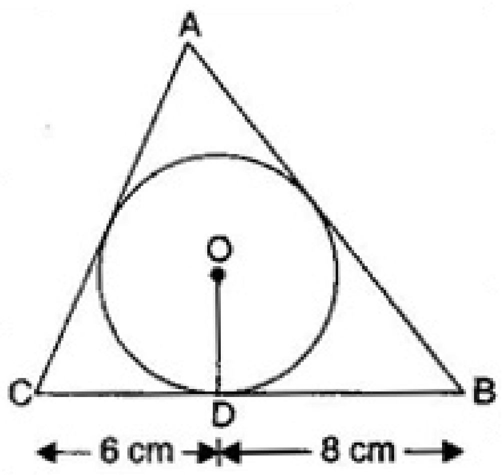
But AB = CD and AD = BC

[Opposite sides of || gm]

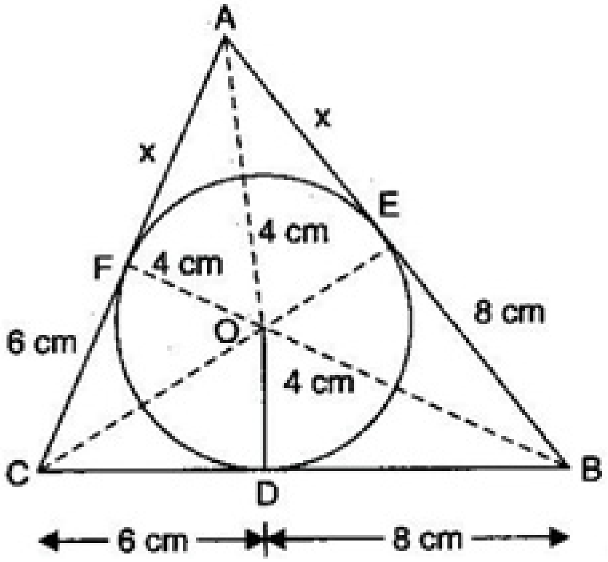
∴ AB = BC = CD = AD

∴ Parallelogram ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Ans. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

∴ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

∴ Semi-perimeter of ΔABC = $\dfrac{(x+6)+(x+8)+(6+8)}{2}$ = $(x+14)$ cm

∴ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

= $\sqrt{(x+14)(x)(6)(8)}$ cm^2

Now, Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $\dfrac{(6+8)4}{2}+\dfrac{(x+6)4}{2}+\dfrac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $28+2x+12+2x+16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$ = $4x+56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$ = $4(x+14)$

Squaring both sides,

$$(x+14)(x)(6)(8)=16(x+14)^2$$

$\Rightarrow\ 3x=x+14$

$\Rightarrow\ 2x=14$

$\Rightarrow\ x=7$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

And AC = $x+6$ = 7 + 6 = 13 cm

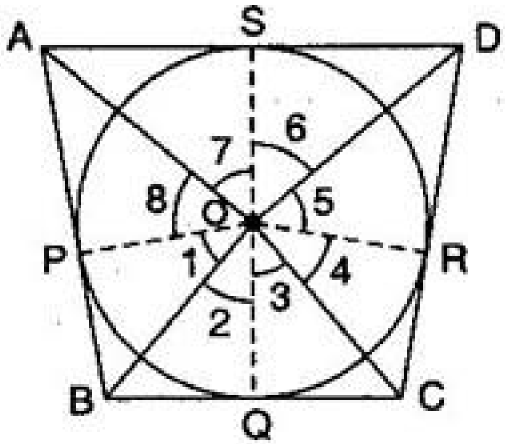
---

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = 180° (ii) $\angle$ BOC + $\angle$ AOD = 180°

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ ....(i)

CQ = CR

DR = DS

In ΔOBP and ΔOBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore$ ΔOPB $\cong$ Δ OBQ [By SSS congruence criterion]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360°$.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360°$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180°$

$\Rightarrow$ $\angle$ AOB + $\angle$ COD = $180°$

Similarly, we can prove that

$\angle$ BOC + $\angle$ AOD = $180°$

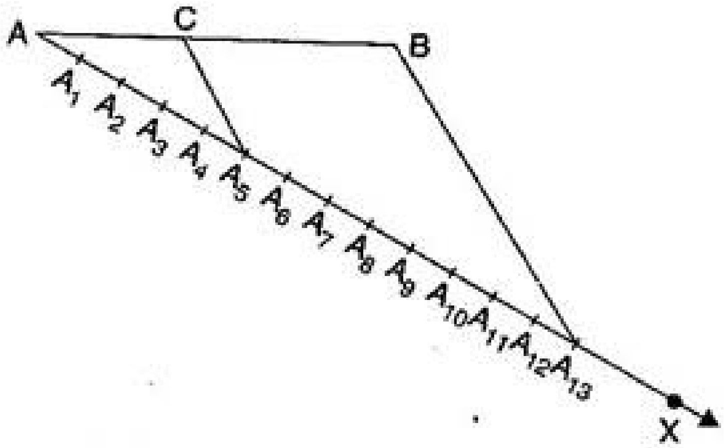
CBSE Class–10 Mathematics
NCERT solution
Chapter - 11
Constructions - Exercise 11.1

In each of the following, give the justification of the construction also:

1. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.

Ans. Given: A line segment of length 7.6 cm.

To construct: To divide it in the ratio 5 : 8 and to measure the two parts.



Steps of construction:

(a) From a point A, draw any ray AX, making an acute angle with AB.

(b) Locate 13 (=5 + 8) points $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}$ and A_{13} on AX such that

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9 = A_9A_{10} = A_{10}A_{11} = A_{11}A_{12} = A_{12}A_{13}$$

(c) Join BA_{13}.

(d) Through the point A_5, draw a line parallel to $A_{13}B$ intersecting AB at the point C.

Then, AC : CB = 5 : 8

On measurement we get, AC = 3.1 cm and CB = 4.5 cm

Justification:

$\because A_5C \parallel A_{13}B$ [By construction]

$$\therefore \frac{AA_5}{A_5A_{13}} = \frac{AC}{CB}$$

[By Basic Proportionality Theorem]

But $\frac{AA_5}{A_5A_{13}} = \frac{5}{8}$ [By construction]

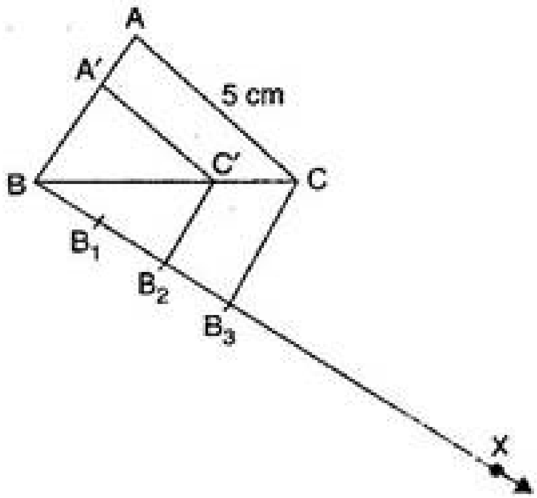
Therefore, $\frac{AC}{CB} = \frac{5}{8}$

$\Rightarrow$ AC : CB = 5 : 8

---

2. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Ans. To construct: To construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.



Steps of construction:

(a) Draw a triangle ABC with sides AB = 4 cm, AC = 5 cm and BC = 6 cm.
(b) From point B, draw any ray BX, making an acute angle with BC on the side opposite to the

vertex A.

(c) Locate 3 points B_1, B_2 and B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$.

(d) Join B_3C and draw a line through the point B_2, draw a line parallel to B_3C intersecting BC at the point C'.

(e) Draw a line through C' parallel to the line CA to intersect BA at A'.

Then, A'BC' is the required triangle.

Justification:

$\because B_3C \parallel B_2C'$ [By construction]

$$\therefore \frac{BB_2}{B_2B_3} = \frac{BC'}{C'C}$$

[By Basic Proportionality Theorem]

But $\frac{BB_2}{B_2B_3} = \frac{2}{1}$ [By construction]

Therefore, $\frac{BC'}{C'C} = \frac{2}{1}$

$$\Rightarrow \frac{C'C}{BC'} = \frac{1}{2}$$

$$\Rightarrow \frac{C'C}{BC'} + 1 = \frac{1}{2} + 1$$

$$\Rightarrow \frac{C'C + BC'}{BC'} = \frac{1+2}{2}$$

$$\Rightarrow \frac{BC}{BC'} = \frac{3}{2}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{2}{3} \quad \text{.........(i)}$$

$\because$ CA $\|$ C'A' [By construction]

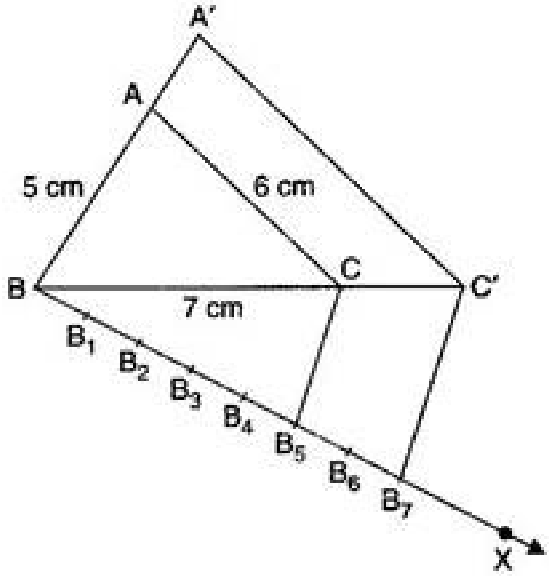
$\therefore$ ΔBC'A' $\sim$ ΔBCA [AA similarity]

$\therefore \dfrac{AB'}{AB} = \dfrac{A'C'}{AC} = \dfrac{BC'}{BC} = \dfrac{2}{3}$ [From eq. (i)]

---

3. Construct a triangle with sides 6 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Ans. To construct: To construct a triangle of sides 5 cm, 6 cm and 7 cm and then a triangle similar to it whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.



Steps of construction:

(a) Draw a triangle ABC with sides AB = 5 cm, AC = 6 cm and BC = 7 cm.

(b) From the point B, draw any ray BX, making an acute angle with BC on the side opposite to the vertex A.

(c) Locate 7 points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

(d) Join B_5C and draw a line through the point B_7, draw a line parallel to B_5C intersecting BC at the point C'.

(e) Draw a line through C' parallel to the line CA to intersect BA at A'.

Then, A'BC' is the required triangle.

Justification:

$\because$ C'A' || CA [By construction]

$\therefore$ ΔABC ~ Δ A'BC' [AA similarity]

$$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC}$$

[By Basic Proportionality Theorem]

$\because B_7C' \parallel B_5C$ [By construction]

$\therefore \Delta BB_7C' \sim \Delta BB_5C$ [AA similarity]

But $\frac{BB_5}{BB_7} = \frac{5}{7}$ [By construction]

Therefore, $\frac{BC}{BC'} = \frac{5}{7}$

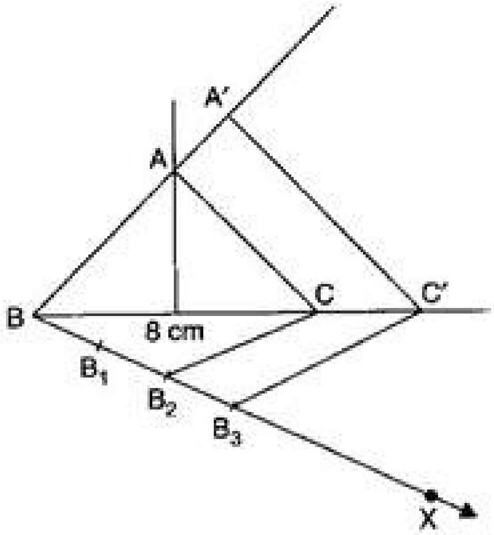
$$\Rightarrow \frac{BC'}{BC} = \frac{7}{5}$$

$$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{7}{5}$$

---

4. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Ans. To construct: To construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then a triangle similar to it whose sides are $1\frac{1}{2}\left(\text{or } \frac{3}{2}\right)$ of the corresponding sides of the first triangle.



Steps of construction:

(a) Draw BC = 8 cm

(b) Draw perpendicular bisector of BC. Let it meets BC at D.

(c) Mark a point A on the perpendicular bisector such that AD = 4 cm.

(d) Join AB and AC. Thus ΔABC is the required isosceles triangle.

(e) From the point B, draw a ray BX, making an acute angle with BC on the side opposite to the vertex A.

(f) Locate 3 points $B_1, B_2 and B_3$ on BX such that $BB_1 = B_1B_2 = B_2B_3$.

(g) Join B_2C and draw a line through the point B_3, draw a line parallel to B_2C intersecting BC at the point C'.

(h) Draw a line through C' parallel to the line CA to intersect BA at A'.

Then, A'BC' is the required triangle.

Justification:

$\because$ C'A' || CA [By construction]

$\therefore$ ΔABC ~ ΔA'BC' [AA similarity]

$$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC}$$

[By Basic Proportionality Theorem]

$\because B_3C' \parallel B_2C$ [By construction]

$\therefore \Delta BB_3C' \sim \Delta BB_2C$ [AA similarity]

But $\frac{BB_3}{BB_2} = \frac{3}{2}$ [By construction]

Therefore,

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{2}$$

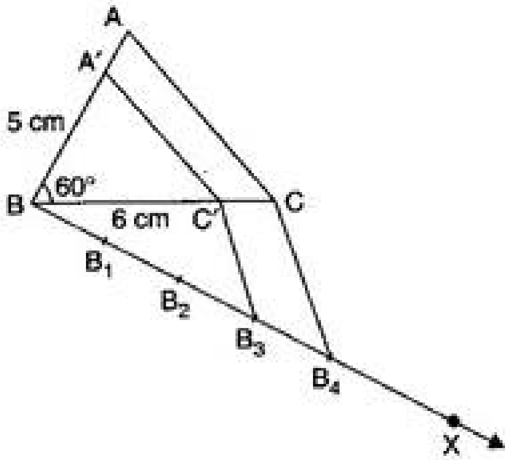
$$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{2}$$

Hence, we get the new triangle similar to the given triangle whose sides are equal to $\frac{3}{2}$ i.e., $1\frac{1}{2}$ times of corresponding sides of triangle ABC.

---

5. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle$ ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC.

Ans. To construct: To construct a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle$ ABC = 60° and then a triangle similar to it whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle ABC.



Steps of construction:

(a) Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle$ABC = 60°.

(b) From the point B, draw a ray BX, making an acute angle with BC on the side opposite to the vertex A.

(c) Locate 4 points $B_1, B_2, B_3 \text{and } B_4$ on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(d) Join B_4C and draw a line through the point B_3, draw a line parallel to B_4C intersecting BC at the point C'.

(e) Draw a line through C' parallel to the line CA to intersect BA at A'.

Then, A'BC' is the required triangle.

Justification:

$\because B_4C \parallel B_3C'$ [By construction]

$$\therefore \frac{BB_3}{BB_4} = \frac{BC'}{BC}$$

[By Basic Proportionality Theorem]

But $\frac{BB_3}{BB_4} = \frac{3}{4}$ [By construction]

Therefore, $\frac{BC'}{BC} = \frac{3}{4}$ ....(i)

$\because$ CA $\parallel$ C'A' [By construction]

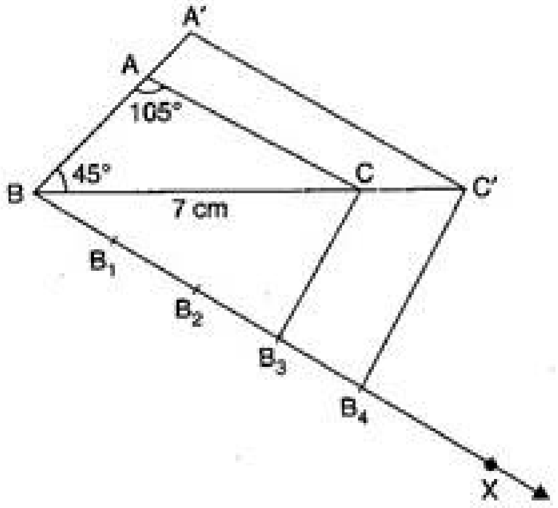
$\therefore \triangle$BC'A' ~ $\triangle$BCA [AA similarity]

$\therefore \frac{AB'}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$ [From eq. (i)]

Hence, we get the new triangle similar to the given triangle whose sides are equal to $\frac{3}{4}$th of corresponding sides of triangle ABC.

6. Draw a triangle ABC with side BC = 7 cm, $\angle$ B = 45°, $\angle$ A = 105°. Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

Ans. To construct: To construct a triangle ABC with side BC = 7 cm, $\angle$ B = 45° and $\angle$ C = 105° and then a triangle similar to it whose sides are $\frac{4}{3}$ of the corresponding sides of the first triangle ABC.



Steps of construction:

(a) Draw a triangle ABC with side BC = 7 cm, $\angle$ B = 45° and $\angle$ C = 105°.
(b) From the point B, draw a ray BX, making an acute angle with BC on the side opposite to the vertex A.
(c) Locate 4 points $B_1, B_2, B_3 and\ B_4$ on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
(d) Join B_3C and draw a line through the point B_4, draw a line parallel to B_3C intersecting BC at the point C'.
(e) Draw a line through C' parallel to the line CA to intersect BA at A'.

Then, A'BC' is the required triangle.

Justification:

$\because B_4C' \parallel B_3C$ [By construction]

$\therefore \Delta BB_4C' \sim \Delta BB_3C$ [AA similarity]

$\therefore \frac{BB_4}{BB_3} = \frac{BC'}{BC}$

[By Basic Proportionality Theorem]

But $\frac{BB_4}{BB_3} = \frac{4}{3}$ [By construction]

Therefore, $\frac{BC'}{BC} = \frac{4}{3}$ ....(i)

$\because$ CA $\|$ C'A' [By construction]

$\therefore$ ΔBC'A' $\sim$ ΔBCA [AA similarity]

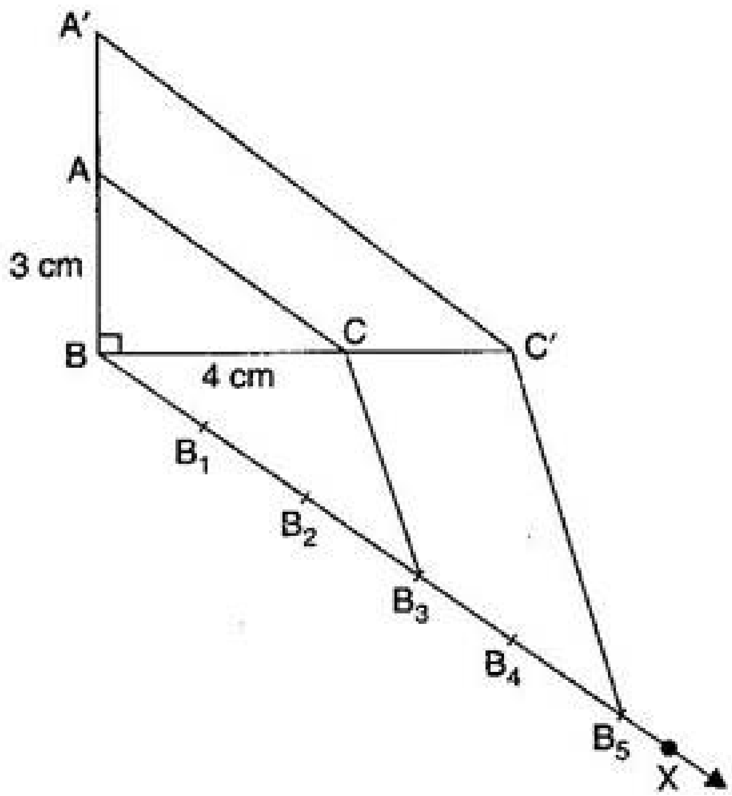
$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{4}{3}$ [From eq. (i)]

Hence, we get the new triangle similar to the given triangle whose sides are equal to $\frac{4}{3}$ times of corresponding sides of triangle ABC.

---

7. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

Ans. To construct: To construct a right triangle in which sides (other than hypotenuse) are of lengths 4 cm and 3 cm and then a triangle similar to it whose sides are $\frac{5}{3}$ of the corresponding sides of the first triangle ABC.



Steps of construction:

(a) Draw a right triangle in which sides (other than hypotenuse) are of lengths 4 cm and 3 cm, right angled at B.

(b) From the point B, draw a ray BX, making an acute angle with BC on the side opposite to the vertex A.

(c) Locate 5 points $B_1, B_2, B_3, B_4 and B_5$ on BX such that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

(d) Join B_3C and draw a line through the point B_5, draw a line parallel to B_3C intersecting BC at the point C'.

(e) Draw a line through C' parallel to the line CA to intersect BA at A'.

Then, A'BC' is the required triangle.

Justification:

$\because B_5C' \parallel B_3C$ [By construction]

$\therefore \Delta BB_5C' \sim \Delta BB_3C$ [AA similarity]

$$\therefore \frac{BB_5}{BB_3} = \frac{BC'}{BC}$$

[By Basic Proportionality Theorem]

But $\frac{BB_5}{BB_3} = \frac{5}{3}$ [By construction]

Therefore, $\frac{BC'}{BC} = \frac{5}{3}$ ....(i)

$\because$ CA $\parallel$ C'A' [By construction]

$\therefore$ $\triangle$BC'A' $\sim$ $\triangle$BCA [AA similarity]

$\therefore \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{5}{3}$ [From eq. (i)]

Hence, we get the new triangle similar to the given triangle whose sides are equal to $\frac{5}{3}$ times of corresponding sides of triangle ABC.

CBSE Class–10 Mathematics

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Chapter - 11

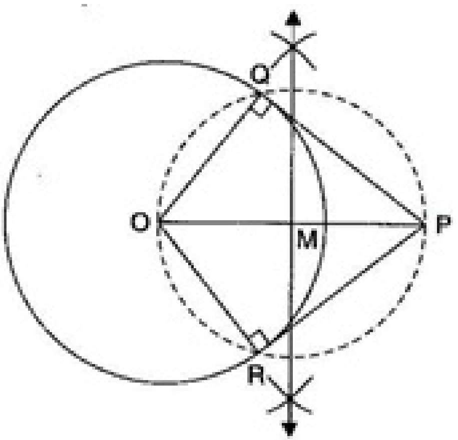
Constructions - Exercise 11.2

In each of the following, give the justification of the construction also:

1. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Ans. Given: A circle whose centre is O and radius is 6 cm and a point P is 10 cm away from its centre.

To construct: To construct the pair of tangents to the circle and measure their lengths.



Steps of Construction:

(a) Join PO and bisect it. Let M be the mid-point of PO.
(b) Taking M as centre and MO as radius, draw a circle. Let it intersects the given circle at the points Q and R.
(c) Join PQ and PR.

Then PQ and PR are the required two tangents.
By measurement, PQ = PR = 8 cm

Justification: Join OQ and OR.

Since $\angle$OQP and $\angle$ORP are the angles in semicircles.

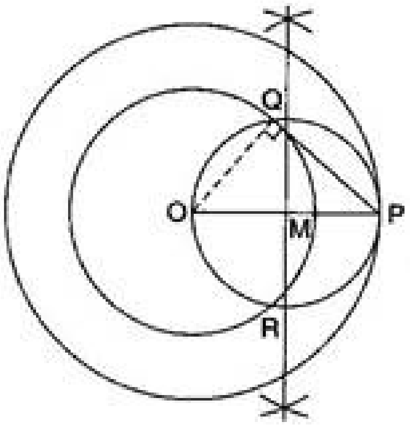
$\therefore \angle OQP = 90^\circ = \angle ORP$

Also, since OQ, OR are radii of the circle, PQ and PR will be the tangents to the circle at Q and R respectively.

$\therefore$ We may see that the circle with OP as diameter increases the given circle in two points. Therefore, only two tangents can be draw.

2. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Ans. To construct: To construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its lengths. Also to verify the measurements by actual calculation.



Steps of Construction:
(a) Join PO and bisect it. Let M be the mid-point of PO.
(b) Taking M as centre and MO as radius, draw a circle. Let it intersects the given circle at the point Q and R.
(c) Join PQ.

Then PQ is the required tangent.

By measurement, PQ = 4.5 cm
By actual calculation,

PQ = $\sqrt{(OP)^2 + (OQ)^2}$

$= \sqrt{6^2 - 4^2} = \sqrt{36-16}$

$= \sqrt{20} = 4.47$ cm = 4.5 cm

Justification: Join OQ. Then $\angle$ PQO is an angle in the semicircle and therefore,

$\angle PQO = 90^\circ$

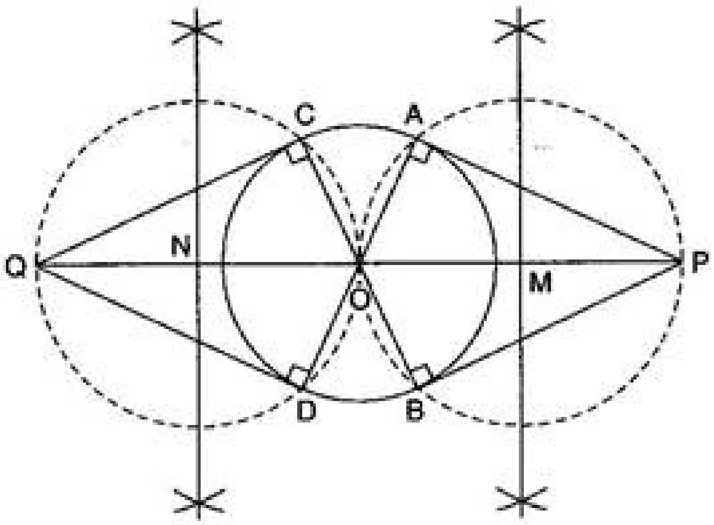
$\Rightarrow PQ \perp OQ$

Since, OQ is a radius of the given circle, PQ has to be a tangent to the circle.

---

3. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

Ans. To construct: A circle of radius 3 cm and take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre and then draw tangents to the circle from these two points P and Q.



Steps of Construction:
(a) Bisect PO. Let M be the mid-point of PO.
(b) Taking M as centre and MO as radius, draw a circle. Let it intersects the given circle at the points A and B.
(c) Join PA and PB. Then PA and PB are the required two tangents.
(d) Bisect QO. Let N be the mid-point of QO.

(e) Taking N as centre and NO as radius, draw a circle. Let it intersects the given circle at the points C and D.
(f) Join QC and QD.

Then QC and QD are the required two tangents.

Justification: Join OA and OB.

Then $\angle$ PAO is an angle in the semicircle and therefore $\angle$ PAO = 90°.

$\Rightarrow$ PA $\perp$ OA

Since OA is a radius of the given circle, PA has to be a tangent to the circle. Similarly, PB is also a tangent to the circle.

Again join OC and OD.

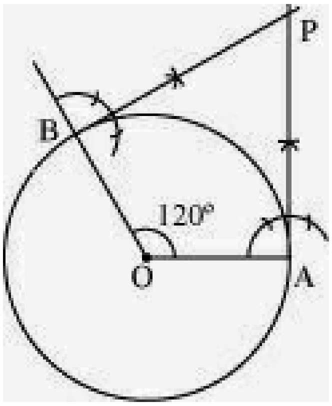
Then $\angle$ QCO is an angle in the semicircle and therefore $\angle$ QCO = 90°.

Since OC is a radius of the given circle, QC has to be a tangent to the circle. Similarly, QD is also a tangent to the circle.

---

4. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Ans. To construct: A pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.



Steps of Construction:

(a) Draw a circle of radius 5 cm and with centre as O.

(b) Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A with the help of compass.

(c) Draw a radius OB, making an angle of 120° (180° − 60°) with OA.

(d) Draw a perpendicular to OB at point B with the help of compass. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°.

Justification: The construction can be justified by proving that $\angle APB = 60°$

By our construction

$\angle OAP = 90°$

$\angle OBP = 90°$

And $\angle AOB = 120°$

We know that the sum of all interior angles of a quadrilateral = 360°

$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360°$

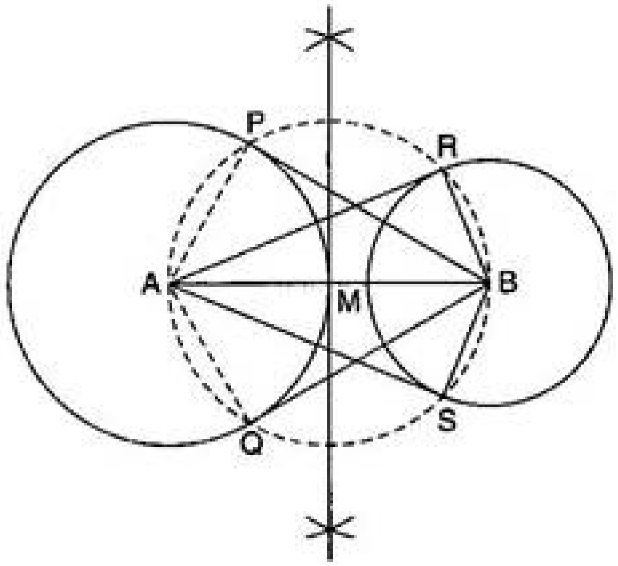
$90° + 120° + 90° + \angle APB = 360°$

$\angle APB = 60°$

---

5. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

Ans. To construct: A line segment of length 8 cm and taking A as centre, to draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Also, to construct tangents to each circle from the centre to the other circle.



Steps of Construction:

(a) Bisect BA. Let M be the mid-point of BA.
(b) Taking M as centre and MA as radius, draw a circle. Let it intersects the given circle at the points P and Q.
(c) Join BP and BQ. Then, BP and BQ are the required two tangents from B to the circle with centre A.
(d) Again, Let M be the mid-point of AB.
(e) Taking M as centre and MB as radius, draw a circle. Let it intersects the given circle at the points R and S.
(f) Join AR and AS.

Then, AR and AS are the required two tangents from A to the circle with centre B.

Justification: Join BP and BQ.

Then $\angle$ APB being an angle in the semicircle is 90°.

$\Rightarrow$ BP $\perp$ AP

Since AP is a radius of the circle with centre A, BP has to be a tangent to a circle with centre A. Similarly, BQ is also a tangent to the circle with centre A.

Again join AR and AS.

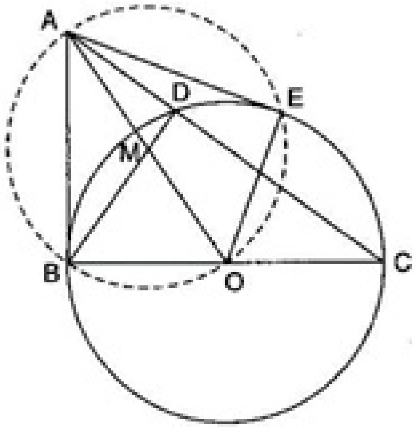
Then $\angle$ ARB being an angle in the semicircle is 90°.

$\Rightarrow$ AR $\perp$ BR

Since BR is a radius of the circle with centre B, AR has to be a tangent to a circle with centre B. Similarly, AS is also a tangent to the circle with centre B.

6. Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle$ B = 90°. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

Ans. To construct: A right triangle ABC with AB = 6 cm, BC = 8 cm and $\angle$ B = 90°. BD is the perpendicular from B on AC and the tangents from A to this circle.



Steps of Construction:
(a) Draw a right triangle ABC with AB = 6 cm, BC = 8 cm and $\angle$ B = 90°. Also, draw perpendicular BD on AC.
(b) Join AO and bisect it at M (here O is the centre of circle through B, C, D).
(c) Taking M as centre and MA as radius, draw a circle. Let it intersects the given circle at the points B and E.
(d) Join AB and AE.

Then AB and AE are the required two tangents.

Justification: Join OE.

Then, $\angle$ AEO is an angle in the semicircle.

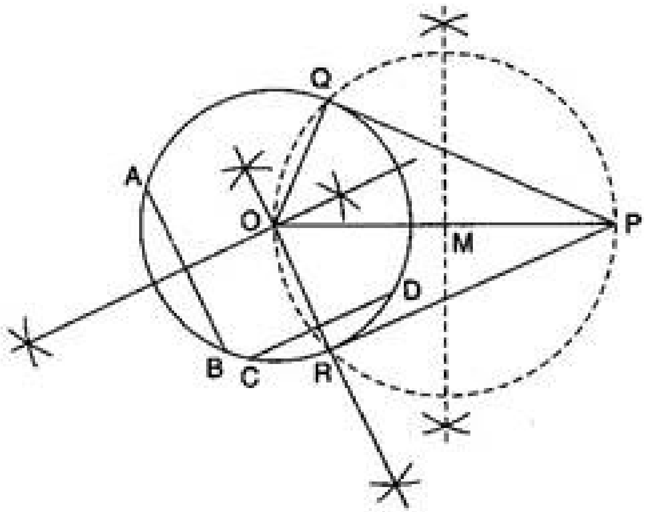
$\Rightarrow \angle$ AEO = 90°

$\Rightarrow$ AE $\perp$ OE

Since OE is a radius of the given circle, AE has to be a tangent to the circle. Similarly, AB is also a tangent to the circle.

7. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

Ans. To construct: A circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.



Steps of Construction:
(a) Draw a circle with the help of a bangle.
(b) Take two non-parallel chords AB and CD of this circle.
(c) Draw the perpendicular bisectors of AB and CD. Let these intersect at O. Then O is the centre of the circle draw.
(d) Take a point P outside the circle.
(e) Join PO and bisect it. Let M be the mid-point of PO.
(f) Taking M as centre and MO as radius, draw a circle. Let it intersects the given circle at the points Q and R.
(g) Join PQ and PR.

Then PQ and PR are the required two tangents.

Justification: Join OQ and OR.

Then, $\angle$ PQO is an angle in the semicircle.

$\Rightarrow \angle PQO = 90^\circ$

$\Rightarrow PQ \perp OQ$

Since OQ is a radius of the given circle, PQ has to be a tangent to the circle. Similarly, PR is also a tangent to the circle.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Ans. Let R be the radius of the circle which has circumference equal to the sum of circumferences of the two circles, then according to question,

$$2\pi R = 2\pi(19) + 2\pi(9)$$

$$\Rightarrow 2\pi R = 2\pi(19 + 9)$$

$\Rightarrow$ R = 19 + 9

$\Rightarrow$ R = 28 cm

---

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Ans. Let R be the radius of the circle which has area equal to the sum of areas of the two circles, then

According to the question,

$$\pi R^2 = \pi(8)^2 + \pi(6)^2$$

$$\Rightarrow \pi R^2 = \pi\left[(8)^2 + (6)^2\right]$$

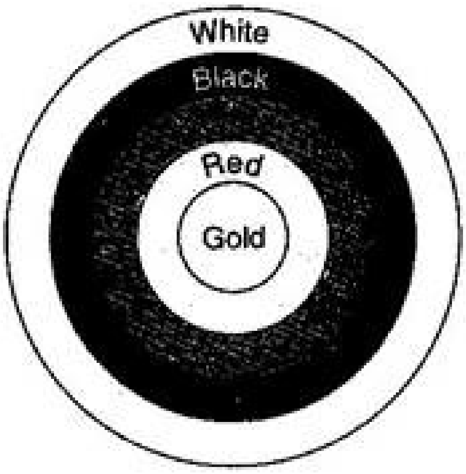
$$\Rightarrow R^2 = (8)^2 + (6)^2$$

$\Rightarrow$ R^2 = 64 + 36

$\Rightarrow R^2 = 100$

$\Rightarrow R = 10$ cm

3. Figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of the five scoring regions.



Ans. Gold: Diameter = 21 cm

$\Rightarrow$ Radius = $\frac{21}{2}$ cm

Area of gold scoring region = $\pi\left(\frac{21}{2}\right)^2$

= $\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$ = 346.5 cm^2

Red: Area of red scoring region = $\pi\left(\frac{21}{2}+10.5\right)^2-\pi\left(\frac{21}{2}\right)^2$

= $\pi(21)^2-346.5$

= $\frac{22}{7}\times 21\times 21 - 346.5$

$= 1386 - 346.5 = 1039.5\ cm^2$

Blue: Area of blue scoring region = $\pi(21 + 10.5)^2 - (1039.5 + 346.5)$

$= \pi(31.5)^2 - 1386$

$= \frac{22}{7} \times 31.5 \times 31.5 - 1386$

$= 3118.5 - 1386 = 1732.5\ cm^2$

Black: Area of black scoring region = $\pi(31.5 + 10.5)^2 - (1732.5 + 1039.5 + 346.5)$

$= \pi(42)^2 - 3118.5$

$= \frac{22}{7} \times 42 \times 42 - 3118.5$

$= 5544 - 3118.5 = 2425.5\ cm^2$

White: Area of white scoring region =
$\pi(42 + 10.5)^2 - (2425.5 + 1732.5 + 1039.5 + 346.5)$

$= \pi(52.5)^2 - 5544$

$= \frac{22}{7} \times 52.5 \times 52.5 - 5544$

$= 8662.5 - 5544 = 3118.5\ cm^2$

---

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Ans. Diameter of wheel = 80 cm

$\Rightarrow$ Radius of wheel (r) = 40 cm

Distance covered by wheel in one revolution = $2\pi r$ = $2\times\frac{22}{7}\times 40$ = $\frac{1760}{7}$ cm

$\because$ Distance covered by wheel in 1 hour = 66 km = 66000 m = 6600000 cm

$\therefore$ Distance covered by wheel in 10 minutes = $\frac{6600000}{60}\times 10$ = 1100000 cm

$\therefore$ No. of revolutions = $\frac{\text{Total distance}}{\text{distance of one revolution}}$

= $\frac{1100000\times 7}{1760}$ = 4375

---

5. Tick the correct answer in the following and justify your choice: If the perimeter and area of a circle are numerically equal, then the radius of the circle is:

(A) 2 units
(B) π units
(C) 4 units
(D) 7 units

Ans. (A)

Circumference of circle = Area of circle

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r$ = 2 units

CBSE Class–10 Mathematics

NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

Ans. Here, r = 6 cm and $\theta = 60^\circ$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$ = $\frac{132}{7}$ cm^2

---

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Ans. Given, $2\pi r$ = 22 cm

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{7}{2}$ cm

We know that for quadrant of circle, $\theta = 90^\circ$

$\therefore$ Area of quadrant = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ = $\frac{77}{8}$ cm^2

---

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Ans. Here, r = 14 cm and since the minute hand rotates through $\frac{360^\circ}{60^\circ} = 6^\circ$ in one minute, therefore, angle swept by minute hand in 5 minutes = $\theta = 6^\circ \times 5 = 30^\circ$.

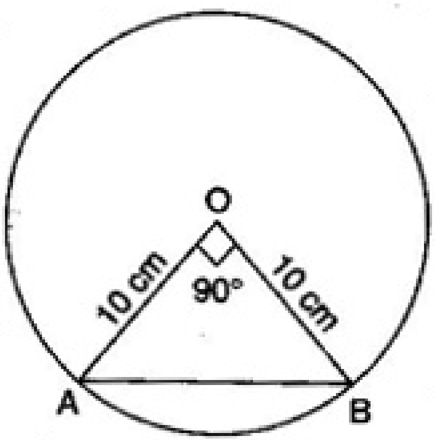
$\therefore$ Area swept = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$ = $\frac{154}{3} cm^2$

---

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment, (ii) major segment. $\left(\text{Use } \pi = 3.14\right)$

Ans. (i) Here, r = 10 cm and $\theta = 90^\circ$



$\therefore$ Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10$ = $78.5\ cm^2$

Area of ΔOAB = $\frac{1}{2} \times Base \times Height$

= $\frac{1}{2} \times 10 \times 10$ = $50\ cm^2$

$\therefore$ Area of minor segment = Area of minor sector – Area of Δ OAB

= 78.5 – 50 = $28.5\ cm^2$

(ii) For major sector, radius = 10 cm and $\theta = 360^\circ - 90^\circ = 270^\circ$

$\therefore$ Area of major sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{270^\circ}{360^\circ} \times 3.14 \times 10 \times 10$ = $235.5\ cm^2$

---

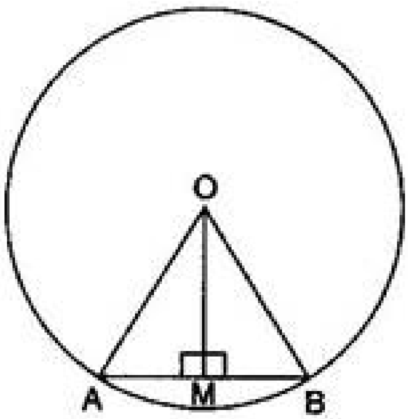
5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc.

(ii) area of the sector formed by the arc.

(iii) area of the segment formed by the corresponding chord.

Ans. Given, r = 21 cm and $\theta = 60^\circ$



(i) Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

= $\frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$ = 22 cm

(ii) Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$ = $231\ cm^2$

(iii) Area of segment formed by corresponding chord

= $\frac{\theta}{360^\circ} \times \pi r^2 -$ Area of ΔOAB

$\Rightarrow$ Area of segment = 231 – Area of ΔOAB……….(i)

In right angled triangle OMA and OMB,

OM = OB [Radii of the same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\therefore$ M is the mid-point of AB and $\angle$AOM = $\angle$BOM

$\Rightarrow$ $\angle$AOM = $\angle$BOM

= $\frac{1}{2}\angle$AOB = $\frac{1}{2} \times 60^\circ = 30^\circ$

Therefore, in right angled triangle OMA,

$\cos 30^\circ = \frac{OM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{21}$

$\Rightarrow$ OM = $\frac{21\sqrt{3}}{2}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA} \Rightarrow \frac{1}{2} = \frac{AM}{21}$

$\Rightarrow$ AM = $\frac{21}{2}$ cm

$\therefore$ AB = 2 AM = $2\times\frac{21}{2}$ = 21 cm

$\therefore$ Area of ΔOAB = $\frac{1}{2}\times AB\times OM$

= $\frac{1}{2}\times 21\times\frac{21\sqrt{3}}{2}$ = $\frac{441\sqrt{3}}{4}cm^2$

Using eq. (i),

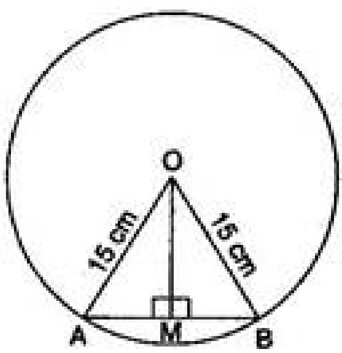
Area of segment formed by corresponding chord = $\left(231-\frac{441\sqrt{3}}{4}\right)cm^2$

---

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor and major segment of the circle.

$\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$

Ans. Here, r = 15 cm and $\theta = 60^\circ$



Area of minor sector= $\frac{\theta}{360^\circ}\times\pi r^2$

= $\frac{60^\circ}{360^\circ}\times 3.14\times 15\times 15$

= $117.75\ cm^2$

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and

$\angle$AOM = $\angle$BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2}\times 60^\circ = 30^\circ$

In right angled triangle OMA, $\cos 30^\circ = \frac{OM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{15}$

$\Rightarrow$ OM = $\frac{15\sqrt{3}}{2}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AM}{15} \Rightarrow$ AM = $\frac{15}{2}$ cm

$\Rightarrow$ 2 AM = $2\times\frac{15}{2}$ = 15 cm

$\Rightarrow$ AB = 15 cm

$\therefore$ Area of ΔAOB = $\frac{1}{2} \times AB \times OM$

= $\frac{1}{2} \times 15 \times \frac{15\sqrt{3}}{2}$ = $\frac{225\sqrt{3}}{3}$

= $\frac{225 \times 1.73}{4}$ = $97.3125\ cm^2$

$\therefore$ Area of minor segment = Area of minor sector – Area of ΔAOB

= 117.75 – 97.3125 = $20.4375\ cm^2$

And, Area of major segment = πr^2 – Area of minor segment

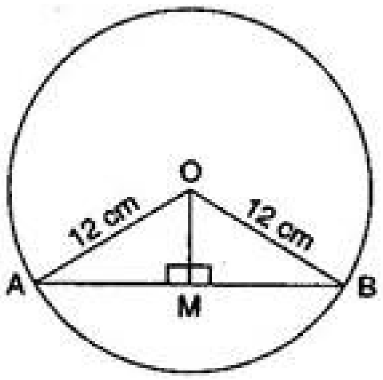
= 3.14 x 15 x 15 - 20.4375 = 706.5 – 20.4375 = $686.0625\ cm^2$

---

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$

Ans. Here, r = 12 cm and $\theta = 120^\circ$

Area of corresponding sector= $\frac{\theta}{360^\circ} \times \pi r^2$



= $\frac{120^\circ}{360^\circ} \times 3.14 \times 12 \times 12$

= $150.72\ cm^2$

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and

$\angle$AOM = $\angle$BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2}\times 120^\circ = 60^\circ$

In right angled triangle OMA, $\cos 60^\circ = \frac{\text{OM}}{\text{OA}}$

$\Rightarrow \frac{1}{2} = \frac{\text{OM}}{12}$

$\Rightarrow$ OM = 6 cm

Also, $\sin 60^\circ = \frac{\text{AM}}{\text{OA}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\text{AM}}{12}$

$\Rightarrow$ AM = $6\sqrt{3}$ cm

$\Rightarrow$ 2 AM = $2\times 6\sqrt{3}$ = $12\sqrt{3}$ cm

$\Rightarrow$ AB = $12\sqrt{3}$ cm

$\therefore$ Area of ΔAOB = $\frac{1}{2} \times AB \times OM$

= $\frac{1}{2} \times 12\sqrt{3} \times 6$ = $36\sqrt{3}$

= 36×1.73 = $62.28\ cm^2$

$\therefore$ Area of corresponding segment = Area of corresponding sector – Area of ΔAOB

= 150.72 – 62.28 = $88.44\ cm^2$

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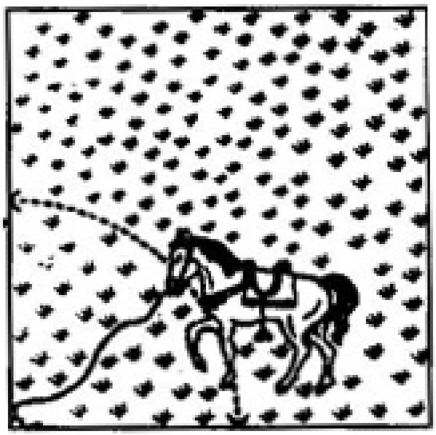
8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:



(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 cm.

$(\text{Use } \pi = 3.14)$

Ans. (i) Area of quadrant with 5 m rope

= $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 5 \times 5 = 19.625\ m^2$

(ii) Area of quadrant with 10 m rope

$= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 78.5\ m^2$

$\therefore$ The increase in grazing area

= 78.5 – 19.625

$= 58.875\ m^2$

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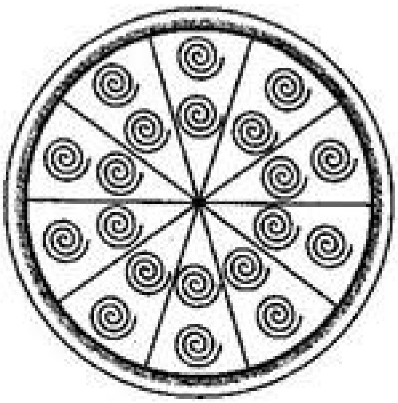
9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:



(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Ans. (i) Diameter of wire = 35 mm

$\Rightarrow$ Radius = $\frac{35}{2}$ mm

Circumference = $2\pi r = 2\times\frac{22}{7}\times\frac{35}{2}$

= 110 mm..........(i)

Length of 5 diameters = 35×5 = 175 mm..........(ii)

$\therefore$ Total length of the silver wire required

= 110 + 175 = 285 mm

(ii) $r=\frac{35}{2}$ mm and $\theta=\frac{360^\circ}{10}=36^\circ$

$\therefore$ The area of each sector of the brooch = $\frac{\theta}{360^\circ}\times\pi r^2$

= $\frac{36^\circ}{360^\circ}\times\frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}$ = $\frac{385}{4}mm^2$

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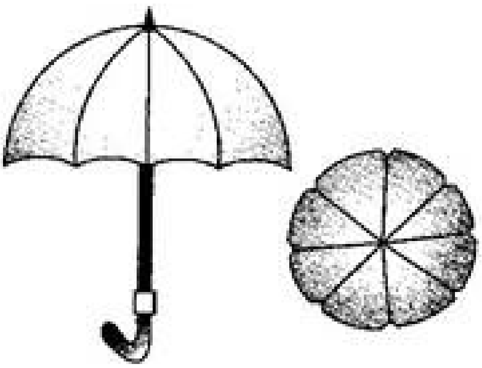
10. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Ans. Here, r = 45 cm and

$\theta=\frac{360^\circ}{8}=45^\circ$

Area between two consecutive ribs of the umbrella

$= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45$

$= \frac{22275}{28} cm^2$

---

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Ans. Here, r = 25 cm and $\theta = 115^\circ$

The total area cleaned at each sweep of the blades

$= 2 \times \left(\frac{\theta}{360^\circ} \times \pi r^2 \right)$

$= 2 \times \left(\frac{115^\circ}{360^\circ} \times \frac{22}{7} \times 25 \times 25 \right)$

$= \frac{158125}{126} cm^2$

---

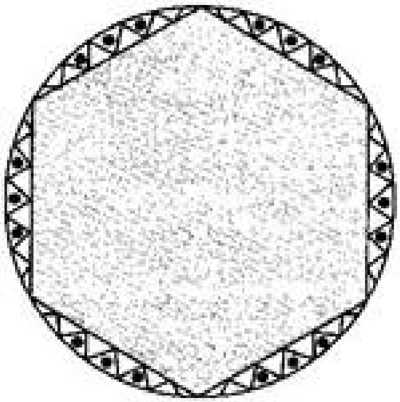
12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. $(\text{Use } \pi = 3.14)$

Ans. Here, r = 16.5 km and $\theta = 80^\circ$

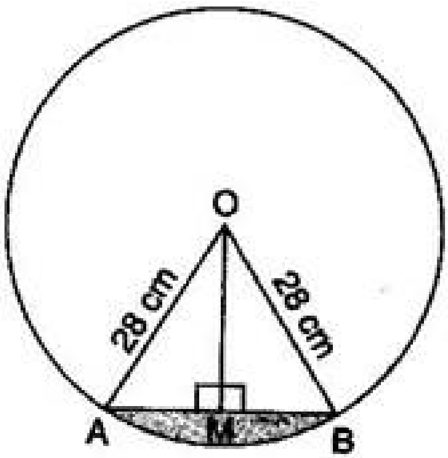
The area of sea over which the ships are warned = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{80^\circ}{360^\circ} \times 3.14 \times 16.5 \times 16.5$ = $189.97\ km^2$

13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. $\big($Use $\sqrt{}$



Ans. r = 28 cm and $\theta = \frac{360^\circ}{6} = 60^\circ$



Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28$

= $\frac{1232}{3}$ = $410.67\ cm^2$

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and $\angle$ AOM = $\angle$ BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2}\times 60^\circ = 30^\circ$

In right angled triangle OMA,

$$\cos 30^\circ = \frac{OM}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$$

$\Rightarrow$ OM = $14\sqrt{3}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA}$

$$\Rightarrow \frac{1}{2} = \frac{AM}{28}$$

$\Rightarrow$ AM = 14 cm

$\Rightarrow$ 2 AM = 2×14 = 28 cm

$\Rightarrow$ AB = 28 cm

$\therefore$ Area of ΔAOB = $\frac{1}{2}\times AB\times OM$

= $\frac{1}{2}\times 28\times 14\sqrt{3}$ = $196\sqrt{3}$

$= 196 \times 1.7 = 333.2\ cm^2$

$\therefore$ Area of minor segment = Area of minor sector – Area of ΔAOB

$= 410.67 - 333.2 = 77.47\ cm^2$

$\therefore$ Area of one design = $77.47\ cm^2$

$\therefore$ Area of six designs = $77.47 \times 6 = 464.82\ cm^2$

Cost of making designs = 464.82×0.35 = Rs. 162.68

---

14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is:

(A) $\frac{p}{180^\circ} \times 2\pi R$

(B) $\frac{p}{180^\circ} \times \pi R^2$

(C) $\frac{p}{360^\circ} \times 2\pi R$

(D) $\frac{p}{720^\circ} \times 2\pi R^2$

Ans. (D) Given, r = R and $\theta = p$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{p}{360^\circ} \times \pi R^2$

$= \frac{p}{2 \times 360^\circ} \times 2\pi R^2$

$= \frac{p}{720^\circ} \times 2\pi R^2$

CBSE Class–10 Mathematics

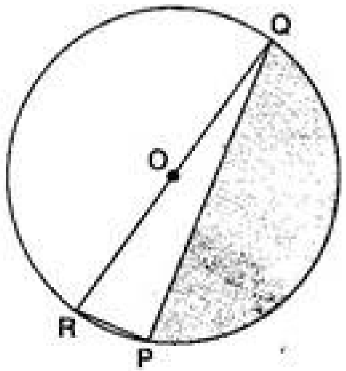
NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.3

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. Find the area of the shaded region in figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



Ans. In the given figure, $\angle$RPQ = 90° [Angle in semi-circle is 90°]

$\therefore\ RQ^2 = PR^2 + PQ^2$

= $(7)^2 + (24)^2$ = 49 + 576 = 625

$\Rightarrow$ RQ = 25 cm

$\Rightarrow$ Diameter of the circle = 25 cm

$\therefore$ Radius of the circle = $\frac{25}{2}$ cm

Area of the semicircle = $\frac{1}{2}\pi r^2$

= $\frac{1}{2}\times\frac{22}{7}\times\frac{25}{2}\times\frac{25}{2}$ = $\frac{6875}{28}\ cm^2$

Area of right triangle RPQ = $\frac{1}{2}$ x PQ x PR

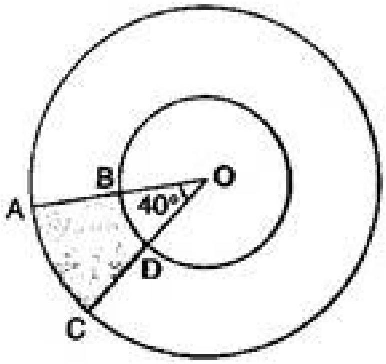
= $\frac{1}{2}\times 24\times 7$ = $84\ cm^2$

Area of shaded region = Area of semicircle – Area of right triangle RPQ

= $\frac{6875}{28}-84=\frac{6875-2352}{28}$ = $\frac{4523}{28}cm^2$

---

2. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle$ AOC = 40°.



Ans. Area of shaded region = Area of sector OAC – Area of sector OBD

= $\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times(14)^2-\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times(7)^2$

= $\frac{40^\circ}{360^\circ}\times\frac{22}{7}\left[(14)^2-(7)^2\right]$

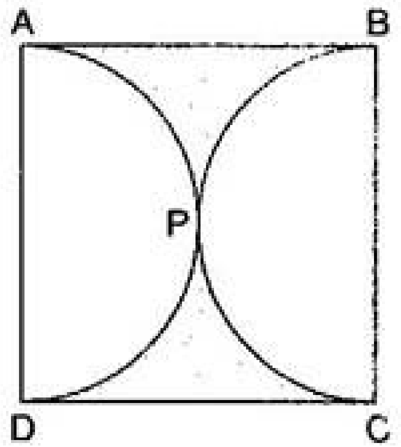
= $\frac{40^\circ}{360^\circ}\times\frac{22}{7}(14-7)(14+7)$

= $\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times 7\times 21$

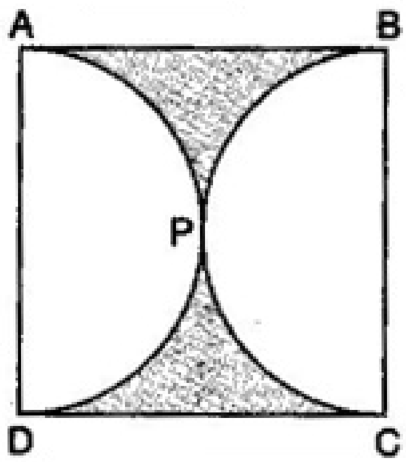
= $\frac{154}{3}cm^2$

---

3. Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Ans. Area of shaded region



= Area of square ABCD – (Area of semicircle APD + Area of semicircle BPC)

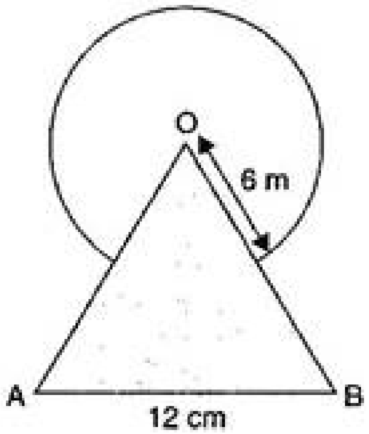
$= 14 \times 14 - \left[\frac{1}{2} \times \frac{22}{7}\left(\frac{14}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7}\left(\frac{14}{2}\right)^2\right]$

$= 196 - \left[\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right]$

$= 196 - \frac{22}{7} \times 7 \times 7$

$= 196 - 154 = 42\ cm^2$

4. Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans. Area of shaded region

= Area of circle + Area of equilateral triangle OAB – Area common to the circle and the triangle (Area of sector)

$= \pi r^2 + \frac{\sqrt{3}}{4}(a)^2 - \frac{\theta}{360^\circ} \times \pi r^2$

$= \pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - \frac{60^\circ}{360^\circ} \times \pi(6)^2$

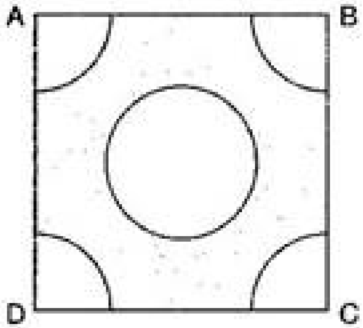
$= 36\pi + 36\sqrt{3} - 6\pi$

$= 30\pi + 36\sqrt{3} = 30 \times \frac{22}{7} + 36\sqrt{3}$

$= \left(\frac{660}{7} + 36\sqrt{3}\right) cm^2$

---

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the figure.



Ans. Area of remaining portion of the square

= Area of square – [(4 $\times$ Area of a quadrant + Area of a circle)]

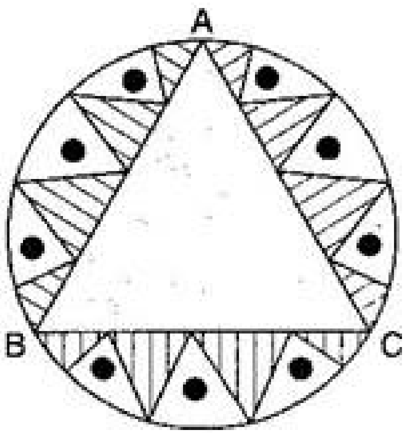
$$= 4 \times 4 - \left[4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (1)^2 + \frac{22}{7} \times \left(\frac{2}{2}\right)^2\right]$$

$$= 16 - \left[\frac{22}{7}\left(4 \times \frac{1}{4} + 1\right)\right]$$

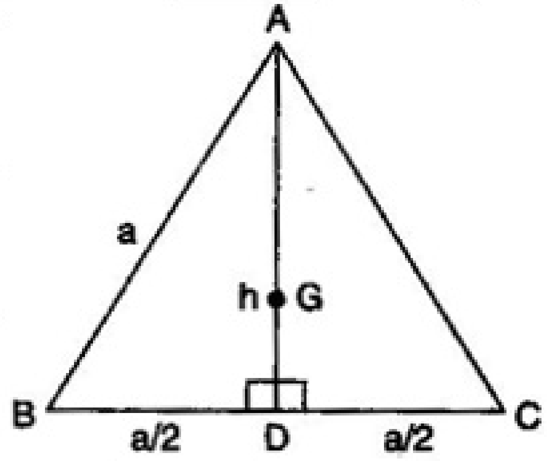
$$= 16 - 2 \times \frac{22}{7} = \frac{68}{7} cm^2$$

---

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design (shaded region).



Ans. Area of design = Area of circular table cover – Area of the equilateral triangle ABC



= $\pi(32)^2 - \frac{\sqrt{3}}{4}a^2$.........(i)

$\because$ G is the centroid of the equilateral triangle.

$\therefore$ radius of the circumscribed circle = $\frac{2}{3}h$ cm

According to the question, $\frac{2}{3}h = 32$

$\Rightarrow h$ = 48 cm

Now, $a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow a^2 = h^2 + \frac{a^2}{4}$

$\Rightarrow a^2 - \frac{a^2}{4} = h^2$

$\Rightarrow \frac{3a^2}{4} = h^2$

$\Rightarrow a^2 = \frac{4h^2}{3}$

$\Rightarrow a^2 = \frac{4(48)^2}{3}$ = 3072

$\Rightarrow a = \sqrt{3072}$ cm

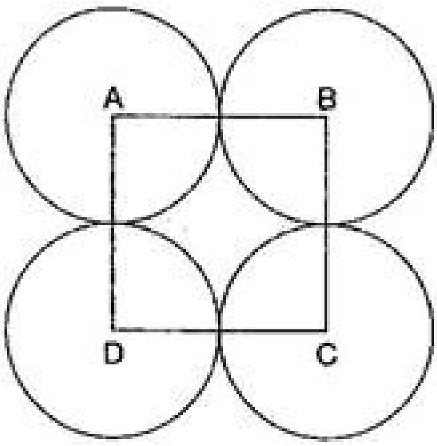
$\therefore$ Required area = $\pi(32)^2 - \frac{\sqrt{3}}{4} \times 3072$ [From eq. (i)]

= $\frac{22}{7} \times 1024 - 768\sqrt{3}$

= $\left(\frac{22528}{7} - 768\sqrt{3}\right) cm^2$

---

7. In figure ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Ans. Area of shaded region = Area of square – 4 $\times$ Area of sector

= $14 \times 14 - 4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2$

= $196 - \frac{22}{7} \times 7 \times 7$ = 196 – 154 = $42\ cm^2$

---

8. Figure depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inner edge.

(ii) the area of the track.

Ans. (i) Distance around the track along its inner edge

$$= 106+106+2\times\left[\frac{180^\circ}{360^\circ}\times 2\times\frac{22}{7}\times\left(\frac{60}{2}\right)\right]$$

$$= 212 + 2 \times \left[\frac{1}{2} \times 2 \times \frac{22}{7} \times \frac{60}{2}\right]$$

$$= 212+60\times\frac{22}{7} = 212+\frac{1320}{7} = \frac{2804}{7} \text{ m}$$

(ii) Area of track $= 106\times 10+106\times 10+2\times\left[\frac{1}{2}\times\frac{22}{7}(30+10)^2-\frac{1}{2}\times\frac{22}{7}(30)^2\right]$

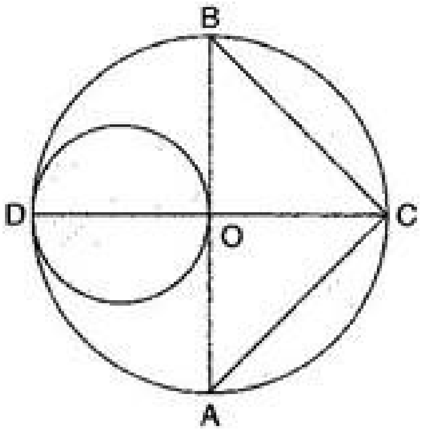
$$= 1060+1060+\frac{22}{7}\left[(40)^2-(30)^2\right]$$

$$= 2120 + \frac{22}{7}(40+30)(40-30)$$

$$= 2120+\frac{22}{7}\times 700 = 4320\ m^2$$

---

9. In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, Find the area of the shaded region.



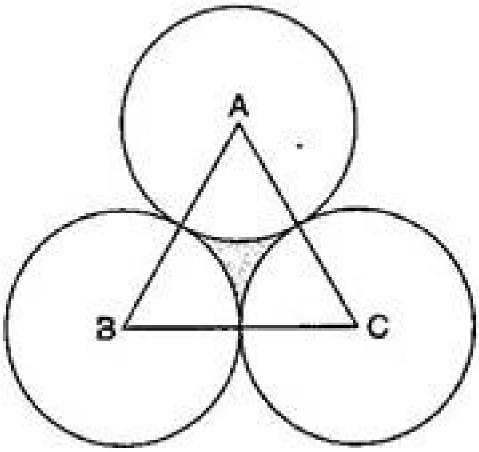
Ans. Area of shaded region = Area of circle with diameter OD + Area of semicircle ACB – Area of $\triangle$ ACB

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7} \times (7)^2 - \left(\frac{1}{2} \times 7 \times 7 + \frac{1}{2} \times 7 \times 7\right)$$

$$= \frac{77}{2} + 187 - 49 = \frac{133}{2} = 66.5\ cm^2$$

---

10. The area of an equilateral triangle ABC is $17320.5\ \text{cm}^2$. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see figure). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.$



Ans. Given: Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2 = 17320.5$

$$\Rightarrow a^2 = \frac{17320.5 \times 4}{\sqrt{3}}$$

$$\Rightarrow a^2 = \frac{17320.5 \times 4}{1.73205}$$

$$\Rightarrow a^2 = 40000$$

$$\Rightarrow a = 200 \text{ cm}$$

Area of shaded region = Area of ΔABC - Area of 3 sectors

$$= 17320.5 - 3\left[\frac{60^\circ}{360^\circ} \times 3.14 \times \left(\frac{200}{2}\right)^2\right]$$

$$= 17320.5 - 3\left[\frac{1}{6} \times 3.14 \times 100 \times 100\right]$$

= 17320.5 - 3 x 5233.33

= 17320.5 – 15700 = $1620.5\ cm^2$

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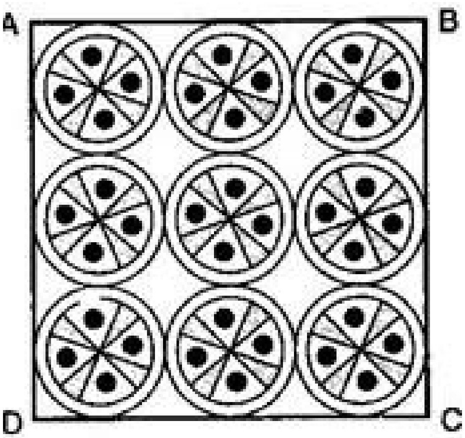
11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.



Ans. Radius of each design = 7 cm, then Diamter = 7 x 2 = 14 cm

Therefore, side of square = 14 + 14 + 14 = 42 cm

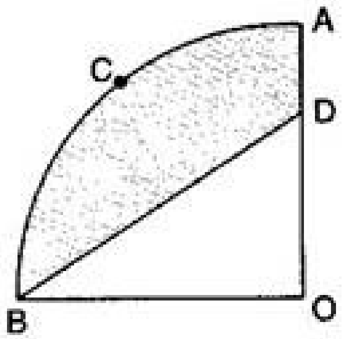
Area of remaining portion of handkerchief = Area of square ABCD – Area of 9 circular designs

$= 42 \times 42 - 9 \times \frac{22}{7} \times 7 \times 7$

$= 1764 - 1386 = 378\ cm^2$

---

12. In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the:



(i) quadrant OACB

(ii) shaded region

Ans. (i) Area of quadrant OACB = $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (3.5)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} = \frac{77}{8}\ cm^2$

(ii) Area of shaded region = Area of quadrant OACB – Area of ΔOBD

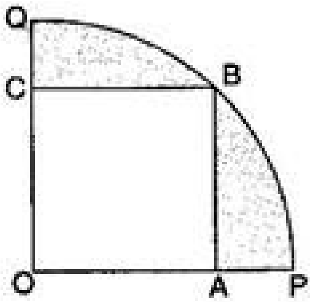
$= \frac{77}{8} - \frac{1}{2} \times \text{OB} \times \text{OD}$

$= \frac{77}{8} - \frac{3.5 \times 2}{2}$

$= \frac{77}{8} - \frac{35}{10} = \frac{49}{8}$ cm^2

---

13. In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. $(\text{Use } \pi = 3.14)$



Ans. OB = $\sqrt{OA^2 + AB^2}$ [Using Pythagoras theorem]

= $\sqrt{OA^2 + OA^2}$

= $\sqrt{2}$ OA = $\sqrt{2} \times 20 = 20\sqrt{2}$ cm

Area of shaded region = Area of quadrant OPBQ – Area of square OABC

= $\frac{90^\circ}{360^\circ} \times 3.14\left(20\sqrt{2}\right)^2 - 20 \times 20$

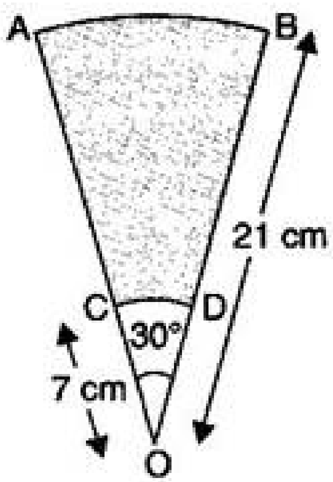
= $\frac{1}{4} \times 3.14 \times 800 - 400$

= $200 \times 3.14 - 400$

= $228\ cm^2$

---

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If $\angle$AOB = 30°, find the area of the shaded region.



Ans. Area of shaded region = Area of sector OAB – Area of sector OCD

$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 - \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$$

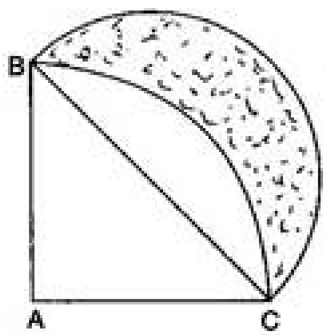
$$= \frac{1}{12} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{12} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{231}{2} - \frac{77}{6} = \frac{693 - 77}{6}$$

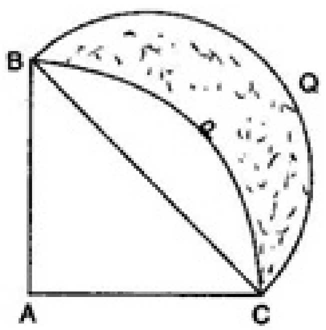
$$= \frac{616}{6} = \frac{308}{3}\ cm^2$$

---

15. In figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Ans. In right triangle BAC, $BC^2 = AB^2 + AC^2$ [Pythagoras theorem]



$\Rightarrow BC^2 = (14)^2 + (14)^2 = 2(14)^2$

$\Rightarrow$ BC = $14\sqrt{2}$ cm

$\therefore$ Radius of the semicircle = $\dfrac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

$\therefore$ Required area = Area BPCQB

= Area BCQB – Area BCPB

= Area BCQB – (Area BACPB – Area ΔBAC)

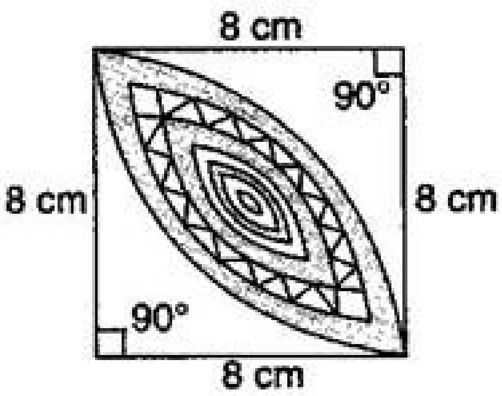
$$= \frac{180^\circ}{360^\circ}\times\frac{22}{7}\left(7\sqrt{2}\right)^2 - \left[\frac{90^\circ}{360^\circ}\times\frac{22}{7}\times(14)^2 - \frac{14\times14}{2}\right]$$

$$= \frac{1}{2}\times\frac{22}{7}\times 98 - \left(\frac{1}{4}\times\frac{22}{7}\times 196 - 98\right)$$

= 154 – (154 – 98) = $98\ cm^2$

---

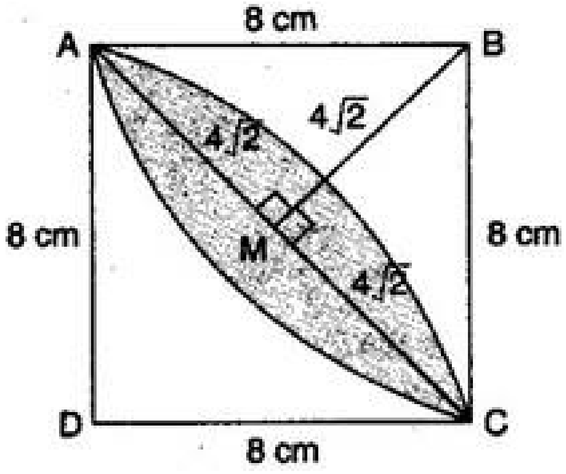
16. Calculate the area of the designed region in figure common between the two quadrants of circles of radius 8 cm each.



Ans. In right triangle ADC, $AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$\Rightarrow AC^2 = (8)^2 + (8)^2 = 2(8)^2$

$\Rightarrow$ AC = $\sqrt{128} = 8\sqrt{2}$ cm



Draw BM $\perp$ AC.

Then AM = MC = $\frac{1}{2}$ AC = $\frac{1}{2} \times 8\sqrt{2}$ = $4\sqrt{2}$ cm

In right triangle AMB,

$AB^2 = AM^2 + BM^2$ [Pythagoras theorem]

$\Rightarrow (8)^2 = \left(4\sqrt{2}\right)^2 + \mathrm{BM}^2$

$\Rightarrow BM^2 = 64 - 32 = 32$

$\Rightarrow$ BM = $4\sqrt{2}$ cm

$\therefore$ Area of ΔABC = $\frac{1}{2} \times AC \times BM$

= $\frac{8\sqrt{2} \times 4\sqrt{2}}{2}$ = $32\ cm^2$

$\therefore$ Half Area of shaded region

= $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 - 32$

= $16 \times \frac{22}{7} - 32$ = $\frac{128}{7} cm^2$

$\therefore$ Area of designed region

= $2 \times \frac{128}{7}$ = $\frac{256}{7} cm^2$

CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Ans. Volume of cube = $(Side)^3$

According to question, $(Side)^3$ = 64

$\Rightarrow (Side)^3 = 4^3$

$\Rightarrow$ Side = 4 cm

For the resulting cuboid, length (l) = 4 + 4 = 8 cm, breadth (b) = 4 cm and height (h) = 4 cm

Surface area of resulting cuboid = $2(lb + bh + hl)$

= $2(8\times4 + 4\times4 + 4\times8)$

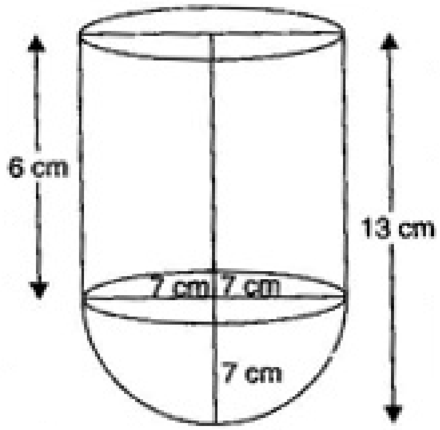
= 2 (32 + 16 + 32)

= $2\times80 = 160\ cm^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Ans. $\because$ Diameter of the hollow hemisphere = 14 cm

$\therefore$ Radius of the hollow hemisphere = $\frac{14}{2}$ = 7 cm



Total height of the vessel = 13 cm

∴ Height of the hollow cylinder = 13 – 7 = 6 cm

∴ Inner surface area of the vessel

= Inner surface area of the hollow hemisphere + Inner surface area of the hollow cylinder

= $2\pi(7)^2 + 2\pi(7)(6)$

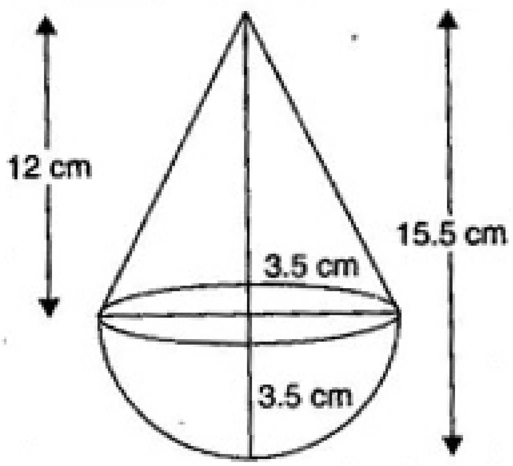
= $98\pi + 84\pi = 182\pi$

= $182 \times \frac{22}{7}$ = $26 \times 22 = 572\ cm^2$

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Ans. Radius of the cone = 3.5 cm

∴ Radius of the hemisphere = 3.5 cm



Total height of the toy = 15.5 cm

∴ Height of the cone = 15.5 – 3.5 = 12 cm

Slant height of the cone = $\sqrt{(3.5)^2+(12)^2}$

= $\sqrt{12.25+144}$

= $\sqrt{156.25}$ = 12.5 cm

∴ TSA of the toy = CSA of hemisphere + CSA of cone

= $2\pi r^2+\pi rl$

= $2\pi(3.5)^2+\pi(3.5)(12.5)$

= $24.5\pi+43.75\pi$ = 68.25π

= $68.25\times\frac{22}{7}$ = $214.5\ cm^2$

---

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Ans. Greatest diameter of the hemisphere = Side of the cubical block = 7 cm

∴ TSA of the solid = External surface area of the cubical block + CSA of hemisphere

$= \left\{6(7)^2 - \pi\left(\frac{7}{2}\right)^2\right\} + 2\pi\left(\frac{7}{2}\right)^2$

$=> \left(294 - \frac{49}{4}\pi\right) + \frac{49}{2}\pi$

$= 294 + \frac{49}{4}\pi$

$= 294 + \frac{49}{4} \times \frac{22}{7}$

$= 294 + \frac{77}{2}$

= 294 + 38.5 = $332.5\ cm^2$

---

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Ans. $\because$ Diameter of the hemisphere = l, therefore radius of the hemisphere = $\frac{l}{2}$

Also, length of the edge of the cube = l

$\therefore$ Surface area of the remaining solid = total surface area of cubical block + curved surface area of hemispherical - area of circular base

$= 2\pi\left(\frac{l}{2}\right)^2 + 6l^2 - \pi\left(\frac{l}{2}\right)^2$

$= \pi\left(\frac{l}{2}\right)^2 + 6l^2$

$= \frac{\pi l^2}{4} + 6l^2$

$= \frac{1}{4}l^2(\pi + 24)$

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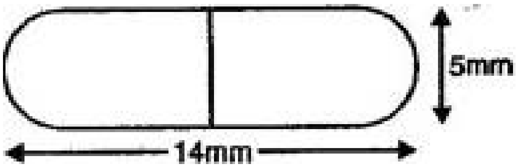
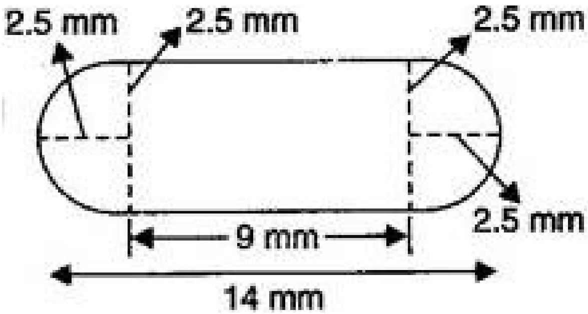
6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each

of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.**



Ans. Radius of the hemisphere = $\frac{5}{2}$ mm

Let radius = r = 2.5 mm
Cylindrical height = Total height – Diameter of sphere = h = 14 – (2.5 + 2.5) = 9 mm
Surface area of the capsule = CSA of cylinder + curved Surface area of 2 hemispheres



= $2\pi rh + 2\left(2\pi r^2\right)$

= $2\pi\left(\frac{5}{2}\right)(9) + 2\left\{2\pi\left(\frac{5}{2}\right)^2\right\}$

= $45\pi + 25\pi$

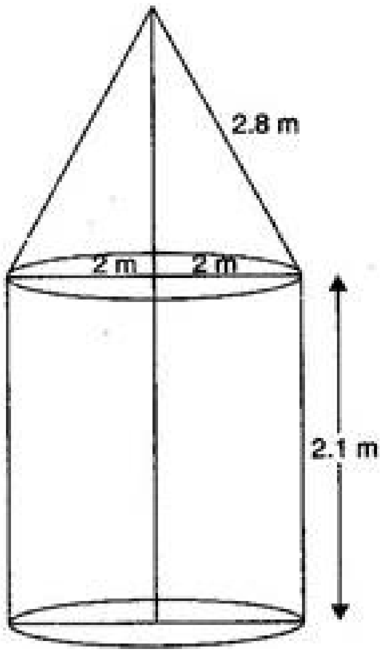
= $70\pi = 70 \times \frac{22}{7}$ = $220\ mm^2$

7. A tent is in the shape of a cylinder surmounted buy a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Ans. Diameter of the cylindrical part = 4 cm

∴ Radius of the cylindrical part = 2 cm

TSA of the tent = CSA of the cylindrical part + CSA of conical top



= $2\pi(2)(2.1)+\pi(2)(2.8)$

= $8.4\pi+5.6\pi$

= 14π

= $14\times\frac{22}{7}$

= $44\ m^2$

∴ Cost of the canvas of the tent of 1 m^2 = Rs. 500

cost of canvas of the tent of 44 m^2 =

= 44×500 = Rs. 22000

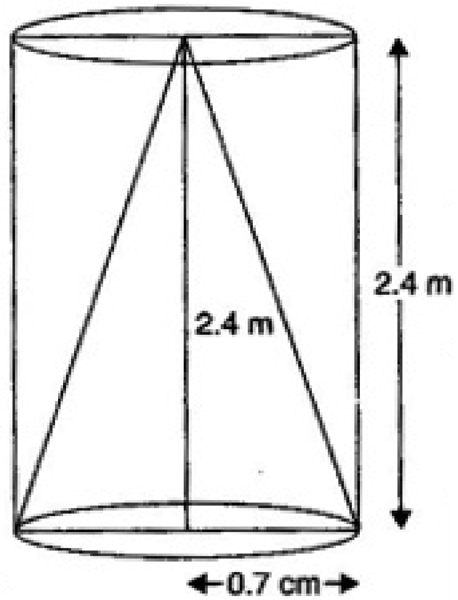
---

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Ans. Diameter of the solid cylinder = 1.4 cm

∴ Radius of the solid cylinder = 0.7 cm

∴ Radius of the base of the conical cavity = 0.7 cm



Height of the solid cylinder = 2.4 cm

∴ Height of the conical cavity = 2.4 cm

∴ Slant height of the conical cavity = $\sqrt{(0.7)^2+(2.4)^2}$

= $\sqrt{0.49+5.76}$

= $\sqrt{6.25}$ = 2.5 cm

∴ TSA of remaining solid = curved surface area of cylinder + area of upper circular part + curved surface area of conical part

= $2\pi(0.7)(2.4)+\pi(0.7)^2+\pi(0.7)(2.5)$

= $3.36\pi+0.49\pi+1.75\pi$

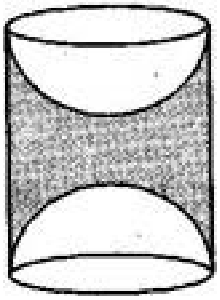
= 5.6π

= $5.6\times\frac{22}{7}$ = 17.6 cm^2

= $18\ cm^2$ (to the nearest cm^2)

9. A wooden article was made by scooping out a hemisphere from each end of a solid

cylinder as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



Ans. TSA of the article = $2\pi rH + 2\left(2\pi r^2\right)$ = curved surface area of cylinder + curved surface area of 2 hemispheres

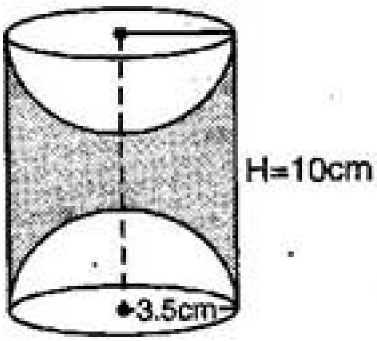
= $2\pi(3.5)(10)+2\left[2\pi(3.5)^2\right]$

= $70\pi + 49\pi$

= 119π

= $119 \times \frac{22}{7}$

= $374\ cm^2$



CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

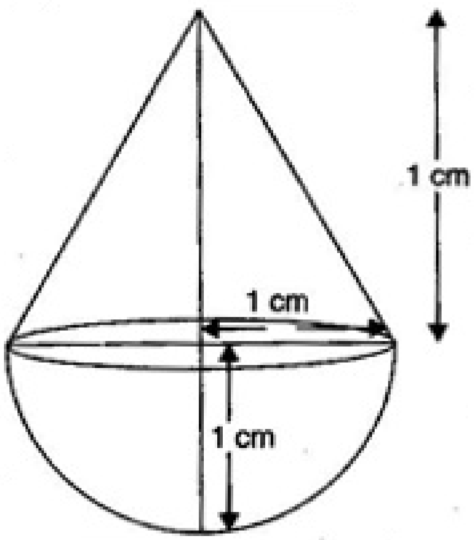
1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Ans. For hemisphere, Radius (r) = 1 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi (1)^3$

= $\frac{2}{3}\pi$ cm^3



For cone, Radius of the base (r) = 1 cm

Height (h) = 1 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(1)^2 \times 1$

= $\frac{1}{3}\pi$ cm^3

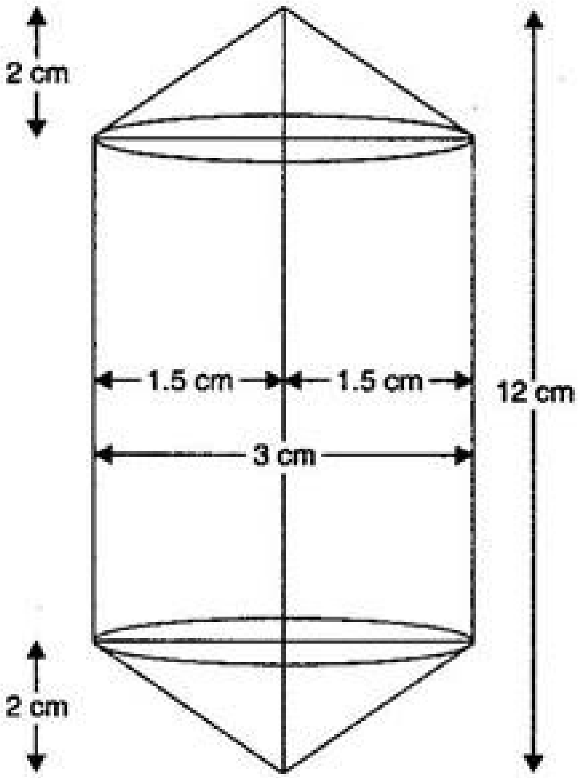
$\therefore$ Volume of the solid = Volume of hemisphere + Volume of cone

= $\frac{2}{3}\pi + \frac{1}{3}\pi = \pi\ cm^3$

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Ans. For upper conical portion, Radius of the base (r) = 1.5 cm

Height (h_1) = 2 cm



Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(1.5)^2 \times 2$

= $1.5\pi cm^3$

For lower conical portion, Volume = $1.5\pi cm^3$

For central cylindrical portion:

Radius of the base (r) = 1.5 cm

Height (h_2) = 12 – (2 + 2) = 8 cm

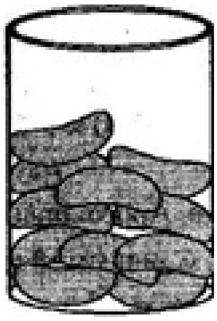
Volume = $\pi r^2 h_2$ = $\pi(1.5)^2 \times 8$ = $18\pi cm^3$

$\therefore$ Volume of the model = 1.5π + 1.5π + 18π = volume of top cone + volume of bottom cone + volume of cylindrical part

= 21π

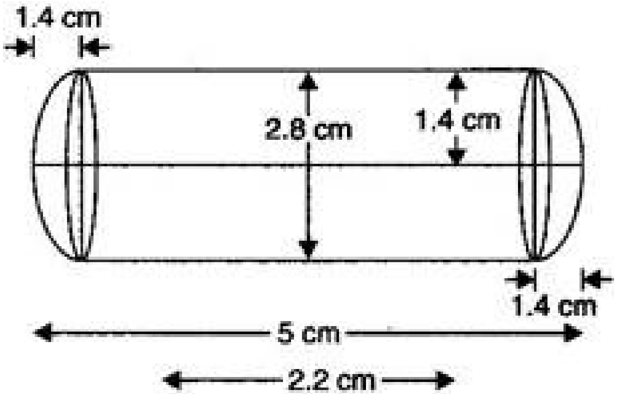
$= 21\times\frac{22}{7} = 66\ cm^3$

3. A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends, with length 5 cm and diameter 2.8 cm (see figure).



Ans. Volume of a gulab jamun = $\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$ = volume of 2hemisphere + volume of cylinder

$= \frac{2}{3}\pi(1.4)^3 + \pi(1.4)^2\times 2.2 + \frac{2}{3}\pi(1.4)^3$



$= \frac{4}{3}\pi(1.4)^3 + \pi(1.4)^2\times 2.2$

$= \pi(1.4)^2\left[\frac{4\times 1.4}{3} + 2.2\right]$

$= \pi \times 1.96\left[\frac{5.6+6.6}{3}\right] = \frac{1.96\times 12.2}{3}\pi\, cm^3$

$\therefore$ Volume of 45 gulab jamuns

$= 45\times\frac{1.96\times 12.2}{3}\pi$

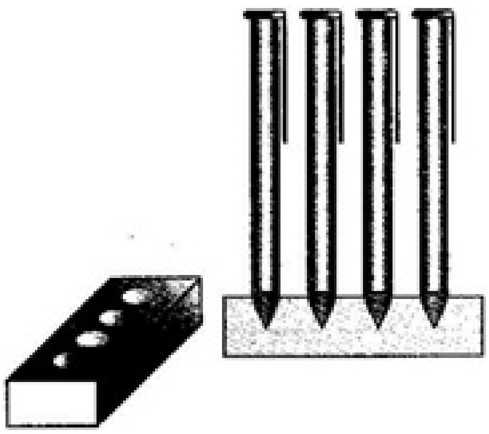
$= 15\times 1.96\times 12.2\times\frac{22}{7}$

$= 1127.28\ cm^3$

$\therefore$ Volume of syrup = $1127.28\times\frac{30}{100}$ = 30% of volume of 45 gulab jamun

$= 338.184\ cm^3 = 338\ cm^3$ (approx.)

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth id 1.4 cm. Find the volume of wood in the entire stand (see figure).



Ans: For Cuboid:

l=15 cm

b=10 cm

h=3.5 cm

Volume of the cuboid = $l \times b \times h$

= $15 \times 10 \times 3.5$

= $525\ cm^3$

For Cone: r = 0.5 cm

h = 1.4 cm

Volume of conical depression = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

= $\frac{11}{30} cm^3$

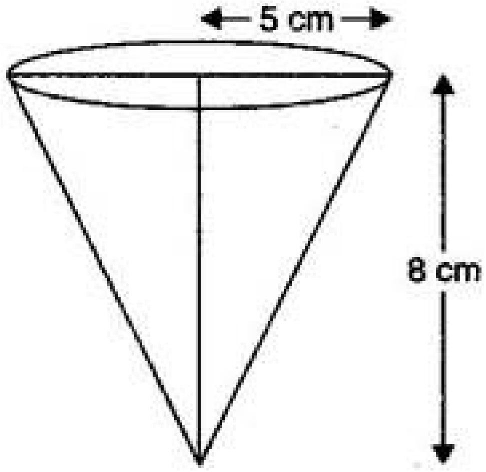
$\therefore$ Volume of four conical depressions = $4 \times \frac{11}{30}$ = $1.47\ cm^3$

$\therefore$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depression =525 – 1.47 = $523.53\ cm^3$

---

5. A vessel is in the form of inverted cone. Its height is 8 cm and the radius of the top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Ans. For cone, Radius of the top (r) = 5 cm and height (h) = 8 cm



Volume of cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(5)^2 \times 8$

= $\frac{200}{3}\pi$ cm^3

For spherical lead shot, Radius (R) = 0.5 cm

Volume of spherical lead shot = $\frac{4}{3}\pi R^3$

= $\frac{4}{3}\pi(0.5)^3$

= $\frac{\pi}{6} cm^3$

Volume of water that flows out = $\frac{1}{4}$ Volume of the cone

= $\frac{1}{4} \times \frac{200\pi}{3} = \frac{50\pi}{3} cm^3$

Let the number of lead shots dropped in the vessel be n.

n * volume of spherical shot = volume of water flows out

$\therefore n \times \frac{\pi}{6} = \frac{50\pi}{3}$

$\Rightarrow n = \frac{50\pi}{3} \times \frac{6}{\pi}$

$\Rightarrow n$ = 100

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that $1\ \text{cm}^3$ of iron has approximately 8 g mass. $(\text{Use } \pi = 3.14)$

Ans. For lower cylinder, Base radius (r) = $\frac{24}{2}$ = 12 cm

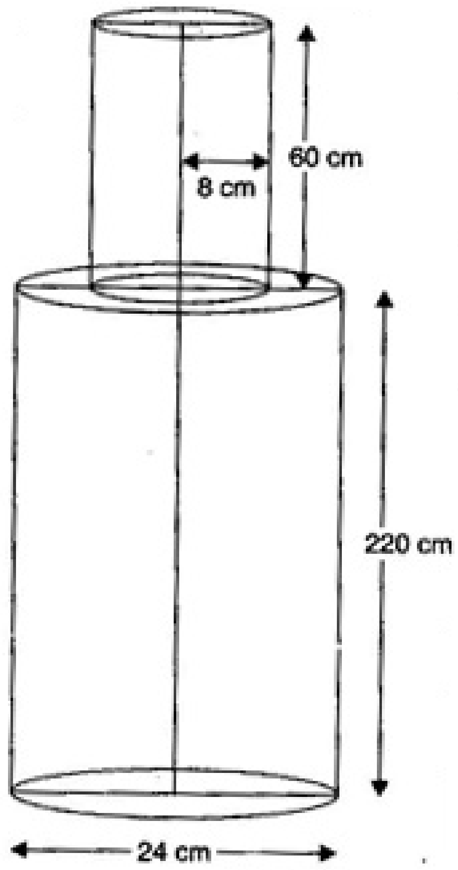
And Height (h) = 220 cm

Volume = $\pi r^2 h$

= $\pi(12)^2 \times 220$

= $31680\pi\ cm^3$

For upper cylinder, Base Radius (R) = 8 cm



And Height (H) = 60 cm

Volume = $\pi R^2 H$

= $\pi(8)^2 \times 60$

= $3840\pi\ cm^3$

$\therefore$ Volume of the solid Iron pole

= V of lower cylinder + V of upper cylinder

= $31680\pi + 3840\pi = 35520\pi$

= 35520×3.14 = 111532.8 cm^3

mass of 1 cm^3 iron = 8 gm

mass of 111532.8 cm^3 iron = 8* 111532.8 = 892262.4 gm = 892.2624 kg

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Ans. For right circular cone, Radius of the base (r) = 60 cm

And Height (h_1) = 120 cm

Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(60)^2 \times 120$

= $144000\pi cm^3$

For Hemisphere, Radius of the base (r) = 60 cm

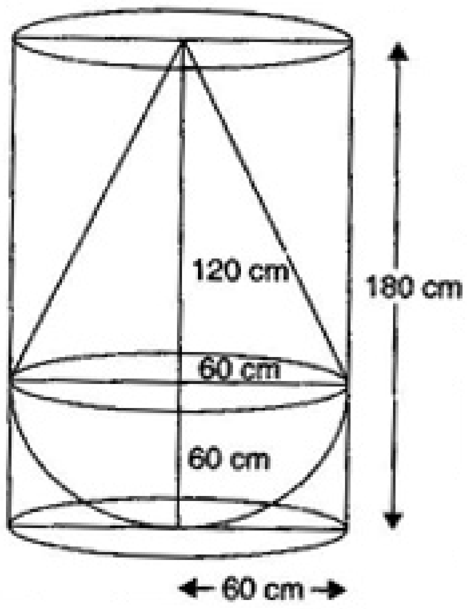
Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(60)^3$

= $144000\pi cm^3$

For right circular cylinder, Radius of the base (r) = 60 cm

And Height (h_2) = 180 cm



Volume = $\pi r^2 h_2$

$= \pi(60)^2 \times 180$

$= 648000\pi \ cm^3$

Now, V of water left in the cylinder

= V of right circular cylinder – (V of right circular cone + V of hemisphere)

$= 648000\pi - (144000\pi + 144000\pi)$

$= 360000\pi \ cm^3$

$= \dfrac{360000}{100 \times 100 \times 100} \pi \ m^3$

$= 0.36 \times \dfrac{22}{7} = 1.131 \ m^3$ (approx.)

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be $345 \ cm^3$. Check whether she is correct, taking the above as the inside measurements and π = 3.14.

Ans.

For Cylinder: diameter of cylin. = 2 cm, height of cylin. = 8 cm

For Sphere : diameter of sphere = 8.5 cm

Amount of water it holds = $\frac{4}{3}\pi r^3 + \pi r^2 h$ = volume of sphere + volume of cylinder

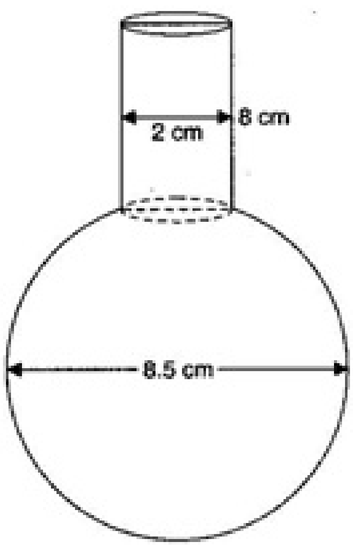
= $\frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi\left(\frac{2}{2}\right)^2 \times 8$

= $\frac{4}{3}\times 3.14\times 4.25\times 4.25\times 4.25+8\times 3.14$

= 321.39 + 25.12

= $346.51\ cm^3$

Hence, she is not correct. The correct volume is $346.51\ cm^3$.



Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Ans. For sphere, Radius (r) = 4.2 cm

Volume = $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi(4.2)^3\ cm^3$

For cylinder, Radius (R) = 6 cm

Let the height of the cylinder be H cm.

Then, Volume = $\pi R^2 H$ = $\pi(6)^2 H\ cm^3$

According to question, Volume of sphere = Volume of cylinder

$\Rightarrow \frac{4}{3}\pi(4.2)^3 = \pi(6)^2 H$

$\Rightarrow$ H = $\frac{4(4.2)^3}{3(6)^2}$

$\Rightarrow$ H = 2.744 cm

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Ans. Let the volume of resulting sphere be r cm.

According to question,

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3$$

$$\Rightarrow r^3 = (6)^3 + (8)^3 + (10)^3$$

$\Rightarrow r^3$ = 216 + 512 + 1000

$\Rightarrow r^3$ = 1728

$\Rightarrow r$ = 12 cm

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Ans. Diameter of well = 7 m

$\therefore$ Radius of well (r) = $\frac{7}{2}$ m

And Depth of earth dug (h) = 20 m

Length of platform (l) = 22 m, Breadth of platform (b) = 14 m

Let height of the platform be h' m

According to question,

Volume of earth dug = Volume of platform

$$\Rightarrow \pi r^2 h = l \times b \times h'$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h'$$

$$\Rightarrow h' = \frac{22 \times 7 \times 7 \times 20}{28 \times 22 \times 14}$$

$\Rightarrow h'$ = 2.5 m

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Ans. Diameter of well = 3 m

$\therefore$ Radius of well (r) = $\frac{3}{2}$ m and Depth of earth dug (h) = 14 m

Width of the embankment = 4 m

$\therefore$ Radius of the well with embankment r' = $\frac{3}{2}+4=\frac{11}{2}$ m

Let the height of the embankment be h' m

According to the question,

Volume of embankment = Volume of the earth dug

$\Rightarrow \pi\left[(r')^2-r^2\right]h' = \pi r^2 h$

$\Rightarrow \left[\left(\frac{11}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]h' = \left(\frac{3}{2}\right)^2\times 14$

$\Rightarrow \left[\frac{121}{4}-\frac{9}{4}\right]h'=\frac{9}{4}\times 14$

$\Rightarrow \frac{112}{4}\times h'=\frac{9}{4}\times 14$

$\Rightarrow h'=\frac{9\times 14\times 4}{4\times 112}$

$\Rightarrow h'$ = 1.125 m

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15

cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans. For right circular cylinder, Diameter = 12 cm

$\therefore$ Radius (r) = $\frac{12}{2}$ = 6 cm and height (h) = 15 cm

For cone & Hemisphere , Diameter = 6 cm

$\therefore$ Radius (r_1) = $\frac{6}{2}$ = 3 cm and height (h_1) = 12 cm

Let n cones be filled with ice cream.

Then, According to question,

Volume of n (cones +Hemisphere) = Volume of right circular cylinder

$\Rightarrow n \times \left(\frac{1}{3}\pi(r_1)^2(h) + \frac{2}{3}\pi(r_1)^3\right) = \pi r^2 h$

$\Rightarrow n\left(\frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3\right) = \frac{22}{7} \times (6)^2 \times 15$

$\Rightarrow n = \frac{22\times36\times15\times3\times7}{(7\times22\times9\times12+7\times44\times27)} = \frac{249480}{24948}$

$\Rightarrow n$ = 10

6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5 \text{ cm} \times 10 \text{ cm} \times 3.5 \text{ cm}$?

Ans. For silver coin, Diameter = 1.75 cm

$\therefore$ Radius $(r) = \frac{1.75}{2} = \frac{7}{8}$ cm and Thickness (h) = 2 mm = $\frac{1}{5}$ cm

For cuboid, Length (l) = 5.5 cm, Breadth (b) = 10 cm and Height (h') = 3.5 cm

Let n coins be melted.

Then, According to question,

Volume of n coins = Volume of cuboid

$\Rightarrow n \times \pi r^2 h = l \times b \times h'$

$\Rightarrow n \times \pi \left(\frac{7}{8}\right)^2 \times \left(\frac{1}{5}\right) = 5.5 \times 10 \times 3.5$

$\Rightarrow n \times \frac{22}{7} \times \frac{49}{64} \times \frac{1}{5} = 5.5 \times 10 \times 3.5$

$\Rightarrow n = \frac{5.5 \times 10 \times 3.5 \times 7 \times 64 \times 5}{22 \times 49}$

$\Rightarrow n = 400$

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Ans. For cylindrical bucket, Radius of the base (r) = 18 cm and height (h) = 32 cm

$\therefore$ Volume = $\pi r^2 h = \pi (18)^2 \times 32$

$= 10368\pi \ cm^3$

For conical heap, Height (h') = 24 cm

Let the radius be r_1 cm.

Then, Volume = $\frac{1}{3}\pi r_1^2 h'$

$= \frac{1}{3} \times \pi \times r_1^2 \times 24 = 8\pi r_1^2 \ cm^3$

According to question, Volume of bucket = Volume of conical heap

$\Rightarrow 10368\pi = 8\pi r_1^2$

$\Rightarrow r_1^2 = \dfrac{10368\pi}{8\pi} = 1296$

$\Rightarrow r_1 = 36$ cm

Now, Slant height $(l) = \sqrt{(r_1)^2 + (h')^2}$

$= \sqrt{(36)^2 + (24)^2} = \sqrt{1296 + 576}$

$= \sqrt{1872} = 12\sqrt{13}$ cm

8. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Ans. For canal, Width = 6 m and Depth = 1.5 m = $\dfrac{3}{2}$ m

Speed of flow of water = 10 km/h

$= 10 \times 1000\ m/h$ = 10000 m/h

$= \dfrac{10000}{60}$ m/min = $\dfrac{500}{3}$ m/min

$\therefore$ Speed of flow of water in 30 minutes

$= \dfrac{500 \times 30}{3}$ m/min = 5000 m/min

$\therefore$ Volume of water that flows in 30 minutes

$= 6 \times \dfrac{3}{2} \times 5000 = 45000\ m^3$

$\therefore$ The area it will irrigate = $\dfrac{45000}{\left(\dfrac{8}{100}\right)} = \dfrac{4500000}{8}$

= $562500\, m^2$

= $\dfrac{562500}{10000}$ hectares = 56.25 hectares

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9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Ans. For cylindrical tank, Diameter = 10 m

$\therefore$ Radius $(r) = \dfrac{10}{2}$ = 5 m and Depth (h) = 2 m

$\therefore$ Volume = $\pi r^2 h$ = $\pi(5)^2 \times 2$ = $50\pi m^3$

Rate of flow of water (h') = 3 km/h = 3000 m/h = $\dfrac{3000}{60}$ m/min = 50 m/min

For pipe, Internal diameter = 20 cm, therefore radius (r_1) = 10 cm = 0.1 m

$\therefore$ Volume of water that flows per minute = $\pi(r_1)^2 h'$

= $\pi(0.1)^2 \times 50$ = $\dfrac{\pi}{2} m^3$

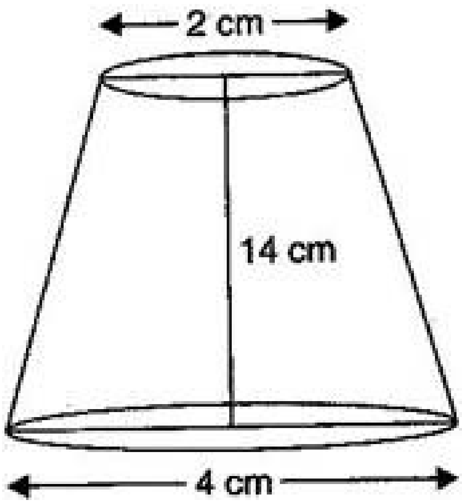
$\therefore$ Required time = $\dfrac{50\pi}{\pi/2}$ = 100 minutes

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Ans. Here, $r_1 = \frac{4}{2}$ = 2 cm,

$r_2 = \frac{2}{2}$ = 1 cm and h = 14 cm



$\therefore$ Capacity of the glass = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

= $\frac{1}{3} \times \frac{22}{7} \times 14\left(2\times 2 + 1\times 1 + 2\times 1\right)$

= $\frac{1}{3} \times \frac{22}{7} \times 14 \times 7$

= $\frac{308}{3} = 102\frac{2}{3} cm^3$

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Ans. Let r_1 cm and r_2 cm be the radii of the ends $(r_1 > r_2)$ of the frustum of the cone.

Then, l = 4 cm

$2\pi r_1$ = 18 cm

$\Rightarrow \pi r_1$ = 9 cm

$2\pi r_2$ = 6 cm

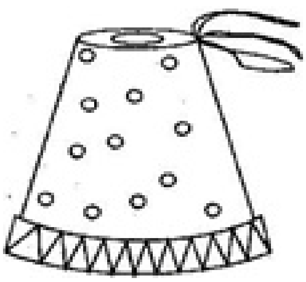
$\Rightarrow \pi r_2$ = 3 cm

Now, CSA of the frustum = $\pi(r_1 + r_2)l$

= $(\pi r_1 + \pi r_2)l$

= $(9+3)\times 4 = 48\ cm^2$

3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Ans. 

Here, r_1 = 10 cm,

r_2 = 4 cm and l = 15 cm

$\therefore$ Surface area = $\pi(r_1+r_2)l+\pi r_2^2$

= $\frac{22}{7}(10+4)\times 15+\frac{22}{7}(4)^2$

= $660+\frac{352}{7}$ = $\frac{4972}{7}=710\frac{2}{7}cm^2$

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the total cost of milk which can completely fill the container at the rate of Rs. 20 per liter. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm^2. $(\text{Take } \pi=3.14)$

Ans. Here, r_1 = 20 cm,

r_2 = 8 cm and h = 16 cm

$\therefore$ Volume of container = $\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$

= $\frac{1}{3}\times 3.14\times 16\left\{(20)^2+(8)^2+20\times 8\right\}$

= $\frac{1}{3}\times 3.14\times 16(400+64+160)$

= $\frac{1}{3}\times 3.14\times 16\times 624$

= $10449.92\ cm^3$ = 10.44992 liters

$\therefore$ Cost of the milk = 10.44992×20

= Rs. 208.9984 = Rs. 209

Now, surface area = $\pi(r_1+r_2)l$ + πr_2^2

$= \pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2} + \pi r_2^2$

$= 3.14(20+8)\sqrt{(16)^2 + (20-8)^2} + 3.14(8)^2$

$= 3.14 \times 28\sqrt{256 + 144} + 3.14 \times 64$

= 1758.4 + 200.96

$= 1959.36\ cm^2$

$\therefore$ Area of the metal sheet used = $1959.36\ cm^2$

$\therefore$ Cost of metal sheet = $1959.36 \times \frac{8}{100}$

= 156.7488 = Rs. 156.75

---

5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

Ans. $\tan 30^\circ = \frac{r_2}{10}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_2}{10}$

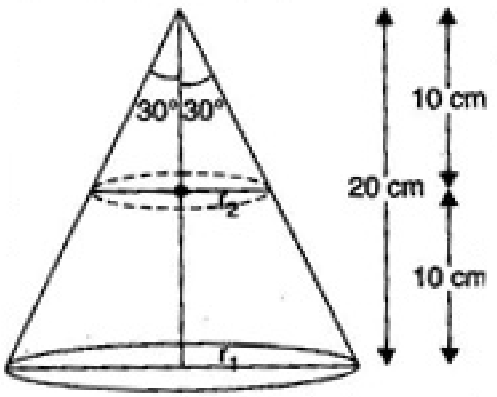
$\Rightarrow r_2 = \frac{10}{\sqrt{3}}$ cm

$\tan 30^\circ = \frac{r_1}{20}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_1}{20}$

$\Rightarrow r_1 = \dfrac{20}{\sqrt{3}}$ cm

h = 10 cm



$\therefore$ Volume = $\dfrac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \frac{1}{3}\times\frac{22}{7}\times 10\left\{\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)\left(\frac{10}{\sqrt{3}}\right)\right\}$$

$$= \frac{1}{3}\times\frac{22}{7}\times 10\times\left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 10\times\frac{700}{3} = \frac{22000}{9}\,cm^3$$

Diameter of the wire = $\dfrac{1}{16}$ cm

$\therefore$ Radius of the wire = $\dfrac{1}{32}$ cm

Let the length of the wire be l cm.

Then, Volume of the wire = $\pi r^2 l = \frac{22}{7}\left(\frac{1}{32}\right)^2 l = \frac{11l}{3584}\ cm^3$

According to the question,

$$\frac{11l}{3584} = \frac{22000}{9}$$

$$\Rightarrow l = \frac{22000 \times 3584}{11 \times 9}$$

$$\Rightarrow l = \frac{2000 \times 3584}{9}$$

$\Rightarrow l =$ 796444.44 cm = 7964.4 m

CBSE Class–10 Mathematics

NCERT solution

Chapter - 13

Surface Areas and Volumes - Exercise 13.5

1. A copper wire, 3 mm in diameter is wound about a cylinder whose length is 12 cm and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Ans. Number of rounds to cover 12 cm, i.e. 120 mm = $\frac{120}{3}$ = 40

Here, Diameter = 10 cm, Radius $(r)=\frac{10}{2}$ cm

Length of the wire used in taking one round

= $2\pi r = 2\pi \times 5 = 10\pi$ cm

Length of the wire used in taking 40 rounds

= $10\pi \times 40 = 400\pi$ cm

Radius of the copper wire = $\frac{3}{2}$ mm

= $\frac{3}{20}$ cm

$\therefore$ Volume of wire = $\pi\left(\frac{3}{20}\right)^2(400\pi)$

= $9\pi^2 cm^3$--------------***

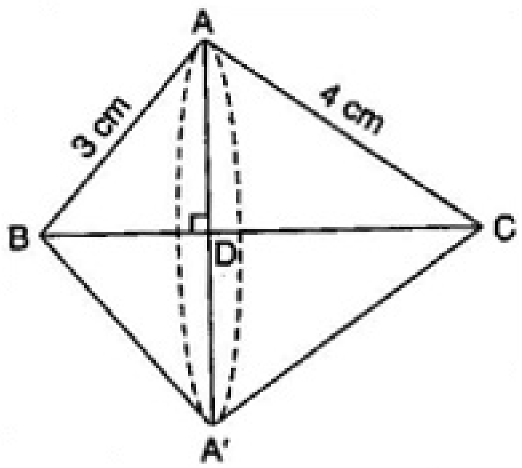
$\therefore$ Mass of the wire = $9\times(3.14)^2\times 8.88$

= 787.98 gm

2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate)

Ans. Hypotenuse = $\sqrt{3^2+4^2}$ = 5 cm

In figure, $\triangle$ ADB $\sim$ $\triangle$ CAB [AA similarity]



$\therefore \dfrac{AD}{CA} = \dfrac{AB}{CB}$

$\Rightarrow \dfrac{AD}{4} = \dfrac{3}{5}$

$\Rightarrow$ AD = $\dfrac{12}{5}$ cm

Also, $\dfrac{DB}{AB} = \dfrac{AB}{CB}$

$\Rightarrow \dfrac{DB}{3} = \dfrac{3}{5}$

$\Rightarrow$ DB = $\dfrac{9}{5}$ cm

$\therefore$ CD = BC – DB = $5-\frac{9}{5}=\frac{16}{5}$ cm

Volume of the double cone

= $\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{9}{5}\right)+\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{16}{5}\right)$

= $\frac{1}{3}\times3.14\times\frac{12}{5}\times\frac{12}{5}\times5$ = $30.14\ cm^3$

Surface area of the double cone

= $\pi\times\frac{12}{5}\times3+\pi\times\frac{12}{5}\times4$

= $\pi\times\frac{12}{5}(3+4)$ = $3.14\times\frac{12}{5}\times7$

= $52.75\ cm^2$

---

3. A cistern, internally measuring $150\text{ cm}\times120\text{ cm}\times110\text{ cm}$ has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being $22.5\text{ cm}\times7.5\text{ cm}\times6.5\text{ cm}$?

Ans. Volume of cistern = $150\times120\times110$ = $1980000\ cm^3$

Volume of water = $129600\ cm^3$

$\therefore$ Volume of cistern to be filled

= 1980000 – 129600 = $1850400\ cm^3$

Volume of a brick = $22.5\times7.5\times6.5$

= $1096.875\ cm^3$

Let n bricks be needed.

Then, water absorbed by n bricks = $n \times \frac{1096.875}{17} cm^3$

$\therefore \ n = \frac{1850400 \times 17}{16 \times 1096.875}$ = 1792 (approx.)

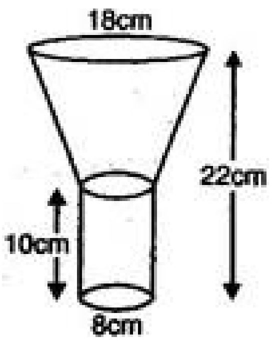
4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Ans. Volume of rainfall =$7280 \times \frac{10}{100 \times 1000}$ = 0.728 km^2

Volume of three rivers = $3 \times 1072 \times \frac{75}{1000} \times \frac{3}{1000}$ = 0.7236 km^2

Hence, the amount of rainfall is approximately equal to the amount of water in three rivers.

5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see figure).



Ans. Slant height of the frustum of the cone

$(l) = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(22-10)^2 + \left(\frac{18}{2} - \frac{8}{2}\right)^2}$ = 13 cm

Area of the tin sheet required

= CSA of cylinder + CSA of the frustum

$= 2\pi(4)(10) + \pi(4+9)13$

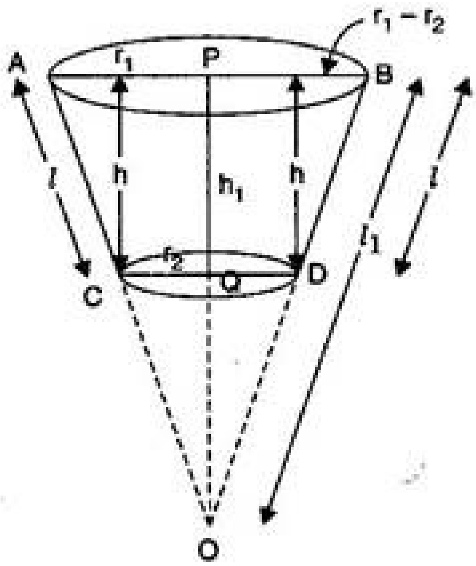
$= 80\pi + 169\pi$

$= 249\pi = 249 \times \frac{22}{7} = 782\frac{4}{7} cm^2$

---

6. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. According to the question, the frustum is the difference of the two cones OAB and OCD (in figure).



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

$\therefore$ Height of the cone OCD = $h_1 - h$

$\because$ Δ OQD ~ Δ OPB [By, AA similarity]

$$\therefore \frac{h_1 - h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow 1 - \frac{r_2}{r_1} = \frac{h}{h_1}$$

$$\Rightarrow h_1 = \frac{hr_1}{r_1 - r_2} \quad ....(i)$$

$\therefore$ height of the cone OCD = $h_1 - h$

$$= \frac{hr_1}{r_1 - r_2} - h = \frac{hr_2}{r_1 - r_2} \quad ....(ii)$$

$\therefore$ V of the frustum

= V of cone OAB – V of cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$= \frac{\pi}{3}\left[r_1^2 \cdot \frac{hr_1}{r_1 - r_2} - r_2^2 \cdot \frac{hr_2}{r_1 - r_2}\right]$$

[From eq. (i) & (ii)]

$$= \frac{\pi h}{3}\left(\frac{r_1^3 - r_2^3}{r_1 - r_2}\right)$$

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

If $A_1 and\ A_2$ are the surface areas of two circular bases, then

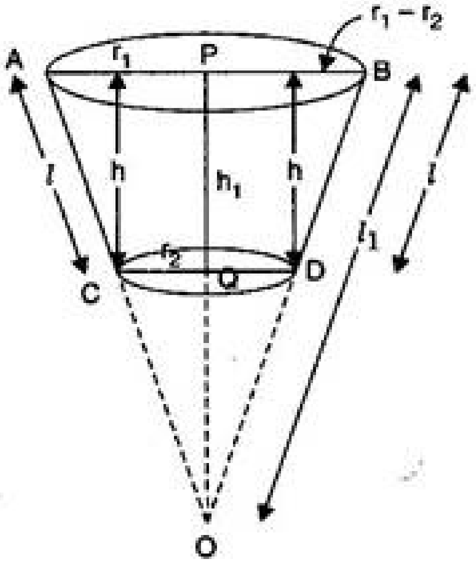
$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

∴ V of the frustum

$$= \frac{h}{3}\left(\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2}.\sqrt{\pi r_2^2}\right)$$

$$= \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

7. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. 

For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

Again, from ΔDEB, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\because$ Δ OQD ~ Δ OPB [AA similarity]

$$\therefore \frac{l_1 - l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow l_1 = \frac{lr_1}{r_1 - r_2} \text{ ....(iii)}$$

$$\therefore l_1 - l = \frac{lr_1}{r_1 - r_2} - l = \frac{lr_2}{r_1 - r_2} \text{ ....(iv)}$$

Hence, CSA of the frustum of the cone = $\pi r_1 l_1 - \pi r_2 (l_1 - l)$

$$= \pi r_1 . \frac{lr_1}{r_1 - r_2} - \pi r_2 \frac{lr_2}{r_1 - r_2} \quad \text{[From eq. (i) and (ii)]}$$

$$= \pi l \left(\frac{r_1^2 - r_2^2}{r_1 - r_2} \right) = \pi l (r_1 + r_2),$$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\therefore$ TSA of the frustum of the cone

$$= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$

CBSE Class–10 Mathematics

NCERT solution

Chapter - 14

Statistics - Exercise 14.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean and why?

Ans. Since, number of plants and houses are small in their values, so we use direct method.

Number of plants	Number of houses (f_i)	Class Marks (x_i)	$f_i x_i$
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	5	7	35
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
Total	$\sum f_i = 20$		$\sum f_i x_i = 162$

Mean $\left(\bar{x}\right) = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{162}{20} = 8.1$

Hence mean number of plants per house is 8.1.

2. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in Rs.)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Ans.

Daily wages (in Rs.)	No. of workers (f_i)	Class Marks (x_i)	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
100 – 120	12	110	– 2	– 24
120 – 140	14	130	– 1	– 14
140 – 160	8	150	0	0
160 – 180	6	170	1	6
180 – 200	10	190	2	20
	$\sum f_i$ =50			$\sum f_i u_i = -12$

From given data, Assume mean (a) = 150, Width of the class (h) = 20

$$\therefore \ \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-12}{50} = -0.24$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = $150 + 20(-0.24)$ = 150 – 4.8 = 145.2

Hence mean daily wages of the workers of factory is Rs. 145.20.

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency (f).

Daily pocket allowance (in Rs.)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of houses	7	6	9	13	f	5	4

Ans.

Daily pocket allowance (in Rs.)	No. of children (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$f_i d_i$
11 – 13	7	12	– 6	– 42
13 – 15	6	14	– 4	– 24
15 – 17	9	16	– 2	– 18
17 – 19	13	18	0	0
19 – 21	f	20	2	$2f$
21 – 23	5	22	4	20
23 – 25	4	24	6	24
	$\sum f_i = 44 + f$			$\sum f_i d_i = 2f - 40$

From given data, Assume mean (a) = 18

$$\therefore\ (\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$\Rightarrow\ 18 = 18 + \frac{2f - 40}{44 + f}$$

$$\Rightarrow\ \frac{2f - 40}{44 + f} = 0$$

$$\Rightarrow\ 2f - 40 = 0$$

$$\Rightarrow\ 2f = 40$$

$$\Rightarrow\ f = 20$$

Hence missing frequency is 20.

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows:

Number of heart beats per minute	65 – 68	68 – 71	71 – 74	74 – 77	77 – 80	80 – 83	83 – 86
Number of women	2	4	3	8	7	4	2

Ans.

No. of heart beats per min.	No. of women (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
65 – 68	2	66.5	– 3	– 6
68 – 71	4	69.5	– 2	– 8
71 – 74	3	72.5	– 1	– 3
74 – 77	8	75.5	0	0
77 – 80	7	78.4	1	7
80 – 83	4	81.5	2	8
83 – 86	2	84.5	3	6
	$\sum f_i = 30$			$\sum f_i u_i = 4$

(in the class interval 77-80 , 78.4 changes to 78.5)

From given data, Assume mean (a) = 75.5, Width of the class (h) = 3

$\therefore\ \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{4}{30} = 0.13$ (approx.)

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 75.5 + 3 (0.13) = 75.5 + 0.39 = 75.89

Hence mean heart beat per minute for women is 75.89.

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5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	12	14	8	6	10

{change the frequency in above table as: 50-52 (15) 53-55 (110) 56-58 (135) 59-61 (115) 62-64 (25)}

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Ans. Since value of number of mangoes and number of boxes are large numerically. So we use step-deviation method

we convert the class interval firstly into exclusive form given as

True Class Interval	No. of boxes (fi)	Class mark (xi)	$u_i = \frac{x_i - a}{h}$	fiui
49.5-52.5	15	51	-2	-30
52.5-55.5	110	54	-1	-110
55.5-58.5	135	57	0	0
58.5-61.5	115	60	1	115
61.5-64.5	25	63	2	50
	$\sum f_i = 400$			$\sum f_i u_i = 25$

From given data, Assume mean (a) = 57, Width of the class (h) = 3

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{25}{400} = 0.0625 \text{ (approx.)}$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 57 + 3 (0.0625) = 57 + 0.1875 = 57.1875 = 57.19 (approx.)

Hence mean number of mangoes kept in a packing box is 57.19.

6. The table below shows the daily expenditure on food of 25 households in a locality:

Daily expenditure (in Rs.)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Ans.

Daily expenditure	**No. of households** (f_i)	**Class Marks** (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100 – 150	4	125	– 2	– 8
150 – 200	5	175	– 1	– 5
200 – 250	12	225	0	0
250 – 300	2	275	1	2
300 – 350	2	325	2	4
	$\sum f_i = 25$			$\sum f_i u_i = -7$

From given data, Assume mean (a) = 225, Width of the class (h) = 50

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-7}{25} = -0.28$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 225 + 50 (– 0.28) = 225 – 14 = 211

Hence mean daily expenditure on food is Rs. 211.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO^2 (in ppm)	0.00 – 0.04	0.04 – 0.08	0.08 – 0.12	0.12 – 0.16	0.16 – 0.20	0.20 – 0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

Ans.

Concentration of SO^2 (in ppm)	**Frequency** (f_i)	**Class Marks** (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0.00 – 0.04	4	0.02	– 2	– 8
0.04 – 0.08	9	0.06	– 1	– 9
0.08 – 0.12	9	0.10	0	0
0.12 – 0.16	2	0.14	1	2
0.16 – 0.20	4	0.18	2	8
0.20 – 0.24	2	0.20	3	6
	$\sum f_i = 30$			$\sum f_i u_i = -1$

From given data, Assume mean (a) = 0.10, Width of the class (h) = 0.04

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-1}{30} = -0.033 \text{ (approx.)}$$

Using formula, Mean $\left(\bar{x}\right) = a + h\bar{u}$ = 0.10 + 0.04 (– 0.033) = 0.10 – 0.0013 = 0.0987 (approx.)

Hence mean concentration of SO_2 in air is 0.0987 ppm.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 – 6	6 – 10	10 – 14	14 – 20	20 – 28	28 – 38	38 – 40
Number of students	11	10	7	4	4	3	1

Ans.

Number of days	No. of students (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$f_i d_i$
0 – 6	11	3	– 14	– 154
6 – 10	10	8	– 9	– 90
10 – 14	7	12	– 5	– 35
14 – 20	4	17	0	0
20 – 28	4	24	7	28
28 – 38	3	33	16	48
38 – 40	1	39	22	22
	$\sum f_i = 40$			$\sum f_i d_i = -181$

From given data, Assume mean (a) = 17

$$\therefore \left(\bar{x}\right) = a + \frac{\sum f_i d_i}{\sum f_i} = 17 + \frac{(-181)}{40} = 17 - 4.52 = 12.48$$

Hence mean 12.48 number of days a student was absent.

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in percentage)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Ans.

Literacy rate (in %)	No. of cities (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
45 – 55	3	50	– 2	– 6
55 – 65	10	60	– 1	– 10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
	$\sum f_i = 35$			$\sum f_i u_i = -2$

From given data, Assume mean (a) = 70, Width of the class (h) = 10

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-2}{35} = -0.057$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 70 + 10 (– 0.057) = 70 – 0.57 = 69.43

Hence mean literacy rate is 69.43%.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 14

Statistics - Exercise 14.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Ans. For Mode: In the given data, maximum frequency is 23 and it corresponds to the class interval 35 – 45.

$\therefore$ Modal class = 35 – 45

And $l = 35$, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$ and $h = 10$

$\because$ Mode = $l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= $35 + \left[\dfrac{23-21}{2(23)-21-14}\right] \times 10$

= $35 + \dfrac{2}{46-35} \times 10$

= $35 + \dfrac{20}{11}$

= 35 + 1.8

= 36.8

For Mean:

Age (in years)	No. of patients (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
5 – 15	6	10	– 2	– 12
15 – 25	11	20	– 1	– 11
25 – 35	21	30	0	0
35 – 45	23	40	1	23
45 – 55	14	50	2	28
45 – 65	5	60	3	15
	$\sum f_i = 80$			$\sum f_i u_i = 43$

From given data, Assume mean (a) = 30, Width of the class (h) = 10

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{43}{80} = 0.5375$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 30 + 10 (0.5375) = 30 + 5.375 = 35.37

Hence mode of given data is 36.8 years and mean of the given data is 35.37 years.

Also, it is clear from above discussion that average age of a patient admitted in the hospital is 35.37 years and maximum number of patients admitted in the hospital are of age 36.8 years.

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Life times (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Ans. Given: Maximum frequency is 61 and it corresponds to the class interval 60 – 80.

$\therefore$ Modal class = 60 – 80

And $l = 60$, $f_1 = 61$, $f_0 = 52$, $f_2 = 38$ and $h = 20$

$\because$ Mode = $l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 60 + \left[\dfrac{61-52}{2(61)-52-38}\right] \times 20$

$= 60 + \dfrac{9}{122-52-38} \times 20$

$= 60 + \dfrac{9}{32} \times 20$

= 60 + 5.625

= 65.625

Hence modal lifetimes of the components is 65.625 hours.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also find the mean monthly expenditure:

Expenditure (in Rs.)	Number of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Ans. For Mode: Here, Maximum frequency is 40 and it corresponds to the class interval 1500 – 2000.

$\therefore$ Modal class = 1500 – 2000

And $l = 1500$, $f_1 = 40$, $f_0 = 24$, $f_2 = 33$ and $h = 500$

$\because$ Mode = $l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 1500 + \left[\dfrac{40-24}{2(40)-24-33}\right] \times 500$

$= 1500 + \dfrac{16}{80-24-33} \times 500$

$= 1500 + \dfrac{8000}{23}$

= 1500 + 347.83

= 1847.83

For Mean:

Expenditure (in Rs.)	**No. of families** (f_i)	**Class Marks** (x_i)	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
1000 – 1500	24	1250	– 3	– 72
1500 – 2000	40	1750	– 2	– 80
2000 – 2500	33	2250	– 1	– 33
2500 – 3000	28	2750	0	0
3000 – 3500	30	3250	1	30
3500 – 4000	22	3750	2	44
4000 – 4500	16	4250	3	48
4500 – 5000	7	4750	4	28
	$\sum f_i$ = 200			$\sum f_i u_i = -35$

From given data, Assume mean (a) = 2750, Width of the class (h) = 500

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-35}{200} = -0.175$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 2750 + 500 (– 0.175) = 2750 – 87.50 = 2662.50

Hence the modal monthly expenditure of family is Rs. 1847.83 and the mean monthly expenditure is Rs. 2662.50.

---

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

No. of students per teacher	Number of states / U.T.
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

Ans. For Mode: Here, Maximum frequency is 10 and it corresponds to the class interval 30 – 35.

$\therefore$ Modal class = 30 – 35

And $l = 30$, $f_1 = 10$, $f_0 = 9$, $f_2 = 3$ and $h = 5$

$\because$ Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ = $30 + \left[\frac{10-9}{2(10)-9-3}\right] \times 5$

$= 30 + \frac{1}{20-12} \times 5 = 30 + \frac{5}{8}$ = 30 + 0.625 = 30.63 (approx.)

For Mean:

Expenditure (in Rs.)	No. of families (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
15 – 20	3	17.5	– 3	– 9
20 – 25	8	22.5	– 2	– 16
25 – 30	9	27.5	– 1	– 9
30 – 35	10	32.5	0	0
35 – 40	3	37.5	1	3
40 – 45	0	42.5	2	0
45 – 50	0	47.5	3	0
50 – 55	2	52.5	4	8
	$\sum f_i$ = 35			$\sum f_i u_i = -23$

From given data, Assume mean (a) = 32.5, Width of the class (h) = 5

$$\therefore \ \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-23}{35} = -0.65$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 32.5 + 5 (– 0.65) = 32.5 – 3.25 = 29.25 (approx.)

Hence mode and mean of given data is 30.63 and 29.25. Also from above discussion, it is clear that states/U.T. have students per teacher is 30.63 and on average, this ratio is 29.25.

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day cricket matches:

Runs scored	Number of batsmen
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	7
7000 – 8000	6
8000 – 9000	3
9000 – 10000	1
10000 – 11000	1

Find mode of the data.

Ans. In the given data, maximum frequency is 18 and it corresponds to the class interval 4000 – 5000.

$\therefore$ Modal class = 4000 – 5000

And $l = 4000$, $f_1 = 18$, $f_0 = 4$, $f_2 = 9$ and $h = 1000$

$\because$ Mode = $l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

= $4000 + \left[\dfrac{18 - 4}{2(18) - 4 - 9} \right] \times 1000$

= $4000 + \dfrac{14}{36 - 13} \times 1000$

= $4000 + \dfrac{14000}{23}$

= 4000 + 608.6956

= 4608.7 (approx.)

Hence, mode of the given data is 4608.7 runs.

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarized it in the table given below:

Number of cars	Frequency
0 – 10	7
10 – 20	14
20 – 30	13
30 – 40	12
40 – 50	20
50 – 60	11
60 – 70	15
70 – 80	8

Find the mode of the data.

Ans. In the given data, maximum frequency is 20 and it corresponds to the class interval 40 – 50.

$\therefore$ Modal class = 40 – 50

And $l = 40$, $f_1 = 20$, $f_0 = 12$, $f_2 = 11$ and $h = 10$

$\because$ Mode = $l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= $40 + \left[\dfrac{20-12}{2(20)-12-11}\right] \times 10$

= $40 + \dfrac{8}{40-23} \times 10$

= $40 + \dfrac{80}{17}$ = 40 + 4.70588 = 44.7 (approx.)

Hence, mode of the given data is 44.7 cars.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 14

Statistics - Exercise 14.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

Ans. For Median:

Monthly consumption (in units)	Number of consumers (f_i)	Cumulative Frequency
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	8	64
185 – 205	4	68
Total	$\sum f_i = n = 68$	

Here, $\sum f_i = n = 68$, then $\frac{n}{2} = \frac{68}{2} = 34$, which lies in interval 125 – 145.

$\therefore$ Median class = 125 – 145

So, $l = 125$, $n = 68$, $f = 20$, $cf = 22$ and $h = 20$

Now, Median = $l + \left[\dfrac{\frac{n}{2} - cf}{f}\right] \times h$

= $125 + \left[\dfrac{\frac{68}{2} - 22}{20}\right] \times 20$

= $125 + \frac{34-22}{20} \times 20$ = 125 + 12 = 137

For Mean:

Monthly consumption (in units)	No. of consumers (f_i)	Class Marks (x_i)	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
65 – 85	4	75	– 3	– 12
85 – 105	5	95	– 2	– 10
105 – 125	13	115	– 1	– 13
125 – 145	20	135	0	0
145 – 165	14	155	1	14
165 – 185	8	175	2	16
185 – 205	4	195	3	12
	$\sum f_i = 68$			$\sum f_i u_i = 7$

From given data, Assume mean (a) = 135, Width of the class (h) = 20

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i}$$

$= \frac{7}{68} = 0.102$

Using formula, Mean $\left(\bar{x}\right) = a + h\bar{u}$ = 135 + 20 (0.102)

= 135 + 2.04 = 137.04

For Mode:

In the given data, maximum frequency is 20 and it corresponds to the class interval 125 – 145.

∴ Modal class = 125 – 145

And $l = 125$, $f_1 = 20$, $f_0 = 13$, $f_2 = 14$ and $h = 20$

∵ Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 125 + \left[\frac{20-13}{2(20)-13-14}\right] \times 20$

$= 125 + \frac{7}{40-27} \times 20$

$= 125 + \frac{140}{13}$

= 125 + 10.76923

= 125 + 10.77

= 135.77

Hence, median, mean and mode of given data is 137 units, 137.04 units and 135.77 units.

2. If the median of the distribution given below is 28.5, then find the values of x and y.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	Total
Frequency	5	x	20	15	y	5	60

Ans.

Monthly consumption (in units)	Number of consumers (f_i)	Cumulative Frequency
0 – 10	5	5
10 – 20	x	$5+x$
20 – 30	20	$25+x$
30 – 40	15	$40+x$
40 – 50	y	$40+x+y$
50 – 60	5	$45+x+y$
Total	$\sum f_i = n = 60$	

Here, $\sum f_i = n = 60$, then $\frac{n}{2} = \frac{60}{2} = 30$, also, median of the distribution is 28.5, which lies in interval 20 – 30.

$\therefore$ Median class = 20 – 30

So, $l = 20$, $n = 60$, $f = 20$, $cf = 5 + x$ and $h = 10$

$\because$ $45 + x + y = 60$

$\Rightarrow$ $x + y = 15$ ....(i)

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$\Rightarrow$ $28.5 = 20 + \left[\frac{30 - (5 + x)}{20}\right] \times 10$

$$\Rightarrow 28.5 = 20 + \frac{30-5-x}{2}$$

$$\Rightarrow 28.5 = \frac{40+25-x}{2}$$

$$\Rightarrow 2(28.5) = 65 - x$$

$$\Rightarrow 57.0 = 65 - x$$

$$\Rightarrow x = 65 - 57 = 8$$

$$\Rightarrow x = 8$$

Putting the value of x in eq. (i), we get,

$$8 + y = 15$$

$$\Rightarrow y = 7$$

Hence the value of x and y are 8 and 7 respectively.

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3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Ages (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Ans.

Ages (in years)	Cumulative Frequency	Number of consumers (f_i)
Below 20	2	2
20 – 25	6	6 – 2 = 4
25 – 30	24	24 – 6 = 18
30 – 35	45	45 – 24 = 21
35 – 40	78	78 – 45 = 33
40 – 45	89	89 – 78 = 11
45 – 50	92	92 – 89 = 3
50 – 55	98	98 – 92 = 6
55 – 60	100	100 – 98 = 2
Total		$\sum f_i = n = 100$

Here, $\sum f_i = n = 100$, then $\frac{n}{2} = \frac{100}{2} = 50$, which lies in interval 35 – 40.

$\therefore$ Median class = 35 – 40

So, $l = 35$, $n = 100$, $f = 33$, $cf = 45$ and $h = 5$

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

= $35 + \left[\frac{\frac{100}{2} - 45}{33}\right] \times 5$

= $35 + \frac{50 - 45}{33} \times 5$

= $35 + \frac{25}{33}$

= 35 + 0.7575

= 35 + 0.76 (approx.)

= 35.76

Hence median age of given data is 35.76 years.

4. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter and data obtained is represented in the following table. Find the median length of the leaves.

Length (in mm)	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Ans. Since the frequency distribution is not continuous, so firstly we shall make it continuous.

Length (in mm)	Number of leaves (f_i)	Cumulative Frequency
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40
Total	$\sum f_i = n = 40$	

Here, $\sum f_i = n = 40$, then $\frac{n}{2} = \frac{40}{2} = 20$, which lies in interval 144.5 – 153.5.

$\therefore$ Median class = 144.5 – 153.5

So, $l = 144.5$, $n = 40$, $f = 12$, $cf = 17$ and $h = 9$

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

= $144.5 + \left[\frac{20 - 17}{12}\right] \times 9$

= $144.5 + \frac{3 \times 9}{12}$

= 144.5 + 2.25

= 146.75

Hence median length of the leaves is 146.75 mm.

5. The following table gives the distribution of the life time of 400 neon lamps. Find the median life time of the lamps.

Life time (in hours)	Number of lamps
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	85
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

(change the frequency of class interval 3000-3500 from 85 to 86)

Ans.

Life time (in hours)	Number of lamps (f_i)	Cumulative Frequency
1500 – 2000	14	14
2000 – 2500	56	70
2500 – 3000	60	130
3000 – 3500	86	216
3500 – 4000	74	290
4000 – 4500	62	352
4500 – 5000	48	400
Total	$\sum f_i = n = 400$	

Here, $\sum f_i = n = 400$, then $\frac{n}{2} = \frac{400}{2} = 200$, which lies in interval 3000 – 3500.

∴ Median class = 3000 – 3500

So, $l = 3000$, $n = 400$, $f = 86$, $cf = 130$ and $h = 500$

Now, Median = $l + \left[\dfrac{\frac{n}{2} - cf}{f}\right] \times h$

= $3000 + \left[\dfrac{200 - 130}{86}\right] \times 500$

= $3000 + \dfrac{70 \times 500}{86}$

= 3000 + 406.9767441

= 3000 + 406.98 (approx.)

= 3406.98

Hence median life time of a lamp is 3406.98 hours.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

No. of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
No. of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also find the modal size of the surnames.

Ans. For Median:

No. of letters	Number of surnames (f_i)	Cumulative Frequency
1 – 4	6	6
4 – 7	30	36
7 – 10	40	76
10 – 13	16	92
13 – 16	4	96
16 – 19	4	100
Total	$\sum f_i = n = 100$	

Here, $\sum f_i = n = 100$, then $\frac{n}{2} = \frac{100}{2} = 50$, which lies in interval 7 – 10.

$\therefore$ Median class = 7 – 10

So, $l = 7$, $n = 100$, $f = 40$, $cf = 36$ and $h = 3$

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 7 + \left[\frac{50 - 36}{40}\right] \times 3$

$= 7 + \frac{14 \times 3}{40}$

= 7 + 1.05

= 8.05

For Mean:

No. of letters	(f_i)	Class Marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1 - 4	6	2.5	- 2	- 12
4 - 7	30	5.5	- 1	- 30
7 - 10	40	8.5	0	0
10 - 13	16	11.5	1	16
13 - 16	4	14.5	2	8
16 - 19	4	17.5	3	12
	$\sum f_i$ = 100			$\sum f_i u_i = -6$

From given data, Assume mean (a) = 8.5, Width of the class (h) = 3

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-6}{100} = 0.06$$

Using formula, Mean $(\bar{x}) = a + h\bar{u}$ = 8.5 + 3 (– 0.06) = 8.5 – 0.18 = 8.32

For Mode:

In the given data, maximum frequency is 40 and it corresponds to the class interval 7 – 10.

$\therefore$ Modal class = 7 – 10

And $l = 7$, $f_1 = 40$, $f_0 = 30$, $f_2 = 16$ and $h = 3$

$\because$ Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$= 7 + \left[\frac{40-30}{2(40)-30-16}\right] \times 3$$

$$= 7 + \frac{10}{80-46} \times 3$$

$$= 7 + \frac{30}{34}$$

= 7 + 0.88 (approx.)

= 7.88

Hence, median, mean and mode of given data is 8.05 letters, 8.32 letters and 7.88 letters respectively.

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
No. of students	2	3	8	6	6	3	2

Ans.

Weight (in kg)	Number of students (f_i)	Cumulative Frequency
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
Total	$\sum f_i = n = 30$	

Here, $\sum f_i = n = 30$, then $\frac{n}{2} = \frac{30}{2} = 15$, which lies in interval 55 – 60.

$\therefore$ Median class = 55 – 60

So, $l = 55$, $n = 30$, $f = 6$, $cf = 13$ and $h = 5$

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

= $55 + \left[\frac{15 - 13}{6}\right] \times 5$

= $55 + \frac{2 \times 5}{6}$

= 55 + 1.66666

= 5 + 1.67 (approx.)

= 56.67

Hence median weight of the students are 56.67 kg.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 14

Statistics - Exercise 14.4

1. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in Rs.)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
No. of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

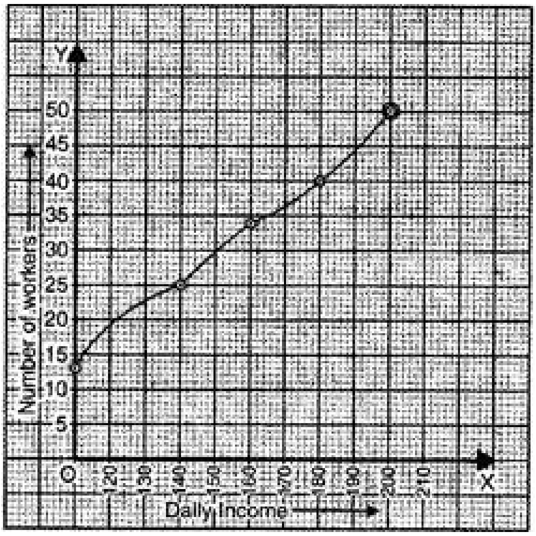
Ans.

Daily income (in Rs.)	Number of workers (f_i)	Cumulative Frequency Less than type (x_i)
100 – 120	12	12
120 – 140	14	26
140 – 160	8	34
160 – 180	6	40
180 – 200	10	50
Total	$\sum f_i = n = 50$	

Now, by drawing the points on the graph,

i.e., (120, 12); (140, 26); (160, 34); (180, 40); (200, 50)

Scale: On $x-$axis 10 units = Rs. 10 and on $y-$axis 10 units = 5 workers



(start the graph from 120 correspond to 12 on y axis)

2. During the medical checkup of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	No. of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

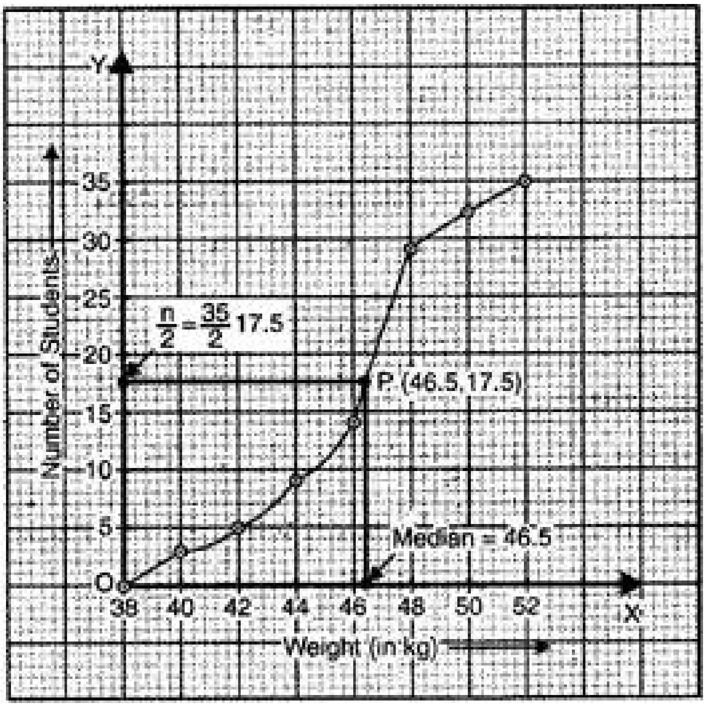
Ans.

Weight (in kg)	No. of students (f_i)	Class interval	Cumulative frequency Less than type
Less than 38	0	36 – 38	0
Less than 40	3 – 0 = 3	38 – 40	3
Less than 42	5 – 3 = 2	40 – 42	5
Less than 44	9 – 5 = 4	42 – 44	9
Less than 46	14 – 9 = 5	44 – 46	14
Less than 48	28 – 14 = 14	46 – 48	28
Less than 50	32 – 28 = 4	48 – 50	32
Less than 52	35 – 32 = 3	50 – 52	35
Total	$\sum f_i = n = 35$		

Hence, the points for graph are:

(38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32), (52, 35)

Scale: On $x-$axis, 10 units = 2 kg and on $y-$axis, 10 units = 5 students



change : (in graph :38 is plotted wrongly on graph on 38 its zero and at 40 38 is there)

From the above graph, Median = 46.5 kg, which lies in class interval 46 – 48.

Here, $\sum f_i = n = 35$, then $\frac{n}{2} = \frac{35}{2} = 17.5$, which lies in interval 46 – 48.

$\therefore$ Median class = 46 – 48

So, $l = 46$, $n = 35$, $f = 14$, $cf = 14$ and $h = 2$

Now, Median = $l + \left[\dfrac{\frac{n}{2} - cf}{f}\right] \times h$

= $46 + \left[\frac{17.5 - 14}{14}\right] \times 2$

= $46 + \frac{7}{14}$

= 46 + 0.5

= 46.5

Hence median weight of students is 46.5 kg.

3. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
No. of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution and draw its ogive.

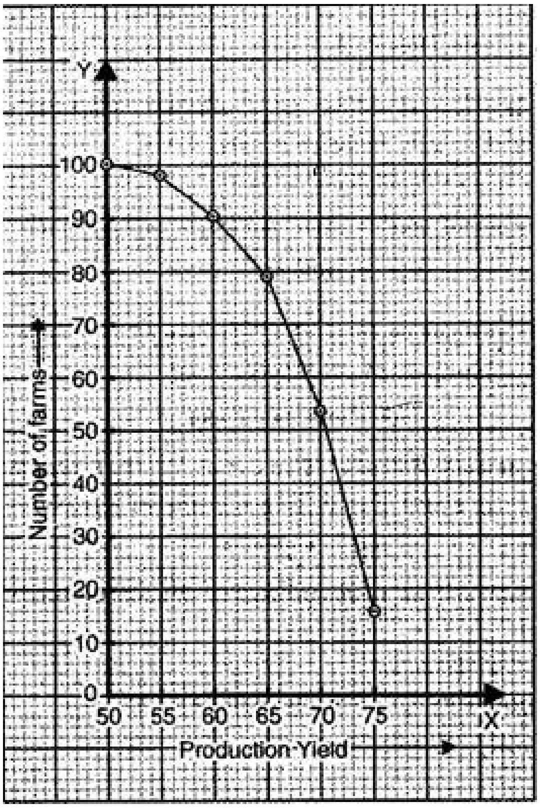
Ans.

Production yield (in kg/ha)	Number of farms (f_i)	Cumulative Frequency Less than type (x_i)
50 – 55	2	100
55 – 60	8	100 – 2 = 98
60 – 65	12	98 – 8 = 90
65 – 70	24	90 – 12 = 78
70 – 75	38	78 – 24 = 54
75 – 80	16	54 – 38 = 16
Total	$\sum f_i = n = 100$	

The points for the graph are:

(50, 100), (55, 98), (60, 90), (65, 78), (70, 54), (75, 16)

Scale: On $x-$ axis, 10 units = 5 kg/ha and on $y-$axis, 10 units = 10 forms.



(change the place of 50 in the graph it must be at 55)

CBSE Class–10 Mathematics

NCERT solution

Chapter - 15

Probability - Exercise 15.1

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1. Complete the statements:

(i) Probability of event E + Probability of event "not E" = _______________

(ii) The probability of an event that cannot happen is _______________. Such an event is called _______________.

(iii) The probability of an event that is certain to happen is _______________. Such an event is called _______________.

(iv) The sum of the probabilities of all the elementary events of an experiment is _______________.

(v) The probability of an event is greater than or equal to _______________ and less than or equal to _______________.

Ans. (i) 1

(ii) 0, impossible event

(iii) 1, sure or certain event

(iv) 1

(v) 0, 1

---

2. Which of the following experiments have equally likely outcomes? Explain.

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong.

(iv) A baby is born. It is a boy or a girl.

Ans. (i) In the experiment, "A driver attempts to start a car. The car starts or does not start", we are not justified to assume that each outcome is as likely to occur as the other. Thus, the experiment has no equally likely outcomes.

(ii) In the experiment, "A player attempts to shoot a basketball. She/he shoots or misses the shot", the outcome depends upon many factors e.g. quality of player. Thus, the experiment has no equally likely outcomes.

(iii) In the experiment, "A trial is made to answer a true-false question. The answer is right or wrong." We know, in advance, that the result can lead to one of the two possible ways – either right or wrong. We can reasonably assume that each outcome, right or wrong, is likely to occur as the other. Thus, the outcomes right or wrong are equally likely.

(iv) In the experiment, "A baby is born, It is a boy or a girl, we know, in advance that there are only two possible outcomes – either a boy or a girl. We are justified to assume that each outcome, boy or girl, is likely to occur as the other. Thus, the outcomes boy or girl are equally likely.

---

3. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Ans. The tossing of a coin is considered to be a fair way of deciding which team should get the ball at the beginning of a football game as we know that the tossing of the coin only land in one of two possible ways – either head up or tail up. It can reasonably be assumed that each outcome, head or tail, is as likely to occur as the other, i.e., the outcomes head and tail are equally likely. So the result of the tossing of a coin is completely unpredictable.

---

4. Which of the following cannot be the probability of an event:

(A) $\frac{2}{3}$

(B) -1.5

(C) 15%

(D) 0.7

Ans. (B) Since the probability of an event E is a number P(E) such that

$0 \leq P(E) \leq 1$

$\therefore -1.5$ cannot be the probability of an event.

5. If P(E) = 0.05, what is the probability of 'not E'?

Ans. Since P(E) + P (not E) = 1

$\therefore$ P (not E) = 1 – P(E) = 1 – 0.05 = 0.95

6. A bag contains lemon flavoured candles only. Malini takes out one candy without looking into the bag. What is the probability that she takes out:

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Ans. (i) Consider the event related to the experiment of taking out of an orange flavoured candy from a bag containing only lemon flavoured candies. Since no outcome gives an orange flavoured candy, therefore, it is an impossible event. So its probability is 0.

(ii) Consider the event of taking a lemon flavoured candy out of a bag containing only lemon flavoured candies. This event is a certain event. So its probability is 1.

7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Ans. Let E be the event of having the same birthday

$\Rightarrow$ P(E) = 0.992

But $P(E) + P\left(\overline{E}\right) = 1$

$\therefore$ $P\left(\overline{E}\right) = 1 - P(E) = 1 - 0.992 = 0.008$

8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:

(i) red?

(ii) not red?

Ans. There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways.

$\therefore$ Total number of elementary events = 8

(i) Since the bag contains 3 red balls, therefore, one red ball can be drawn in 3 ways.

$\therefore$ Favourable number of elementary events = 3

Hence P (getting a red ball) = $\frac{3}{8}$

(ii) Since the bag contains 5 black balls along with 3 red balls, therefore one black (not red) ball can be drawn in 5 ways.

$\therefore$ Favourable number of elementary events = 5

Hence P (getting a black ball) = $\frac{5}{8}$

9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be:

(i) red?

(ii) white?

(iii) not green?

Ans. Total number of marbles in the box = 5 + 8 + 4 = 17

∴ Total number of elementary events = 17

(i) There are 5 red marbles in the box.

∴ Favourable number of elementary events = 5

∴ P (getting a red marble) = $\frac{5}{17}$

(ii) There are 8 white marbles in the box.

∴ Favourable number of elementary events = 8

∴ P (getting a white marble) = $\frac{8}{17}$

(iii) There are 5 + 8 = 13 marbles in the box, which are not green.

∴ Favourable number of elementary events = 13

∴ P (not getting a green marble) = $\frac{13}{17}$

---

10. A piggy bank contains hundred 50 p coins, fifty Re. 1 coins, twenty Rs. 2coins and ten Rs. 5 coins. If it is equally likely that of the coins will fall out when the bank is turned upside down, what is the probability that the coin:

(i) will be a 50 p coin?

(ii) will not be a Rs.5 coin?

Ans. Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180

∴ Total number of elementary events = 180

(i) There are one hundred 50 coins in the piggy bank.

∴ Favourable number of elementary events = 100

∴ P (falling out of a 50 p coin) = $\frac{100}{180} = \frac{5}{9}$

(ii) There are 100 + 50 + 20 = 170 coins other than Rs. 5 coin.

∴ Favourable number of elementary events = 170

∴ P (falling out of a coin other than Rs. 5 coin) = $\frac{170}{180} = \frac{17}{18}$

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fishes and 8 female fishes (see figure). What is the probability that the fish taken out is a male fish?

Ans. Total number of fish in the tank = 5 + 8 = 13

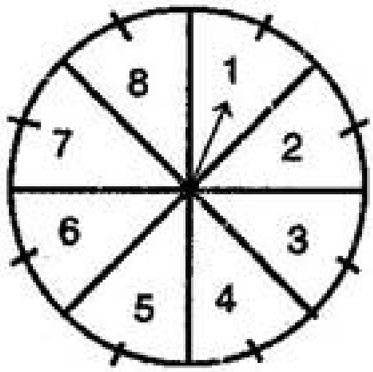
∴ Total number of elementary events = 13

There are 5 male fishes in the tank.

∴ Favourable number of elementary events = 5

Hence, P (taking out a male fish) = $\frac{5}{13}$

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure) and these are equally likely outcomes. What is the probability that it will point at:



(i) 8?

(ii) an odd number?

(iii) a number greater than 2?

(iv) a number less than 9?

Ans. Out of 8 numbers, an arrow can point any of the numbers in 8 ways.

$\therefore$ Total number of possible outcomes = 8

(i) Favourable number of outcomes = 1

Hence, P (arrow points at 8) = $\frac{1}{8}$

(ii) Favourable number of outcomes = 4

Hence, P (arrow points at an odd number) = $\frac{4}{8} = \frac{1}{2}$

(iii) Favourable number of outcomes = 6

Hence, P (arrow points at a number > 2) = $\frac{6}{8} = \frac{3}{4}$

(iv) Favourable number of outcomes = 8

Hence, P (arrow points at a number < 9) = $\frac{8}{8}$ =1

13. A dice is thrown once. Find the probability of getting:

(i) a prime number.

(ii) a number lying between 2 and 6.

(iii) an odd number.

Ans. Total number of Possible outcomes of throwing a dice = 6

(i) On a dice, the prime numbers are 2, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting a prime number) = $\frac{3}{6} = \frac{1}{2}$

(ii) On a dice, the number lying between 2 and 6 are 3, 4, 5.

Therefore, favourable outcomes = 3

Hence P (getting a number lying between 2 and 6) = $\frac{3}{6} = \frac{1}{2}$

(iii) On a dice, the odd numbers are 1, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting an odd number) = $\frac{3}{6} = \frac{1}{2}$

---

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds.

Ans. Total number of possible outcomes = 52

(i) There are two suits of red cards, i.e., diamond and heart. Each suit contains one king.

$\therefore$ Favourable outcomes = 1

Hence, P (a king of red colour) = $\frac{2}{52} = \frac{1}{26}$

(ii) There are 12 face cards in a pack.

$\therefore$ Favourable outcomes = 12

Hence, P (a face card) = $\frac{12}{52} = \frac{3}{13}$

(iii) There are two suits of red cards, i.e., diamond and heart. Each suit contains 3 face cards.

$\therefore$ Favourable outcomes = 2×3 = 6

Hence, P (a red face card) = $\frac{6}{52} = \frac{3}{26}$

(iv) There are only one jack of heart.

$\therefore$ Favourable outcome = 1

Hence, P (the jack of hearts) = $\frac{1}{52}$

(v) There are 13 cards of spade.

$\therefore$ Favourable outcomes = 13

Hence, P (a spade) = $\frac{13}{52} = \frac{1}{4}$

(vi) There is only one queen of diamonds.

∴ Favourable outcome = 1

Hence, P (the queen of diamonds) = $\frac{1}{52}$

---

15. Five cards – then ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Ans. Total number of possible outcomes = 5

(i) There is only one queen.

∴ Favourable outcome = 1

Hence, P (the queen) = $\frac{1}{5}$

(ii) In this situation, total number of favourable outcomes = 4

(a) Favourable outcome = 1

Hence, P (an ace) = $\frac{1}{4}$

(b) There is no card as queen.

∴ Favourable outcome = 0

Hence, P (the queen) = $\frac{0}{4} = 0$

16. 12 defective pens are accidently mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Ans. Total number of possible outcomes = 132 + 12 = 144

Number of favourable outcomes = 132

Hence, P (getting a good pen) = $\frac{132}{144} = \frac{11}{12}$

17. (i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Ans. (i) Total number of possible outcomes = 20

Number of favourable outcomes = 4

Hence P (getting a defective bulb) = $\frac{4}{20} = \frac{1}{5}$

(ii) Now total number of possible outcomes = 20 – 1 = 19

Number of favouroable outcomes = 19 – 4 = 15

Hence P (getting a non-defective bulb) = $\frac{15}{19}$

18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

Ans. Total number of possible outcomes = 90

(i) Number of two-digit numbers from 1 to 90 are 90 – 9 = 81

$\therefore$ Favourable outcomes = 81

Hence, P (getting a disc bearing a two-digit number) = $\frac{81}{90} = \frac{9}{10}$

(ii) From 1 to 90, the perfect squares are 1, 4, 9, 16, 25, 36, 49, 64 and 81.

$\therefore$ Favourable outcomes = 9

Hence P (getting a perfect square) = $\frac{9}{90} = \frac{1}{10}$

(iii) The numbers divisible by 5 from 1 to 90 are 18.

$\therefore$ Favourable outcomes = 18

Hence P (getting a number divisible by 5) = $\frac{18}{90} = \frac{1}{5}$

19. A child has a die whose six faces show the letters as given below:

A B C D E A

The die is thrown once. What is the probability of getting:

(i) A?

(ii) D?

Ans. Total number of possible outcomes = 6

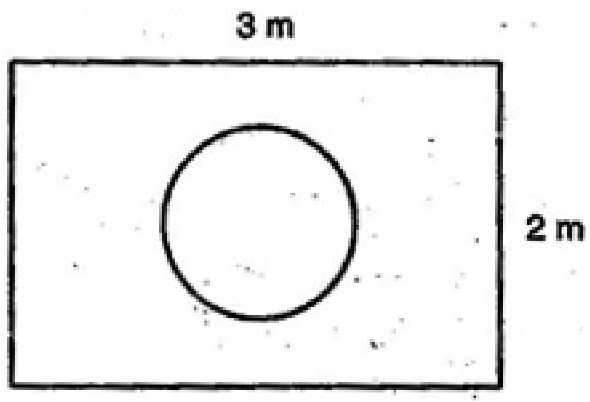
(i) Number of favourable outcomes = 2

Hence P (getting a letter A) = $\frac{2}{6} = \frac{1}{3}$

(ii) Number of favourable outcomes = 1

Hence P (getting a letter D) = $\frac{1}{6}$

20. Suppose you drop a die at random on the rectangular region shown in the figure given on the next page. What is the probability that it will land inside the circle with diameter 1 m?



Ans. Total area of the given figure (rectangle) = $3 \times 2 = 6\ m^2$

And Area of circle = $\pi r^2 = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}\ \text{m}^2$

Hence, P (die to land inside the circle) = $\frac{\pi/4}{6} = \frac{\pi}{24}$

---

21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that:

(i) she will buy it?

(ii) she will not buy it?

Ans. Total number of possible outcomes = 144

(i) Number of non-defective pens = 144 – 20 = 124

∴ Number of favourable outcomes = 124

Hence P (she will buy) = P (a non-defective pen) = $\frac{124}{144} = \frac{31}{36}$

(ii) Number of favourable outcomes = 20

Hence P (she will not buy) = P (a defective pen) = $\frac{20}{144} = \frac{5}{36}$

22. Refer to example 13.

(i) Complete the following table:

Event: Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Ans. Total possible outcomes of throwing two dice are:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$\therefore$ Total number of favourable outcomes = 36

(i) Favourable outcomes of getting the sum as 3 = 2

Hence P (getting the sum as 3) = $\frac{2}{36} = \frac{1}{18}$

Favourable outcomes of getting the sum as 4 = 3

Hence P (getting the sum as 4) = $\frac{3}{36} = \frac{1}{12}$

Favourable outcomes of getting the sum as 5 = 4

Hence P (getting the sum as 5) = $\frac{4}{36} = \frac{1}{9}$

Favourable outcomes of getting the sum as 6 = 5

Hence P (getting the sum as 6) = $\frac{5}{36}$

Favourable outcomes of getting the sum as 7 = 6

Hence P (getting the sum as 7) = $\frac{6}{36} = \frac{1}{6}$

Favourable outcomes of getting the sum as 9 = 4

Hence P (getting the sum as 9) = $\frac{4}{36} = \frac{1}{9}$

Favourable outcomes of getting the sum as 10 = 3

Hence P (getting the sum as 10) = $\frac{3}{36} = \frac{1}{12}$

Favourable outcomes of getting the sum as 11 = 2

Hence P (getting the sum as 11) = $\frac{2}{36} = \frac{1}{18}$

Event: Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) I do not agree with the argument given here. Justification has already been given in part (i).

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.

Ans. The outcomes associated with the experiment in which a coin is tossed thrice:

HHH, HHT, HTH, THH, TTH, HTT, THT, TTT

Therefore, Total number of possible outcomes = 8

Number of favourable outcomes = 6

Hence required probability = $\frac{6}{8} = \frac{3}{4}$

24. A die is thrown twice. What is the probability that:

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

Ans. (i) The outcomes associated with the experiment in which a dice is thrown is twice:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Therefore, Total number of possible outcomes = 36

Now consider the following events:

A = first throe shows 5 and B = second throw shows 5

Therefore, the number of favourable outcomes = 6 in each case.

$\therefore$ P(A) = $\frac{6}{36}$ and P(B) = $\frac{6}{36}$

$\Rightarrow P(\overline{A}) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$ and $P(\overline{B}) = \frac{5}{6}$

$\therefore$ Required probability = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

(ii) Let S be the sample space associated with the random experiment of throwing a die twice. Then, n(S) = 36

$\therefore$ $A \cap B$ = first and second throw shoe 5, i.e. getting 5 in each throw.

We have, A = (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

And B = (1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)

$\therefore$ P(A) = $\frac{6}{36}$, P(B) = $\frac{6}{36}$ and $P(A \cap B) = \frac{1}{36}$

$\therefore$ Required probability = Probability that at least one of the two throws shows 5

= $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

= $\frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

25. Which of the following arguments are correct and which are not correct? Give reasons for your answer:

(i) If two coins are tossed simultaneously there are three possible outcomes – two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes – an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Ans. (i) Incorrect: We can classify the outcomes like this but they are not then, 'equally likely'. Reason is that 'one of each' can result in two ways – from a head on first coin and tail on the second coin or from a tail on the first coin and head on the second coin. This makes it twice as likely as two heads (or two tails).

(ii) Correct: The two outcomes considered in the question are equally likely.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 15

Probability - Exercise 15.2

1. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?

Ans. Total favourable outcomes associated to the random experiment of visiting a particular shop in the same week (Tuesday to Saturday) by two customers Shyam and Exta are:

(T, T) (T, W) (T, TH) (T, F) (T, S)

(W, T) (W, W) (W, TH) (W, F) (W, S)

(TH, T) (TH, W) (TH, TH) (TH, F) (TH, S)

(F, T) (F, W) (F, TH) (F, F) (F, S)

(S, T) (S, W) (S, TH) (S, F) (S< S)

where T = Tuesday, W = Wednesday, Th = Thursday, F = Friday, S = Saturday

$\therefore$ Total number of favourable outcomes = 5 x 5 = 25

(i) The favourable outcomes of visiting on the same day are (T, T), (W, W), (TH, TH), (F, F) and (S, S).

$\therefore$ Number of favourable outcomes = 5

Hence required probability = $\frac{5}{25} = \frac{1}{5}$

(ii) The favourable outcomes of visiting on consecutive days are (T, W), (W, T), (W, TH), (TH, W), (TH, F), (F, TH), (S, F) and (F, S).

∴ Number of favourable outcomes = 8

Hence required probability = $\frac{8}{25}$

(iii) Number of favourable outcomes of visiting on different days are 25 – 5 = 20

∴ Number of favourable outcomes = 20

Hence required probability = $\frac{20}{25} = \frac{4}{5}$

---

2. A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:

Number in first throw

+ (Number in second throw)	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2					5	
3						
3			5			9
6	7	8	8	9	9	12

What is the probability that the total score is

What is the probability that the total score is:

(i) even

(ii) 6

(iii) at least 6?

Ans. Complete table is as under:

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

(Columns: Number in first throw; Rows: Number in second throw)

It is clear that total number of favourable outcomes = 6×6 = 36

(i) Even scores are: 2, 4, 4, 4, 4, 8, 4, 4, 8, 4, 6, 4, 6, 6, 8, 8, 12

Number of favourable outcomes of getting total score even are 18

Hence P (getting total score even) = $\frac{18}{36} = \frac{1}{2}$

(ii) Number of favourable outcomes of getting total score 6 are 4

Hence P (getting total score 6) = $\frac{4}{36} = \frac{1}{9}$

(iii) Total score at least 6 = 7, 8, 8, 6, 6, 9, 6, 6, 9, 7, 8, 8, 9, 9, 12

Number of favourable outcomes of getting total score at least 6 are 15

Hence P (getting total score at least 6) = $\frac{15}{36} = \frac{5}{12}$

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3. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

Ans. Let there be x blue balls in the bag.

$\therefore$ Total number of balls in the bag = $5 + x$

Now, P_1 = Probability of drawing a blue ball = $\frac{x}{5+x}$

And P_2 = Probability of drawing a red ball = $\frac{5}{5+x}$

But according to question, $P_1 = 2P_2$

$$\Rightarrow \frac{x}{5+x} = 2 \times \frac{5}{5+x}$$

$$\Rightarrow \frac{x}{5+x} \times \frac{5+x}{5} = 2$$

$$\Rightarrow x = 10$$

Hence, there are 10 blue balls in the bag.

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4. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

Ans. There are 12 balls in the box.

Therefore, total number of favourbale outcomes = 12

The number of favourable outcomes (Black balls) = x

Therefore P_1 = P (getting a black ball) = $\frac{x}{12}$

If 6 more balls put in the box, then

Total number of favourable outcomes = 12 + 6 = 18

And Number of favourable outcomes = $x+6$

$\therefore$ P_2 = P (getting a black ball) = $\frac{x+6}{18}$

According to question, $P_2 = 2P_1$

$\Rightarrow \frac{x+6}{18} = 2\times\frac{x}{12}$

$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$

$\Rightarrow 6x + 36 = 18x$

$\Rightarrow 18x - 6x = 36$

$\Rightarrow 12x = 36$

$\Rightarrow x = 3$

5. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number if blue balls in the jar.

Ans. Here, Total number of favourable outcomes = 24

Let there be x green marbles.

Therefore, Favourable number of outcomes = x

$\therefore$ P(Green ball) = $\frac{x}{24}$

But P(Green ball) = $\frac{2}{3}$

$\therefore \frac{x}{24} = \frac{2}{3}$

$\Rightarrow x$ = 16

Therefore, number of green marbles are 16

And number of blue marbles = 24 – 16 = 8