**CBSE Class–10 Mathematics**

**NCERT solution**

**Chapter - 1**

**Real Numbers - Exercise 1.1**

---

**1. Use Euclid's division algorithm to find the HCF of:**

**(i) 135 and 225**

**(ii) 196 and 38220**

**(iii) 867 and 255**

**Ans. (i)** 135 and 225

We have 225 > 135,

So, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Here remainder 90 ≠ 0, we apply the division lemma again to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45 ≠ 0, and apply the division lemma to obtain

$$90 = 2 \times 45 + 0$$

Since at this time the remainder is zero, the process is stopped.

The divisor at this stage is 45

Therefore, the HCF of 135 and 225 is 45.

**(ii)** 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

As the remainder is zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

**(iii)** 867 and 255

We have 867 > 255,

So, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Here remainder 102 ≠ 0, we apply the division lemma again to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

Here remainder 51 ≠ 0, we apply the division lemma again to 102 and 51 to obtain

$$102 = 51 \times 2 + 0$$

As the remainder is zero, the process stops.

The divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

---

**2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.**

**Ans.** Let $a$ be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where $k_1$ is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where $k_2$ is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where $k_3$ is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where $k$ is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$,

or $6q + 5$

---

**3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?**

**Ans.** We have to find the HCF (616, 32) to find the maximum number of columns in which they can march.

To find the HCF, we can use Euclid's algorithm.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

Since, the last divisor is 8.

Therefore, the HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

---

**4. Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer $m$. [Hint: Let $x$ be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]**

---

**Ans.** Let $a$ be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 or (3q+1)^2 or (3q+2)^2$$
$$a^2 = (9q)^2 or 9q^2 + 6q + 1 or 9q^2 + 12q + 4$$
$$= 3 \times (3q^2) or 3(3q^2 + 2q) + 1 or 3(3q^2 + 4q + 1) + 1$$
$$= 3k_1 or 3k_2 + 1 or 3k_3 + 1$$

Where $k_1, k_2, and k_3$ are some positive integers.

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

---

**5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.**

**Ans.** Let $a$ be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $r = 0, 1, 2$ because $0 \leq r < 3$

a = 3q or 3q + 1 or 3q + 2

Therefore, every number can be represented as these three forms.

We have three cases.

**Case 1**: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Where $m$ is an integer such that $m = 3q^3$

---

**Case 2**: When $a = 3q + 1$,

$$a^3 = (3q + 1)^3$$

$$a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$a^3 = 9\left(3q^3 + 3q^2 + q\right) + 1$$

$$a^3 = 9m + 1$$

Where $m$ is an integer such that $m = (3q^3 + 3q^2 + q)$

**Case 3**: When $a = 3q + 2$,

$$a^3 = (3q + 2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9\left(3q^3 + 6q^2 + 4q\right) + 8$$

$$a^3 = 9m + 8$$

Where $m$ is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

**CBSE Class–10 Mathematics**

**NCERT solution**

**Chapter - 1**

**Real Numbers -Exercise 1.2**

---

**1. Express each number as product of its prime factors:**

**(i) 140**

**(ii) 156**

**(iii) 3825**

**(iv) 5005**

**(v) 7429**

**Ans. (i)** 140 = $2\times2\times5\times7 = 2^2\times5\times7$

**(ii)** 156 = $2\times2\times3\times13 = 2^2\times3\times13$

**(iii)** 3825 = $3\times3\times5\times5\times17 = 3^2\times5^2\times17$

**(iv)** 5005 = $5\times7\times11\times13$

**(v)** 7429 = $17\times19\times23$

---

**2. Find the LCM and HCF of the following pairs of integers and verify that $\mathrm{LCM}\times\mathrm{HCF}$ = product of the two numbers.**

**(i) 26 and 91**

**(ii) 510 and 92**

**(iii) 336 and 54**

**Ans. (i)** 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

HCF (26, 91) = 13

LCM (26, 91) = $2 \times 7 \times 13$ = 182

Product of two numbers 26 and 91 $= 26 \times 91$ = 2366

$HCF \times LCM = 13\times182$ = 2366

Hence, product of two numbers = $HCF \times LCM$

---

**(ii)** 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF (510, 92) = 2

LCM (510, 92) = $2 \times 2 \times 3 \times 5 \times 17 \times 23$ = 23460

Product of two numbers 510 and 92 = $510 \times 92$ = 46920

$HCF \times LCM = 2 \times 23460$ = 46920

Hence, product of two numbers = $HCF \times LCM$

**(iii)** 336 and 54

336= $2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

54 = $2 \times 3 \times 3 \times 3 = 2 \times 3^3$

HCF (336, 54) = $2 \times 3 = 6$

LCM (336, 54) = $2^4 \times 3^3 \times 7$ = 3024

Product of two numbers 336 and 54 = $336 \times 54$ = 18144

$HCF \times LCM = 6 \times 3024$ = 18144

Hence, product of two numbers = $HCF \times LCM$

---

**3. Find the LCM and HCF of the following integers by applying the prime factorisation method.**

**(i) 12, 15 and 21**

**(ii) 17, 23 and 29**

**(iii) 8, 9 and 25**

**Ans. (i)** 12, 15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF (12, 15, 21) = 3

LCM (12, 15, 21) = $2^2 \times 3 \times 5 \times 7$ = 420

**(ii)** 17, 23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

HCF (17, 23, 29) = 1

LCM (17, 23, 29) = $17 \times 23 \times 29$ = 11339

**(iii)** 8, 9 and 25

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5^2$

HCF (8, 9, 25) = 1

LCM (8, 9, 25) = $2^3 \times 3^2 \times 5^2$ = 1800

---

**4. Given that HCF (306, 657) = 9, find LCM (306, 657).**

**Ans.** HCF (306, 657) = 9

We know that, LCM × HCF = Product of two numbers

$LCM \times HCF = 306 \times 657$

$$LCM = \frac{306 \times 657}{HCF} = \frac{306 \times 657}{9}$$

LCM (306, 657) = 22338

---

**5. Check whether $6^n$ can end with the digit 0 for any natural number *n*.**

**Ans.** If any number ends with the digit 0, it should be divisible by 10.

In other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of $6^n$ .

Hence, for any value of *n*, $6^n$ will not be divisible by 5.

Therefore, $6^n$ cannot end with the digit 0 for any natural number *n*.

---

**6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.**

**Ans.** Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78 = 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors other than 1 and number itself.

Therefore, it is a composite number.

$7\times6\times5\times4\times3\times2\times1+5$

$= 5\times(7\times6\times4\times3\times2\times1+1)$

$= 5 \times (1008 + 1) = 5 \times 1009$

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors other than 1 and number itself.

Hence, it is a composite number.

---

**7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?**

**Ans.** It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2\times3\times3$ And, $12 = 2\times2\times3$

LCM of 12 and $18 = 2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

**CBSE Class–10 Mathematics**

**NCERT solution**

**Chapter - 1**

**Real Numbers - Exercise 1.3**

---

**1. Prove that $\sqrt{5}$ is irrational.**

**Ans.** Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ ... **(1)**

It means that 5 is factor of $a^2$

Hence, **5 is also factor of *a*** by Theorem. ... **(2)**

If, **5 is factor of *a***, it means that we can write ***a*** = **5*c*** for some integer ***c***.

Substituting value of ***a*** in **(1)**,

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

It means that 5 is factor of $b^2$.

Hence, **5 is also factor of *b*** by Theorem. ... **(3)**

From **(2)** and **(3)**, we can say that **5** is factor of both ***a* and *b***.

But, ***a* and *b*** are **co-prime**.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

---

**2. Prove that (3 + 2 $\sqrt{5}$ ) is irrational.**

**Ans.** We will prove this by contradiction.

Let us suppose that (3+2 $\sqrt{5}$ ) is rational.

It means that we have co-prime integers ***a* and *b*** (*b* ≠ 0) such that

$$\frac{a}{b}=3+2\sqrt{5} \Rightarrow \frac{a}{b}-3=2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b}=2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b}=\sqrt{5} \ldots \textbf{(1)}$$

***a* and *b*** are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational. It is not possible. Therefore, our supposition is wrong. (3+2 $\sqrt{5}$ ) cannot be rational.

Hence, (3+2 $\sqrt{5}$ ) is irrational.

---

**3. Prove that the following are irrationals.**

**(i)** $\frac{1}{\sqrt{2}}$

**(ii)** $7\sqrt{5}$

**(iii)** $6+\sqrt{2}$

**Ans. (i)** We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers $a$ and $b$ ($b \neq 0$) such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$ ... **(1)**

**R.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

Hence, it is irrational.

**(ii)** We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers $a$ and $b$ ($b \neq 0$) such that

$7\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ... **(1)**

**R.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

**(iii)** We will prove $6+\sqrt{2}$ irrational by contradiction.

Let us suppose that ($6+\sqrt{2}$) is rational.

It means that we have co-prime integers ***a* and *b*** (*b* ≠ 0) such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{a}{b}-6$$

$$\Rightarrow \sqrt{2}=\frac{a-6b}{b} \quad \dots \textbf{(1)}$$

***a* and *b*** are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. ($6+\sqrt{2}$) cannot be rational.

Hence, ($6+\sqrt{2}$) is irrational.

**CBSE Class–10 Mathematics**

**NCERT solution**

**Chapter - 1**

**Real Numbers - Exercise 1.4**

---

**1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating decimal expansion.**

**(i)** $\frac{13}{3125}$

**(ii)** $\frac{17}{8}$

**(iii)** $\frac{64}{455}$

**(iv)** $\frac{15}{1600}$

**(v)** $\frac{29}{343}$

**(vi)** $\frac{23}{2^3 \times 5^2}$

**(vii)** $\frac{129}{2^2 \times 5^7 \times 7^5}$

**(viii)** $\frac{6}{15}$

**(ix)** $\frac{35}{50}$

**(x)** $\frac{77}{210}$

**Ans.** According to Theorem, any given rational number of the form $\frac{p}{q}$ **where *p* and *q* are co-prime**, has a terminating decimal expansion if q is of the form $2^n \times 5^m$, where m and n are non-negative integers.

**(i)** $\frac{13}{3125}$

$q = 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 2^0 \times 5^5$

Here, denominator is of the form $2^n \times 5^m$, where m = 5 and n = 0.

It means rational number $\frac{13}{3125}$ has a **terminating** decimal expansion.

**(ii)** $\frac{17}{8}$

$q = 8 = 2 \times 2 \times 2 = 2^3 = 2^3 \times 5^0$

Here, denominator is of the form $2^n \times 5^m$, where m = 0 and n = 3.

It means rational number $\frac{17}{8}$ has a **terminating** decimal expansion.

**(iii)** $\frac{64}{455}$

$q = 455 = 5 \times 91$

Here, denominator is not of the $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{64}{455}$ has a **non-terminating repeating** decimal expansion.

**(iv)** $\frac{15}{1600} = \frac{3}{320}$

$q = 320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 6.

It means rational number $\frac{15}{1600}$ has a **terminating** decimal expansion.

**(v)** $\frac{29}{343}$

$q = 343 = 7 \times 7 \times 7 = 7^3$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{29}{343}$ has **non-terminating repeating** decimal expansion.

**(vi)** $\frac{23}{2^3 \times 5^2}$

$q = 2^3 \times 5^2$

Here, denominator is of the form $2^n \times 5^m$, where m = 2 and n = 3 are non-negative integers.

It means rational number $\frac{23}{2^3 \times 5^2}$ has **terminating** decimal expansion.

**(vii)** $\frac{129}{2^2 \times 5^7 \times 7^5}$

$q = 2^2 \times 5^7 \times 7^5$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{129}{2^2 \times 5^7 \times 7^5}$ has **non-terminating repeating** decimal expansion.

**(viii)** $\frac{6}{15} = \frac{2}{5}$

$q = 5 = 5^1 = 2^0 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 0.

It means rational number $\frac{6}{15}$ has **terminating** decimal expansion.

**(ix)** $\frac{35}{50} = \frac{7}{10}$

$q = 10 = 2 \times 5 = 2^1 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 1.

It means rational number $\frac{35}{50}$ has **terminating decimal** expansion.

**(x)** $\frac{77}{210} = \frac{11}{30}$

$q = 30 = 5 \times 3 \times 2$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{77}{210}$ has **non-terminating repeating** decimal expansion.

---

**2. Write down the decimal expansions of those rational numbers in Question 1 which**

**have terminating decimal expansions.**

**Ans. (i)** $\frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{416}{10^5} = 0.00416$

**(ii)** $\frac{17}{8} = \frac{17}{2^3} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.215$

**(iv)** $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^2 \times 5^4} = \frac{15 \times 5^4}{10^6} = \frac{9375}{10^6} = 0.009375$

**(vi)** $\frac{23}{2^3 \times 5^2} = \frac{23 \times 5^1}{2^3 \times 5^2 \times 5^1} = \frac{23 \times 5^1}{10^3} = \frac{115}{10^3} = 0.115$

**(viii)** $\frac{6}{15} = \frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$

**(ix)** $\frac{35}{50} = \frac{7}{10} = 0.7$

---

**3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If, they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?**

**(i) 43.123456789**

**(ii) 0.120120012000120000...**

**(iii)** $43.\overline{123456789}$

**Ans. (i)** 43.123456789

It is rational because decimal expansion is terminating. Therefore, it can be expressed in $\frac{p}{q}$ form where q = $10^9$ and factors of *q* are of the form $2^n \times 5^m$ where *n* and *m* are non-negative integers

**(ii)** 0.120112001200012000...

It is irrational because decimal expansion is neither terminating nor non-terminating repeating.

**(iii)** $43.\overline{123456789}$

It is rational because decimal expansion is non-terminating repeating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of *q* **are not** of the form $2^n \times 5^m$ where *n* and *m* are non-negative integers.

Thus, $43.123456789 = \frac{p}{q}$, where q = 999999999