CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.1

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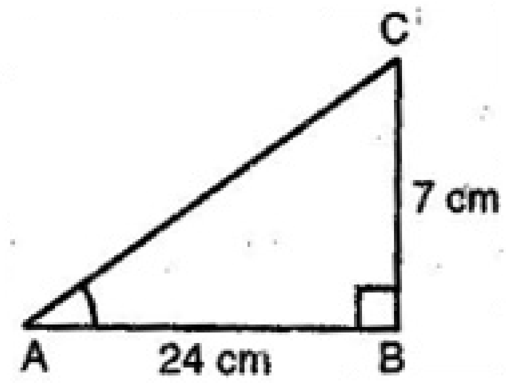
1. In $\triangle$ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$

Ans. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,



Let AC = 24k and BC = 7k

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

= $(24)^2 + (7)^2$ = 576 + 49 = 625

$\Rightarrow$ AC = 25 cm

(i) $\sin \mathrm{A} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{7}{25}$, $\cos \mathrm{A} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{24}{25}$

(ii) $\sin \mathrm{C} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{24}{25}$, $\cos \mathrm{C} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{7}{25}$

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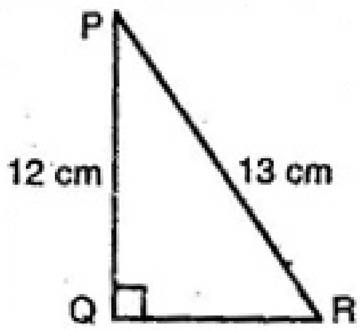
2. In adjoining figure, find $\tan P - \cot R$ **:**



Ans. In triangle PQR, Using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow$ QR = 5 cm

$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{12} - \frac{5}{12} = 0$

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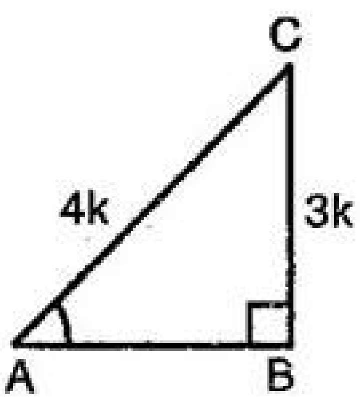
3. If $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$



Let BC = $3k$ and AC = $4k$

Then, Using Pythagoras theorem,

AB = $\sqrt{(AC)^2-(BC)^2}$ = $\sqrt{(4k)^2-(3k)^2}$

= $\sqrt{16k^2-9k^2}$ = $k\sqrt{7}$

$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

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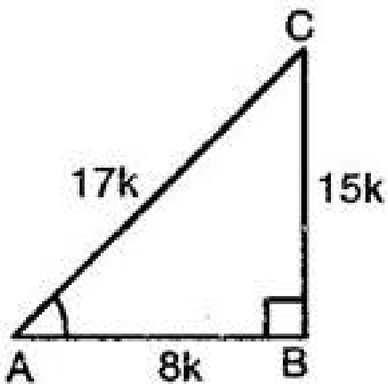
4. Given $15\cot A = 8$, find $\sin A$ and $\sec A$

Ans. Given: A triangle ABC in which $\angle$ B = 90°

$15\cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$



Let AB = $8k$ and BC = $15k$

Then using Pythagoras theorem,

AC = $\sqrt{(AB)^2+(BC)^2}$

= $\sqrt{(8k)^2+(15k)^2}$

= $\sqrt{64k^2+225k^2}$

= $\sqrt{289k^2}$ = $17k$

$\therefore \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{H}{B} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

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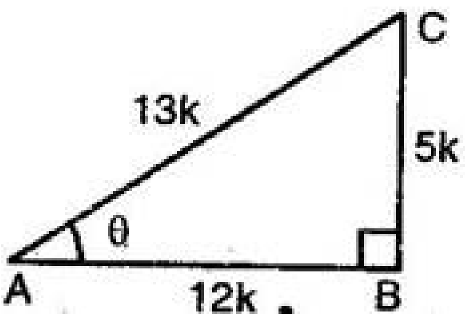
5. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°



Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2 - (AB)^2}$

= $\sqrt{(13k)^2 - (12k)^2}$

= $\sqrt{169k^2 - 144k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin\theta = \frac{P}{H} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{B}{H} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

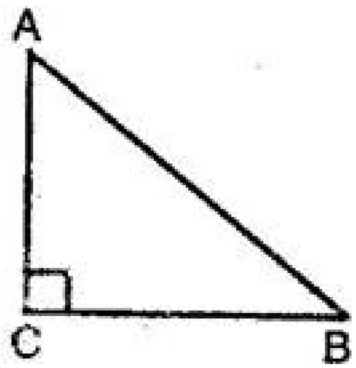
$\tan\theta = \frac{P}{B} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{B}{P} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$\operatorname{cosec}\theta = \frac{\mathrm{H}}{\mathrm{P}} = \frac{\mathrm{AC}}{\mathrm{BC}} = \frac{13k}{5k} = \frac{13}{5}$

6. If $\angle$ And $\angle$ B are acute angles such that $\cos A = \cos B$, then show that $\angle$ A = $\angle$ B.

Ans. In right triangle ABC,



$\cos A = \frac{\mathrm{AC}}{\mathrm{AB}}$ and $\cos B = \frac{\mathrm{BC}}{\mathrm{AB}}$

But $\cos A = \cos B$ [Given]

$\Rightarrow \frac{\mathrm{AC}}{\mathrm{AB}} = \frac{\mathrm{BC}}{\mathrm{AB}}$

$\Rightarrow$ AC = BC

$\Rightarrow$ $\angle$ A = $\angle$ B

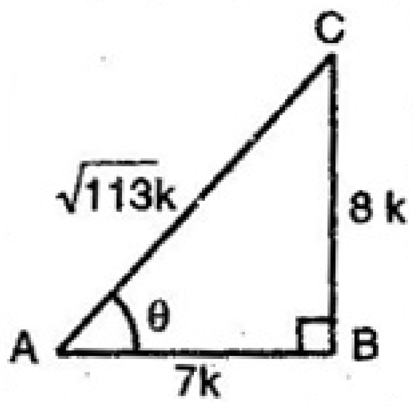
[Angles opposite to equal sides are equal]

7. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°



Let AB = $7k$ and BC = $8k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2+(AB)^2}$

= $\sqrt{(8k)^2+(7k)^2}$

= $\sqrt{64k^2+49k^2}$

= $\sqrt{113k^2}$ = $\sqrt{113}k$

$$\therefore \sin\theta=\frac{BC}{AC}=\frac{8k}{\sqrt{113}k}=\frac{8}{\sqrt{113}}$$

$$\cos\theta=\frac{AB}{AC}=\frac{7k}{\sqrt{113}k}=\frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ = $\frac{1-\sin^2\theta}{1-\cos^2\theta}$

= $\frac{1-\frac{64}{113}}{1-\frac{49}{113}}$ = $\frac{113-64}{113-49}$ = $\frac{49}{64}$

(ii) $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{49/113}{64/113} = \frac{49}{64}$

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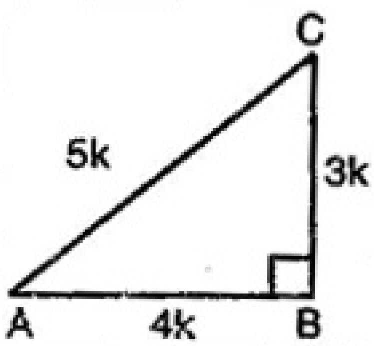
8. If $3\cot A = 4$, **check whether** $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Ans. Consider a triangle ABC in which $\angle B = 90^\circ$.



And $3\cot A = 4$

$\Rightarrow \cot A = \frac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(3k)^2 + (4k)^2}$

= $\sqrt{16k^2 + 9k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

And $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

= $\frac{16-9}{16+9}$ = $\frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A$ = $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

= $\frac{16}{25} - \frac{9}{25}$ = $\frac{7}{25}$

$\because$ L.H.S. = R.H.S.

$\therefore$ $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\cos^2 A - \sin^2 A$

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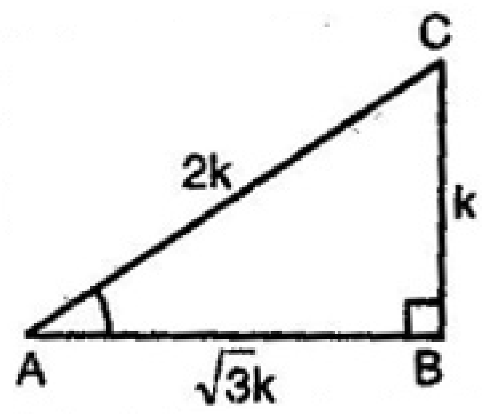
9. In $\triangle$ ABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, find value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Ans. Consider a triangle ABC in which $\angle$ B = 90°.

Let BC = k and AB = $\sqrt{3}k$



Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$

= $\sqrt{k^2 + 3k^2}$ = $\sqrt{4k^2}$ = $2k$

$\therefore \sin A = \dfrac{BC}{AC} = \dfrac{k}{2k} = \dfrac{1}{2}$

$\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

For $\angle$C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$\therefore \sin C = \dfrac{AB}{AC} = \dfrac{\sqrt{3}k}{2k} = \dfrac{\sqrt{3}}{2}$

$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$ = $\dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2}$

= $\dfrac{1}{4} + \dfrac{3}{4} = \dfrac{4}{4}$ = 1

(ii) $\cos A \cos C - \sin A \sin C$ = $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

= $\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

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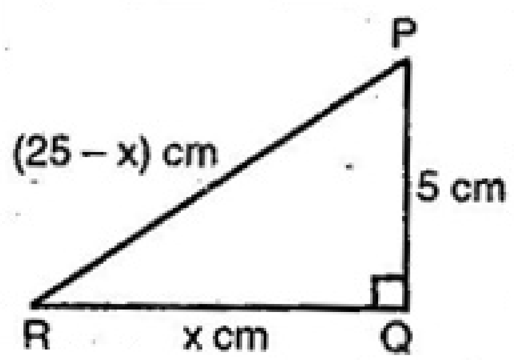
10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P$, $\cos P$ **and** $\tan P$.

Ans. In ΔPQR, right angled at Q.



PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm, then PR = $(25 - x)$ cm

Using Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25 - x)^2 = (x)^2 + (5)^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = 12$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$\therefore \sin P = \frac{RQ}{RP} = \frac{12}{13}$

$\cos P = \frac{PQ}{RP} = \frac{5}{13}$

And $\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

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11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1.

CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.2

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1. Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Ans. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}$

$= \dfrac{3}{4} + \dfrac{1}{4} = \dfrac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= 2 + \dfrac{3}{4} - \dfrac{3}{4} = 2$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2+2\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)}$

$= \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\times 2\left(3-1\right)}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{4\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$= \dfrac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \dfrac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4}$

$= \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \dfrac{3\sqrt{3}-4}{3\sqrt{3}-4}$

$= \dfrac{27+16-24\sqrt{3}}{27-16}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \dfrac{43-24\sqrt{3}}{11}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\times\frac{1}{4} + 4\times\frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}}$$

$$= \frac{15+64-12}{12} = \frac{67}{12}$$

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2. Choose the correct option and justify:

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ =

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

(iii) $\sin 2A = 2\sin A$ **is true when A** =

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ =

(A) $\cos 60^\circ$

(B) $\sin 60^\circ$

(C) $\tan 60^\circ$

(D) None of these

Ans. (i) (A) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$$= \frac{2\times {}^{1}\!/\!_{\sqrt{3}}}{1+\left({}^{1}\!/\!_{\sqrt{3}}\right)^2}$$

$= \frac{2}{\sqrt{3}} \times \frac{3}{3+1}$ $= \frac{\sqrt{3}}{2}$ $= \sin 60^\circ$

(ii) (D) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ $= \frac{1-1}{1+1}$ $= \frac{0}{2}$ = 0

(iii) (A) Since A = 0, then

$\sin 2A = \sin 0^\circ = 0$ and $2\sin A = 2\sin 0^\circ$

$= 2\times 0 \ = \ 0$

$\therefore \ \sin 2A = \sin A$ when A = 0

(iv) (C) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$= \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ $= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{3-1}$ $= \sqrt{3} = \tan 60^\circ$

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3. If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^\circ < A+B \le 90^\circ$; $A > B$, **find A and B.**

Ans. $\tan(A+B) = \sqrt{3}$

$\Rightarrow \ \tan(A+B) = \tan 60^\circ$

$\Rightarrow$ A + B = 60° ....(i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^\circ$

$\Rightarrow$ A – B = 30°……….(ii)

On adding eq. (i) and (ii), we get,

2A = $90^\circ \Rightarrow$ A = 45°

On Subtracting eq. (i) and eq. (ii), we get

2B = $30^\circ \Rightarrow$ B = 15°

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4. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0^\circ$.

Ans. (i) False, because, let A = 60° and B = 30°

Then, $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ$ = 1

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$ = $\frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

(ii) True, because it is clear from the table below:

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Therefore, it is clear, the value of $\sin\theta$ increases as θ increases.

(iii) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

(v) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

$= \frac{1}{0}$ i.e. undefined.

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Chapter - 8

Introduction to Trigonometry - Exercise 8.3

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1. Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\cos ec 31^\circ - \sec 59^\circ$

Ans. (i) $\frac{\sin 18^\circ}{\cos 72^\circ}$ = $\frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

= $\frac{\cos 72^\circ}{\cos 72^\circ}$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

= 1

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$ = $\frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

= $\frac{\cot 64^\circ}{\cot 64^\circ}$ [Since $\tan(90^\circ - \theta) = \cot\theta$]

= 1

(iii) $\cos 48^\circ - \sin 42^\circ$

= $\cos(90^\circ - 42^\circ) - \sin 42^\circ$

= $\sin 42° - \sin 42°$ [Since $\cos(90° - \theta) = \sin\theta$]

= 0

(iv) $\operatorname{cosec} 31° - \sec 59°$

= $\operatorname{cosec}(90° - 59°) - \sec 59°$

= $\sec 59° - \sec 59°$ [Since $\operatorname{cosec}(90° - \theta) = \sec\theta$]

=0

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2. Show that:

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$

Ans. (i) L.H.S. $\tan 48° \tan 23° \tan 42° \tan 67°$

= $\tan(90° - 42°)\tan(90° - 67°)\tan 42° \tan 67°$

= $\cot 42° \cot 67° \tan 42° \tan 67°$

= $\frac{1}{\tan 42°}.\frac{1}{\tan 67°}.\tan 42°.\tan 67°$ = 1 = R.H.S.

(ii) R.H.S. $\cos 38° \cos 52° - \sin 38° \sin 52°$

= $\cos(90° - 52°).\cos(90° - 38°) - \sin 38°.\sin 52°$

= $\sin 52° \sin 38° - \sin 38° \sin 52°$ = 0 = R.H.S.

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3. If $\tan 2A = \cot(A - 18°)$, where 2A is an acute angle, find the value of A.

Ans. Given: $\tan 2A = \cot(A - 18°)$

$\Rightarrow \cot(90° - 2A) = \cot(A - 18°)$ [Since $\tan(90° - \theta) = \cot\theta$]

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow -2A - A = -18^\circ - 90^\circ$

$\Rightarrow -3A = -108^\circ$

$\Rightarrow$ A = 36°

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4. If $\tan A = \cot B$, **prove that** $A + B = 90^\circ$.

Ans. Given: $\tan A = \cot B$

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow \quad 90^\circ = A + B$

$\Rightarrow$ A + B = 90°

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5. If $\sec 4A = \operatorname{cos}ec(A - 20^\circ)$, **where 4A is an acute angle, find the value of A.**

Ans. Given: $\sec 4A = \operatorname{cos}ec(A - 20^\circ)$

$\Rightarrow \operatorname{cos}ec(90^\circ - 4A) = \operatorname{cos}ec(A - 20^\circ)$ [Since $\sec(90^\circ - \theta) = \cos ec\theta$]

$\Rightarrow 90^\circ - 4A = A - 20^\circ$

$\Rightarrow -4A - A = -20^\circ - 90^\circ$

$\Rightarrow -5A = -110^\circ$

$\Rightarrow$ A = 22°

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6. If A, B and C are interior angles of a Δ **ABC, then show that** $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Ans. Given: A, B and C are interior angles of a ΔABC.

$\therefore$ A + B + C = 180° [Triangle sum property]

Dividing both sides by 2, we get

$$\Rightarrow \frac{A+B+C}{2} = 90^\circ$$

$$\Rightarrow \frac{A}{2} + \frac{B+C}{2} = 90^\circ$$

$$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

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7. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Ans. $\sin 67^\circ + \cos 75^\circ$

= $\sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$ [Since $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$]

= $\cos 23^\circ + \sin 15^\circ$

CBSE Class–10 Mathematics

NCERT solution

Chapter - 8

Introduction to Trigonometry - Exercise 8.4

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1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans. For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

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2. Write the other trigonometric ratios of A in terms of $\sec A$

Ans. For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,

$$\cos A = \frac{1}{\sec A}$$

For $\tan A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\cos ecA$,

$$\cos ecA = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$\Rightarrow \operatorname{cosec} A = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

For $\cot A$,

$\cot A = \dfrac{1}{\tan A}$

$\Rightarrow \cot A = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

---

3. Evaluate:

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

= $\dfrac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$

= $\dfrac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$

$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$

= $\dfrac{1}{1} = 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

= $\sin 25°.\cos(90° - 25°) + \cos 25°.\sin(90° - 25°)$

= $\sin 25°.\sin 25° + \cos 25°.\cos 25°$

$\left[\because \sin(90^\circ-\theta)=\cos\theta, \cos(90^\circ-\theta)=\sin\theta\right]$

$= \sin^2 25^\circ + \cos^2 25^\circ = 1$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

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4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta) =$

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $(\sec A+\tan A)(1-\sin A) =$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$ =

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans. (i) (B) $9\sec^2 A-9\tan^2 A$

= $9\left(\sec^2 A-\tan^2 A\right)$

= $9\times1=9$ [Since $\sec^2\theta-\tan^2\theta=1$]

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

= $\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

= $\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

= $\frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$ = 2

(iii)(D) $(\sec A+\tan A)(1-\sin A)$

$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

$= \frac{1-\sin^2 A}{\cos A}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \frac{\cos^2 A}{\cos A}$

$= \cos A \left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv)(D) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A-\tan^2 A+\tan^2 A}{\operatorname{cosec}^2 A-\cot^2 A+\cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

---

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A$**, using the identity** $\operatorname{cosec}^2 A=1+\cot^2 A$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$

$=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)$

$=\dfrac{1}{\tan A+\cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Ans. (i) L.H.S. $(\operatorname{cosec}\theta-\cot\theta)^2$

$=\operatorname{cosec}^2\theta+\cot^2\theta-2\operatorname{cosec}\theta\cot\theta$ [Since $(a-b)^2=a^2+b^2-2ab$]

$=\dfrac{1}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}-2\times\dfrac{1}{\sin\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1+\cos^2\theta}{\sin^2\theta} - \dfrac{2\cos\theta}{\sin^2\theta}$

$= \dfrac{1+\cos^2\theta-2\cos\theta}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$ $\left[\because a^2+b^2-2ab=(a-b)^2\right]$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$ = R.H.S.

(ii) L.H.S. $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$= \dfrac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$

$= \dfrac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$

$= \dfrac{1+1+2\sin A}{(1+\sin A)\cos A}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \dfrac{2+2\sin A}{(1+\sin A)\cos A}$ = $\dfrac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \dfrac{2}{\cos A}$ = $2\sec A$ = R.H.S

(iii) L.H.S. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$\left[\because a^3-b^3=(a-b)(a^2+b^2+ab)\right]$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}\left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$= \frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}+1 = 1+\frac{1}{\sin\theta\cos\theta}$$

$$= 1+\sec\theta\,\mathrm{cosec}\,\theta$$

(iv) L.H.S. $\frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \times \frac{\cos A}{1} = 1+\cos A$

$= 1+\cos A \times \frac{1-\cos A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos A}$ [Since $(a+b)(a-b) = a^2 - b^2$]

$= \frac{\sin^2 A}{1-\cos A}$ = R.H.S.

(v) L.H.S. $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$= \frac{\cot A-1+\cos ecA}{\cot A+1-\cos ecA} = \frac{\cot A+\cos ecA-1}{\cot A-\cos ecA+1}$

$= \frac{(\cot A+\cos ecA)-(\cos ec^2A-\cot^2 A)}{(1+\cot A-\cos ecA)}$ [Since $\cos ec^2\theta - \cot^2\theta = 1$]

$= \frac{(\cot A+\cos ecA)+(\cot^2 A-\cos ec^2 A)}{(1+\cot A-\cos ecA)}$

$= \frac{(\cot A+\cos ecA)+(\cot A+\cos ecA)(\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \frac{(\cot A+\cos ecA)(1+\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \cot A+\cos ecA$ = R.H.S.

(vi) L.H.S. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

= $\sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$

= $\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$ $\left[\because (a+b)(a-b) = a^2 - b^2\right]$

= $\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$ $\left[\because 1-\sin^2\theta = \cos^2\theta\right]$

= $\frac{1+\sin A}{\cos A}$ = $\frac{1}{\cos A} + \frac{\sin A}{\cos A}$

= $\sec A + \tan A$ = R.H.S.

(vii) L.H.S. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

= $\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$

= $\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}$

$\left[\because 1-\sin^2\theta = \cos^2\theta\right]$

= $\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$

= $\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$ = $\frac{\sin\theta}{\cos\theta}$

$= \tan\theta$ = R.H.S

(viii) L.H.S. $(\sin A + \cos ecA)^2 + (\cos A + \sec A)^2$

$$= \left(\sin A + \frac{1}{\sin A}\right)^2 + \left(\cos A + \frac{1}{\cos A}\right)^2$$

$$= \sin^2 A + \frac{1}{\sin^2 A} + 2.\sin A.\frac{1}{\sin A} + \cos^2 A + \frac{1}{\cos^2 A} + 2.\cos A.\frac{1}{\cos A}$$

$$= 2 + 2 + \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$$

$$= 4 + 1 + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$$

$$= 5 + \cos ec^2 A + \sec^2 A$$

$$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$$

$$\left[\because \cos ec^2\theta = 1 + \cot^2\theta, \sec^2\theta = 1 + \tan^2\theta\right]$$

$$= 7 + \tan^2 A + \cot^2 A$$

= R.H.S.

(ix) L.H.S. $(\cos ecA - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A.\cos A$$

$$= \frac{\sin A.\cos A}{\sin^2 A + \cos^2 A} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

Dividing all the terms by $\sin A.\cos A$,

$$= \frac{\frac{\sin A.\cos A}{\sin A.\cos A}}{\frac{\sin^2 A}{\sin A.\cos A} + \frac{\cos^2 A}{\sin A.\cos A}}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A} = \text{R.H.S.}$$

(x) L.H.S. $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{\sec^2 A}{\cos ec^2 A}$

$[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta]$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A = \text{R.H.S.}$$

Now, Middle side = $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right)^2 = (-\tan A)^2$$

$= \tan^2 A$ = R.H.S.