CBSE Class–10 Mathematics

NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Ans. Let R be the radius of the circle which has circumference equal to the sum of circumferences of the two circles, then according to question,

$$2\pi R = 2\pi(19) + 2\pi(9)$$

$$\Rightarrow 2\pi R = 2\pi(19 + 9)$$

$\Rightarrow$ R = 19 + 9

$\Rightarrow$ R = 28 cm

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2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Ans. Let R be the radius of the circle which has area equal to the sum of areas of the two circles, then

According to the question,

$$\pi R^2 = \pi(8)^2 + \pi(6)^2$$

$$\Rightarrow \pi R^2 = \pi\left[(8)^2 + (6)^2\right]$$

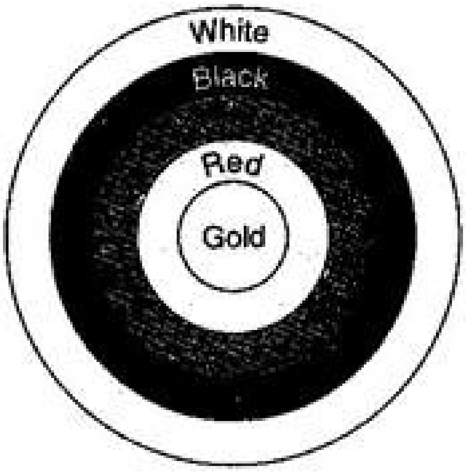
$$\Rightarrow R^2 = (8)^2 + (6)^2$$

$\Rightarrow$ R^2 = 64 + 36

$\Rightarrow R^2 = 100$

$\Rightarrow R = 10$ cm

3. Figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of the five scoring regions.



Ans. Gold: Diameter = 21 cm

$\Rightarrow$ Radius = $\frac{21}{2}$ cm

Area of gold scoring region = $\pi\left(\frac{21}{2}\right)^2$

= $\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$ = 346.5 cm^2

Red: Area of red scoring region = $\pi\left(\frac{21}{2}+10.5\right)^2-\pi\left(\frac{21}{2}\right)^2$

= $\pi(21)^2-346.5$

= $\frac{22}{7}\times 21\times 21-346.5$

$= 1386 - 346.5 = 1039.5\ cm^2$

Blue: Area of blue scoring region = $\pi(21 + 10.5)^2 - (1039.5 + 346.5)$

$= \pi(31.5)^2 - 1386$

$= \frac{22}{7} \times 31.5 \times 31.5 - 1386$

$= 3118.5 - 1386 = 1732.5\ cm^2$

Black: Area of black scoring region = $\pi(31.5 + 10.5)^2 - (1732.5 + 1039.5 + 346.5)$

$= \pi(42)^2 - 3118.5$

$= \frac{22}{7} \times 42 \times 42 - 3118.5$

$= 5544 - 3118.5 = 2425.5\ cm^2$

White: Area of white scoring region =
$\pi(42 + 10.5)^2 - (2425.5 + 1732.5 + 1039.5 + 346.5)$

$= \pi(52.5)^2 - 5544$

$= \frac{22}{7} \times 52.5 \times 52.5 - 5544$

$= 8662.5 - 5544 = 3118.5\ cm^2$

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4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Ans. Diameter of wheel = 80 cm

$\Rightarrow$ Radius of wheel (r) = 40 cm

Distance covered by wheel in one revolution = $2\pi r$ = $2 \times \frac{22}{7} \times 40$ = $\frac{1760}{7}$ cm

$\because$ Distance covered by wheel in 1 hour = 66 km = 66000 m = 6600000 cm

$\therefore$ Distance covered by wheel in 10 minutes = $\frac{6600000}{60} \times 10$ = 1100000 cm

$\therefore$ No. of revolutions = $\frac{\text{Total distance}}{\text{distance of one revolution}}$

= $\frac{1100000 \times 7}{1760}$ = 4375

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5. Tick the correct answer in the following and justify your choice: If the perimeter and area of a circle are numerically equal, then the radius of the circle is:

(A) 2 units
(B) π units
(C) 4 units
(D) 7 units

Ans. (A)

Circumference of circle = Area of circle

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r$ = 2 units

CBSE Class–10 Mathematics

NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

Ans. Here, r = 6 cm and $\theta = 60^\circ$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$ = $\frac{132}{7}$ cm^2

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2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Ans. Given, $2\pi r$ = 22 cm

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{7}{2}$ cm

We know that for quadrant of circle, $\theta = 90^\circ$

$\therefore$ Area of quadrant = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ = $\frac{77}{8}$ cm^2

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3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Ans. Here, r = 14 cm and since the minute hand rotates through $\frac{360^\circ}{60^\circ} = 6^\circ$ in one minute, therefore, angle swept by minute hand in 5 minutes = $\theta = 6^\circ \times 5 = 30^\circ$.

$\therefore$ Area swept = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$ = $\frac{154}{3} cm^2$

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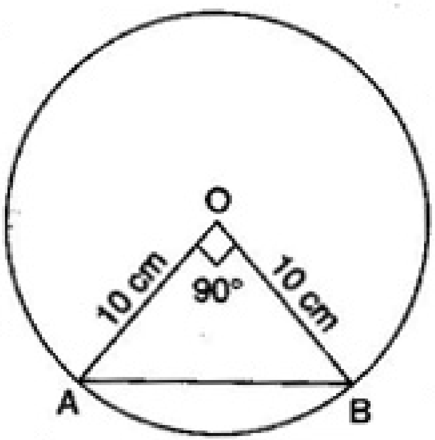
4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment, (ii) major segment. $(\text{Use } \pi = 3.14)$

Ans. (i) Here, r = 10 cm and $\theta = 90^\circ$



$\therefore$ Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10$ = $78.5\ cm^2$

Area of ΔOAB = $\frac{1}{2} \times Base \times Height$

= $\frac{1}{2} \times 10 \times 10$ = $50\ cm^2$

$\therefore$ Area of minor segment = Area of minor sector – Area of ΔOAB

= 78.5 – 50 = $28.5\ cm^2$

(ii) For major sector, radius = 10 cm and $\theta = 360^\circ - 90^\circ = 270^\circ$

$\therefore$ Area of major sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{270^\circ}{360^\circ} \times 3.14 \times 10 \times 10$ = $235.5\ cm^2$

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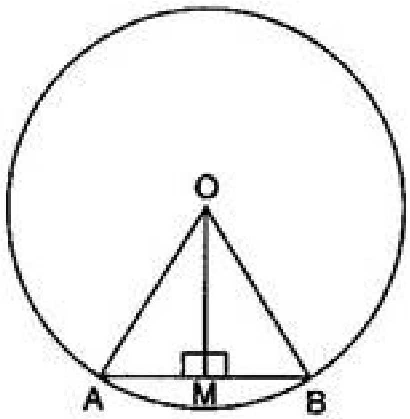
5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc.

(ii) area of the sector formed by the arc.

(iii) area of the segment formed by the corresponding chord.

Ans. Given, r = 21 cm and $\theta = 60^\circ$



(i) Length of arc = $\frac{\theta}{360^\circ} \times 2\pi r$

= $\frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$ = 22 cm

(ii) Area of the sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$ = $231\ cm^2$

(iii) Area of segment formed by corresponding chord

= $\frac{\theta}{360^\circ} \times \pi r^2 -$ Area of ΔOAB

$\Rightarrow$ Area of segment = 231 – Area of ΔOAB..........(i)

In right angled triangle OMA and OMB,

OM = OB [Radii of the same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\therefore$ M is the mid-point of AB and $\angle$AOM = $\angle$BOM

$\Rightarrow$ $\angle$AOM = $\angle$BOM

= $\frac{1}{2}\angle$AOB = $\frac{1}{2} \times 60^\circ = 30^\circ$

Therefore, in right angled triangle OMA,

$\cos 30^\circ = \frac{OM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{21}$

$\Rightarrow$ OM = $\frac{21\sqrt{3}}{2}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA} \Rightarrow \frac{1}{2} = \frac{AM}{21}$

$\Rightarrow$ AM = $\frac{21}{2}$ cm

$\therefore$ AB = 2 AM = $2 \times \frac{21}{2}$ = 21 cm

$\therefore$ Area of ΔOAB = $\frac{1}{2} \times AB \times OM$

= $\frac{1}{2} \times 21 \times \frac{21\sqrt{3}}{2}$ = $\frac{441\sqrt{3}}{4} cm^2$

Using eq. (i),

Area of segment formed by corresponding chord = $\left(231 - \frac{441\sqrt{3}}{4}\right) cm^2$

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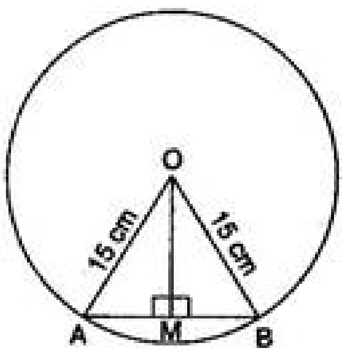
6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor and major segment of the circle.

$\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$

Ans. Here, r = 15 cm and $\theta = 60^\circ$



Area of minor sector= $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times 3.14 \times 15 \times 15$

= $117.75\ cm^2$

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and

$\angle$AOM = $\angle$BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2}\times 60^\circ = 30^\circ$

In right angled triangle OMA, $\cos 30^\circ = \frac{\text{OM}}{\text{OA}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\text{OM}}{15}$

$\Rightarrow$ OM = $\frac{15\sqrt{3}}{2}$ cm

Also, $\sin 30^\circ = \frac{\text{AM}}{\text{OA}}$

$\Rightarrow \frac{1}{2} = \frac{\text{AM}}{15} \Rightarrow$ AM = $\frac{15}{2}$ cm

$\Rightarrow$ 2 AM = $2\times\frac{15}{2}$ = 15 cm

$\Rightarrow$ AB = 15 cm

$\therefore$ Area of ΔAOB = $\frac{1}{2} \times AB \times OM$

= $\frac{1}{2} \times 15 \times \frac{15\sqrt{3}}{2}$ = $\frac{225\sqrt{3}}{3}$

= $\frac{225 \times 1.73}{4}$ = $97.3125\ cm^2$

$\therefore$ Area of minor segment = Area of minor sector – Area of ΔAOB

= 117.75 – 97.3125 = $20.4375\ cm^2$

And, Area of major segment = $\pi r^2 -$ Area of minor segment

= 3.14 x 15 x 15 - 20.4375 = 706.5 – 20.4375 = $686.0625\ cm^2$

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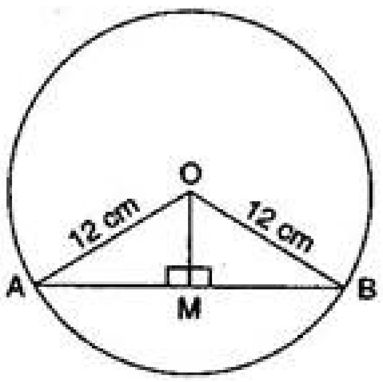
7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.73\right)$

Ans. Here, r = 12 cm and $\theta = 120^\circ$

Area of corresponding sector= $\frac{\theta}{360^\circ} \times \pi r^2$



= $\frac{120^\circ}{360^\circ} \times 3.14 \times 12 \times 12$

= $150.72\ cm^2$

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and

$\angle$AOM = $\angle$BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2}\times 120^\circ = 60^\circ$

In right angled triangle OMA, $\cos 60^\circ = \frac{\text{OM}}{\text{OA}}$

$\Rightarrow \frac{1}{2} = \frac{\text{OM}}{12}$

$\Rightarrow$ OM = 6 cm

Also, $\sin 60^\circ = \frac{\text{AM}}{\text{OA}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\text{AM}}{12}$

$\Rightarrow$ AM = $6\sqrt{3}$ cm

$\Rightarrow$ 2 AM = $2\times 6\sqrt{3}$ = $12\sqrt{3}$ cm

$\Rightarrow$ AB = $12\sqrt{3}$ cm

$\therefore$ Area of ΔAOB = $\frac{1}{2} \times AB \times OM$

= $\frac{1}{2} \times 12\sqrt{3} \times 6$ = $36\sqrt{3}$

= 36×1.73 = $62.28\ cm^2$

$\therefore$ Area of corresponding segment = Area of corresponding sector – Area of ΔAOB

= 150.72 – 62.28 = $88.44\ cm^2$

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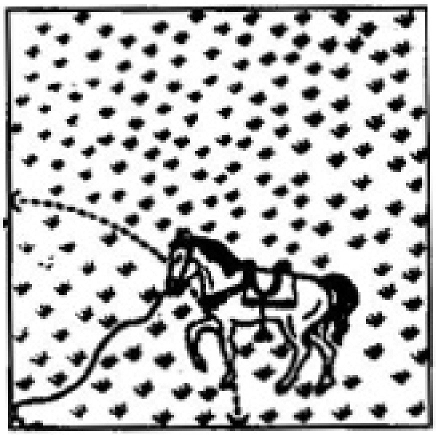
8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:



(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 cm.

$(\text{Use } \pi = 3.14)$

Ans. (i) Area of quadrant with 5 m rope

= $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 5 \times 5 = 19.625\ m^2$

(ii) Area of quadrant with 10 m rope

$= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 78.5\ m^2$

$\therefore$ The increase in grazing area

= 78.5 – 19.625

$= 58.875\ m^2$

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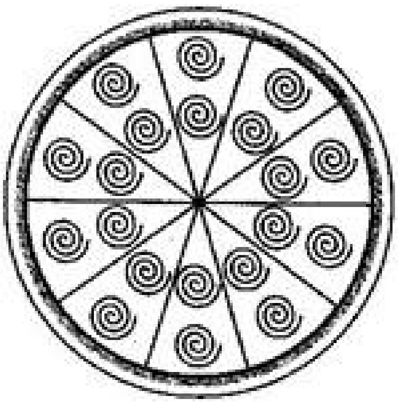
9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:



(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Ans. (i) Diameter of wire = 35 mm

$\Rightarrow$ Radius = $\frac{35}{2}$ mm

Circumference = $2\pi r$ = $2\times\frac{22}{7}\times\frac{35}{2}$

= 110 mm……….(i)

Length of 5 diameters = 35×5 = 175 mm……….(ii)

$\therefore$ Total length of the silver wire required

= 110 + 175 = 285 mm

(ii) $r=\frac{35}{2}$ mm and $\theta=\frac{360^\circ}{10}=36^\circ$

$\therefore$ The area of each sector of the brooch = $\frac{\theta}{360^\circ}\times\pi r^2$

= $\frac{36^\circ}{360^\circ}\times\frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}$ = $\frac{385}{4}\,mm^2$

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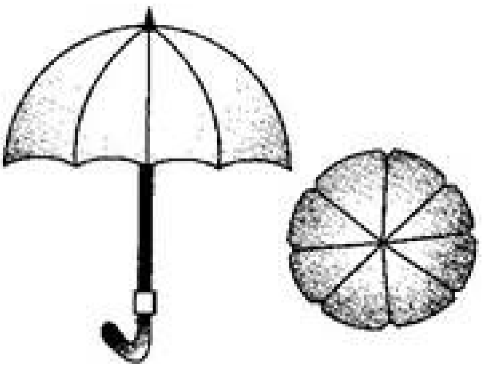
10. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Ans. Here, r = 45 cm and

$$\theta=\frac{360^\circ}{8}=45^\circ$$

Area between two consecutive ribs of the umbrella

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 45 \times 45$$

$$= \frac{22275}{28} cm^2$$

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11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Ans. Here, r = 25 cm and $\theta = 115^\circ$

The total area cleaned at each sweep of the blades

$$= 2 \times \left(\frac{\theta}{360^\circ} \times \pi r^2 \right)$$

$$= 2 \times \left(\frac{115^\circ}{360^\circ} \times \frac{22}{7} \times 25 \times 25 \right)$$

$$= \frac{158125}{126} cm^2$$

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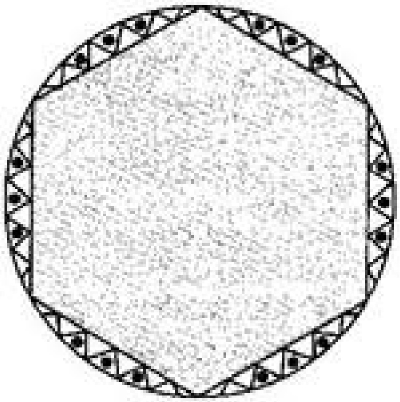
12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. $(\text{Use } \pi = 3.14)$

Ans. Here, r = 16.5 km and $\theta = 80^\circ$

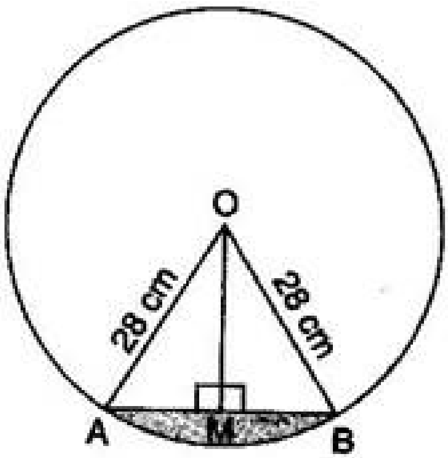
The area of sea over which the ships are warned = $\frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{80^\circ}{360^\circ} \times 3.14 \times 16.5 \times 16.5 = 189.97 \ km^2$$

13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (Use $\sqrt{}$



Ans. r = 28 cm and $\theta = \frac{360^\circ}{6} = 60^\circ$



Area of minor sector = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28$

= $\frac{1232}{3}$ = $410.67\ cm^2$

For, Area of ΔAOB,

Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore$ ΔOMA $\cong$ ΔOMB [RHS congruency]

$\therefore$ AM = BM [By C.P.C.T.]

$\Rightarrow$ AM = BM = $\frac{1}{2}$ AB and $\angle$ AOM = $\angle$ BOM = $\frac{1}{2}\angle$AOB = $\frac{1}{2}\times 60^\circ = 30^\circ$

In right angled triangle OMA,

$$\cos 30^\circ = \frac{OM}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$$

$\Rightarrow$ OM = $14\sqrt{3}$ cm

Also, $\sin 30^\circ = \frac{AM}{OA}$

$$\Rightarrow \frac{1}{2} = \frac{AM}{28}$$

$\Rightarrow$ AM = 14 cm

$\Rightarrow$ 2 AM = 2×14 = 28 cm

$\Rightarrow$ AB = 28 cm

$\therefore$ Area of ΔAOB = $\frac{1}{2}\times AB\times OM$

= $\frac{1}{2}\times 28\times 14\sqrt{3}$ = $196\sqrt{3}$

$= 196 \times 1.7 = 333.2\ cm^2$

$\therefore$ Area of minor segment = Area of minor sector – Area of Δ AOB

= 410.67 – 333.2 = $77.47\ cm^2$

$\therefore$ Area of one design = $77.47\ cm^2$

$\therefore$ Area of six designs = $77.47 \times 6 = 464.82\ cm^2$

Cost of making designs = 464.82×0.35 = Rs. 162.68

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14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is:

(A) $\frac{p}{180^\circ} \times 2\pi R$

(B) $\frac{p}{180^\circ} \times \pi R^2$

(C) $\frac{p}{360^\circ} \times 2\pi R$

(D) $\frac{p}{720^\circ} \times 2\pi R^2$

Ans. (D) Given, r = R and $\theta = p$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{p}{360^\circ} \times \pi R^2$

$= \frac{p}{2 \times 360^\circ} \times 2\pi R^2$

$= \frac{p}{720^\circ} \times 2\pi R^2$

CBSE Class–10 Mathematics

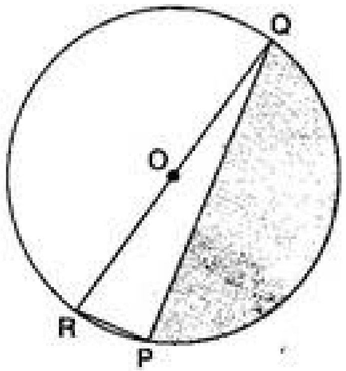
NCERT solution

Chapter - 12

Area Related to Circles - Exercise 12.3

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. Find the area of the shaded region in figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



Ans. In the given figure, $\angle$RPQ = 90° [Angle in semi-circle is 90°]

$\therefore\ RQ^2 = PR^2 + PQ^2$

= $(7)^2 + (24)^2$ = 49 + 576 = 625

$\Rightarrow$ RQ = 25 cm

$\Rightarrow$ Diameter of the circle = 25 cm

$\therefore$ Radius of the circle = $\frac{25}{2}$ cm

Area of the semicircle = $\frac{1}{2}\pi r^2$

= $\frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$ = $\frac{6875}{28}\ cm^2$

Area of right triangle RPQ = $\frac{1}{2}$ x PQ x PR

$$= \frac{1}{2} \times 24 \times 7 = 84\ cm^2$$

Area of shaded region = Area of semicircle – Area of right triangle RPQ

$$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28} = \frac{4523}{28} cm^2$$

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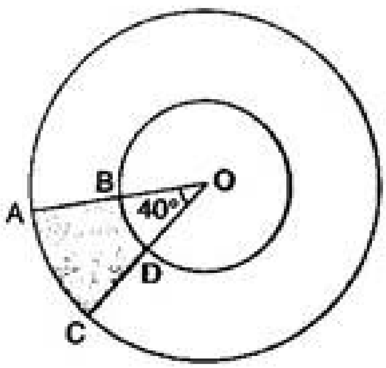
2. Find the area of the shaded region in figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle$ AOC = 40°.



Ans. Area of shaded region = Area of sector OAC – Area of sector OBD

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 - \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times (7)^2$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7}\left[(14)^2 - (7)^2\right]$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7}(14 - 7)(14 + 7)$$

$$= \frac{40^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 21$$

$$= \frac{154}{3} cm^2$$

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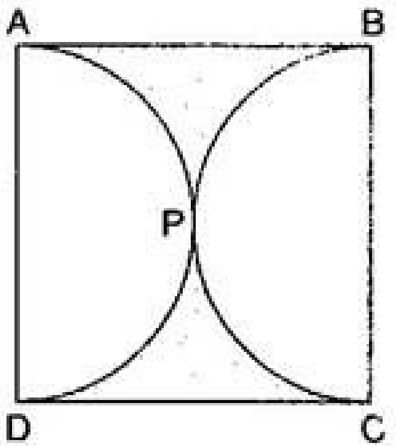
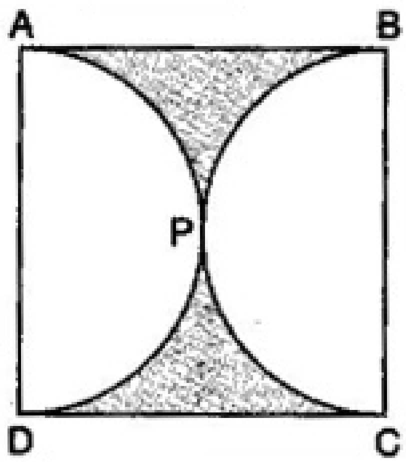
3. Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Ans. Area of shaded region



= Area of square ABCD – (Area of semicircle APD + Area of semicircle BPC)

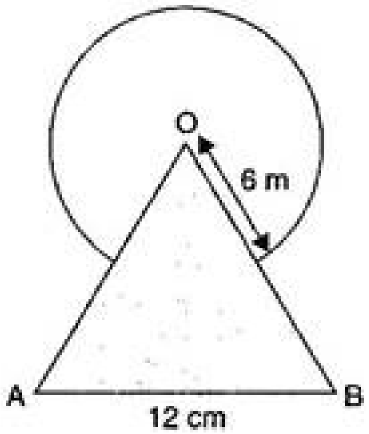
$$= 14\times14-\left[\frac{1}{2}\times\frac{22}{7}\left(\frac{14}{2}\right)^2+\frac{1}{2}\times\frac{22}{7}\left(\frac{14}{2}\right)^2\right]$$

$$= 196 - \left[\tfrac{1}{2}\times\tfrac{22}{7}\times7\times7+\tfrac{1}{2}\times\tfrac{22}{7}\times7\times7\right]$$

$$= 196-\frac{22}{7}\times7\times7$$

= 196 – 154 = $42\ cm^2$

4. Find the area of the shaded region in figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans. Area of shaded region

= Area of circle + Area of equilateral triangle OAB – Area common to the circle and the triangle (Area of sector)

= $\pi r^2 + \frac{\sqrt{3}}{4}(a)^2 - \frac{\theta}{360^\circ} \times \pi r^2$

= $\pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - \frac{60^\circ}{360^\circ} \times \pi(6)^2$

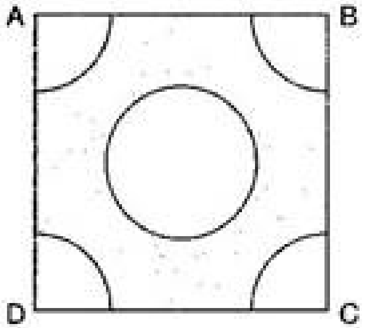
= $36\pi + 36\sqrt{3} - 6\pi$

= $30\pi + 36\sqrt{3}$ = $30 \times \frac{22}{7} + 36\sqrt{3}$

= $\left(\frac{660}{7} + 36\sqrt{3}\right) cm^2$

---

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the figure.



Ans. Area of remaining portion of the square

= Area of square – [(4 $\times$ Area of a quadrant + Area of a circle)]

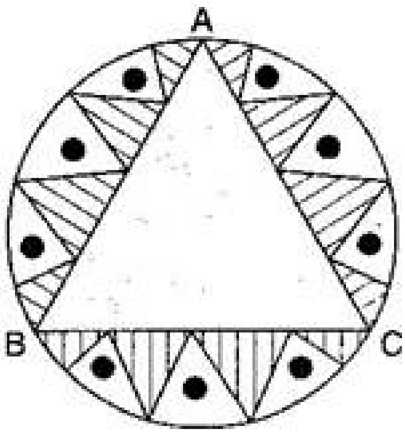
$$= 4\times4-\left[4\times\frac{90^\circ}{360^\circ}\times\frac{22}{7}\times(1)^2+\frac{22}{7}\times\left(\frac{2}{2}\right)^2\right]$$

$$= 16-\left[\frac{22}{7}\left(4\times\frac{1}{4}+1\right)\right]$$

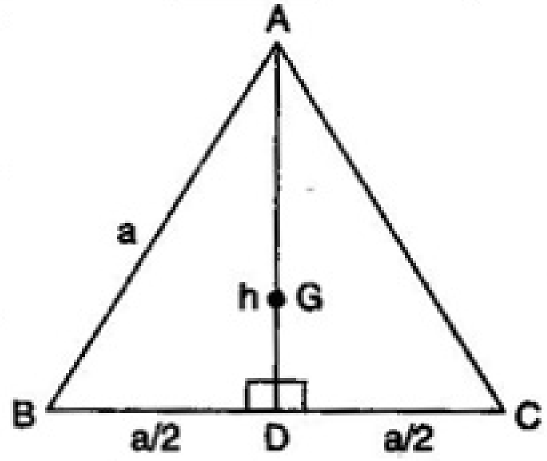
$$= 16-2\times\frac{22}{7} = \frac{68}{7}cm^2$$

---

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design (shaded region).



Ans. Area of design = Area of circular table cover – Area of the equilateral triangle ABC



= $\pi(32)^2 - \frac{\sqrt{3}}{4}a^2$.........(i)

∵ G is the centroid of the equilateral triangle.

∴ radius of the circumscribed circle = $\frac{2}{3}h$ cm

According to the question, $\frac{2}{3}h = 32$

$\Rightarrow h$ = 48 cm

Now, $a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow a^2 = h^2 + \frac{a^2}{4}$

$\Rightarrow a^2 - \frac{a^2}{4} = h^2$

$\Rightarrow \frac{3a^2}{4} = h^2$

$\Rightarrow a^2 = \frac{4h^2}{3}$

$\Rightarrow a^2 = \frac{4(48)^2}{3}$ = 3072

$\Rightarrow a = \sqrt{3072}$ cm

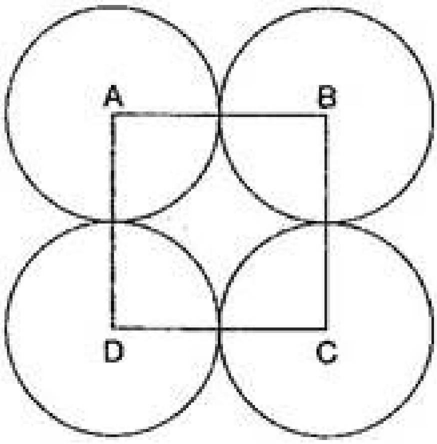
$\therefore$ Required area = $\pi(32)^2 - \frac{\sqrt{3}}{4} \times 3072$ [From eq. (i)]

= $\frac{22}{7} \times 1024 - 768\sqrt{3}$

= $\left(\frac{22528}{7} - 768\sqrt{3}\right) cm^2$

---

7. In figure ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Ans. Area of shaded region = Area of square – 4 $\times$ Area of sector

= $14 \times 14 - 4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2$

= $196 - \frac{22}{7} \times 7 \times 7$ = 196 – 154 = $42\ cm^2$

---

8. Figure depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inner edge.

(ii) the area of the track.

Ans. (i) Distance around the track along its inner edge

$= 106+106+2\times\left[\frac{180^\circ}{360^\circ}\times 2\times\frac{22}{7}\times\left(\frac{60}{2}\right)\right]$

$= 212 + 2 \times \left[\frac{1}{2} \times 2 \times \frac{22}{7} \times \frac{60}{2}\right]$

$= 212+60\times\frac{22}{7} = 212+\frac{1320}{7} = \frac{2804}{7}$ m

(ii) Area of track $= 106\times 10+106\times 10+2\times\left[\frac{1}{2}\times\frac{22}{7}(30+10)^2-\frac{1}{2}\times\frac{22}{7}(30)^2\right]$

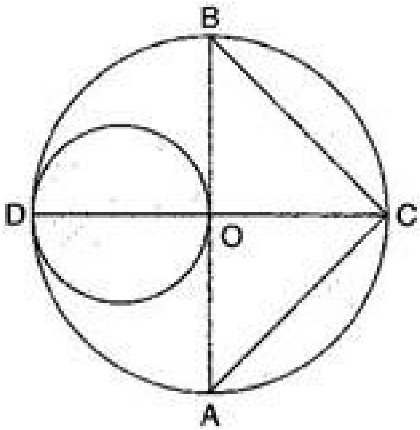
$= 1060+1060+\frac{22}{7}\left[(40)^2-(30)^2\right]$

$= 2120 + \frac{22}{7}(40+30)(40-30)$

$= 2120+\frac{22}{7}\times 700 = 4320\, m^2$

---

9. In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, Find the area of the shaded region.



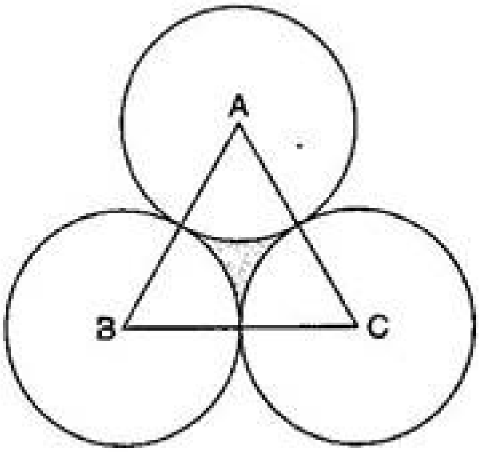
Ans. Area of shaded region = Area of circle with diameter OD + Area of semicircle ACB – Area of Δ ACB

= $\frac{22}{7} \times \left(\frac{7}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7} \times (7)^2 - \left(\frac{1}{2} \times 7 \times 7 + \frac{1}{2} \times 7 \times 7\right)$

= $\frac{77}{2} + 187 - 49$ = $\frac{133}{2}$ = $66.5\ cm^2$

---

10. The area of an equilateral triangle ABC is $17320.5\ \text{cm}^2$. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see figure). Find the area of the shaded region. $\left(\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.\right.$



Ans. Given: Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$ = 17320.5

$$\Rightarrow a^2 = \frac{17320.5 \times 4}{\sqrt{3}}$$

$$\Rightarrow a^2 = \frac{17320.5 \times 4}{1.73205}$$

$$\Rightarrow a^2 = 40000$$

$$\Rightarrow a = 200 \text{ cm}$$

Area of shaded region = Area of ΔABC - Area of 3 sectors

$$= 17320.5 - 3\left[\frac{60^\circ}{360^\circ} \times 3.14 \times \left(\frac{200}{2}\right)^2\right]$$

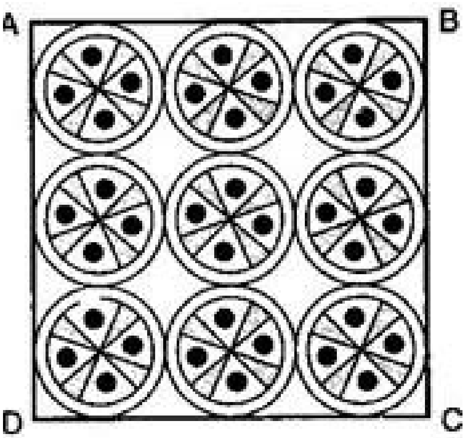
$$= 17320.5 - 3\left[\frac{1}{6} \times 3.14 \times 100 \times 100\right]$$

= 17320.5 - 3 x 5233.33

= 17320.5 – 15700 = $1620.5\ cm^2$

---

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see figure). Find the area of the remaining portion of the handkerchief.



Ans. Radius of each design = 7 cm, then Diamter = 7 x 2 = 14 cm

Therefore, side of square = 14 + 14 + 14 = 42 cm

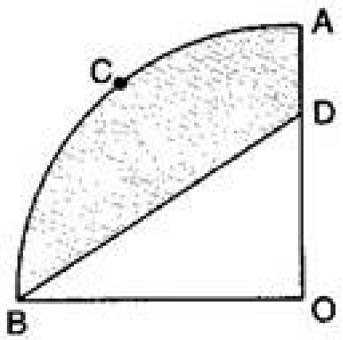
Area of remaining portion of handkerchief = Area of square ABCD – Area of 9 circular designs

$= 42 \times 42 - 9 \times \frac{22}{7} \times 7 \times 7$

$= 1764 - 1386 = 378\ cm^2$

---

12. In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the:



(i) quadrant OACB

(ii) shaded region

Ans. (i) Area of quadrant OACB = $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (3.5)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} = \frac{77}{8}\ cm^2$

(ii) Area of shaded region = Area of quadrant OACB – Area of ΔOBD

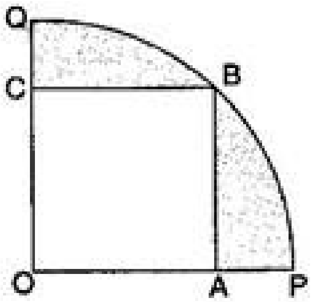
$= \frac{77}{8} - \frac{1}{2} \times \text{OB} \times \text{OD}$

$= \frac{77}{8} - \frac{3.5 \times 2}{2}$

$= \frac{77}{8} - \frac{35}{10} = \frac{49}{8}\ \text{cm}^2$

---

13. In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. $(\text{Use } \pi = 3.14)$



Ans. OB = $\sqrt{OA^2 + AB^2}$ [Using Pythagoras theorem]

= $\sqrt{OA^2 + OA^2}$

= $\sqrt{2}$ OA = $\sqrt{2} \times 20 = 20\sqrt{2}$ cm

Area of shaded region = Area of quadrant OPBQ – Area of square OABC

= $\frac{90^\circ}{360^\circ} \times 3.14 \left(20\sqrt{2}\right)^2 - 20 \times 20$

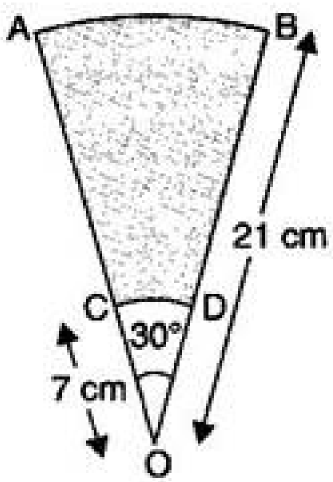
= $\frac{1}{4} \times 3.14 \times 800 - 400$

= $200 \times 3.14 - 400$

= $228\ cm^2$

---

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see figure). If $\angle$AOB = 30°, find the area of the shaded region.



Ans. Area of shaded region = Area of sector OAB – Area of sector OCD

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 - \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7$

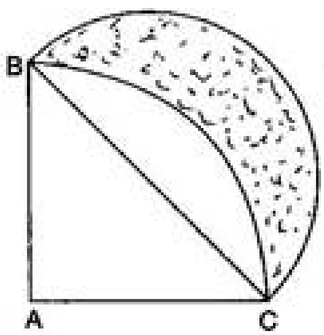
$= \frac{1}{12} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{12} \times \frac{22}{7} \times 7 \times 7$

$= \frac{231}{2} - \frac{77}{6} = \frac{693-77}{6}$

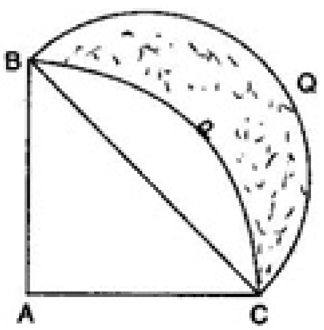
$= \frac{616}{6} = \frac{308}{3}\ cm^2$

---

15. In figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Ans. In right triangle BAC, $BC^2 = AB^2 + AC^2$ [Pythagoras theorem]



$\Rightarrow BC^2 = (14)^2 + (14)^2 = 2(14)^2$

$\Rightarrow$ BC = $14\sqrt{2}$ cm

$\therefore$ Radius of the semicircle = $\frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

$\therefore$ Required area = Area BPCQB

= Area BCQB – Area BCPB

= Area BCQB – (Area BACPB – Area ΔBAC)

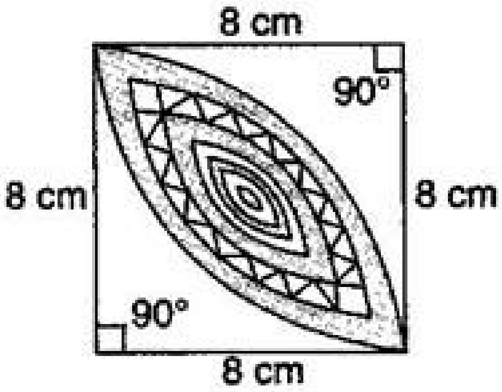
$$= \frac{180^\circ}{360^\circ} \times \frac{22}{7}\left(7\sqrt{2}\right)^2 - \left[\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 - \frac{14 \times 14}{2}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 98 - \left(\frac{1}{4} \times \frac{22}{7} \times 196 - 98\right)$$

= 154 – (154 – 98) = $98\ cm^2$

---

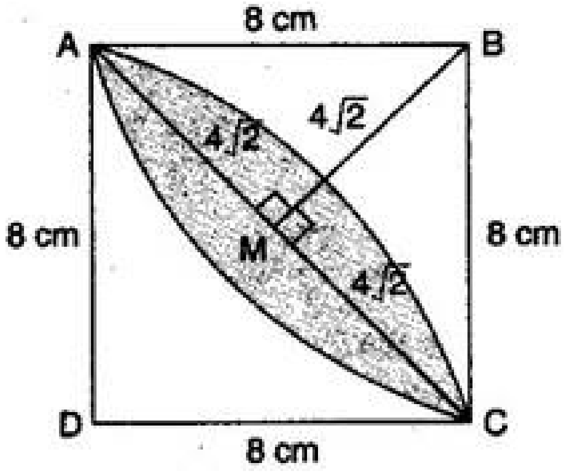
16. Calculate the area of the designed region in figure common between the two quadrants of circles of radius 8 cm each.



Ans. In right triangle ADC, $AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$\Rightarrow AC^2 = (8)^2 + (8)^2 = 2(8)^2$

$\Rightarrow$ AC = $\sqrt{128} = 8\sqrt{2}$ cm



Draw BM $\perp$ AC.

Then AM = MC = $\frac{1}{2}$ AC = $\frac{1}{2} \times 8\sqrt{2}$ = $4\sqrt{2}$ cm

In right triangle AMB,

$AB^2 = AM^2 + BM^2$ [Pythagoras theorem]

$\Rightarrow (8)^2 = \left(4\sqrt{2}\right)^2 + \mathrm{BM}^2$

$\Rightarrow BM^2 = 64 - 32 = 32$

$\Rightarrow$ BM = $4\sqrt{2}$ cm

$\therefore$ Area of ΔABC = $\frac{1}{2} \times AC \times BM$

= $\frac{8\sqrt{2} \times 4\sqrt{2}}{2}$ = $32\ cm^2$

$\therefore$ Half Area of shaded region

= $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2 - 32$

= $16 \times \frac{22}{7} - 32$ = $\frac{128}{7} cm^2$

$\therefore$ Area of designed region

= $2 \times \frac{128}{7}$ = $\frac{256}{7} cm^2$