CBSE Class–10 Mathematics

NCERT solution

Chapter - 7

Coordinate Geometry - Exercise 7.1

1. Find the distance between the following pairs of points:

(i) (2, 3), (4,1)

(ii) (–5, 7), (–1, 3)

(iii) (a, b), (–a, –b)

Ans. (i) Applying Distance Formula to find distance between points (2, 3) and (4,1), we get

d = $\sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(ii) Applying Distance Formula to find distance between points (–5, 7) and (–1, 3), we get

d = $\sqrt{[-1-(-5)]^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

(iii) Applying Distance Formula to find distance between points (a, b) and (–a, –b), we get

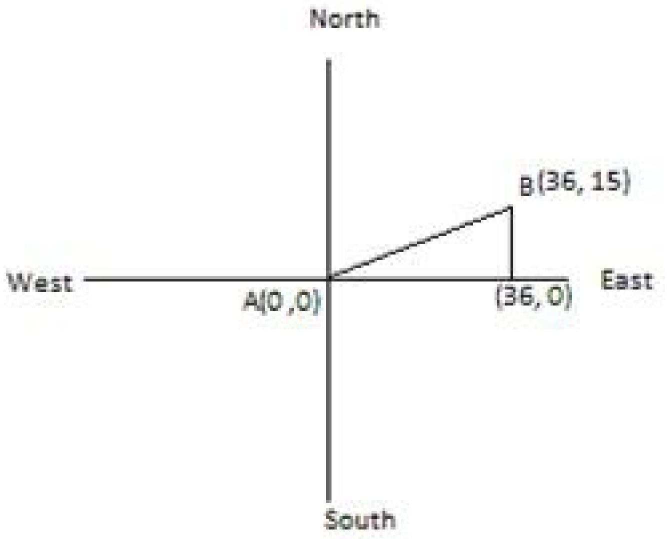
d = $\sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2} = \sqrt{4a^2+4b^2} = \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$

2. Find the distance between the points (0, 0) and (36, 15). Also, find the distance between towns A and B if town B is located at 36 km east and15 km north of town A.

Ans. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

d = $\sqrt{(36-0)^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ units

Town B is located at 36 km east and15 km north of town A. So, the location of town A and B can be shown as:



Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

d = $\sqrt{[36-0]^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ Km

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3. Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Ans. Let A = (1, 5), B = (2, 3) and C = (–2, –11)

Using Distance Formula to find distance AB, BC and CA.

AB = $\sqrt{[2-1]^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{[-2-2]^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

CA = $\sqrt{[-2-1]^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+256} = \sqrt{265}$

Since AB + AC ≠ BC, BC + AC $\neq$ AB and AC $\neq$ BC.

Therefore, the points A, B and C are not collinear.

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4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

Ans. Let A = (5, –2), B = (6, 4) and C = (7, –2)

Using Distance Formula to find distances AB, BC and CA.

AB = $\sqrt{[6-5]^2+[4-(-2)]^2} = \sqrt{(1)^2+(6)^2} = \sqrt{1+36} = \sqrt{37}$

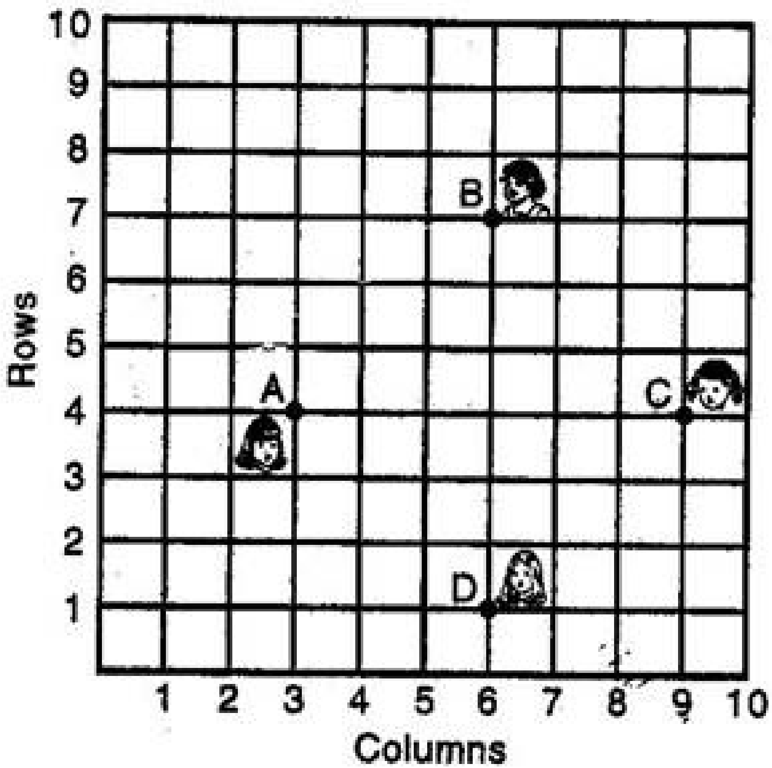
BC = $\sqrt{[7-6]^2+(-2-4)^2} = \sqrt{(1)^2+(-6)^2} = \sqrt{1+36} = \sqrt{37}$

CA = $\sqrt{[7-5]^2+[-2-(-2)]^2} = \sqrt{(2)^2+(0)^2} = \sqrt{4+0} = \sqrt{4} = 2$

Since AB = BC.

Therefore, A, B and C are vertices of an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. "Don't you think ABCD is a square?"Chameli disagrees. Using distance formula, find which of them is correct.



Ans. We have A = (3, 4), B = (6, 7), C = (9, 4) and D = (6, 1)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[6-3]^2+[7-4]^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

BC = $\sqrt{[9-6]^2+[4-7]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[6-9]^2+[1-4]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

DA = $\sqrt{[6-3]^2+[1-4]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Therefore, All the sides of ABCD are equal here. … (1)

Now, we will check the length of its diagonals.

AC = $\sqrt{[9-3]^2+[4-4]^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36+0} = 6$

BD = $\sqrt{[6-6]^2+[1-7]^2} = \sqrt{(0)^2+(-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$

So, Diagonals of ABCD are also equal. … (2)

From (1) and (2), we can definitely say that ABCD is a square.

Therefore, Champa is correct.

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6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (–3, 0)

(ii) (–3, 5), (3, 1), (0, 3), (–1, –4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Ans. (i) Let A = (–1, –2), B = (1, 0), C= (–1, 2) and D = (–3, 0)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[1-(-1)]^2+[0-(-2)]^2} = \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{[-1-1]^2+[2-0]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $\sqrt{[-3-(-1)]^2+[0-2]^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{[-3-(-1)]^2+[0-(-2)]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Therefore, all four sides of quadrilateral are equal. ... (1)

Now, we will check the length of diagonals.

AC = $\sqrt{[-1-(-1)]^2+[2-(-2)]^2} = \sqrt{(0)^2+(4)^2} = \sqrt{0+16} = \sqrt{16} = 4$

BD = $\sqrt{[-3-1]^2+[0-0]^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16+0} = \sqrt{16} = 4$

Therefore, diagonals of quadrilateral ABCD are also equal. ... (2)

From (1) and (2), we can say that ABCD is a square.

(ii) Let A = (–3, 5), B= (3, 1), C= (0, 3) and D= (–1, –4)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[3-(-3)]^2+[1-5]^2} = \sqrt{(6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

BC = $\sqrt{[0-3]^2+[3-1]^2} = \sqrt{(-3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

CD = $\sqrt{[-1-0]^2+[-4-3]^2} = \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

DA = $\sqrt{[-1-(-3)]^2+[-4-5]^2} = \sqrt{(2)^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD.

(iii) Let A = (4, 5), B= (7, 6), C= (4, 3) and D= (1, 2)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[7-4]^2+[6-5]^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{[4-7]^2+[3-6]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[1-4]^2+[2-3]^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

DA = $\sqrt{[1-4]^2+[2-5]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Here opposite sides of quadrilateral ABCD are equal. … (1)

We can now find out the lengths of diagonals.

AC = $\sqrt{[4-4]^2+[3-5]^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

BD = $\sqrt{[1-7]^2+[2-6]^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Here diagonals of ABCD are not equal. … (2)

From (1) and (2), we can say that ABCD is not a rectangle therefore it is a parallelogram.

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7. Find the point on the x–axis which is equidistant from (2, –5) and (–2, 9).

Ans. Let the point be (x, 0) on x–axis which is equidistant from (2, –5) and (–2, 9).

Using Distance Formula and according to given conditions we have:

$$\sqrt{[x-2]^2+[0-(-5)]^2} = \sqrt{[x-(-2)]^2+[(0-9)]^2}$$

$\Rightarrow \sqrt{x^2+4-4x+25} = \sqrt{x^2+4+4x+81}$

Squaring both sides, we get

$\Rightarrow x^2+4-4x+25 = x^2+4+4x+81$

$\Rightarrow -4x + 29 = 4x + 85$

$\Rightarrow 8x = -56$

$\Rightarrow x = -7$

Therefore, point on the x–axis which is equidistant from (2, –5) and (–2, 9) is (–7, 0)

8. Find the values of y for which the distance between the points P (2, –3) and Q (10, y) is 10 units.

Ans. Using Distance formula, we have

$$10 = \sqrt{(2-10)^2 + (-3-y)^2}$$

$$\Rightarrow 10 = \sqrt{(-8)^2 + 9 + y^2 + 6y}$$

$$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$$

Squaring both sides, we get

$$100 = 73 + y^2 + 6y$$

$$\Rightarrow y^2 + 6y - 27 = 0$$

Solving this Quadratic equation by factorization, we can write

$$\Rightarrow y^2 + 9y - 3y - 27 = 0$$

$\Rightarrow y (y + 9) – 3 (y + 9) = 0$

$\Rightarrow (y + 9) (y − 3) = 0$

$\Rightarrow y = 3, −9$

9. If, Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Ans. It is given that Q is equidistant from P and R. Using Distance Formula, we get

PQ = RQ

$\Rightarrow \sqrt{(0-5)^2+[1-(-3)]^2} = \sqrt{(0-x)^2+(1-6)^2}$

$\Rightarrow \sqrt{(-5)^2+[4]^2} = \sqrt{(-x)^2+(-5)^2}$

$\Rightarrow \sqrt{25+16} = \sqrt{x^2+25}$

Squaring both sides, we get

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4, -4$

Thus, Q is (4, 6) or (–4, 6).

Using Distance Formula to find QR, we get

Using value of x = 4 QR = $\sqrt{(4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Using value of x = –4 QR = $\sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Therefore, QR = $\sqrt{41}$

Using Distance Formula to find PR, we get

Using value of x = 4 PR = $\sqrt{(4-5)^2+[6-(-3)]^2} = \sqrt{1+81} = \sqrt{82}$

Using value of x =–4 PR = $\sqrt{(-4-5)^2+[6-(-3)]^2} = \sqrt{81+81} = \sqrt{162}$ = $9\sqrt{2}$

Therefore, x = 4, –4

QR = $\sqrt{41}$, PR = $\sqrt{82}, 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (–3, 4).

Ans. It is given that (x, y) is equidistant from (3, 6) and (–3, 4).

Using Distance formula, we can write

$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{[x-(-3)]^2+(y-4)^2}$$

$$\Rightarrow \sqrt{x^2+9-6x+y^2+36-12y}=\sqrt{x^2+9+6x+y^2+16-8y}$$

Squaring both sides, we get

$$\Rightarrow x^2+9-6x+y^2+36-12y = x^2+9+6x+y^2+16-8y$$

$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$

$\Rightarrow 12x + 4y = 20$

$\Rightarrow 3x + y = 5$

CBSE Class–10 Mathematics

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Chapter - 7

Coordinate Geometry - Exercise 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2:3.

Ans. Let $x_1 = -1,\ x_2 = 4,\ y_1 = 7$ and $y_2 = -3,\ m_1 = 2 \, and \, m_2 = 3$

Using Section Formula to find coordinates of point which divides join of (–1, 7) and (4, –3) in the ratio 2:3, we get

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2\times4 + 3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times(-3) + 3\times7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Therefore, the coordinates of point are (1, 3) which divides join of (–1, 7) and (4, –3) in the ratio 2:3.

2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Ans.

We want to find coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

We are given AC = CD = DB

We want to find coordinates of point C and D.

Let coordinates of point C be (x_1, y_1) and let coordinates of point D be (x_2, y_2).

Clearly, point C divides line segment AB in 1:2 and point D divides line segment AB in 2:1.

Using Section Formula to find coordinates of point C which divides join of (4, –1) and (–2, –3) in the ratio 1:2, we get

$$x_1 = \frac{1\times(-2)+2\times4}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$y_1 = \frac{1\times(-3)+2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

Using Section Formula to find coordinates of point D which divides join of (4, –1) and (–2, –3) in the ratio 2:1, we get

$$x_2 = \frac{2\times(-2)+1\times4}{1+2} = \frac{-4+4}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2\times(-3)+1\times(-1)}{1+2} = \frac{-6-1}{3} = \frac{-7}{3}$$

Therefore, coordinates of point C are $(2, -\frac{5}{3})$ and coordinates of point D are $(0, -\frac{7}{3})$

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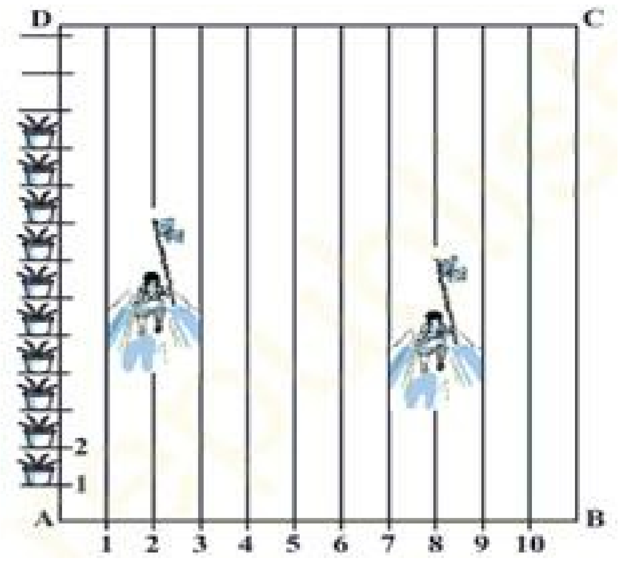
3. To conduct sports day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag. Preet runs 15th of the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Ans. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag.

There are 100 flower pots. It means, she stops at 25th flower pot.

Therefore, the coordinates of point where she stops are (2 m, 25 m).

Preet runs 15th of the distance AD on the eighth line and posts a red flag. There are 100 flower pots. It means, she stops at 20th flower pot.

Therefore, the coordinates of point where she stops are (8, 20).

Using Distance Formula to find distance between points (2 m, 25 m) and (8 m, 20 m), we get

$$d = \sqrt{(2-8)^2 + (25-20)^2} = \sqrt{(-6)^2 + 5^2} = \sqrt{36+25} = \sqrt{61}m$$

Rashmi posts a blue flag exactly halfway the line segment joining the two flags.

Using section formula to find the coordinates of this point, we get

$$x = \frac{2+8}{2} = \frac{10}{2} = 5$$

$$y = \frac{25+20}{2} = \frac{45}{2}$$

Therefore, coordinates of point, where Rashmi posts her flag are $(5, \frac{45}{2})$ (5, $\frac{45}{2}$).

It means she posts her flag in 5th line after covering $\frac{45}{2}$ = 22.5 m of distance.

4. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6).

Ans. Let (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in k:1.

Using Section formula, we get

$$-1 = \frac{(-3)\times 1 + 6\times k}{k+1}$$

$\Rightarrow -k - 1 = (-3 + 6k)$

$\Rightarrow -7k = -2$

$\Rightarrow k = \frac{2}{7}$

Therefore, the ratio is $\frac{2}{7}$:1 which is equivalent to 2:7.

Therefore, (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in 2:7.

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5. Find the ratio in which the line segment joining A (1, –5) and B (–4, 5) is divided by the x–axis. Also find the coordinates of the point of division.

Ans. Let the coordinates of point of division be (x, 0) and suppose it divides line segment joining A (1, –5) and B (–4, 5) in k:1.

According to Section formula, we get

$$x = \frac{1\times 1+(-4)\times k}{k+1} = \frac{1-4k}{k+1} \text{ and } 0 = \frac{(-5)\times 1+5k}{k+1} \quad \ldots (1)$$

$$0 = \frac{(-5)\times 1+5k}{k+1}$$

$\Rightarrow 5 = 5k$

$\Rightarrow k = 1$

Putting value of k in (1), we get

$$x = \frac{1\times 1+(-4)\times 1}{1+1} = \frac{1-4}{2} = \frac{-3}{2}$$

Therefore, point ($\frac{-3}{2}$, 0) on x–axis divides line segment joining A (1, –5) and B (–4, 5) in 1:1.

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6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Ans. Let A = (1, 2), B = (4, y), C = (x, 6) and D = (3, 5)

We know that diagonals of parallelogram bisect each other. It means that coordinates of midpoint of diagonal AC would be same as coordinates of midpoint of diagonal BD. ... (1)

Using Section formula, the coordinates of midpoint of AC are:

$$\frac{1+x}{2}, \frac{2+6}{2} = \frac{1+x}{2}, 4$$

Using Section formula, the coordinates of midpoint of BD are:

$$\frac{4+3}{2}, \frac{5+y}{2} = \frac{7}{2}, \frac{5+y}{2}$$

According to condition (1), we have

$$\frac{1+x}{2} = \frac{7}{2}$$

$\Rightarrow (1 + x) = 7$

$\Rightarrow x = 6$

Again, according to condition (1), we also have

$$4 = \frac{5+y}{2}$$

$\Rightarrow 8 = 5 + y$

$\Rightarrow y = 3$

Therefore, x = 6 and y = 3

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7. **Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).**

Ans. We want to find coordinates of point A. AB is the diameter and coordinates of center are (2, –3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$$2 = \frac{x+1}{2}$$

⇒ 4 = x + 1

⇒ x = 3

Using section formula, we get

$$-3 = \frac{4+y}{2}$$

⇒ −6 = 4 + y

⇒ y = −10

Therefore, Coordinates of point A are (3, –10).

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8. If A and B are (–2, –2) and (2, –4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

Ans. A = (–2, –2) and B = (2, –4)

It is given that AP= $\frac{3}{7}$ AB

PB = AB – AP = AB − $\frac{3}{7}$ AB = $\frac{4}{7}$ AB

So, we have AP: PB = 3: 4

Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

$$x = \frac{(-2)\times 4 + 2\times 3}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{(-2)\times 4 + (-4)\times 3}{3+4} = \frac{-8-12}{7} = \frac{-20}{7}$$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

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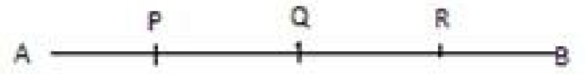
9. Find the coordinates of the points which divides the line segment joining A (–2, 2) and B (2, 8) into four equal parts.

Ans. A = (–2, 2) and B = (2, 8)



Let P, Q and R are the points which divide line segment AB into 4 equal parts.

Let coordinates of point $P = (x_1, y_1)$, $Q = (x_2, y_2)$ *and* $R = (x_3, y_3)$

We know AP = PQ = QR = RS.

It means, point P divides line segment AB in 1:3.

Using Section formula to find coordinates of point P, we get

$$x_1 = \frac{(-2)\times 3 + 2\times 1}{1+3} = \frac{-6+2}{4} = \frac{-4}{4} = -1$$

$$y_1 = \frac{2\times 3 + 8\times 1}{1+3} = \frac{6+8}{4} = \frac{14}{4} = \frac{7}{2}$$

Since, AP = PQ = QR = RS.

It means, point Q is the mid-point of AB.

Using Section formula to find coordinates of point Q, we get

$$x_2 = \frac{(-2)\times 1 + 2\times 1}{1+1} = \frac{-2+2}{2} = \frac{0}{2} = 0$$

$$y_2 = \frac{2\times 1 + 8\times 1}{1+1} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Because, AP = PQ = QR = RS.

It means, point R divides line segment AB in 3:1

Using Section formula to find coordinates of point P, we get

$$x_3 = \frac{(-2)\times 1 + 2\times 3}{1+3} = \frac{-2+6}{4} = \frac{4}{4} = 1$$

$$y_3 = \frac{2\times 1 + 8\times 3}{1+3} = \frac{2+24}{4} = \frac{26}{4} = \frac{13}{2}$$

Therefore, P = (–1, $\frac{7}{2}$), Q= (0, 5)and R = (1, $\frac{13}{2}$)

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10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (–2, –1) taken in order. {Hint: Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)}

Ans. Let A = (3, 0), B = (4, 5), C = (–1, 4) and D = (–2, –1)

Using Distance Formula to find length of diagonal AC, we get

$$AC = \sqrt{[3-(-1)]^2 + (0-4)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Using Distance Formula to find length of diagonal BD, we get

$$BD = \sqrt{[4-(-2)]^2 + [5-(-1)]^2} = \sqrt{6^2 + 6^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

∵ Area of rhombus = ½ (product of its diagonals)

= $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ = 24 sq. units

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Chapter - 7

Coordinate Geometry - Exercise 7.3

1. Find the area of the triangle whose vertices are:

(i) (2, 3), (–1, 0), (2, –4)

(ii) (–5, –1), (3, –5), (5, 2)

Ans. (i) (2, 3), (–1, 0), (2, –4)

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [2 {0 − (−4)} – 1 (−4 − 3) + 2 (3 − 0)]

= $\frac{1}{2}$ [2 (0 + 4) – 1 (−7) + 2 (3)]

= $\frac{1}{2}$ (8 + 7 + 6) = $\frac{21}{2}$ sq. units

(ii) (–5, –1), (3, –5), (5, 2)

Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [−5 (−5 − 2) + 3 {2 − (−1)} + 5 {−1 − (−5)}]

= $\frac{1}{2}$ [−5 (−7) + 3 (3) + 5 (4)]

= $\frac{1}{2}$ (35 + 9 + 20)

$= \frac{1}{2}$ (64) = 32 sq. units

2. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, –2), (5, 1), (3, k)

(ii) (8, 1), (k, –4), (2, –5)

Ans. (i) (7, –2), (5, 1), (3, k)

Since, the given points are collinear, it means the area of triangle formed by them is equal to zero.

Area of Triangle = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

$\Rightarrow \frac{1}{2}$ [7 (1 − k) + 5 {k − (−2)} + 3 (−2 − 1)] = 0

$= \frac{1}{2}$ (7 − 7k + 5k + 10 − 9) = 0

$\Rightarrow \frac{1}{2}$ (7 − 7k + 5k + 1) = 0

$\Rightarrow \frac{1}{2}$ (8 − 2k) = 0

⇒ 8 − 2k = 0

⇒ 2k = 8

⇒ k = 4

(ii) (8, 1), (k, –4), (2, –5)

Since, the given points are collinear, it means the area of triangle formed by them is equal to zero.

Area of Triangle = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

$\Rightarrow \frac{1}{2}$ [8 {−4 − (−5)} + k (−5 − 1) + 2 {1 − (−4)}] = 0

$\Rightarrow \frac{1}{2}$ (8 − 6k + 10) = 0

$\Rightarrow \frac{1}{2}$ (18 − 6k) = 0

⇒18 − 6k = 0

⇒ 18 = 6k

⇒ k = 3

---

3. Find the area of the triangle formed by joining the mid–points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Ans. Let A = (0, –1) = (x_1, y_1), B = (2, 1) = (x_2, y_2) and

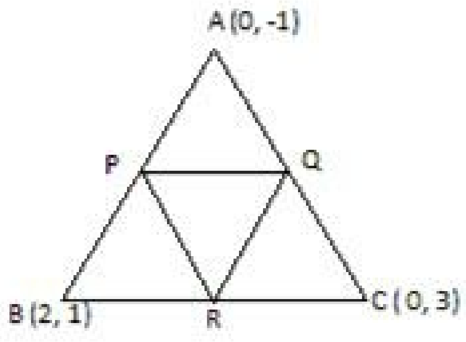
C = (0, 3) = (x_3, y_3)

Area of △ABC = $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

⇒ Area of △ABC

= $\frac{1}{2}$ [0 (1 − 3) + 2 {3 − (−1)} + 0 (−1 − 1)] = $\frac{1}{2} \times 8$

= 4 sq. units



P, Q and R are the mid–points of sides AB, AC and BC respectively.

Applying Section Formula to find the vertices of P, Q and R, we get

$$P=\frac{0+2}{2},\frac{1-1}{2}=(1,0)$$

$$Q=\frac{0+0}{2},\frac{-1+3}{2}=(0,1)$$

$$R=\frac{2+0}{2},\frac{1+3}{2}=(1,2)$$

Applying same formula, Area of $\triangle$PQR = $\frac{1}{2}$ [1 (1 − 2) + 0 (2 − 0) + 1 (0 − 1)] = $\frac{1}{2}$ $|-2|$

= 1 sq. units (numerically)

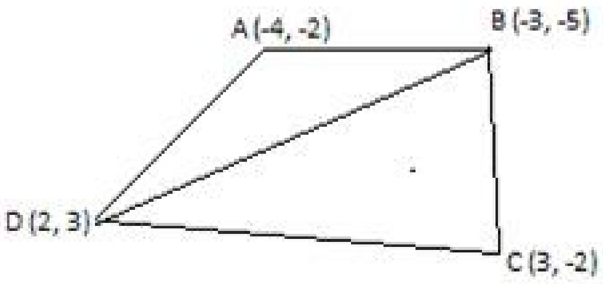
Now, $\frac{Area\ of\ \Delta PQR}{Area\ of\ \Delta ABC}=\frac{1}{4}=1:4$

4. Find the area of the quadrilateral whose vertices taken in order are (–4, –2), (–3, –5), (3, –2) and (2, 3).

Ans. Area of Quadrilateral ABCD

= Area of Triangle ABD +

Area of Triangle BCD … (1)



Using formula to find area of triangle:

Area of ΔABD

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}$ [−4 (−5 − 3) – 3 {3 − (−2)} + 2 {−2 − (−5)}]

$= \frac{1}{2}$ (32 – 15 + 6)

$= \frac{1}{2}$ (23) = 11.5 sq units ... (2)

Again using formula to find area of triangle:

Area of $\triangle$BCD $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}$ [−3 (−2 − 3) + 3 {3 − (−5)} + 2 {−5 − (−2)}]

$= \frac{1}{2}$ (15 + 24 − 6)

$= \frac{1}{2}$ (33) = 16.5 sq units ... (3)

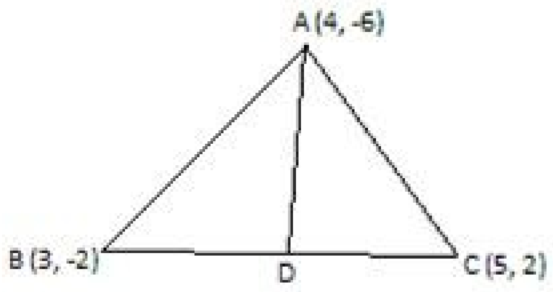
Putting (2) and (3) in (1), we get

Area of Quadrilateral ABCD = 11.5 + 16.5 = 28 sq units

5. We know that median of a triangle divides it into two triangles of equal areas. Verify this result for △ABC whose vertices are A (4, –6), B (3, –2) and C (5, 2).

Ans. We have △ABC whose vertices are given.

We need to show that ar(△ABD) = ar(△ACD).



Let coordinates of point D are (x, y)

Using section formula to find coordinates of D, we get

$$x = \frac{3+5}{2} = \frac{8}{2} = 4$$

$$y = \frac{-2+2}{2} = \frac{0}{2} = 0$$

Therefore, coordinates of point D are (4, 0)

Using formula to find area of triangle:

Area of △ABD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}$ [4 (−2 − 0) + 3 {0 − (−6)} + 4 {−6 − (−2)}]

$= \frac{1}{2}$ (−8 + 18 −16)

$= \frac{1}{2}$ (−6) = −3 sq units

Area cannot be in negative.

Therefore, we just consider its numerical value.

Therefore, area of △ABD = 3 sq units … (1)

Again using formula to find area of triangle:

Area of △ACD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}$ [4 (2 − 0) + 5 {0 − (−6)} + 4 {−6 −2)}]

$= \frac{1}{2}$ (8 + 30 − 32) = ½ (6) = 3 sq units … (2)

From (1) and (2), we get ar(△ABD) = ar(△ACD)

Hence Proved.

CBSE Class–10 Mathematics

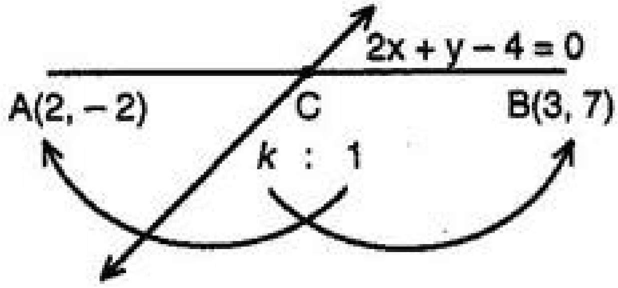
NCERT solution

Chapter - 7

Coordinate Geometry - Exercise 7.4

1. Determine the ratio in which the line $2x+y-4=0$ divides the line segment joining the points A$(2,-2)$ and B$(3,7)$.

Ans. Let the line $2x+y-4=0$ divides the line segment joining A$(2,-2)$ and B (3, 7) in the ratio $k:1$ at point C. Then, the coordinates of C are $\left(\frac{3k+2}{k+1},\frac{7k-2}{k+1}\right)$.



But C lies on $2x+y-4=0$, therefore

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow 6k+4+7k-2-4k-4=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Hence, the required ratio if 2: 9 internally.

2. Find a relation between x and y if the points (x,y),$(1,2)$ and $(7,0)$ are collinear.

Ans. The points A(x,y), B (1, 2) and C (7, 0) will be collinear if

Area of triangle = 0

$$\Rightarrow \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]=0$$

$$\Rightarrow 2x-y+7y-14=0$$

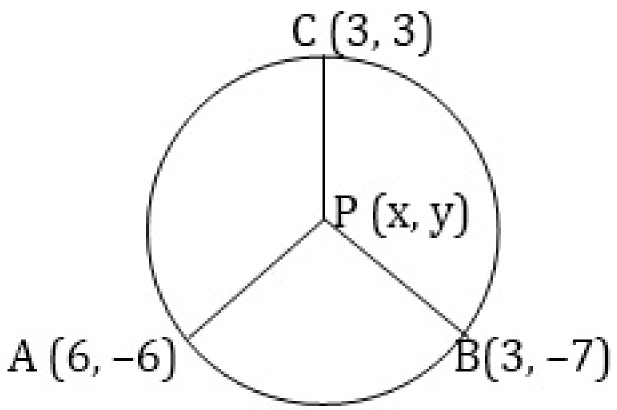
$$\Rightarrow 2x+6y-14=0$$

$$\Rightarrow x+3y-7=0$$

3. Find the centre of a circle passing through the points $(6,-6)$, $(3,-7)$ and $(3,3)$.

Ans. Let P(x,y), be the centre of the circle passing through the points A$(6,-6)$, B$(3,-7)$ and C (3, 3). Then AP = BP = CP.

Taking AP = BP



$$\Rightarrow AP^2 = BP^2$$

$$\Rightarrow (x-6)^2+(y+6)^2=(x-3)^2+(y+7)^2$$

$$\Rightarrow x^2-12x+36+y^2+12y+36=x^2-6x+9+y^2+14y+49$$

$$\Rightarrow -12x+6x+12y-14y+72-58=0$$

$$\Rightarrow -6x-2y+14=0$$

$\Rightarrow 3x+y-7=0$ ....(i)

Again, taking BP = CP

$\Rightarrow BP^2 = CP^2$

$\Rightarrow (x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow x^2-6x+9+y^2+14y+49=x^2-6x+9+y^2-6y+9$

$\Rightarrow -6x+6x+14y+6y+58-18=0$

$\Rightarrow 20y+40=0$

$\Rightarrow y=-2$

Putting the value of y in eq. (i),

$3x+y-7=0$

$\Rightarrow 3x=9$

$\Rightarrow x=3$

Hence, the centre of the circle is $(3,-2)$.

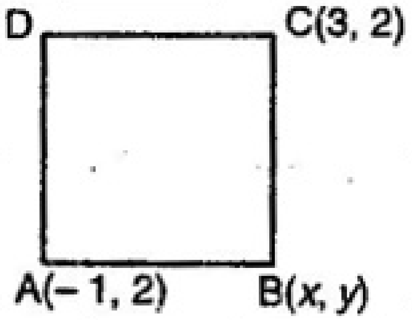
---

4. The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.

Ans. Let ABCD be a square and B(x,y) be the unknown vertex.

AB = BC

$\Rightarrow AB^2 = BC^2$



$\Rightarrow (x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow x^2+1+2x+y^2+4-4y=x^2+9-6x+y^2+4-4y$

$\Rightarrow 2x+1=-6x+9$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ....(i)

In ΔABC, $AB^2+BC^2=AC^2$

$\Rightarrow (x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2=(3+1)^2+(2-2)^2$

$\Rightarrow 2(y-2)^2+\left[(x+1)^2+(x-3)^2\right]=16+0$

$\Rightarrow 2(y-2)^2+\left[(1+1)^2+(1-3)^2\right]=16$

$\Rightarrow 2\left(y^2-4y+4\right)+8=16$

$\Rightarrow y^2-4y+4=4$

$\Rightarrow y(y-4)=0$

$\Rightarrow y$ = 0 or 4

Hence, the required vertices of the square are (1, 0) and (1, 4).

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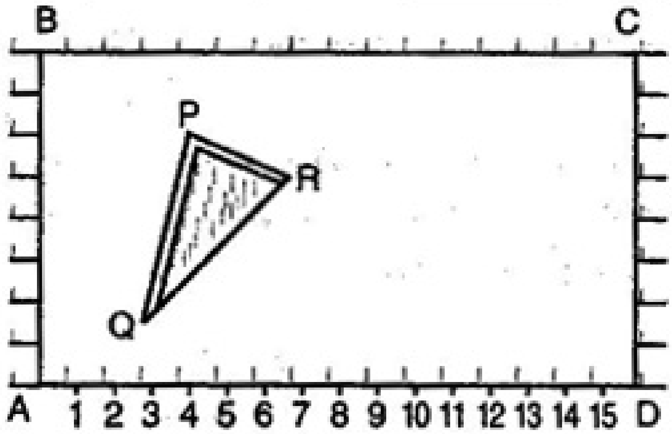
5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted

on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.**



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of $\triangle$PQR if C is the origin? Also calculate the area of the triangle in these cases. What do you observe?

Ans. (i) Taking A as the origin, AD and AB as the coordinate axes. Clearly, the points P, Q and R are (4, 6), (3, 2) and (6, 5) respectively.

(ii) Taking C as the origin, CB and CD as the coordinate axes. Clearly, the points P, Q and R are given by (12, 2), (13, 6) and (10, 3) respectively.

We know that the area of the triangle = $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$\therefore$ Area of $\triangle$PQR (First case) = $\frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$

= $\frac{1}{2}\left[4(-3)+3(-1)+6(4)\right]$

= $\frac{1}{2}\left[-12-3+24\right]$ = $\frac{9}{2}$ sq. units

And Area of $\triangle$ PQR (Second case) = $\frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$

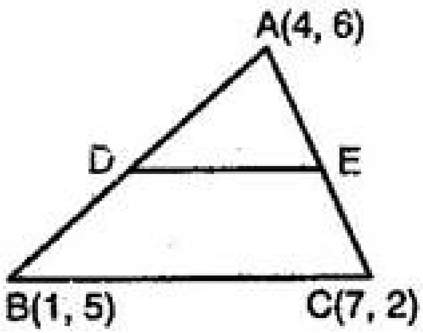
$= \frac{1}{2}[12(3)+13(1)+10(-4)]$

$= \frac{1}{2}[36+13-40]$ = $\frac{9}{2}$ sq. units

Hence, the areas are same in both the cases.

6. The vertices of a $\triangle$ABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the $\triangle$ADE and compare it with the area of $\triangle$ABC.

Ans. Since, $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$



$\therefore$ DE $\parallel$ BC[By Thales theorem]

$\therefore$ $\triangle$ADE ~ $\triangle$ABC

$$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta ABC)}=\frac{AD^2}{AB^2}$$

$$=\left(\frac{AD}{AB}\right)^2=\left(\frac{1}{4}\right)^2=\frac{1}{16} \text{ ....(i)}$$

Now, Area ($\triangle$ABC) = $\frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$

$= \frac{1}{2}[12-4+7] = \frac{15}{2}$ sq. units..........(ii)

From eq. (i) and (ii),

Area (ΔADE) = $\frac{1}{16} \times$ Area (ΔABC) = $\frac{1}{16} \times \frac{15}{2} = \frac{15}{32}$ sq. units

$\therefore$ Area (ΔADE): Area (ΔABC) = 1: 16

---

7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP: PD = 2: 1.

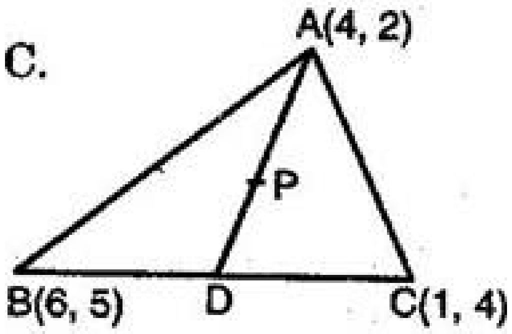
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR : RF = 2 : 1.

(iv) What do you observe?

(Note: The point which is common to all the three medians is called *centroid* and this point divides each median in the ratio 2: 1)

(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Ans. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.



(i) Since AD is the median of ΔABC.

$\therefore$ D is the mid-point of BC.

$\therefore$ Its coordinates are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Since P divides AD in the ratio 2: 1

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Since BE is the median of ΔABC.

$\therefore$ E is the mid-point of AD.

$\therefore$ Its coordinates are $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Since Q divides BE in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Since CF is the median of ΔABC.

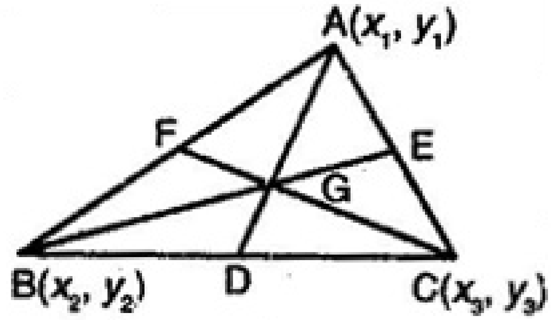
$\therefore$ F is the mid-point of AB.

$\therefore$ Its coordinates are $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Since R divides CF in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) We observe that the points P, Q and R coincide, i.e., the medians AD, BE and CF are concurrent at the point $\left(\frac{11}{3}, \frac{11}{3}\right)$. This point is known as the centroid of the triangle.



(v) According to the question, D, E, and F are the mid-points of BC, CA and AB respectively.

∴ Coordinates of D are $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

Coordinates of a point dividing AD in the ratio 2: 1 are

$$\left(\frac{1.x_1 + 2\left(\frac{x_2 + x_2}{2}\right)}{1+2}, \frac{1.y_1 + 2\left(\frac{y_2 + y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$.

∴ The coordinates of a point dividing BE in the ratio 2: 1 are

$$\left(\frac{1.x_2 + 2\left(\frac{x_1 + x_3}{2}\right)}{1+2}, \frac{1.y_2 + 2\left(\frac{y_1 + y_3}{2}\right)}{1+2}\right) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Similarly, the coordinates of a point dividing CF in the ratio 2: 1 are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Thus, the point $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 2: 1.

$\therefore$ The median of a triangle are concurrent and the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

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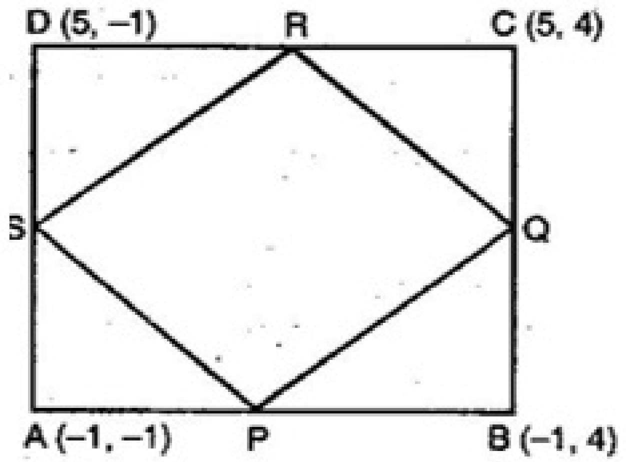
8. ABCD is a rectangle formed by joining points A$(-1, -1)$, B$(-1, 4)$, C$(5, 4)$ and D $(5, -1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? Or a rhombus? Justify your answer.

Ans. Since P is mid-point of AB, therefore, the coordinates of P are $\left(-1, \frac{3}{2}\right)$.

Similarly, the coordinates of Q are (2, 4)., the coordinates of R are $\left(5, \frac{3}{2}\right)$ and the coordinates of S are $(2, -1)$.

Using distance formula, PQ = $\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}$



$= \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

QR = $\sqrt{(5-2)^2+\left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

RS = $\sqrt{(2-5)^2+\left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

SP = $\sqrt{(-1-2)^2+\left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

$\Rightarrow$ PQ = QR = RS = SP

Now, PR = $\sqrt{(5+1)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36} = 6$

And SQ = $\sqrt{(2-2)^2+(4+1)^2} = \sqrt{25} = 5$

$\Rightarrow$ PR $\neq$ SQ

Since all the sides are equal but the diagonals are not equal.

$\therefore$ PQRS is a rhombus.