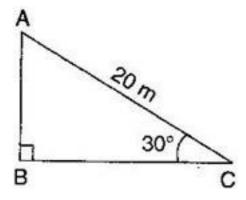
## **CBSE Class–10 Mathematics**

## **NCERT** solution

## Chapter - 9

## Some Applications of Trigonometry - Exercise 9.1

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is  $30^{\circ}$  (see figure).



Ans. In right triangle ABC,

$$\sin 30^{\circ} = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{AB}{20}$$

$$AB = 20/2$$

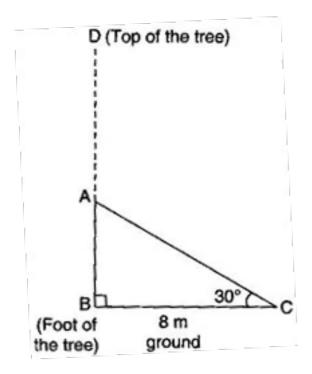
Hence, the height of the pole is 10 m.

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle  $30^{\circ}$  with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Ans. Let AC be the broken part of tree

In right triangle ABC,

$$\cos 30^{\circ} = \frac{BC}{AC}$$



$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$$

$$\Rightarrow$$
 AC =  $\frac{16}{\sqrt{3}}$  m

Again, 
$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$\Rightarrow$$
 AB =  $\frac{8}{\sqrt{3}}$  m

∴ Height of the tree = AB + AD=AB+AC

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$$

$$=\frac{24}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=8\sqrt{3}$$
 m

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m and is inclined at an angle of  $30^{\circ}$  to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m and inclined at an angle of  $60^{\circ}$  to the ground. What should be the length of the slide in each case?

Ans. In right triangle ABC,

$$\sin 30^{\circ} = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{1.5}{AC}$$

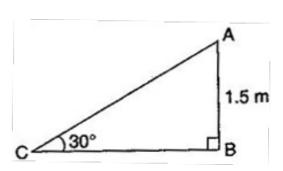
$$\Rightarrow$$
 AC = 3 m

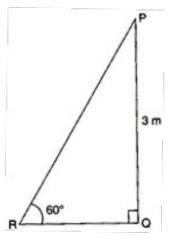
In right triangle PQR,

$$\sin 60^\circ = \frac{PQ}{PR}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{PR}$$

$$\Rightarrow$$
 PR =  $2\sqrt{3}$  m





Hence, the lengths of the slides are 3 m and  $2\sqrt{3}$  m respectively.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is  $30^{\circ}$ . Find the height of the tower.

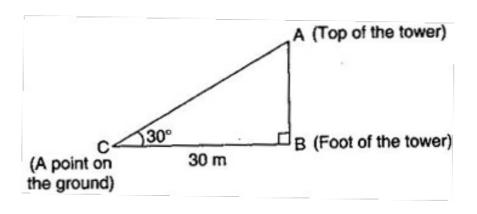
Ans. In right triangle ABC, AB be the height of the tower.

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\Rightarrow$$
 AB =  $\frac{30}{\sqrt{3}}$  m

$$\Rightarrow \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$



5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is  $60^{\circ}$ . Find the length of the string, assuming that there is no slack in the string.

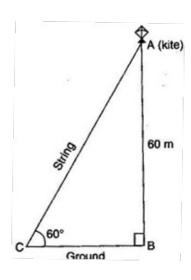
Ans. In right triangle ABC, AC is the length of the string

$$\sin 60^{\circ} = \frac{AB}{AC}$$

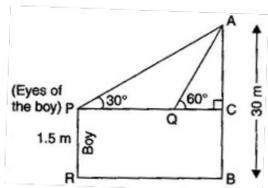
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{\Delta C}$$

$$\Rightarrow$$
 AC =  $40\sqrt{3}$  m

Hence the length of the string is  $40\sqrt{3}$  m.



6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from  $30^{\circ}$  to  $60^{\circ}$  as he walks towards the building. Find the distance he walked towards the building.



$$AC = AB - BC$$

$$= AB - PR$$
 (As,  $BC = PR$ )

$$= 30 - 1.5$$

$$= 28.5 \text{ m}$$

In right triangle ACQ,

$$\tan 60^{\circ} = \frac{AC}{QC}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{QC} \Rightarrow QC = \frac{28.5}{\sqrt{3}} \text{ m}$$

In right triangle ACP,

$$\tan 30^\circ = \frac{AC}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ + QC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ + \frac{28.5}{\sqrt{3}}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5 \times \sqrt{3}}{PQ\sqrt{3} + 28.5}$$

$$\Rightarrow PQ\sqrt{3} + 28.5 = 85.5$$

$$\Rightarrow PQ\sqrt{3} = 57$$

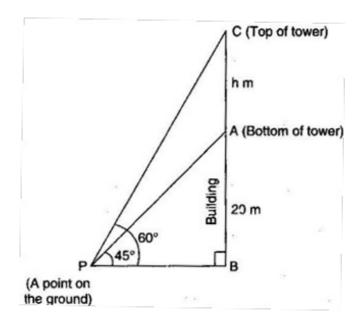
$$\Rightarrow$$
 PQ =  $\frac{57}{\sqrt{3}}$ 

$$\Rightarrow$$
 PQ =  $\frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$  = 19 $\sqrt{3}$  m

Hence, the distance the boy walked towards the building is  $19\sqrt{3}$  m.

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are  $45^{\circ}$  and  $60^{\circ}$  respectively. Find the height of the tower.

**Ans.** Let the height of the tower be h m. Then, in right triangle CBP,



$$\tan 60^\circ = \frac{BC}{BP}$$

$$\Rightarrow \sqrt{3} = \frac{AB + AC}{BP}$$

$$\Rightarrow \sqrt{3} = \frac{20+h}{RP}$$
 .....(i)

In right triangle ABP,

$$\tan 45^\circ = \frac{AB}{BP}$$

$$\Rightarrow 1 = \frac{20}{BP} \Rightarrow BP = 20 \text{ m}$$

Putting this value in eq. (i), we get,

$$\sqrt{3} = \frac{20 + h}{20}$$

$$\Rightarrow 20\sqrt{3} = 20 + h$$

$$\Rightarrow h = 20\sqrt{3} - 20$$

$$\Rightarrow h = 20(\sqrt{3} - 1) \text{ m}$$

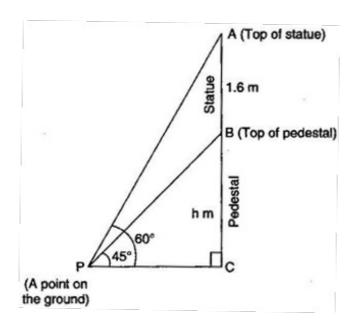
The height of the tower is  $20(\sqrt{3}-1)$  m.

8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is  $60^{\circ}$  and from the same point the angle of elevation of the top of the pedestal is  $45^{\circ}$ . Find the height of the pedestal.

**Ans.** Let the height of the pedestal be h m.

$$\therefore$$
 BC =  $h$  m

In right triangle ACP,



$$\tan 60^\circ = \frac{AC}{PC}$$

$$\Rightarrow \sqrt{3} = \frac{AB + BC}{PC}$$

$$\Rightarrow \sqrt{3} = \frac{1.6 + h}{PC}$$
 .....(i)

In right triangle BCP,

$$\tan 45^{\circ} = \frac{BC}{PC}$$

$$\Rightarrow 1 = \frac{h}{PC} \Rightarrow PC = h$$

$$\therefore \sqrt{3} = \frac{1.6 + h}{h}$$
 [From eq. (i)]

$$\Rightarrow \sqrt{3}h = 1.6 + h$$

$$\Rightarrow h(\sqrt{3}-1)=1.6$$

$$\Rightarrow h = \frac{1.6}{\sqrt{3}-1}$$

$$\Rightarrow h = \frac{1.6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$\Rightarrow h = \frac{1.6\sqrt{3} + 1}{3 - 1}$$

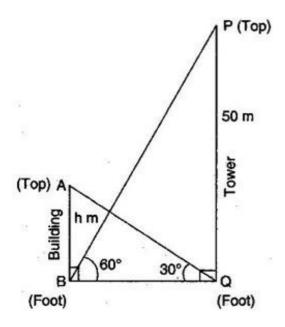
$$\Rightarrow h = \frac{1.6(\sqrt{3}+1)}{2}$$

$$\Rightarrow h = 0.8(\sqrt{3} + 1) \text{ m}$$

Hence, the height of the pedestal is  $0.8(\sqrt{3} + 1)$  m.

9. The angle of elevation of the top of a building from the foot of the tower is  $30^{\circ}$  and the angle of elevation of the top of the tower from the foot of the building is  $60^{\circ}$ . If the tower is 50 m high, find the height of the building.

**Ans.** Let the height of the building be h m.



In right triangle PQB,

$$\tan 60^{\circ} = \frac{PQ}{BQ} \Rightarrow \sqrt{3} = \frac{50}{BQ}$$

$$\Rightarrow$$
 BQ =  $\frac{50}{\sqrt{3}}$  m....(i)

In right triangle ABQ,

$$\tan 30^{\circ} = \frac{AB}{BQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BQ}$$

$$\Rightarrow$$
 BQ =  $h\sqrt{3}$  m....(ii)

From eq. (i) and (ii),

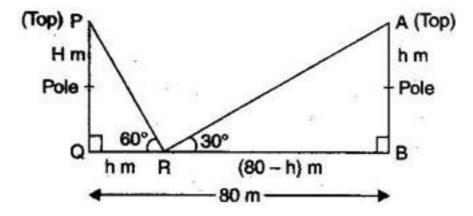
$$h\sqrt{3} = \frac{50}{\sqrt{3}} \implies h = \frac{50}{3} = 16\frac{2}{3} \text{ m}$$

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are  $60^{\circ}$  and  $30^{\circ}$  respectively. Find the height of the poles and the distances of the point from the poles.

Ans. Let the height of each poles be H m

$$AB = PQ = H$$

In right triangle PRQ,



(in figure change AB= H m rather than h m)

$$\tan 60^{\circ} = \frac{PQ}{QR} \Rightarrow \sqrt{3} = \frac{H}{h}$$

$$\Rightarrow$$
 H =  $h\sqrt{3}$  m....(i)

In right triangle ABR,

$$\tan 30^\circ = \frac{AB}{BR}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H}{80 - h}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h\sqrt{3}}{80 - h} [From eq. (i)]$$

$$\Rightarrow$$
 80 - h = 3h

$$\Rightarrow 4h = 80$$

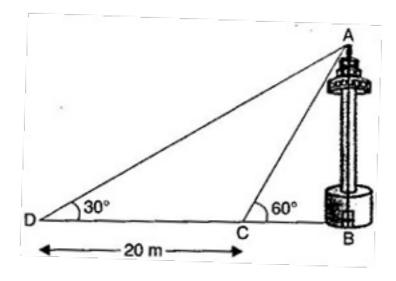
$$\Rightarrow h = 20 \text{ m}$$

∴ H = 
$$h\sqrt{3}$$
 = 20 $\sqrt{3}$  m

Also, BR = 
$$80 - h = 80 - 20 = 60 \text{ m}$$

Hence the heights of the poles are  $20\sqrt{3}$  m each and the distances of the point from poles are 20 m and 60 m respectively.

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is  $60^{\circ}$ . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^{\circ}$  (see figure). Find the height of the tower and the width of the canal.



Ans. Let AB be the TV tower.

In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{BC}$$

$$\Rightarrow$$
 AB = BC $\sqrt{3}$  m....(i)

In right triangle ABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$$

$$\Rightarrow$$
 AB =  $\frac{BC + 20}{\sqrt{3}}$  m....(ii)

From eq. (i) and (ii),

$$BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$$

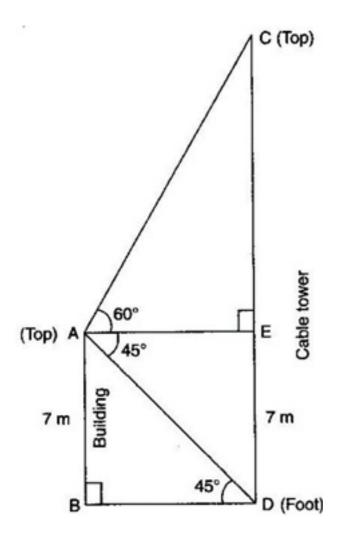
From eq. (i), AB =  $10\sqrt{3}$  m

Hence height of the tower is  $10\sqrt{3}$  m and the width of the canal is 10 m.

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is  $60^{\circ}$  and the angle of depression of its foot is  $45^{\circ}$ . Determine the height of the tower.

Ans. In right triangle ABD,

$$\tan 45^\circ = \frac{AB}{BD}$$



$$\Rightarrow 1 = \frac{7}{BD}$$

$$\Rightarrow$$
 BD = 7 m

$$\implies$$
 AE = 7 m

In right triangle AEC,

$$tan\,60^\circ = \frac{CE}{AE}$$

$$\Rightarrow \sqrt{3} = \frac{\text{CE}}{7}$$

$$\Rightarrow$$
 CE =  $7\sqrt{3}$  m

$$\therefore$$
 CD = CE + ED

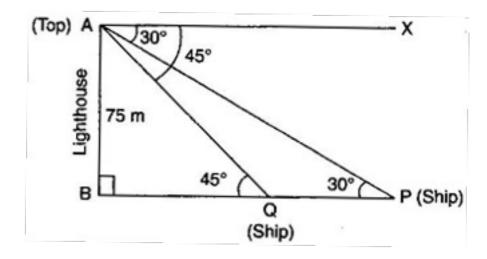
$$= CE + AB (As AB = ED)$$

$$= 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$$
 m

Hence height of the tower is  $7(\sqrt{3}+1)$  m.

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are  $30^{\circ}$  and  $45^{\circ}$ . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.

Ans. In right triangle ABQ,



$$\tan 45^\circ = \frac{AB}{BQ}$$

$$\Rightarrow 1 = \frac{75}{BO}$$

In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BQ + QP}$$

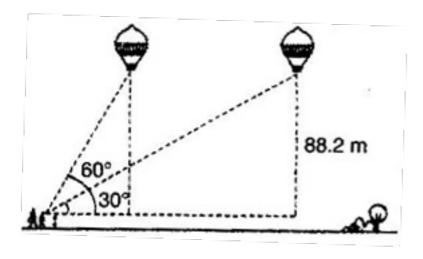
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{75 + QP} \text{ [From eq. (i)]}$$

$$\Rightarrow$$
 75 + QP =  $75\sqrt{3}$ 

$$\Rightarrow$$
 QP =  $75(\sqrt{3}-1)$  m

Hence the distance between the two ships is  $75(\sqrt{3}-1)$  m.

14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any distant is  $60^0$  After some time, the angle of elevation reduces to  $30^0$  (see figure). Find the distance travelled by the balloon during the interval.

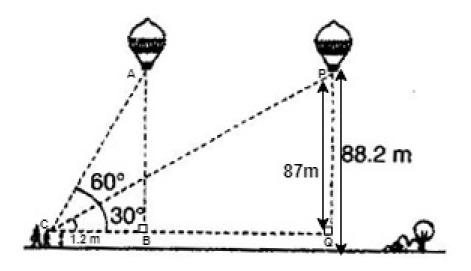


Ans. As, per question;

$$AB = PQ = 88.2 - 1.2 = 87 \text{ m}$$

In right triangle ABC,

$$\tan 60^{\circ} = \frac{AB}{BC}$$



$$\Rightarrow \sqrt{3} = rac{87}{BC}$$
  $\Rightarrow BC = rac{87}{\sqrt{3}} = 29\sqrt{3} \; ext{m}$ 

In right triangle PQC,

$$\tan 30^\circ = \frac{PQ}{CQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{29\sqrt{3} + BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{88.2}{\frac{88.2}{\sqrt{3}} + BQ}$$

$$\Rightarrow 29\sqrt{3} + BQ = 87\sqrt{3}$$

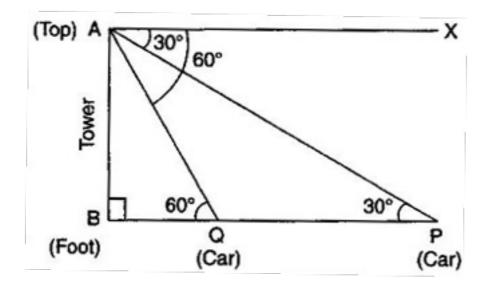
$$\Rightarrow BQ = 58\sqrt{3} \; \mathrm{m}$$

Hence the distance travelled by the balloon during the interval is  $58\sqrt{3}\,$  m.

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of  $30^{\circ}$ , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be  $60^{\circ}$ . Find the time taken by the car to reach the foot of the tower from this point.

Ans. In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BP}$$

$$\Rightarrow$$
 BP = AB  $\sqrt{3}$  .....(i)

In right triangle ABQ,

$$\tan 60^{\circ} = \frac{AB}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$$

$$\Rightarrow$$
 BQ =  $\frac{AB}{\sqrt{3}}$  .....(ii)

$$PQ = BP - BQ$$

$$\therefore PQ = AB \sqrt{3} - \frac{AB}{\sqrt{3}}$$

$$=\frac{3AB-AB}{\sqrt{3}}=\frac{2AB}{\sqrt{3}}=2BQ$$
 [From eq. (ii)]

$$\Rightarrow$$
 BQ =  $\frac{1}{2}$  PQ

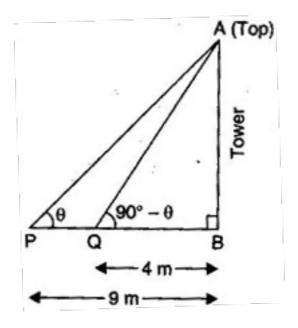
- Time taken by the car to travel a distance PQ = 6 seconds.
- Time taken by the car to travel a distance BQ, i.e.  $\frac{1}{2}$  PQ =  $\frac{1}{2}$  x 6 = 3 seconds.

Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

**Ans.** Let 
$$\angle$$
 APB =  $\theta$ 

Then, 
$$\angle AQB = (90^{\circ} - \theta)$$



[ / APB and / AQB are complementary]

In right triangle ABP,

$$\tan \theta = \frac{AB}{PB}$$

$$\Rightarrow \tan \theta = \frac{AB}{9}$$
 .....(i)

In right triangle ABQ,

$$\tan\left(90^\circ - \theta\right) = \frac{AB}{QB}$$

$$\Rightarrow$$
 cot  $\theta = \frac{AB}{4}$  .....(ii)

Multiplying eq. (i) and eq. (ii),

$$\frac{AB}{9} \cdot \frac{AB}{4} = \tan \theta \cdot \cot \theta$$

$$\Rightarrow \frac{AB^2}{36} = 1 \Rightarrow AB^2 = 36$$

$$\Rightarrow$$
 AB = 6 m

Hence, the height of the tower is 6 m.

Proved.