CBSE Class–10 Mathematics

NCERT solution

Chapter - 10

Circles - Exercise 10.1

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1. How many tangents can a circle have?

Ans. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

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2. Fill in the blanks:

(i) A tangent to a circle intersects it in _______________ point(s).
(ii) A line intersecting a circle in two points is called a _______________.

(iii) A circle can have _______________ parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called _________.

Ans. (i) A tangent to a circle intersects it in exactly one point.
(ii) A line intersecting a circle in two points is called a secant.
(iii) A circle can have two parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called point of contact.

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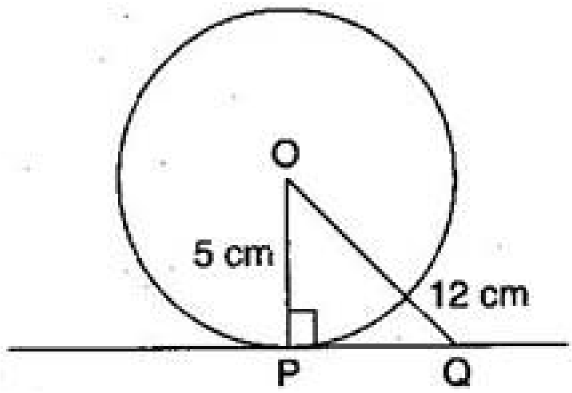
3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Ans. (D) $\because$ PQ is the tangent and OP is the radius through the point of contact.

$\therefore \angle OPQ = 90^\circ$ [The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ In right triangle OPQ,



$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$\Rightarrow (12)^2 = (5)^2 + PQ^2$

$\Rightarrow 144 = 25 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

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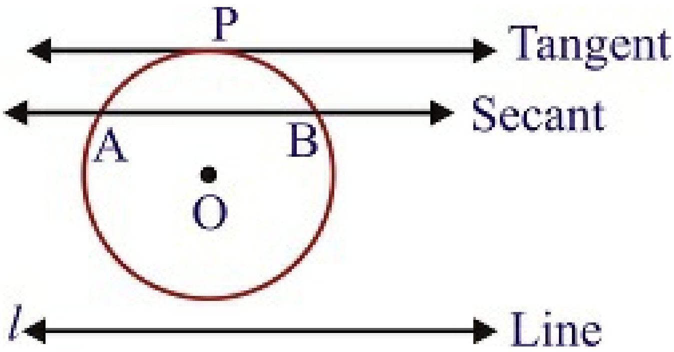
4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans.



CBSE Class–10 Mathematics

NCERT solution

Chapter - 10

Circles - Exercise 10.2

In Q 1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

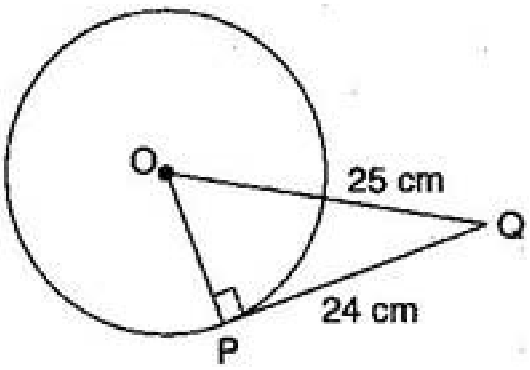
(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Ans. (A)

$\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]



$\therefore$ In right triangle OPQ,

$$OQ^2 = OP^2 + PQ^2$$

[By Pythagoras theorem]

$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

$\Rightarrow$ $OP^2 = 625 - 576 = 49$

$\Rightarrow$ OP = 7 cm

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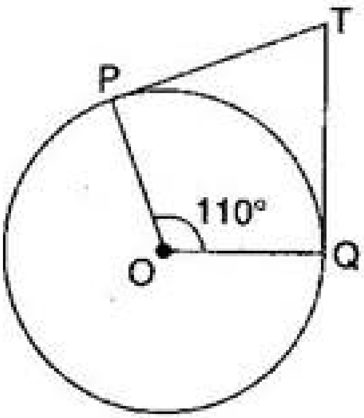
2. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$POQ = 110°, then $\angle$PTQ is equal to:



(A) 60° **(B)** 70° **(C)** 80° **(D)** 90°

Ans. (B)

$\angle$POQ = 110°, $\angle$OPT = 90° and $\angle$OQT = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In quadrilateral OPTQ,

$\angle$POQ + $\angle$OPT + $\angle$OQT + $\angle$PTQ = 360°

[Angle sum property of quadrilateral]

$\Rightarrow 110^\circ + 90^\circ + 90^\circ$ + $\angle$PTQ = 360°

$\Rightarrow$ 290° + $\angle$PTQ = 360°

$\Rightarrow$ $\angle$PTQ = $360^0 - 290^0$

$\Rightarrow$ $\angle$PTQ = 70°

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3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to:

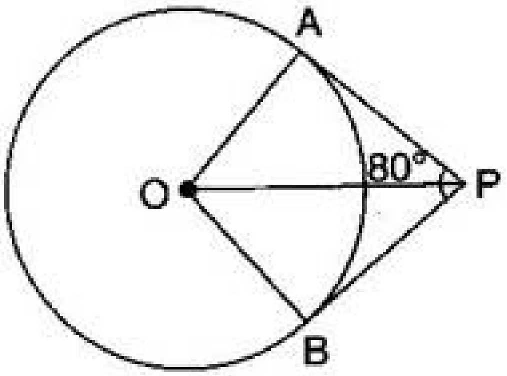
(A) 50° **(B)** 60° **(C)** 70° **(D)** 80°

Ans. (A)

$\because$ $\angle$ OAP = 90°

[The tangent at any point of a circle is $\perp$ to the radius

through the point of contact]



$\angle$ OPA $= \frac{1}{2} \angle$ BPA $= \frac{1}{2} \times 80^\circ = 40^\circ$

[Centre lies on the bisector of the

angle between the two tangents]

In $\triangle$ OPA,

$\angle$ OAP + $\angle$ OPA + $\angle$ POA = 180°

[Angle sum property of a triangle]

$\Rightarrow 90^\circ + 40^\circ$ + $\angle$ POA = 180°

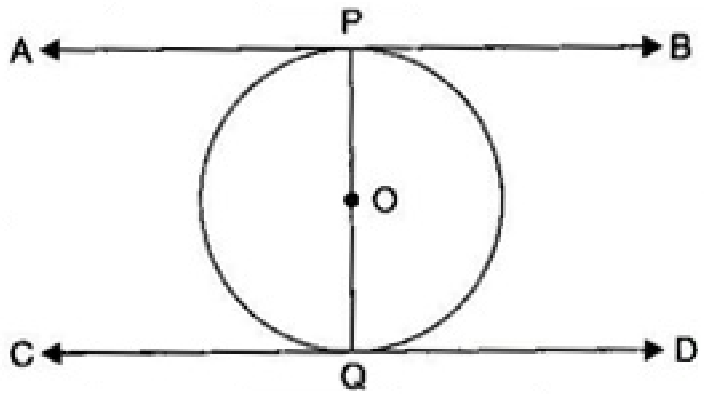
$\Rightarrow$ 130° + $\angle$ POA = 180°

$\Rightarrow$ $\angle$ POA = 50°

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans. Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.



To Prove: AB $\parallel$ CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle OPA = 90^\circ$ ....(i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle OQD = 90^\circ$ ....(ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

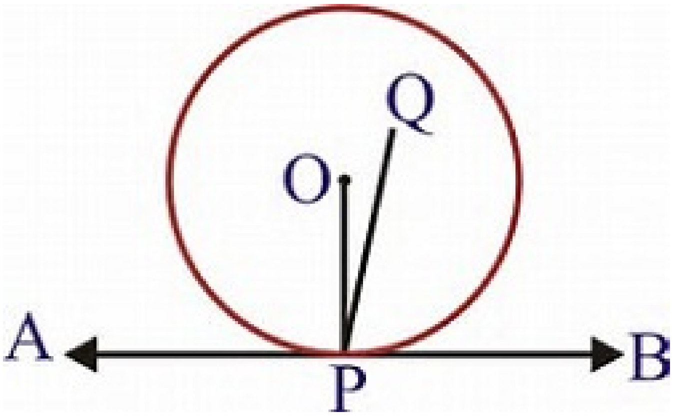
From eq. (i) and (ii), $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

$\therefore$ AB || CD

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans. Let AB be the tangent drawn at the point P on the circle with O.



If possible, let PQ be perpendicular to AB, not passing through O.

Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, $AB \perp OP$ $\Rightarrow$ $\angle OPB = 90^{o}$

Also, $\angle QPB = 90^{o}$ [By construction]

Therefore, $\angle QPB = \angle OPB$, which is not possible as a part cannot be equal to whole.

Thus, it contradicts our supposition.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

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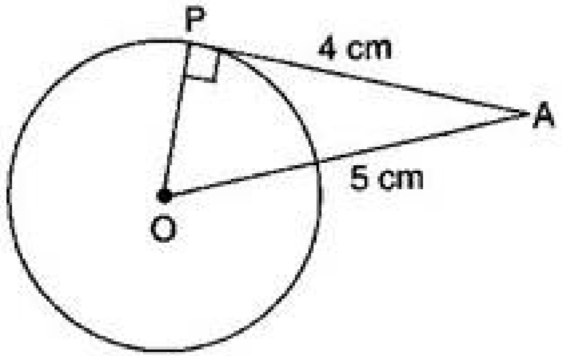
6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Ans. We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.



$\therefore \angle OPA = 90^\circ$

$\therefore OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (OP)^2 + (4)^2$

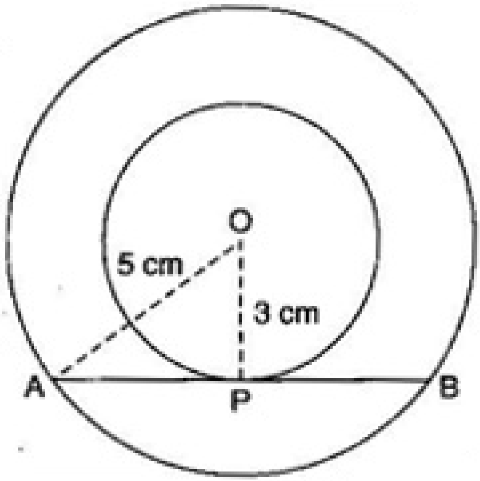
$\Rightarrow 25 = (OP)^2 + 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow$ OP = 3 cm

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + AP^2$

[By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = 4 cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

$\Rightarrow$ AB = AP + BP

= AP + AP = 2AP

= 2×4 = 8 cm

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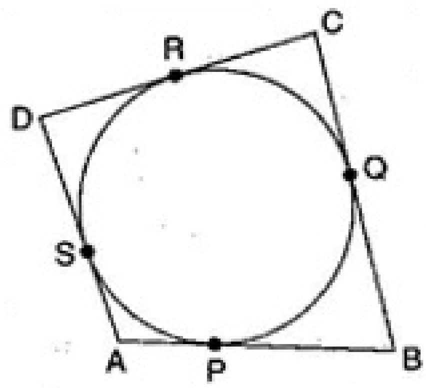
8. A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

AB + CD = AD + BC



Ans. We know that the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

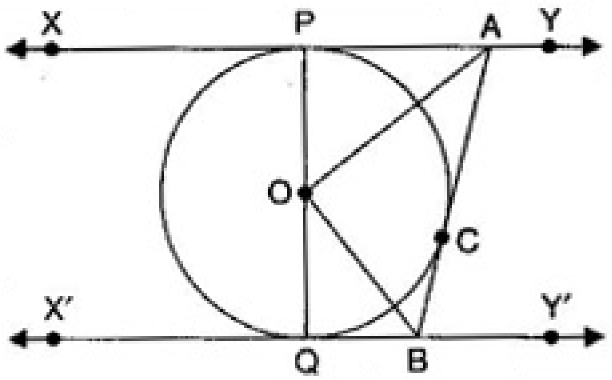
(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

9. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle$ AOB = 90°.

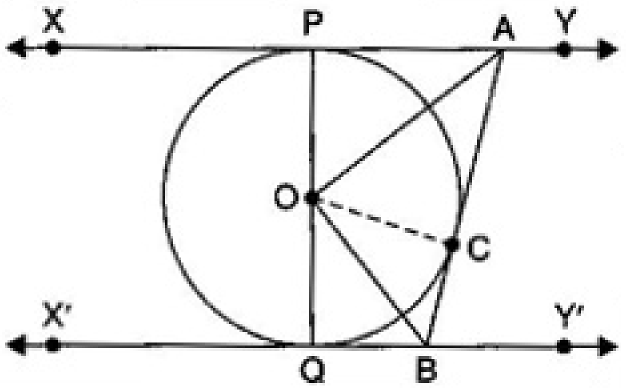


Ans. Given: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove: $\angle$ AOB = 90°

Construction: Join OC

Proof: $\angle$ OPA = 90°……….(i)



$\angle$ OCA = 90°……….(ii)

[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

In right angled triangles OPA and OCA,

$\angle$ OPA = $\angle$ OCA = 90^0

OA = OA [Common]

AP = AC [Tangents from an external

point to a circle are equal]

$\therefore \Delta OPA \cong \Delta OCA$

[RHS congruence criterion]

$\therefore \angle OAP = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \angle OAC = \frac{1}{2}\angle PAB$ ....(iii)

Similarly, $\angle OBQ = \angle OBC$

$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$ ....(iv)

$\because$ XY $\parallel$ X'Y' and a transversal AB intersects them.

$\therefore \angle PAB + \angle QBA = 180^\circ$

[Sum of the consecutive interior angles on the same side of the transversal is 180°]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA$

$= \frac{1}{2}\times 180^\circ$..........(v)

$\Rightarrow \angle OAC + \angle OBC = 90^\circ$

[From eq. (iii) & (iv)]

In ΔAOB,

$\angle OAC + \angle OBC + \angle AOB = 180^\circ$

[Angel sum property of a triangle]

$\Rightarrow 90^\circ + \angle AOB = 180^\circ$ [From eq. (v)]

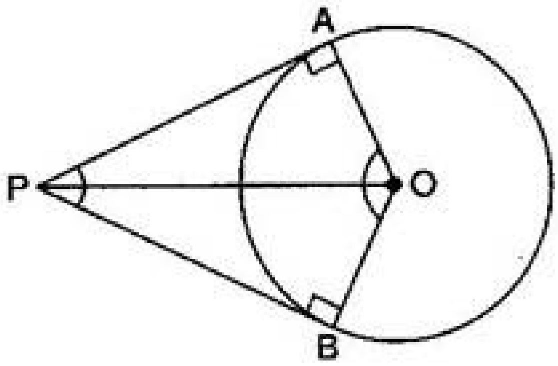
$\Rightarrow \angle AOB = 90^\circ$

Hence proved.

10. Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Ans. $\angle OAP = 90^\circ$ ....(i)

$\angle OBP = 90^\circ$ ....(ii)



[Tangent at any point of a circle is $\perp$ to

the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^\circ$

[Angle sum property of a quadrilateral]

$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$

[From eq. (i) & (ii)]

$\Rightarrow \angle APB + \angle AOB = 180^\circ$

$\therefore \angle APB$ and $\angle AOB$ are supplementary.

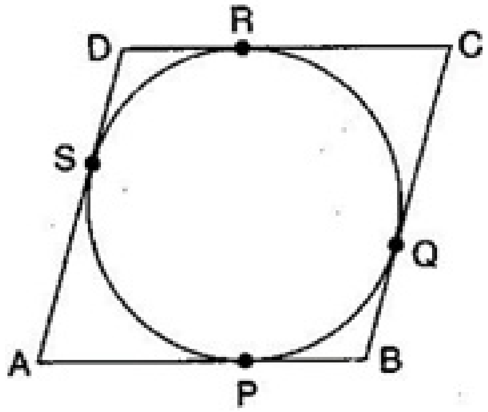
11. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans. Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ....(i)



BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD

[Opposite sides of $\|$gm are equal]

$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB = AD

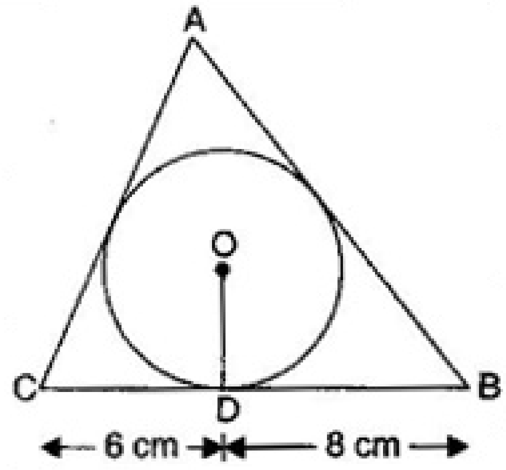
But AB = CD and AD = BC

[Opposite sides of || gm]

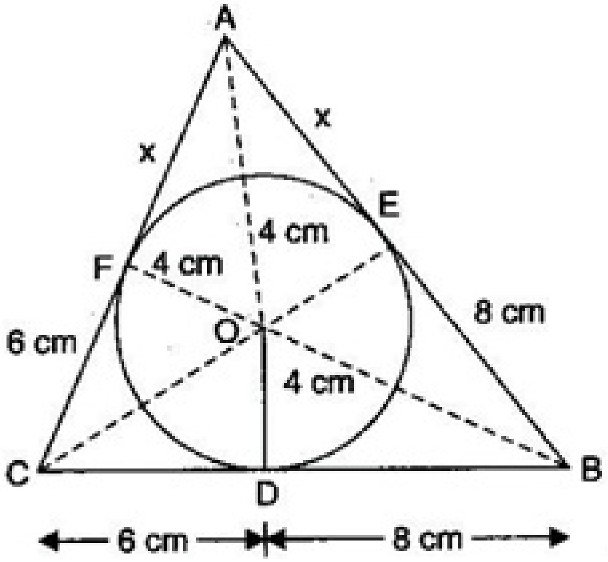
∴ AB = BC = CD = AD

∴ Parallelogram ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Ans. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

∴ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

∴ Semi-perimeter of ΔABC = $\frac{(x+6)+(x+8)+(6+8)}{2}$ = $(x+14)$ cm

∴ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

= $\sqrt{(x+14)(x)(6)(8)}$ cm^2

Now, Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $\frac{(6+8)4}{2}+\frac{(x+6)4}{2}+\frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

= $28+2x+12+2x+16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$ = $4x+56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$ = $4(x+14)$

Squaring both sides,

$$(x+14)(x)(6)(8)=16(x+14)^2$$

$$\Rightarrow 3x = x+14$$

$$\Rightarrow 2x = 14$$

$$\Rightarrow x = 7$$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

And AC = $x+6$ = 7 + 6 = 13 cm

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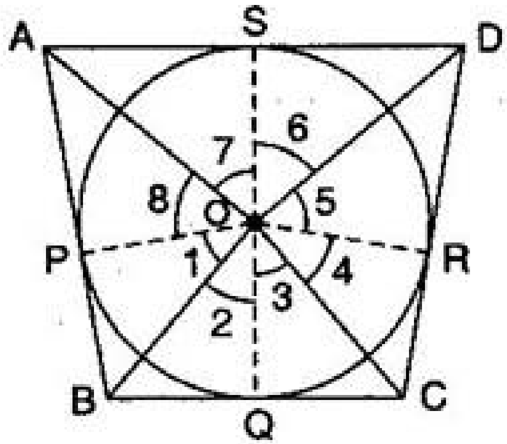
13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = 180° (ii) $\angle$ BOC + $\angle$ AOD = 180°

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ ....(i)

CQ = CR

DR = DS

In ΔOBP and ΔOBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore$ ΔOPB $\cong$ ΔOBQ [By SSS congruence criterion]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to 360°.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^\circ$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180^\circ$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180^\circ$

$\Rightarrow$ $\angle$AOB + $\angle$COD = 180°

Similarly, we can prove that

$\angle$BOC + $\angle$AOD = 180°