



# An Input Linearized Powertrain Model for the Optimal Control of Hybrid Electric Vehicles

**Thomas Steffen** Loughborough University

**Siyuan Zhan** Maynooth University

**James Knowles and Will midgley** Loughborough University

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## Abstract

Models of hybrid powertrains are used to establish the best combination of conventional engine power and electric motor power for the current driving situation. The model is characteristic for having two control inputs and one output constraint: the total torque should be equal to the torque requested by the driver. To eliminate the constraint, several alternative formulations are used, considering engine power or motor power or even the ratio between them as a single control input. From this input and the constraint, both power levels can be deduced. There are different popular choices for this one control input.

This paper presents a novel model based on an input linearizing transformation. It is demonstrably superior to alternative model forms, in that the core dynamics of the model (battery state of energy) are linear, and the non-linearities of the model are pushed into the inputs and outputs in a

Wiener/Hammerstein form. The output non-linearities can be approximated using a quadratic model, which creates a problem in the linear-quadratic framework. This facilitates the direct application of linear control approaches such as LQR control, predictive control, or Model Predictive Control (MPC).

The paper demonstrates the approach using the ELectrified Vehicle library for sImulation and Optimization (ELVIO). It is an open-source MATLAB/Simulink library designed for the quick and easy simulation and optimization of different powertrain and drivetrain architectures. It follows a modelling methodology that combines backward-facing and forward-facing signal path, which means that no driver model is required. The results show that the approximated solution provides a performance that is very close to the solution of the original problem except for extreme parts of the operating range (in which case the solution tends to be driven by constraints anyway).

## Key words

open-source library, Simulink, backward facing simulation, optimal control of hybrid powertrains, powertrain efficiency

modelling, vehicle simulation.

## Introduction

Transport consumes a significant amount of our overall energy demand, and the automotive industry is under increasing pressure to reduce the fuel consumption of vehicles. One very promising approach is the use of hybrid powertrains, which combine an internal combustion engine with one or more electric machines to create a more efficient propulsion system

The optimal control of hybrid powertrains is a very popular control problem, with many thousands of papers written to propose solutions. It captures the imagination of many control engineers, because it is a slightly unusual control problem in object of everyday significance. The problem can be approached in many different ways including both formal approaches and engineering intuition.

This paper considers the control of a series hybrid. This is essentially a pure battery electric vehicle, where an engine is added to generate electricity that can contribute to the electrical power and energy of the battery. This is one of the simplest hybrid control problems, although the same principle applies to all hybrid architectures: the goal is to combine the two sources of energy in the most effective way, reducing overall fuel and electricity consumption, while still satisfying the power or torque request from the driver.

The key contribution of this paper is an input linearizing transformation, that turns this problem of finding two inputs subject to a constraint into a single control input. This transformation differs from existing approaches by using a linearizing input. This input is not a physical variable, but it does determine all physical variables. The result is a linear dynamic

demand, and it is not possible to create a generic optimal solution with it.

The second limitation is that this approach does not deal with variable limits, and especially not with start/stop of the generator engine. This is a serious limitation, because start/stop can significantly improve the economy of the vehicle. Further work in this direction is necessary, and the presented optimization problem is simple enough to solve to combine for example with a collocation-based strategy for optimizing engine on/off periods.

Applications of the same methods to other hybrid powertrain architectures or other hybrid systems is possible, as long as they can fit into the basic structure introduced in [Figure 2](#). For parallel hybrids and powersplit architectures this is obvious, but the same structure can also be used for optimal charging of electric vehicles using a time variable tariff.

## Conclusion and Outlook

As demonstrated, the linearizing input transformation of a hybrid powertrain model makes the problem much easier to solve. Using based linear quadratic optimal control and via simple algebraic insertion it is possible to find an optimal control law with a fixed end state or control of the battery energy level. The former has been demonstrated in simulation. The control law is much simpler than competing prediction optimization approaches based on MPC, dynamic programming or PMP.

Further work is needed to look at the impact of range limits and state transitions, specifically the engine on/off choice used by start/stop systems. Turning the engine off when not needed is essential for reaching the best fuel economy, so this will make a big difference to the practical performance. The control law presented here is simple enough to be used together with an optimization scheme for finding the best periods for turning the engine off. This is subject to future research.

All the tools developed in this paper are published as part of the ELVIO library, and this makes it easy to replicate or extend the results presented. The library is available on GitHub [\[24\]](#), and contributions are welcome.

## References

- Gao, D.W., Mi, C., and Emadi, A., "Modeling and Simulation of Electric and Hybrid Vehicles," *Proceedings of the IEEE* 95, no. 4 (2007): 729-745, doi:10.1109/JPROC.2006.890127.
- Sciarretta, A., Serrao, L., Dewangan, P.C., Tona, P., Bergshoeff, E.N.D., Bordons, C., Charmpa, L., Elbert, P., Eriksson, L., Hofman, T., and others, "A Control Benchmark on the Energy Management of a Plug-In Hybrid Electric Vehicle," *Control Engineering Practice* 29: 287-298, 2014.
- Rousseau, A., Pagerit, S., and Gao, D.W., "Plug-In Hybrid Electric Vehicle Control Strategy Parameter Optimization," *Journal of Asian Electric Vehicles* 6, no. 2 (2008): 1125-1133.
- Zhang, C. and Vahidi, A., "Route Preview in Energy Management of Plug-In Hybrid Vehicles," *IEEE Transactions on Control Systems Technology* 20, no. 2 (2011): 546-553.
- Sun, C., Hu, X., Moura, S.J., and Sun, F., "Velocity Predictors for Predictive Energy Management in Hybrid Electric Vehicles," *IEEE Transactions on Control Systems Technology* 23, no. 3 (2014): 1197-1204.
- Kumar, V., Zhu, D., and Dadam, S.R., "Intelligent Auxiliary Battery Control - A Connected Approach," SAE Technical Paper 2021-01-1248 (2021), doi:10.4271/2021-01-1248.
- Zhu, D., Pritchard, E., Dadam, S., Kumar, V. et al., "Optimization of Rule-Based Energy Management Strategies for Hybrid Vehicles Using Dynamic Programming," *Combustion Engines* 184, no. 1: 2021, doi:10.19206/ce-131967.
- Tribioli, L., Barbieri, M., Capata, R., Sciubba, E. et al., "A Real Time Energy Management Strategy for Plug-In Hybrid Electric Vehicles Based on Optimal Control Theory," *Energy Procedia* 45, no. 2014 (2014): 949-958.
- Larsson, V., Johannesson, L., Egardt, B., and Larsson, A., "Benefit of Route Recognition in Energy Management of Plug-In Hybrid Electric Vehicles," in *2012 American Control Conference (ACC)*, 1314-1320, 2012.
- Martinez, C.M., Hu, X., Cao, D., Velenis, E. et al., "Energy Management in Plug-In Hybrid Electric Vehicles: Recent Progress and a Connected Vehicles Perspective," *IEEE Transactions on Vehicular Technology* 66, no. 6 (2016): 4534-4549, doi:10.1109/TVT.2016.2582721.
- Bertsekas, D.P., *Dynamic Programming and Optimal Control*, 3rd ed. Vol. II (Belmont, MA: Athena Scientific, 2011)
- Sun, C., Moura, S.J., Hu, X., Hedrick, J.K. et al., "Dynamic Traffic Feedback Data Enabled Energy Management in Plug-In Hybrid Electric Vehicles," *IEEE Transactions on Control Systems Technology* 23, no. 3 (2014): 1075-1086.
- Karbowsky, D., Rousseau, A., Pagerit, S., and Sharer, P., "Plug-In Vehicle Control Strategy: From Global Optimization to Real Time Application," in *22th International Electric Vehicle Symposium (EVS22)*, 1-12, 2006.
- Technology, G., "Gamma Technologies | Engine and Vehicle Simulation," <https://www.gtisoft.com/>, Aug. 2018.
- Ricardo, "WAVE - Global Engineering, Environmental and Strategic Consultancy," <https://software.ricardo.com/products/wave>, Aug. 2018.
- AVL, "AVL CRUISE™ - avl.com," <https://www.avl.com/cruise>, Aug. 2018.
- Siemens, "Hybrid and Electric Vehicle: Siemens PLM Software," <https://www.plm.automation.siemens.com/fr/products/lms/imagine-lab/automotive/vehicle-integration/hybrid-electric-vehicles.shtml>, Aug. 2018.
- Mathworks, "Simscape Driveline - MATLAB & Simulink," <https://uk.mathworks.com/products/simdrive.html>, Aug. 2018.
- Systèmes, Dassault, "Dymola," <https://www.3ds.com/products-services/catia/products/dymola/>, Aug. 2018.
- CMU, "Vehicle Simulation Software - DDL Wiki," [https://wiki.ece.cmu.edu/ddl/index.php/Vehicle\\_simulation\\_software](https://wiki.ece.cmu.edu/ddl/index.php/Vehicle_simulation_software), Aug. 2018.

21. Zhu, D., Pritchard, E.G.D., and Silverberg, L.M., "A New System Development Framework Driven by a Model-Based Testing Approach Bridged by Information Flow," *IEEE Systems Journal* 12, no. 3 (2018), doi:[10.1109/JSYST.2016.2631142](https://doi.org/10.1109/JSYST.2016.2631142).
22. Teratani, T., Kuramochi, K., Nakao, H., Tachibana, T., Yagi, K., and Abou, S., "Development of Toyota Mild Hybrid System (THS-M) with 42V PowerNet," in *IEEE International Electric Machines and Drives Conference, 2003. IEMDC'03*, IEEE, ISBN 0-7803-7817-2, doi:[10.1109/IEMDC.2003.1211235](https://doi.org/10.1109/IEMDC.2003.1211235).
23. Styler, A., Sauer, A., Nourbakhsh, I., and Rottengruber, H., "Learned Optimal Control of a Range Extender in a Series Hybrid Vehicle," 2015 in *IEEE 18th International Conference on Intelligent Transportation Systems*, IEEE, ISBN 978-1-4673-6596-3, 2015, doi:[10.1109/ITSC.2015.420](https://doi.org/10.1109/ITSC.2015.420).
24. Steffen, T., "Electrified Vehicle Library for Simulation and Optimisation," <https://github.com/tsteffenlboro/elvio>, 2017.
25. Paganelli, G., Ercole, G., Brahma, A., Guezennec, Y. et al., "General Supervisory Control Policy for the Energy Optimization of Charge-Sustaining Hybrid Electric Vehicles," *JSAE Review* 22, no. 4 (2001): 511-518.
26. Paganelli, G., Delprat, S., Guerra, T.M., Rimaux, J., and Santin, J.J., "Equivalent Consumption Minimization Strategy for Parallel Hybrid Powertrains," in *Vehicular Technology Conference. IEEE 55th Vehicular Technology Conference. VTC Spring 2002* (Cat. No.02CH37367), IEEE, ISBN 0-7803-7484-3, doi:[10.1109/VTC.2002.1002989](https://doi.org/10.1109/VTC.2002.1002989).
27. US Department of Energy, "2011 Nissan Leaf - VIN 0356: Advanced Vehicle Testing - Beginning-of-Test Battery Testing Results," 2011.

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## Contact Information

The ELVIO library is available as open source on GitHub [24]. If you have any comments, recommendations, or contributions, please contact the first and correspondence author at [t.steffen@lboro.ac.uk](mailto:t.steffen@lboro.ac.uk) or [t.steffen@ieee.org](mailto:t.steffen@ieee.org). See <http://www.lboro.ac.uk/departments/aae/staff/thomas-steffen/> for more details.