AUTONOMOUS AGENTS ASSIGNMENT 2

Single Agent Learning

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October 4, 2013

Abstract

The aim of this assignment is to focus on the research of a learning scenario in which the agent does not know the transition probabilities, nor the reward structure. For completing that, we have implemented and tested various methods, namely Q-learning, On-policy Monte Carlo Control, Off-policy Monte Carlo Control and Sarsa in order to explain their theoretical differences, to compare their results and to conclude which one is the optimal high-reward policy which yields from the fastest moves of the predator in order to catch the prey. (Conclusion)

1 INTRODUCTION

In this research, we have considered that the model is unknown and, in this situation, two approaches can be pursued. A model-based approach tries to learn the model explicitly and then use methods like Dynamic Programming to compute the optimal policy with respect to the estimate of the model. On the other hand, a model-free approach concentrates on learning the state value function (Q-value function) directly and obtaining the optimal policy from this estimates.
We shall try to focus on model-free methods for learning in MDPs by making use of:
• Q-learning with ϵ-greedy action selection;

- Q-learning with Softmax action selection;
- On-policy Monte-Carlo Control;
- Off-policy Monte-Carlo Control;
- Sarsa.

In this assignment, we have explored the learning methods for estimating value functions and discovering optimal policies.
Monte Carlo methods (MC methods) require only experience through sample sequences of states, actions and rewards from on-line or simulated interaction with an environment. They do not require prior knowledge of the environment's dynamics, yet can still achieve an optimal behavior. Concerning the disadvantages of these methods, Monte Carlo method is conceptually simple and do not require any model, but is not suited for step-by-step incremental computation.
Temporal-Difference Learning (TD Learning) represents a combination of Monte Carlo and Dynamic Programming ideas. Like MC methods, TD methods do not require a model, while like DP, TD methods bootstrap; they update estimates based on other learned estimates, without waiting for a final outcome like MC:

$$V(s_t) \leftarrow V(s_t) + \alpha[R_t - V(s_t)] \qquad \text{MC: update target is } R_t \tag{1.1}$$

$$V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \qquad \text{TD: update target is } r_{t+1} + \gamma V_t(s_{t+1}) \tag{1.2}$$

During this research, the main focus is to understand better the idea of each model-free method and to highlight some demands like: What are the differences, both theoretical and practical, between all the implemented methods? Which method converges to the most optimal policy faster?

In order to do that, we have structured the report in 4 sections comprising: section 2, where we explain the theoretical background of each method and learn how to implement the pseudo-code; in section 3 we present some of the theoretical differences between the methods and we show the experiments through results and graphics, afterwards in section 4 we discuss about the brief conclusion of this assignment.

1.1 Environment: Should we keep this? Maybe the 1st paragraph?

The environment is a 11x11 grid which is toroidal and each state is encoded as the positions of the two agents, predator and prey. The simulator is defined through a while-loop of a hundred runs where the initial position of the predator is (0,0) and that of the prey (5,5). In each transition, the predator will make a move first and then the prey, according to their respective policies. Each run is composed of an episode, where the goal of the predator is to capture the prey, which can be interpreted as the end state of this particular episode.

Both the predator and the prey are initialized with a random policy in the beginning where for the predator it is equiprobable to choose any of the possible actions in any state. The prey has a fixed policy and hence it can be modelled as part of the environment. It has 0.8 probability of waiting and 0.2 probability of moving to any of the adjacent squares, unless this square is occupied, thereby changing the probability distribution.

2 Methods and Procedures

2.1 Action selection

ϵ-greedy action selection

Using ϵ-greedy method underlines the idea of the agent choosing the action that it believes to have the best long-term effect with probability $1-\epsilon$, and it chooses an action uniformly at random. The action is selected independently of the action-value estimates and the highest estimated reward is called the greediest action. This method ensures that if enough trials are done, each action will be tried an infinite number of times, thus ensuring optimal actions are discovered.

Softmax action selection

Considering the Softmax Action Selection, we have chosen an action a on the t-th play with the probability that depends on the value of Q and τ, which represents the temperature and is defined as a positive parameter. Therefore, we can assign the distribution in the following way:

$$P(a) = \frac{e^{Q_t(a)/\tau}}{\sum_{b=1}^{n} e^{Q_t(b)/\tau}} \tag{2.1}$$

It is important to note that high temperatures cause the actions to be all (nearly) equiprobable, whereas low temperatures cause a greater difference in selection probability for actions that differ in their value estimates. In the limit as $\tau \to 0$, softmax action selection becomes the same as greedy action selection.
A random action is selected with regards to the weight associated with each action, meaning the worst actions are unlikely to be chosen. This is a good approach to take where the worst actions are very unfavourable.

2.2 On-Policy Monte Carlo

For on-policy Monte Carlo method we have considered the ϵ-greedy policies, meaning that most of the time the chosen action has the maximal estimated action value, but while considering the probability ϵ they instead select an action at random. The on-policy method attempt to estimate the action-value function with respect to the current policy.

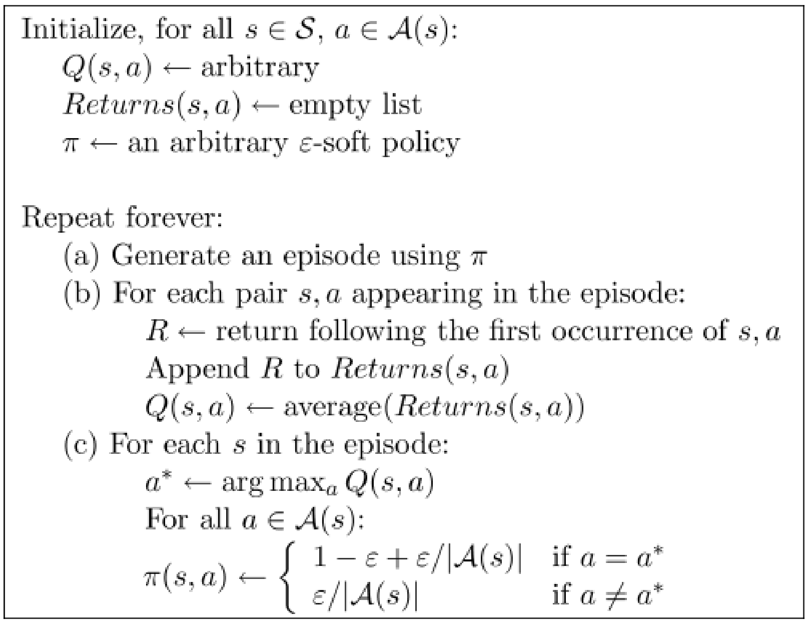
On-Policy Evaluation

On-policy evaluates and improves the policy that is already used by the agent in order to decide which action to select at a specific state. The policy that is generally used in on-policy control methods is a soft one, where the probability is $\pi(s,a) > 0$ for all states and possible actions. The on-policy method implemented in this assignment uses an ϵ-greedy policy, which is an example of an ϵ-soft policy, and this translates into that most of the times the greedy action will be selected, except of some small random probability $\epsilon > 0$ to select a different random action.

On-policy Monte Carlo control behaves like a Generalized Policy Iteration (GPI). The evaluation step is composed of generating random episodes, estimating the action-value function in each one, and then averaging over all of them. First-visit Monte Carlo methods are used in order to estimate the action-value function for each episode derived from the current policy. The returns are computed using the equation 2.2, where t is the state from where we begin to calculate, k is the number of the subsequent states, r is the immediate reward and γ is the discount factor.

$$R_t = \sum_{k=0}^{K} \gamma^k r_{t+k+1} \tag{2.2}$$

After the processing of each episode, the improvement of the policy occurs by getting the action that produces the highest reward in a state, and updates the probabilities in the ϵ-soft policy according to that action, which becomes the new greedy action. The pseudo code is illustrated in Figure 2.1.



```
Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
    $Q(s,a) \leftarrow$ arbitrary
    $Returns(s,a) \leftarrow$ empty list
    $\pi \leftarrow$ an arbitrary $\varepsilon$-soft policy

Repeat forever:
    (a) Generate an episode using $\pi$
    (b) For each pair $s,a$ appearing in the episode:
            $R \leftarrow$ return following the first occurrence of $s,a$
            Append $R$ to $Returns(s,a)$
            $Q(s,a) \leftarrow$ average($Returns(s,a)$)
    (c) For each $s$ in the episode:
            $a^* \leftarrow \arg\max_a Q(s,a)$
            For all $a \in \mathcal{A}(s)$:
            $\pi(s,a) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(s)| & \text{if } a = a^* \\ \varepsilon/|\mathcal{A}(s)| & \text{if } a \neq a^* \end{cases}$
```

Figure 2.1: On-policy Monte Carlo Control pseudo-code

2.3 OFF-POLICY MONTE CARLO

The off-policy Monte Carlo control method follows the behavior policy while learning and improving the estimation policy. This method requires that the behavior policy should have a non-zero probability of selecting all actions that might be selected by the estimation policy. Separating these two functions, namely behavior policy and estimation policy, represents an advantage in establishing that the behavior policy continues to sample all possible actions, while the estimation policy may be deterministic.

Off-Policy Evaluation

In many cases, V^{π} or Q^{π} can be estimated from episodes generated by policy π' as long as all taken actions under π are also taken under π'. Therefore, it is required that $\pi(s,a) > 0$ implies that $\pi'(s,a) > 0$, thus π' must be stochastic. π is *a target policy* because the target of the learning process is to estimate its value function, and π' is *a behavior policy* because it controls the agent and hence generates the behavior.

To compute the return by following the first-visit state s in an i-th episode, the observed return should be weighted by the relative probability of the complete sequence of the episode occurred under π called $p_i(s)$ and also under π' which is $p'_i(s)$. Therefore, the Monte Carlo estimation after observing n_s returned from state s is given by:

$$V(s) = \frac{\sum_{i=1}^{n_s} \frac{p_i(s)}{p'(s)} R_i(s)}{\sum_{i=1}^{n_s} \frac{p_i(s)}{p'(s)}} \tag{2.3}$$

Then, the relative probability $p_i(s)/p'_i(s)$ is obtained from:

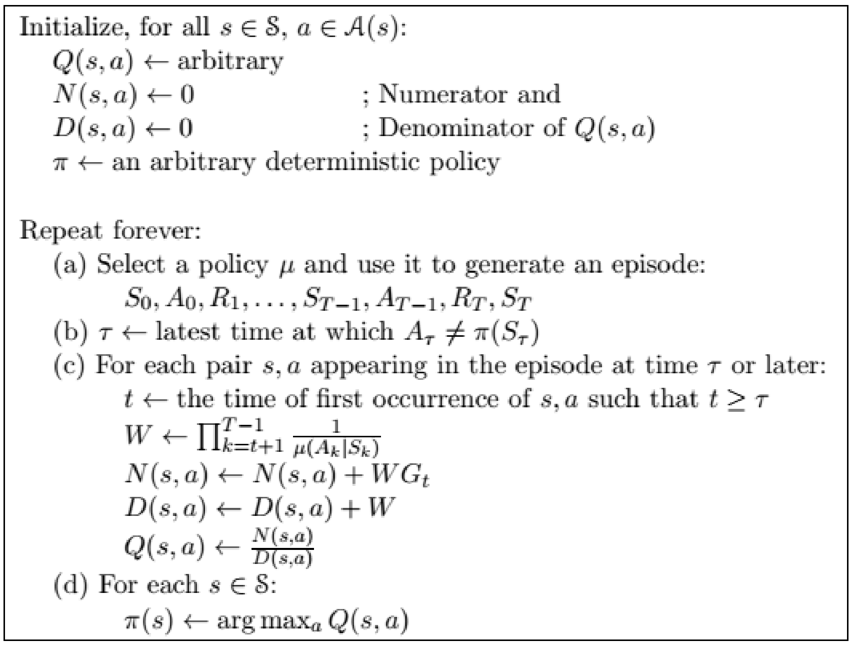
$$\frac{p_i(s_t)}{p'_i(s_t)} = \frac{\prod_{k=t}^{T_i(s)-1} \pi(s_k, a_k) P_{s_k s_{k+1}}}{\prod_{k=t}^{T_i(s)-1} \pi'(s_k, a_k) P_{s_k s_{k+1}}} \tag{2.4}$$

$$= \prod_{k=t}^{T_i(s)-1} \frac{\pi(s_k, a_k)}{\pi'(s_k, a_k)} \tag{2.5}$$

where $T_i(s)$ is the termination time of ith episode involving s, and t is the time of first occurrence of state s in the episode. The transition probability in (2.4) is unknown, but it can be eliminated since it belongs to the same environment. Thus, the weight for the return depends only on the two policies.

Off-Policy Monte Carlo Control

The Off-Policy Monte Carlo Control uses off-policy evaluation as mentioned before to estimate the action value function, but it improves the *target policy* instead of the *behavior policy*. An advantage of this method is that the *target policy* may be deterministic whereas the *behavior policy* remains stochastic in order to keep the chance of exploring all possible actions. The pseudo-code of the whole process of Off-Policy Monte Carlo Control is given by Figure 2.2:



```
Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
    $Q(s,a) \leftarrow$ arbitrary
    $N(s,a) \leftarrow 0$                ; Numerator and
    $D(s,a) \leftarrow 0$                ; Denominator of $Q(s,a)$
    $\pi \leftarrow$ an arbitrary deterministic policy

Repeat forever:
    (a) Select a policy $\mu$ and use it to generate an episode:
            $S_0, A_0, R_1, \dots, S_{T-1}, A_{T-1}, R_T, S_T$
    (b) $\tau \leftarrow$ latest time at which $A_\tau \neq \pi(S_\tau)$
    (c) For each pair $s, a$ appearing in the episode at time $\tau$ or later:
            $t \leftarrow$ the time of first occurrence of $s, a$ such that $t \geq \tau$
            $W \leftarrow \prod_{k=t+1}^{T-1} \frac{1}{\mu(A_k|S_k)}$
            $N(s,a) \leftarrow N(s,a) + WG_t$
            $D(s,a) \leftarrow D(s,a) + W$
            $Q(s,a) \leftarrow \frac{N(s,a)}{D(s,a)}$
    (d) For each $s \in \mathcal{S}$:
            $\pi(s) \leftarrow \arg\max_a Q(s,a)$
```

Figure 2.2: Off-Policy Monte Carlo Control Pseudo-code

2.4 Sarsa

Sarsa is an on-policy method based on model-free action policy estimation. It determines the optimal policy while controlling the MDP with respect to the algorithm which behaves according to the same policy that has been improved.

Sarsa On-Policy TD Control

Sarsa is an on-policy method of TD learning, whose name (State-Action-Reward-State-Action) is derived from the quintuple $Q(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$, where $\mathbf{s_t}, \mathbf{a_t}$ are the original state and action, $\mathbf{r_{t+1}}$ is the reward observed in the following state and $\mathbf{s_{t+1}}, \mathbf{a_{t+1}}$ are the new state-action pair. It expresses the transition from one state-action pair to the next and its update rule is as follow:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \tag{2.6}$$

The algorithm for Sarsa is presented in Figure 2.3 below:

```
Initialize $Q(s, a)$ arbitrarily
Repeat (for each episode):
    Initialize $s$
    Choose $a$ from $s$ using policy derived from $Q$ (e.g., $\varepsilon$-greedy)
    Repeat (for each step of episode):
        Take action $a$, observe $r$, $s'$
        Choose $a'$ from $s'$ using policy derived from $Q$ (e.g., $\varepsilon$-greedy)
        $Q(s, a) \leftarrow Q(s, a) + \alpha\big[r + \gamma Q(s', a') - Q(s, a)\big]$
        $s \leftarrow s'$; $a \leftarrow a'$;
    until $s$ is terminal
```

Figure 2.3: Sarsa Pseudo-code

As in all on-policy methods, we continually estimate Q^{π} for the behavior policy π, and at the same time change π toward greediness with respect to Q^{π}. The update is done after every transition from a nonterminal state s. If s' is terminal, then $Q(s', a')$ will be defined as zero. Regarding both Q-learning and Sarsa methods, the major difference between them is that the maximum reward for the next state is not necessarily used for updating the Q-values.

2.5 **Q-LEARNING**

Q-learning is a model-free method which works by learning an action-value function that ultimately gives the expected utility of taking a given action in a given state and following the optimal policy thereafter. The optimal policy can be constructed by selecting the action with the highest value in each state.

Q-LEARNING OFF-POLICY TD CONTROL

Q-learning is an off-policy TD control algorithm. It directly approximates the optimal policy Q^* independent from the policy being followed. Its update rule is as follows:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)] \tag{2.7}$$

The influence of the policy is still in determining which state-action pairs are visited and updated. However, as long as all pairs continue to be updated, which is the minimal requirement to find optimal behavior in the general case, correct convergence shall be reached. Convergence to Q^* with a probability of 1, requires stochastic approximation conditions on the sequence of step-size parameters, i.e. discounted ϵ, α. The algorithm is presented in Figure 2.4 below:

```
Initialize $Q(s,a)$ arbitrarily
Repeat (for each episode):
    Initialize $s$
    Repeat (for each step of episode):
        Choose $a$ from $s$ using policy derived from $Q$ (e.g., $\varepsilon$-greedy)
        Take action $a$, observe $r$, $s'$
        $Q(s,a) \leftarrow Q(s,a) + \alpha\big[r + \gamma \max_{a'} Q(s',a') - Q(s,a)\big]$
        $s \leftarrow s'$;
    until $s$ is terminal
```

Figure 2.4: Q-learning: An off-policy TD control algorithm.

3 Experiments and Results

3.1 Theoretical Differences

As we have seen from the previous section, the Sarsa' s learning function depends on the action taken, therefore on the agent's decision policy, essentially on the exploration factor. On the other hand, Q learning does not care about what is the next action that was taken, but rather about what is the optimal action that could have been taken. Sarsa is useful when you want to optimize the value of an agent that is exploring. If you want to do off-line learning, and then use that policy in an agent that does not explore, Q-learning may be more appropriate. The differences that appear while considering the off-policy Monte Carlo and on-policy Monte Carlo methods are related to the theoretical contrast between an off-policy and an on-policy. In the on-policy method, the agent commits to always exploring and tries to find the best policy that still explores. In off-policy method, the agent also explores, but learns a deterministic optimal policy that may be unrelated to the policy followed.If the on-policy method is generally soft (e.g. ϵ-soft policy) and improves the policy that is used to make decision, then the off-policy method is composed of two separated functions (behavior policy and target policy) which can be considered an advantage, because the target policy may be deterministic (e.g. greedy), while the behavior policy can continue to sample all possible actions.

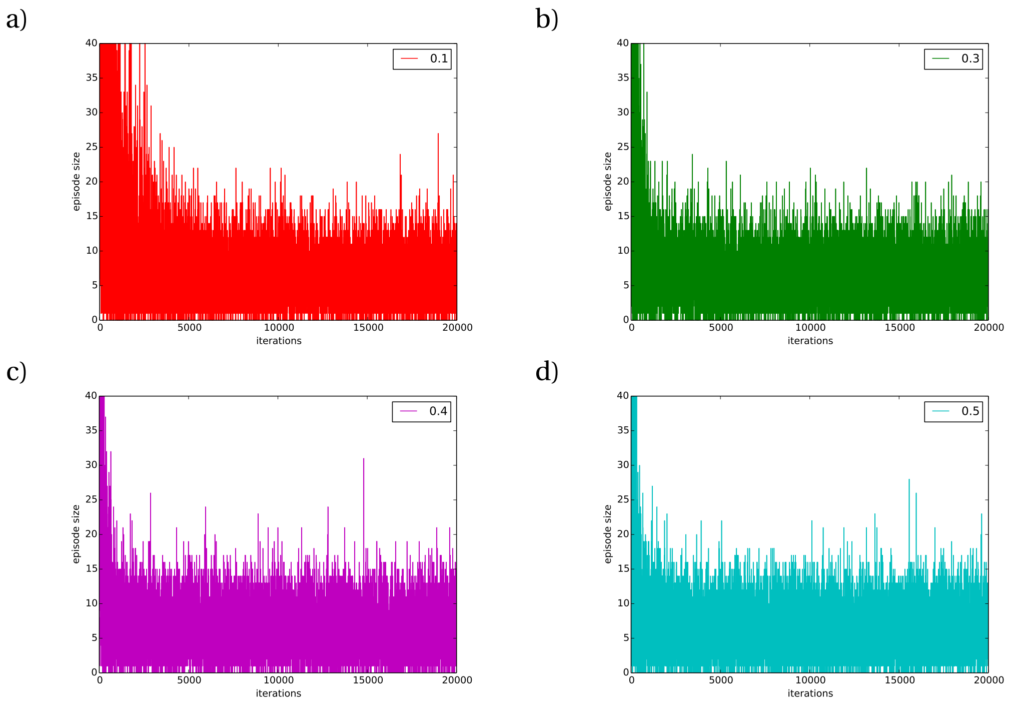


Table 3.1: Convergence for different values of α

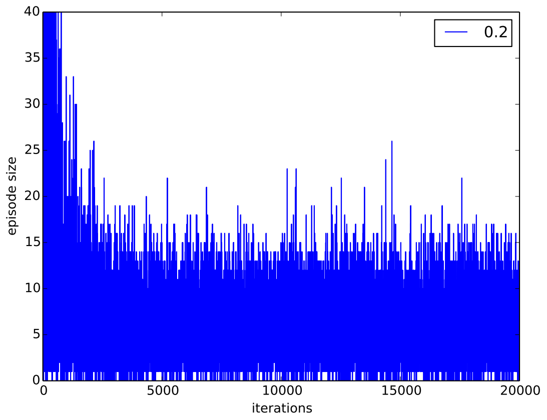


Figure 3.1: Optimal convergence for $\alpha = 0.2$

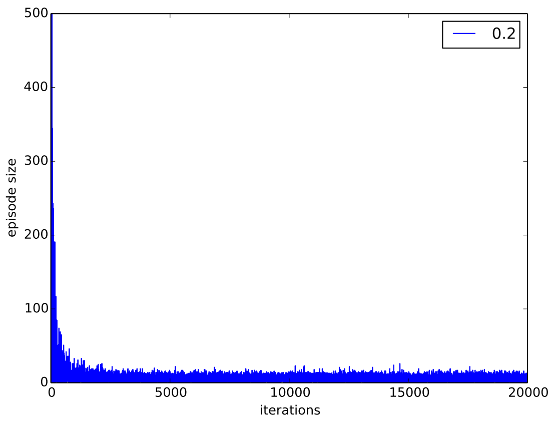


Figure 3.2: Optimal convergence for $\alpha = 0.2$ on bigger y-axis

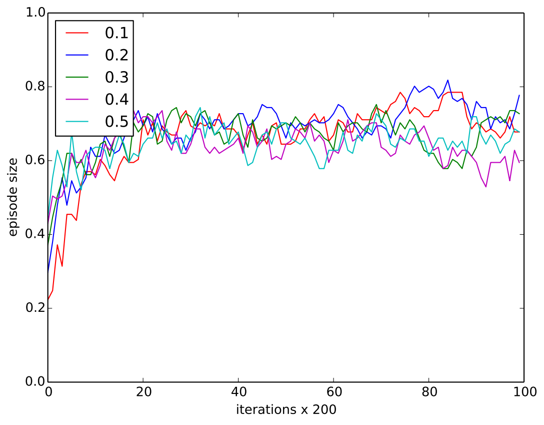
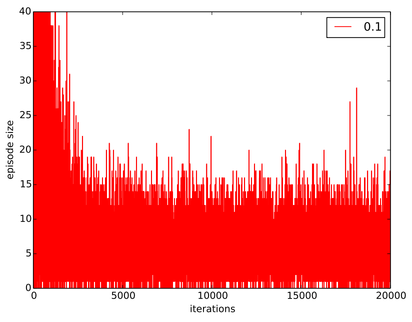


Figure 3.3: Optimality for different α as a ratio to the optimal policy

e)



f)

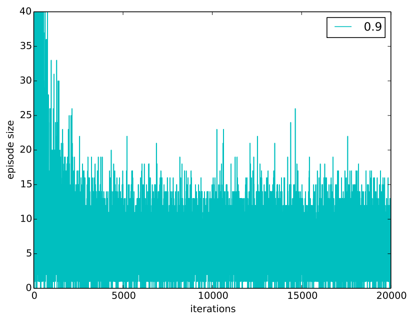


Table 3.2: Convergence for different values of γ

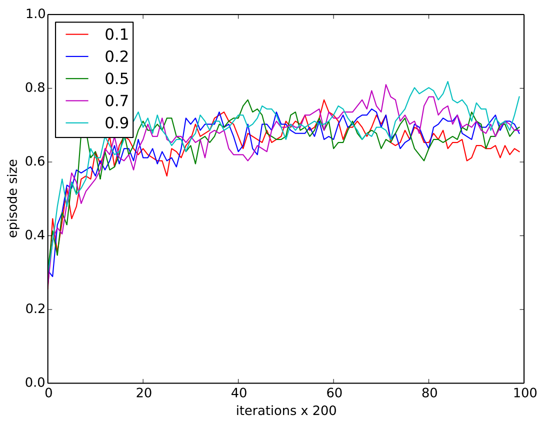
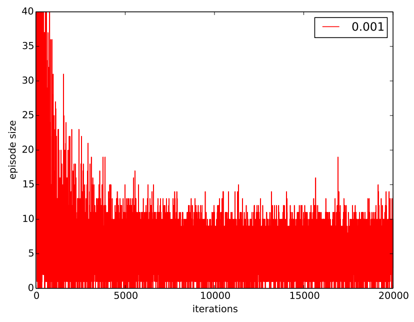
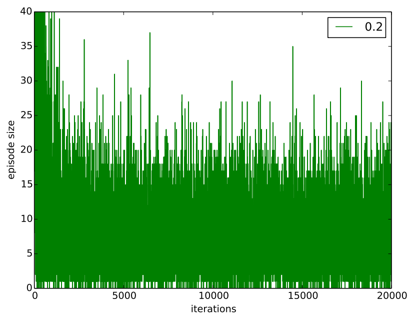


Figure 3.4: Optimality for different γ as a ratio to the optimal policy

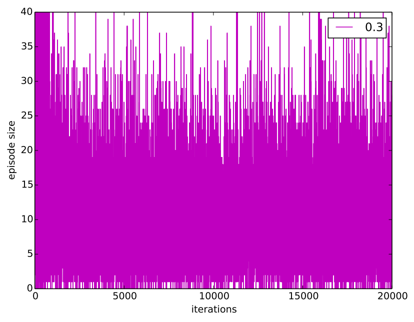
g)



h)



i)



j)

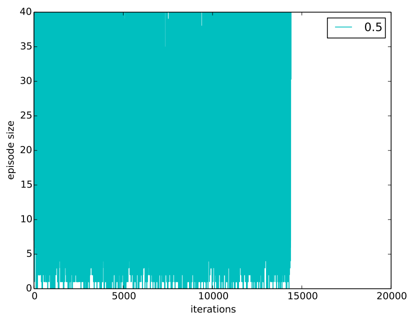


Table 3.3: Convergence for different values of ϵ

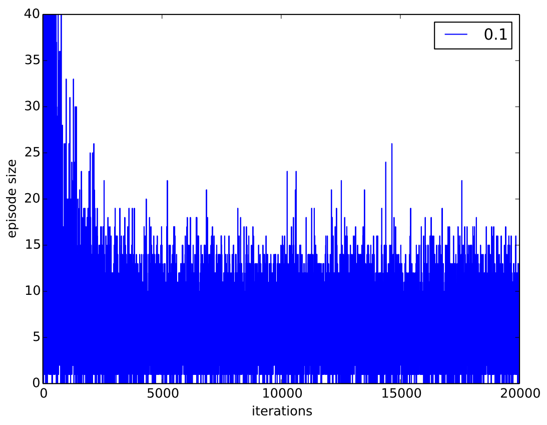


Figure 3.5: Optimal convergence for $\epsilon = 0.1$

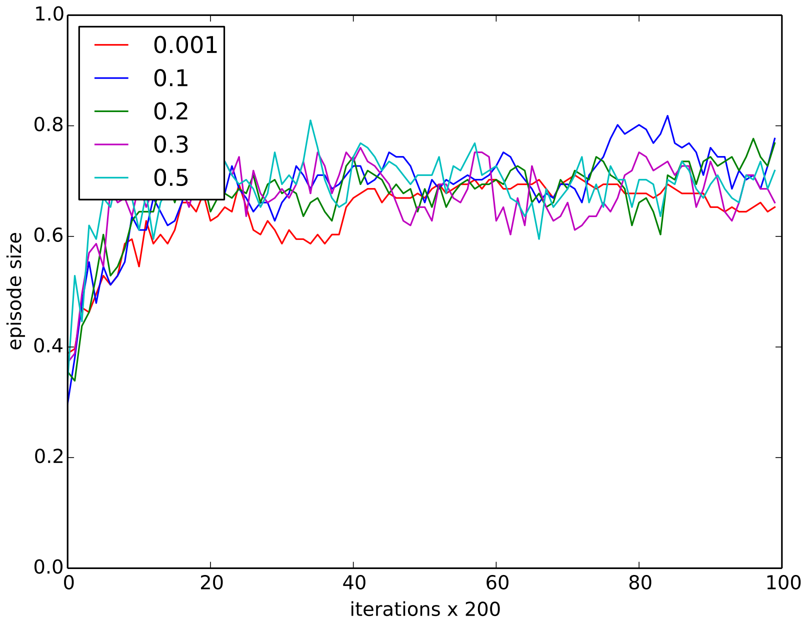


Figure 3.6: Optimality for different ϵ as a ratio to the optimal policy

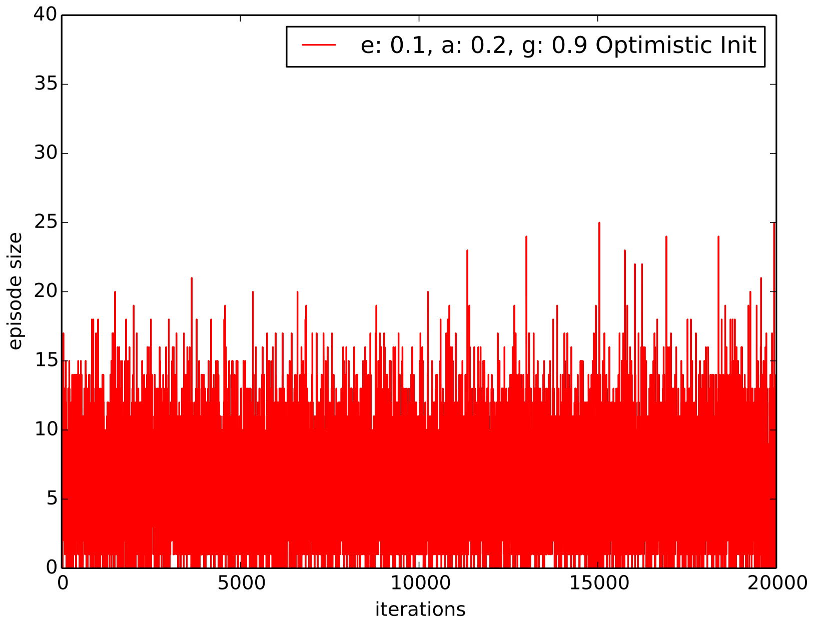


Figure 3.7: Divergence after optimistic initialization

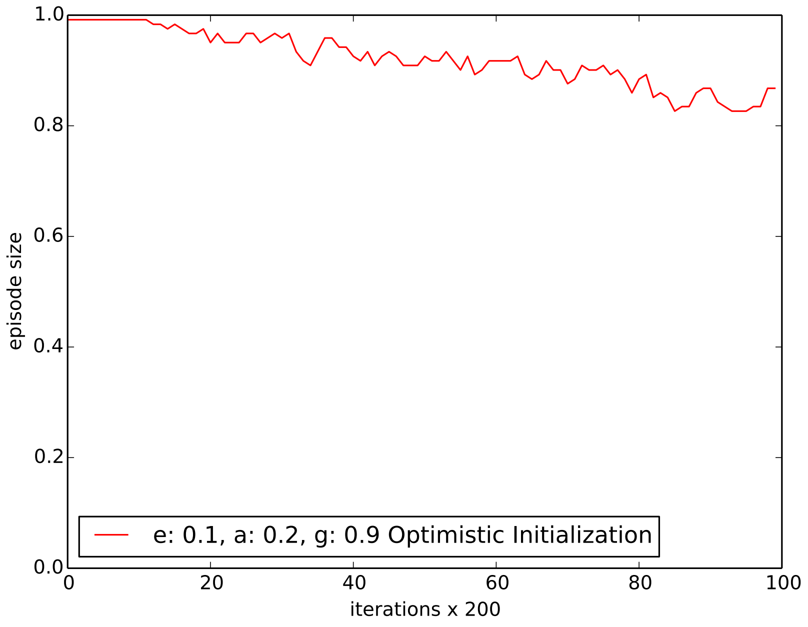
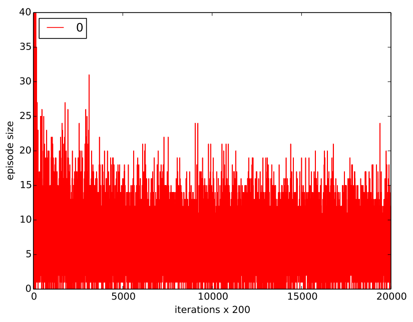
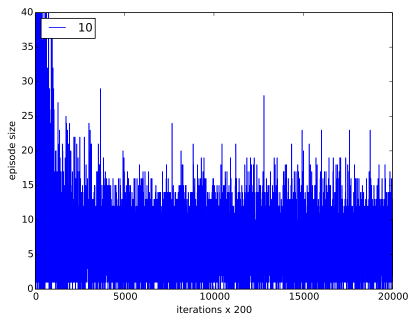


Figure 3.8: Divergence from the optimal policy after optimistic initialization

k)



l)



m)

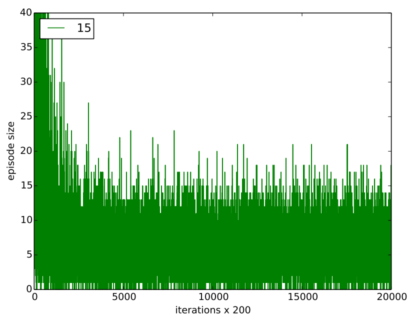


Table 3.4: Convergence for different values of initialization

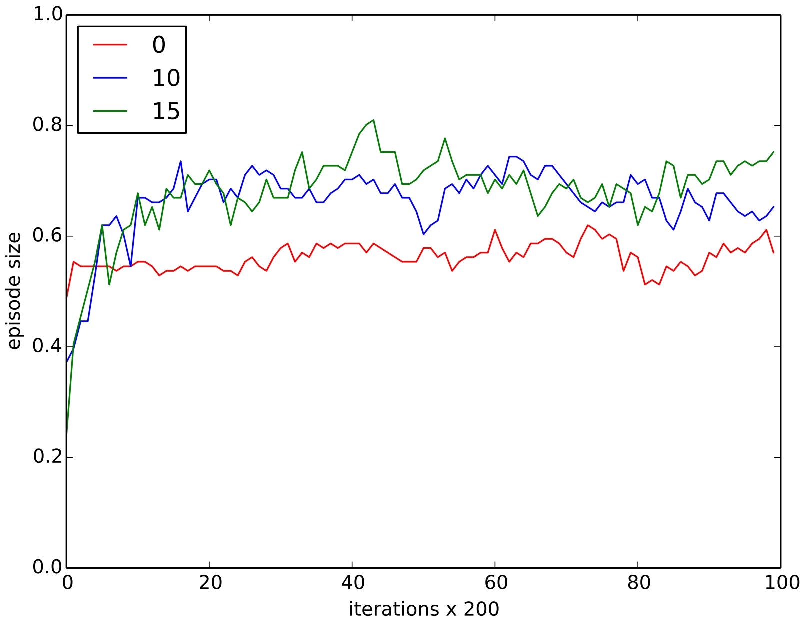


Figure 3.9: Convergence to the optimal policy after different initialization values

3.3 On-Policy Monte Carlo Control

By performing this experiment the goal was to see the performance of the On-policy Monte Carlo control in the predator prey environment. The maximum number of control iterations was fixed at 20000 for every run with different parameters. Additionally the final outcome of the optimal policy was recorded as well as the final values of the function $Q(s,a)$. The metric used in order to determine the performance between different approaches was the size of the episode that was generated each time in the algorithm. A smaller size, between $[1,15]$, shows that the predator catches the prey faster, whereas a bigger size denotes the opposite. Different values for the ϵ parameter for the ϵ-soft policy were tested, since ϵ directly influences the amount of exploring and exploiting the agent performs. Additionally we also experimented on runs with optimistic initial values, namely 15, for the state-action function $Q(s,a)$, since it also influences the amount of exploration. Only fixed ϵ values were tested with optimistic initial values of $Q(s,a)$ since they performed better than the adaptive ones, as we will see in the following section. The discount rate γ for the returns, was fixed at 0.8.

Experiment with different values of ϵ

The values that we tested were fixed ϵ at 0.7, 0.5, 0.3, and 0.1, as well as adaptive ϵ that decreases by 20% every 100 iterations, with different initial value. The results are shown in figure 3.10 below. The total number of iterations is 20000, where we take the average episode size every 100 episodes, in order for the visualization to be clearer.

Figure 3.10: On Policy MC catching time with different epsilon value

The value of ϵ directly influences the amount of time it takes for the agent to find a good estimate for the $Q(s, a)$ value. Fixed ϵ performs better than adaptive ϵ and this is expected since the degree of exploration is minimized as the number of iterations increases. The effect becomes worser the lower the initial value of ϵ is, since the degree of exploration is limited quite early, hence the agent cannot explore and get a better estimate for the Q value. However higher values allow more exploring, thus getting better results. Fixed ϵ achieves higher performance since the ratio of exploration/exploitation is consistenly kept during all the control iterations, essentially making the agent explore even in later iterations of the control loop. We can see from 3.10 that higher values of ϵ achieve a sharper decrease in the episode size in the beggining, since there is a bigger episode size which derives from the high exploration probability, but they perform worse in later iterations since they don't exploit as much as lower values of ϵ. The overal best choice seems to be a fixed ϵ of 0.1 since it might converge slower to a better policy, but it produces more consistent results in the later iterations, due to the increased rate of exploitation.

Experiment with optimistic initial value of $Q(s, a)$

Having an optimistic initial value of Q just forces the agent to explore in the begining which creates episodes with an extremely big size, which results in a sharper drop after that since the Q values have a reasonable value again. This approach didn't provide better results though since the estimated values were not obtained through episodes of a reasonable size, hence they were not very accurate. The results can be seen in figure 3.11 where we performed an average of the episode size for every 100 iterations again, in order to produce a clearer visualization.

Figure 3.11: On Policy MC catching time with optimistic initial Q value

Convergence

As we have seen in figure 3.10 the algorithm seems to converge in an optimal policy in about $100 * 100 = 10000$ iterations, which explains the consistent low episode size and faster catch times for the predator. The optimal policy table derived with the best choice of epsilon is shown in 3.5 and the $Q(s, a)$ values that the policy was derived from are show in table 3.6. The values are reasonable since they follow the pattern which implies that the further away the predator is from the prey, then the lower the value will be.

	0	1	2	3	4	5	6	7	8	9	10
0	→	↓	↓	→	↓	↓	↓	←	↓	←	↓
1	↓	→	↓	→	↓	↓	↓	↓	←	←	↓
2	→	→	→	→	→	↓	←	←	↓	↓	←
3	→	→	→	→	→	↓	←	←	↓	←	←
4	→	→	→	→	→	↓	←	←	←	←	↑
5	↑	↑	→	→	→	*P*	←	←	←	↑	→
6	↑	↓	→	↑	↑	↑	←	←	↓	↓	↑
7	→	→	↑	↑	→	↑	←	←	↓	←	←
8	←	→	→	←	→	↑	←	↑	↑	↑	←
9	↑	↑	↓	→	↑	↑	←	↓	↑	↑	↑
10	←	→	→	↓	↓	↑	↑	←	←	↑	↑

Table 3.5: Target Policy Off-Policy Monte Carlo Control

	0	1	2	3	4	5	6	7	8	9	10
0	1.013	1.466	1.659	2.305	2.934	3.626	2.935	2.368	1.861	1.576	1.245
1	1.386	1.898	2.229	2.922	3.681	4.589	3.731	2.906	2.340	1.752	1.549
2	1.939	2.421	2.946	3.666	4.599	5.904	4.588	3.657	3.028	1.864	1.855
3	1.832	2.494	3.138	4.734	5.880	7.545	5.882	4.669	3.737	2.593	2.038
4	2.922	3.805	4.873	6.069	7.489	10.000	7.551	6.054	4.814	3.744	1.985
5	1.785	2.977	5.805	7.533	10.000	0.000	10.000	6.836	5.345	2.817	1.902
6	1.237	2.008	4.417	5.889	6.917	10.000	7.489	6.030	2.870	2.188	1.518
7	2.354	3.050	3.710	4.843	5.984	7.452	5.705	4.581	2.119	2.756	1.972
8	1.382	1.929	2.768	1.074	4.633	5.838	4.490	3.654	2.716	2.216	1.758
9	1.157	1.827	1.079	2.040	1.196	4.588	2.791	1.472	2.377	1.804	1.402
10	0.676	1.172	1.493	1.845	2.257	3.569	2.727	2.102	1.182	0.966	0.943

Table 3.6: Optimal state value function for each state given prey at (5,5)

3.4 Off-Policy Monte Carlo Control

The purpose of this experiment was to implement Off-Policy Monte Carlo Control algorithm and to observe the results in terms of the target policy, maximum action value function, and also the number of steps the predator take to catch the prey. We tried two different approaches of setting the ϵ of the behavior policy, fixed value and dynamic value which will be explained in the next part. Different fixed ϵ value of ϵ-greedy policy also were tested to show how this parameter, which represents the probability of exploring and exploiting, affect the learning processes of action value function. We organize this subsection starting from presenting the results of using different approach of setting ϵ value of behavior policy, then we show the result of using several different fixed values of ϵ, and finally we will show the convergence of the target policy.

Experiment with different approaches of setting ϵ value

Firstly we set the ϵ value in ϵ-greedy policy as a fix number, then we tried a different approach by making it dynamic by reducing the value after each fix number of iterations. The idea behind this approach was to put more chance in exploration at the beginning and slowly reduced the chance thus it started to exploit more time after time, but also maintained it to be stochastic in order to keep the learning process run.

Figure 3.12 shows the number of steps the predator took to catch the prey in every iteration governed by the target policy. The ϵ value was set to be fix of 0.1. In order to get a consistent measurement, the episodes were generated from fix start points of the predator and the prey, which were (0,0) and (5,5) respectively. The time that the predator took to catch the prey can be used to measure the performance of Off-Policy Monte Carlo Learning after each iteration, because the target policy uses the Q value immediately after each iteration. Since the target policy was greedy, then it always take best action based on the Q value. The better the Q value, the better target policy's performance should be.

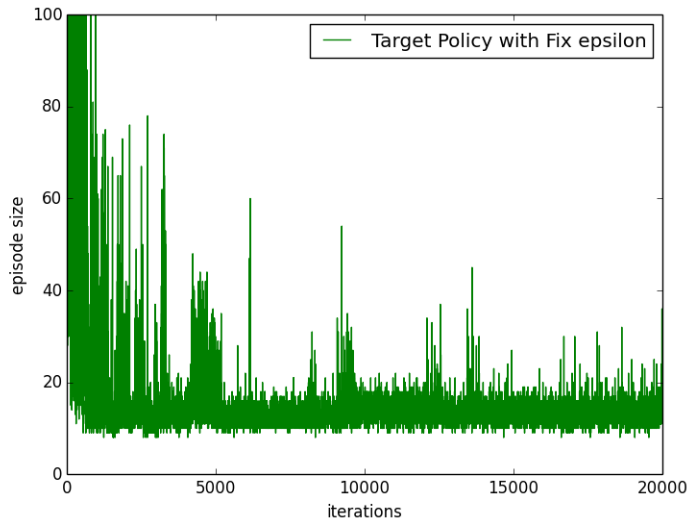


Figure 3.12: Catching time with fix epsilon value

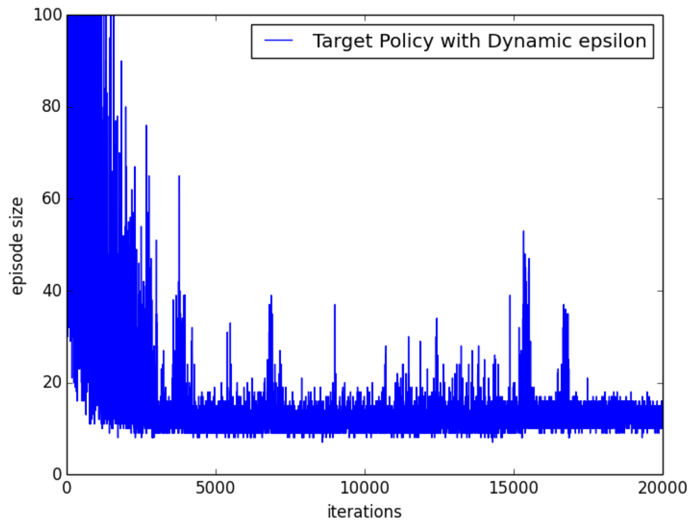


Figure 3.13: Catching time with dynamic epsilon value

Figure 3.13 was generated from the same setting as before except the value of ϵ was set to dynamically changed over iteration. First we set the value to 0.5 then after every 500 iterations it was reduced by multiplying with 0.8. In order to maintain the value not become too small, we set the minimum value to 0.1, because if ϵ is too small then it will become greedy and almost never explore anymore. As a consequence, learning processes will stop because both behavior and target policy always agree on actions taken in the generated episodes.

Even though the difference between the two figures above is not significant, but we can still see the influence of setting the ϵ value to a fix and to a dynamic value. For fix value approach, the number of time drop quicker than the one from dynamic approach. This is because the fix value approach exploited more even from beginning due to the probability of exploiting much larger than exploring. On the other hand,the dynamic approach give the same probability between exploiting and exploring, so at the beginning it explored more than the fix value approach. Now let us see the last 3000 iterations from the two figures, it can be seen that dynamic value approach shows a slightly better performance, it fluctuates less than that in the other figure. Since the difference is not significant then we can not claim which one is better. The general statement is that they both can converge with anough number of iterations.

Experiment with different values of ϵ

In this part we made some experiments by changing the fix value of ϵ. Since the epsilon represents the chance of exploring and exploiting in learning, then we will make a conclusion related to that matter.

The values that we tested was 0.5, 0.3, 0.1, and 0.001, and the result is shown by figure 3.14 below. The number of iterations is 10000, to make a clearer figure, we devided it into 200 parts, and averaged each part, so 1 iteration in the figure is average of 50 iterations.

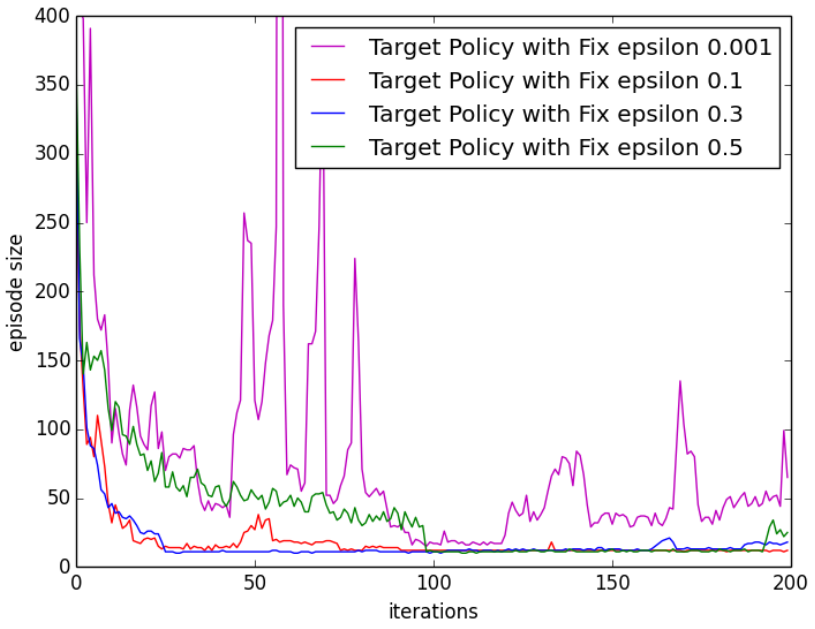


Figure 3.14: Catching time with different fixed epsilon value

The figure above tells us that in our test case, ϵ value of 0.1 and 0.3 give the best performance, while 0.5 result shows a slower convergence than the previous values and 0.001 become the worst. This result confirm that exploration and exploitation have a significant role in learning process. Setting the exploitation probability value too small makes the model hard to learn, it could be trapped to exploit wrong actions, where setting the value too high means small chance to exploiting and then leads to a late convergence.

Convergence

In this section we will show the convergence of the Off-Policy Monte Carlo Control method. Table 3.7 and table 3.8 show the optimal value function and the optimal target policy's actions respectively. Even though this method use sampling to estimate the target policy, but with enough number of iteration the convergence to the optimal value function can be reach. From several experiments, we know that as long as the behavior policy is stochastic and has reasonable ϵ value, like what we have explained in the previous part, then it will converge. The results below were generated from 100.000 iterations with fixed ϵ 0.3.

	0	1	2	3	4	5	6	7	8	9	10
0	1.282	1.501	2.068	2.514	3.149	3.673	2.949	2.488	2.028	1.626	1.324
1	1.540	2.055	2.469	3.208	3.974	4.694	3.951	3.185	2.573	2.006	1.791
2	1.896	2.691	3.198	3.987	4.939	6.054	4.958	3.999	3.201	2.576	1.965
3	2.622	3.233	3.985	4.943	6.150	7.611	6.198	4.971	4.027	3.191	2.395
4	3.058	4.004	4.960	6.156	7.609	10.000	7.631	6.192	4.712	3.837	2.962
5	3.763	4.738	6.005	7.633	10.000	0.000	10.000	7.616	6.026	4.729	3.721
6	3.207	3.967	4.928	6.159	7.620	10.000	7.597	5.981	4.852	3.699	3.007
7	2.587	3.240	4.007	4.987	6.011	7.624	6.164	4.887	3.912	3.144	2.540
8	2.129	2.202	3.174	4.005	4.830	6.037	4.749	3.781	3.073	2.494	2.073
9	1.695	2.091	2.624	3.240	3.747	4.752	3.845	2.947	2.564	2.021	1.511
10	1.199	1.706	1.964	2.349	2.925	3.733	3.038	2.451	1.971	1.715	1.378

Table 3.7: Optimal state value function for each state given prey at (5,5)

	0	1	2	3	4	5	6	7	8	9	10
0	$\rightarrow$	$\downarrow$	$\downarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$
1	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\leftarrow$	$\downarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$
2	$\downarrow$	$\rightarrow$	$\downarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\leftarrow$	$\leftarrow$	$\downarrow$
3	$\downarrow$	$\downarrow$	$\rightarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\downarrow$	$\downarrow$
4	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\leftarrow$	$\leftarrow$
5	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	P	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$
6	$\rightarrow$	$\rightarrow$	$\uparrow$	$\rightarrow$	$\rightarrow$	$\uparrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\uparrow$	$\leftarrow$
7	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\uparrow$	$\rightarrow$	$\uparrow$	$\uparrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$
8	$\uparrow$	$\rightarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$
9	$\rightarrow$	$\uparrow$	$\uparrow$	$\rightarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\leftarrow$	$\leftarrow$	$\uparrow$	$\uparrow$
10	$\uparrow$	$\uparrow$	$\rightarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$

Table 3.8: Optimal target policy's actions for each state given the prey at (5,5)

3.5 Comparison of all approaches to the learning problem

Regarding on and off policy Monte Carlo control, in the case of the best parameters ϵ for both of the algorithms, the On-policy Monte Carlo performs better. It converges faster to an optimal policy that produces faster catch times for the predator and manages to keep them consistent in the later episodes. On the contrary Off-policy seems to converge slower and has some fluctuations on the catch times which was derived from the fact that the optimization of the target policy was done through a different behaviour policy. Furthermore, in the off-policy method, state-action pair that will be used to estimate is only pairs that occur after the last disagree action between the two policies. So it is likely to use fewer samples than those in on-policy with the same number of iterations. The average catch times for the last 100 episodes are shown in table 3.9 and show the fact that on-policy Monte Carlo produces a better policy.

Figure 3.15: Catching times for best choice of ϵ for On and Off policy MC

Q-Learning	0
Sarsa	0
On-policy MC	7
Off-policy MC	12

Table 3.9: Average catching time for all algorithms at convergence

4 CONCLUSION

REFERENCES

[1] Richard S. Sutton, Andrew G. Barto, *Reinforcement Learning: An Introduction.* MIT Press, Cambridge, MA, A Bradford Book, 1998.