Lesson 1: Linear Programming Review, Intro to Integer Programming

1 Goals

- Formulate a concrete linear programming model.

- Introduce an integrality requirement on variables.

- Convert the integer program to parameterized form.

2 Review of SA305 Formulations

The five components of formulating an optimization model are:

1.

2.

3.

4.

5.

What are the 3 assumptions/characteristics that make an optimization model a linear program?

1.

2.

3.

3 Concrete Model

Chelsea is heading out on a camping trip, and she wants to carry only one pack that has 5.3 ft^3 of volumetric space. To keep from hurting her back, she needs to make sure that the contents of her backpack weighs no more than 12.5 lbs. You can assume the backpack weight is negligible. See the list of items that she is able to bring:

ID	Item	Volume (ft^3)	Usefulness Factor	Weight (lbs.)
1	Rope	2	1	3
2	Matches	0.01	5	0.1
3	Tent	3	7	10
4	Sleeping bag	2	6	4
5	Hammock	0.4	4.5	4
6	Granola bars	0.67	8	2

This problem is referred to as the **knapsack** problem and is a very widely used type of integer program. We'll see why we need IP shortly. We will first formulate it as an LP.

Problem 1. Write a concrete linear program whose solution maximizes the usefulness of the contents of Chelsea's bag given volume and weight requirements.

a) Define decision variables and then describe the objective function and the role of each constraint.

b) Write the concrete model.

4 Integrality Restrictions

Suppose we solve this LP and get the following solution:

$$x_{granola} = x_{hammock} = x_{matches} = x_{sleeping} = 1$$

$$x_{tent} = 0.24$$

$$x_{rope} = 0$$

What is the objective function value of this solution? Denote this solution as z_{LP}^*.

Is this a reasonable solution?

Suppose we change the problem so that x_i is **binary** instead of continuous. A binary variable is only allowed to take the value of 0 or 1. That is, the variables would be changed to:

Our problem has now become a **integer program**. Why?

Let z_{IP}^* be the optimal objective function value of the integer program. Would we expect z_{IP}^* to be smaller, larger, or the same as z_{LP}^*?

5 Convert to Parameterized Models

Problem 2. Assuming integrality restrictions, convert your model to a parameterized model. Clearly define all sets, parameters, and decision variables.