Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 4

Name: _____

You're going to Six Flags this weekend and are a roller coaster enthusiast. Six Flags has 7 total roller coasters. You label them 1 to 7 and plot out the distance you'd have to walk in order to get to each roller-coaster. The following matrix gives these distances (note it's symmetric):

$$\begin{bmatrix} - & 10 & 12 & 7 & 13 & 8 & 9 \\ & - & 10 & 13 & 25 & 6 & 7 \\ & & - & 21 & 23 & 12 & 10 \\ & & & - & 12 & 3 & 9 \\ & & & & - & 20 & 8 \\ & & & & & - & 3 \\ & & & & & & - \end{bmatrix}$$

You decide that you want to walk as little as possible; so you model this problem as a minimum spanning tree (MST) in order to find the path you'd have to take to walk as little as possible.

Suppose you use the variables $x_{i,j} = 1$ if edge (i, j) is part of the tree and 0 otherwise.

1. (20 points) When modeling this problem as a MST you need to include a constraint that says you visit each roller coaster. Using the $x_{i,j}$ variables, write a concrete constraint that ensures at least one edge connected to roller coaster 3 is selected.

   $$x_{1,3} + x_{2,3} + x_{3,4} + x_{3,5} + x_{3,6} + x_{3,7} \geq 1$$

2. After implementing this model in python, your solver returns the following solution:

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   The optimal solution is to select edges [1,2], [2,3], [3,5], [4,6], [4,7], [6,7]
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   (a) (15 points) What are the values of the $x_{i,j}$ variables corresponding to this solution?

   $$x_{1,2} = x_{2,3} = x_{3,5} = x_{4,6} = x_{4,7} = x_{6,7} = 1$$

   All other variables equal to 0.

   (b) (15 points) What is the total distanced traveled by this solution?

   $$10 + 10 + 23 + 3 + 9 + 3 = 58$$

(c) (10 points) Is this solution optimal to your MST problem? No, there is a cycle on nodes 4, 6, and 7

(d) (20 points) If this solution is optimal explain why. If the solution is not optimal, write which constraint(s) you would add to your model to exclude this solution.

$$x_{4,6} + x_{4,7} + x_{6,7} \leq 2$$

3. (10 points) Given a graph on n nodes, how many edges do I need to choose to form a cycle?

   n nodes form a cycle

4. (10 points) True or False, when solving a MST problem in python, we should implement **every** constraint in the model.

   False, there are an exponential number of subtour elimination constraints, we only implement them when we need them.