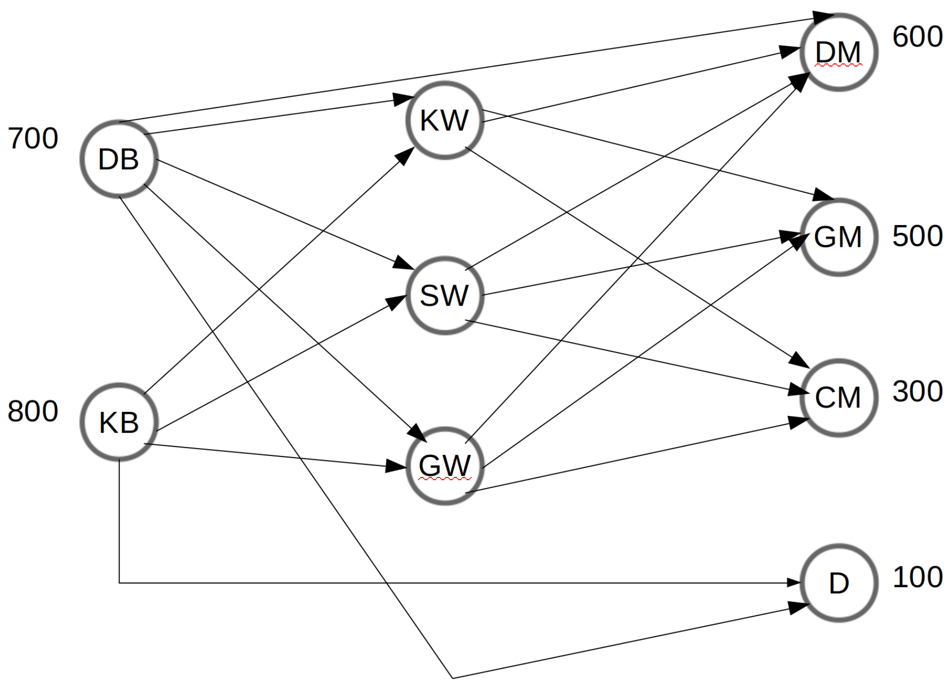
Project 1: Guiness

Part 1: Solution

You may use my solution for your python implementation for part 2. If your own model is already correct, you can use it instead or make the necessary corrections.

1 Network Diagram



2 Concrete Model

Decision Variables

Let $x_{i,j}$ be the number of cases to transport from i to j for each edge (i,j)

Let $z_i = 1$ if we use warehouse i or 0 if we do not for each $i \in \{KW, SW, GW\}$

Objective Function

$$\text{minimize cost: } 15x_{DB,KW} + 10x_{DB,SW} + \cdots + 12x_{GW,CM} + 240z_{KW} + 450x_{SW} + 320z_{GW}$$

Constraints

$$
\begin{array}{rrcll}
\text{st} & x_{DB,KW} + x_{KB,KW} = x_{KW,DM} + x_{KW,GM} + x_{KW,CM} & & & \text{(Flow in = Flow out at Kilgore)} \\
& x_{DB,SW} + x_{KB,SW} = x_{SW,DM} + x_{SW,GM} + x_{SW,CM} & & & \text{(Flow in = Flow out at Sligo)} \\
& x_{DB,GW} + x_{KB,GW} = x_{GW,DM} + x_{GW,GM} + x_{GW,CM} & & & \text{(Flow in = Flow out at Galway)} \\
& x_{DB,KW} + x_{DB,SW} + x_{DB,GW} + x_{DB,DM} + x_{DB,D} & = & 700 & \text{(Supply at Dublin-B)} \\
& x_{KB,KW} + x_{KB,SW} + x_{KB,GW} + x_{KB,D} & = & 800 & \text{(Supply at Kilarny)} \\
& x_{DB,DM} + x_{KW,DM} + x_{SW,DM} + x_{GW,DM} & = & 600 & \text{(Demand at Dublin-M)} \\
& x_{KW,GM} + x_{SW,GM} + x_{GW,GM} & = & 500 & \text{(Demand at Galway)} \\
& x_{KW,CM} + x_{SW,CM} + x_{GW,CM} & = & 300 & \text{(Demand at Cork)} \\
& x_{DB,D} + x_{KB,D} & = & 100 & \text{(Demand at Dummy)} \\
& x_{KW,DM} + x_{KW,GM} + x_{KW,CM} & \leq & 400z_{KW} & \text{(Weak forcing constraint on Kilgore W)} \\
& x_{SW,DM} + x_{SW,GM} + x_{SW,CM} & \leq & 800z_{SW} & \text{(Weak forcing constraint on Sligo W)} \\
& x_{GW,DM} + x_{GW,GM} + x_{GW,CM} & \leq & 600z_{GW} & \text{(Weak forcing constraint on Galway W)} \\
& x_{DB,KW}, x_{DB,SW}, \cdots, x_{GW,CM} & \in & \mathbb{Z}^+ & \text{(Integrality)} \\
& z_{KW}, z_{SW}, z_{GW} & \in & \{0,1\} & \text{(Binary)}
\end{array}
$$

3 Parameterized Model

Sets

Let E be the set of edges
Let W be the set of warehouses
Let N be the set of nodes

Decision Variables

Let $x_{i,j}$ be the units shipped along edge $(i,j) \in E$
Let $z_w = 1$ if warehouse w is open and 0 otherwise for all $w \in W$

Parameters

Let $c_{i,j}$ be the cost of sending one unit along edge (i,j) for all $(i,j) \in E$.
Let f_w be the fixed cost of opening warehouse w for all $w \in W$
Let s_i be the supply of node i for all $i \in N$
Let d_i be the demand of node i for all $i \in N$
Let M_w be the capacity of warehouse w for all $w \in W$

Objective Function

$$\text{minimize cost:} \quad \sum_{(i,j)\in E} c_{i,j} x_{i,j} + \sum_{w \in W} f_w z_w$$

Constraints

$$\begin{array}{llcll}
\text{st} & \displaystyle\sum_{(i,n)\in E} x_{i,n} + s_n & = & \displaystyle\sum_{(n,j)\in E} x_{n,j} + d_n & \text{for } n \in N & \text{(Flow balance of each node)} \\
& \displaystyle\sum_{(i,w)\in E} x_{i,w} & \leq & M_w z_w & \text{for all } w \in W & \text{(forcing and capacity constraints)} \\
& x_{i,j} & \in & \mathbb{Z}^+ & \text{for all } (i,j) \in E & \text{(Integrality)} \\
& z_w & \in & \{0,1\} & \text{for all } w \in W & \text{(Binary)}
\end{array}$$

Note: You can replace the weak forcing constraints with strong forcing constraints. But if you do this, you still have to account for the capacities by putting a bound on the flow in or flow out of each warehouse.