HW9: Branch and Bound Part 1

Solve the following problem using Branch and Bound

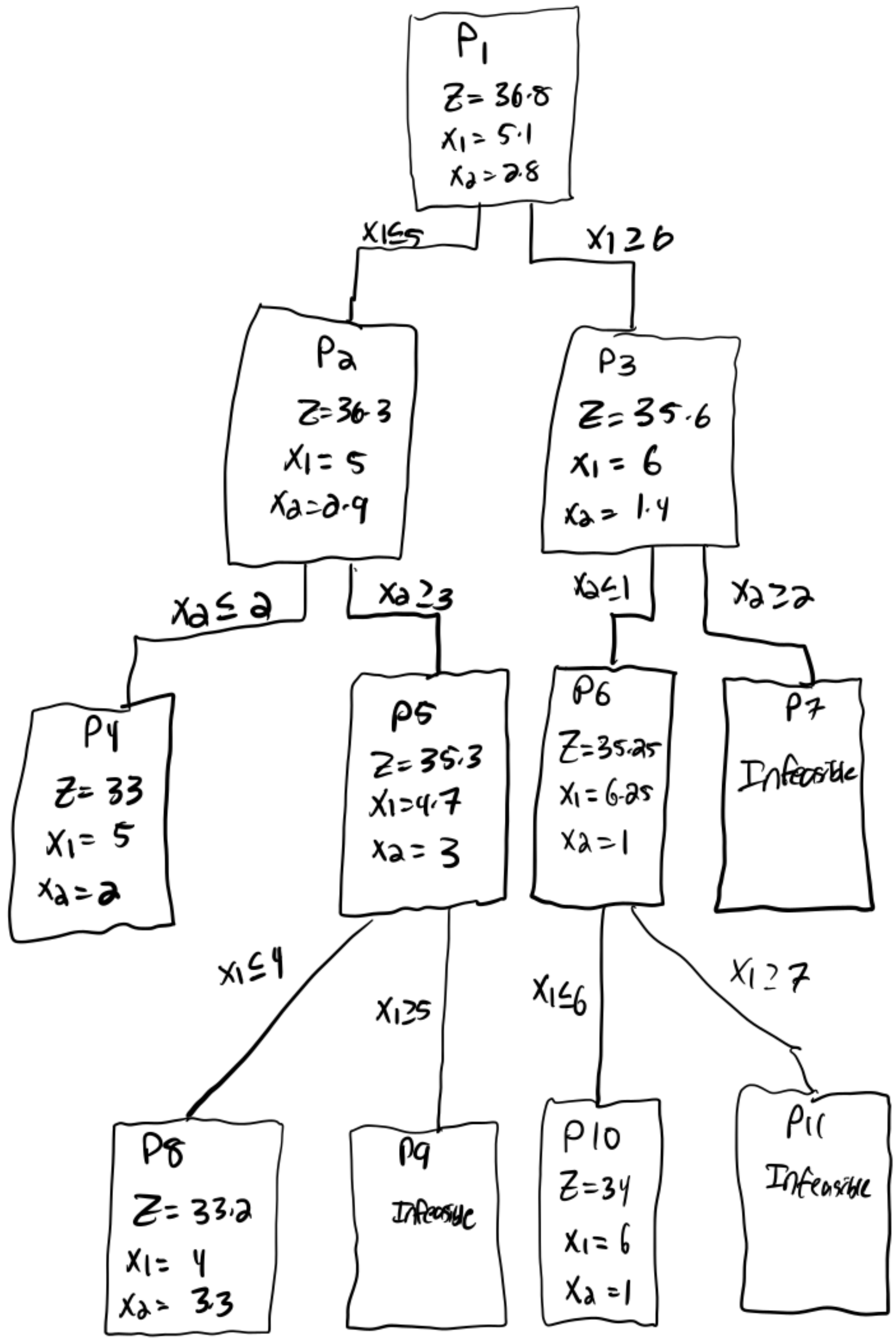
$$\begin{aligned} \max \quad & 5x_1 + 4x_2 \\ \text{st} \quad & 6x_1 + 13x_2 \leq 67 \\ & 8x_1 + 5x_2 \leq 55 \\ & x_1, x_2 \in \mathbb{Z}^+ \end{aligned}$$

Solve each subproblem graphically or with python.

I solved each subproblem in Python. The branch and bound tree is on the next page. The logic is as follows.

- Step 1, solve P1. Initial solution $z = 36.8$, $x = (5.1, 2.8)$. At this point, I know $z_{IP} \leq 36$. I can branch on either x_1 or x_2, I choose x_1
- Step 2, solve P2 and P3. For P2, $z = 36.3$ and x_2 is fractional. For P3, $z = 35.6$, x_2 is fractional. I can branch on either P2 or P3. I choose P2 because $36.3 > 35.6$.
- Step 3: Solve P4 and P5. For P4, $z = 33$ and solution is integer. For P5, $z = 35.3$ and x_1 is fractional. So at this point, I know $33 \leq z_{IP} \leq 36$. I can choose to branch on either P3 or P5. I choose P3
- Step 4: Solve P6 and P7. For P6, $z = 35.25$ and x_1 is fractional. P7 is infeasible. At this point, my active nodes are P5 and P6. Note that both have z values between 35 and 36. So I can update my bounds to be $33 \leq z_{IP} \leq 35$. I choose to branch on P5.
- Step 5: Solve P8 and P9. For P8, $z = 33.2$ and x_2 is fractional. P9 is infeasible. I eliminate node P8 because 33.2 is equal to my current lower bound. All that's left is to branch on P6.
- Step 6: Solve P10 and P11. For P10, $z = 34$ and is integral. P11 is infeasible. No active nodes are left. P10 becomes my current solution because $34 > 33$. I stop, P10 is optimal.

The optimal solution is $x^* = (6, 1)$ with objective value $z^* = 34$.


P1
Z= 36.8
X1= 5.1
X2= 2.8
X1≤5
X1≥6
P2
Z=36.3
X1= 5
X2=2.9
P3
Z= 35.6
X1 = 6
X2 = 1.4
X2≤2
X2≥3
X2≤1
X2≥2
P4
Z= 33
X1= 5
X2= 2
P5
Z= 35.3
X1=4.7
X2 = 3
P6
Z=35.25
X1=6.25
X2=1
P7
Infeasible
X1≤4
X1≥5
X1≤6
X1≥7
P8
Z= 33.2
X1= 4
X2= 3.3
P9
Infeasible
P10
Z=34
X1= 6
X2 =1
P11
Infeasible