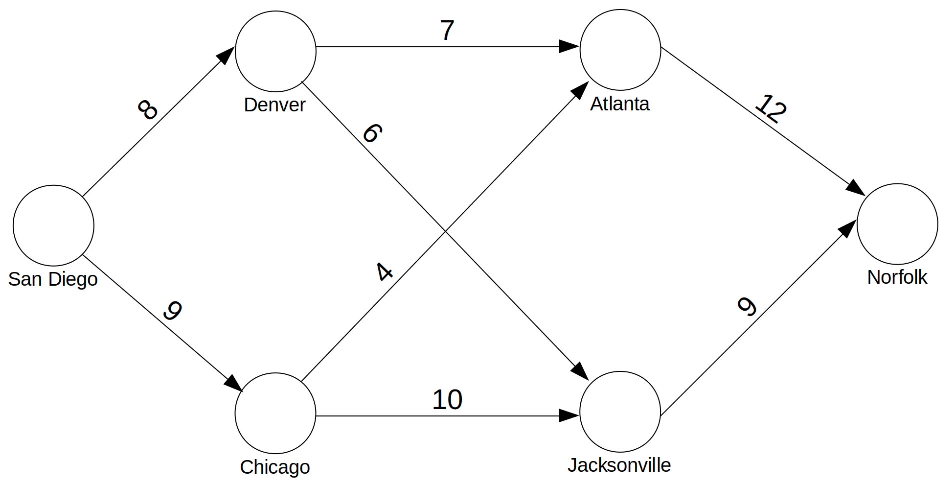
Lesson 5: Max Flow

The Navy is trying to transport a large number of sailors from San Diego to Norfolk using airplanes. Unfortunately, they can not do direct flights, so they must make several stops along the way. The following diagram shows the flights available and the capacity (in hundreds) of each flight. Formulate a linear program that would allow the Navy to send as many sailors as possible from San Diego to Norfolk.



Notice that these problems have a special structure. Specifically, max flow problems generally have two special nodes called the **source** and **sink** nodes.

- **Source node:**

- **Sink node:**

1. Based on the network diagram, is there an intuitive way to find the optimal objective value for this problem?

2. Can you describe a minimization problem that has the same objective value as this max flow problem? (This is the “dual” problem.)

3. Does this problem specify supply and demand? If not, how can we write our balance of flow constraints for the start and end nodes? (Hint: there are two correct ways to do this).

Important Network Problem Theorems

Before we formulate this problem, there's a couple of Theorems that are important to conclude our study of basic network problems.

- Max Flow-Min Cut Theorem

- Max Flow Integrality Theorem

- Min Cost Integrality Theorem

> Note that these theorems mean that each of the 3 network problems we've formulated so far as integer-programs (transportation, minimum cost network flow, and shortest path) as well as the max flow problem we are formulating here are **guaranteed** to have an integer solution even if you formulate them as an LP.
> The remainder of the network problems that we study do not have this nice property so they must be formulated as an IP.

4. Formulate the concrete LP associated with this max flow model.

5. Generalize your LP model to a parameterized model.