Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 5

Name: _____

You are continuing your roller coaster enthusiasm by planning a trip to Busch Gardens. Busch Gardens has 8 roller coasters and, yet again, you label them 1 to 8 and plot the distance you'd have to walk in order to get to each roller coaster. The following matrix gives these distances (in hundreds of feet).

$$
\begin{bmatrix}
- & 6 & 8 & 3 & 12 & 5 & 7 & 6 \\
 & - & 6 & 4 & 6 & 4 & 3 & 2 \\
 & & - & 13 & 10 & 2 & 7 & 10 \\
 & & & - & 12 & 3 & 9 & 13 \\
 & & & & - & 6 & 9 & 12 \\
 & & & & & - & 7 & 4 \\
 & & & & & & - & 1 \\
 & & & & & & & -
\end{bmatrix}
$$

You've refined your model and decided to model this problem as a traveling salesperson (TSP) in order to walk as little as possible.

Suppose you use the variables $x_{i,j} = 1$ if edge (i, j) is part of the tour and 0 otherwise.

1. (15 points) Give the objective function of this model in both concrete and parameterized form. For the parameterized form, make sure you define any new parameters used.

2. (15 points) In order to obtain a tour of the graph, how many edges must be selected? Write a constraint, in either concrete or parameterized form, which enforces that this number of edges is selected from the graph.

3. You implement this model in python, solve it, and get the following solution:

```
The optimal solution is to select cycles 2-1-5-3-2 and 4-7-8-6-4
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   (a) (15 points) What are the values of the $x_{i,j}$ variables corresponding to this solution?

   (b) (15 points) What is the total distanced traveled by this solution?

   (c) You know that this is not the optimal solution to your problem because it is not a tour of the entire graph, but instead is two cycles of size 4. You decide to eliminate the first cycle `2-1-5-3-2`. You recall that the general subtour elimination constraints for TSP were:

   $$\sum_{(i,j)\in E:i\in S,j\in S} x_{i,j} \leq |S| - 1 \text{ for all } S \subset N, |S| \geq 3$$

   i. (10 points) For the cycle `2-1-5-3-2` what is the set S?

   ii. (20 points) What is the concrete constraint you would add to your model to properly eliminate the cycle `2-1-5-3-2`?

   iii. (10 points) If you wanted to eliminate **all** cycles of size 4 from this graph, how many constraints would you have to write?