

## Branch-and-bound Python Example (Accompanying Lesson 16)

### 1 Today...

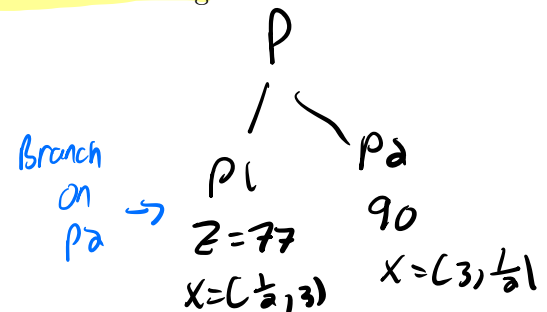
- This example is part of Lesson 16, Branch-and-bound.

### 2 Branch-and-bound Example

Solve the following IP using branch-and-bound.

$$\begin{aligned}
 \text{(P1)} \quad & z_{IP}^* = \max 8x + 7y \\
 \text{s.t.} \quad & -18x + 38y \leq 133 \\
 & 13x + 11y \leq 125 \\
 & 10x - 8y \leq 55 \\
 & x, y \in \mathbb{Z}^{\geq 0}
 \end{aligned}$$

- Use Python to solve LP relaxations of subproblems
- Branching Rules
  - Always select the active node with the largest upperbound for branching.
  - Branch on  $x$  if it is fractional. Otherwise branch on  $y$ .
- Book-keeping
  - Keep track of the:
    - ◇ incumbent solution  $\underline{x}$ ,
    - ◇ global lower bound  $\underline{z}$ , and
    - ◇ list of active nodes.
  - Draw the branch-and-bound tree:
    - ◇ Record the local upper bound ( $z$ ) and relaxed optimal solution ( $x$ ) for each subproblem.
    - ◇ Label each edge with the constraint that is added to form the child subproblem.
    - ◇ X-out fathomed nodes. Circle incumbent solution nodes.
  - Use the provided diagram to illustrate the (relaxed) feasible region of each subproblem.

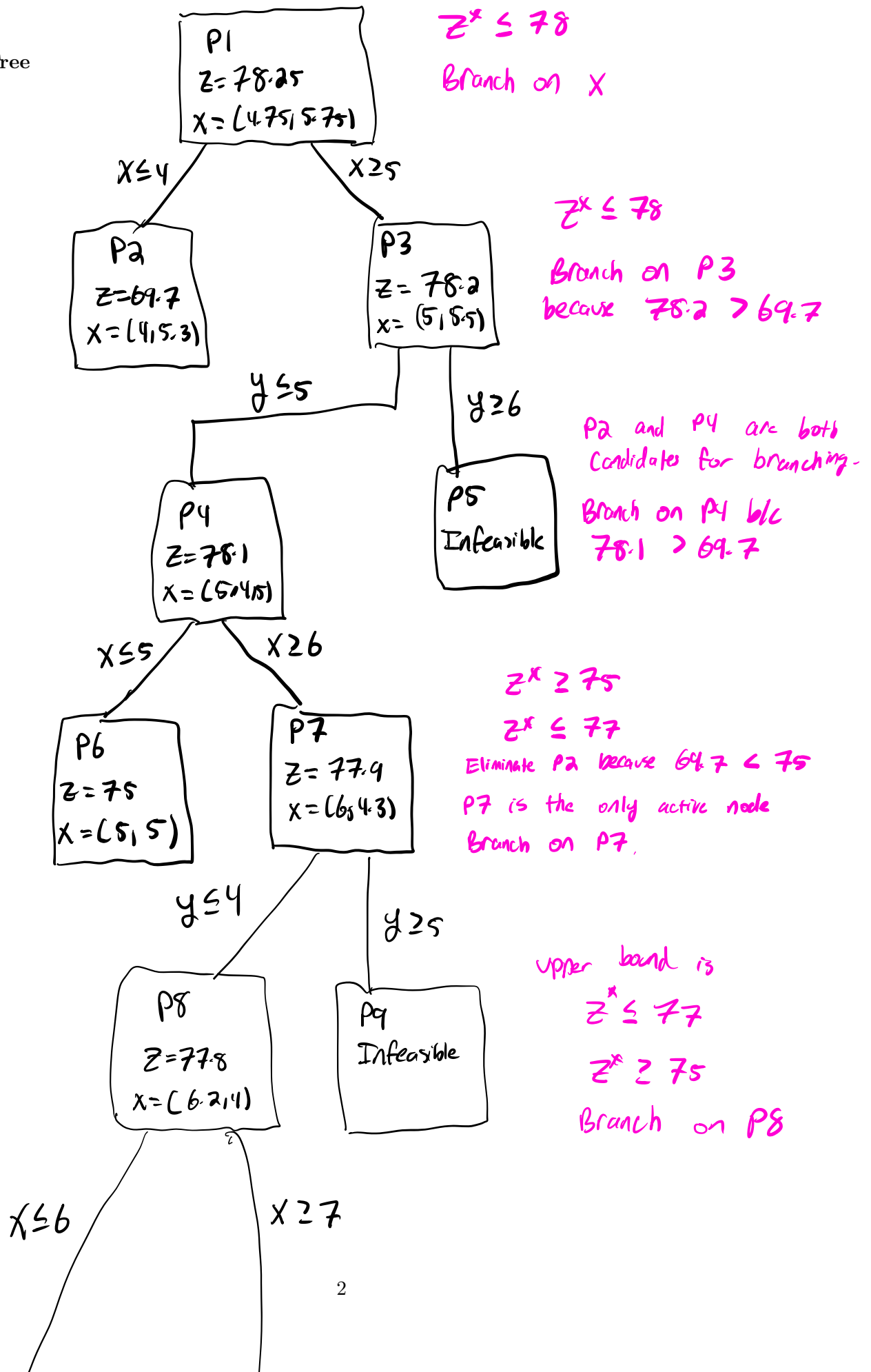


incumbent solution  $\underline{x}$

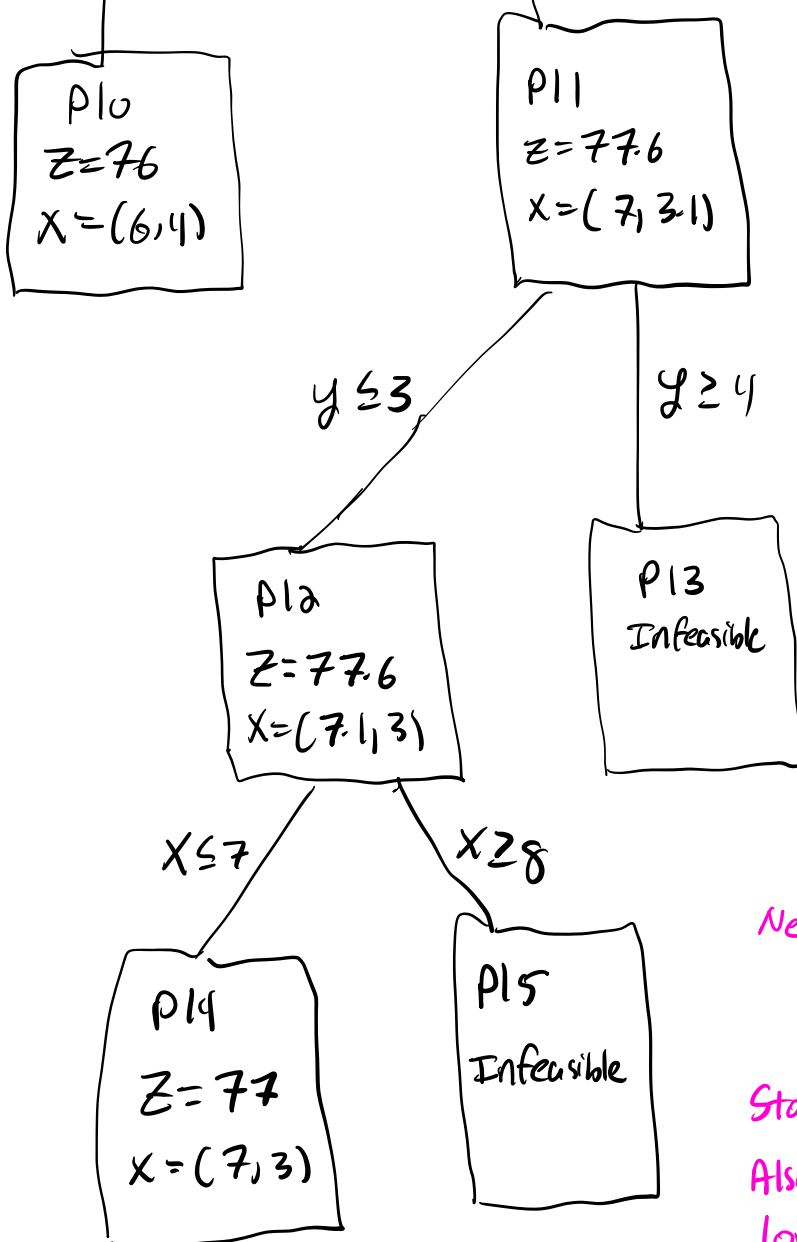
global lower bound  $\underline{z}$

active nodes

# B & B Tree



(B&B Tree Continued)



$$z^* \leq 77$$

$$z^* \geq 76$$

P10 is new incumbent solution because  $76 > 75$ .

Eliminate P6 and branch on P11

Bounds stay same

$$z^* \leq 77$$

$$z^* \geq 76$$

Branch on P12

New lower bound of

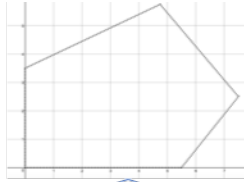
$$z^* \geq 77$$

Stop because no active nodes

Also can stop because lower bound = upper bound

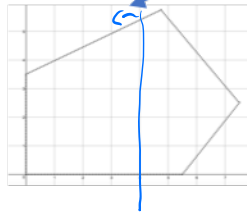
Optimal solution  $z^* = 77$   
 $x^* = (7, 3)$

$p_1$

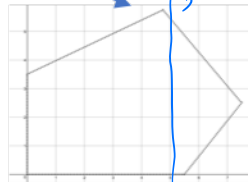


$p_2$

$x \leq 4$

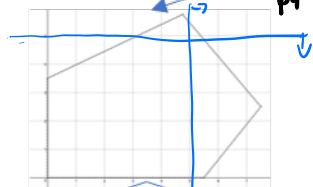


$p_3$



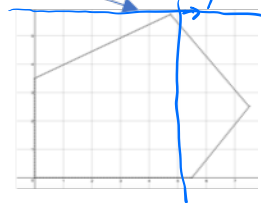
$y \leq 5$

$p_4$



$y \geq 6$

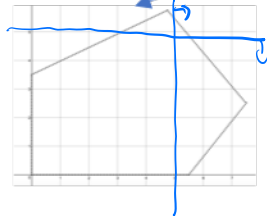
$p_5$



$x \leq 5$

$x \geq 6$

$p_6$



$p_7$

