Branch-and-bound Graphical Example (Accompanying Lesson 16)

1 Branch-and-bound Example

Solve the following IP using branch-and-bound. Solve each sub problem graphically.

$$\begin{aligned}
\text{(P1)} \quad z_{IP}^* = \max \ & 4x_1 - x_2 \\
\text{s.t.} \quad & 7x_1 - 2x_2 \le 14 \\
& 2x_1 - 2x_2 \le 3 \\
& x_2 \le 3 \\
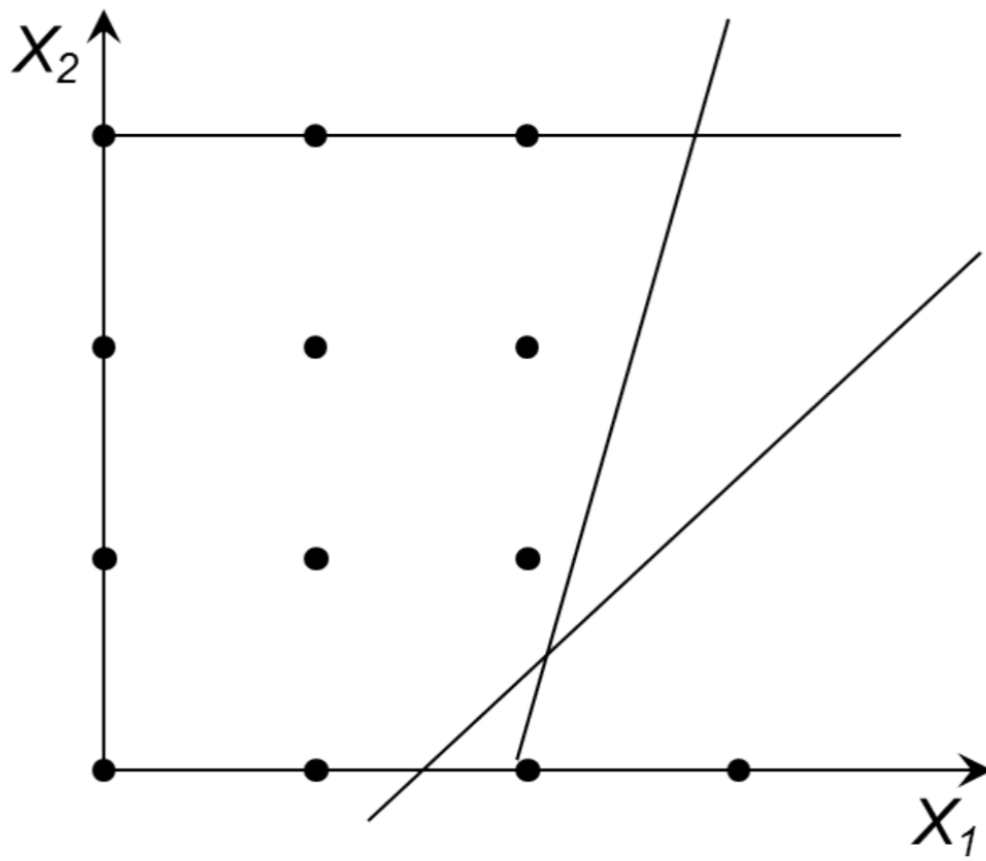& x_1, x_2 \in \mathbb{Z}^{\ge 0}
\end{aligned}$$

- Solve each sub-problem graphically

- Branching Rules

  - Always select the active node with the largest upperbound for branching.
  - Branch on x_1 if it is fractional. Otherwise branch on x_2.

- Book-keeping

  - Keep track of the:
    - incumbent solution $\underline{x}$,
    - global lower bound $\underline{z}$, and
    - list of active nodes.
  - Draw the branch-and-bound tree:
    - Record the local upper bound (z) and relaxed optimal solution (x) for each subproblem.
    - Label each edge with the constraint that is added to form the child subproblem.
    - X-out fathomed nodes. Circle incumbent solution nodes.
  - Use the provided diagram to illustrate the (relaxed) feasible region of each subproblem.

incumbent solution $\underline{x}$ **global lower bound $\underline{z}$**   **active nodes**

Feasible Region

B & B Tree