Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 2

Name: ______________________________

Luxurious Pillows INC has two fulfillment centers in Seattle and Los Angeles. They ship pillows to the DMV area through either a warehouse in Chicago or Dallas for the costs given in the table below.

	Transportation Costs				
	Chicago	Dallas	Washington DC	Baltimore	Annapolis
Seattle	15	25	-	-	-
Los Angeles	20	18	-	-	-
Chicago	-	-	17	15	20
Dallas	-	-	22	13	12

The fulfillment centers in Seattle and Los Angeles have supplies of 500 and 600 pillows, respectively. Likewise, DC, Baltimore, and Annapolis have demands of 300, 600, and 400; respectively. They want to meet demand at as low of a cost as possible.

1. (25 points) Draw the network diagram for this problem. Be sure to indicate the supply/demand of each node.

2. For the next questions, consider the following sets and variables:

   Sets

   Let N be the set of nodes
   Let E be the set of edges

   Variables

   Let $x_{i,j}$ be the flow along edge (i,j) for all $(i,j) \in E$.

   (a) (20 points) With these sets and decision variables, write the concrete objective function for this model. Be sure to clearly define any new sets, variables, or parameters used.

   $$\text{min cost: } 15x_{s,c} + 25x_{s,d} + \cdots + 12x_{d,a}$$

   (b) (15 points) With these decision variables, write the (concrete) flow balance constraint for Dallas.

   $$x_{S,D} + x_{LA,D} = x_{D,DC} + x_{D,B} + x_{D,A}$$

   (c) (20 points) With these sets and decision variables, write the (parameterized) balance of flow constraints for this model. Be sure to clearly define any new sets, variables, or parameters used.

   Let b_n be the demand$-$ supply of node n for each $n \in N$

   $$\sum_{(i,n)\in E} x_{i,n} - \sum_{(n,j)\in E} x_{n,j} = b_n$$

3. (20 points) Suppose that $I = \{1,2,3\}$ and $C = \{A,B\}$. Expand the following set of constraints so that they are written with no index sets (i.e., convert these constraints from parameterized to concrete form).

   $$\sum_{i\in I} r_c x_{i,c} \geq u_c \text{ for all } c \in C$$

   $$r_A x_{1,A} + r_A x_{2,A} + r_A x_{3,A} \geq u_A$$

   $$r_B x_{1,B} + r_B x_{2,B} + r_B x_{3,B} \geq u_B$$