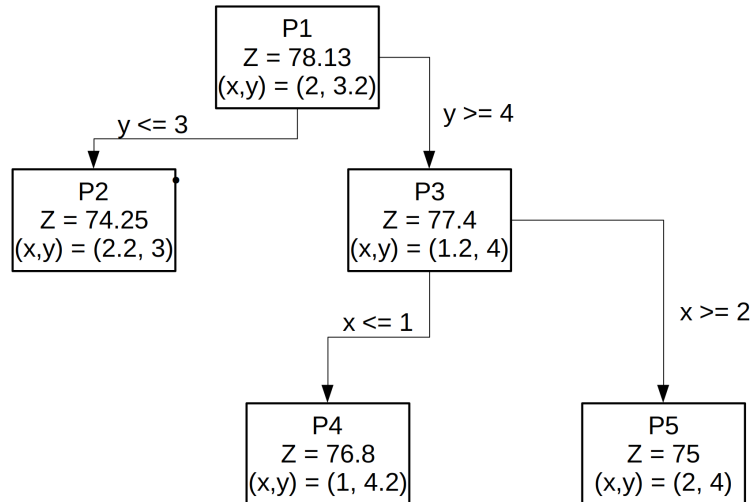# Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 7

Name: solution

Suppose you are solving a maximization integer program via the branch and bound algorithm and at this iteration you have the following tree. Note that at this point, **you have just finished solving problem P5**. This means that you have not yet made any decisions based on the solution of problem P5!

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                        P1
                      Z = 78.13
                     (x,y) = (2, 3.2)
        y <= 3                          y >= 4

      P2                              P3
    Z = 74.25                       Z = 77.4
   (x,y) = (2.2, 3)                (x,y) = (1.2, 4)

                          x <= 1                    x >= 2

                        P4                        P5
                      Z = 76.8                  Z = 75
                     (x,y) = (1, 4.2)          (x,y) = (2, 4)
```

Answer the following questions.

1. (10 points) Which nodes in the tree are currently candidates for branching?

   Nodes P2 and P4 are candidates for branching

2. Let z_{IP}^* denote the optimal objective function value of the integer program.

   (a) (10 points) What is the current best upper bound on z_{IP}^*?

   $$z_{IP}^* \leq 76$$

   (b) (10 points) What is the current best lower bound on z_{IP}^*?

   $$z_{IP}^* \geq 75$$

3. (10 points) What is the current/incumbent solution in the tree?

Current solution is $(x, y) = (2, 4)$

(a) (5 points) Is this current solution optimal? (YES   NO   DON'T KNOW YET).

Dont know yet

(b) (5 points) If we allow a MIP gap of 5% is this current solution optimal? (YES   NO)

Yes, $76 - 75/75 = 1/75 < 0.05$

4. (10 points) After solving problem P5, are there any nodes that can be eliminated from the tree?

yes can eliminate node P2

5. Suppose we are using a strategy where we explore the region with the highest upper bound first.

(a) (10 points) Which of the active nodes should we branch on?

Branch on node P4

(b) (15 points) What constraints would we add to this problem in order to generate our two new nodes (i.e., what variables are we branching on and what are the limits)

We'd add constraints $y \leq 4$ and $y \geq 5$

6. (15 points) Based on this tree, which of the following is **possible** for our final IP solution?

(a) Unique optimal solution? (YES   NO) Yes

(b) Multiple optimal solutions? (YES   NO) Yes

(c) Infeasible? (YES   NO) No

(d) Unbounded? (YES   NO) No

(e) Our final solution is $x = (2, 3.2)$ (YES   NO) No