SA405

HW2: Intro to Network Models-Solution

1. Graham Krackers has 3 plants where it produces (you guessed it) graham crackers that it then ships to 5 warehouses. The table below gives the plant and warehouse locations as well as the cost to ship between each plant and warehouse.

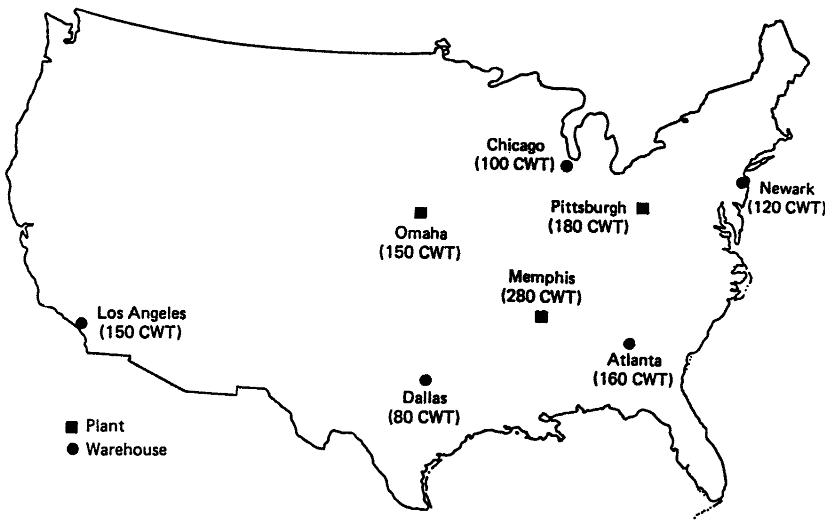
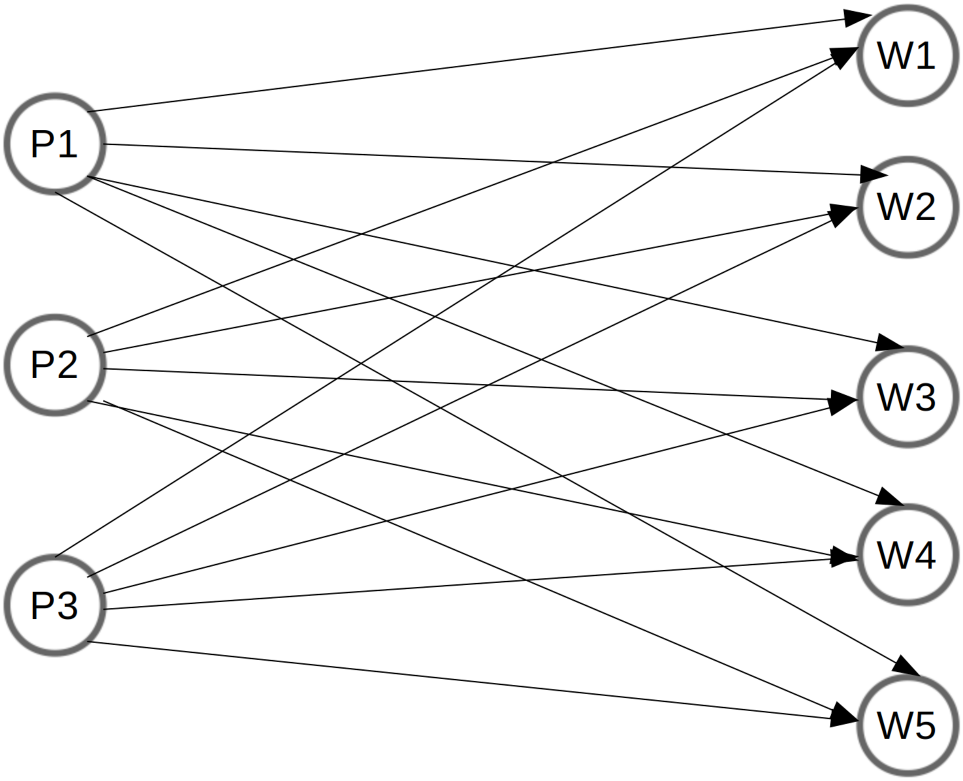


Figure 1: Locations of Graham Krackers plants and warehouses

$/ton	Warehouse 1 (Newark)	Warehouse 2 (Chicago)	Warehouse 3 (Atlanta)	Warehouse 4 (Dallas)	Warehouse 5 (Los Angeles)	Supply
Plant 1 (Pittsburgh)	4	6	5	12	19	180 tons
Plant 2 (Memphis)	10	4	8	5	14	280 tons
Plant 3 (Omaha)	13	9	3	6	10	150 tons
Demand	120 tons	100 tons	160 tons	80 tons	150 tons	

a) Draw the network diagram for this problem.



b) Formulate a concrete model that would allow Graham Krackers to satisfy demand of the warehouses at minimum cost.

Decision Variables

Let $x_{1,1}$ be the number of crackers sent from plant 1 to warehouse 1

Let $x_{1,2}$ be the number of crackers sent from plant 1 to warehouse 2

$\vdots$

Let $x_{3,5}$ be the number of crackers sent from plant 3 to warehouse 5

Objective Function

$$\text{min cost: } 4x_{1,1} + 6x_{1,2} + \cdots + 10x_{3,5}$$

Constraints

$$
\begin{aligned}
\text{st} \quad x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} &\leq 180 && \text{(supply of plant 1)} \\
x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} &\leq 280 && \text{(supply of plant 2)} \\
x_{3,1} + x_{3,2} + x_{3,3} + x_{3,4} + x_{3,5} &\leq 150 && \text{(supply of plant 3)} \\
x_{1,1} + x_{2,1} + x_{3,1} &= 120 && \text{(demand of warehouse 1)} \\
x_{1,2} + x_{2,2} + x_{3,2} &= 100 && \text{(demand of warehouse 2)} \\
x_{1,3} + x_{2,3} + x_{3,3} &= 160 && \text{(demand of warehouse 3)} \\
x_{1,4} + x_{2,4} + x_{3,4} &= 80 && \text{(demand of warehouse 4)} \\
x_{1,5} + x_{2,5} + x_{3,5} &= 150 && \text{(demand of warehouse 5)} \\
x_{1,1}, x_{1,2}, \ldots, x_{3,5} &\geq 0 \text{ integer} && \text{(non-negativity)}
\end{aligned}
$$

c) Formulate a parameterized model that would allow Graham Krackers to satisfy demand of the warehouses at minimum cost.

Sets

Let S be the set of supply nodes

Let D be the set of demand nodes

Let E be the set of edges

Decision Variables

Let $x_{i,j}$ be the number of crackers sent along edge (i, j) for all $(i, j) \in E$

Parameters

Let $c_{i,j}$ be the cost of edge (i, j) for all $(i, j) \in E$

Let s_i be the supply of node i for all $i \in S$

Let d_j be the demand of node j for all $j \in D$

Objective Function

$$\text{min cost: } \sum_{(i,j) \in E} c_{i,j} x_{i,j}$$

Constraints

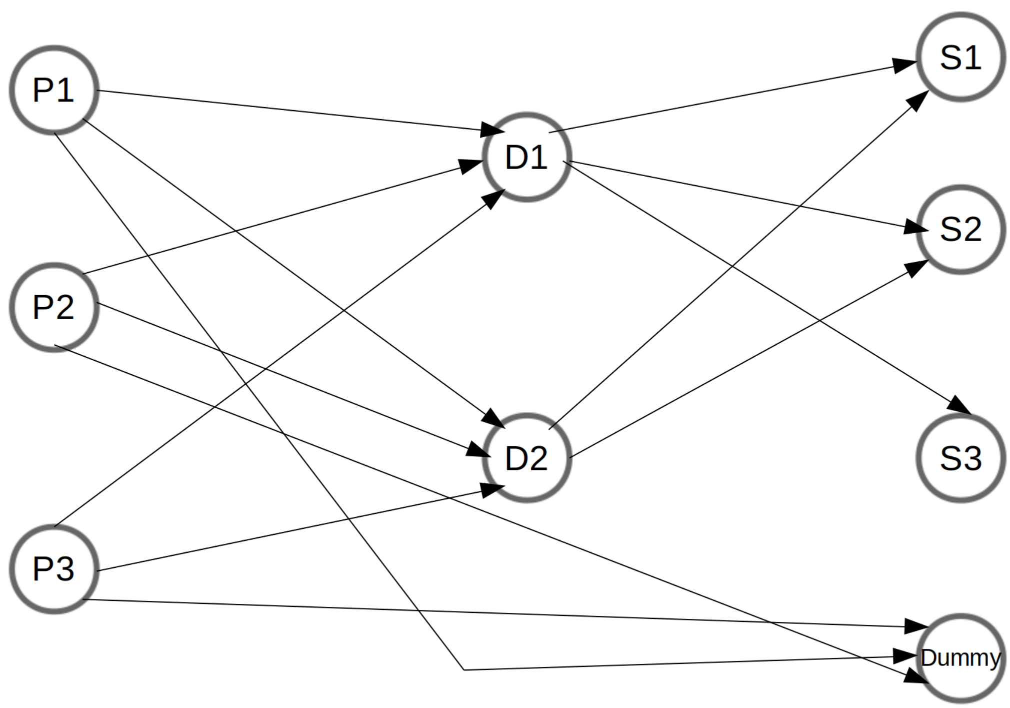
$$
\begin{aligned}
\text{st} \quad \sum_{j \in D} x_{i,j} &\leq s_i && \text{for all } i \in S && \text{(supply constraints)} \\
\sum_{i \in S} x_{i,j} &= d_j && \text{for all } j \in D && \text{(demand constraints)} \\
x_{i,j} &\in \mathbb{Z}^+ && \text{for all } (i, j) \in E && \text{(non-neg)}
\end{aligned}
$$

2. (slightly modified problem 2.36 from the book) Velvet Ale is produced by a local brewer. Currently, it has three production plants in town, one that can produce 1000 bottles a day, another that produces 750 bottles per day, while the third produces only 500 bottles per day. The brewer uses two distributors to deliver its beer to the three stores that sell it. The (per day) demand for the three stores is 700, 600, and 800, respectively. In addition, cost (in cents per bottle) to ship the beer between locations is given in the table below.

	Distributor 1	Distributor 2	Store 1	Store 2	Store 3
Plant 1	8	14	-	-	-
Plant 2	12	10	-	-	-
Plant 3	16	12	-	-	-
Distributor 1	-	-	10	8	12
Distributor 2	-	-	6	15	-

a) Draw the network diagram for this problem.

Note that there is a dummy demand node because the network is unbalanced. The demand of this dummy node is the difference between supply and demand: $(1000 + 750 + 500) - (700 + 600 + 800) = 150$



b) Formulate a concrete model that would allow the brewer to satisfy demand at minimum cost.

Decision Variables

Let $x_{P1,D1}$ be the number of beer bottles sent from plant 1 to distribution center 1

Let $x_{P1,D2}$ be the number of beer bottles sent from plant 1 to distribution center 2

$\vdots$

Let $x_{D2,S2}$ be the number of beer bottles sent from distribution center 2 to store 2

Objective Function

$$\text{min cost: } 8x_{P1,D1} + 14x_{P1,D2} + \cdots + 15x_{D2,S2} + 0x_{P1,D} + 0x_{P2,D} + 0x_{P3,D}$$

Constraints

$$\begin{aligned}
\text{st} \quad x_{P1,D1} + x_{P1,D2} + x_{P1,D} &= 1000 && \text{(supply of plant 1)} \\
x_{P2,D1} + x_{P2,D2} + x_{P1,D} &= 750 && \text{(supply of plant 2)} \\
x_{P3,D1} + x_{P3,D2} + x_{P3,D} &= 500 && \text{(supply of plant 3)} \\
x_{P1,D1} + x_{P2,D1} &= x_{D1,S1} + x_{D1,S2} + x_{D1,S3} && \text{(flow balance DC 1)} \\
x_{P1,D2} + x_{P2,D2} &= x_{D2,S1} + x_{D2,S2} + x_{D2,S3} && \text{(flow balance DC 2)} \\
x_{D1,S1} + x_{D2,S1} &= 700 && \text{(demand of store 1)} \\
x_{D1,S2} + x_{D2,S2} &= 600 && \text{(demand of store 2)} \\
x_{D1,S3} &= 800 && \text{(demand of store 3)} \\
x_{P1,D} + x_{P2,D} + x_{P3,D} &= 150 && \text{(demand of dummy)} \\
x_{P1,D1}, x_{P1,D2}, \ldots, x_{D2,S2} &\geq 0 \text{ integer} && \text{(non-negativity)}
\end{aligned}$$

c) Formulate a parameterized model that would allow the brewer to satisfy demand at minimum cost.

Sets

Let N be the set of nodes

Let E be the set of edges

Decision Variables

Let $x_{i,j}$ be the number of beer bottles sent along edge (i, j) for all $(i, j) \in E$

Parameters

Let $c_{i,j}$ be the cost of edge (i, j) for all $(i, j) \in E$

Let b_n be demand $-$ supply at node n for all $n \in N$

Objective Function

$$\text{min cost: } \sum_{(i,j)\in E} c_{i,j} x_{i,j}$$

Constraints

$$\text{st} \quad \sum_{(i,n)\in E} x_{i,n} - \sum_{(n,j)\in E} x_{n,j} = b_n \text{ for all } n \in N \text{ (flow balance)}$$

$$x_{i,j} \in \mathbb{Z}^+ \qquad \text{for all } (i, j) \in E \quad \text{(non-neg)}$$