

## Lesson 3: Network Flows: Shortest Path

### 1 Print Shop – Copier Purchase Plan

In Scranton, PA, Dunder Mifflin prints high volumes of photocopying to meet their high demand. The office manager, Michael Scott, is interested in determining when to purchase a new high-speed copier over the next 5 years. During the years that a copier is not purchased, maintenance must be performed. The maintenance cost depends on the age of the copier. The table below provides estimated maintenance cost per age of machine.

Age at Beginning of Year	Maintenance Cost for the Coming Year
0	\$2000
1	\$3500
2	\$6000
3	\$9500

The cost (in today's dollars) of purchasing copiers at the beginning of each year is given below.

Year	Purchase Cost
1	\$10,000
2	\$13,000
3	\$16,500
4	\$20,000

Determine the years in which a new copier should be purchased in order to minimize the cost (purchase + maintenance) of having a machine for 5 years.

- Draw a node for each year, 1 through 5, from left to right. Draw every possible directed arc from a year to a later year; e.g., (1, 2) and (1, 3), but not (3, 1).

- Arc  $(i, j)$  represents the cost of purchasing a copier at the beginning of year  $i$  and maintaining it until the beginning of year  $j$ . For example, the cost incurred by selecting arc  $(1, 4)$  (in thousands) is  $\$10 + \$2 + \$3.5 + \$6 = \$21.5$ : which is the cost of purchasing a new copier in year 1, then maintaining it through years 1, 2, and 3. Add arc costs to the network diagram.

## 2 How is this a shortest path problem?

A **path** is an ordered sequence of connected arcs such that any node is “visited” at most once.

1. In this problem, the minimum cost strategy corresponds to the minimum cost *path* from where to where?
2. What are the decision variables for this problem?

3. Write the concrete model for this problem

4. Write the parameterized model for this problem