HW7: Facility Location Models

A county is deciding where to place police precincts in order to respond to potential emergencies. There are 10 locations which need police protection and 5 of them are candidates to place a police station. The county has a budget of $2,500,000 to place the police stations. You are given the following information:

- C is the set of locations that need police protection $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- S is the set of potential locations to place a police station $S = \{1, 4, 6, 8, 9\}$.
- Each zone in C has a demand of 150. Additionally, the demand of each zone must be fully served by exactly 1 police station.
- Each potential location has a fixed cost to open and capacity. The table below gives these values:

Location	Fixed Cost	Capacity
1	$800,000	500
4	$750,000	450
6	$1,200,000	700
8	$1,000,000	600
9	$400,000	325

- Lastly, the distance between each potential police location and each zone in the city is given in the table below:

Station	1	2	3	4	5	6	7	8	9	10
1	0	7	6	2	2	5	3	4	7	9
4	2	3	4	0	6	5	7	9	1	3
6	5	4	3	5	2	0	1	8	6	5
8	4	3	5	9	5	8	3	7	9	9
9	7	4	6	1	9	6	4	9	0	1

Part 1: Formulate a concrete model whose solution would minimize the total distance traveled by the police officers while remaining within budget.

We need two types of variables in this problem

Decision Variables

Let $x_1 = 1$ if a station is placed at location 1 and 0 otherwise
$\vdots$
Let $x_9 = 1$ if a station is placed at location 9 and 0 otherwise
Let $y_{1,1} = 1$ if location 1 is assigned to station 1 and 0 otherwise

$\vdots$

Let $y_{10,9} = 1$ if location 10 is assigned to station 9 and 0 otherwise

Objective Function

$$\text{min distance: } 0y_{1,1} + 7y_{2,1} + \cdots + 1y_{10,9}$$

Constraints

There are 4 types of constraints:

- Can't exceed budget of $2,500,000
- Every location is assigned to exactly 1 station
- No station exceeds its maximum capacity
- All variables are binary

$$\begin{aligned}
0.8x_1 + 0.75x_4 + 1.2x_6 + 1x_8 + 0.4x_9 &\leq 2.5 && \text{(budget)} \\
y_{1,1} + y_{1,4} + y_{1,6} + y_{1,8} + y_{1,9} &= 1 && \text{(location 1 has 1 station)} \\
&\vdots \\
y_{10,1} + y_{10,4} + y_{10,6} + y_{10,8} + y_{10,9} &= 1 && \text{(location 10 has 1 station)} \\
150y_{1,1} + 150y_{2,1} + \ldots + 150y_{10,1} &\leq 500x_1 && \text{(capacity of 1)} \\
&\vdots \\
150y_{1,9} + 150y_{2,9} + \ldots + 150y_{10,9} &\leq 325x_9 && \text{(capacity of 9)} \\
y_{1,1}, \ldots, y_{10,9} &\in \{0,1\} && \text{(binary)} \\
x_1, \ldots, x_9 &\in \{0,1\} && \text{(binary)}
\end{aligned}$$

Part 2: Parameterize your model from part 1.

Sets

Let C be the set of locations
Let S be the set of potential stations

Variables

Let $y_{c,s} = 1$ if location c is assigned to station s for all $c \in C$ and $s \in S$
Let $x_s = 1$ if station s is opened and 0 otherwise for all $s \in S$

Parameters

Let $d_{c,s}$ be the distance between customer c and station s for all $c \in C$ and $s \in S$
Let u_s be the capacity of station s for all $s \in S$
Let f_s be the fixed cost of station s for all $s \in S$
Let h_c be the demand of customer c for all $c \in C$

Objective Function

$$\text{min: } \sum_{c \in C} \sum_{s \in S} d_{c,s} y_{c,s}$$

Constraints

$$\begin{aligned}
\sum_{s \in S} f_s x_s &\leq 2.5 & & \text{(budget)} \\
\sum_{s \in S} y_{c,s} &= 1 & \text{for } c \in C & \quad \text{(each location assigned to 1 station)} \\
\sum_{c \in C} h_c y_{c,s} &\leq u_s x_s & \text{for } s \in S & \quad \text{(Capacity)} \\
y_{c,s} &\in \{0,1\} & \text{for } c \in C \text{ and } s \in S & \quad \text{(binary)} \\
x_s &\in \{0,1\} & \text{for } s \in S & \quad \text{(binary)}
\end{aligned}$$