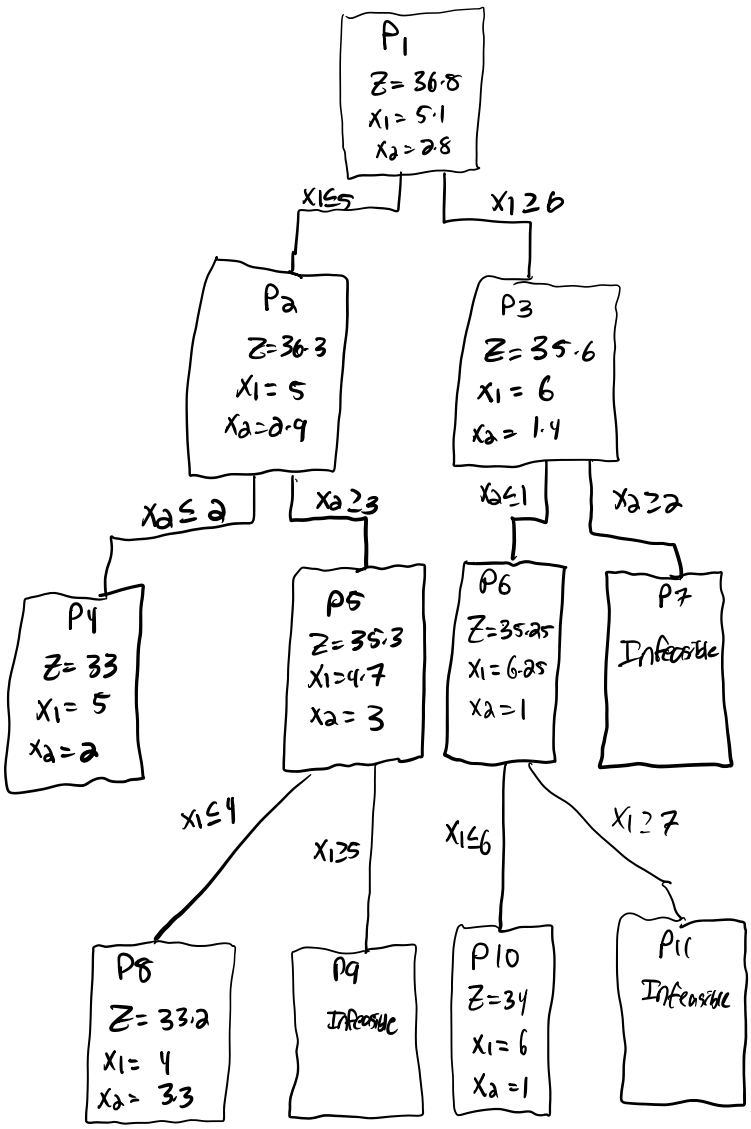
B&B HW solution

① max $5x_1 + 4x_2$

s.t. $6x_1 + 13x_2 \leq 67$

$8x_1 + 5x_2 \leq 55$

$x_1, x_2 \in \mathbb{Z}^+$



Will solve each problem in python

P_1: $Z = 36.8$ $(x_1, x_2) = (5.1, 2.8)$

Both fractional, Branch on x_1

I know $Z^*_{IP} \leq 36.8$

P_2: $Z = 36.3$ $(x_1, x_2) = (5, 2.9)$

x_2 fractional

P_3: $Z = 35.6$, $(x_1, x_2) = (6, 1.4)$

x_2 fractional

At this point, I know

$Z^*_{IP} \leq 36$.

I can branch on either P_2 or P_3. Will choose P_2

P_4: $Z = 3$, $(x_1, x_2) = (5, 2)$

Integer

P_5: $Z = 35.3$, $(x_1, x_2) = (4.7, 3)$

At this point, I know

$33 \leq Z^*_{IP} \leq 36$

Can branch on either P_3 or P_5, Choose P_3

P_6: $Z = 35.25$, $(x_1, x_2) = (6.25, 1)$

P_7: Infeasible

2 Active nodes, P_5 and P_6

Can branch on either. Choose P_5.

P_8: $Z = 33.2$ $(x_1, x_2) = (4, 3.3)$

Prune P_8 because bound = current LB

P_9: Infeasible

P_{10}: $Z = 34$ $(x_1, x_2) = (6, 1)$

We know $34 \leq Z^*_{IP} \leq 36$

(Note, our upper bound can be 35 here)

P_{11}: Infeasible

No Active nodes, P_{10} optimal

$Z^* = 34$

$x^* = (6, 1)$

(2) Max $20x_1 + 16x_2 + 25x_3 + 19x_4 + 9x_5$

s.t. $3x_1 + 2x_2 + 5x_3 + 4x_4 + 2x_5 \leq 13$

$x_1, x_2, x_3, x_4, x_5 \in \{0,1\}$

P1: $Z = 73.5$, x_4 is fractional

We know $Z^*_{IP} \leq 73.5$

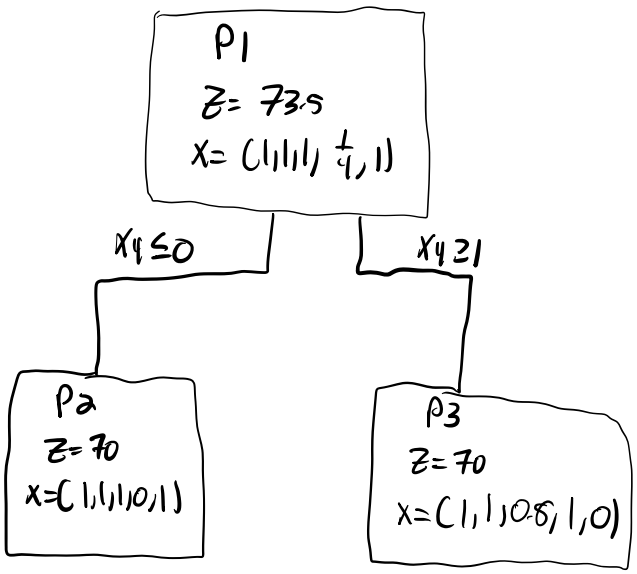
Branch on x_4

P2: $Z = 70$, integer solution

We know $70 \leq Z^*_{IP} \leq 73$

P3: $Z = 70$, x_3 fractional

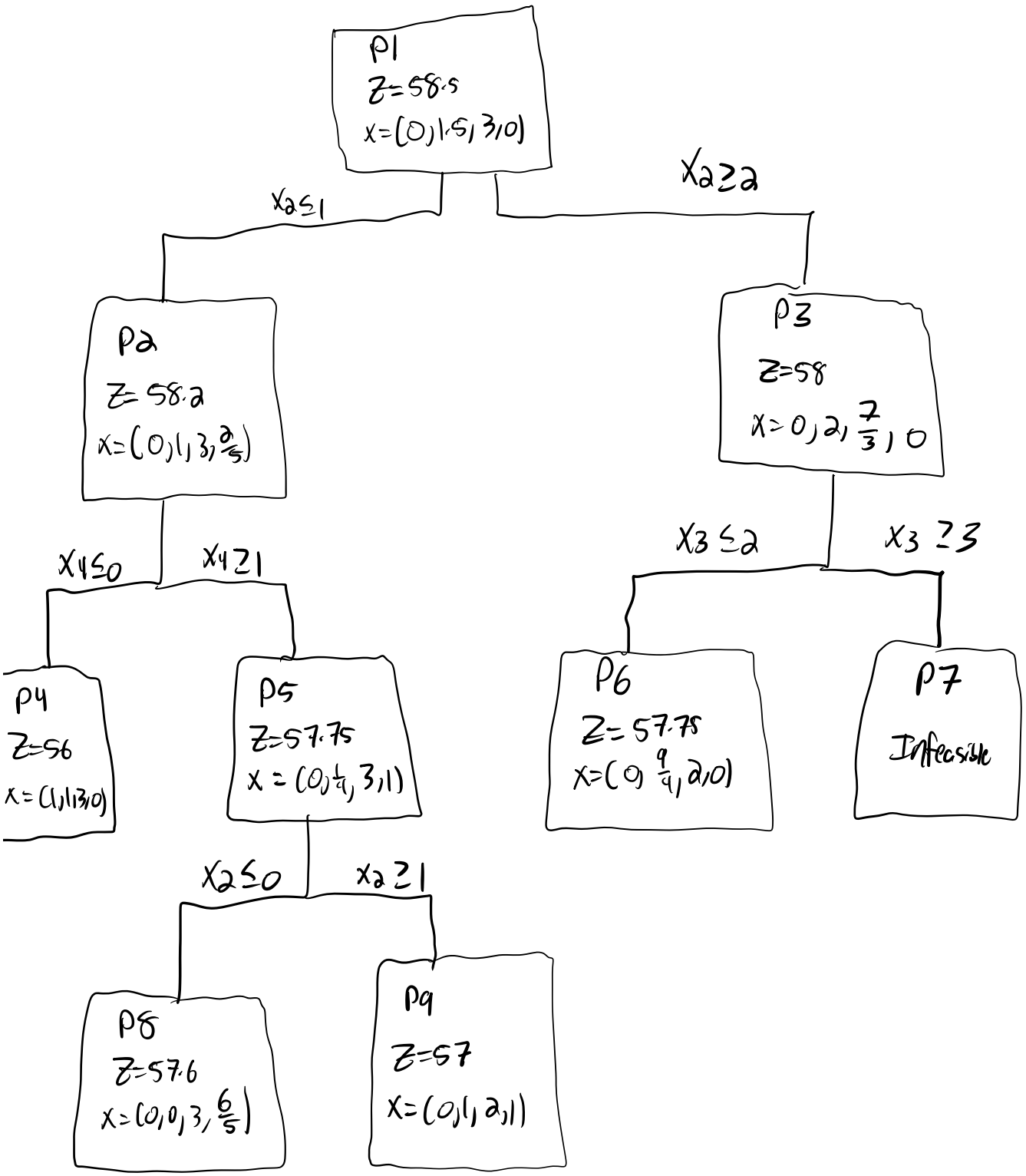
Stop! 70 = lower bound, so no value in exploring this region



P2 optimal

$Z^* = 70$

$x^* = (1,1,1,1,0,1)$


P1
Z=58.5
x=(0,1.5,3,0)
x2≤1
x2≥2
P2
Z=58.2
x=(0,1,3,2/5)
P3
Z=58
x=0,2,7/3,0
x4≤0
x4≥1
x3≤2
x3≥3
P4
Z=56
x=(1,1,3,0)
P5
Z=57.75
x=(0,1/4,3,1)
P6
Z=57.75
x=(0,9/4,2,0)
P7
Infeasible
x2≤0
x2≥1
P8
Z=57.6
x=(0,0,3,6/5)
P9
Z=57
x=(0,1,2,1)