HW8: IP Formulations

1. Consider the following integer program.

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 - 4x_3 \\ \text{st} \quad & x_1 + x_2 + 2x_3 \le 7 \\ & x_2 + x_3 \ge 1.25 \\ & x_1 \le 5 \\ & x_1 \ge 0,\text{integer} \\ & x_2, x_3 \in \{0, 1\} \end{aligned}$$

   (a) An inequality is called **valid** if adding it does not violate any of the constraints of the model. In other words, adding a valid inequality does not remove any current integer feasible points. Is the inequality $x_2 + x_3 \le 2$ a valid inequality? Why or why not? Yes it is valid because it does not restrict the values x_2 and x_3 can take.

   (b) Is the inequality $x_1 + x_2 + x_3 \le 3$ a valid inequality? Why or why not? No it is not a valid inequality because it restricts the values that x_1 can take.

   (c) Suppose you solve the LP relaxation and obtain the solution $(x_1, x_2, x_3) = (4.75, 1, 0.25)$

      i. What is the objective function value associated with this solution? 11.5

      ii. Is this solution optimal for your IP? No, it's not integer

   (d) Consider the solution $(4, 1, 1)$.

      i. Is this solution feasible? Yes it does not violate any constraints.

      ii. What is the objective function value of this solution? 7

   (e) Using all of the information you've obtained so far, what are lower and upper bounds for the optimal objective function value of the IP, z_{IP}?

$$7 \le z_{IP} \le 11$$

   (f) Suppose you replace the constraint $x_2 + x_3 \ge 1.25$ with $x_2 + x_3 \ge 2$

      i. Explain why this is an appropriate constraint substitution. Because they are both binary, ≥ 1.25 is the same as ≥ 2

      ii. Suppose after solving the LP relaxation, you now obtain the solution $(4, 1, 1)$. Is this point optimal for the original IP? (Yes, No, Don't Know Yet). Why or why not? Yes it is because the new inequality is valid so our new solution is the real upper bound and the upper bound matches the lower bound so it must be optimal.

2. Is the set $S = \{(x, y) : x^2 + y^2 \le 4\}$ a convex set? Why or why not?

   Yes it is. If you graph it, the graph is a filled in circle.

3. Is the set $S = \{(x, y) : 2x + y \le 3, x \le 2, x \ge 0 \text{ integer}, y \ge 0 \text{ integer}\}$ a convex set? Why or why not? No it's not. This is the feasible region of an IP and IPs are not convex sets.