

Lesson 11: Scheduling Problems

1 Overview

Scheduling models are a type of optimization problem that determine how jobs/people/etc should be assigned given scheduling requirements while optimizing a given objective.

Just like in other models that we've seen, in many cases we also need to include **logical constraints** to account for complicating factors in the models.

2 Problem Description

The USS George H.W. Bush (CVN-77) is the tenth and final Nimitz-class supercarrier of the United States Navy. You've been tasked with scheduling the following test flights on the USS Bush:

Flights	Total Flight Time (hours)	Flight Importance
1	2	4
2	5	3
3	1	2
4	4	5
5	3	1
6	7	1.5

For this problem, you can to assume that each flight must take off and land before the next flight can leave. In other words, the test flights cannot commence simultaneously. Your ship captain has tasked you with scheduling the in order in which these test flights should commence order to minimizing the total weighted (by importance) cumulative flight completion time.

3 Concrete Model

To model this problem, it is convenient to define a set T which is the time horizon. To solve this problem, we need to plan for hours. Our variables can then be like the typical assignment variable. Specifically, we'll define:

$$x_{f,t} = \begin{cases} 1 & \text{if} \\ 0 & \text{otherwise} \end{cases}$$

Objective Function: We want to minimize the weighted flight completion time. Suppose $x_{2,1} = 1$. What is the completion time of flight 2?

Likewise, suppose $x_{4,6} = 1$. What is the completion time of flight 4?

So what is the completion time of flight 4 if it can start at any time t ?

If we weight this by importance, we obtain the following objective function:

Constraints

- Each flight must be completed.
 - Write a constraint that ensures flight 1 is completed in some time slot.

 - Likewise, write a constraint which ensures that flight 6 is scheduled in some time slot.

- No two flights can occupy the carrier at the same time.
 - Write a constraint which ensures that no two flights can be scheduled at time 1.
 - Using the same style, write a constraint that ensures no two flights can be scheduled at time 2.
 - Why does the constraint above not work for time 2?
 - Correct the constraint for time 2. We would need a similar constraint like this for each other time slot.
- Which constraint is remaining?

4 Parameterized Model

Before we parameterize the model, let's clearly define the sets, parameters, and variables and explain what they do.

Sets

Parameters

Variables

Objective Function

- What is the completion time of each flight?
- What is the objective function?

Constraints

- a. To ensure that every flight must be scheduled on the Bush, write a constraint that every flight must be scheduled to begin in some time slot:

- b. Write a constraint to ensure that no two flights simultaneously occupy the carrier in any time slot. There are several ways to do so, but we can define a set F_k , for each $k \in \mathbf{T}$, that represents all combinations of flights f and time slots t that would occupy the carrier at time k if flight f is scheduled at time t . Formally:

$$F_k = \{(f, t) : f \in F \text{ and } t \in \{\max\{0, k - p_f + 1\}, \dots, \min\{k, T - p_f\}\}\} \quad \forall k \in \mathbf{T}.$$

What are the elements of F_k for $k = 3$?

Using the set F_k above, write a constraint to ensure that every time slot must be assigned to one flight:

- c. Write the final binary constraints.

5 Multiple Carriers

Now, it turns out that we can use any of the 11 carriers in the fleet to complete our flights. We are now going to convert the parameterized version of this model for the single carrier case to a parameterized version of the model with multiple carriers.

New Parameters and Sets:

- Suppose now that there exists a set of m carriers.

New Variables

- Variables $x_{ftc} \in \{0, 1\}$ equal 1 if flight $f \in F$ is scheduled at time $t \in T$ on carrier $c \in \{1, \dots, m\}$.

Notes:

In this model, we want to allow idleness (especially at the end of the time horizon) on the carriers. This is necessary because when multiple carriers are used, we typically do not use each carrier over the entire time horizon, nor do we know how long each carrier will be utilized before solving the scheduling problem.

How would you revise your constraints from the single carrier example?

The new objective weighted completion time objective becomes:

6 Alternative Objectives:

Minimize weighted tardiness

For *weighted tardiness*, the completion time of flight f is $(t + p_f)$ if $x_{ftc} = 1$ for some carrier c . In that case, the tardiness of flight f would be given by $\max\{0, t + p_f - d_f\}$. Before solving the model, we can simply compute tardiness parameters $\tau_{ft} = \max\{0, t + p_f - d_f\}$ for each $f \in F$ and $t \in \{0, \dots, T - p_f\}$. The objective would then be:

Minimize the number of late flights

A similar approach also handles the case in which we *minimize the number of late flights*. Define binary parameters $\ell_{ft} = 1$ if $(t + p_f) > d_f$ (and thus scheduling flight f at time t would cause it to be late), and $\ell_{ft} = 0$ otherwise, for all $f \in F$ and $t \in \{0, \dots, T - p_f\}$. To minimize the number of late flights, we would use the objective function:

Minimize makespan

Minimizing makespan requires a slightly different approach. Define a variable, z , that represents the maximum completion time of any flight. Because the completion time of flight f is given by $\sum_{t=0}^{T-p_f} \sum_{c=1}^m (t + p_f)x_{ftc}$, the following set of constraints guarantees that z is *at least* as large as the makespan:

$$z \geq \sum_{t=0}^{T-p_f} \sum_{c=1}^m (t + p_f)x_{ftc} \quad \forall f \in F. \quad (1)$$

Minimizing the makespan requires the following simple objective:

$$\text{Min } z. \quad (2)$$

Because z is being minimized, an optimal solution will exist in which z equals the completion time of the latest flight to finish on a carrier. This is very similar idea to the p-center facility location.