HW8: IP Formulations

1. Consider the following integer program.

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 - 4x_3 \\ \text{st} \quad & x_1 + x_2 + 2x_3 \leq 7 \\ & x_2 + x_3 \geq 1.25 \\ & x_1 \leq 5 \\ & x_1 \geq 0\text{,integer} \\ & x_2, x_3 \in \{0, 1\} \end{aligned}$$

   (a) An inequality is called **valid** if adding it does not violate any of the constraints of the model. In other words, adding a valid inequality does not remove any current integer feasible points. Is the inequality $x_2 + x_3 \leq 2$ a valid inequality? Why or why not?

   (b) Is the inequality $x_1 + x_2 + x_3 \leq 3$ a valid inequality? Why or why not?

   (c) Suppose you solve the LP relaxation and obtain the solution $(x_1, x_2, x_3) = (4.75, 1, 0.25)$
      i. What is the objective function value associated with this solution?
      ii. Is this solution optimal for your IP?

   (d) Consider the solution $(4, 1, 1)$.
      i. Is this solution feasible?
      ii. What is the objective function value of this solution?

   (e) Using all of the information you've obtained so far, what are lower and upper bounds for the optimal objective function value of the IP, z_{IP}?

   (f) Suppose you replace the constraint $x_2 + x_3 \geq 1.25$ with $x_2 + x_3 \geq 2$
      i. Explain why this is an appropriate constraint substitution.
      ii. Suppose after solving the LP relaxation, you now obtain the solution $(4, 1, 1)$. Is this point optimal for the original IP? (Yes, No, Don't Know Yet). Why or why not?

2. Is the set $S = \{(x, y) : x^2 + y^2 \leq 4\}$ a convex set? Why or why not?

3. Is the set $S = \{(x, y) : 2x + y \leq 3, x \leq 2, x \geq 0 \text{ integer}, y \geq 0 \text{ integer}\}$ a convex set? Why or why not?