

HW10: Branch and Bound part 2

1. Solve the following problem using Branch and Bound

$$\begin{array}{ll}\max & 20x_1 + 16x_2 + 25x_3 + 14x_4 + 9x_5 \\ \text{st} & 3x_1 + 2x_2 + 5x_3 + 4x_4 + 2x_5 \leq 13 \\ & x_1, x_2, x_3, x_4, x_5 \in \{0, 1\}\end{array}$$

Solve each subproblem either using python or using the following algorithm:

- Compute the ratio r_i for each variable where

$$r_i = \frac{\text{coefficient of } x_i \text{ in objective function}}{\text{coefficient of } x_i \text{ in constraint}}$$

- Order r_i from largest to smallest
- Starting with the largest r_i , set x_i to its largest possible (continuous) value without violating the existing constraints.

For example, this algorithm to solve the initial problem would compute r_i as:

$$r_i = \left\{ \frac{20}{3}, 8, 5, \frac{7}{2}, \frac{9}{2} \right\}$$

The largest of these is 8 so we would set $x_2 = 1$. The RHS of the constraint is now 11. The next largest is $\frac{20}{3}$, so we would set $x_1 = 1$ and the RHS of the constraint is now 8. The next largest is 5, so we would set $x_3 = 1$ and the RHS of the constraint becomes 3. The next largest is $\frac{9}{2}$, so we would set $x_5 = 1$ and the RHS of the constraint becomes 1. All that's left is x_4 . We can't set it equal to 1, instead we maximize it and set $x_4 = \frac{1}{4}$. So the initial solution is $x = (1, 1, 1, 0.25, 1)$.

2. Solve the following problem using Branch and Bound:

$$\begin{array}{ll}\max & 5x_1 + 15x_2 + 12x_3 + 18x_4 \\ \text{st} & 20x_1 + 40x_2 + 30x_3 + 50x_4 \leq 150 \\ & x_1 \leq 5 \\ & x_2 \leq 3 \\ & x_3 \leq 3 \\ & x_4 \leq 2 \\ & x_1, x_2, x_3, x_4 \in \mathbb{Z}^+\end{array}$$

Solve each subproblem either using python or using the same algorithm as problem 2 (notice that this time the variables are not binary so you'd maximize the value based on their upper bounds).