Branch-and-bound Easy Example (Accompanying Lesson 16)

1 Branch-and-bound Example

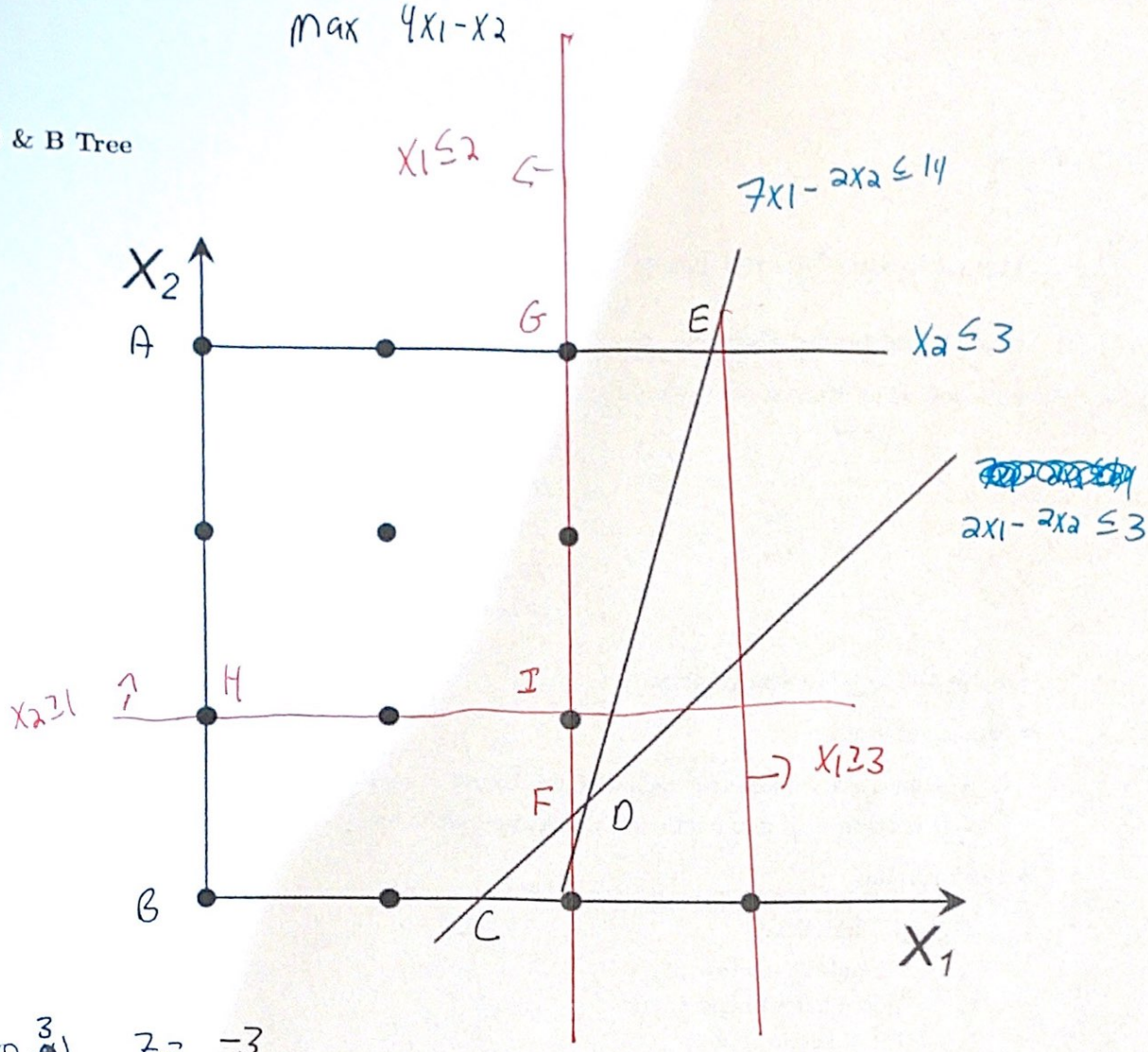
Solve the following IP using branch-and-bound. Solve each sub problem graphically.

$$\begin{aligned} \text{(P1)} \quad z^*_{IP} = \max\ & 4x_1 - x_2 \\ \text{s.t.}\ & 7x_1 - 2x_2 \leq 14 \\ & 2x_1 - 2x_2 \leq 3 \\ & x_2 \leq 3 \\ & x_1, x_2 \in \mathbb{Z}^{\geq 0} \end{aligned}$$

- Solve each sub-problem graphically
- Branching Rules
  - Always select the active node with the largest upperbound for branching.
  - Branch on x_1 if it is fractional. Otherwise branch on x_2.
- Book-keeping
  - Keep track of the:
    - incumbent solution $\underline{x}$,
    - global lower bound $\underline{z}$, and
    - list of active nodes.
  - Draw the branch-and-bound tree:
    - Record the local upper bound (z) and relaxed optimal solution (x) for each subproblem.
    - Label each edge with the constraint that is added to form the child subproblem.
    - X-out fathomed nodes. Circle incumbent solution nodes.
  - Use the provided diagram to illustrate the (relaxed) feasible region of each subproblem.

incumbent solution $\underline{x}$ **global lower bound $\underline{z}$** **active nodes**

B & B Tree



A: $(0, \frac{3}{2})$, $z = -3$

B: $(0,0)$, $z = 0$

C: $(\frac{3}{2}, 0)$, $z = 6$

D: $(\frac{11}{5}, \frac{7}{10})$, $z = 8.1$

E: $(\frac{20}{7}, 3)$, $z = 8.4$

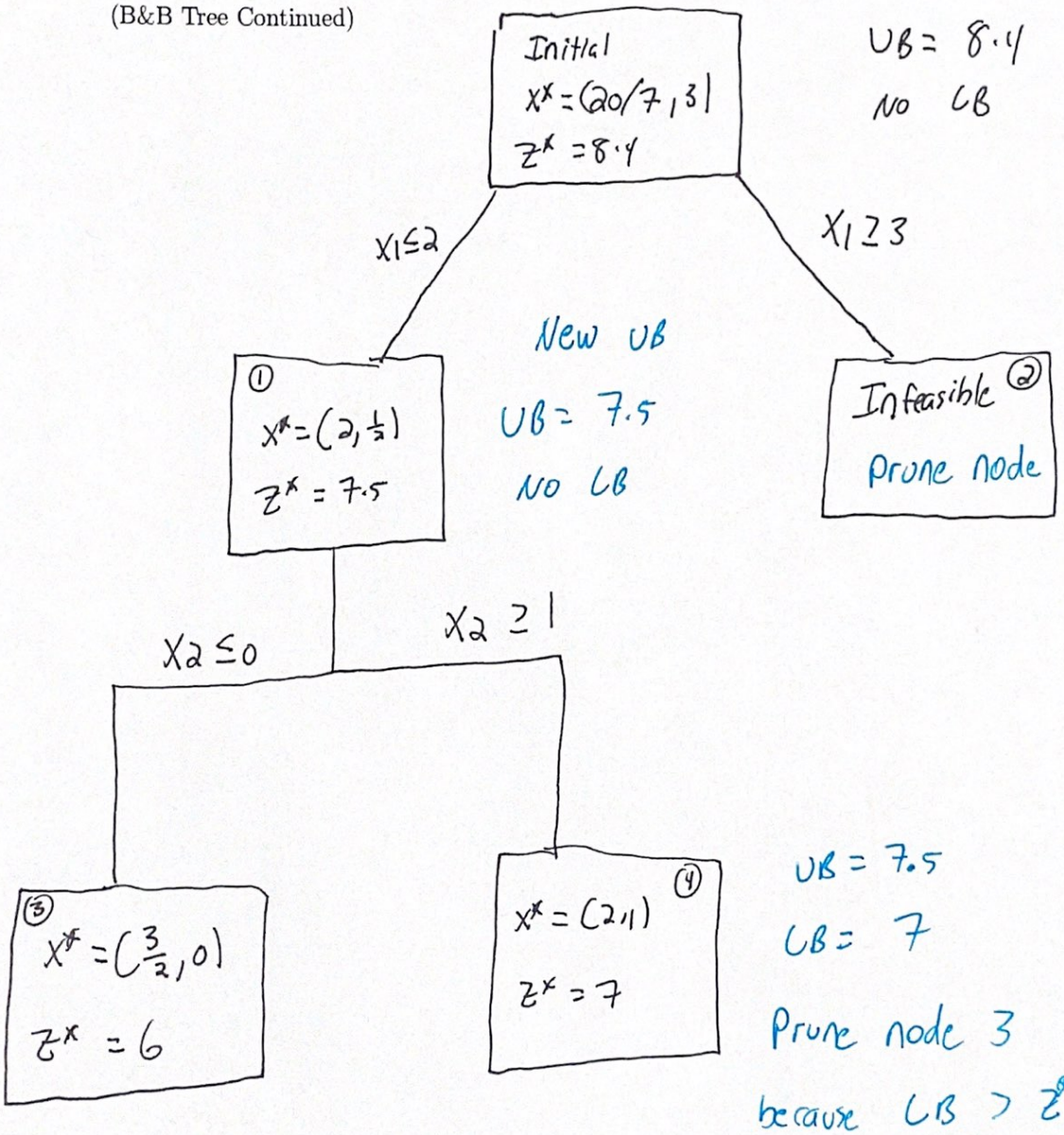
F: $(2, \frac{1}{2})$, $z = 7.5$

G: $(2,3)$, $z = 5$

H: $(0,1)$, $z = -1$

I: $(2,1)$, $z = 7$

(B&B Tree Continued)



UB = 7.5

LB = 7

Prune node 3

because LB > z^*_3

No further nodes to branch on

$z^*_{IP} = 7$

$x^*_{IP} = (2, 1)$