**SA405, Fall 2021**

**Exam 2**
**4 November 2021**

**Name:** ____________________________

- **One sided** 8.5 by 11 inch formula/note sheet is allowed.
- Show work clearly and neatly.
- Define all notation used.
- Please read each question carefully. If you are not sure what a question is asking, ask for clarification.
- If you start over on a problem, please CLEARLY indicate what your final answer is, along with its accompanying work.

Grade Table (for teacher use only)

| Question | Points | Score |
|---|---|---|
| 1 | 45 | |
| 2 | 55 | |
| Total: | 100 | |

*Please write the following statement and sign it.*
The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

1. Anne Arundel County is determining locations for COVID booster shot clinics. They have enough vaccines to open two clinics, each of which would have an allotment of 100,000 booster shots. The potential locations for clinics are Annapolis, Crofton, Severn, and Glen Burnie. The distance (in miles) between each of these potential vaccine clinics and cities they must serve, as well as the demand of each city, is given in the table below.

| | Distances (miles) | | | | | | |
|---|---|---|---|---|---|---|---|
| Clinic Locations | Annapolis | Crofton | Edgewater | Glen Burnie | Odenton | Parole | Severn |
| Annapolis | 0 | 10 | 15 | 13 | 17 | 6 | 22 |
| Crofton | 10 | 0 | 17 | 9 | 11 | 16 | 27 |
| Glen Burnie | 17 | 11 | 12 | 0 | 7 | 14 | 11 |
| Severn | 22 | 27 | 6 | 8 | 11 | 20 | 0 |
| Demand | 35,000 | 15,000 | 12,000 | 50,000 | 7,000 | 11,000 | 47,000 |

To get started, the county has defined the following:

**Sets**

Let $S$ be the set of possible clinic locations, $S = \{A, C, G, S\}$

Let $C$ be the set of cities needing vaccines, $C = \{A, C, E, G, O, P, S\}$

**Variables**

Let $x_s = 1$ if a vaccine clinic is placed in location $s$ and 0 otherwise for all $s \in S$

Let $y_{c,s} = 1$ if city $c$ is assigned to vaccine clinic $s$ for all $c \in C$ and all $s \in S$, and 0 otherwise.

(a) (15 points) The county has decided that they want to place the clinics in order to minimize the total distance traveled by the people in each city (for example, if Crofton is assigned to Annapolis, then 15,000 people must travel 10 miles to a clinic leading to a cumulative distance traveled of 150,000).

Assume that if a city is assigned to a clinic, then that clinic will satisfy the entire demand for that city. Write an objective function, in both concrete and parameterized form, that would minimize this total distance traveled. Be sure to define any new parameters used.

**Solution:** Concrete form:

$$\min 35000 * 0 * y_{A,A} + 35000 * 10 * y_{A,C} + 35000 * 17 * y_{A,G} + \cdots + 47,000 * 0 * y_{S,S}$$

Parameterized form:
Let $d_{c,s}$ be the distance from customer $c$ to clinic $s$ for all $c \in C$ and $s \in S$
Let $h_c$ be the demand of customer $c$

$$\min \sum_{c \in C} \sum_{s \in S} d_{c,s} h_c y_{c,s}$$

(b) (10 points) Since the county only has an allotment for 200,000 total vaccines, they can only open 2 clinics. Using the variables defined above, write a concrete constraint that ensures only two clinics are open.

**Solution:**

$$x_A + x_G + x_P + x_S = 2$$

(c) (10 points) Since each clinic has an allotment of 100,000 vaccines, you realize that a capacity constraint also needs to be enforced. If the clinic is open, it should be allowed to serve only up to 100,000 people. If the clinic is closed, it should not be able to serve any customers.

Provide a parameterized constraint that enforces that if a clinic is open, it can service up to 100,000 units of total demand and if it's closed it can't service any demand. Be sure to clearly define any new parameters used.

**Solution:** Let $h_c$ be the demand of customer $c$

$$\sum_{c \in C} h_c y_{c,s} \leq 100,000 x_s \text{ for all } s \in S$$

(d) (10 points) Lastly, you realize that you are missing a constraint linking your $x_s$ variables and $y_{c,s}$ variables. Specifically, you should only be able to assign a city to a clinic if that clinic is open.

Write a concrete constraint which enforces that, if Annapolis is assigned to Crofton, then the clinic at Crofton must be open.

**Solution:**

$$y_{A,C} \leq x_C$$

2. Dwight Schrute has been tasked by the Dant with visiting the set $R$ of restaurants in order to rate them.

$$R = \{\text{McDonald's, Burger King, Popeye's, ChickFilA, PF Changs, Basmati, El Cabrito, Chevy's, Pizza Hut, Domino's Pizza}\}$$

The Dant wants Dwight Schrute, Kevin Malone, and Oscar Martinez to review these restaurants in order to minimize the similarities between consecutively visited restaurants (e.g., He wants you to avoid visiting McDonald's and Burger King one after the other because they taste so similar).

The following table displays the similarity factor $s$ between each pair of restaurants. A value of 0 means that the restaurants have exactly the same menu, and a value of 10 means that they are polar opposite restaurants. The restaurants are all numbered 0 (USNA) to 10 (Domino's).

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | - | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 1 | - | - | 1 | 4 | 5 | 8 | 8 | 9 | 9 | 7 | 7 |
| 2 | - | - | - | 4 | 6 | 7 | 8 | 10 | 9 | 7 | 7 |
| 3 | - | - | - | - | 1 | 7 | 9 | 9 | 9 | 7 | 7 |
| 4 | - | - | - | - | - | 8 | 4 | 9 | 8 | 7 | 8 |
| 5 | - | - | - | - | - | - | 3 | 6 | 7 | 4 | 4 |
| 6 | - | - | - | - | - | - | - | 9 | 10 | 3 | 6 |
| 7 | - | - | - | - | - | - | - | - | 1 | 6 | 9 |
| 8 | - | - | - | - | - | - | - | - | - | 4 | 6 |
| 9 | - | - | - | - | - | - | - | - | - | - | 1 |
| 10 | - | - | - | - | - | - | - | - | - | - | - |

(a) (20 points) The Dant is allowing the three drivers to split up the task of visiting these restaurants among the three of them. Assume that each driver must start at USNA and you must return after they've visited every restaurant, provide the (abbreviated) concrete integer programming formulation in order to complete the Dant's assignment. If an exponential number of constraints exist in your model, provide an example of at least two of them. Clearly define all decision variables required to model this problem.

**Solution:** **Decision Variables**

Let $x_{0,1} = 1$ if edge $(0,1)$ is included in a route

$\vdots$

Let $x_{9,10} = 1$ if edge $(9,10)$ is included in a route

**Objective Function**

$$\text{maximize } 10x_{0,1} + 10x_{0,2} + \dots 1x_{9,10}$$

**OR**

$$\text{minimize } 1/10x_{0,1} + 1/10x_{0,2} + \dots 1x_{9,10}$$

**Constraints**

$$\begin{aligned}
x_{0,1} + x_{0,2} + \dots x_{0,10} &= 2 * 3 && \text{(node 0)} \\
x_{0,1} + x_{1,2} + \dots x_{1,10} &= 2 && \text{(node 1)} \\
&\vdots \\
x_{0,10} + x_{1,10} + \dots x_{9,10} &= 2 && \text{(node 10)} \\
x_{1,2} + x_{1,3} + x_{2,3} &\leq 2 && \text{(example subtour for 1,2,3)} \\
x_{1,2} + x_{1,3} + x_{1,4} + x_{2,3} + x_{2,4} + x_{3,4} &\leq 3 && \text{(example subtour for 1,2,3,4)} \\
x_{0,1}, x_{0,2}, \dots, x_{9,10} &\in \{0,1\} && \text{(binary)}
\end{aligned}$$

(b) (20 points) For the same problem, provide the parameterized integer programming model for solving the Dant's assignment. Clearly define all required decision variables, sets, and parameters not already provided to you.

**Solution:** <u>**Sets**</u> Let $N$ be the set of nodes

Let $R$ be the set of restaurants

Let $E$ be the set of edges

<u>**Decision Variables**</u>

Let $x_{i,j} = 1$ if edge $(i,j)$ is included in a route for all $(i,j) \in E$

<u>**Parameters**</u>

Let $s_{i,j}$ be the similarity factor of edge $(i,j)$ for all $(i,j) \in E$

<u>**Objective Function**</u>

$$\text{maximize} \sum_{(i,j)\in E} s_{i,j} x_{i,j}$$

**OR**

$$\text{minimize} \sum_{(i,j)\in E} 1/s_{i,j} x_{i,j}$$

<u>**Constraints**</u>

$$\begin{aligned}
\sum_{(0,j)\in E} x_{0,j} &= 2*3 && && \text{(node 0)} \\
\sum_{(i,n)\in E} x_{i,n} + \sum_{(n,j)\in E} x_{n,j} &= 2 && \text{for all } n \in R && \text{(all other nodes)} \\
\sum_{(i,j)\in E: i\in S, j\in S} x_{i,j} &\le |S| - 1 && \text{for all } S \subset R,\ |S| \ge 3 && \text{(subtour)} \\
x_{i,j} &\in \{0,1\} && \text{for all } (i,j) \in E && \text{(binary)}
\end{aligned}$$

(c) (15 points) The Dant now realizes that he wants to make sure that Dwight, Kevin, and Oscar don't exceed 2500 calories while visiting their sequence of restaurants. The following table below includes the number calories that would be consumed if visited by one of the three reviewers:

| | McD(1) | BK(2) | Pop(3) | CFA(4) | PF(5) | B(6) | EC(7) | C(8) | PH(9) | DP(10) |
|---|---|---|---|---|---|---|---|---|---|---|
| Cal. | 800 | 750 | 850 | 600 | 550 | 700 | 900 | 450 | 550 | 650 |

Given this extra constraint, Dwight computed the following solution:

- Dwight visits restaurants in the order {USNA, McD, PF, B}
- Oscar visits restaurants in the order {USNA, BK, EC, Pop, CFA}
- Kevin visits restaurants in the order {USNA, C, PH, and DP}

Is this solution feasible? If not, provide a concrete inequality that eliminates this solution as well as all other similar ones.

**Solution:** This solution is not feasible, Oscar exceeds daily amount.

$$x_{BK,EC} + x_{BK,Pop} + x_{BK,CFA} + x_{EC,Pop} + x_{EC,CFA} + x_{Pop,CFA} \leq 4 - 2$$