Lesson 13. Facility Location

1 Facility Location Introduction

The **facility location** problem is another famous OR problem which is very applied for industry/military applications. In this lesson, we will look at 3 different types of formulations of this problem.

In these problems, the **input data** is...

- a network of "customers" (demand nodes),
- a set of *possible* "facilities" (supply nodes),
- a set of edges between customers and facilities that *could* serve them
- distances on the edges (which could represent distance, time, cost, or some combination of these factors)

The **goal** is to choose a set of supply facilities to serve the customers' demand based on some metric.

- For example: minimize the number of supply facilities opened while requiring that all customers are served.
- The different problem types result from varied metrics and/or requirements.
- Real world problems of this type include locating
  - military installations,
  - fire/police stations,
  - cell phone towers,
  - retail distribution centers and stores,
  - schools,
  - vaccine clinics.

2 *General* Facility Location Problems

Goal: Choose a set of supply facilities to meet customer demand according to some metric.

Notation:

Sets:

C = set of customer nodes

S = set of possible supply nodes

E = edges (c, s) connecting a customer c with a supply facility s that *could* serve the customer

Parameters:

$d_{c,s}$ = distance (or cost or time) between customer c and supply location s, for $(c, s) \in E$

h_c = demand of customer c, for $c \in C$

Decision Variables:

$$x_s = \begin{cases} 1 \text{ if } \boxed{} \\ 0 \text{ otherwise} \end{cases}, \text{ for all } s \in S$$

Problem 1. We will use the network and data on the following page for all of our example problems.

(a) *All* vertices represent customers. *Boxed* vertices represent possible supply locations. Use set notation to list the elements of the sets C and S.

(b) The distance between a customer c and a supplier s is the length of the shortest path between them. Find $d_{1,1}$, $d_{4,1}$, and $d_{8,5}$. (Note that the "edges" in the model do not correspond directly to the edges in the graph.)

(c) How should the columns and rows in the distance matrix be labeled? Do your answers in part (a) agree with the corresponding values in the distance matrix?

DATA for FACILITY LOCATION EXAMPLES:

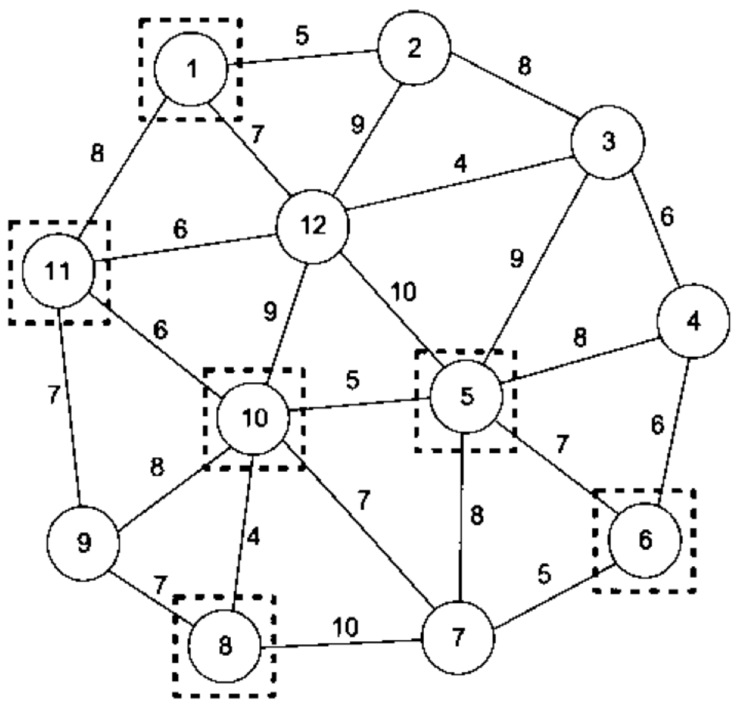

FIGURE 4.1 Example problem used for facility location models.

$$\mathbf{d} = \begin{bmatrix} 0 & 17 & 23 & 18 & 14 & 8 \\ 5 & 17 & 20 & 22 & 18 & 13 \\ 11 & 9 & 12 & 17 & 13 & 10 \\ 17 & 8 & 6 & 17 & 13 & 16 \\ 17 & 0 & 7 & 9 & 5 & 11 \\ 23 & 7 & 0 & 15 & 12 & 18 \\ 21 & 8 & 5 & 10 & 7 & 13 \\ 18 & 9 & 15 & 0 & 4 & 10 \\ 15 & 13 & 20 & 7 & 8 & 7 \\ 14 & 5 & 12 & 4 & 0 & 6 \\ 8 & 11 & 18 & 10 & 6 & 0 \\ 7 & 10 & 16 & 13 & 9 & 6 \end{bmatrix}.$$

$\mathbf{h} = (100,\ 90,\ 110,\ 120,\ 80,\ 100,\ 95,\ 75,\ 110,\ 90,\ 120,\ 85).$

3 *Set Covering* Facility Location Problem

Given a set of potential facilities to open, open the fewest number of facilities such that each customer is "covered" by at least one facility.

Problem 2. Find the minimum number of facilities required to serve all customers. A facility must be within $D = 9$ miles of a customer in order to serve the customer.

(a) We define a new set for each customer, referred to as the neighborhood of customer c, N_c. N_c is the set of facilities that can cover customer c:

$$N_c = \left\{ s \in S : \boxed{} \right\}, \text{ for all } c \in C.$$

(b) Complete the missing neighborhoods.

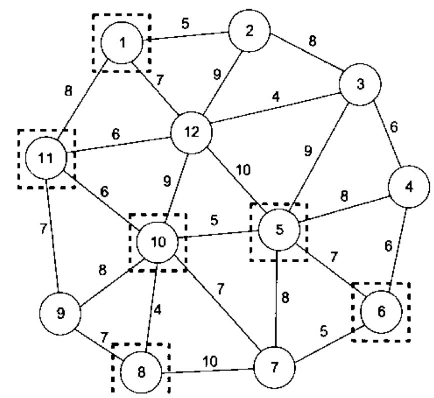

FIGURE 4.1 Example problem used for facility location models.

$N_1 = \{1, 11\}$

$N_2 = \{1\}$

$N_3 = \boxed{}$

$N_4 = \{5, 6\}$

$N_5 = \{5, 6, 8, 10\}$

$N_6 = \{5, 6\}$

$N_7 = \{5, 6, 10\}$

$N_8 = \boxed{}$

$N_9 = \{8, 10, 11\}$

$N_{10} = \boxed{}$

$N_{11} = \{1, 10, 11\}$

$N_{12} = \boxed{}$

(c) Write an abbreviated version of the concrete model using the x_s variables defined above.

(d) Using the sets and variables defined below, complete the parameterized set covering facility location model.

Sets

Let S be the set of possible supply locations
Let C be the set of all customers
Let N_c be the neighborhood of customer c for all $c \in C$

Variables

Let $x_s = 1$ if a facility is placed at location s and 0 otherwise for all $s \in S$.

4 *Maximal Covering* Location Problem

The **maximal covering location problem**: Given p facilities to open, maximize the customer demand that is covered. (A customer, c, can only be covered by a supply facility, s, in its neighborhood: $s \in N_c$.)

Maximal covering facility location model

***New* Parameters:**

$h_c =$ the demand at customer c, for all $c \in C$

$p =$ the number of facilities to open

***New* Decision Variables:**

$$y_c = \begin{cases} 1 \text{ if } \boxed{} \\ 0 \text{ otherwise} \end{cases}, \text{ for all } c \in C$$

Problem 3. Assuming that 2 facilities can be opened, write a concrete mode for the maximal covering facility location using the data from page 3.

Problem 4. Write the parameterized version of the maximal covering facility location model.

4.1 Example: Maximal Covering Facility Location Problem

Problem 5. Suppose that we can only afford to build and maintain $p = 2$ facilities, and the demand values for the customers (in order) are

$$\mathbf{h} = (100, 90, 110, 120, 80, 100, 95, 75, 110, 90, 120, 85).$$

(a) The optimal solution is to choose facilities 5 and 11. List the values of the decision variables x_s and y_c in the optimal solution. Illustrate the solution on the graph of the network.

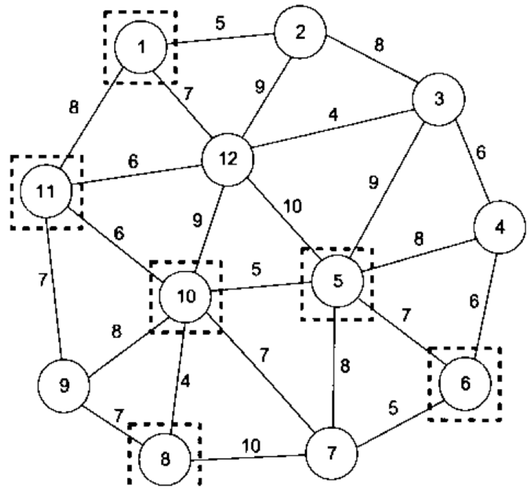

FIGURE 4.1 Example problem used for facility location models.

(b) Find the optimal objective function value. What does it mean?

(c) Suppose we allow $p = 3$

i Which new facility would we open to cover the most demand?

ii What would be our new objective function value?

5 *p-Center* Facility Location Problem: Minimize Maximum Distance

Goal of the p**-center problem**: Choose p supply facilities to open in order to minimize the maximum distance between any customer and the supply facility that serves it.

Problem 6. Complete the p-center facility location formulation below by filling in the missing constraints and descriptions.

p-center facility location model

***New* Decision Variables:**

$$z_{c,s} = \begin{cases} 1 \text{ if } \boxed{} \\ 0 \text{ otherwise} \end{cases}, \text{ for all } c \in C,\ s \in S$$

W = the maximum distance between a customer and the facility chosen to serve it

Notice that by the way $z_{c,s}$ is defined, we assume that each customer could be served by any supplier.

Objective and constraint descriptions:

(1) and (2) $\boxed{}$

(3) Exactly p facilities are opened

(4) $\boxed{}$

(5) Customer c cannot be assigned to a facility that is not open

$$
\begin{aligned}
\text{minimize} \quad & W && (1)\\
\text{subject to} \quad & \sum_{s \in S} d_{c,s} z_{c,s} \leq W, \text{ for } c \in C && (2)\\
& \boxed{} && (3)\\
& \sum_{s \in S} z_{c,s} = 1, \text{ for } c \in C && (4)\\
& \boxed{} && (5)\\
& x_s \in \{0,1\}, \text{ for } s \in S\\
& z_{c,s} \in \{0,1\}, \text{ for } c \in C,\ s \in S
\end{aligned}
$$

5.1 Example: p-Center Facility Location Problem

Problem 7. Assume the same network, distances, and demand values that we have already been using. We wish to find the $p = 2$ facilities that can serve all of the customers so that the maximum distance between a customer and the facility it is served by is minimized.

(a) Again, the optimal solution is to choose facilities 5 and 11. Facility 5 serves the customers 3, 4, 5, 6, 7, and 8. Facility 11 serves the rest. Write the values of the decision variables for this optimal solution and draw the solution on the network.

$$\mathbf{d} = \begin{bmatrix} 0 & 17 & 23 & 18 & 14 & 8 \\ 5 & 17 & 20 & 22 & 18 & 13 \\ 11 & 9 & 12 & 17 & 13 & 10 \\ 17 & 8 & 6 & 17 & 13 & 16 \\ 17 & 0 & 7 & 9 & 5 & 11 \\ 23 & 7 & 0 & 15 & 12 & 18 \\ 21 & 8 & 5 & 10 & 7 & 13 \\ 18 & 9 & 15 & 0 & 4 & 10 \\ 15 & 13 & 20 & 7 & 8 & 7 \\ 14 & 5 & 12 & 4 & 0 & 6 \\ 8 & 11 & 18 & 10 & 6 & 0 \\ 7 & 10 & 16 & 13 & 9 & 6 \end{bmatrix}.$$

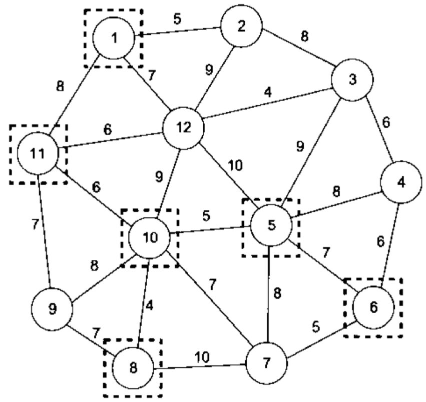

FIGURE 4.1 Example problem used for facility location models.

(b) Find the optimal objective function value. What does it mean?

(c) Write concrete versions of the following:

i constraint (2) for customers 2 and 3:

ii constraint (4) for customer 2:

iii constraints (5) for supplier 5 and all of its potential customers:

6 Summary

We discuss 3 common types of facility location models. These models, and their goals were:

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This is **not** an all inclusive list of models. Typically, one could use one of these as a starting point and then expand from there. Other things to model include:

- A fixed charge associated with each facility (think back to Lesson 6)
- Incorporating logical constraints like in Lesson 8
- Multiperioud facility location (can be quite difficult)
- Different objective functions such as:
  - Minimizing the average distance traveled
  - Minimizing a weighted distance (i.e., prioritize customers with high demand)
  - Opening facilities based on need (i.e., some areas have a priority)
  - etc...