Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 3

Name: ______________________________

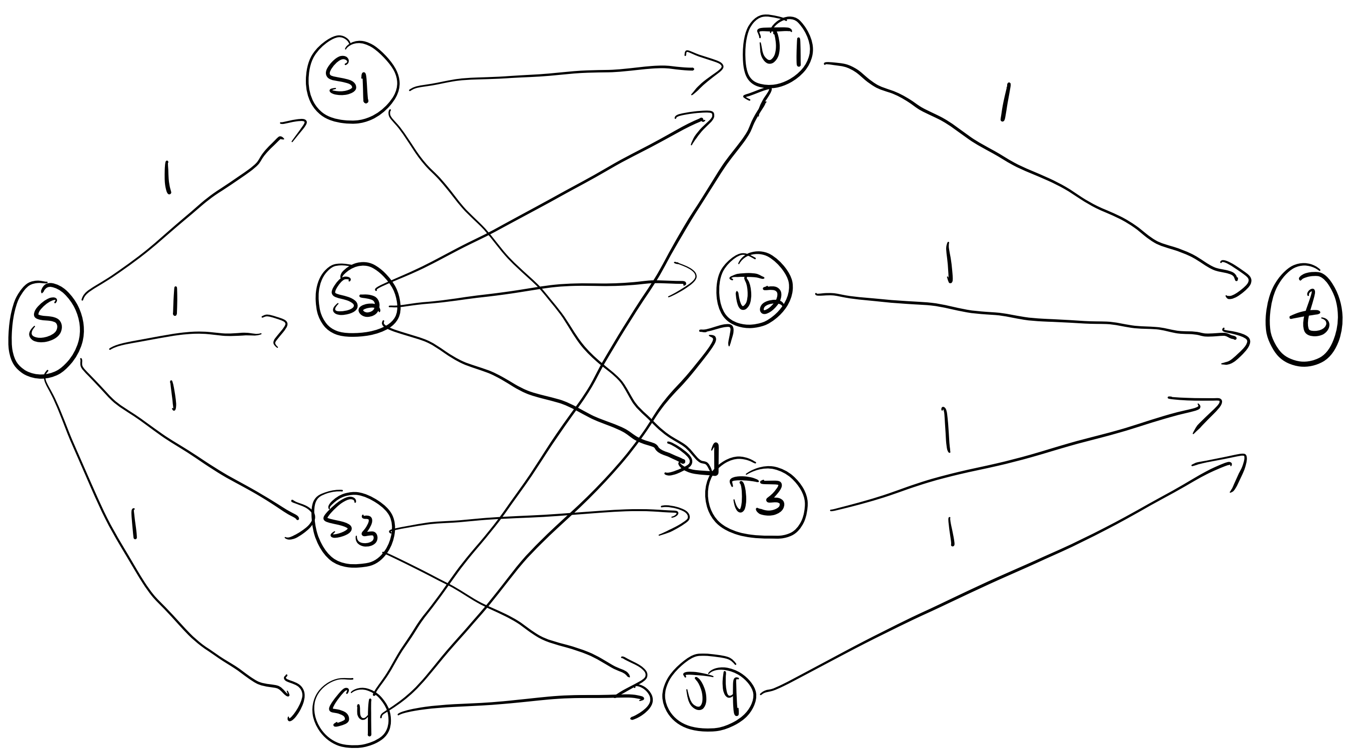
The Navy is assigning 4 sailors to 4 different jobs. Unfortunately, each sailor is not qualified to do each job. Specifically, a Q in the table below indicates that sailor i is qualified for job j.

	Job 1	Job 2	Job 3	Job 4
Sailor 1	Q		Q	
Sailor 2	Q	Q	Q	
Sailor 3			Q	Q
Sailor 4	Q	Q		Q

The goal is to assign sailors to jobs so that as many jobs are done as possible. Each worker can do at most one job. This problem can be formulated as a **max flow** model.

Part 1:

1. (30 points) Draw a max flow network diagram that models this problem. *Hint: Draw a source and sink node. Think of which nodes should be connected to the source and which should be connected to the sink.*



↑ All have capacity = 1

2. (10 points) Each edge in this network should have the same capacity. What is the capacity of each edge?

   1

3. (10 points) What is the value of the flow in the network to ensure every job gets done?

   4

Part 2: For part 2, consider the following sets, decision variables, and parameters.

Sets:

Let N be the sets of nodes, $n \in N$

Let E be the set of edges, $(i, j) \in E$

Decision variables:

Let $x_{i,j}$ be the flow on edge (i, j), $\forall (i, j) \in E$

Parameters:

Let $c_{i,j}$ be the capacity of edge (i, j), $\forall (i, j) \in E$

4. (15 points) Using the sets, variables, and parameters defined above, write the objective function for this max flow model.

   Let s be the source node

$$\max \sum_{(s,i)\in E} x_{s,i}$$

5. (15 points) One type of constraint in max flow models is capacity constraints. Using the sets, variables, and parameters above, write the capacity constraints for this model.

$$x_{i,j} \leq c_{i,j} \text{ for all } (i, j) \in E$$

6. (20 points) Another constraint that arises in all network models is flow balance. Using the sets, variables, and parameters defined above, write a general flow balance constraint that works for **any** network flow model. Be sure to clearly define any new parameters used.

   Let b_n be demand$-$supply of each node n for all $n \in N$ (notice that for max flow $b_n = 0$)

$$\sum_{(i,n)\in E} x_{i,n} - \sum_{(n,j)\in E} x_{n,j} = b_n \text{ for all } n \in N$$