Branch-and-bound Python Example (Accompanying Lesson 16)

1 Today...

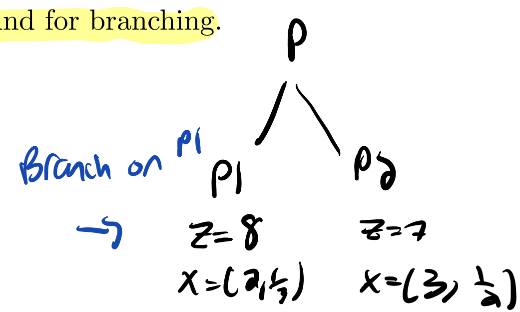
- This example is part of Lesson 16, Branch-and-bound.

2 Branch-and-bound Example

Solve the following IP using branch-and-bound.

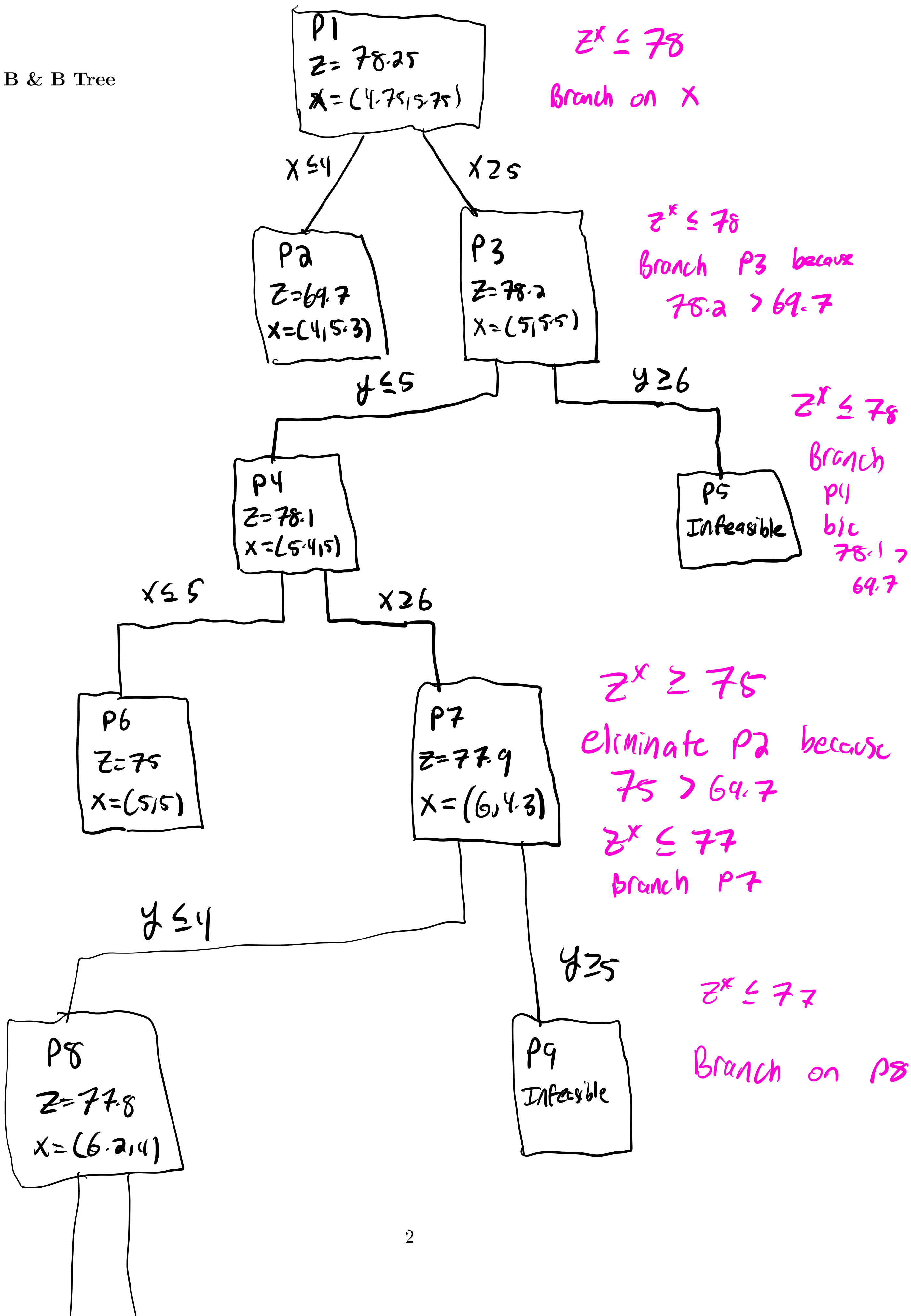
$$
\begin{aligned}
\text{(P1)} \quad z_{IP}^* = \max\ & 8x + 7y \\
\text{s.t.}\quad & -18x + 38y \leq 133 \\
& 13x + 11y \leq 125 \\
& 10x - 8y \leq 55 \\
& x, y \in \mathbb{Z}^{\geq 0}
\end{aligned}
$$

- Use Python to solve LP relaxations of subproblems
- Branching Rules
  - Always select the active node with the largest upperbound for branching.
  - Branch on x if it is fractional. Otherwise branch on y.
- Book-keeping
  - Keep track of the:
    - incumbent solution $\underline{x}$,
    - global lower bound $\underline{z}$, and
    - list of active nodes.
  - Draw the branch-and-bound tree:
    - Record the local upper bound (z) and relaxed optimal solution (x) for each subproblem.
    - Label each edge with the constraint that is added to form the child subproblem.
    - X-out fathomed nodes. Circle incumbent solution nodes.
  - Use the provided diagram to illustrate the (relaxed) feasible region of each subproblem.



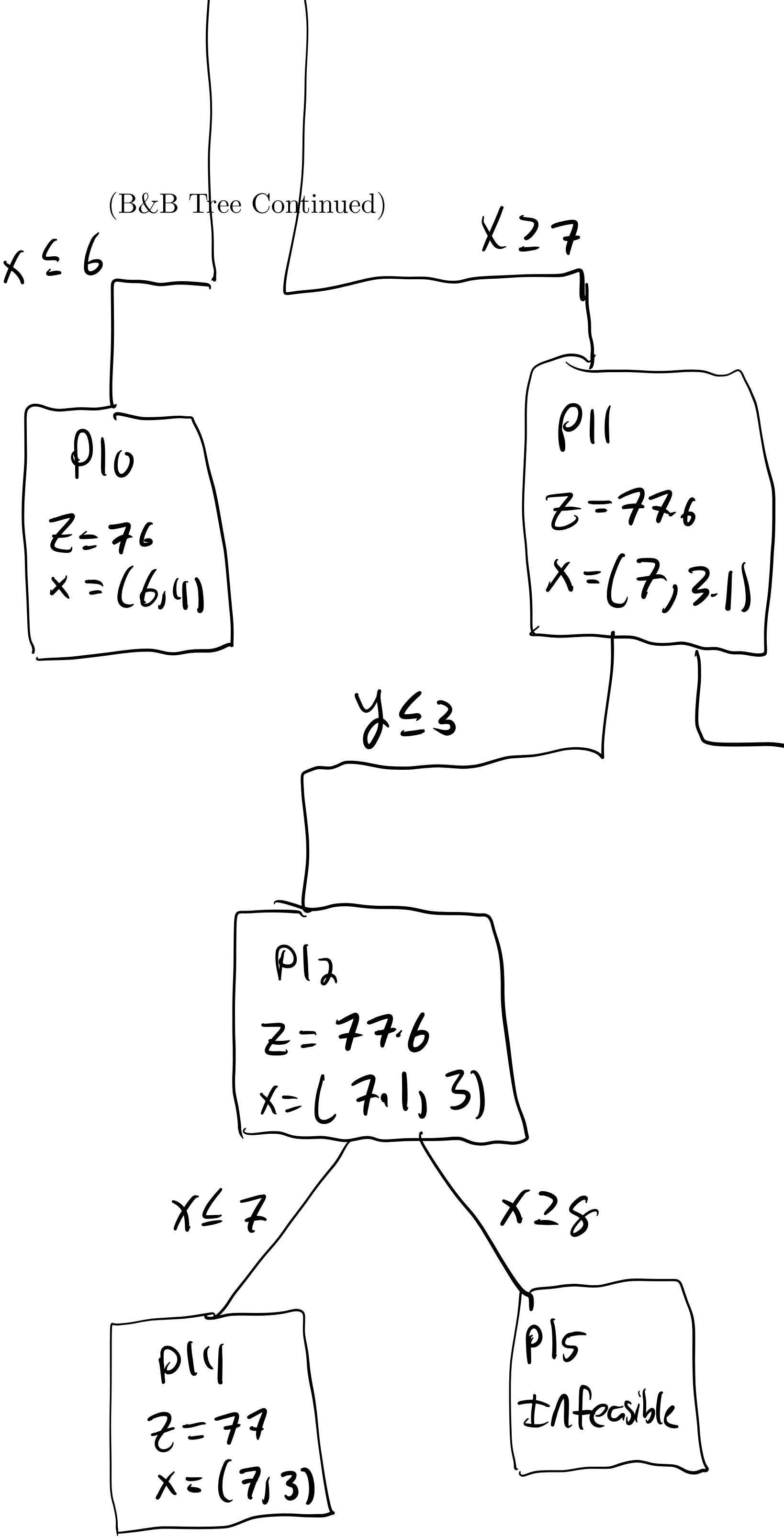
incumbent solution $\underline{x}$ **global lower bound $\underline{z}$** **active nodes**

B & B Tree



P1
Z= 78.25
X=(4.75,5.75)
Z* ≤ 78
Branch on X
X ≤ 4
X ≥ 5
P2
Z=69.7
X=(4,5.3)
P3
Z=78.2
X=(5,5.5)
Z* ≤ 78
Branch P3 because
78.2 > 69.7
y ≤ 5
y ≥ 6
P4
Z=78.1
X=(5.4,5)
P5
Infeasible
Z* ≤ 78
Branch
P4
b/c
78.1 >
69.7
X ≤ 5
X ≥ 6
P6
Z=75
X=(5,5)
P7
Z=77.9
X=(6,4.3)
Z* ≥ 75
eliminate P2 because
75 > 69.7
Z* ≤ 77
Branch P7
y ≤ 4
y ≥ 5
P8
Z=77.8
X=(6.2,4)
P9
Infeasible
Z* ≤ 77
Branch on P8

(B&B Tree Continued)



Old bound $z \ge 75$

New int solution with $z = 76$

New incumbent solution is $(6,4)$

$z \ge 76$

$z \le 77$

$z \ge 76$

$z \le 77$

Branch on x

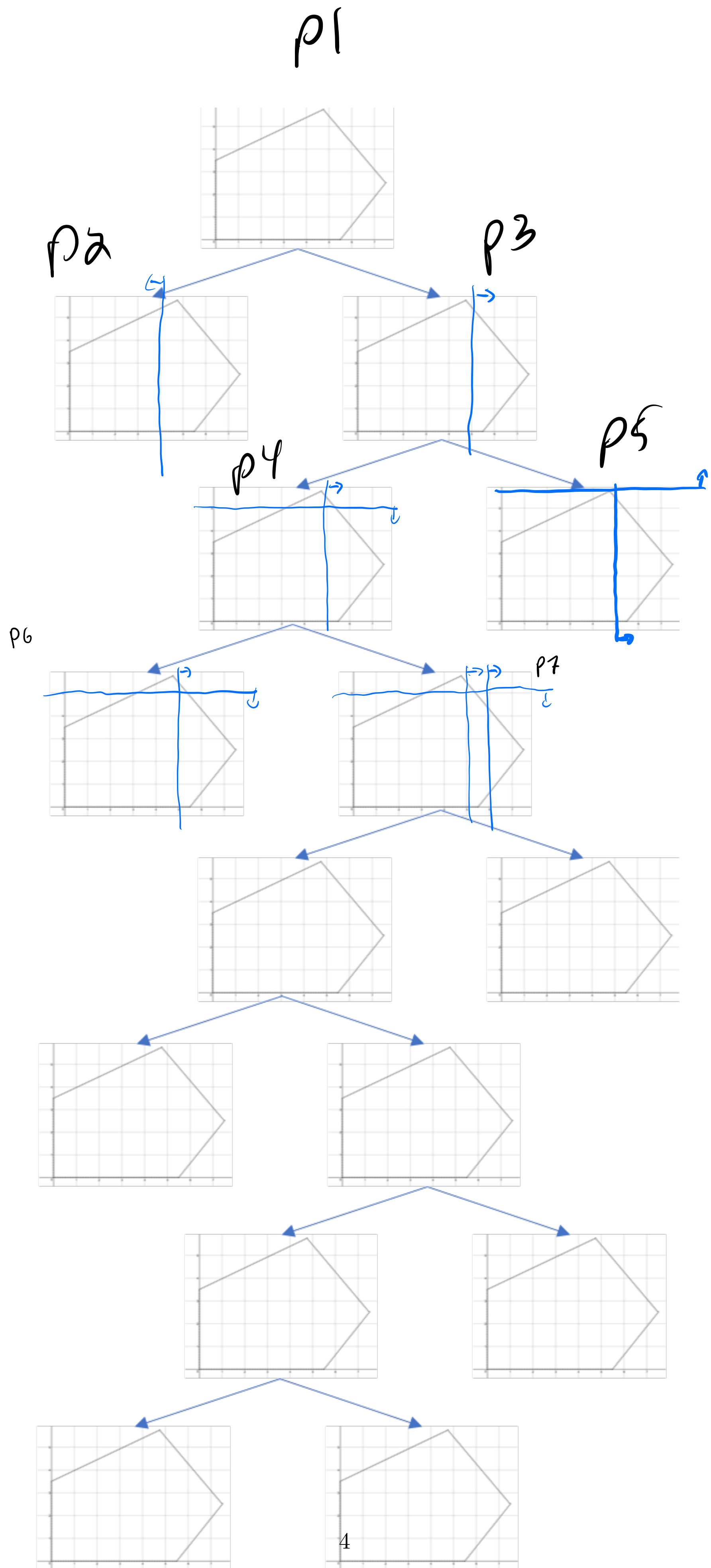
P14 is new solution because $77 > 76$

so $z \ge 77$

No nodes left so stop

Also can stop because lower bound = upper bound.

$z^* = 77 \quad x^* = (7,3)$


p1
p2
p3
p4
p5
p6
p7