Practice Questions for Exam 1

Name: _____

- This practice exam consists of 6 problems. **Please note that clearly this is much longer than the actual exam 1 so that you can have more practice problems**
- No credit will be given to unjustified answers.
- Please read the entire exam carefully. Good Luck!

1. Walmart's shipping network in our area has 3 warehouses located in Baltimore, Laurel, and DC. Each warehouse has a capacity of goods it produces and each city it ships to has a demand. Walmart must ship goods throughout the area for the costs illustrated in the below table.

	Destinations				
Warehouse	Annapolis	Fort Meade	Ocean City	Arlington	Capacity
Baltimore	5	2	9	12	30000
Laurel	8	4	7	9	35000
DC	7	5	12	5	20000
Demand	10000	15000	25000	25000	

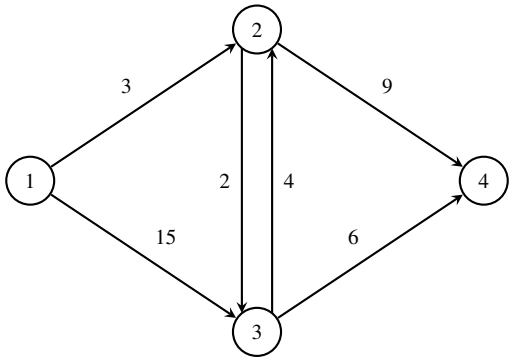
a. Draw the network diagram associated with this problem.

b. What type of network model is this?

c. Formulate a concrete model that will allow Walmart to ship its goods at minimum cost.

d. Formulate an parameterized model that will allow Walmart to ship its goods at minimum cost. Clearly label all sets, parameters and variables used.

2. Consider the directed network shown below, where the numbers on the arcs represent cost, $c_{i,j}$, to send one unit of flow along the arc. Use the start of a formulation given below to answer the following questions.



Sets

$N =$ the set of nodes i for all $i \in N$
$E =$ the set of edges (i, j) for all $(i, j) \in E$

Decision Variables

Let $x_{i,j}$ be the flow along edge (i, j) for all $(i, j) \in E$

Parameters

Let $c_{i,j}$ be the cost to send 1 unit of flow along edge (i, j) for all $(i, j) \in E$

a. We place one unit of supply at node 1, 1 unit of demand at node 4 and zero units of supply at all other nodes. Formulate an objective function to compute the minimum cost strategy for meeting the demand at node 4.

b. What type of network problem is this?

c. Write the constraints for this problem using concrete form.

d. Write the constraints for this problem using parameterized form.

3. You have been asked to assign 1000 plebes to 50 different Calculus 1 sections. This problem can be modeled as a network. Specifically, the network has 1000 supply nodes and 50 demand nodes. Each student ranks their section choices from 1 to 50. For example, a plebe would be happiest with choice 1 and most unhappy with choice 50. Each section must have exactly 20 plebes assigned.

Sets

Let P be the set of plebes
Let S be the set of calculus sections

a. Using these sets, formulate the parameterized model associated with this problem.

4. A clothing company manufactures shirts, shorts, and pants. Each shirt takes 3 hours of labor and 4 yards of cloth, each pair of shorts takes 2 hours of labor and 3 units of cloth, and each pair of pants takes 6 hours of labor and 4 units of cloth. Each week the company has 150 hours of labor and 160 units of cloth. The profit of selling a shirt is \$12, shorts is \$8, and pants are \$15. They formulate the following concrete model to maximize their profits:

Decision Variables

Let x_1 be the number of shirts made each week
Let x_2 be the number of shorts made each week
Let x_3 be the number of pants made each week

Parameters

None

Objective Function

$$\text{maximize profit: } 12x_1 + 8x_2 + 15x_3$$

Constraints

$$\begin{aligned} \text{subject to:} \quad 3x_1 + 2x_2 + 6x_3 &\leq 150 \quad \text{(Labor)} \\ 4x_1 + 3x_2 + 4x_3 &\leq 160 \quad \text{(Cloth)} \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

a. Convert their model to parameterized form. *Use only one parameter of the form $a_{i,j}$ for the left hand side of the constraints*

b. The suppliers for the clothing company have changed their method of charging. Specifically, if the company wants shirts, they must order at least 10 shirts from the supplier and pay a fixed charge of \$50. If they want shorts, they must order at least 8 pairs of shorts from the supplier and pay a fixed charge of \$60. If they want pants, they must order at least 15 pairs from the supplier and pay a fixed charge of \$100.

   (a) What new variable(s) would you need to introduce to model this new charging scheme?

   (b) Modify your objective function so that it reflects these new fixed charges.

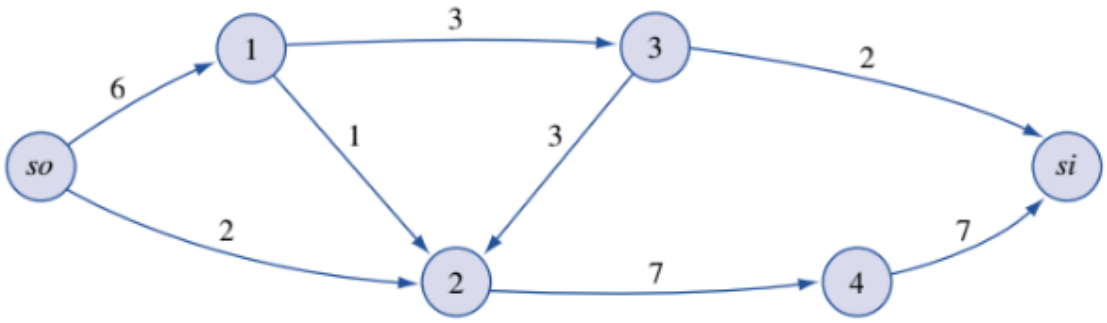
   (c) Add a constraint to the concrete model which says if they order shirts, they must order at least 10 of them from the supplier.

5. The Naval academy must purchase 1100 computers and St Johns must purchase 600 computers from three vendors. Vendor 1 charges $500 per computer plus a delivery charge of $5000. Vendor 2 charges $350 per computer plus a delivery charge of $4000. Vendor 3 charges $250 per computer plus a delivery charge of $6000. Vendor 1 can sell at most 800 computers, vendor 2 can sell at most 700 computers, and vendor 3 can sell at most 1000 computers.

a. Formulate a concrete model which allows both the Naval Academy and St Johns to purchase computers at minimum cost.

b. Formulate an parameterized model which allows both the Naval Academy and St Johns to purchase computers at minimum cost.

6. Consider the following network. Note that so is the source node and si is the sink node:



An OR student writes the following model:

Sets

Let N be the set of nodes, $n \in N$

Let E be the set of edges, $(i, j) \in E$

Decision Variables

Let $x_{i,j}$ be the flow along edge (i, j) for all $(i, j) \in E$

Parameters

Let $c_{i,j}$ be the cost of edge (i, j)

Let b_i be the supply/demand at node i. Specifically, at node so, $b_{so} = -1$ at nodes 1, 2, 3, and 4, $b_i = 0$ and at node si, $b_{si} = 1$.

Objective Function

$$\text{minimize cost: } \sum_{(i,j)\in E} c_{i,j}x_{i,j}$$

Constraints

$$\text{subject to: } \sum_{(i,n)\in E} x_{i,n} - \sum_{(n,j)\in E} x_{n,j} = b_n \quad \text{for } n \in N$$

$$x_{i,j} \geq 0 \quad \text{for all } (i, j) \in E$$

a. What type of network problem is this?

b. Why is the supply at node so 1?

c. Using the given network, transform this parameterized model into a concrete model.