Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 5

Name: ______________________________

You are continuing your roller coaster enthusiasm by planning a trip to Busch Gardens. Busch Gardens has 8 roller coasters and, yet again, you label them 1 to 8 and plot the distance you'd have to walk in order to get to each roller coaster. The following matrix gives these distances (in hundreds of feet).

$$\begin{bmatrix} - & 6 & 8 & 3 & 12 & 5 & 7 & 6 \\ & - & 6 & 4 & 6 & 4 & 3 & 2 \\ & & - & 13 & 10 & 2 & 7 & 10 \\ & & & - & 12 & 3 & 9 & 13 \\ & & & & - & 6 & 9 & 12 \\ & & & & & - & 7 & 4 \\ & & & & & & - & 1 \\ & & & & & & & - \end{bmatrix}$$

You've refined your model and decided to model this problem as a traveling salesperson (TSP) in order to walk as little as possible.

Suppose you use the variables $x_{i,j} = 1$ if edge (i, j) is part of the tour and 0 otherwise.

1. (15 points) Give the objective function of this model in both concrete and parameterized form. For the parameterized form, make sure you define any new parameters used.

$$\min 6x_{1,2} + 8x_{1,3} + \cdots + x_{7,8}$$

For the parameterized form, define $d_{i,j}$ as the distance along edge (i, j) for all $(i, j) \in E$. Then, the objective is:

$$\min \sum_{(i,j)\in E} d_{i,j} x_{i,j}$$

2. (15 points) In order to obtain a tour of the graph, how many edges must be selected? Write a constraint, in either concrete or parameterized form, which enforces that this number of edges is selected from the graph.

You must select 8 edges to form a tour since there are 8 nodes. This constraint is:

$$\sum_{(i,j)\in E} x_{i,j} = 8$$

3. You implement this model in python, solve it, and get the following solution:

```
The optimal solution is to select cycles 2-1-5-3-2 and 4-7-8-6-4
```

(a) (15 points) What are the values of the $x_{i,j}$ variables corresponding to this solution?

$$x_{1,2} = x_{1,5} = x_{3,5} = x_{2,3} = x_{4,7} = x_{7,8} = x_{6,8} = x_{4,6} = 1$$

All other $x_{i,j}$ are 0.

(b) (15 points) What is the total distanced traveled by this solution?

$$6+12+10+6+9+1+4+3 = 51$$

(c) You know that this is not the optimal solution to your problem because it is not a tour of the entire graph, but instead is two cycles of size 4. You decide to eliminate the first cycle `2-1-5-3-2`. You recall that the general subtour elimination constraints for TSP were:

$$\sum_{(i,j)\in E: i\in S, j\in S} x_{i,j} \leq |S| - 1 \text{ for all } S \subset N,\ |S| \geq 3$$

i. (10 points) For the cycle `2-1-5-3-2` what is the set S?

$$S = \{1,2,3,5\}$$

ii. (20 points) What is the concrete constraint you would add to your model to properly eliminate the cycle `2-1-5-3-2`?

$$x_{1,2} + x_{1,3} + x_{1,5} + x_{2,3} + x_{2,5} + x_{3,5} \leq 3$$

iii. (10 points) If you wanted to eliminate **all** cycles of size 4 from this graph, how many constraints would you have to write? Have to eliminate 8 choose 4 cycles which is 70