HW6: Branch and Bound

1. Solve the following problem using Branch and Bound

$$\begin{array}{rrcl} \max & 5x_1 + 4x_2 & & \\ \text{st} & 6x_1 + 13x_2 & \leq & 67 \\ & 8x_1 + 5x_2 & \leq & 55 \\ & x_1, x_2 & \in & \mathbb{Z}^+ \end{array}$$

   Solve each subproblem graphically or with python.

2. Solve the following problem using Branch and Bound

$$\begin{array}{rrcl} \max & 20x_1 + 16x_2 + 25x_3 + 14x_4 + 9x_5 & & \\ \text{st} & 3x_1 + 2x_2 + 5x_3 + 4x_4 + 2x_5 & \leq & 13 \\ & x_1, x_2, x_3, x_4, x_5 & \in & \{0, 1\} \end{array}$$

   Solve each subproblem either using python or using the following algorithm:

   - Compute the ratio r_i for each variable where

$$r_i = \frac{\text{coefficient of } x_i \text{ in objective function}}{\text{coefficient of } x_i \text{ in constraint}}$$

   - Order r_i from largest to smallest
   - Starting with the largest r_i, set x_i to its largest possible (continuous) value without violating the existing constraints.

   For example, this algorithm to solve the initial problem would compute r_i as:

$$r_i = \{\frac{20}{3}, 8, 5, \frac{7}{2}, \frac{9}{2}\}$$

   The largest of these is 8 so we would set $x_2 = 1$. The RHS of the constraint is now 11. The next largest is $\frac{20}{3}$, so we would set $x_1 = 1$ and the RHS of the constraint is now 8. The next largest is 5, so we would set $x_3 = 1$ and the RHS of the constraint becomes 3. The next largest is $\frac{9}{2}$, so we would set $x_5 = 1$ and the RHS of the constraint becomes 1. All that's left is x_4. We can't set it equal to 1, instead we maximize it and set $x_4 = \frac{1}{4}$. So the initial solution is $x = (1, 1, 1, 0.25, 1)$.

3. Solve the following problem using Branch and Bound:

$$
\begin{aligned}
\max \quad & 5x_1 + 15x_2 + 12x_3 + 18x_4 \\
\text{st} \quad & 20x_1 + 40x_2 + 30x_3 + 50x_4 \le 150 \\
& x_1 \le 5 \\
& x_2 \le 3 \\
& x_3 \le 3 \\
& x_4 \le 2 \\
& x_1, x_2, x_3, x_4 \in \mathbb{Z}^+
\end{aligned}
$$

Solve each subproblem either using python or using the same algorithm as problem 2 (notice that this time the variables are not binary so you'd maximize the value based on their upper bounds).