(7.12)

$\because$ independant, $f(x_1, x_2) = f(x_1)f(x_2) = e^{-(x_1+x_2)}, \quad x_1>0, x_2>0$

$y_2 = \frac{x_1}{x_1+x_2} = \frac{x_1}{y_1} \Rightarrow x_1 = y_1 y_2$

$x_2 = y_1 - x_1 = y_1 - y_1y_2 = y_1(1-y_2)$

$\Rightarrow \begin{cases} x_1 = y_1 y_2 \\ x_2 = y_1(1-y_2) \end{cases}$, for $y_1 > 0$, $0 < y_2 < 1$

$J = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = -y_1y_2 - y_1 + y_2y_1 = -y_1$

$g(y_1, y_2) = f(y_1y_2, y_2 - y_1y_2)\,|-y_1| = e^{-(y_1)}y_1$, for $y_1>0$, $0<y_2<1$

$\Rightarrow g(y_1) = \int_0^1 y_1 e^{-y_1} dy_2 = y_1 e^{-y_1}, \quad y_1 > 0$

$g(y_2) = \int_0^\infty y_1 e^{-y_1} dy_1 = \Gamma(2) = 1, \quad 0<y_2<1$

Since $g(y_1, y_2) = g(y_1)g(y_2)$, the random variables Y_1 and Y_2 are independent #

(7.14)

$x_1 = \sqrt{y}, \quad J_1 = \frac{1}{2\sqrt{y}}$

$x_2 = -\sqrt{y}, \quad J_2 = -\frac{1}{2\sqrt{y}}$

$\boxed{g(y) =} f(\sqrt{y})|J_1| + f(-\sqrt{y})\left|-\frac{1}{2\sqrt{y}}\right| = \frac{1+\sqrt{y}}{2}\frac{1}{2\sqrt{y}} + \frac{1-\sqrt{y}}{2}\frac{1}{2\sqrt{y}} = \frac{2}{4\sqrt{y}} \boxed{= \frac{1}{2\sqrt{y}}, \text{ for } 0<y<1}$

$\Rightarrow g(y) = \frac{1}{2\sqrt{y}}$, for $0<y<1$ #

(7.18)

$M_X(t) = E(e^{tX}) = p\sum_{x=1}^{\infty} e^{tx} q^{x-1} = \frac{p}{q}\sum_{x=1}^{\infty}(e^t q)^x = \frac{pe^t}{1-qe^t}, \quad t < \ln q$ #

$\mu = M'_X(0) = \left.\frac{pe^t(1-qe^t) + pe^t qe^t}{(1-qe^t)^2}\right|_{t=0} = \frac{p(1-q)+pq}{(1-q)^2} = \frac{1}{p}$

$\mu'_2 = M''_X(0) = \left.\frac{(1-qe^t)^2 pe^t + 2pqe^{2t}(1-qe^t)}{(1-qe^t)^4}\right|_{t=0} = \frac{(1-q)^2 p + 2pq(1-q)}{(1-q)^4} = \frac{2-p}{p^2}$

$\sigma^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2} = \frac{q}{p^2}$ #, $\mu = \frac{1}{p}$ #

(7.22) $M_X(t) = E(e^{tX}) = \frac{1}{\Gamma(\frac{v}{2})2^{\frac{v}{2}}}\int_0^\infty x^{\frac{v}{2}-1} e^{-(\frac{1}{2}-t)x} dx = \frac{(\frac{1}{2}-t)^{-\frac{v}{2}}}{2^{\frac{v}{2}}}\int_0^\infty \frac{x^{\frac{v}{2}-1}e^{-(\frac{1}{2}-t)x}}{\Gamma(\frac{v}{2})(\frac{1}{2}-t)^{-\frac{v}{2}}} dx = (1-2t)^{-\frac{v}{2}}$

$\mu = \left. -\frac{v}{2}(1-2t)^{-\frac{v}{2}-1}(-2)\right|_{t=0} = v$

$\mu'_2 = M''_X(0) = \left(v(1-2t)^{-\frac{v}{2}-1}\right)' = \left.\left(-\frac{v}{2}-1\right)v(1-2t)^{-\frac{v}{2}-2}(-2)\right|_{t=0} = v(v+2)$

$\mu = v$ #, $\sigma^2 = v(v+2) - v^2 = 2v$ #

1.(b)

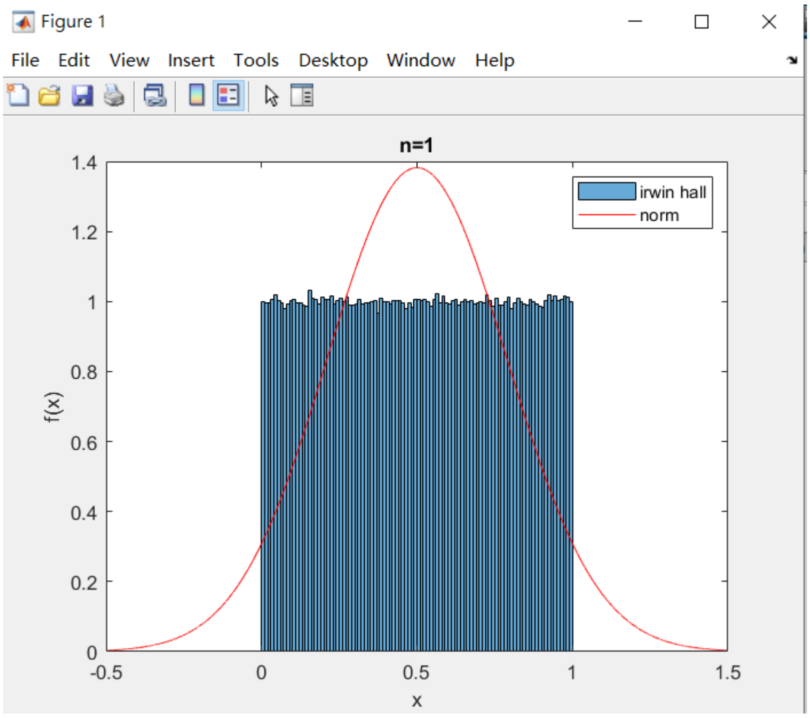


Figure 1
File Edit View Insert Tools Desktop Window Help
n=1
irwin hall
norm
f(x)
x

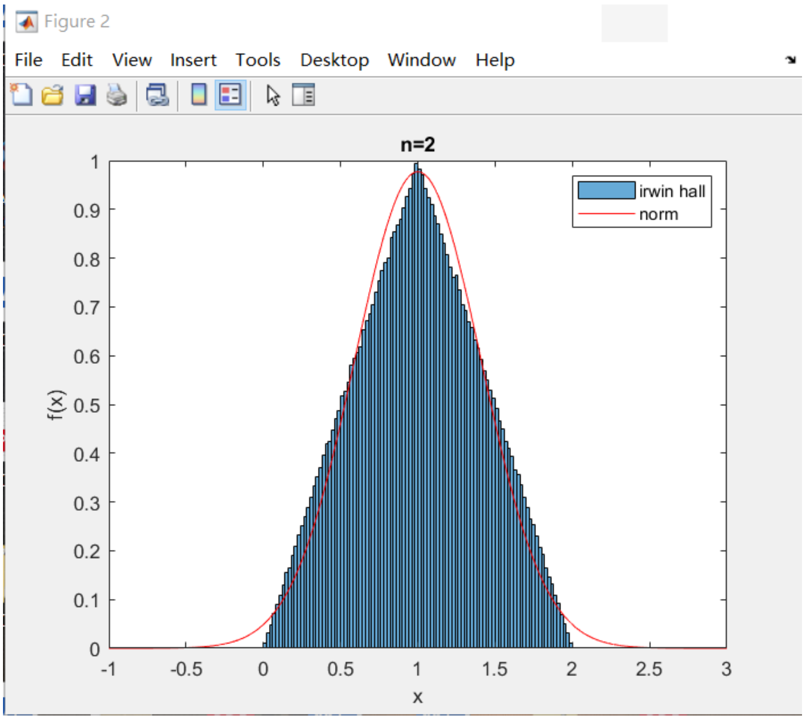
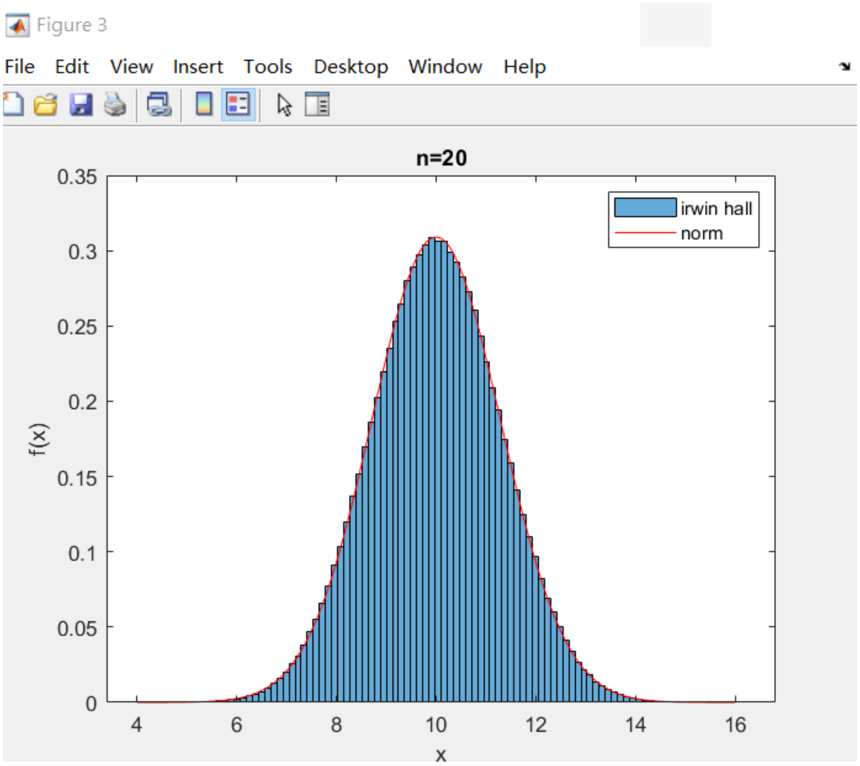


Figure 2
File Edit View Insert Tools Desktop Window Help
n=2
irwin hall
norm
f(x)
x



n 越大，irwin-hall distribution 近似 normal distribution 近似的越好