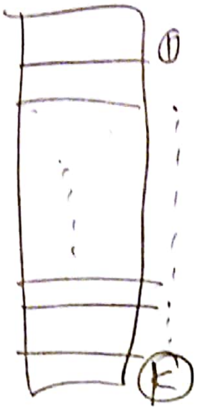
~~1, 3, 4, 6, 8, 10, 11, 13, 16, 18, 21, 24, 26, 29, 37, 39~~

#1

a)



False

$\Rightarrow$ we get new $\hat{\beta}$ by using $k-1$ folds

b)

Null dev $\Rightarrow -2\sum (y_i \log \hat{p}_i + (1-y_i) \log (1-\hat{p}_i))$

$= -2\sum [y_i \log \bar{y} + (1-y_i) \log (1-\bar{y})]$

Residual dev $= -2n[\bar{y} \log \bar{y} + (1-\bar{y}) \log (1-\bar{y})]$

$\Downarrow$

$-2\sum [y_i \log \hat{p}_i + (1-y_i) \log (1-\hat{p}_i)]$

when you add variable ~~Res~~ deviance $\downarrow$/same.

True

c)

$-2 \log$(max value of likelihood in m)

False $\Rightarrow$ It says general...

Null deviance is when model includes only an intercept...

#3)

prove

② $1-\frac{RSS}{TSS}\cdot\frac{n-1}{n-p-1}\leq 1$

$\hookrightarrow 0\leq \frac{RSS}{TSS}\frac{n-1}{n-p-1}$

$\hookrightarrow$ prove $0\leq\frac{RSS}{TSS}$

$\hookrightarrow$ Always True

$$adj\ R^2 = 1-\frac{RSS/n-p-1}{TSS/n-1}$$

$$=1-\frac{\frac{RSS}{n-p-1}}{\frac{TSS}{n-1}}$$

$$=1-\frac{RSS}{TSS}\cdot\frac{n-1}{n-p-1} = 1-\frac{RSS}{RSS+RegSS}\cdot\frac{n-1}{n-p-1}$$

~~$0\leq R^2\leq 1$~~
~~$-\infty\leq adj\ R^2\leq 1$~~

~~$adj\ R^2\leq R^2\leq 1$~~

$\hookrightarrow [-\infty, 1]$

~~$TSS = RegSS + RSS$~~

~~$R^2 = 1-\frac{RSS}{TSS} = \frac{RegSS}{TSS}$~~

~~$\hookrightarrow TSS = \frac{RegSS}{R^2}$~~

~~$\left(\frac{RSS}{TSS}+R^2\right) - \frac{RSS}{TSS}\cdot\frac{n-1}{n-p-1}$~~

~~$=\frac{RSS}{TSS}\left(1-\frac{n-1}{n-p-1}\right)+R^2$~~

~~$=\frac{RSS}{TSS}\cdot\frac{-p}{n-p-1}+R^2$~~

~~$\frac{RSS}{TSS}\left(1-\frac{n-1}{n-p-1}\right)+R^2$~~

~~$\frac{RSS(m)-RSS(M)}{RegSS(M)/p}$~~

~~$RSS/n-p-1$~~

$\frac{RSS}{n-p-1}=\hat{\sigma}$

① $1-\frac{n-1}{TSS}\hat{\sigma}^2 = 1-\frac{\hat{\sigma}^2}{\hat{\sigma}^2_{int}}$

$\hookrightarrow \hat{\sigma}^2$ intercept

$\hookrightarrow$ X explain model well... so bigger than $\hat{\sigma}^2$

$0\leq \cancel{x}\leq 1$

$\hookrightarrow -1\leq // \leq 0$

$0\leq // +1\leq 1$

$\hookrightarrow$ ~~(#)~~ No! can work... m?

#4

$Y_i \sim pos(\mu_i)$

$$f(x;\theta_i,\phi_i) = h(x,\phi_i)\exp\left(\frac{x\theta_i - b(\theta_i)}{a(\phi_i)}\right)$$

① $\Rightarrow f(x) = e^{-\mu_i}\frac{\mu_i^x}{x!}$

$$= \frac{1}{x!} e^{-\mu_i} e^{x\log\mu_i}$$

$$= \frac{1}{x!}\exp(x\log\mu_i - \mu_i)$$

(?) Is this what they're asking for when they say 'set up GLM problem for estimation of β_i'?

$\theta_i = \log(\mu_i)$

$b(\theta_i) = e^{\theta_i}$

$\phi_i = 1$

$(b')^{-1}(\mu_i) = \theta_i$

$\mu_i = e^{\beta_1 + \beta_2 x_i}$

① we want to estimate μ_i
② set up $g(\mu_i)$ as a linear model
③ find g !!! link function
④ get $\hat\beta$ with MLE.

② $\log(\mu_i) = \beta_1 + \beta_2 X_i$ is the link function

$g(\mu_i) = x_i'\beta$

$\mu_i = x_i'\beta$

③ $y_i \sim pois(\mu_i)$

$$\prod_{i=1}^{n} e^{-\mu_i}\frac{\mu_i^{y_i}}{y_i!} \quad \text{~~where~~ } \mu_i = \beta_1 + \beta_2 x_i$$

$$\sum_{i=1}^{n} -\mu_i + y_i\log\mu_i - \log y_i! = l(\beta)$$

plug $\mu_i = \beta_1 + \beta_2 x_i$

$$\sum [-(\beta_1 + \beta_2 x_i) + y_i\log(\beta_1 + \beta_2 x_i) - \log y_i!] = l(\beta)$$

$\nabla l(\beta) = 0$ or Newton Raphson

Scanned by CamScanner

#5

a) $Y_i \sim Ber(\pi_i)$

$0 \le \pi_i = \Phi(\beta_1 + \beta_2 x_i) \le 1 \Rightarrow$ probit

$g(\pi_i) = \Phi^{-1}(\pi_i) = \beta_1 + \beta_2 x_i \Rightarrow$ How to set up GLM here...?

Link function g

$(b')^{-1}\mu_i = \theta_i$

$\mu_i = E(Y_i) = b'(\theta_i)$

$(b')^{-1}(\pi_i) = \theta_i$

b) $\prod_{i=1}^{n} \pi_i^{y_i}(1-\pi_i)^{1-y_i}$ → pg 446

$= \sum (y_i \log \pi_i + (1-y_i)\log(1-\pi_i))$

$= \sum [y_i \log[\Phi(\beta_1+\beta_2 x_i)] + (1-y_i)\log(1-\Phi(\beta_1+\beta_2 x_i))]$

$y_i \log \pi_i + \log(1-\pi_i) - y_i \log(1-\pi_i)$

$\Downarrow$

$y_i\left(\log \frac{\pi_i}{1-\pi_i}\right) + \log(1-\pi_i)$

Find $D\,l = 0$, $H\,l = 0$

$\Downarrow$

not possible
do Newton-Raphson

#6.

$\frac{1}{n}\left[n\,I(G) - n_1 \right.$

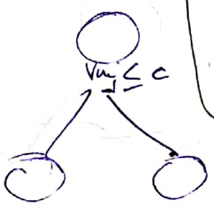
$RSS(j, c) = \sum_{i \in G_1} (y_i - \bar{y}_1)^2 + \sum (y$

(b) I'm not sure how to prove when it's the first split...

$RSS(j, c) \le \cancel{RSS(T)} \le TSS(T)$

variable j | cut off c

~~only one variable used to explain model~~



→ Analogous to Anova RSS

~~all variables to explain model.~~

☆ Maybe solve it with induction???

prove $\boxed{TSS} \ge \sum_{i \in G_1} (y_i - \bar{y}_1)^2 + \sum_{i \in G_2} (y_i - \bar{y}_2)^2$

(a)

$$TSS = \sum (y_i - \bar{y})^2 = \sum_{i \in G_1} (y_i - \bar{y})^2 + \sum_{i \in G_2} (y_i - \bar{y})^2$$

(a) Approach ① ⇒ $\sum_{i \in G_1} (y_i - \bar{y})^2 \ge \sum_{i \in G_1} (y_i - \bar{y}_1)^2$ and prove with $\sum (y_i - \bar{y})^2 = \sum (y_i - \bar{y}_1 + \bar{y}_1 - \bar{y})^2$

Approach ② ⇒ $\sum (y_i - \beta_0)^2$ is minimized by $\beta_0 = \bar{y}_1$

Approach ③ ⇒ show $RSS(j, c)$ is the RSS of a one-way Anova model with 2 groups G_1 & G_2. Then recall $RSS \le TSS$

#7

a) $E(y) = (X^TX+\lambda I)^{-1}X^TX\beta = \beta$

$\min \|Y - X\beta\|^2 + \lambda\|\beta\|^2$

☆ $X \in R^{n\times 1}$ where there are n 1's.

Intercept only model as $E(Y) = X^T\beta = \beta$ ☆☆

$\tilde{\beta} = (n+\lambda)^{-1} \sum_{i=1}^{n} y_i$

$= \boxed{\frac{1}{n+\lambda} \cdot n\bar{Y}}$

$\tilde{\beta} = \frac{n}{n+\lambda}\bar{Y} \le \bar{Y}$

shrinked

LS solution

$1\times n \; n\times 1$

$\frac{1}{n}\sum y$

b) $E(\tilde{\beta})$

$= E\left(\frac{n}{n+\lambda}\bar{Y}\right) = \frac{n}{n+\lambda}E(\bar{Y}) = \frac{n}{n+\lambda}E\left(\frac{1}{n}\sum y_i\right)$

$= \frac{n}{n+\lambda} \cdot \frac{1}{n} E(y_1 + \cdots + y_n)$

$= \frac{1}{n+\lambda} \cdot n\beta = \boxed{\frac{n\beta}{n+\lambda}}$

assumed Y_i is identical.

$\mathrm{var}(\tilde{\beta}) = \left(\frac{n}{n+\lambda}\right)^2 \mathrm{var}\left(\frac{1}{n}\sum y_i\right)$

$= \frac{n^2}{(n+\lambda)^2} \cdot \frac{1}{n^2} n\,\mathrm{var}(y) = \boxed{\frac{n}{(n+\lambda)^2}\sigma^2}$

#C)

$$MSE(\tilde{\beta}) = E(\tilde{\beta} - \overline{\beta})^2$$

$$= \boxed{bias(\tilde{\beta})}^2 + var(\tilde{\beta})$$

$$\boxed{\beta - E(\tilde{\beta})}$$

→ random involved

$$\Rightarrow bias(\tilde{\beta}) = \beta - \frac{n\beta}{n+\lambda}$$

$$var(\tilde{\beta}) = \frac{n}{(n+\lambda)^2}\sigma^2$$

$$\Rightarrow \text{so, } \left(\frac{\beta(n+\lambda) - n\beta}{n+\lambda}\right)^2 + \frac{n\sigma^2}{(n+\lambda)^2}$$

$$= \frac{1}{(n+\lambda)^2}\left[(\beta\lambda)^2 + n\sigma^2\right]$$

$$= \frac{1}{(n+\lambda)^2}\beta^2\lambda^2 + \frac{1}{(n+\lambda)^2}n\sigma^2$$

$$= \frac{\lambda^2}{(n+\lambda)^2}\beta^2 + \frac{n^2}{(n+\lambda)^2}\cdot\frac{\sigma^2}{n}$$

$$= \boxed{\left(1 - \frac{n}{n+\lambda}\right)^2\beta^2 + \left(\frac{n}{n+\lambda}\right)^2\frac{\sigma^2}{n}}$$

☆ $MSE(\hat{\beta}_{OLS})$

$$= var(\hat{\beta}_{OLS})$$

$$= var[(X^TX)^{-1}X^TY]$$

$$= \sigma^2(X^TX)^{-1}$$

so,

$$\frac{(\beta\lambda)^2 + n\sigma^2}{(n+\lambda)^2} < \sigma^2(X^TX)^{-1}$$

$$\sigma^2(X^TX)^{-1} = \frac{\sigma^2}{n}$$

$$\Downarrow$$

$$(1-\alpha)^2\beta^2 + \alpha^2\frac{\sigma^2}{n} < \frac{\sigma^2}{n}$$

$$\Downarrow$$

$$(1-\alpha)^2\beta^2 < (1-\alpha^2)\frac{\sigma^2}{n}$$

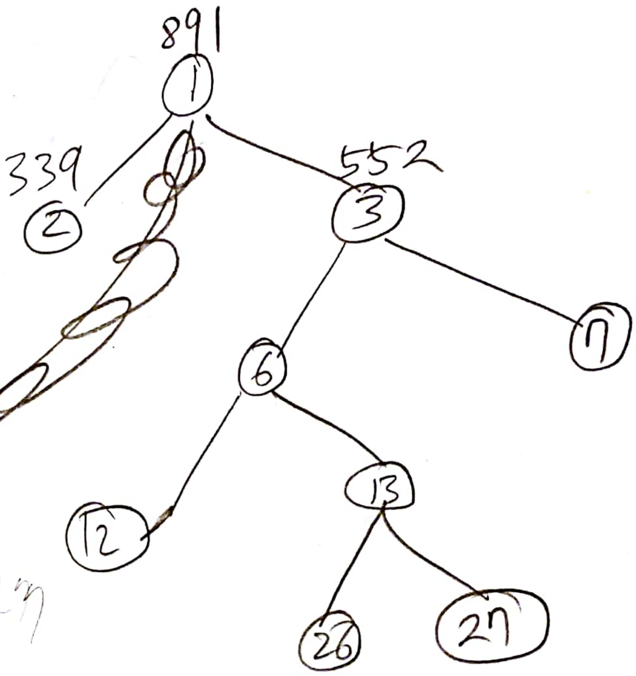
$$\Downarrow$$

$$\boxed{\frac{\beta^2}{\sigma^2} < \frac{(1-\alpha)(1+\alpha)}{(1-\alpha)(1-\alpha)}\frac{1}{n}}$$

☆ Although Bias ↑, it's small compared to var ↓.

#8

a)
1)
2)
3)
6)
12)
13)
26)
27)
7)



① ⇒ 891 < 1: 342 / 0: 549

② ⇒ 339 < 1: 67 / 0: 272

③ ⇒ 552 < 1: 275 / 0: 277

⑥ ⇒ 455 < 1: 201 / $\boxed{0}$: 254

$\boxed{\text{loss}}$

⑦ ⇒ 552-455 < 1: / 0: 23

⇒ $\boxed{97}$

$\left(\frac{171}{403}\right)\left(\frac{403-171}{403}\right)$

↓

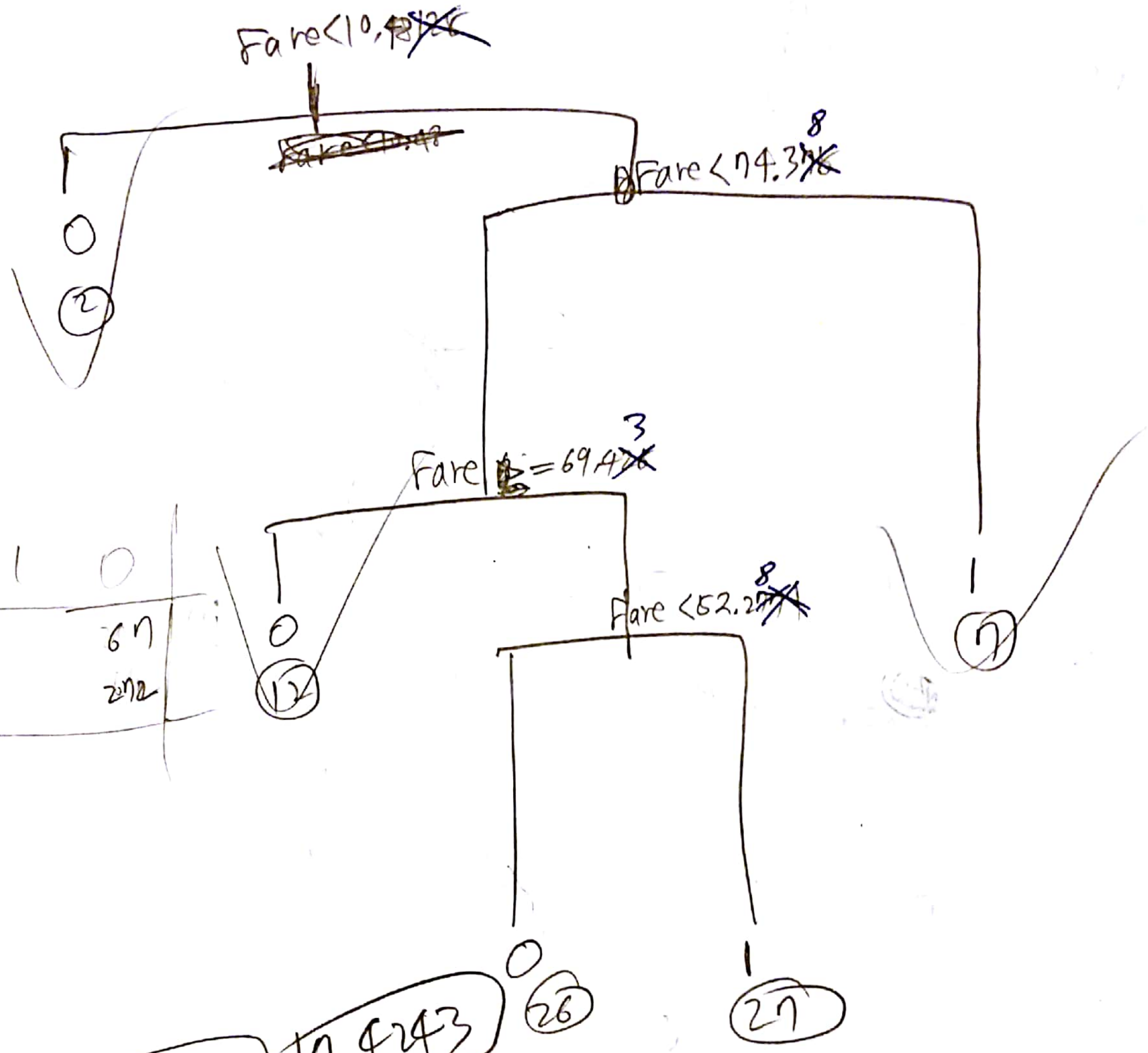
1

⑫ ⇒ 15 < 1: 2 / 0: 13

⑬ ⇒ 440 < 1: 199 / 0: 241

Scanned by CamScanner

b)



	1	0
1		67
0		272

c) $\boxed{\frac{171}{403}}$ => $\boxed{0.4243}$ => survival prob

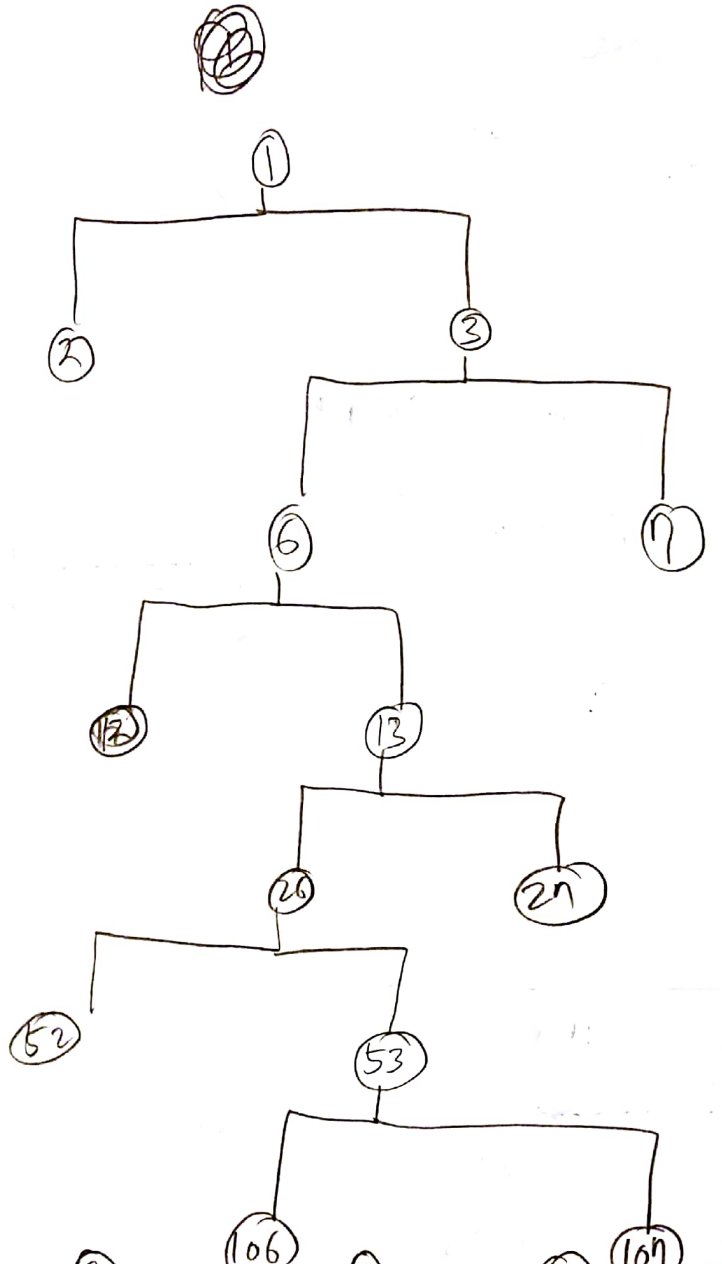
d) precision => positive predicted value => $\boxed{\frac{74+28}{(74+28)+(23+9)}}$

Real => sensitivity

$\boxed{\frac{74+28}{342}}$

obs / pred	1	0	
1	74 + 28	23 + 9	
0	67+2+171	272+13 +232	
	342	549	891

e)



3 6 13 & 53 & 107 26

Ans ⇒ Female & pclass 3 & Fare = 15 & Embark = Southhampton & ~~2 siblings~~

X included

Not importance variable....

∴ 0.8126

3 7

f) Female & pclass 1 & fare = 18 & South & Spouse

∴ 0.947

$\boxed{14.2}$

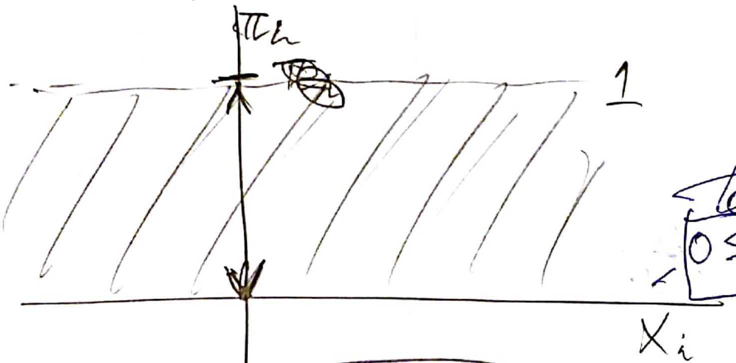
$0 \le \pi_i = \Phi(\mu_i) = \Phi(\alpha+\beta X_i) \le 1$

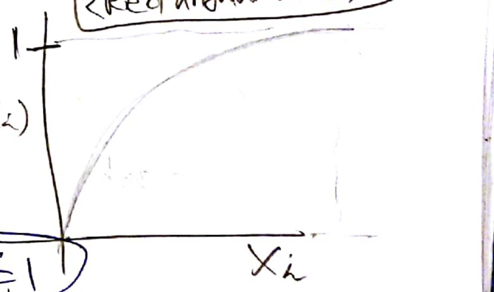
Here p is $\boxed{g^{-1}}$

Again, $\pi_i \in [0,1]$ and $-\infty < \alpha+\beta X_i < \infty$.

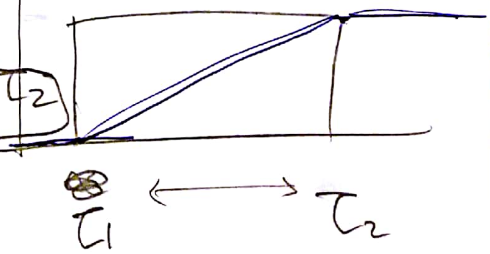
And, by taking CDF Φ on $\alpha+\beta X_i$

<Rectangular dist>





$0 \le \Phi(X_i) \le 1$



If we say X_i is defined between τ_1 and τ_2

when $X_i < \tau_1$,

$\pi_i = 0$

when $\tau_1 \le X_i \le \tau_2$,

$\pi_i = \Phi(\alpha+\beta X_i) = \left[\frac{(\tau_2-\tau_1)(\alpha+\beta X_i)}{2}\right] \Big/ (\tau_2-\tau_1)(\alpha+\beta\tau_2)$

assuming it's increasing linear.

when $X_i > \tau_2$

$\pi_i = 1$

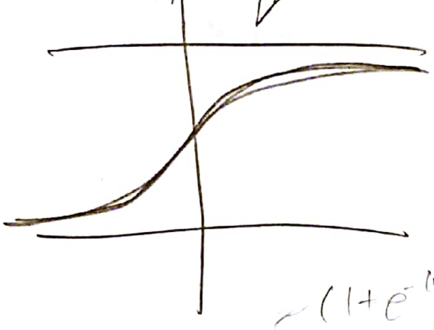
Remember $\alpha+\beta X_i$ is strictly increasing ~~monotone~~ and Φ: cdf is always between 0 and 1

→ so transformation $\Phi^{-1}(\pi_i) = \alpha+\beta X_i$ is one-to-one

It is linear-probability model!!!

14.3

$$\pi = \frac{1}{1+e^{-(\alpha+\beta X)}} = \frac{e^{(\alpha+\beta X)}}{1+e^{(\alpha+\beta X)}}$$



$$\frac{d\pi}{dX} = \cancel{\ln\left|1+e^{-(\alpha+\beta X)}\right| \cdot e^{-(\alpha+\beta X)} \cdot -\beta}$$

$$\cancel{= -\beta \cdot e^{-(\alpha+\beta X)} \cdot \ln\left|1+e^{-(\alpha+\beta X)}\right|}$$

$$\cancel{= -\beta \cdot \left(\frac{1}{\pi}-1\right) \cdot \ln\left|\frac{1}{\pi}\right|}$$

$$\boxed{(\ln x)' = \frac{1}{x}}$$

$-(1+e^{-(\alpha+\beta X)})$

$$\frac{d\pi}{dx} \Rightarrow \frac{d\left[\left(1+e^{-(\alpha+\beta X)}\right)^{-1}\right]}{dx}$$

$$= -1\left(1+e^{-(\alpha+\beta X)}\right)^{-2} \cdot -\beta e^{-(\alpha+\beta X)}$$

$$= \beta \cdot e^{-(\alpha+\beta X)} \cdot \frac{1}{\left(1+e^{-(\alpha+\beta X)}\right)^2}$$

$$= \beta \cdot \left(\frac{1}{\pi}-1\right) \cdot \pi^2 = \beta \cdot \frac{1-\pi}{\pi}\pi^2$$

$$= \boxed{\beta(1-\pi)\pi}$$

#14.4

$p(y_i) = \pi_i^{y_i}(1-\pi_i)^{1-y_i}$

probability when $y_i = 1$

Bernoulli where $Y_i \sim Ber(\pi_i)$

$p(y_i = 0) = (1-\pi_i)$

$p(y_i = 1) = \pi_i$

So we use

$f(x) = \pi_i^{x}(1-\pi_i)^{1-x}$

and get GLM set-up.

$\Downarrow$

$\prod_{i=1}^{n} p(y_i)$ to get MLE

#14.9

Don't confuse with logistic curve regression and logit!!!

logit ⇒ $p \to \log\left(\frac{p}{1-p}\right)$

probit ⇒ $p \to \Phi^{-1}(p)$

$$\prod_{i=1}^{n}\left[y_i \log p_i + (1-y_i)\log(1-p_i)\right] \quad \& \quad \log\left(\frac{p_i}{1-p_i}\right) = x_i^T\beta$$

⇒ Understand what they are asking!!!

⇒ MLE for logit model is

$$\beta^{(m+1)} = \beta^{(m)} + (X^TWX)^{-1}X^T(Y-p)$$

$$= (X^TWX)^{-1}X^TWZ$$

where $Z = X\beta^{(m)} + W^{-1}(Y-p)$

Where $W_i = \mathrm{diag}(p_i(1-p_i))$

#15.6

$$p(y;\theta,\phi) = \exp\left[\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right]$$

$$= \exp[c(y,\phi)] \cdot \exp\left[\frac{y\theta - b(\theta)}{a(\phi)}\right]$$

$$= h(y,\phi) \exp\left(\frac{y\theta - b(\theta)}{a(\phi)}\right)$$

→ what we have in the lecture note!!

① poisson ⟹ Assume $y_i \sim Pois(\mu_i)$

$$f(x) = e^{-\mu_i} \frac{\mu_i^{x}}{x!}$$

$$= \frac{1}{x!} e^{-\mu_i} e^{x \ln \mu_i}$$

$$= \frac{1}{x!} e^{(x \ln \mu_i - \mu_i)}$$

$\boxed{\theta_i = \ln \mu_i}$ $\quad b(\theta_i) = b(\ln \mu_i) = \mu_i$

→ so $\boxed{b(\theta_i) = e^{\theta_i}}$

$\boxed{a(\phi) = 1}$

$\frac{1}{x!} = \exp[c(y,\phi_i)]$ ⟹ so, $\ln\left(\frac{1}{x!}\right) = c(y,\phi_i)$

→ $\boxed{c(y,\phi_i) = -\ln x!}$

② Gamma

$$\frac{e^{-\lambda x}(\lambda x)^{\alpha-1}\lambda}{\Gamma(\alpha)} \sim \text{Gamma}(\alpha,\lambda)$$

→ works not really good

Inversed

$$= \frac{1}{\Gamma(\alpha)}\cdot\frac{1}{\lambda^{\alpha}}\cdot x^{\alpha-1}\cdot e^{-\frac{x}{\lambda}}$$

→ works better

$$h(y,\phi)\exp\left(\frac{y\theta-b(\theta)}{\alpha(\phi)}\right)$$

↳ $\exp[c(y,\phi)]$

↳ pg. 422

$$\cancel{\frac{1}{\Gamma(\alpha)}\cdot e^{-\lambda x}\cdot e^{\alpha\ln\lambda}\cdot e^{(\alpha-1)\ln x}}$$

$$\cancel{= \frac{1}{\Gamma(\alpha)}\exp[-\lambda x+\alpha\ln\lambda+(\alpha-1)\ln x] = \frac{x^{\alpha-1}}{\Gamma(\alpha)}\exp[x(-\lambda)+\alpha\ln\lambda]}$$

$$\cancel{\exp[-\ln(\Gamma(\alpha)) - \lambda x + \alpha\ln\lambda + (\alpha-1)\ln x]}$$

= ☆ Goal is making this part look similar ☆ Key point!!

$$\exp\left[-\ln(\Gamma(\alpha)) - \alpha\ln\lambda + (\alpha-1)\ln x - \frac{x}{\lambda}\right]$$

how to separate it out ----

$$= \exp\Big[\underbrace{x\left(-\tfrac{1}{\lambda}\right)}_{\approx y(\theta)} - \boxed{\alpha\ln\lambda} \underbrace{- \ln(\Gamma(\alpha)) + (\alpha-1)\ln x}_{\approx h(y,\phi)}\Big]$$

$\boxed{\theta = -\frac{1}{\lambda}}$ → Then, $b(\theta)$ not written in terms of θ

Also, I will get $b(\theta)=\alpha\ln(-\frac{1}{\lambda})$ and it depends on α, which is not allowed

$$\Rightarrow \exp\left[\alpha\cdot x\left(-\frac{1}{\alpha\lambda}\right) - \alpha\ln\lambda - \ln(\Gamma(\alpha)) + (\alpha-1)\ln x - \alpha\ln\alpha + \alpha\ln\alpha\right]$$

$$= \exp\left[\alpha\left[x\left(-\frac{1}{\alpha\lambda}\right) - \ln\lambda - \ln\alpha\right] - \ln(\Gamma(\alpha)) + (\alpha-1)\ln x + \alpha\ln\alpha\right]$$

$$= \exp[\alpha\ln\alpha + (\alpha-1)\ln x - \ln(\Gamma(\alpha))]\cdot\exp\left[\frac{x\left(-\frac{1}{\alpha\lambda}\right) - \ln(\alpha\lambda)}{1/\alpha}\right]$$

$\Rightarrow \theta = -\frac{1}{\alpha\lambda}$

$\boxed{b(\theta) = -\ln(-\theta)}$

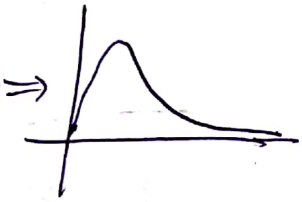
$\boxed{\alpha(\phi) = \phi}$

↳ $\boxed{\phi = \frac{1}{\alpha}}$

$\alpha\ln(\alpha x) - \ln x - \ln(\Gamma(\alpha))$

$$\boxed{c(y,\phi) = \frac{1}{\phi}\ln\left(\frac{x}{\phi}\right) - \ln x - \ln(\Gamma(\phi^{-1}))}$$

(ch 6.4) # 2, (n a ... ~~15, 18, 21, 24, 26, 29, 37, 39~~ ...

③ Inverse − Gaussian = Wald dist ⇒ 

↳ It's not actually inverse of normal

$$f(x;\mu,\lambda)=\left(\frac{\lambda}{2\pi x^3}\right)^{1/2}\exp\left(\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right)$$

x, λ

$$f(x;\theta,\phi)=h(x,\phi)\exp\left(\frac{x\theta-b(\theta)}{a(\phi)}\right)$$

↳ $\exp[c(x,\phi)]$

$$\Rightarrow \frac{\lambda^{1/2}}{(2\pi x^3)^{1/2}}\exp\left[\frac{-\lambda(x^2-2\mu x+\mu^2)}{2\mu^2 x}\right]$$

☆ Goal is making this part similar ☆

$$=\frac{\lambda^{1/2}}{(2\pi x^3)^{1/2}}\exp\left[\frac{-\lambda x^2+2\lambda\mu x-\lambda\mu^2}{2\mu^2 x}\right]=\ //\cdot\exp\left[-\frac{\lambda x}{2\mu^2}+\frac{\lambda}{\mu}-\frac{\lambda}{2x}\right]$$

$$=\ //\cdot\exp\left[x\left(-\frac{\lambda}{2\mu^2}\right)+\mu\left(\frac{\lambda}{\mu^2}\right)\right]\exp\left[-\frac{\lambda}{2x}\right]$$

$$=\frac{\lambda^{1/2}}{(2\pi x^3)^{1/2}}\cdot\exp\left[\frac{-\lambda}{2x}\right]\cdot\exp\left[\frac{x\left(-\frac{1}{2\mu^2}\right)+\left(\frac{1}{\mu}\right)}{1/\lambda}\right]$$

$$\Rightarrow \theta=-\frac{1}{2\mu^2}$$

$$\boxed{b(\theta)=-\sqrt{-2\theta}}$$

$$\boxed{a(\phi)=\phi}$$

$$\phi=\frac{1}{\lambda}$$

$$-\frac{1}{2}\left[\ln(2\pi x^3\phi)+\frac{1}{\phi x}\right]$$

Also, // is $\exp\left[\ln\left(\frac{\lambda^{1/2}}{(2\pi x^3)^{1/2}}\right)\right]\cdot\exp\left[-\frac{\lambda}{2x}\right]$

$$=\exp\left[\ln\left(\frac{\lambda^{1/2}}{(2\pi x^3)^{1/2}}\right)-\frac{\lambda}{2x}\right]$$

$$\text{So, } c(x,\phi)=\frac{1}{2}\left[\ln\lambda-\ln(2\pi x^3)-\frac{\lambda}{x}\right]$$

$$=\frac{1}{2}\left[\ln\left(\frac{1}{\phi}\right)-\ln(2\pi x^3)-\frac{1}{x\cdot\phi}\right]$$

$$=\frac{1}{2}\left[-\ln(\phi)-\ln(2\pi x^3)-\frac{1}{\phi x}\right]$$

Ch 6.4) # 3, 6, 7, 9, 10, 12, 13, 15, 19, 33, 36, 3[illegible] 8

#15.7

$V(y_i) = b''(\theta_i)\,\alpha(\phi_i)$ ⇒ Formula in the class

① Gaussian ⇒ $\alpha(\phi) = \phi$, $b(\theta) = \frac{\theta^2}{2}$

$\Downarrow$

$b''(\theta) = 1$ ⇒ $b''(\theta)\,\alpha(\phi) = \boxed{\phi}$

$\boxed{\sigma^2}$

② Binomial ⇒ $\alpha(\phi) = \frac{1}{n}$, $b(\theta) = \ln(1+e^\theta)$ ⇒ $b'(\theta) = \frac{e^\theta}{1+e^\theta}$

$$\Rightarrow b''(\theta) = \frac{e^\theta\cdot(1+e^\theta) - e^\theta\cdot(e^\theta)}{(1+e^\theta)^2} = \frac{e^\theta + (e^{2\theta}) - (e^{2\theta})}{(1+e^\theta)^2}$$

$$= \frac{e^\theta}{(1+e^\theta)^2}$$

so, $b''(\theta)\cdot\alpha(\phi) = \frac{e^\theta}{(1+e^\theta)^2}\cdot\frac{1}{n}$

$$\Rightarrow \frac{\frac{p_i}{1-p_i}}{\left(1+\frac{p_i}{1-p_i}\right)^2}\cdot\frac{1}{n}$$

$$= \frac{\frac{p_i}{1-p_i}}{\left(\frac{1-p_i+p_i}{1-p_i}\right)^2}\cdot\frac{1}{n} = \frac{\frac{p_i}{1-p_i}}{\left(\frac{1}{1-p_i}\right)^2}\cdot\frac{1}{n} = \boxed{p_i(1-p_i)\cdot\frac{1}{n}}$$

3) ~~Gauss~~ poisson

$\alpha(\phi) = 1$

$b(\theta) = e^{\theta} \rightarrow b''(\theta) = e^{\theta}$

$\alpha(\phi) \cdot b''(\theta) = e^{\theta} = e^{\log \lambda_i} = \lambda_i = \boxed{\mu}$

4) Gamma

$\alpha(\phi) = \phi$

$b(\theta) = -\log(-\theta)$

$\downarrow$

$b'(\theta) = - \frac{-1}{-\theta} = \frac{-1}{\theta} = -\theta^{-1}$

$b''(\theta) = -1 \cdot -1 \cdot \theta^{-2} = \theta^{-2}$

so, $\alpha(\phi) \cdot b''(\theta) = \phi \cdot \theta^{-2} \Rightarrow \begin{cases} \phi = \frac{1}{\alpha} \\ \theta = -\frac{1}{\alpha\lambda} \end{cases}$

$\Downarrow$

$\frac{1}{\alpha} \cdot \left(-\frac{1}{\alpha\lambda}\right)^{-2}$

$= \frac{1}{\alpha}\alpha^2\lambda^2 = \boxed{\alpha\lambda^2}$

5) $\alpha(\phi) = \phi$ | $b(\theta) = -\sqrt{-2\theta} \rightarrow b'(\theta) = (-2\theta)^{-1/2}$

$\downarrow$

$\alpha(\phi) \cdot b''(\theta) = \phi \cdot (-2\theta)^{-3/2}$ $\qquad b''(\theta) = (-2\theta)^{-\frac{3}{2}}$

$= \boxed{(-2\theta)^{-\frac{3}{2}}}$

$= \phi\left(\frac{1}{\lambda}\right)\left(-2 \cdot -\frac{1}{2\mu^2}\right)^{-3/2}$

$= \frac{1}{\lambda} \cdot (\mu^2)^{3/2}$

$\boxed{\phi\mu^3}$

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