Physics 121:
2D Motion, Projectiles, Relative Motion

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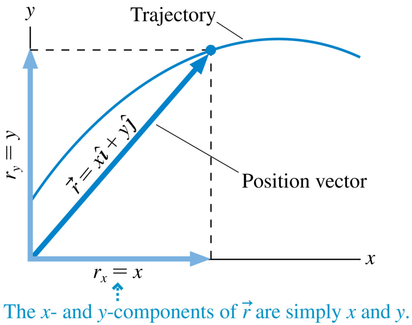
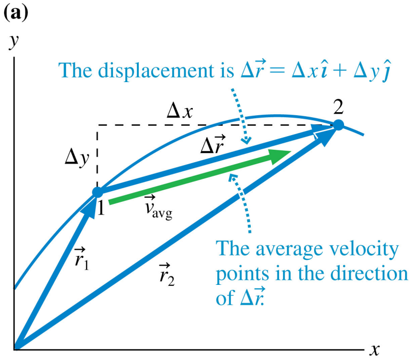
Reminders

- The first exam is this coming Tuesday (19 Sep).
- Does anybody not have a book yet?
- Today we are going to apply the vector stuff we learned to motion!

2D Motion



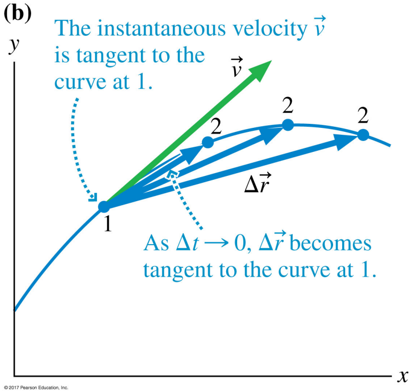
- This is the trajector of a particle in the xy-plane. This is a "picture" of the motion.
- Notice that any at any time the motion can be described by a position vector $\vec{r}$, just like we talked about last week.



- This is the trajector of a particle going from position $\vec{r}_1$ to position $\vec{r}_2$.
- Notice that the average velocity points in the direction of the displacement vector

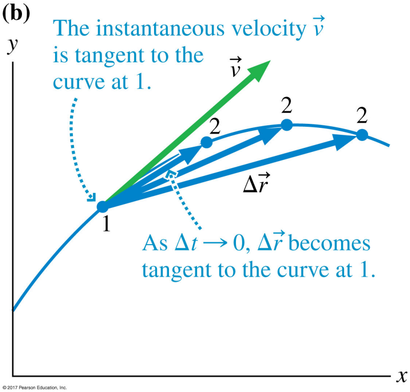
$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

2D Motion



- The average velocity approaches the instantaneous velocity as the spacing decreases, which you can see above.
- The instantaneous velocity is the limit (derivative)

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$



- The instantaneous velocity can also be written as

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

where

$$v_x = \frac{dx}{dt} \quad \text{and} \quad v_y = \frac{dy}{dt}$$

Quick Check

What is the instantaneous velocity at time t of the object that follows the trajectory $\vec{r}(t) = (4 + 2t^2 - t^3)\hat{i} - (3\pi t^5)\hat{j}$?

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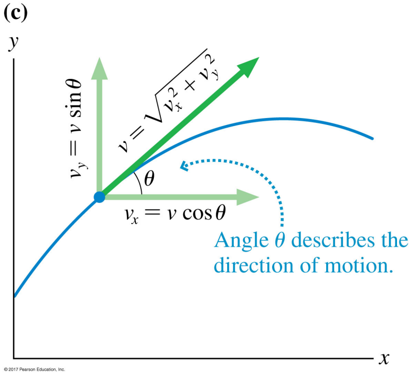
$$\vec{v}(t) = \frac{d\vec{r}}{dt} = (4t - 3t^2)\hat{i} - (15\pi t^4)\hat{j}$$



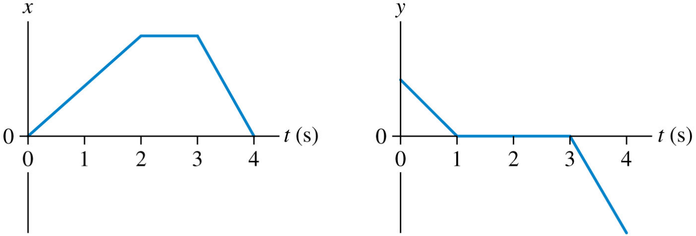
- If we know the angle of the motion with respect to the positive x-axis we can calculate the components and speed.
- Conversely, if we know the components we can determine the direction of motion with

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Quick Check

During which time interval or intervals is the particle described by these position graphs at rest? More than one may be correct

a. 0-1 s
b. 1-2 s
c. 2-3 s
d. 3-4 s



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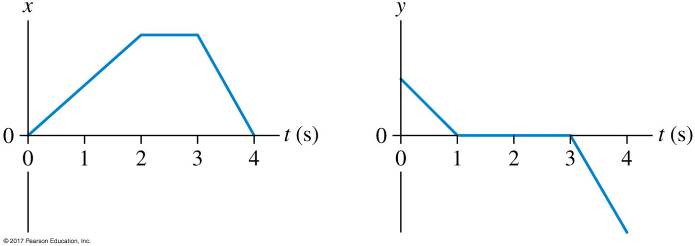

2D Acceleration

- The average acceleration is defined just like it was before in 1D, but now we need to take 2 dimensions into account

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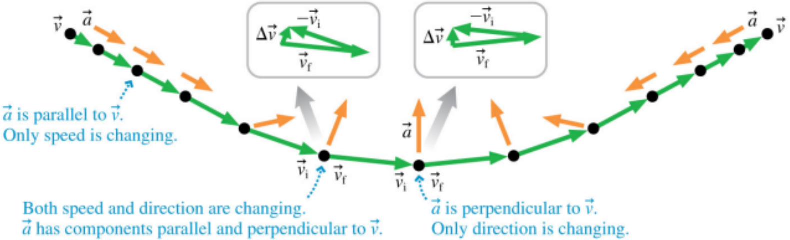
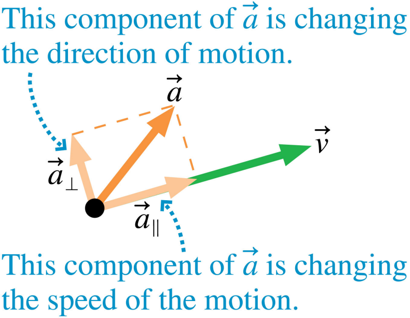
- Here is an example of a 2D acceleration graphically.

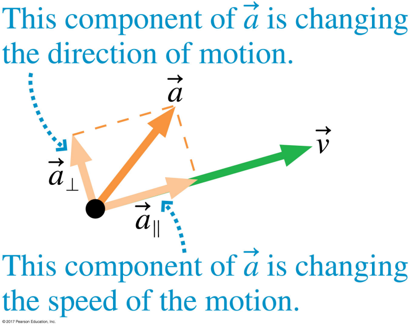


2D Acceleration



- There are two parts to acceleration, one parallel to the velocity $\vec{a}_\perp$, and one parallel to the velocity, $\vec{a}_\parallel$.
- Which one causes an object to change speed?

2D Acceleration



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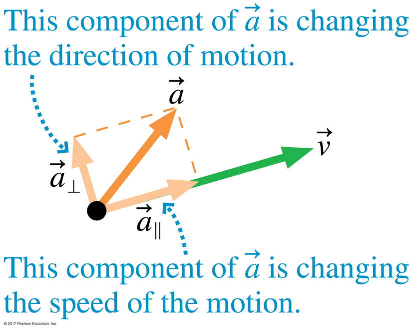

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- Which one causes an object to change direction?

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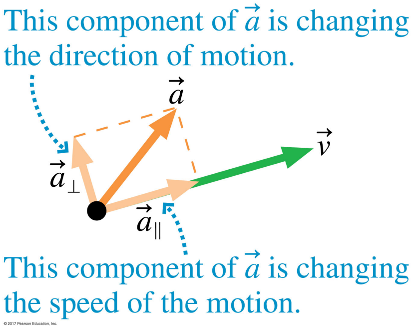

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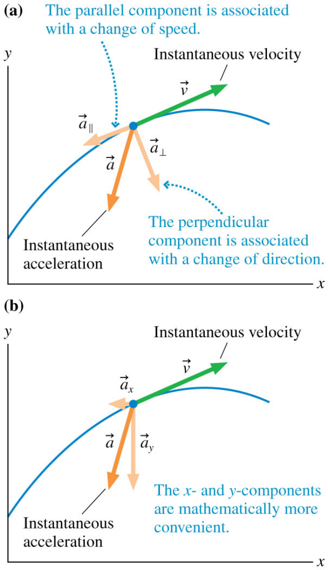
  - $\vec{a}_\parallel$
- Which one causes an object to change direction?
  - $\vec{a}_\perp$

You can maybe see this better in this figure



2D Accleration

- The x- and y-components look the same as they did for velocity.

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

where

$$a_x = \frac{dv_x}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt}$$

2D CONSTANT Acceleration

- When we had constant acceleration in 1D motion what equations did we use?

2D CONSTANT Acceleration

- When we had constant acceleration in 1D motion what equations did we use?
- They were called kinematic equations. In 2D you just have two sets, one for each dimension.

$$x_f = x_i + v_{ix} t + \frac{1}{2} a_x t^2 \qquad y_f = y_i + v_{iy} t + \frac{1}{2} a_y t^2$$

$$v_{fx} = v_{ix} + a_x t \qquad v_{fy} = v_{iy} + a_y t$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \qquad v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

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- You have to keep track of MANY variables in 2D problems. Make sure to be more careful when setting up problems.

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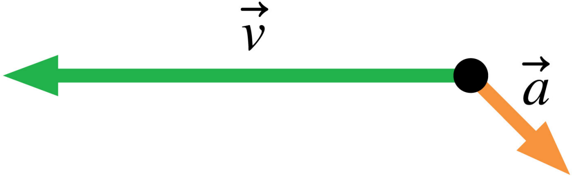
This acceleration will cause the particle to



a. Speed up and curve upward
b. Slow down and curve upward
c. Move to the right and down
d. Speed up and curve downward
e. Slow down and curve downward
f. Reverse direction

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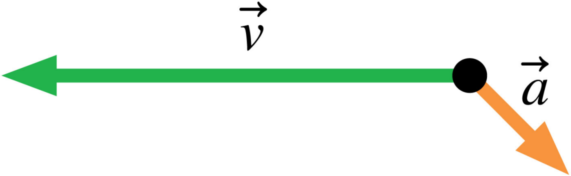

a. Speed up and curve upward
b. Slow down and curve upward
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d. Speed up and curve downward
✓ e. Slow down and curve downward
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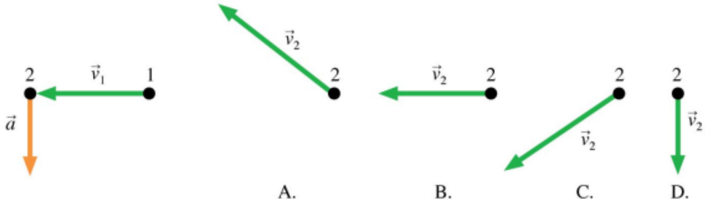
A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the velocity vector $\vec{v}_2$ as the object moves away from point 2?



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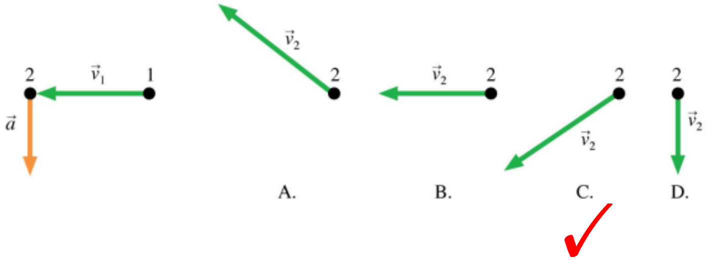
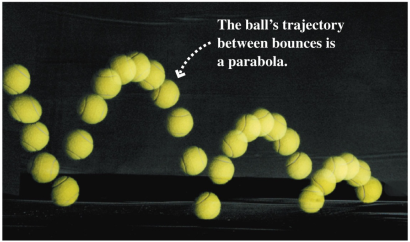
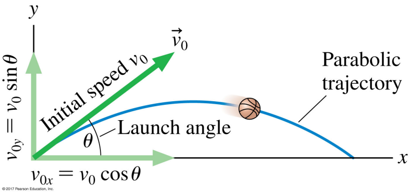


Projectile Motion



- Projectile Motion is 2D motion where an object is only influenced by gravity. It's like 2D free fall motion.

Projectile Motion



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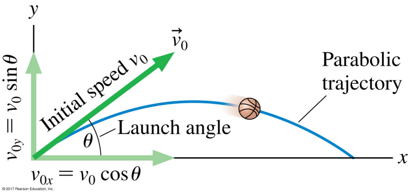

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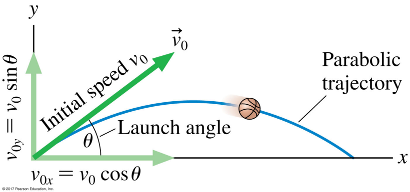

- You can solve for the x and y components of the initial velocity, v_{0x} and v_{0y}, given the angle, θ.
- What are the objects horizontal and vertical accelerations?
  - $a_x = 0\ \mathrm{m/s^2}$ and $a_y = -9.80\ \mathrm{m/s^2}$
- Notice that just like with free fall $a_y = -g$.

Projectile Motion

- If I have an objects that is launched at $t = 0$ s and has the initial velocity $\vec{v}_0 = (9.8\hat{i} + 19.6\hat{j})$ m/s. What is the velocity at $t = 1$ s and $t = 2$ s?

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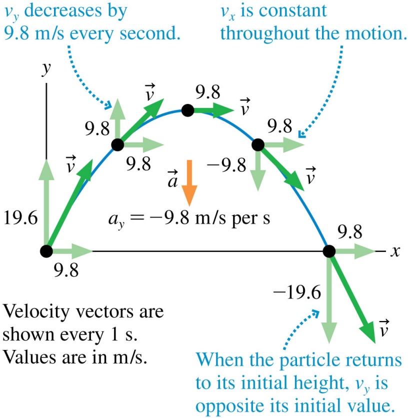
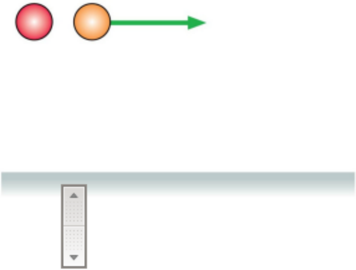


- v_x never changes because there is no horizontal acceleration.
- v_y decreases by 9.8 m/s every second.

A heavy red ball is released from rest 2.0 m above a flat, horizontal surface. At exactly the same instant, a yellow ball with the same mass is fired horizontally at 3.0 m/s. Which ball hits the ground first?



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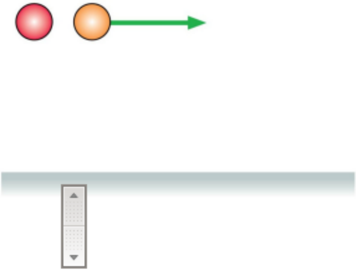

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A 100 g ball rolls off a table and hits 2.0 m from the base of the table. A 200 g ball rolls off the same table with the same speed. It lands at distance

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B. Between 1 m and 2 m.
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A hunter points his rifle directly at a coconut that he wishes to shoot off a tree. It so happens that the coconut falls from the tree at the exact instant the hunter pulls the trigger. Consequently,

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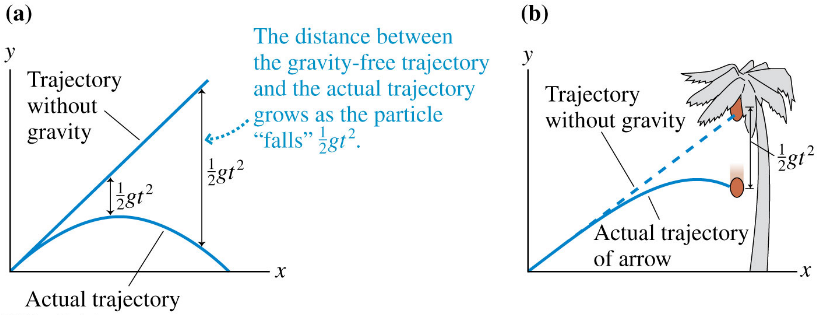

(a)
y
Trajectory without gravity
$\frac{1}{2}gt^2$
$\frac{1}{2}gt^2$
x
Actual trajectory
The distance between the gravity-free trajectory and the actual trajectory grows as the particle "falls" $\frac{1}{2}gt^2$.
(b)
y
Trajectory without gravity
$\frac{1}{2}gt^2$
Actual trajectory of arrow
x

Projectile Range

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Notice that $x_i = a_x = y_f = y_i = 0$, $a_y = -g$, $v_{ix} = v_0 \cos(\theta)$ and $v_{iy} = v_0 \sin(\theta)$ which leaves you with

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$$x_f = v_0\cos(\theta)t$$
$$0 = v_0\sin(\theta)t - 1/2gt^2$$

Now solve for t in the y equation and plug it into the x equation, noticing and $\cos(\theta)\sin(\theta) = \sin(2\theta)$, which gives

$$xf = \frac{2v_0^2}{g}\sin(2\theta) = \text{range}$$

Projectile Range

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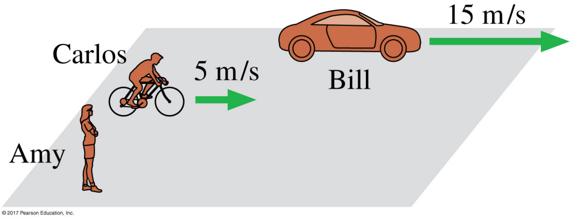
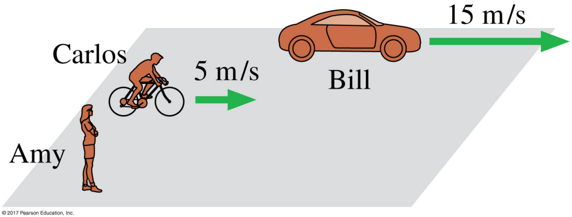
Notice that the maximum of the range is then at 45°.

Relative Motion

Relative Motion



- The figure below shows Amy and Bill watching Carlos on his bicycle. According to Amy, Carlos's velocity is $(v_x)_{CA} = +5$ m/s.
  - The *CA* subscript means "C relative to A."



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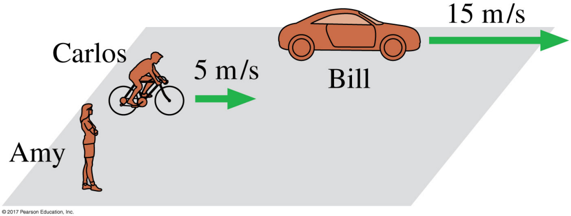
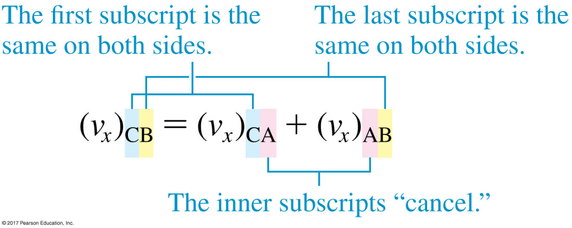
Relative Motion



- The figure below shows Amy and Bill watching Carlos on his bicycle. According to Amy, Carlos's velocity is $(v_x)_{CA} = +5$ m/s.
  - The *CA* subscript means "C relative to A."
- According to Bill, Carlos's velocity is $(v_x)_{CB} = -10$ m/s.
- Every velocity is measured relative to a certain observer. There is no "true" velocity.

Relative Motion

- The velocity of C relative to B is the velocity of C relative to A plus the velocity of A relative to B.



- If B is moving to the right relative to A, then A is moving to the left relative to B.

$$(v_x)_{AB} = -(v_x)_{BA}$$

Mythbusters - Relative Motion

Relative Motion

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Relative Motion

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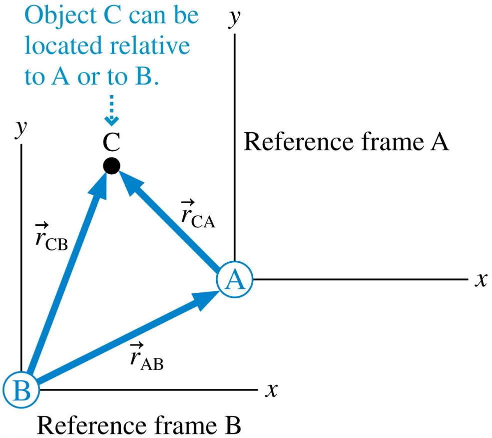
- In the figure, Object C is measured in two different reference frames, A and B.



Relative Motion

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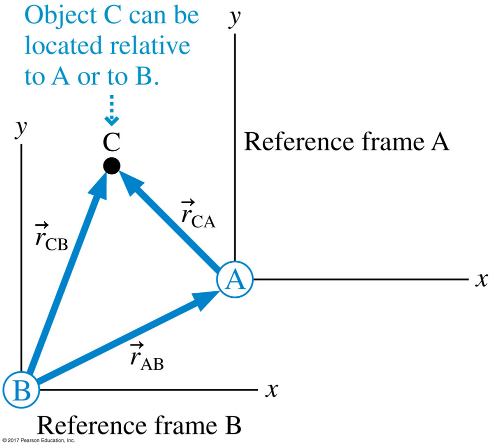

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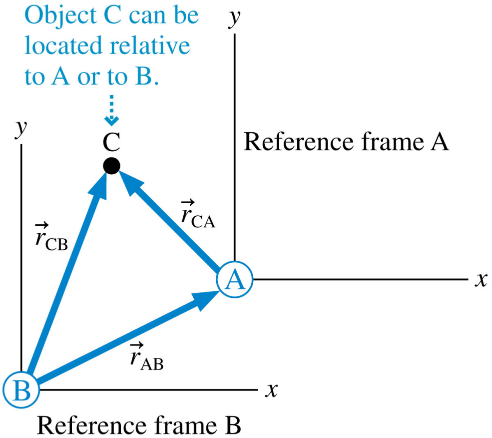
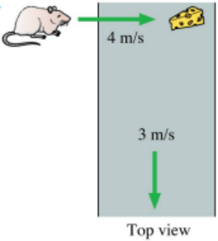


- Notice here that $\vec{r}_{CB} = \vec{r}_{CA} + \vec{r}_{AB}$
- Also, since the relative velocities are just the derivatives of the relative positions we can say $\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$. This is called a **Galilean Transformation**.

Quick Check

A factory conveyor belt rolls at 3 m/s. A mouse sees a piece of cheese directly across the belt and heads straight for the cheese at 4 m/s. What is the mouse's speed relative to the factory floor?



A. 1 m/s
B. 2 m/s
C. 3 m/s
D. 4 m/s
E. 5 m/s

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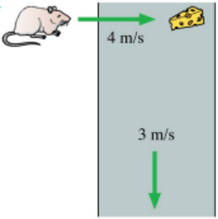
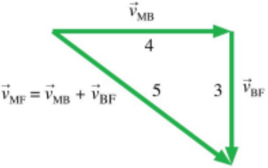


Top view

A. 1 m/s
B. 2 m/s
C. 3 m/s
D. 4 m/s
✓E. 5 m/s



3-4-5 right triangle

M = mouse
B = belt
F = floor

Picture References

None yet