Physics 121: Circular Motion

Cody Petrie

Mesa Community College

Quiz

1. Which of these accelerations will always cause an object to change direction?

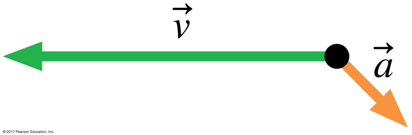
A. $\vec{a}_x$

B. $\vec{a}_y$

C. $\vec{a}_\perp$

D. $\vec{a}_{||}$

2. This acceleration will cause the particle to



A. Speed up and curve upward

B. Speed up and curve downward

C. Slow down and curve upward

D. Slow down and curve downward

E. Move to the right and down

F. Reverse direction

Reminders

- HW is due on Saturday.
- The first exam is this coming Tuesday (19 Sep).
    - Covers materials from chapters 1-4.
    - Bring pencil and calculator. You cannot use a cell phone as a calculator as they need to be put away during the exam.
    - You will get an hour to take the exam.
    - We will have a question led review before the exam.
    - Bring a standard sized notecard with any equaitons or information that you want on it. You can't share cards so make sure you bring your own!

Uniform Circular Motion

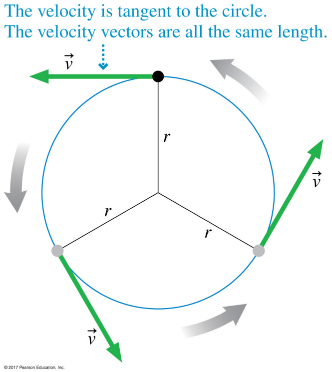
- What are some examples of circular motion?

Uniform Circular Motion

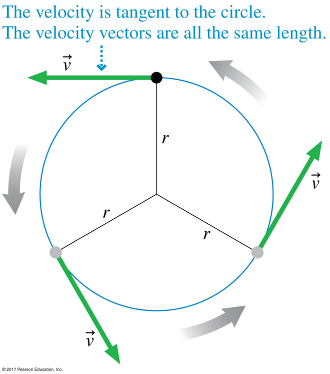
- What are some examples of circular motion?
  - Ball on the end of a string being swung around
  - Planet or Satellite orbiting a celestial object
  - **NOT** a spinning top, unless you're talking about a single point on the top.

Uniform Circular Motion

- What are some examples of circular motion?
  - Ball on the end of a string being swung around
  - Planet or Satellite orbiting a celestial object
  - **NOT** a spinning top, unless you're talking about a single point on the top.
- If the *speed* around the circle is constant then the motion is called **uniform circular motion**.

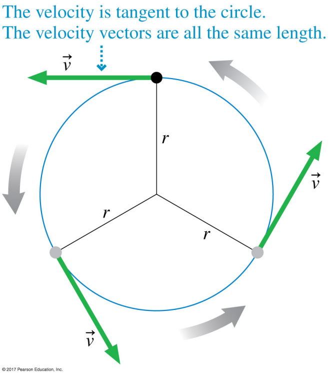


Uniform Circular Motion



- If the radius of motion is r and the constant speed is v then what is the **period** of the motion?

Uniform Circular Motion



- If the radius of motion is r and the constant speed is v then what is the **period** of the motion?
- $v = \frac{\text{1 circumference}}{\text{1 period}} = \frac{2\pi r}{T}$

Quick Check

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

Quick Check

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

1. Convert 2400 rpm to revolution per second

$$\frac{2400 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 40 \text{ rev/sec}$$

Quick Check

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

1. Convert 2400 rpm to revolution per second

$$\frac{2400 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 40 \text{ rev/sec}$$

2. How long it takes to do one turn is measured in seconds per revolution, s/rev

$$T = \frac{1}{40} \text{ s} = 0.0025 \text{ s}$$

Quick Check

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

1. Convert 2400 rpm to revolution per second

$$\frac{2400 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 40 \text{ rev/sec}$$

2. How long it takes to do one turn is measured in seconds per revolution, s/rev

$$T = \frac{1}{40} \text{ s} = 0.0025 \text{ s}$$

3. Now plug in what we know to the equation we just found to get the speed

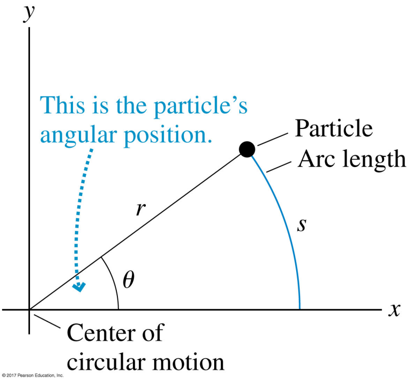
$$v = \frac{2\pi r}{T} = \frac{2\pi(0.020 \text{ m})}{0.025 \text{ s}} = 5.0 \text{ m/s}$$

Angular Position

- Which coordinates would be the easiest to use when describing the position vectors in circular motion, (x, y) or (r, θ)? Why?

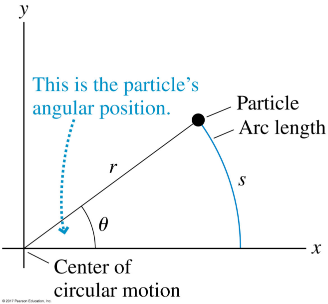
Angular Position

- Which coordinates would be the easiest to use when describing the position vectors in circular motion, (x, y) or (r, θ)? Why?
    - r is often constant in circular motion and so the only thing that changes is θ.



θ is called the **angular position** of the particle.

Angular Position

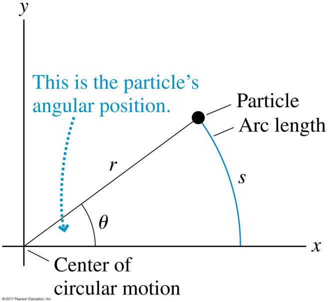


- Angular position is defined (in radians) as

$$\theta(\text{radians}) \equiv \frac{s}{r}$$

- This is one of the reasons to use radians because $s = r\theta$ if θ is measured in radians.

Angular Position



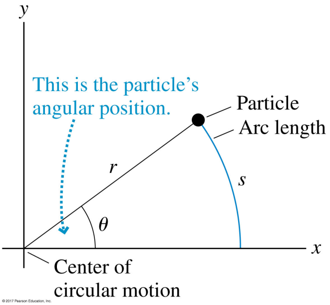
- Angular position is defined (in radians) as

$$\theta(\text{radians}) \equiv \frac{s}{r}$$

- This is one of the reasons to use radians because $s = r\theta$ if θ is measured in radians.

- Radians are the SI unit for angle. How many radians are in a full circle?

Angular Position



- Angular position is defined (in radians) as

$$\theta(\text{radians}) \equiv \frac{s}{r}$$

- This is one of the reasons to use radians because $s = r\theta$ if θ is measured in radians.

- Radians are the SI unit for angle. How many radians are in a full circle?
  - $\theta_{\text{full circle}} = \frac{2\pi r}{r} = 2\pi$ rad $\qquad$ 1 rev $= 360^\circ = 2\pi$ rad

Quick Check

Convert 1 rad to degrees?

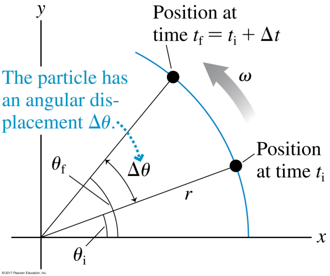
Quick Check

Convert 1 rad to degrees?

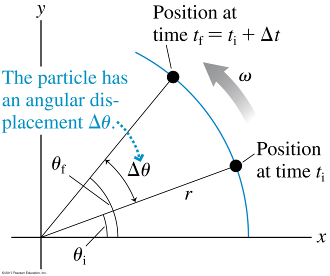
$$1 \text{ rad} = 1 \text{ rad} \times \frac{360^\circ}{2\pi \text{ rad}} = 57.3^\circ$$

Note that 1 rad is $\approx 60^\circ$, this can make quick calculations easier to do in your head.

Angular Velocity



- We can define an *angular* displacement, $\Delta\theta$ (just like Δr) just like we did before.



- We can define an *angular* displacement, $\Delta\theta$ (just like Δr) just like we did before.
- From this we can define an average **angular velocity**

$$\text{angular velocity} = \omega \equiv \frac{\Delta\theta}{\Delta t},$$

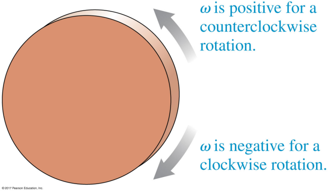
which is the rate at which the angular position changes with respect to time

*ω is the Greek leter "omega" not just w

Angular Velocity

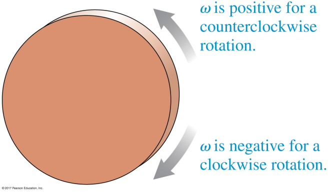
- How do you know if ω is positive or negative? (think right hand rule)

- How do you know if ω is positive or negative? (think right hand rule)



Angular Velocity

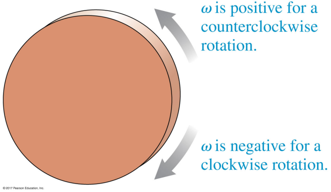
- How do you know if ω is positive or negative? (think right hand rule)



- So if we have uniform circular motion what does that tell us about ω?

Angular Velocity

- How do you know if ω is positive or negative? (think right hand rule)



- So if we have uniform circular motion what does that tell us about ω?
  - Uniform circular motion means that ω is constant (just like uniform motion meant that v was constant).

Angular Velocity

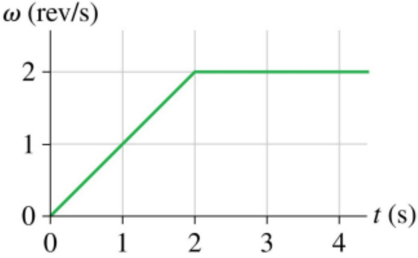
- Just as before if we take the limit of the average angular velocity you get a derivative

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Velocity

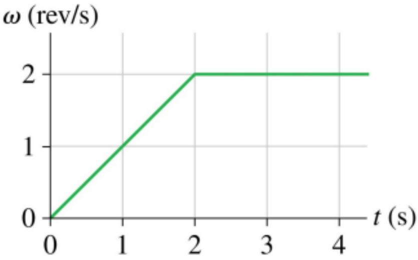
- Just as before if we take the limit of the average angular velocity you get a derivative
$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$
- Since everything for ω and θ parallel s and v we can draw some general conclusions about the graphical representations.
  - $\omega =$ slope of the θ-versus-t graph at time t
  - $\theta_f = \theta_i +$ area under the ω-versus-t curve between t_i and $t_f = \theta_i + \omega \Delta t$

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?



A. 1
B. 2
C. 4
D. 6
E. 8

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?



A. 1
B. 2
C. 4
✓D. 6
E. 8

Angular Velocity

- The angular velocity and period are related, can you figure out how?

Angular Velocity

- The angular velocity and period are related, can you figure out how?
- The period is the time it takes to do one full revolution (i.e. $\Delta\theta = 2\pi$ rad). So we get

$$|\omega| = \frac{2\pi \text{ rad}}{T} \quad \rightarrow \quad T = \frac{2\pi \text{ rad}}{|\omega|}$$

Quick Check

A ball rolls around a circular track with an angular velocity of 4π rad/s. What is the period of the motion?

A. $\frac{1}{2}$ s
B. 1 s
C. 2 s
D. $\frac{1}{2\pi}$ s
E. $\frac{1}{4\pi}$ s

A ball rolls around a circular track with an angular velocity of 4π rad/s. What is the period of the motion?

✓A. $\frac{1}{2}$ s

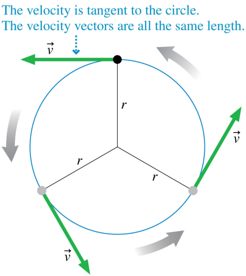
B. 1 s

C. 2 s

D. $\frac{1}{2\pi}$ s

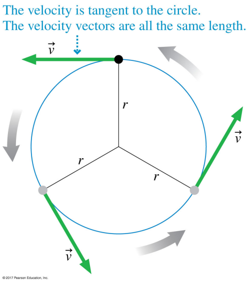
E. $\frac{1}{4\pi}$ s

Tangential Velocity



- The **tangential velocity** of an object is the speed at which the object moves around the circle, ds/dt, where s is the arc length.

Tangential Velocity

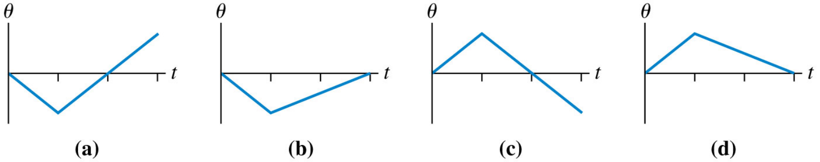


- The **tangential velocity** of an object is the speed at which the object moves around the circle, ds/dt, where s is the arc length.
- Just like arc length could be described in terms of the angle (in radians) the tangential velocity can be descrived in terms of the angular momentum (in rad/s).

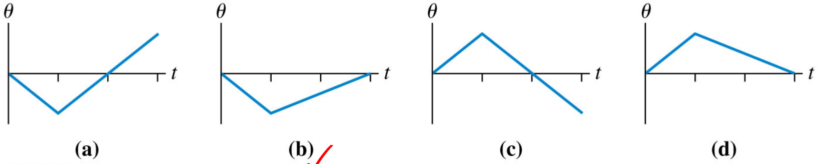
$$v_t = \omega r$$

Quick Check

A particle moves cw around a circle at constant speed for 2.0 s. It then reverses direction and moves ccw at half the original speed until it has traveled through the same angle. Which is the particle's angle-versus-time graph?

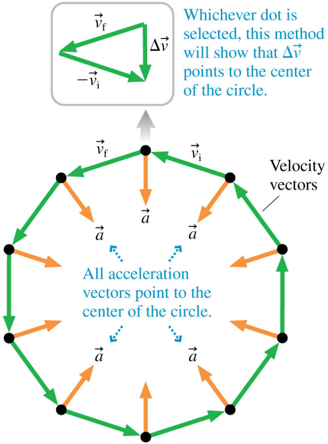


A particle moves cw around a circle at constant speed for 2.0 s. It then reverses direction and moves ccw at half the original speed until it has traveled through the same angle. Which is the particle's angle-versus-time graph?



✓

Centripital Acceleration



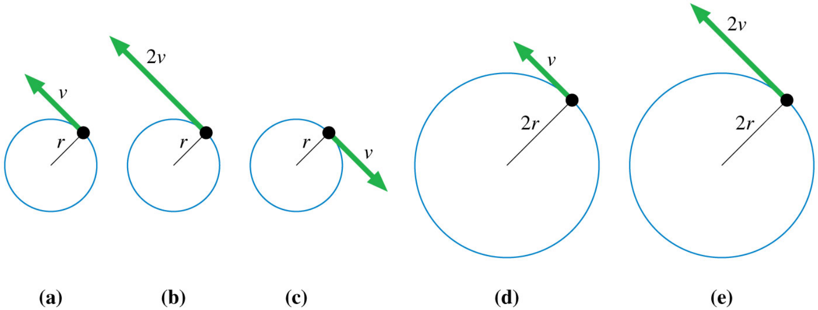
Maria's acceleration is an acceleration of changing direction, not of changing speed.

- This is the motion of a car on a ferris wheel.
- The acceleration direction is always pointing to the center. This is **centripital acceleration** (Greek for "center seeking")
- This is the acceleration of uniform circular motion.
- The magnitude of a_c is v^2/r, where v is the tangential velocity.

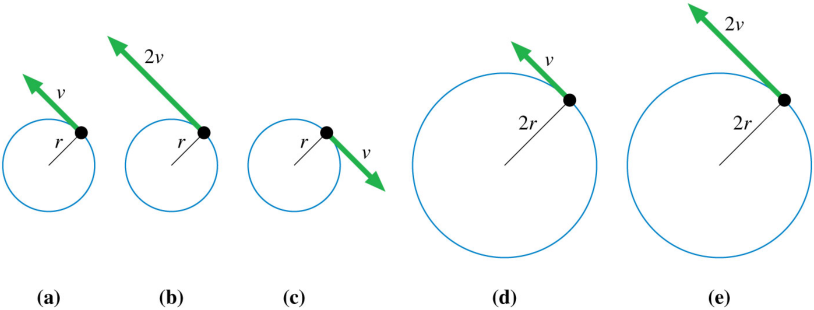
$$\vec{a}_c = \left(\frac{v^2}{r}, \text{ toward center of circle} \right)$$

Quick Check

Rank in order, from largest to smallest, the centripetal accelerations of these particles.



Rank in order, from largest to smallest, the centripetal accelerations of these particles.



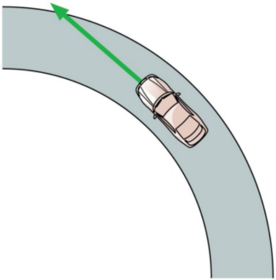
b,e,(a,c),d

Quick Check

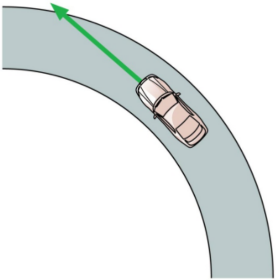
A car is traveling around a curve at a steady 45 mph. Is the car accelerating?

A. Yes

B. No



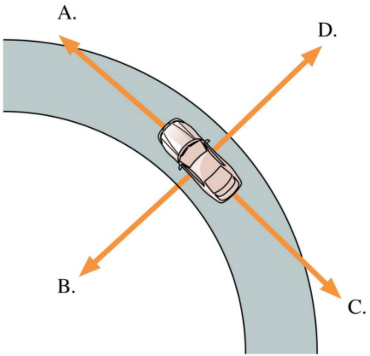
A car is traveling around a curve at a steady 45 mph. Is the car accelerating?



A. Yes

B. No

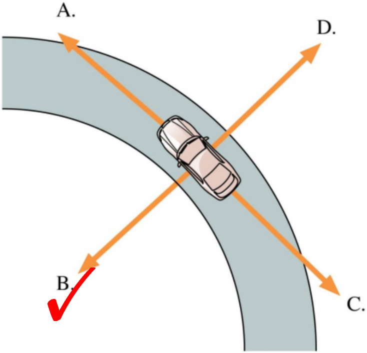
A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?



E. The acceleration is zero.

Quick Check

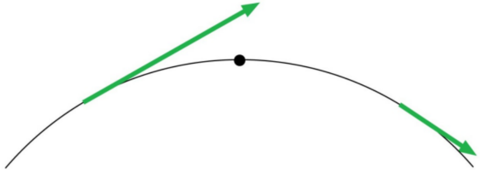
A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?



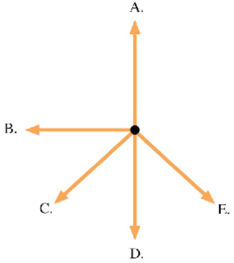
E. The acceleration is zero.

Quick Check

A car is slowing down as it drives over a circular hill.

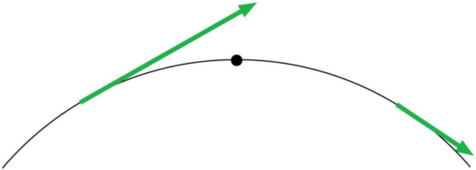


Which of these is the acceleration vector at the highest point?

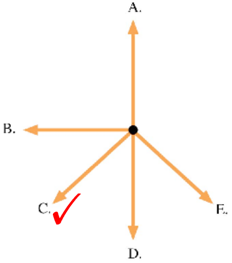


Quick Check

A car is slowing down as it drives over a circular hill.



Which of these is the acceleration vector at the highest point?



Angular Acceleration

- If an object is speeding up as it is going about it's circular orbit (like a car speeding around a turn or a roller coaster slowing down then speeding up on a loop) this motion is called **nonlinear circular motion**.

Angular Acceleration

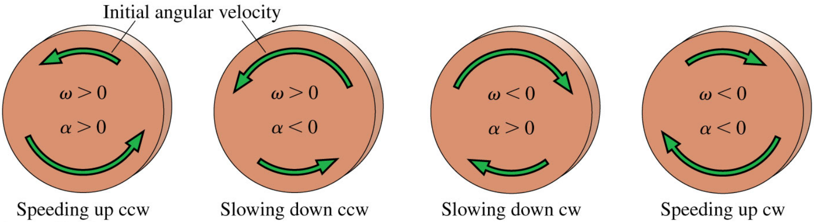
- If an object is speeding up as it is going about it's circular orbit (like a car speeding around a turn or a roller coaster slowing down then speeding up on a loop) this motion is called **nonlinear circular motion**.
- The angular acceleration (rate of change of ω) is given the Greek symbol alpha and is defined as you might imagine

$$\alpha \equiv \frac{d\omega}{dt}$$

- If an object is speeding up as it is going about it's circular orbit (like a car speeding around a turn or a roller coaster slowing down then speeding up on a loop) this motion is called **nonlinear circular motion**.
- The angular acceleration (rate of change of ω) is given the Greek symbol alpha and is defined as you might imagine

$$\alpha \equiv \frac{d\omega}{dt}$$

- The units for α are rad/s^2 and you need to remember that the sign doesn't mean slowing down or speeding up.

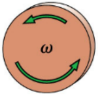


MODEL 4.3

Constant angular acceleration

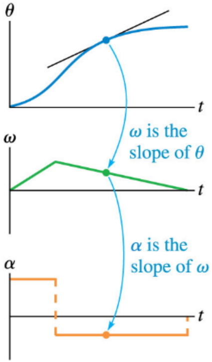
For motion with constant angular acceleration α.

- Applies to particles with circular trajectories and to rotating solid objects.



- Mathematically: The graphs and equations for this circular/rotational motion are analogous to linear motion with constant acceleration.
  - Analogs: $s \longrightarrow \theta$ $v_s \longrightarrow \omega$ $a_s \longrightarrow \alpha$

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\Delta t$
$\theta_f = \theta_i + \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$



The fan blade is slowing down. What are the signs of ω and α?

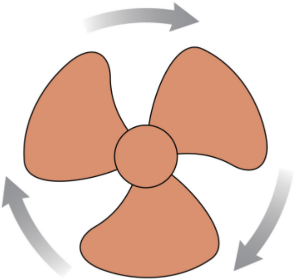
A. ω is positive and α is positive.

B. ω is positive and α is negative.

C. ω is negative and α is positive.

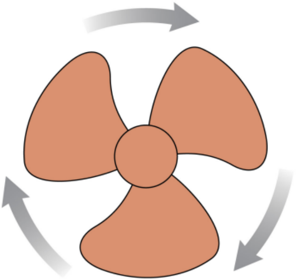
D. ω is negative and α is negative.

E. ω is positive and α is zero.



Quick Check

The fan blade is slowing down.
What are the signs of ω and α?



A. ω is positive and α is positive.

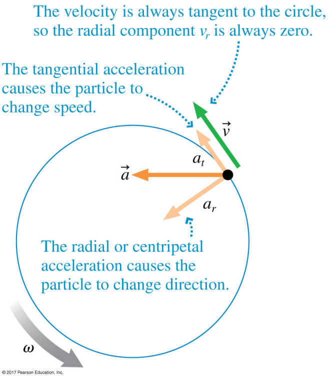
B. ω is positive and α is negative.

✓ C. ω is negative and α is positive.

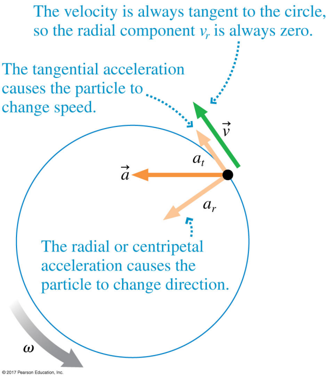
D. ω is negative and α is negative.

E. ω is positive and α is zero.

Tangential Acceleration

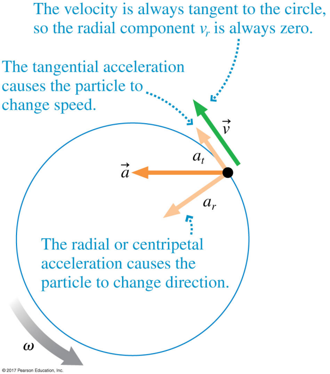


- Just like with velocity we can define a **tangential acceleration**.
- This is the same as the parralel part of the acceleration, the part that causes it to change speed, $\vec{a}_{\parallel} = \vec{a}_t$.



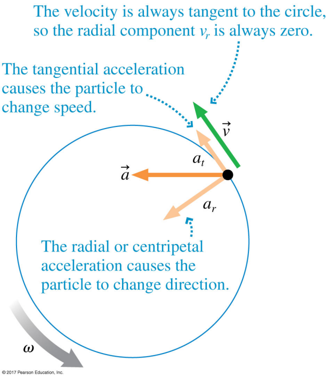
- Just like with velocity we can define a **tangential acceleration**.
- This is the same as the parralel part of the acceleration, the part that causes it to change speed, $\vec{a}_{\parallel} = \vec{a}_t$.
- That means that the perpendicular part is the **radial acceleration** (similar to centripital for uniform circular motion), $\vec{a}_{\perp} = \vec{a}_r = v_t^2/r = \omega^2/r$.

Tangential Acceleration



- Just like with velocity we can define a **tangential acceleration**.
- This is the same as the parralel part of the acceleration, the part that causes it to change speed, $\vec{a}_{\|} = \vec{a}_t$.
- That means that the perpendicular part is the **radial acceleration** (similar to centripital for uniform circular motion), $\vec{a}_{\perp} = \vec{a}_r = v_t^2/r = \omega^2/r$.
- The magnitude of a is given by $a = \sqrt{a_r^2 + at^2}$.

Tangential Acceleration



- Just like with velocity we can define a **tangential acceleration**.
- This is the same as the parralel part of the acceleration, the part that causes it to change speed, $\vec{a}_{||} = \vec{a}_t$.
- That means that the perpendicular part is the **radial acceleration** (similar to centripital for uniform circular motion), $\vec{a}_{\perp} = \vec{a}_r = v_t^2/r = \omega^2/r$.
- The magnitude of a is given by $a = \sqrt{a_r^2 + at^2}$.
- And just like with v_t we can write $a_t = \alpha r$.

Picture References

None yet