Physics 121: Equilibrium, N2L, Weight

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Quiz

1. Which of these is(are) a true statement(s) for mechanical equilibrium? (There may be more than one correct answer)

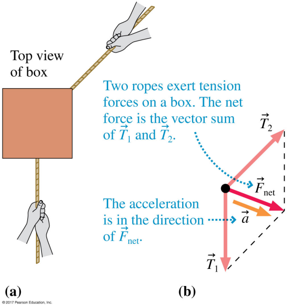
A. $\vec{F}_{net} = m\vec{a}$ ($\vec{a} \neq 0$)

B. $\vec{a} = 0$

C. $\vec{F}_{net} = 0$

D. $\vec{v} = 0$

2. Consider the situation shown in the figure. What is the acceleration of the box?



A. $(T_1 + T_2)/m$

B. 9.8 m/s^2

C. 0

D. None of the above

Equilibrium

- When an object is at rest or is moving with constant velocity ($\vec{a} = 0$) we say that the object is in mechanical equilibrium.

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- That seems like a boring physical system, so why would we want to study it? Can you think of examples of when it might be useful?

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- That seems like a boring physical system, so why would we want to study it? Can you think of examples of when it might be useful?
- What about engineering stationary objects like bridges?

Equilibrium

Write down Newton's second law equation(s) for a 2 dimensional problem in mechanical equilibrium.

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Quick Check

A ring, seen from above, is pulled on by three forces. The ring is not moving. How big is the force F?

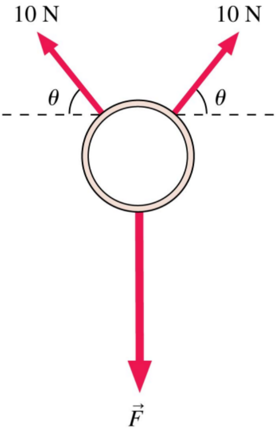
A. 20 N

B. $10\cos\theta$ N

C. $10\sin\theta$ N

D. $20\cos\theta$ N

E. $20\sin\theta$ N



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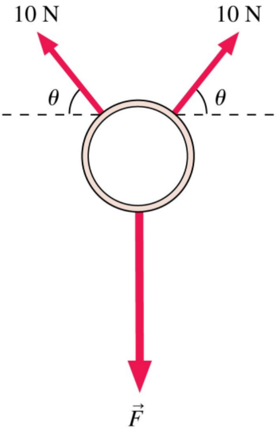

Example

EXAMPLE 6.2 | **Towing a car up a hill**

A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N maximum tension. Will it break?

MODEL Model the car as a particle in equilibrium.

Draw a picture, build a free body diagram and write down unknowns and things you need to find.

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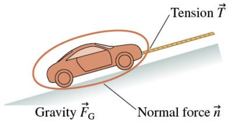

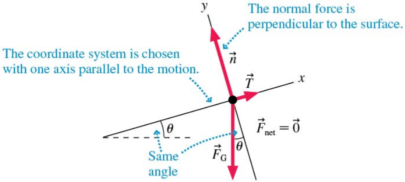

Known

$\theta = 20°$

$F_G = 15{,}000$ N

Find

T

Example

Now write down the Newton's second law equations

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$$T_x = T,\, T_y = 0,\, n_x = 0,\, n_y = n,\, (F_G)_x = -F_G \sin\theta,\, (F_G)_y = -F_G \cos\theta$$
$$T - F_G \sin\theta = 0$$
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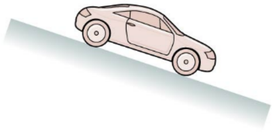
The first equation can be used to solve for the tension

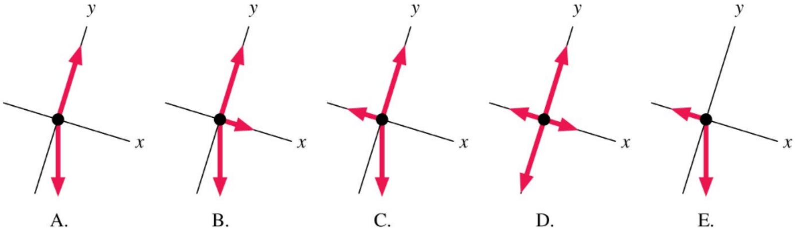
$$T = F_G \sin\theta = (15,000 \text{ N}) \sin 20^\circ = 5100 \text{ N}$$

$5100 \text{ N} < 6000 \text{ N}$ so **the tow rope holds**!

Quick Check

A car is parked on a hill.
Which is the correct
free-body diagram?





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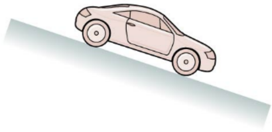

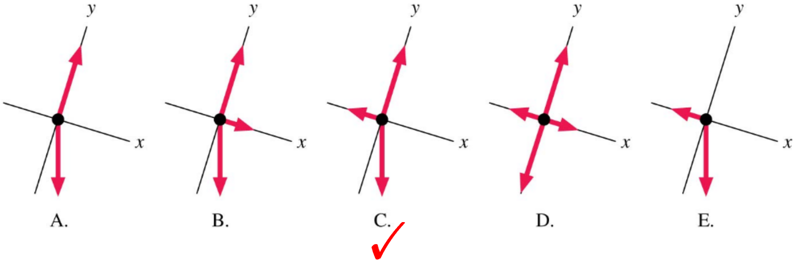

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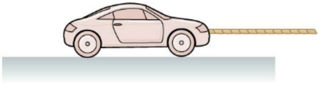

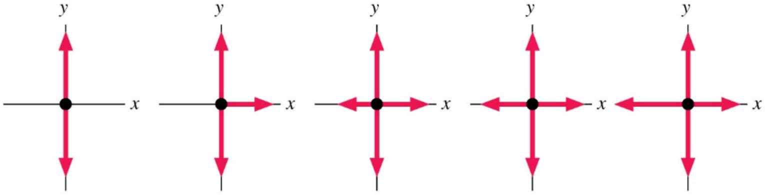

A. B. C. D. E.

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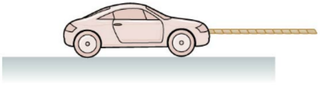
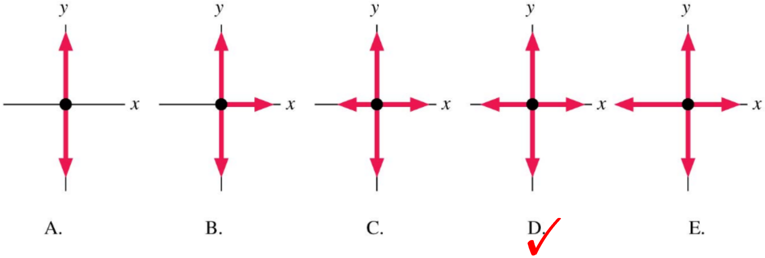




- Alright, now we've been doing a lot stuff that hasn't been connected yet. So how do you use Newton's second law to solve physics problems?
- There is a long version is the book that I'll show here and then I'll break it down to the most important parts.

PROBLEM-SOLVING STRATEGY 6.1

MP

Newtonian mechanics

MODEL Model the object as a particle. Make other simplifications depending on what kinds of forces are acting.

VISUALIZE Draw a **pictorial representation**.

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$. The acceleration is zero for an object in equilibrium.
- Identify all forces acting on the object *at this instant* and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

SOLVE The mathematical representation is based on Newton's second law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

The forces are "read" directly from the free-body diagram. Depending on the problem, either

- Solve for the acceleration, then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, then solve for unknown forces.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 23

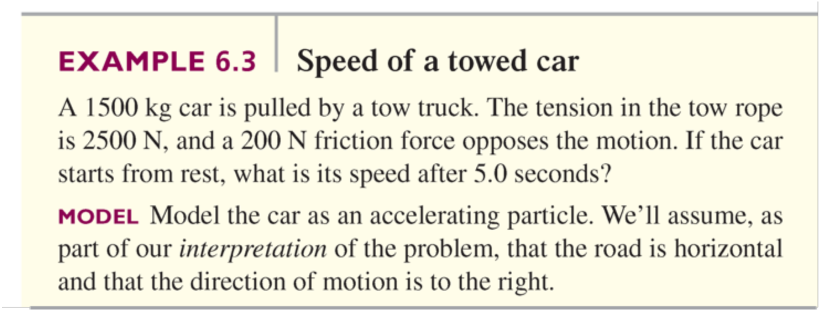
Using Newton's Second Law

1. Draw a picture.
    1. Choose a coordinate system and simplify the model as much as possible (but not more) (particle model)
    2. Make the drawing a free-body diagram including ALL forces acting on the object.
2. Write down Newton's second law equation(s)

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$$

3. Do one of the following
    1. Solve for the acceleration, then use kinematics to find velocities and positions OR
    2. Use kinematics to determine the acceleration, then solve for unknown forces.

Example



EXAMPLE 6.3 | Speed of a towed car

A 1500 kg car is pulled by a tow truck. The tension in the tow rope is 2500 N, and a 200 N friction force opposes the motion. If the car starts from rest, what is its speed after 5.0 seconds?

MODEL Model the car as an accelerating particle. We'll assume, as part of our *interpretation* of the problem, that the road is horizontal and that the direction of motion is to the right.

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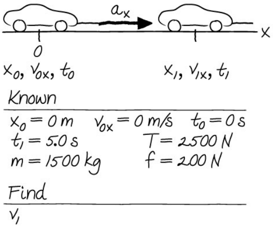
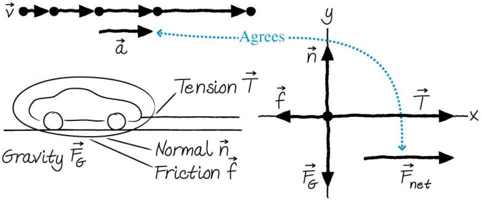
Sketch



Motion diagram and forces



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$$T_x = +T, T_y = 0, n_x = 0, n_y = +n$$
$$f_x = -f, f_y = 0, (F_G)_x = 0, (F_G)_y = -F_G$$

$$\begin{aligned} a_x &= \frac{1}{m}(T - f) \\ &= \frac{1}{1500 \text{ kg}}(2500 \text{ N} - 200 \text{ N}) = 1.53 \text{ m/s}^2 \\ a_y &= \frac{1}{m}(n - F_G) \end{aligned}$$

Example

Now we have the (constant) acceleration and so use kinematics to solve for v_{1x}

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$$
\begin{aligned}
v_{1x} &= v_{0x} + a_x \Delta t \\
&= 0 + (1.53 \text{ m/s}^2)(5.0 \text{ s}) = 7.7 \text{ m/s}^2 \approx 15 \text{ mph}
\end{aligned}
$$

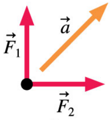
Is that a reasonable speed to be at after accelerating for 5 s?

MODEL 6.2

Constant force

For objects on which the net force is constant.

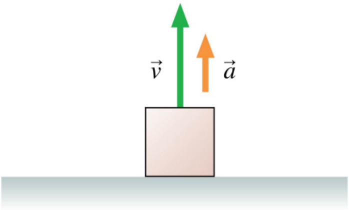
- Model the object as a particle with uniform acceleration.
  - The particle accelerates in the direction of the net force.
- Mathematically:
  - **Newton's second law** is $\vec{F}_{\text{net}} = \sum_i \vec{F}_{\text{i}} = m\vec{a}$.
  - Use the kinematics of constant acceleration.
- Limitations: Model fails if the forces aren't constant.



$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

The object undergoes uniform acceleration.

The box is sitting on the floor of an elevator. The elevator is accelerating upward. The magnitude of the normal force on the box is



A. $n > mg$.

B. $n = mg$.

C. $n < mg$.

D. $n = 0$.

E. Not enough information to tell.

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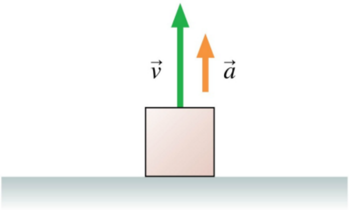

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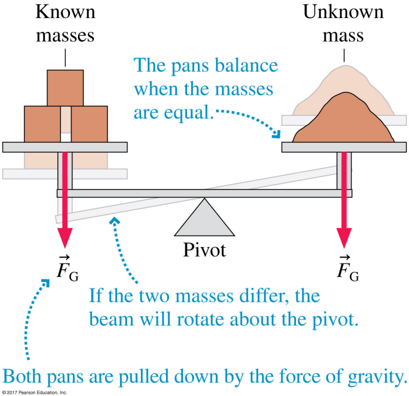
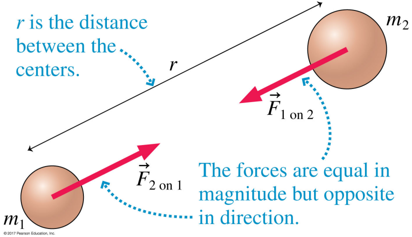
Mass, Gravity and Weight

Mass - An Intrinsic Property

- A pan balance, shown in the figure, is a device for measuring mass.
- The measurement does not depend on the strength of gravity.
- Mass is a scalar quantity that describes an object's inertia.
- Mass describes the amount of matter in an object.
- Mass is an intrinsic property of an object.





- Gravity is an attractive, long-range force between any two objects.
- The figure shows two objects with masses m_1 and m_2 whose centers are separated by distance r.
- Each object pulls on the other with a force:

$$F_{12} = F_{21} = \frac{Gm_1 m_2}{r^2} \ (\textbf{Newton's Law of Gravity})$$

where $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$ is the gravitational constant.

- Why does it seem that there is no gravitational force between any two of us?

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- Notice that the acceleration due to gravity g doesn't depend on the mass of the object m, so it's always constant assuming the mass and radius of the planet don't change ...EVERYTHING FALLS AT THE SAME RATE, and now you know why :)

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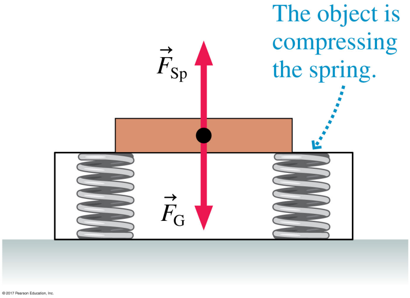
- Show this...For earth $g = \frac{GM}{R^2} = 9.8\text{m}/\text{s}^2$.

Weight - A Measurement

- Weight is a measurement of gravitational force.
- It is usually measured by measuring another force that is equal to the gravitational force with $\vec{a} = \vec{F}_{net} = \vec{0}$ (think Newton's 2st law and equilibrium :))



Quick Check

An astronaut takes her bathroom scales to the moon, where $g = 1.6\ \mathrm{m/s^2}$. On the moon, compared to at home on earth,

A. Her weight is the same and her mass is less.
B. Her weight is less and her mass is less.
C. Her weight is less and her mass is the same.
D. Her weight is the same and her mass is the same.
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- $w = 0$ N??? They are weightless, but they are still on earth? They still weight SOMETHING right? Is that right or is something wrong?

Weightlessness

- That is in fact true, an object in free fall has no weight. This is the case for astronauts as well.

- In fact $g_{iss} \approx 8.7\text{m/s}^2$ and $g_{moon} \approx 1.6\text{m/s}^2$ so they should fall right? They do fall, orbiting objects are in "free fall" and are thus weightless.

A 50-kg student ($mg = 490$ N) gets in a 1000-kg elevator at rest and stands on a metric bathroom scale. Sadly, the elevator cable breaks. What is the student's weight during the few second it takes the student to plunge to his doom?

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Picture References

Nothing this time