

Physics 121: Vectors and Coordinate Systems

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Mesa Community College

① What is the magnitude of the gravitational acceleration near the surface of the earth?

A. 4.98 m/s^2

B. 9.80 m/s

C. 9.80 m/s^2

D. It depends on the velocity

② Which of these is **NOT** a kinematic equation?

A. $s_f = s_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$

B. $v_f = v_i + a \Delta t$

C. $a_f = a_i + \int_{t_i}^{t_f} v_s dt$

D. $v_f^2 = v_i^2 + 2a \Delta s$

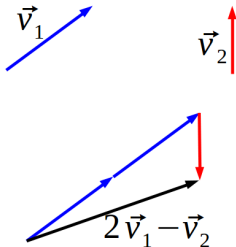
- The next HW will be due on Saturday and will include all the material we cover in chapters 3 and 4.
- The exam is coming up on Tuesday 19 Sep. The exam will be taken in class. It will cover all material from chapters 1-4.

Before we dive anymore into physics and motion in multiple dimensions let's do a thorough review of vectors!

- In your words what is a vector?

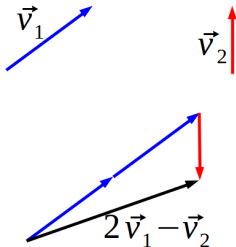
Vectors

- In your words what is a vector?
- You have learned how to add vectors as pictures so I won't bore you with that.



Vectors

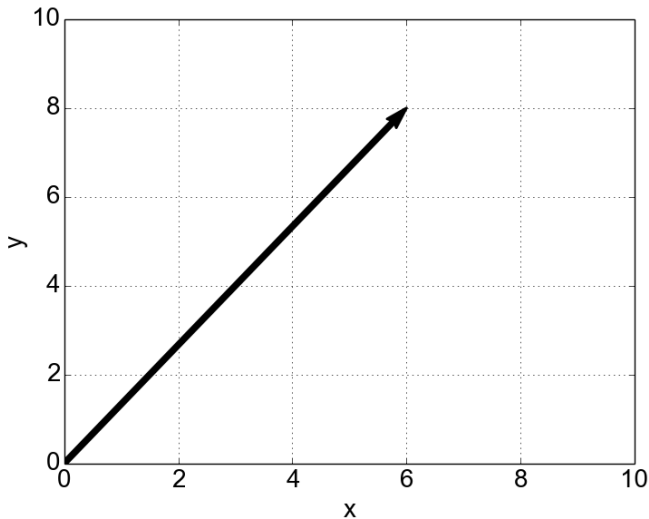
- In your words what is a vector?
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- However we haven't talked about their components in detail much.

Vectors

Write down how you would describe this vector

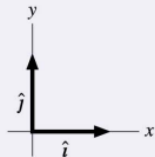


What are unit vectors?

Unit vectors define what we *mean* by the $+x$ - and $+y$ -directions in space.

- A unit vector has magnitude 1.
- A unit vector has no units.

Unit vectors simply point.



- What is the difference between a scalar and a vector?

- What is the difference between a scalar and a vector?
- The magnitude (length) of a vector is a scalar (ex. $\vec{V} = X\hat{i} + Y\hat{j}$).

$$V = |\vec{V}| = \sqrt{X^2 + Y^2}$$

Think of the pythagorean theorem

- What is the difference between a scalar and a vector?
- The magnitude (length) of a vector is a scalar (ex. $\vec{V} = X\hat{i} + Y\hat{j}$).

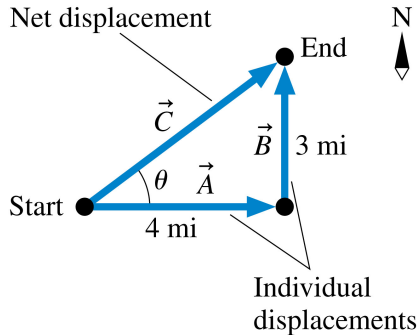
$$V = |\vec{V}| = \sqrt{X^2 + Y^2}$$

Think of the pythagorean theorem

- If the vector was the velocity then notice that the magnitude would be the speed. Also, the magnitude cannot be negative (just like speed can't be negative, but velocity can).

Vectors

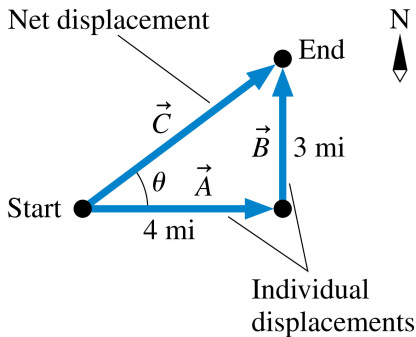
Add the two displacement vectors, \vec{A} and \vec{B} to get the **resultant vector** (or total displacement) \vec{C} .



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Vectors

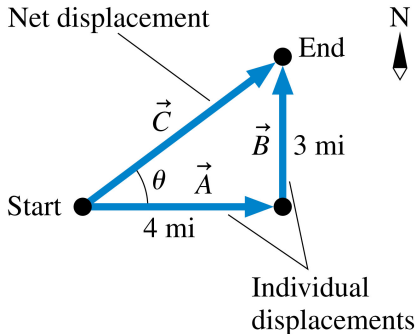
Add the two displacement vectors, \vec{A} and \vec{B} to get the **resultant vector** (or total displacement) \vec{C} .



$$\vec{C} = 4 \text{ mi } \hat{i} + 3 \text{ mi } \hat{j}$$

Vectors

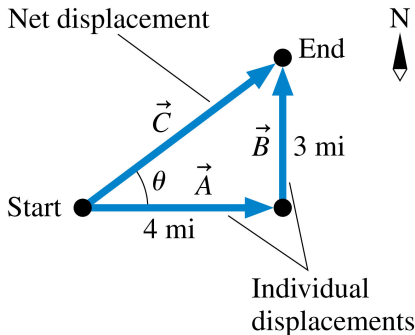
This time use \vec{A} and \vec{B} to get \vec{C} in terms of the magnitude C and the angle as measured from the x axis, θ .



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Vectors

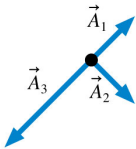
This time use \vec{A} and \vec{B} to get \vec{C} in terms of the magnitude C and the angle as measured from the x axis, θ .



$$C = \sqrt{A^2 + B^2} = \sqrt{(4 \text{ mi})^2 + (3 \text{ mi})^2} = 5 \text{ mi}$$
$$\theta = \tan^{-1} \left(\frac{B}{A} \right) = \tan^{-1} \left(\frac{3 \text{ mi}}{4 \text{ mi}} \right) = 37^\circ$$

Quick Check

What is $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



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(a)



(b)



(c)



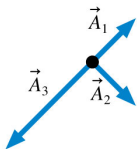
(d)



(e)

Quick Check

What is $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



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(a)



(b)



(c)



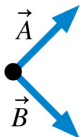
(d)



(e)

Quick Check

Which figure shows $2\vec{A} - \vec{B}$?



(a)



(b)



(c)



(d)

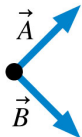


(e)

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Quick Check

Which figure shows $2\vec{A} - \vec{B}$?



(a)



(b)



(c)



(d)



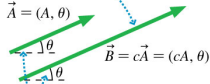
(e)

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Vectors

More vector math

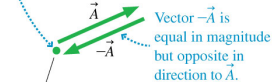
The length of \vec{B} is "stretched" by the factor c . That is, $B = cA$.



\vec{B} points in the same direction as \vec{A} .

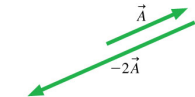
Multiplication by a scalar

$\vec{A} + (-\vec{A}) = \vec{0}$. The tip of $-\vec{A}$ returns to the starting point.



The **zero vector** $\vec{0}$ has zero length

The negative of a vector

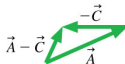


Multiplication by a negative scalar

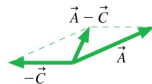


Vector subtraction: What is $\vec{A} - \vec{C}$?
 Write it as $\vec{A} + (-\vec{C})$ and add!

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Tip-to-tail subtraction using $-\vec{C}$



Parallelogram subtraction using $-\vec{C}$

Coordinate Systems

- We have talked about coordinate systems and origins (on the first day)
- Which of these things depends on the coordinate system and origin?
 - 1 Vector components
 - 2 Magnitude of vector
 - 3 Angle of vector
 - 4 Displacement vector (i.e. $\Delta\vec{r} = \vec{r}_1 - \vec{r}_0$)

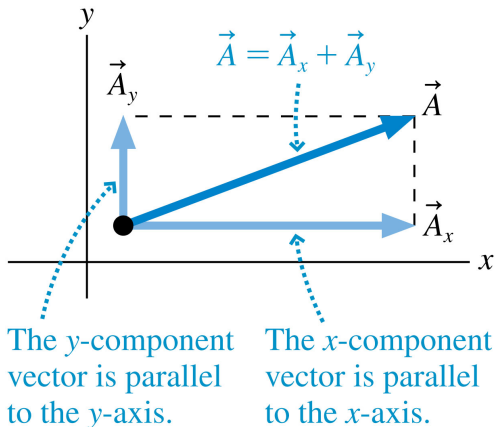
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1 and 3

Coordinate Systems

- Once we have picked a suitable coordinate system we can then take any vector and decompose it into its **component vectors**. This is called **decomposition** of a vector into its component vectors.



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Just as a warning \vec{A}_x is not the same as the component A_x .

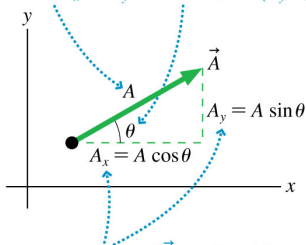
$$\vec{A}_x = A_x \hat{i}.$$

Warning #2

As a second warning when going between component and geometric forms of vectors make sure you get signs right (sometimes you have to put them in by hand) and make sure you know where angles are being measured from.

The magnitude and direction of \vec{A} are found from the components. In this example,

$$A = \sqrt{A_x^2 + A_y^2} \quad \theta = \tan^{-1} (A_y/A_x)$$

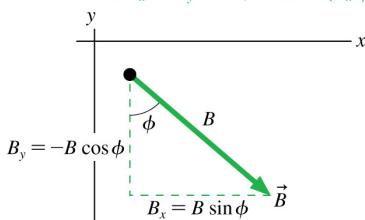


The components of \vec{A} are found from the magnitude and direction.

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The angle is defined differently. In this example, the magnitude and direction are

$$B = \sqrt{B_x^2 + B_y^2} \quad \phi = \tan^{-1} (B_x/|B_y|)$$

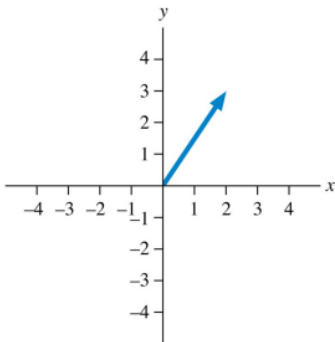


Minus signs must be inserted manually, depending on the vector's direction.

Quick Check

What are the x - and y -components of this vector?

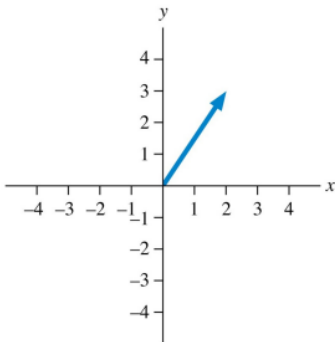
- A. 3, 2
- B. 2, 3
- C. -3, 2
- D. 2, -3
- E. -3, -2



Quick Check

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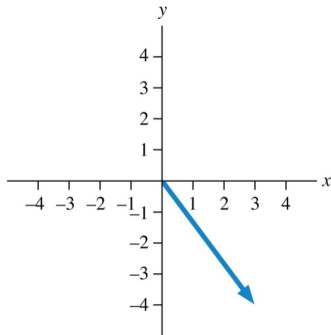
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Quick Check

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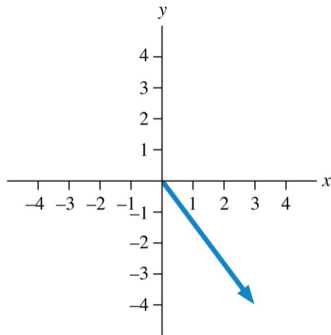
- A. 3, 4
- B. 4, 3
- C. -3, 4
- D. 4, -3
- E. 3, -4



Quick Check

What are the x - and y -components of this vector?

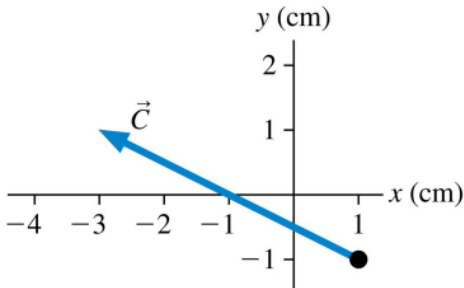
- A. 3, 4
- B. 4, 3
- C. -3, 4
- D. 4, -3
- ✓ E. 3, -4



Quick Check

What are the x - and y -components of vector \vec{C} ?

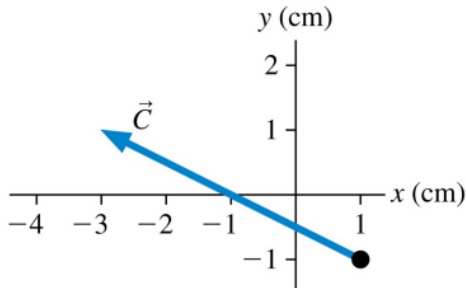
- A. 1, -3
- B. -3, 1
- C. 1, -1
- D. -4, 2
- E. 2, -4



Quick Check

What are the x - and y -components of vector \vec{C} ?

- A. 1, -3
- B. -3, 1
- C. 1, -1
- ✓ D. -4, 2
- E. 2, -4



- How would you add three vectors $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ where the vectors are given by $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and similar for \vec{B} and \vec{C} ?

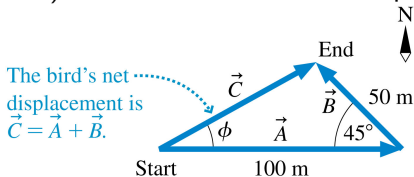
- How would you add three vectors $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ where the vectors are given by $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and similar for \vec{B} and \vec{C} ?
$$\vec{D} = (A_x + B_x + C_x)\hat{i} + (A_y + B_y + C_y)\hat{j}$$

Quick Check

A bird flies 100 m due east from a tree, then 50 m northwest (45° north of west). What is the bird's net displacement?

Quick Check

A bird flies 100 m due east from a tree, then 50 m northwest (45° north of west). What is the bird's net displacement?



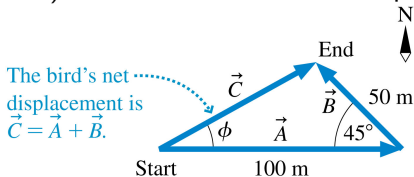
$$\vec{A} = 100\hat{i}$$

$$\vec{B} = -50 \sin(45^\circ)\hat{i} + 50 \sin(45^\circ)\hat{j} = -35.4\hat{i} + 35.4\hat{j}$$

$$\vec{C} = 64.6\hat{i} + 35.4\hat{j}$$

Quick Check

A bird flies 100 m due east from a tree, then 50 m northwest (45° north of west). What is the bird's net displacement?



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$$\vec{B} = -50 \sin(45^\circ)\hat{i} + 50 \cos(45^\circ)\hat{j} = -35.4\hat{i} + 35.4\hat{j}$$

$$\vec{C} = 64.6\hat{i} + 35.4\hat{j}$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{64.6^2 + 35.4^2} = 73.6 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{35.4}{64.6}\right) = 28.7^\circ$$

$$\vec{C} = (73.6 \text{ m}, 28.7^\circ)$$

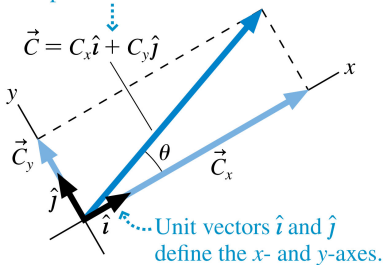
Tilted Axes

- Let's say I have an inclined plane. Can I set my axes such that \hat{i} points along (parallel) to the plane and \hat{j} points perpendicular to the plane?

Tilted Axes

- Let's say I have an inclined plane. Can I set my axes such that \hat{i} points along (parallel) to the plane and \hat{j} points perpendicular to the plane?
- Of course you can, we talked about how that could be useful last time! This can help you determine the component vectors that are “parallel” (make object move) and “perpendicular” (doesn't make the object move).

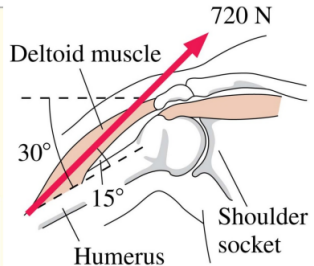
The components of \vec{C} are found with respect to the tilted axes.



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EXAMPLE 3.7 Muscle and bone

The deltoid—the rounded muscle across the top of your upper arm—allows you to lift your arm away from your side. It does so by pulling on an attachment point on the humerus, the upper arm bone, at an angle of 15° with respect to the humerus. If you hold your arm at an angle 30° below horizontal, the deltoid must pull with a force of 720 N to support the weight of your arm, as shown in [FIGURE 3.21a](#). (You'll learn in Chapter 5 that force is a vector quantity measured in units of *newtons*, abbreviated N.) What are the components of the muscle force parallel to and perpendicular to the bone?



Tilted Axes

EXAMPLE 3.7 Muscle and bone

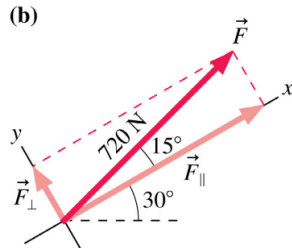
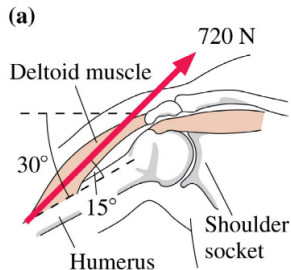
VISUALIZE FIGURE 3.21b shows a tilted coordinate system with the x -axis parallel to the humerus. The force \vec{F} is shown 15° from the x -axis. The component of force parallel to the bone, which we can denote F_{\parallel} , is equivalent to the x -component: $F_{\parallel} = F_x$. Similarly, the component of force perpendicular to the bone is $F_{\perp} = F_y$.

SOLVE From the geometry of Figure 3.21b, we see that

$$F_{\parallel} = F \cos 15^\circ = (720 \text{ N}) \cos 15^\circ = 695 \text{ N}$$

$$F_{\perp} = F \sin 15^\circ = (720 \text{ N}) \sin 15^\circ = 186 \text{ N}$$

ASSESS The muscle pulls nearly parallel to the bone, so we expected $F_{\parallel} \approx 720 \text{ N}$ and $F_{\perp} \ll F_{\parallel}$. Thus our results seem reasonable.



Picture References

None yet