Physics 121:
Instantaneous Velocity, Kinematic Equations

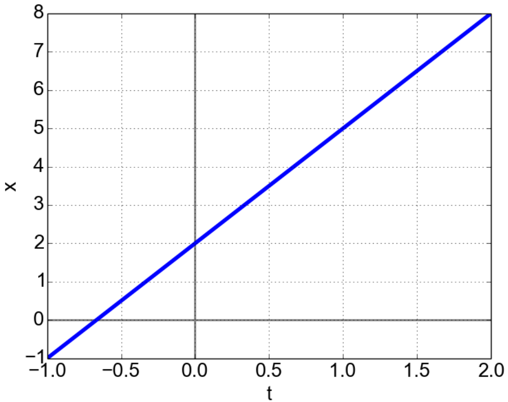
Cody Petrie

Mesa Community College

Quiz

1. True/False: If an object has a negative acceleration it must be slowing down.

2. What is the velocity of the object whose x(t) graph is shown below? Don't just write it, explain **briefly** how you got it.



How is HW going?

Instantaneous Velocity

- Instantaneous and average velocities are slightly different. Which is displayed on the speedometer of your car?

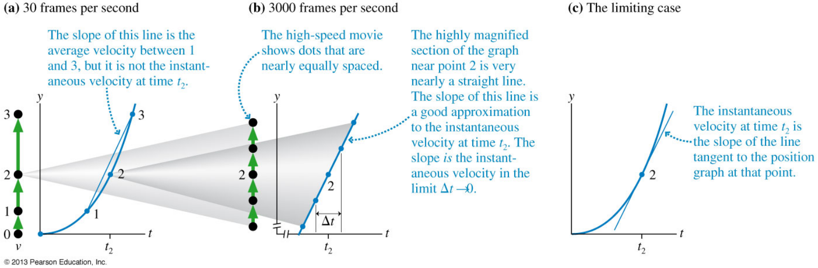
Instantaneous Velocity

- Instantaneous and average velocities are slightly different. Which is displayed on the speedometer of your car?
  - Instantaneous velocity

Instantaneous Velocity

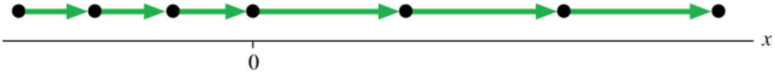
- Instantaneous and average velocities are slightly different. Which is displayed on the speedometer of your car?
  - Instantaneous velocity
- Let's try to illustrate the difference and when it's appropriate to use average velocities.

This is an accelerating rocket (not uniform motion anymore)

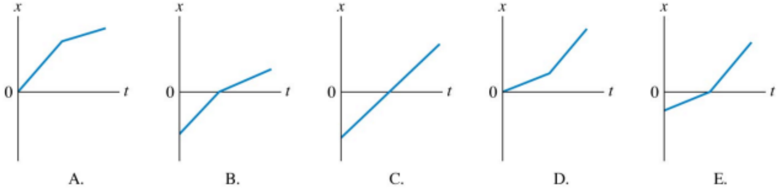


Quick Check

Here is a motion diagram of a car moving along a straight road:

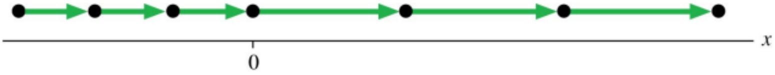


Which position-versus-time graph matches this motion diagram?

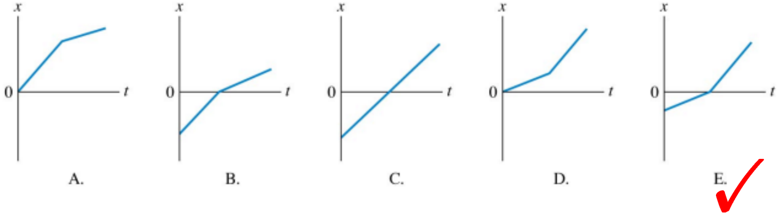


Quick Check

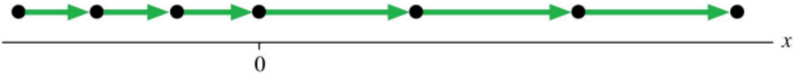
Here is a motion diagram of a car moving along a straight road:



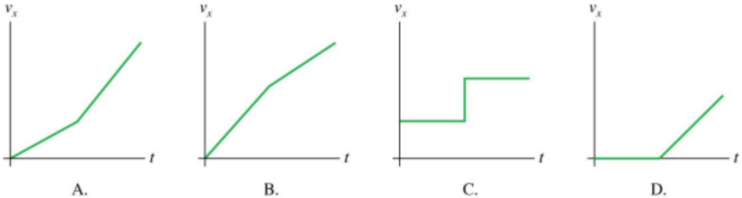
Which position-versus-time graph matches this motion diagram?



Here is a motion diagram of a car moving along a straight road:



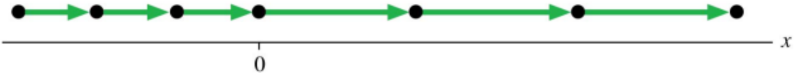
Which velocity-versus-time graph matches this motion diagram?



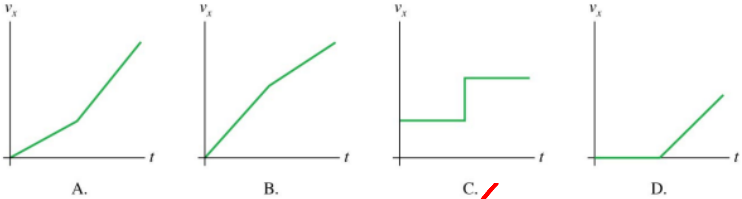
E. None of the above.

Quick Check

Here is a motion diagram of a car moving along a straight road:



Which velocity-versus-time graph matches this motion diagram?

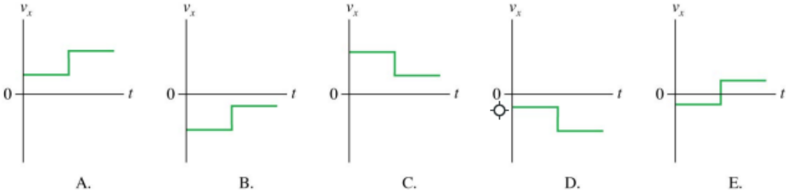


E. None of the above.

Quick Check

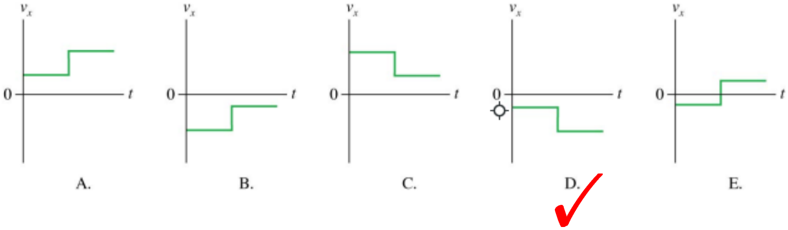
Here is a motion diagram of a car moving along a straight road:

Which velocity-versus-time graph matches this motion diagram?



Here is a motion diagram of a car moving along a straight road:

Which velocity-versus-time graph matches this motion diagram?



Instantaneous Velocity

- As you zoom in more and more on the graph it's like taking smaller and smaller Δt steps. The truly instantaneous velocity is when you take $\Delta t \to 0$.

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(Instantaneous Velocity)}$$

$$v_s = \text{slope of the position-versus-time graph at time } t$$

Instantaneous Velocity

- As you zoom in more and more on the graph it's like taking smaller and smaller Δt steps. The truly instantaneous velocity is when you take $\Delta t \to 0$.

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(Instantaneous Velocity)}$$

$$v_s = \text{slope of the position-versus-time graph at time } t$$

- If v is the slope of x(t) then how to we get x from v(t)?

Instantaneous Velocity

- As you zoom in more and more on the graph it's like taking smaller and smaller Δt steps. The truly instantaneous velocity is when you take $\Delta t \to 0$.

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(Instantaneous Velocity)}$$

$$v_s = \text{slope of the position-versus-time graph at time } t$$

- If v is the slope of x(t) then how to we get x from v(t)?
  - Some of you might recognize that as an integral.

$$x(t) = \int v(t)dt + x_0 \tag{1}$$

Derivatives Review

- $\frac{ds}{dt}$ is called *derivative of s with respect to t*.
- The derivative is the slope of the line that is tangent to the to the position vs. time graph.
- The book (and probably I) will only deal with derivaties of powers and polynomials so let's review those.

Derivatives Review

If $u(t) = ct^n$ then $\frac{du}{dt} =$

Derivatives Review

If $u(t) = ct^n$ then $\frac{du}{dt} = nct^{n-1}$

Derivatives Review

If $u(t) = ct^n$ then $\frac{du}{dt} = nct^{n-1}$

Let's try this with a specific example then. Let the trajectory of the particle be $s(t) = 2t^2$. Now solve for the velocity of the particle at any time along the path.

$$\frac{ds}{dt} =$$

Derivatives Review

If $u(t) = ct^n$ then $\frac{du}{dt} = nct^{n-1}$

Let's try this with a specific example then. Let the trajectory of the particle be $s(t) = 2t^2$. Now solve for the velocity of the particle at any time along the path.

$$\frac{ds}{dt} = 4t$$

Here are a couple more that might be useful to remember.

$$\frac{d}{dt}C =$$

$$\frac{d}{dt}(u(t) + w(t)) =$$

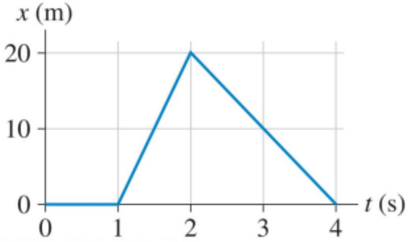
Derivatives Review

Here are a couple more that might be useful to remember.

$$\frac{d}{dt}C = 0$$

$$\frac{d}{dt}(u(t) + w(t)) = \frac{du}{dt} + \frac{dw}{dt}$$

Here is a position graph of an object:

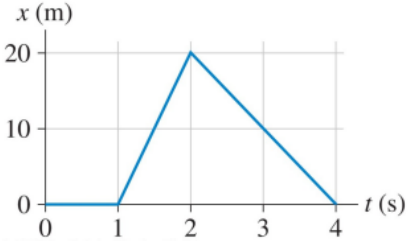


At t = 1.5 s, the object's velocity is

A. 40 m/s.
B. 20 m/s.
C. 10 m/s.
D. –10 m/s.
E. None of the above.

Quick Check

Here is a position graph of an object:



At t = 1.5 s, the object's velocity is

A. 40 m/s.

✓B. 20 m/s.

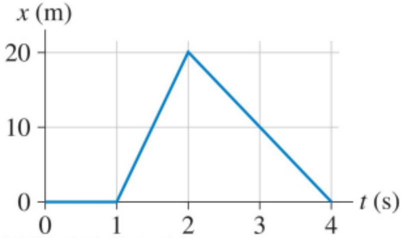
C. 10 m/s.

D. –10 m/s.

E. None of the above.

Quick Check

Here is a position graph of an object:

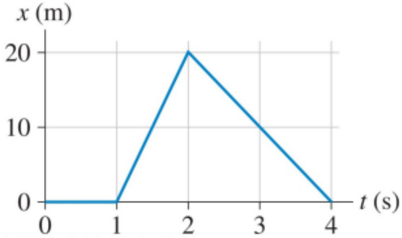


At $t = 3.0$ s, the object's velocity is

A. 40 m/s.
B. 20 m/s.
C. 10 m/s.
D. –10 m/s.
E. None of the above.

Quick Check

Here is a position graph of an object:

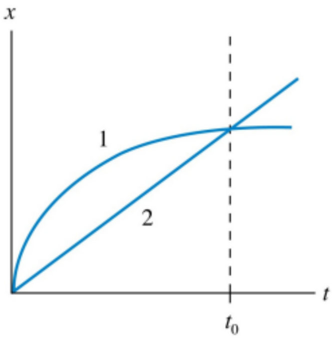


At $t = 3.0$ s, the object's velocity is

A. 40 m/s.
B. 20 m/s.
C. 10 m/s.
D. –10 m/s.
E. None of the above.

Quick Check

When do objects 1 and 2 have the same velocity?



A. At some instant before time t_0.

B. At time t_0.

C. At some instant after time t_0.

D. Both A and B.

E. Never.

Quick Check

When do objects 1 and 2 have the same velocity?

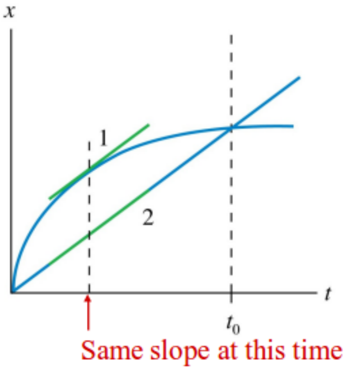
✓A. At some instant before time t_0.

B. At time t_0.

C. At some instant after time t_0.

D. Both A and B.

E. Never.



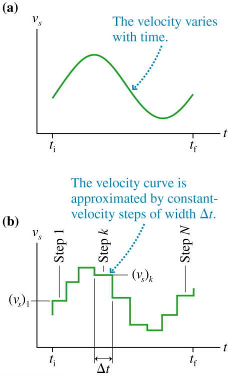
Now Integrals

Position from Velocity

- Can we use our knowledge of the velocity to predict the position at a future time like we did with the uniform motion equation, $s_f = s_i + v_s \Delta t$?

Position from Velocity

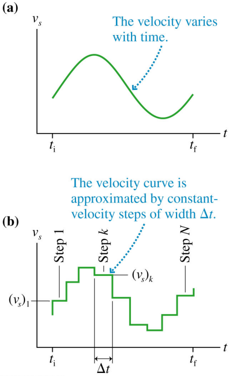
- Can we use our knowledge of the velocity to predict the position at a future time like we did with the uniform motion equation, $s_f = s_i + v_s \Delta t$?
- Let's start by breaking up a $v(t)$ curve into N steps.



- Notice that the motion at each step is uniform motion so we can use ol' faithful, $s_f = s_i + v_s \Delta t \rightarrow \Delta s = v_s \Delta t$.

Position from Velocity

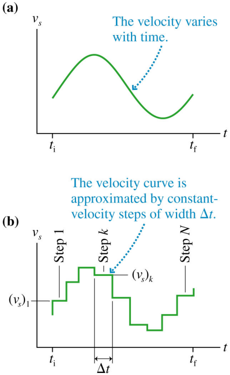
- Can we use our knowledge of the velocity to predict the position at a future time like we did with the uniform motion equation, $s_f = s_i + v_s \Delta t$?
- Let's start by breaking up a $v(t)$ curve into N steps.



- Notice that the motion at each step is uniform motion so we can use ol' faithful, $s_f = s_i + v_s \Delta t \rightarrow \Delta s = v_s \Delta t$.
- So $v_s(t_k)\Delta t$ is the amount that is moved during between the steps k and $k + 1$.

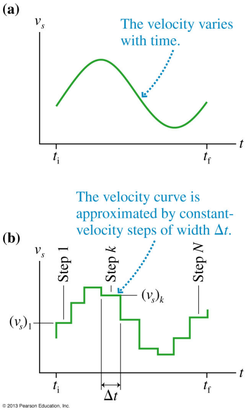
Position from Velocity

- Can we use our knowledge of the velocity to predict the position at a future time like we did with the uniform motion equation, $s_f = s_i + v_s \Delta t$?
- Let's start by breaking up a $v(t)$ curve into N steps.



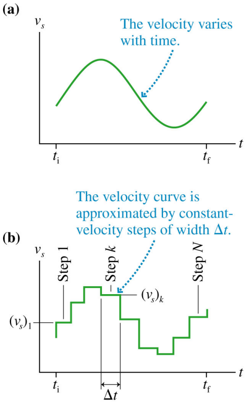
- Notice that the motion at each step is uniform motion so we can use ol' faithful, $s_f = s_i + v_s \Delta t \rightarrow \Delta s = v_s \Delta t$.
- So $v_s(t_k)\Delta t$ is the amount that is moved during between the steps k and $k+1$.
- So if I want $\Delta s = s_f - s_i$ then I want to sum over all of steps,

$$\Delta s \approx \Delta s_1 + \Delta s_2 + \ldots + \Delta s_N = \sum_{k=1}^{N} (v_s)_k \Delta t.$$



- We can then take this and from it determine the final position of a particle if we know it's initial position, s_i and it's velocity at each time, $v(t)$.

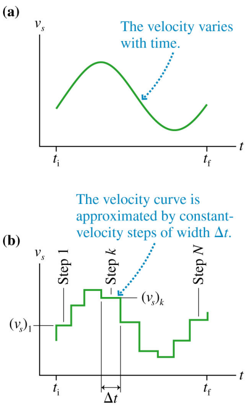
$$s_f \approx s_i + \sum_{k=1}^{N} (v_s)_k \Delta t$$



- We can then take this and from it determine the final position of a particle if we know it's initial position, s_i and it's velocity at each time, $v(t)$.

$$s_f \approx s_i + \sum_{k=1}^{N} (v_s)_k \Delta t$$

- How do we get rid of the pesky approximate sign ($\approx$) though because we can't the true answer?



- We can then take this and from it determine the final position of a particle if we know it's initial position, s_i and it's velocity at each time, $v(t)$.

$$s_f \approx s_i + \sum_{k=1}^{N} (v_s)_k \Delta t$$

- How do we get rid of the pesky approximate sign ($\approx$) though because we can't the true answer?

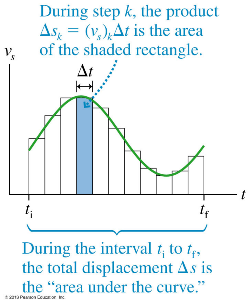
$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s dt \qquad (2)$$

Position from Velocity

- Just like we had a geometrical definition for a derivative let's find one for integrals.

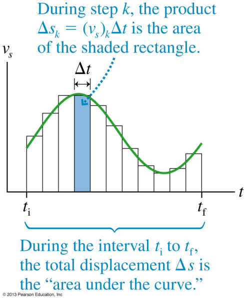
Position from Velocity

- Just like we had a geometrical definition for a derivative let's find one for integrals.



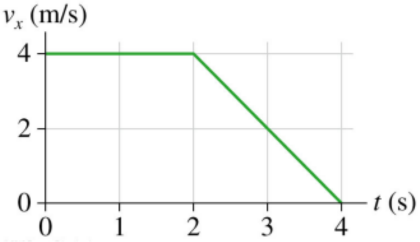
Position from Velocity

- Just like we had a geometrical definition for a derivative let's find one for integrals.



$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \tag{3}$$

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.



At $t = 4.0$ s, the object's position is

A. 20 m.

B. 16 m.

C. 12 m.

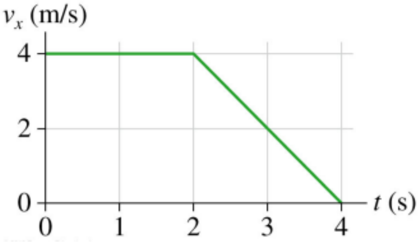
D. 7 m.

E. 4 m.

Quick Check

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.

At $t = 4.0$ s, the object's position is

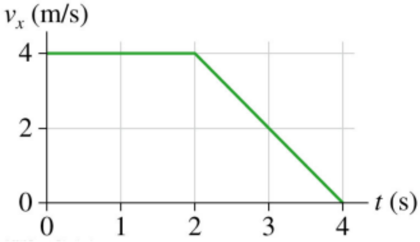


A. 20 m.
B. 16 m.
✓ C. 12 m.
D. 7 m.
E. 4 m.

Quick Check

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.

What is Δs between t=1s and t=3s

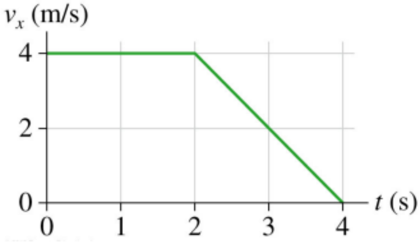


A. 20 m.
B. 16 m.
C. 12 m.
D. 7 m.
E. 4 m.

Quick Check

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.

What is Δs between t=1s and t=3s



A. 20 m.

B. 16 m.

C. 12 m.

✓D. 7 m.

E. 4 m.

Integrals Review

- Again let's look at integrating polynomials. Think of undoing a derivative (multiply by power and subtract 1 from power).

$$\int_{t_i}^{t_f} ct^n dt =$$

Integrals Review

- Again let's look at integrating polynomials. Think of undoing a derivative (multiply by power and subtract 1 from power).

$$\int_{t_i}^{t_f} ct^n dt = \left.\frac{ct^{n+1}}{n+1}\right|_{t_i}^{t_f} = \frac{ct_f^{n+1}}{n+1} - \frac{ct_i^{n+1}}{n+1} \qquad (n \neq -1)$$

Integrals Review

- Again let's look at integrating polynomials. Think of undoing a derivative (multiply by power and subtract 1 from power).

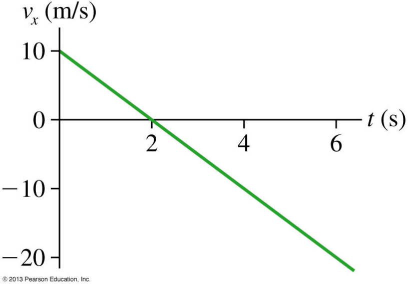
$$\int_{t_i}^{t_f} ct^n dt = \left.\frac{ct^{n+1}}{n+1}\right|_{t_i}^{t_f} = \frac{ct_f^{n+1}}{n+1} - \frac{ct_i^{n+1}}{n+1} \qquad (n \neq -1)$$

- Here is another identity for integrals

$$\int_{t_i}^{t_f} (u(t) + w(t))dt = \int_{t_i}^{t_f} u(t)dt + \int_{t_i}^{t_f} w(t)dt$$

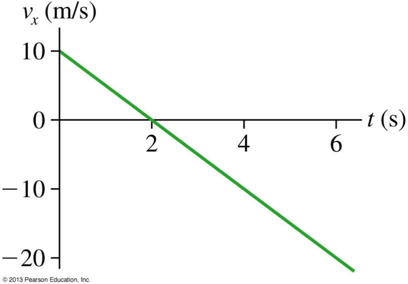
Quick Check

From the v(t) graph find the position of this particle at time *t*.



Quick Check

From the v(t) graph find the position of this particle at time t.



- The first step might be to figure out v(t), v(t)=10-5t (where t is in seconds and v is in m/s).

Position from Velocity

$$v(t) = 10 - 5t \text{ and } x(t = 0) = 30m$$

- Then you might plug $v(t)$ and your initial conditions into the equation $s_f = s_i + \int_{t_i}^{t_f} v_s dt$.

Position from Velocity

$$v(t) = 10 - 5t \text{ and } x(t=0) = 30m$$

- Then you might plug $v(t)$ and your initial conditions into the equation $s_f = s_i + \int\limits_{t_i}^{t_f} v_s dt$.

$$x = x_i + \int\limits_0^t v(t)dt = 30 \text{ m } + \int\limits_0^t (10 - 5t)dt$$

Position from Velocity

$$v(t) = 10 - 5t \text{ and } x(t=0) = 30m$$

- Then you might plug $v(t)$ and your initial conditions into the equation $s_f = s_i + \int_{t_i}^{t_f} v_s dt$.

$$x = x_i + \int_0^t v(t)dt = 30 \text{ m } + \int_0^t (10 - 5t)dt$$

$$= 30 \text{ m } + \int_0^t 10\ dt - \int_0^t 5t\ dt$$

Position from Velocity

$$v(t) = 10 - 5t \text{ and } x(t = 0) = 30m$$

- Then you might plug $v(t)$ and your initial conditions into the equation $s_f = s_i + \int_{t_i}^{t_f} v_s dt$.

$$x = x_i + \int_0^t v(t)dt = 30 \text{ m } + \int_0^t (10 - 5t)dt$$

$$= 30 \text{ m } + \int_0^t 10 \ dt - \int_0^t 5t \ dt$$

$$= 30 \text{ m } + 10t\Big|_0^t - \frac{5}{2}t^2\Big|_0^t$$

Position from Velocity

$$v(t) = 10 - 5t \text{ and } x(t=0) = 30m$$

- Then you might plug $v(t)$ and your initial conditions into the equation $s_f = s_i + \int_{t_i}^{t_f} v_s dt$.

$$x = x_i + \int_0^t v(t)dt = 30 \text{ m } + \int_0^t (10 - 5t)dt$$

$$= 30 \text{ m } + \int_0^t 10 \; dt - \int_0^t 5t \; dt$$

$$= 30 \text{ m } + 10t\Big|_0^t - \frac{5}{2}t^2\Big|_0^t$$

$$= 30 \text{ m } + (10t - 10 * 0) - \left(\frac{5}{2}t^2 - \frac{5}{2}0^2\right)$$

Position from Velocity

$$v(t) = 10 - 5t \text{ and } x(t = 0) = 30m$$

- Then you might plug $v(t)$ and your initial conditions into the equation $s_f = s_i + \int_{t_i}^{t_f} v_s dt$.

$$x = x_i + \int_0^t v(t)dt = 30 \text{ m } + \int_0^t (10 - 5t)dt$$

$$= 30 \text{ m } + \int_0^t 10 \ dt - \int_0^t 5t \ dt$$

$$= 30 \text{ m } + 10t\Big|_0^t - \frac{5}{2}t^2\Big|_0^t$$

$$= 30 \text{ m } + (10t - 10 * 0) - \left(\frac{5}{2}t^2 - \frac{5}{2}0^2\right)$$

$$x = 30 \text{ m } + 10t - \frac{5}{2}t^2$$

Motion with Constant Acceleration

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.
- If my velocity is 5m/s and I accelerate at $a = 2\text{m/s}^2$ for 1 second what is my new velocity?

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.
- If my velocity is 5m/s and I accelerate at $a = 2\text{m/s}^2$ for 1 second what is my new velocity?
  - $v = 7\text{m/s}$

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.
- If my velocity is 5m/s and I accelerate at $a = 2$m/s^2 for 1 second what is my new velocity?
  - $v = 7$m/s
- What about after 2 seconds?

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.
- If my velocity is 5m/s and I accelerate at $a = 2\text{m/s}^2$ for 1 second what is my new velocity?
  - $v = 7\text{m/s}$
- What about after 2 seconds?
  - $v = 9\text{m/s}$

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.
- If my velocity is 5m/s and I accelerate at $a = 2\text{m/s}^2$ for 1 second what is my new velocity?
  - $v = 7\text{m/s}$
- What about after 2 seconds?
  - $v = 9\text{m/s}$
- What $a = -2\text{m/s}^2$ for 2 seconds?

Motion with Constant Acceleration

- **Acceleration**: Accleration is the rate at which velocity changes.
- The SI units for acceleration are (m/s)/s or m/s^2.
- If my velocity is 5m/s and I accelerate at $a = 2\text{m/s}^2$ for 1 second what is my new velocity?
  - $v = 7\text{m/s}$
- What about after 2 seconds?
  - $v = 9\text{m/s}$
- What $a = -2\text{m/s}^2$ for 2 seconds?
  - $v = 1\text{m/s}$

Motion with Constant Acceleration

- Just like the average velocity the **average acceleration** is the slope of v(t).

$$a_{ave} = \frac{\Delta v}{\Delta t}$$

Motion with Constant Acceleration

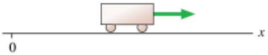
- Just like the average velocity the **average acceleration** is the slope of v(t).

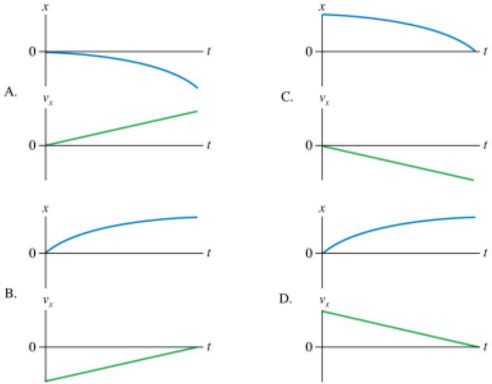
$$a_{ave} = \frac{\Delta v}{\Delta t}$$

- If the acceleration is constant then a_{ave} and the instantaneous acceleration, a_s are the same.

$$a_s = \frac{dv}{dt}$$

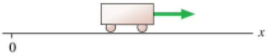
A cart slows down while moving away from the origin. What do the position and velocity graphs look like?

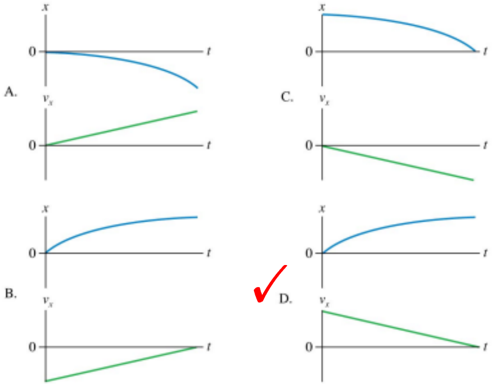




Quick Check

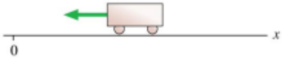
A cart slows down while moving away from the origin. What do the position and velocity graphs look like?

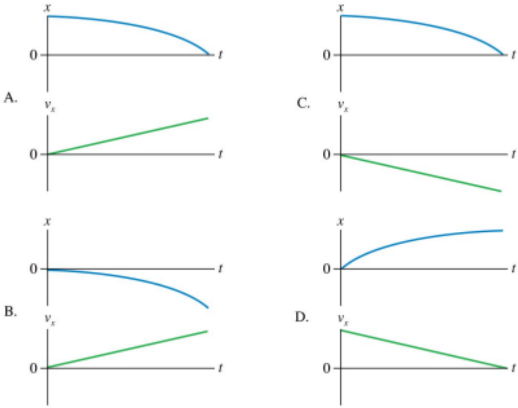




Quick Check

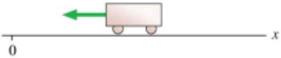
A cart speeds up toward the origin. What do the position and velocity graphs look like?

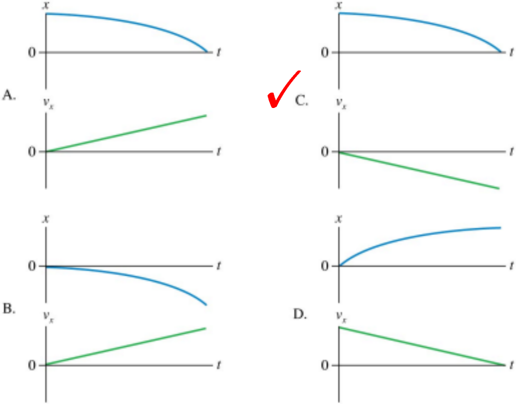




Quick Check

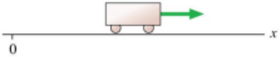
A cart speeds up toward the origin. What do the position and velocity graphs look like?



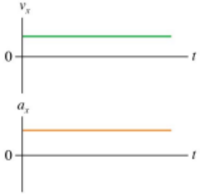


Quick Check

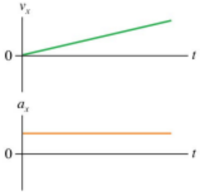
A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?



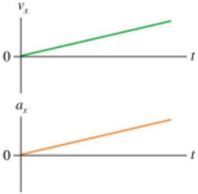
A.



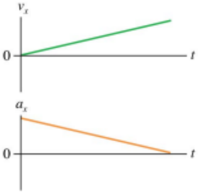
B.



C.

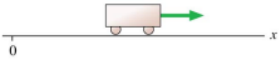


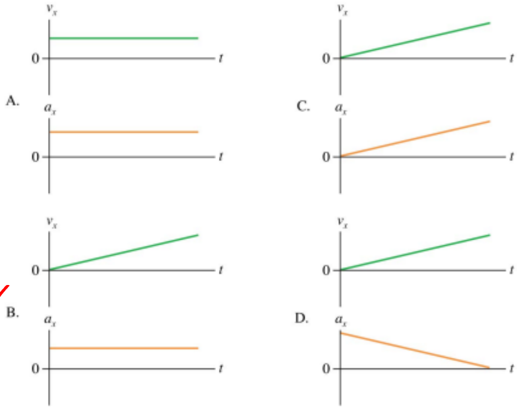
D.



Quick Check

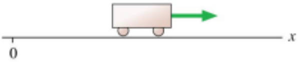
A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?

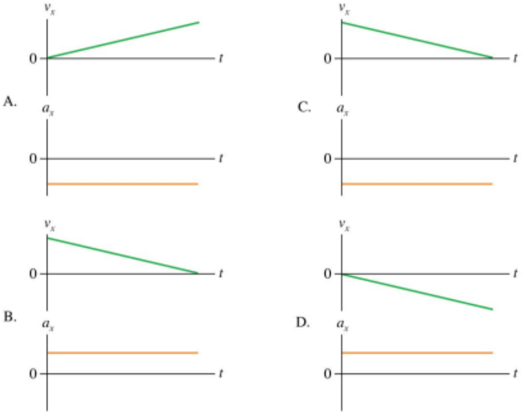




Quick Check

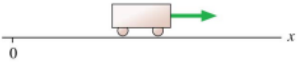
A cart *slows down* while moving away from the origin. What do the velocity and acceleration graphs look like?

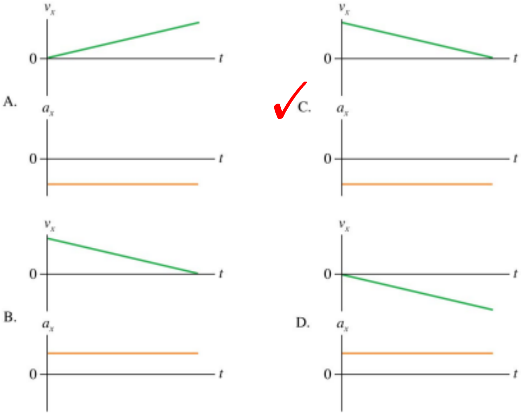




Quick Check

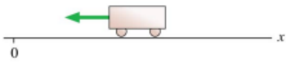
A cart *slows down* while moving away from the origin. What do the velocity and acceleration graphs look like?

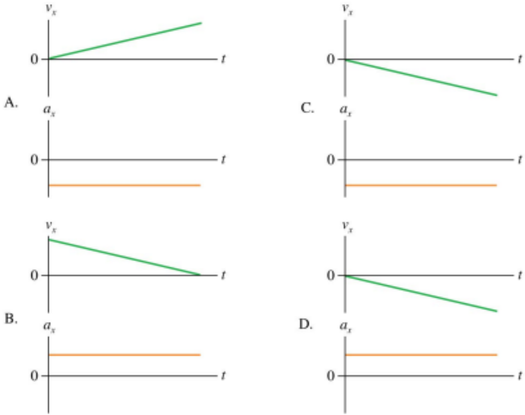




Quick Check

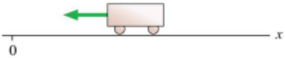
A cart *speeds up* while moving toward the origin. What do the velocity and acceleration graphs look like?

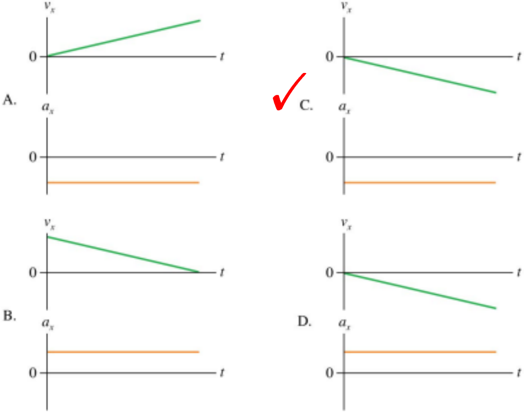




Quick Check

A cart *speeds up* while moving toward the origin. What do the velocity and acceleration graphs look like?





Kinematic Equations

- Remember with the initial position and the velocity we could calculation the positon with $s_f = s_i + \int\limits_{t_i}^{t_f} v_s dt$
- Since we know that the acceleration is the slope of $v(t)$, $a_{ave} = \frac{\Delta v}{\Delta t} \rightarrow v(t) = v_i + a\Delta t$ we can plug this into the integral.

Kinematic Equations

- Remember with the initial position and the velocity we could calculation the positon with $s_f = s_i + \int\limits_{t_i}^{t_f} v_s dt$
- Since we know that the acceleration is the slope of $v(t)$, $a_{ave} = \frac{\Delta v}{\Delta t} \to v(t) = v_i + a\Delta t$ we can plug this into the integral.
- Practice your integration by plugging v(t) into the integral above to find an expression for the positon $s(t)$ in terms of the initial velocity and the acceleration.

$$s_f = s_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

Kinematic Equations

- If a particle is initially at s_i with a velocity of v_i you can figure out what the final velocity is by substituting in $\Delta t = \Delta v / a$ into the previous equation. This gives us the third kinematic equation.

$$v_f = v_i + a\Delta t \tag{4}$$

$$s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \tag{5}$$

$$v_f^2 = v_i^2 + 2a\Delta s \tag{6}$$

A rocket sled accelerates at 50 m/s^2 for 5.0 s, coasts for 3.0 s, then deploys a braking parachute and accelerated at -3.0 m/s^2 (you would call this deceleration) until it comes to a stop.

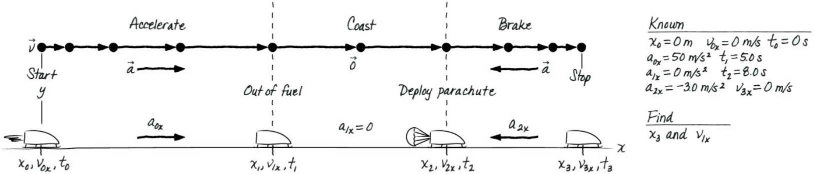
a. What is the maximum velocity of the rocket sled?

b. What is the total distance traveled?

A rocket sled accelerates at 50 m/s^2 for 5.0 s, coasts for 3.0 s, then deploys a braking parachute and accelerated at -3.0 m/s^2 (you would call this deceleration) until it comes to a stop.

a. What is the maximum velocity of the rocket sled?

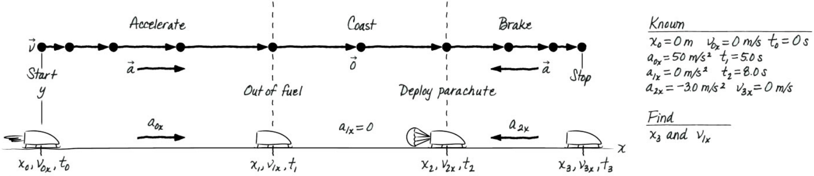
b. What is the total distance traveled?



A rocket sled accelerates at 50 m/s^2 for 5.0 s, coasts for 3.0 s, then deploys a braking parachute and accelerated at -3.0 m/s^2 (you would call this deceleration) until it comes to a stop.

a. What is the maximum velocity of the rocket sled?

b. What is the total distance traveled?



$$v_{1x} = 250 \text{ m/s}$$
$$x_3 = 12000 \text{ m}$$

Picture References

Nothing this time