

Physics 121: Free Fall, Inclined Plane Motion

Cody Petrie

Mesa Community College

- There is a HW assignment due this Thursday on Mastering Physics. How did last week go?
- On Thursday we are doing the V: Vectors lab. This is in the appendix of the labs so make sure to bring it (just in case some of you only bring 2-3 labs at a time like me).

Quick Review - Derivatives

- $v_s = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$ (Instantaneous Velocity)

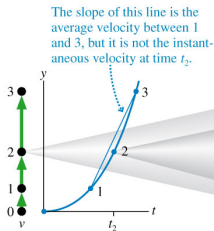
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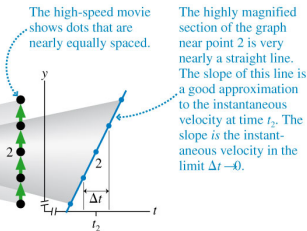
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- The instantaneous velocity is the slope of the tangent line to the $x(t)$ curve.

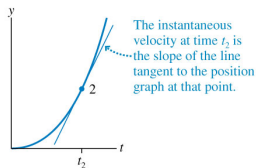
(a) 30 frames per second



(b) 3000 frames per second



(c) The limiting case



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Quick Review - Integrals

- If you want to determine the position at time t given the position at time $t = 0$ and the velocity you use an integral.

$$s_f = s_i + \lim_{\Delta t \rightarrow 0} \sum_{k=1}^N (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s dt$$

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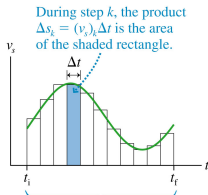
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- This is the area under the $v(t)$ curve.



During the interval t_i to t_f , the total displacement Δs is the "area under the curve."

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- Using these ideas you can write down the kinematic equations (which will be used A LOT in this class to calculation the motion of objects in various situations with constant acceleration)

$$v_f = v_i + a\Delta t$$

$$s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$v_f^2 = v_i^2 + 2a\Delta s$$

FREE FALL MOTION!

Free Fall

- Galileo Galilei (1564-1642) supposedly dropped objects from bell towers in Italy to study object in free fall motion. Here are two of his main discoveries.



- 1 If you drop two objects from the same height, if there is no air resistance, will hit the ground at the same time at the same speed.
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What about a feather and a bowling ball?

Vacuum Chamber

Free Fall

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- Since free fall motion is *constant acceleration* motion you can use the equations mentioned before, but with $a = -g$.
- $g = 9.80 \text{ m/s}^2$ only on earth. We'll see more about this when we talk about gravity.

Quick Check

A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration a_y is

- A. Positive
- B. Negative
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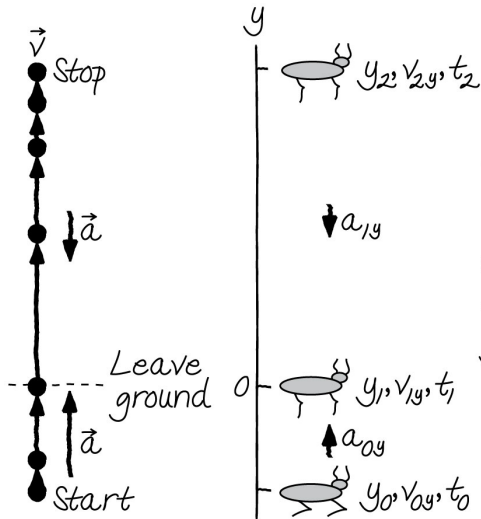
EXAMPLE 2.14

Finding the height of a leap

The springbok, an antelope found in Africa, gets its name from its remarkable jumping ability. When startled, a springbok will leap straight up into the air—a maneuver called a “pronk.” A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at 35 m/s^2 for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air. How high does it go?



Quick Check



Known

$$y_0 = -0.70 \text{ m} \quad t_0 = 0 \text{ s}$$

$$v_{0y} = 0 \text{ m/s} \quad a_{0y} = 35 \text{ m/s}^2$$

$$y_1 = 0 \text{ m} \quad v_{2y} = 0 \text{ m/s}$$

$$a_{1y} = -g = -9.80 \text{ m/s}^2$$

Find

$$y_2$$

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 - Use $v_f = v_i + a\Delta t$ to solve for Δt and then $s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$ to solve for y_2 . OR use one equation, $v_f^2 = v_i^2 + 2a\Delta s$ alone.

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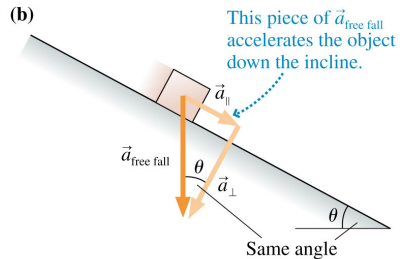
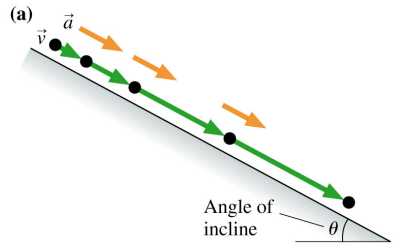
Motion on an Inclined Plane

Inclined Plane

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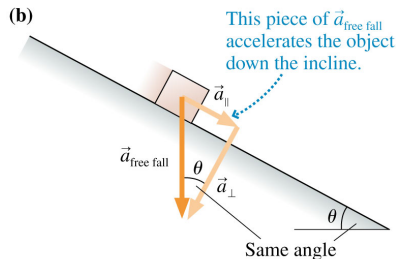
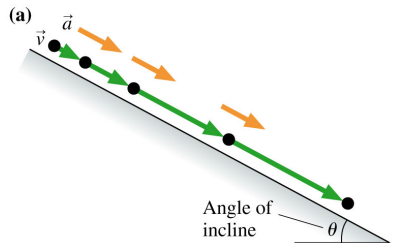


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Inclined Plane

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 - \vec{a} changes and the direction of the motion is down the ramp.
- The vector \vec{a}_{ff} can be broken into two components, \vec{a}_{\perp} and \vec{a}_{\parallel}
- Somehow the surface blocks \vec{a}_{\perp} (which we'll talk about later), which means that the acceleration along the surface is given by

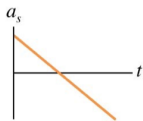
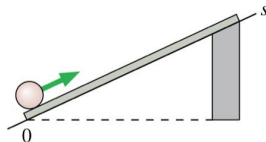
$$a_s = a_{\parallel} = \pm g \sin \theta$$



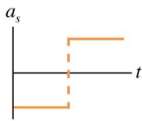
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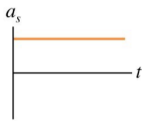
A ball rolls up the ramp, then back down. Which is the correct acceleration graph?



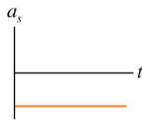
A.



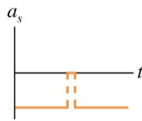
B.



C.



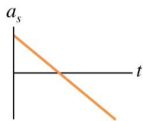
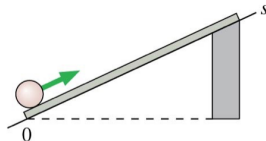
D.



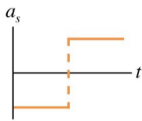
E.

Quick Check

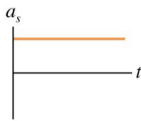
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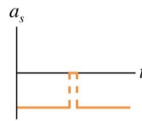
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C.

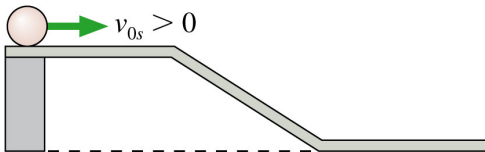


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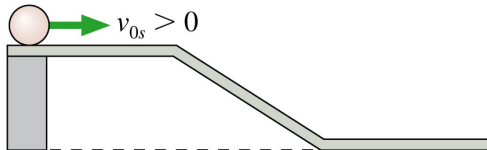
Quick Check



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Draw the x , v_s and a_s vs. t graphs for a ball rolling down the inclined plane here. Include all three sections of motion.

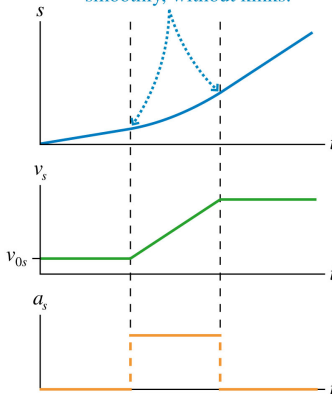
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Draw the x , v_s and a_s vs. t graphs for a ball rolling down the inclined plane here. Include all three sections of motion.

The position graph changes smoothly, without kinks.

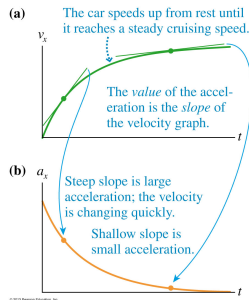


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Instantaneous Acceleration

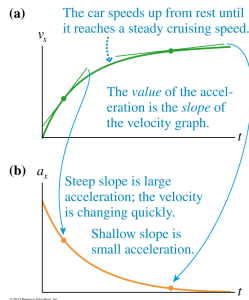
Instantaneous Acceleration

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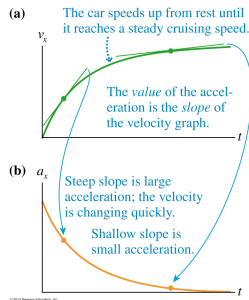


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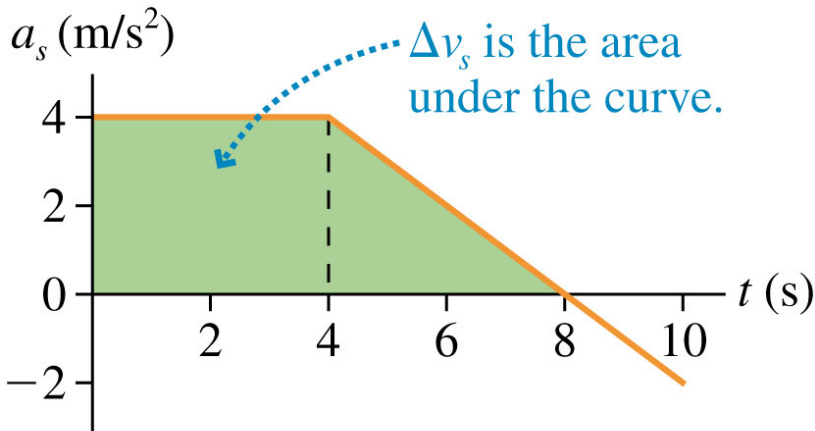
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- Also you can use an integral to determine $v(t)$ from the acceleration and initial $v(t = 0)$

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s dt$$

Instantaneous Acceleration

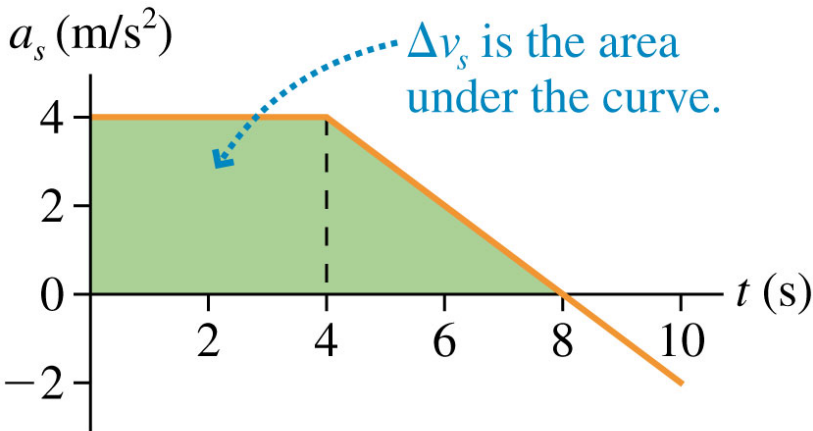
If an object is initially moving at 10 m/s and has this acceleration curve what is the velocity of the object at $t=8$ s?



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$$v_s(t = 8 \text{ s}) = 34 \text{ m/s}$$

Picture References

Galileo Galilei on the tower (accessed 4 Sep 17):

<https://i.pinimg.com/736x/ab/f2/7b/abf27bfdda4a9d19c810ab78964cdff1-galileo-experiment-biology.jpg>