PHY6938 Optics & Thermodynamics Fall 99 to Spring 97

1. A molecule consists of four identical atoms at the corner of a tetrahedron.

a) What is the number of (i) translational, (ii) rotational, and (iii) vibrational degrees of freedom for this molecule?

b) On the basis of the equipartition principle, what are the values of C_v and $\gamma = C_p/C_v$ for one mole of an ideal gas composed of these molecules?

c) Assuming that the vibrations of the molecule are quantized, what is C_v for temperatures such that $k_{\mathrm{B}}T \ll \hbar\omega_v$? Here ω_v is the lowest frequency of vibration of the molecule.

d) At very low T the rotations of the molecule are also quantized, i.e. $E_{\mathrm{rot}} = (\hbar^2/2I)l(l+1)$, where $l = 0, 1, 2...$ and I is the moment of inertia of the molecule. What is the value of C_v at very low temperatures ($k_{\mathrm{B}}T \ll \hbar^2/2I$)? Assume the gas is still ideal at low T.

2. Two slits of width $a = 0.015$ mm are separated by a distance $d = 0.06$ mm and illuminated by light of wavelength $\lambda = 650$ nm. How many bright fringes are seen in the central diffraction maximum?

3. Consider a real gas described by the van der Waals equation of state

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

where P is the pressure, T is the temperature, v is the volume per mole of the gas, R is the universal gas constant, and a and b are constants.

a) Briefly explain the physical origin and meaning of the constants a and b.

b) The coefficient of thermal volume expansion is defined as $\alpha = v^{-1}(\partial v/\partial T)_P$. Show that for the van der Waals gas

$$\alpha = \frac{Rv^2(v-b)}{RTv^3 - 2a(v-b)^2}.$$

c) Show that the isothermal compressibility, $\kappa_T = -v^{-1}(\partial v/\partial P)_T$, for the van der Waals gas is given by

$$\kappa_T = \frac{v^2(v-b)^2}{RTv^3 - 2a(v-b)^2}.$$

d) Obtain $(\partial P/\partial T)_v$ and verify that

$$\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_P \left(\frac{\partial v}{\partial P}\right)_T = -1.$$

4. An object is 15 cm to the left of a thin convex lens of focal length 10 cm. A concave mirror of radius 10 cm is 25 cm to the right of the lens.

a) Find the position of the final image formed by the miror and the lens.

b) Is the image real or virtual, upright or inverted?

c) Show on a diagram where your eye must be to see this image.

4. An object is 15 cm to the left of a thin convex lens of focal length 10 cm. A concave mirror of radius 10 cm is 25 cm to the right of the lens.

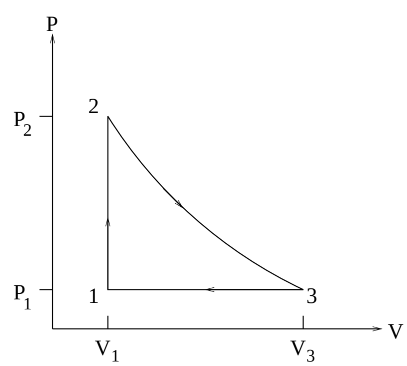
a) Find the position of the final image formed by the miror and the lens.

b) Is the image real or virtual, upright or inverted?

c) Show on a diagram where your eye must be to see this image.

5. A 35 mm camera has a negative size of 24 mm by 36 mm. It is to be used to take a picture of a person 175 cm tall in which the image of the person just fills the height (24 mm) of the film. How far should the person stand from the camera if the focal length of the lens is 50 mm?

6. An ideal gas ($\gamma = 1.5$) follows the cycle shown in the figure.



a) If $P_1 = 1$ atm, $V_1 = 100$ L, and $T_1 = 20°$ C, find the amount of gas considered at point 1.

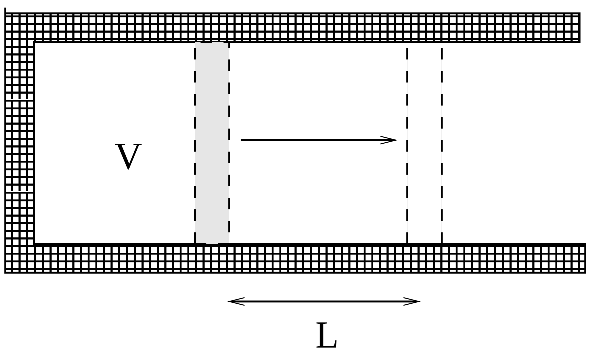
b) The gas is then heated at constant volume to point 2, where the pressure is $P_2 = 2$ atm. What is the temperature of the gas there?

c) Next, the gas is isothermally expanded to point 3, such that $P_3 = P_1 = 1$ atm. What is the work W_{23} done during the isothermal expansion?

d) How is the result modified if an adiabatic rather than isothermal expansion is considered between points 2 and 3?

7. Two lenses are separated by 35 cm. An object is placed 20 cm to the left of the first lens, which is a converging lens of focal length 10 cm. The second lens is a diverging lens of focal length -15 cm. What is the position of the final image? Is the image real or virtual? Erect or inverted? What is the overall magnification of the image?

8. The cylinder shown in the figure has a piston of mass M that can slide without friction. The area of the piston is S and the cylinder is filled with an ideal gas ($\gamma = 1.5$), with an initial volume V, and an initial pressure P. Assume that the outside pressure on the piston is zero (vacuum).



a) Determine the initial acceleration of the piston.

b) Calculate the velocity of the piston after it has moved a distance L, assuming that the gas is thermally isolated (adiabatic).

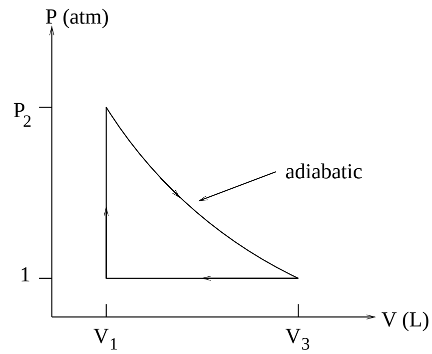
c) What distance would the piston have to move for the temperature of the gas to drop to one half of the original value?

9. A wedge of air is formed between two glass plates held apart at one edge by a sheet of paper whose thickness is 4.1×10^{-5} m. Green light ($\lambda_{\text{vacuum}} = 552$ nm) strikes the glass plates nearly perpendicularly. Assume $n_{\text{glass}} = 1.52$, $n_{\text{air}} = 1.00$.

a) How many bright fringes occur between the place where the plates touch and the edge of the sheet of the paper?

b) Is there a dark fringe or a bright fringe where the plates touch? Why?

10. In the cycle shown in the figure, 1 mol of an ideal gas ($\gamma = 1.4$) is initially at a pressure of 1 atm and a temperature of 0°C. The gas is heated at constant volume to $T_2 = 150$°C amd is then expanded adiabatically until its pressure is again 1 atm. It is then compressed at constant pressure back to its original state. Recall $R = 0.082\ \mathrm{L \cdot atm/(mol \cdot K)}$. Find:



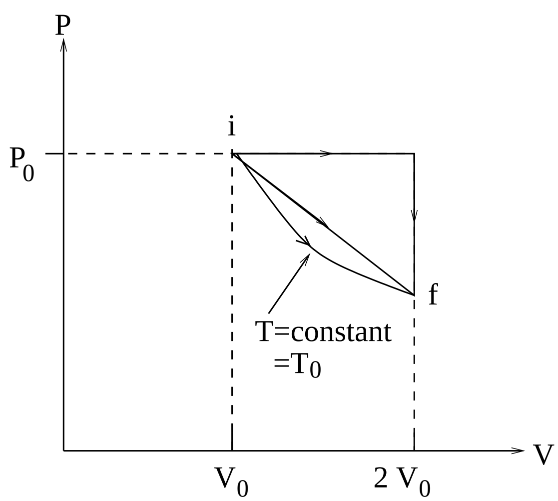
a) the temperature T_3 after the adiabatic expansion

b) the heat entering or leaving the system during each process

c) the efficiency of this cycle

d) the efficiency of a Carnot cycle operating between the temperature extremes of this cycle.

11. The figure shows three reversible processes joining initial and final states of n moles of a monatomic gas. For the three processes calculate:



a) the work done on the system

b) the heat interchanged with the environment

c) the change in internal energy

d) the change in entropy. Express all results in terms of n and the initial temperature T_0.

12. A mixture of red light ($\lambda_{\text{vacuum}} = 661$ nm) and blue light ($\lambda_{\text{vacuum}} = 472$ nm) shines perpendicularly on a thin layer of gasoline ($n_{\text{gas}} = 1.40$ lying on water ($n_{\text{water}} = 1.33$). The gasoline layer has a uniform thickness of 1.69×10^{-7} m. What color is the light reflected from the gasoline? Neglect any effects(s) caused by the material under the water.