PHY6938 E & M Spring 2000

1. A non-uniform, non-conducting disk of mass M, radius R, and total charge Q has a surface charge density $\sigma = \sigma_0 r/R$, where r is the distance from the center of the disk, and a mass per unit area $\sigma_m = M\sigma/Q$. The disk rotates with angular velocity ω about its axis.

a) Calculate the magnetic moment μ of the disk.

b) Show that the magnetic moment $\boldsymbol{\mu}$ and the angular momentum $\mathbf{L}$ are related by $\boldsymbol{\mu} = Q\mathbf{L}/2M$, where $\mathbf{L} = I\boldsymbol{\omega}$ and I is the moment of inertia.

2. A series LCR circuit is driven by an emf with angular frequency ω, so that the voltage across the resistor is

$$V_R(t) = V_0 \sin(\omega t)$$

In answering the following questions, express everything in terms of the given quantities: L, C, R, ω, V_0, and ε_0.

a) Draw a phasor diagram for V_R, V_L, and V_C, and specify the angles between the phasors.

b) What is the current $I_L(t)$ through the inductance?

c) Find the voltage $V_L(t)$ across the inductance.

d) Find the voltage $V_C(t)$ across the capacitance.

e) Determine the angular frequency, ω_0, for which the voltage $V_R(t)$ across the resistance is the same as that across the emf source, i.e. $V_R(t) = \varepsilon(t)$.

f) If the emf source is suddenly short-circuited, what will be the frequency of the current $I_L(t)$? What is $I_L(t)$ long after the emf is short-circuited?

3. A positive charge is uniformly distributed throughout a very long cylindrical volume of radius R. The charge per unit volume is ρ.

a) Find the electric field $\vec{E}$ everywhere as a function of the distance r from the axis of the cylinder.

b) Find the electric potential V everywhere as a function of r. Define $V = 0$ at the surface of the cylinder.

c) Sketch E and V as functions of r, from $r = 0$ to $r = 3R$, showing the values of each at $r = 0$, R, and $3R$.