PHY6938 Modern Physics and Quantum Mechanics, Fall 2000

1. During a photoelectric effect experiment, sodium metal is illuminated with light of wavelength 4.20×10^2 nm. The stopping potential is found to be 0.65 V. When the wavelength is changed to 3.10×10^2 nm, the stopping potential is found to be 1.69 V. Using only these data and the values of the speed of light, $c = 3.00 \times 10^8$ m/s, and the elementary charge, $e = 1.60 \times 10^{-19}$ C, find a value for Planck's constant.

2. The Hamiltonian of a quantum-mechanical harmonic oscillator is given by

$$\mathcal{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 .$$

a) Obtain the value of α for which the wave function

$$\psi(x) = C \exp\left(-\alpha^2 x^2\right) ,$$

where C is the normalization constant, is a solution of Schrödinger's equation. Find the corresponding energy eigenvalue.

b) Obtain the expectation values of x, x^2, p, and p^2 for the wave function in (a).
Hint: The definition of the gamma function,

$$\Gamma(y) = \int_0^\infty \mathrm{d}t\, t^{y-1} e^{-y}, \quad \Gamma(1+y) = y\Gamma(y) ,$$

could be helpful in the integration.

c) Verify that the Heisenberg uncertainty relation for p and x is satisfied.

3. In nuclear beta decay, electrons are observed to be ejected from the atomic nucleus. Assume that electrons are somehow trapped within the nucleus and that occasionally one escapes.

a) Estimate the kinetic energy that such an electron must have. Assume a nuclear diameter of 1.0×10^{-14} m.

b) Electrons emitted from the nucleus in nuclear beta decay typically have kinetic energies of about 1 MeV. Comment on the difference between the actual kinetic energy and your result for part a.