PHY6938 Mechanics Spring 99 January 17, 2000

1. A 1.2 kg block moving in the $+z$ direction at $0.8c$ makes a head-on collision with a 1.6 kg block moving in the $-z$ direction at $0.6c$. They stick together. (*Note*: this is unrealistic; it is far more likely that the blocks would both disintegrate. But carry on anyway!)

a) What is the total momentum of the system (give your answer in units of kg$\cdot$ c)?

b) What is the total energy of the system (give your answer in units of kg$\cdot$ c^2)?

c) What are the mass and speed of the combined object after the collision?

2. Consider the Earth as a frame rotating about its axis with frequency ω. Particles are then subject to the Coriolis force given by $\mathbf{F} = -2m(\boldsymbol{\omega} \times \mathbf{v}_r)$, where $\mathbf{v}_r$ is the velocity of the particle relative to the frame and m is the mass of the particle. Choose the z–axis along the upwards vertical direction, the y–axis pointing North, and the x–axis to the East. Assume you are in the Northern hemisphere at a colatitude θ (angle of the z–axis with $\boldsymbol{\omega}$).

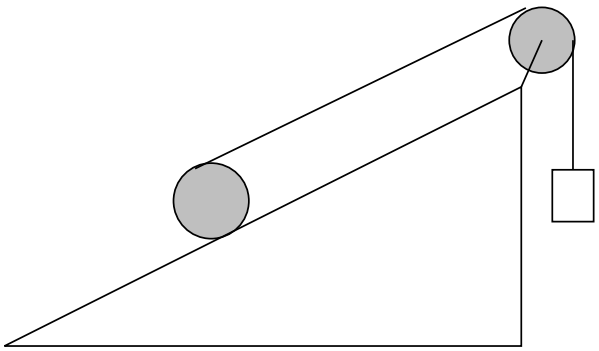
a) A particle moves under the influence of gravitation and the Coriolis force. Write the equations of motion in the x, y, and z directions. Use the approximation that the component of velocity in the z direction is much larger than the components in the x and y directions, i.e. $|v_z| \gg |v_x|,\ |v_y|$.

b) The particle is dropped at rest from a height h above the ground; it arrives at the ground with velocity $v_0 = \sqrt{2gh}$. Find the magnitude and direction of the Coriolis deflection.

c) The particle is now thrown vertically upward with an initial speed v_0, so that it reaches the maximum height h [the same h as in part (b)], and then it falls back to the ground. Find the magnitude and direction of the Coriolis deflection.

d) Compare your results for parts (b) and (c).

3. A cylinder with a mass of 10 kg and a radius of 0.07 cm can roll without slipping on a 30^o ramp. An unstretchable rope is wrapped around the cylinder while the other end is tied to a 2.0 kg block over a massless frictionless pulley, as shown in the diagram.



a) Draw the free-body diagrams for the cylinder and the block.

b) If the block drops down by 1 m, how much does the cylinder move up vertically?

c) Determine the magnitude and direction of the acceleration of the cylinder.

d) What are the magnitude and direction of the static friction between the cylinder and the ramp?