

Study guide for qualifying exams

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1 Classical Mechanics Equations

Newtonian Mechanics

Newton's Laws:

1. An object will maintain it's current motion unless acted upon by an external force.
2. $\vec{F} = m\vec{a}$
3. All forces occur in equal but directionally opposite pairs.

Second Law: $\vec{F} = m\vec{a} = \dot{\vec{p}}$

Angular Position/Velocity/Acceleration: $\theta = s/r$, $\omega = v/r$, $\alpha = a/r$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F} = \dot{\vec{L}}$

Centripital Acceleration: $a_c = v^2/r$

Centrifugal/Coriolis Forces: $\vec{F}_{cent} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})$, $\vec{F}_{cor} = -2m\vec{\omega} \times \dot{\vec{r}}$

Work to go from positions \vec{a} to \vec{b} : $W_{ab} = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{s}$

Conservative Force Field (2 eq): W_{ab} is the same regardless of path so $\oint \vec{F} \cdot d\vec{s} = 0$, and thus we can write the force as $\vec{F} = -\nabla V(\vec{r})$.

Lagrangian Formalism

Functional Derivative: $\frac{\delta F}{\delta u}[x_0] = \lim_{\epsilon \rightarrow 0} \frac{F[x_0 + \epsilon u] - F[x_0]}{\epsilon} \rightarrow \frac{\delta F}{\delta x(t)}[x(t')] = \lim_{\epsilon \rightarrow 0} \frac{F[x(t') + \epsilon \delta(t' - t)] - F[x(t')]}{\epsilon}$

Principle of Least Action: $\delta S = 0$, where $S = \int_{t_i}^{t_f} L(\vec{q}, \dot{\vec{q}}, t) dt$

Lagranges Equation: $\frac{d}{dt} \frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = 0$

Holonomic Constraints: $f_\alpha(x^A, t) = 0$, $L' = L(x^A, \dot{x}^A) + \lambda_\alpha f_\alpha(x^A, t) \rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = \lambda_\alpha \frac{\partial f_\alpha}{\partial x^A}$

Noether's Theorem: A continuous symmetry in the Action (and thus Lagrangian) result in a conserved quantity.

Moment of Inertia Tensor: $\vec{L} = \overleftrightarrow{I} \vec{\omega}$, $T = \frac{1}{2} \omega_a I_{ab} \omega_b$, $I_{ab} = \sum_i m_i ((\vec{r}_i \cdot \vec{r}_i) \delta_{ab} - (\vec{r}_i)_a (\vec{r}_i)_b)$

Euler's Equations: Only look at rotation, not translation. Conservation of Angular Momentum gives $I_i\dot{\omega}_i + \omega_j\omega_k(I_k - I_j) = 0$, for i,j,k being cyclic permutations of 1,2,3.

Hamiltonian Formalism

Generalized Momenta: $p_i = \frac{\partial L}{\partial \dot{q}_i}$, $\dot{p}_i = \frac{\partial L}{\partial q_i}$

Hamiltonian: $H(q_i, p_i, t) = \sum_{i=1}^n p_i \dot{q}_i - L(q_i, \dot{q}_i, t)$

Hamilton's Equations:

1. $\dot{p}_i = -\frac{\partial H}{\partial q_i}$
2. $\dot{q}_i = \frac{\partial H}{\partial p_i}$
3. $-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$

Cyclic/Ignorable Coordinates: q is ignorable if $\frac{\partial L}{\partial q} = 0$, i.e. if q does not appear in L . Thus $p = \frac{\partial L}{\partial \dot{q}}$ is conserved.

Liouville's Theorem: A volume of a region of phase space remains the same, even when the region changes. $V = dq_1 \dots dq_n dp_1 \dots dp_n$.

Poisson Bracket: $\{f, g\} = \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}$.

Constant of Motion from Poisson Bracket: $\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}$. If $I, H = 0$, then I is a constant of motion.

Canonical Transformation: Transformation $(q_i \rightarrow Q_i(q, p), p_i \rightarrow P_i(q, p))$ that leaves Hamilton's equations invariant.

2 Statistical Mechanics Equations

2.1 Thermodynamics

Laws of Thermodynamics:

1. Energy conservation. $dE = dQ - pdV$. dQ just means that the heat is an inexact differential and the integral depends on the path.
2. $\Delta S \geq \int \frac{dQ}{T}$, where equality is for a process that is reversible (never leaves equilibrium).
3. Entropy at zero temperature is zero. In stat mech this means that the ground state is nondegenerate and $S \propto \ln(W)$, where W is the number of available states.

Intensive vs Extensive Variables: Intensive variables do NOT scale with system size (T, p, μ), while extensive do scale (E, S, V, N).

Thermodynamic Potentials:

- Internal Energy: $U(S, V, N)$

- Helmholtz Free Energy: $F(T, V, N) = U - TS$
- Enthalpy: $H(S, p, N) = U + pV$
- Gibbs Free Energy: $G(T, p, N) = U - TS + pV$
- Landau(Grand) Potential: $\Omega(T, V, \mu) = U - TS - \mu_i N_i$

Thermodynamic Ensembles:

1. Microcanonical: Does not exchange energy or particles with environment. Fixed E, N
2. Canonical: Does not exchange particles, but can exchange energy (heat bath). Fixed N, T
3. Grand canonical: Can exchange energy and particles with environment. Fixed T, μ .

Maxwell's Relations (4 main):

- $\frac{\partial^2 U}{\partial S \partial V} = - \left(\frac{\partial p}{\partial S} \right)_V = \left(\frac{\partial T}{\partial V} \right)_S$
- $\frac{\partial^2 F}{\partial T \partial V} = \left(\frac{\partial p}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T$
- $\frac{\partial^2 H}{\partial S \partial p} = \left(\frac{\partial V}{\partial S} \right)_p = \left(\frac{\partial T}{\partial p} \right)_S$
- $\frac{\partial^2 G}{\partial T \partial p} = \left(\frac{\partial V}{\partial T} \right)_p = - \left(\frac{\partial S}{\partial p} \right)_T$

Engine Efficiency: $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{T_{out}}{T_{in}}$

Isobaric Thermal Expansion Coefficient: $\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$, How much the volume changes with a change in temperature.

Isothermal Compressibility: $\kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$, How much the volume changes when the pressure changes.

Isentropic(Adiabatic) Compressibility: $\kappa_S = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_S$, Same as above.

Specific Heat at Constant V: $C_V = \left(\frac{\partial Q}{\partial T} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V$, Amount of heat per unit mass to raise the temp by 1 degree.

Specific Heat at Constant p: $C_p = \left(\frac{\partial Q}{\partial T} \right)_p = \left(\frac{\partial H}{\partial T} \right)_p$, Same as above.

Fermi Energy/Temperature: Chemical potential at $T = 0$. $\epsilon_F = \mu(T = 0)$

2.2 Statistical Mechanics

Number of microstates in a macrostate (ways to get n heads): $\Omega = \frac{N!}{\prod_i n_i!}$

Stirling's Approximation: $\ln n! = n \ln n - n$

How many order important ways to order n things: $n!$

How many order important ways to order n things r at a time: $\frac{n!}{(n-r)!}$

How many NOT order important ways to order n things r at a time: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Microcanonical(Classical) Partition Function: $Z_m = \sum_s g_s e^{-\beta E_s}$

Canonical Partition Function: $Z_c = \text{tr} \left(e^{-\beta \hat{H}} \right)$

Grand Canonical Partition Function: $Z_{gc} = \text{tr} \left(e^{-\beta(\hat{H} - \mu \hat{N})} \right)$

Geometric Series: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

Classical limit of the trace of an operator: $\text{tr}(\mathcal{O}) = \frac{1}{N!(2\pi\hbar)^{3N}} \int d^3r_1 \dots d^3r_N \int d^3p_1 \dots d^3p_N \mathcal{O}$,
 $N!$ is for identical particles.

Thermodynamic Limit: $T \rightarrow \infty, V \rightarrow \infty, N/V = \text{const}$

Expectation value for pure/mixed: $\langle \mathcal{O} \rangle_p = \langle \psi | \mathcal{O} | \psi \rangle, \langle \mathcal{O} \rangle_m = \sum_i P_i \langle \psi_i | \mathcal{O} | \psi_i \rangle$

Density Matrix (ex. Canonical Ensemble): $\rho = \sum_n P_n |\psi_n\rangle \langle \psi_n|, \rho_c = \frac{e^{-\beta \hat{H}}}{\text{tr} e^{-\beta \hat{H}}}$

Expectation value with Density Matrix: $\langle \mathcal{O} \rangle = \text{tr}(\mathcal{O}\rho)$

Trace of Density matrix: $\text{tr}(\rho) = 1$

Time evolution of density matrix: $\frac{\partial}{\partial t} \rho = \frac{1}{i\hbar} [\hat{H}, \hat{\rho}]$

Z_{gc} for an ideal gas: $Z_{gc} = \frac{V^N (2mT\pi)^{3N/2}}{N! (2\pi\hbar)^{3N}} e^{\beta\mu}$

Z_{gc} for ideal fermi gas: $Z_{gc} = \prod_k (1 + e^{-\beta(\epsilon_k - \mu)})$

Z_{gc} for ideal bose gas: $Z_{gc} = \prod_k \frac{1}{(1 - e^{-\beta(\epsilon_k - \mu)})}$

Stuff here for black-body and phonons and bose condensates.

Explain Bose-Condensates with Bose statistics: $\lim_{T \rightarrow 0} n(p) = \lim_{\beta \rightarrow \infty} \frac{1}{1 - e^{\beta(\epsilon - \mu)}} \rightarrow 0$ un-

less $\epsilon \rightarrow \mu$, which happens at the ground state. **Is this true?**

What is cluster expansion used for?: Systems of interacting particles.

3 Quantum Mechanics Equations

Properties of a vector space:

- Sum $|V\rangle + |W\rangle$
- Scalar product with properties
 1. closure: results in another vector in the space.
 2. distributive: $a(|V\rangle + |W\rangle) = a|V\rangle + a|W\rangle, (a+b)|V\rangle = a|V\rangle + b|V\rangle$
 3. associative: $a(b|V\rangle) = ab|V\rangle, |V\rangle + (|W\rangle + |Z\rangle) = (|V\rangle + |W\rangle) + |Z\rangle$
 4. commutative: $|V\rangle + |W\rangle = |W\rangle + |V\rangle$
 5. additive inverse: $|V\rangle + |-V\rangle = |0\rangle$
 6. null vector: $|V\rangle + |0\rangle = |V\rangle$

Hilbert space: Vector space with defined inner product.

Expand in orthonormal basis: $|V\rangle = \sum_i v_i |i\rangle$

Hermitian operator: $\mathcal{O}^\dagger = \mathcal{O}$

Anti-Hermitian operator: $\mathcal{O}^\dagger = -\mathcal{O}$

Unitary operator: $UU^\dagger = \mathbb{I}$

Orthogonality: $\langle i | j \rangle = \delta_{ij}$

Completeness: $\sum_i i = \mathbb{I}$

Postulates of QM:

1. The state of a physical system, at some fixed time, is given by a normalized ray in a Hilbert space over the complex numbers. (ray is vector whose norm doesn't matter)
2. The ray evolves deterministically in time according to Schrödinger's equation.
3. Observables correspond to self-adjoint (hermitian) operators.
4. If a particle is in the state $|\psi\rangle$ then a measurement of \mathcal{O} will yield one of the eigenvalues of \mathcal{O} , ω . The state of the system changes to an eigenstate of \mathcal{O} , $|\omega\rangle$.

Schrödinger equation: $i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$

Free particle ψ_p and E_p : $\psi_p = Ae^{ikx} + Be^{-ikx}$, $k^2 = \frac{2mE_p}{\hbar^2}$, $E_p = \frac{p^2}{2m}$

Particle in a box ψ_n and E_n : $\psi_n = \sqrt{\frac{2}{L}} \sin(k_n x)$, $k_n = \frac{n\pi}{L}$, $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$

Harmonic Oscillator \hat{H} , ψ_n and E_n : $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 \hat{x}^2$, $\psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{2^n n!} H_n(x) e^{-x^2/2}$, $E_n = (n + \frac{1}{2})\hbar\omega$

Raising and lowering operators and how to affect $|n\rangle$ (3-2):

- $a = \left(\frac{m\omega}{2\hbar}\right)^{1/2} \left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$, $a|n\rangle = \sqrt{n}|n-1\rangle$, $a|0\rangle = 0$
- $a^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2} \left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$, $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

\hat{H} in terms of a and a^\dagger : $\hat{H} = \hbar\omega(a^\dagger a + 1/2)$

Commutation relations for \hat{H} , a , a^\dagger :

- $[\hat{H}, a] = -\hbar\omega a$
- $[\hat{H}, a^\dagger] = \hbar\omega a^\dagger$
- $[a, a^\dagger] = 1$

J^2 and J_z on the angular momentum state $|jm_j\rangle$:

- $J^2|jm_j\rangle = j(j+1)\hbar^2|jm_j\rangle$
- $J_z|jm_j\rangle = m_j\hbar|jm_j\rangle$

Commutation relations for J_i and J_j and for J^2 and J_i :

- $[J_i, J_j] = i\hbar J_k$
- $[J^2, J_i] = 0$

J_z and J^2 in position basis:

- $J_z = -i\hbar \frac{\partial}{\partial t}$
- $\mathbf{J}^2 = -\hbar^2 \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$

Raising and Lowering Angular Momentum Operators on $|j, m\rangle$:

$$J_{\pm} |j, m\rangle = \hbar[j(j+1) - m(m \pm 1)]^{1/2} |j, m \pm 1\rangle$$

$$J_x \text{ and } J_y \text{ in terms of } J_+ \text{ and } J_-: J_x = \frac{1}{2}(J_+ + J_-), J_y = \frac{1}{2i}(J_+ - J_-)$$

$$\text{Momentum eigenstate, } \langle x|p\rangle: \langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$$

$$\text{Hydrogen Atom } V(r), \psi_n, E_n(\mathbf{x4}): V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}, \psi_n = \text{stuff} * L_{n-l-1}^{2l+1}(\rho) Y_l^m(\theta, \phi) (\text{Laguerre})$$

$$E_n = -\frac{1}{2n^2} \left(\frac{Ze^2}{4\pi\epsilon_0 \hbar} \right)^2 m_e = -\frac{1}{2n^2} \alpha^2 m_e c^2 = -\frac{1}{n^2} 13.6 \text{ eV} = -\frac{1}{2n^2} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{a_0} \right),$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}, a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$$

Pauli matrices and commutation relations:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_a \sigma_b] = 2i\epsilon_{abc} \sigma_c$$

Non-Deg Time-Ind Perturbation, $E_n^{(1)}, |n^{(1)}\rangle, E_n^{(2)}$:

$$E_n^{(1)} = H'_{nn} = \langle n^{(0)} | H' | n^{(0)} \rangle$$

$$|n^{(1)}\rangle = \sum_{m \neq n} \frac{\langle n^{(0)} | H' | n^{(0)} \rangle}{(E_n^{(0)} - E_m^{(0)})} |m^{(0)}\rangle$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|\langle n^{(0)} | H' | n^{(0)} \rangle|^2}{(E_n^{(0)} - E_m^{(0)})}$$

Deg Time-Ind Perturbation, $E_n^{(1)}$: Diagonalize the perturbation hamiltonian in the degenerate subspace.

$$\text{Time-Dep Perturbation, } P_{i \rightarrow f}(t): P_{i \rightarrow f}(t) = \frac{1}{\hbar^2} \left| \int_0^t dt' \langle f | H'(t') | i \rangle e^{i(E_f - E_i)t'/\hbar} \right|^2$$

$$\text{Fermi's golden rule, and } g(E_f) \text{ as } \delta: R_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2 g(E_f), g(E_f) \approx \delta(E_f^{(0)} - E_f^{(0)} - \hbar\omega)$$

Einstein's Stimulated/Spontaneous emission coefficients:

$$\text{Stimulated: } B_{if} = \frac{\pi e^2}{3\epsilon_0 \hbar^2} |\langle f | \mathbf{r} | i \rangle|^2$$

$$\text{Spontaneous: } A_{if} = \frac{e^2 \omega_{if}^3}{3\pi \epsilon_0 \hbar c^3} |\langle f | \mathbf{r} | i \rangle|^2$$

Total $\psi(\mathbf{r})$ in scattering problem:

$$\psi(\mathbf{r}) = \psi_{inc}(\mathbf{r}) + \psi_s(\mathbf{r}) = \psi_{inc}(\mathbf{r}) + f(\theta, \phi) \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{r}$$

$$f(\theta, \phi) = -\frac{m}{2\pi\hbar^2} \int d^3r' e^{-i\mathbf{k}' \cdot \mathbf{r}'} V(\mathbf{r}') \psi(\mathbf{r}')$$

$$\text{Differential Cross Section: } \frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2$$

Born Approximation: In the above integral for $f(\theta, \phi)$ let $\psi \rightarrow \psi_{inc}$.

Dirac Equation:

$$(i\hbar\gamma^\mu \partial_\mu - mc)\psi = 0$$

$$\gamma^0 = \beta, \gamma^i = \beta\alpha_i$$

$$\beta = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}, \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

4 Electricity and Magnetism Equations

Maxwell's Equations in Vacuum (SI):

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0, \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's Equations in Matter (SI), and \mathbf{D} and \mathbf{H} :

$$\nabla \cdot \mathbf{D} = \rho, \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E}, \mathbf{B} = \mu_0 \mathbf{H}$$

$$\text{Continuity Equation: } \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

$$\text{Lorentz Force: } \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\text{Coulomb's Law (x2): } \mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}, \mathbf{F} = Q\mathbf{E}$$

$$\text{Gauss' Law: } \oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$$

$$\text{Electrostatic Potential (x2): } \mathbf{E} = -\nabla\Phi, \Phi = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$\text{Laplace's Equation \& General Solution(Spherical Coordinates, no } \phi): \nabla^2\Phi = 0$$

$$\Phi(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$\text{Poisson's Equation: } \nabla^2\Phi = -\rho/\epsilon_0$$

Explain the Method of Images: Because of the uniqueness theorem you can add charges OUTSIDE of the computational area to meet the same boundary conditions. A solution to this new configuration is also a solution to the initial configuration.

Method of Images (plane, sphere, hem boss):

plane: add one charge below plane.

sphere: 1 test charge inside sphere.

hem boss: 3 test charges.

$$\text{Multipole Expansion of } V(\mathbf{r}): V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \int d^3r' \frac{P_n(\cos \alpha)}{r^{n+1}} \rho(\mathbf{r}')$$

Work and Energy in Electrostatics: The Energy of a system is the work it requires to assemble the system.

$$\text{Atomic Polarizability } (\alpha): \mathbf{p} = \alpha \mathbf{E}$$

$$\text{Polarization: Electric dipole moment per unit volume. } \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\text{Magnetization: Magnetic dipole moment per unit volume } \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

$$\text{Bound Charge: } \rho_b = -\nabla \cdot \mathbf{P}$$

$$\text{Bound Current: } \mathbf{J}_b = \nabla \times \mathbf{M}$$

$$\text{Linear Media x2: } \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \mathbf{M} = \chi_m \mathbf{H}$$

$$\text{Biot-Savart Law: } \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}') \times |\mathbf{r}-\mathbf{r}'|}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$\text{Ohm's Law: } \mathbf{J} = \sigma \mathbf{E}, \text{ where } \sigma \text{ is the conductivity}$$

$$\text{Resistivity: } \rho = 1/\sigma$$

Boundary Conditions:

$$D_1^\perp - D_2^\perp = \sigma_f$$

$$B_1^\perp - B_2^\perp = 0$$

$$\mathbf{E}_1^\parallel - \mathbf{E}_2^\parallel = 0$$

$$\mathbf{H}_1^\parallel - \mathbf{H}_2^\parallel = \mathbf{K}_f \times \hat{\mathbf{n}}$$

$$\text{Poynting's Theorem, units of } \mathbf{S}: \mathbf{S} = \frac{\text{energy}}{\text{time} \cdot \text{energy}}$$

$$\frac{dW}{dt} = -\frac{d}{dt} \int d^3r \frac{1}{2} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) - \frac{1}{\mu_0} \oint d\mathbf{a} \cdot (\mathbf{E} \times \mathbf{B}) = -\frac{d}{dt} (W_e + W_m) - \oint d\mathbf{a} \cdot \mathbf{S}$$

Maxwell Stress Tensor and Static Force:

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$$

$$\mathbf{F} = \oint d\mathbf{a} \cdot \overleftrightarrow{\mathbf{T}}$$

5 Miscellaneous Physics

$$\text{Taylor Expansion: } f(\vec{x} + \vec{a}) = f(\vec{x}) + a_i \partial_i f(\vec{x}) + \mathcal{O}(\vec{a}^2)$$

$$\text{Gaussian Integral: } \int_{-\infty}^{\infty} dx e^{-ax^2+bx+c} = \sqrt{\frac{\pi}{a}} e^{b^2/(4a)+c}$$

3 types of Boundary Conditions:

$$\text{Dirichlet: } \Phi(\mathbf{a}) = \text{const}$$

$$\text{Neumann: } \frac{\partial \Phi(\mathbf{a})}{\partial \mathbf{n}} = \hat{\mathbf{n}} \cdot \nabla \Phi = \text{const}$$

Robin: Linear combination of the first two

$$\text{Value of fine structure constant: } \alpha \approx \frac{1}{137}$$

$$\text{Mass of electron in eV: } m_e c^2 = 0.511 \text{ eV}$$

$$\text{Value of the Bohr radius: } a_0 = 0.529 \text{ \AA}$$