Study guide for qualifying exams

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1 Classical Mechanics

1. Newtonian Mechanics

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   (b) Energy

   (c) Momentum/Angular Momentum

2. Lagrangian Mechanics

   (a) Calculous of Variations

   (b) Principle of Least Action/Lagranges Equation

   (c) Generalized Coordinates

   (d) Holonomic/Non-Holonomic Constraints

   (e) Noether's Theorem

   (f) Rigid Body Motion

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3. Hamiltonian Formalism

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   (b) Generalized Momenta/Cyclic Coordinates/Conserved Quantities

   (c) Liouville's Theorem

   (d) Poisson Brackets

   (e) Canonical Transformations

2   Statistical Mechanics

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    (c) Thermodynamic Potentials and Ensembles
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    (e) Various Definitions
        i. Compressibility
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    (i) Phonons
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4. Particle in a Box

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7. Hydrogen Atom

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            i. Poisson/Laplace's Equations
    (c) Boundary Conditions
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    (e) Multipole Expansion
    (f) Work and Energy
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3. Electrodynamics

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    (b) Ampere's Law
    (c) Ohm's Law
    (d) Maxwell's Equations
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(h) Maxwell Stress Tensor

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    i. The Wave Equation from Maxwell's Eq.

    ii. EM Waves in Matter

    iii. Wave Guides

4. Scalar and Vector Potentials

5. Coulomb and Lorentz Gauge

6. Retarted Potentials

7. Lienard-Wiechert Potentials

8. Radiation

(a) Electric/Magnetic Dipole Radiation

9. Helmholtz Theorem

10. Special Relativity

(a) Einstein's Postulates

(b) Lorentz Transformation

(c) 4-Vectors

(d) Field Tensor and Transformation

(e) Relativistic Potentials

5 Classical Mechanics Equations

Newtonian Mechanics

Newton's Laws:

1. An object will maintain it's current motion unless acted upon by an external force.

2. $\vec{F} = m\vec{a}$

3. All forces occur in equal but directionally opposite pairs.

Second Law: $\vec{F} = m\vec{a} = \dot{\vec{p}}$
Angular Position/Velocity/Acceleration: $\theta = s/r$, $\omega = v/r$, $\alpha = a/r$
Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$
Torque: $\vec{\tau} = \vec{r} \times \vec{F} = \dot{\vec{L}}$
Centripital Acceleration: $a_c = v^2/r$
Centrifugal/Coriolis Forces: $\vec{F}_{cent} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$, $\vec{F}_{cor} = -2m\vec{\omega} \times \dot{\vec{r'}}$

Work to go from positions $\vec{a}$ to $\vec{b}$: $W_{ab} = \int\limits_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{s}$

Conservative Force Field (2 eq): W_{ab} is the same regardless of path so $\oint \vec{F} \cdot d\vec{s} = 0$, and thus we can write the force as $\vec{F} = -\nabla V(\vec{r})$.

Lagrangian Formalism

Functional Derivative: $\frac{\delta F}{\delta u}[x_0] = \lim\limits_{\epsilon \to 0} \frac{F[x_0 + \epsilon u] - F[x_0]}{\epsilon} \to \frac{\delta F}{\delta x(t)}[x(t')] = \lim\limits_{\epsilon \to 0} \frac{F[x(t') + \epsilon \delta(t' - t)] - F[x(t')]}{\epsilon}$
Principle of Least Action: $\delta S = 0$, where $S = \int_{t_i}^{t_f} L(\vec{q}, \dot{\vec{q}}, t) dt$
Lagranges Equation: $\frac{d}{dt} \frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = 0$
Holonomic Constraints: $f_\alpha(x^A, t) = 0$, $L' = L(x^A, \dot{x}^A) + \lambda_\alpha f_\alpha(x^A, t) \to \frac{d}{dt} \frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = \lambda_\alpha \frac{\partial f_\alpha}{\partial x^A}$
Noether's Theorem: A continuous symmetry in the Action (and thus Lagrangian) result in a conserved quantity.
Moment of Inertia Tensor: $\vec{L} = \overleftrightarrow{I} \vec{\omega}$, $T = \frac{1}{2}\omega_a I_{ab} \omega_b$, $I_{ab} = \sum_i m_i((\vec{r}_i \cdot \vec{r}_i)\delta_{ab} - (\vec{r}_i)_a(\vec{r}_i)_b$
Euler's Equations: Only look at rotation, not translation. Conservation of Angular Momentum gives $I_i \dot{\omega}_i + \omega_j \omega_k (I_k - I_j) = 0$, for i,j,k being cyclic permutations of 1,2,3.

Hamiltonian Formalism

Generalized Momenta: $p_i = \frac{\partial L}{\partial \dot{q}_i}$, $\dot{p}_i = \frac{\partial L}{\partial q_i}$
Hamiltonian: $H(q_i, p_i, t) = \sum\limits_{i=1}^{n} p_i \dot{q}_i - L(q_i, \dot{q}_i, t)$
Hamilton's Equations:

1. $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

2. $\dot{q}_i = \frac{\partial H}{\partial p_i}$

3. $-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$

Cyclic/Ignorable Coordinates: q is ignorable if $\frac{\partial L}{\partial q} = 0$, i.e. if q does not appear in L. Thus $p = \frac{\partial L}{\partial \dot{q}}$ is conserved.

Liousille's Theorem: A volume of a region of phase space remains the same, even when the refion changes. $V = dq_1 \ldots dq_n dp_1 \ldots dp_n$.

Poisson Bracket: $\{f, g\} = \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}$.

Constant of Motion from Poisson Bracket: $\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}$. If $I, H = 0$, then I is a constant of motion.

Canonical Transformation: Transformation $(q_i \to Q_i(q, p), p_i \to P_i(q, p))$ that leaves Hamilton's equations invariant.

6 Statistical Mechanics Equations

6.1 Thermodynamics

Laws of Thermodynamics:

1. Energy conservation. $dE = đQ - pdV$. $đQ$ just means that the heat is an inexact differential and the integral depends on the path.

2. $\Delta S \geq \int \frac{đQ}{T}$, where equality is for a process that is reversible (never leaves equilibrium).

3. Entropy at zero temperature is zero. In stat mech this means that the ground state is nondegenerate and $S \propto \ln(\text{W})$, where W is the number of available states.

Intensive vs Extensive Variables: Intensive variables do NOT scale with system size (T, p, μ), while extensive do scale (E, S, V, N).

Thermodynamic Potentials:

- Internal Energy: $U(S, V, N)$

- Helmholtz Free Energy: $F(T, V, N) = U - TS$

- Enthalpy: $H(S, p, N) = U + pV$

- Gibbs Free Energy: $G(T, p, N) = U - TS + pV$

- Landau(Grand) Potential: $\Omega(T, V, \mu) = U - TS - \mu_i N_i$

Thermodynamic Ensembles:

1. Microcanonical: Does not exchange energy or particles with environment. Fixed E, N

2. Canonical: Does not exchange particles, but can exchange energy (heat bath). Fixed N, T

3. Grand canonical: Can exchange energy and particles with environment. Fixed T, μ.

Maxwell's Relations (4 main):

- $\frac{\partial^2 U}{\partial S \partial V} = -\left(\frac{\partial p}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$

- $\frac{\partial^2 F}{\partial T \partial V} = \left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$

- $\frac{\partial^2 H}{\partial S \partial p} = \left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial T}{\partial p}\right)_S$

- $\frac{\partial^2 G}{\partial T \partial p} = \left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$

Engine Efficience: $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{T_{out}}{T_{in}}$

Isobaric Thermal Expansion Coeffieicnt: $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$, How much the volume changes with a change in termperature.

Isothermal Compressibility: $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$, How much the volume changes when the pressure changes.

Isentropic(Adiabatic) Compressibility: $\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$, Same as above.

Specific Heat at Constant V: $C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$, Amount of heat per unit mass to raise the temp by 1 degree.

Specific Heat at Constant p: $C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial H}{\partial T}\right)_p$, Same as above.

Fermi Energy/Temperature: Chemical potential at $T = 0$. $\epsilon_F = \mu(T = 0)$

6.2 Statistical Mechanics

Number of microstates in a mactostate (ways to get n heads): $\Omega = \frac{N!}{\prod_i n_i!}$

Stirling's Approximation: $\ln n! = n \ln n - n$

How many order important ways to order n things: $n!$

How many order important waus to order n things r at a time: $\frac{n!}{(n-r)!}$

How many NOT order important ways to order n things r at a time: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Microcanonical(Classical) Partition Function: $Z_m = \sum_s g_s e^{-\beta E_s}$

Canonical Partition Function: $Z_c = \text{tr}\left(e^{-\beta \hat{H}}\right)$

Grand Canonical Partition Function: $Z_{gc} = \text{tr}\left(e^{-\beta(\hat{H} - \mu \hat{N})}\right)$

Geometric Series: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

Classical limit of the trace of an operator: $\text{tr}(\mathcal{O}) = \frac{1}{N!(2\pi\hbar)^{3N}} \int d^3r_1 \ldots d^3r_N \int d^3p_1 \ldots d^3p_N \mathcal{O}$, $N!$ is for identical particles.

Thermodynamic Limit: $T \to \infty, V \to \infty, N/V = const$

Expectation value for pure/mixed: $\langle \mathcal{O} \rangle_p = \langle \psi | \mathcal{O} | \psi \rangle$, $\langle \mathcal{O} \rangle_m = \sum_i P_i \langle \psi_i | \mathcal{O} | \psi_i \rangle$

Density Matrix (ex. Canonical Ensemble): $\rho = \sum_n P_n |\psi_n\rangle \langle \psi_n|$, $\rho_c = \frac{e^{-\beta \hat{H}}}{\text{tr} e^{-\beta \hat{H}}}$

Expectation value with Density Matrix: $\langle \mathcal{O} \rangle = \text{tr}(\mathcal{O}\rho)$

Trace of Density matrix: $\text{tr}(\rho) = 1$

Time evolution of density matrix: $\frac{\partial}{\partial t} \rho = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right]$

Z_{gc} **for an ideal gas:** $Z_{gc} = \frac{V^N (2mT\pi)^{3N/2}}{N!(2\pi\hbar)^{3N}} e^{\beta\mu}$

Z_{gc} **for ideal fermi gas:** $Z_{gc} = \prod_k \left(1 + e^{-\beta(\epsilon_k - \mu)}\right)$

Z_{gc} **for ideal bose gas:** $Z_{gc} = \prod_k \frac{1}{\left(1 - e^{-\beta(\epsilon_k - \mu)}\right)}$

Stuff here for black-body and phonons and bose condensates.

Explain Bose-Condensates with Bose statistics: $\lim\limits_{T->0} n(p) = \lim\limits_{\beta->\infty} \frac{1}{1 - e^{\beta(\epsilon - \mu)}} \to 0$ unless $\epsilon \to \mu$, which happens at the ground state. Is this true?

What is cluster expansion used for?: Systems of interacting particles.

7 Quantum Mechanics Equations

Properties of a vector space:

- Sum $|V\rangle + |W\rangle$

- Scalar product with properties

    1. closure: results in another vector in the space.
    2. distributive: $a(|V\rangle + |W\rangle = a|V\rangle + a|W\rangle)$, $(a+b)|V\rangle = a|V\rangle + b|V\rangle$
    3. associative: $a(b|V\rangle) = ab|V\rangle$, $|V\rangle + (|W\rangle + |Z\rangle) = (|V\rangle + |W\rangle) + |Z\rangle$
    4. commutative: $|V\rangle + |W\rangle = |W\rangle + |V\rangle$
    5. addative inverse: $|V\rangle + |-V\rangle = |0\rangle$
    6. null vector: $|V\rangle + |)\rangle = |V\rangle$

Hilbert space: Vector space with defined inner product.

Expand in orthonormal basis: $|V\rangle = \sum\limits_i vi |i\rangle$

Hermitian operator: $\mathcal{O}^\dagger = \mathcal{O}$

Anti-Hermitian operator: $\mathcal{O}^\dagger = \mathcal{O}$

Unitary operator: $UU^\dagger = \mathbb{1}$

Orthogonality: $\langle i|j\rangle = \delta_{ij}$

Completeness: $\sum\limits_i i = \mathbb{1}$

Postulates of QM:

1. The state of a physical system, at some fixed time, is given by a normalized ray in a Hilbert space over the complex numbers. (ray is vector whose norm doesn't matter)

2. The ray evolves deterministically in time according to Schrödingers equation.

3. Observables correspond to self-adjoint (hermitian) operators.

4. If a particle is in the state $|\psi\rangle$ then a measurement of $\mathcal{O}$ will yield one of the eigenvalues of $\mathcal{O}$, ω. The state of the system changes to an eigenstate of $\mathcal{O}$, $|\omega\rangle$.

Schrödinger equation: $i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$

Free particle ψ_p and E_p: $\psi_p = Ae^{ikx} + Be^{-ikx}$, $k^2 = \frac{2mE_n}{\hbar^2}$, $E_p = \frac{p^2}{2m}$

Particle in a box ψ_n and E_n: $\psi_n = \sqrt{\frac{2}{L}}\sin(k_nx)$, $k_n = \frac{n\pi}{L}$, $E_n = \frac{\hbar^2\pi^2n^2}{2mL^2}$

Harmonic Oscillator $\hat{H}$, ψ_n and E_n: $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, $\psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{2^nn!}H_n(x)e^{-x^2/2}$, $E_n = (n + \frac{1}{2})\hbar\omega$

Raising and lowering operators and how to affect $|n\rangle$ (3-2):

- $a = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$, $a\,|n\rangle = \sqrt{n}\,|n-1\rangle$, $a\,|0\rangle = 0$

- $a^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$, $a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$\hat{H}$ **in terms of a and $a^\dagger$:** $\hat{H} = \hbar\omega(a^\dagger a + 1/2)$

Commutation relations for $\hat{H}$, a, $a^\dagger$:

- $[\hat{H}, a] = -a$

- $[\hat{H}, a^\dagger] = a^\dagger$

- $[a, a^\dagger] = 1$

$\mathbf{J}^2$ and J_z on the angular momentum state $|jm_j\rangle$:

- $\mathbf{J}^2\,|=\rangle\, j(j+1)\hbar^2\,|jm_j\rangle$

- $J_z\,|jm_j\rangle = m_j\hbar\,|jm_j\rangle$

Commutation relations for J_i and J_j and for $\mathbf{J}^2$ and J_i:

- $[J_i, J_j] = i\hbar J_k$

- $[\mathbf{J}^2, J_i] = 0$

J_z and $\mathbf{J}^2$ in position basis:

- $J_z = -i\hbar\frac{\partial}{\partial t}$

- $\mathbf{J}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$

Raising and Lowering Angular Momentum Operators on $|j, m\rangle$:
$J_\pm\,|j, m\rangle = \hbar[j(j+1) - m(m\pm1)]^1/2\,|j, m\pm1\rangle$

J_x and J_y in terms of J_+ and J_-: $J_x = \frac{1}{2}(J_+ + J_-)$, $J_y = \frac{1}{2i}(J_+ - J_i)$

Momentum eigenstate, $\langle x|p\rangle$: $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}$

Hydrogen Atom $V(r)$, ψ_n, E_n(x4): $V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$, $\psi_n = stuff*L_{n-l-1}^{2l+1}(\rho)Y_l^m(\theta, \phi)$(Laguerre)

$E_n = -\frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\epsilon_0\hbar}\right)^2 m_e = -\frac{1}{2n^2}\alpha^2 m_ec^2 = -\frac{1}{n^2}13.6eV = -\frac{1}{2n^2}\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{a_0}\right)$,

$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$, $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_ee^2}$

Pauli matricies and commutation relations:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_a \sigma_b] = 2i\epsilon_{abc}\sigma_c$$

Non-Deg Time-Ind Perturbation, $E_n^{(1)}$, $\left|n^{(1)}\right\rangle$, $E_n^{(2)}$:

$$E_n^{(1)} = H'_{nn} = \left\langle n^{(0)}\right| H' \left|n^{(0)}\right\rangle$$

$$\left|n^{(1)}\right\rangle = \sum_{m\neq n} \frac{\left\langle n^{(0)}\left|H'\right|n^{(0)}\right\rangle}{(E_n^{(0)} - E_m^{(0)})} \left|m^{(0)}\right\rangle$$

$$E_n^{(2)} = \sum_{m\neq n} \frac{\left|\left\langle n^{(0)}\left|H'\right|n^{(0)}\right\rangle\right|^2}{(E_n^{(0)} - E_m^{(0)})}$$

Deg Time-Ind Perturbation, $E_n^{(1)}$: Diagonalize the perturbation hamiltonian in the degenerate subspace.

Time-Dep Perturbation, $P_{i\to f}(t)$: $P_{i\to f}(t) = \frac{1}{\hbar^2}\left|\int_0^t dt' \left\langle f\right| H'(t') \left|i\right\rangle e^{i(E_f - E_i)t'/\hbar}\right|^2$

Fermi's golder rule, and $g(E_f)$ as δ: $R_{i\to f} = \frac{2\pi}{\hbar}\left|\left\langle f\right| H' \left|i\right\rangle\right|^2 g(E_f)$, $g(E_f) \approx \delta(E_f^{(0)} - E_f^{(0)} - \hbar\omega)$

Einstein's Stimulated/Spontaneous emission coeffients:

Stimulated: $B_{if} = \frac{\pi e^2}{3\epsilon_0 \hbar^2}\left|\left\langle f\right| \mathbf{r} \left|i\right\rangle\right|^2$

Spontaneous: $A_{if} = \frac{e^2 \omega_{21}^3}{3\pi\epsilon_0 \hbar c^3}\left|\left\langle f\right| \mathbf{r} \left|i\right\rangle\right|^2$

Total $\psi(\mathbf{r})$ in scattering problem:

$$\psi(\mathbf{r}) = \psi_{inc}(\mathbf{r}) + \psi_s(\mathbf{r}) = \psi_{inc}(\mathbf{r}) + f(\theta,\phi)\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r}$$

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int d^3r' e^{-i\mathbf{k'}\cdot\mathbf{r'}}V(\mathbf{r'})\psi(\mathbf{r'})$$

Differential Cross Section: $\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$

Born Approximation: In the above integral for $f(\theta,\phi)$ let $\psi \to \psi_{inc}$.

Dirac Equation:

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

$$\gamma^0 = \beta, \; \gamma^i = \beta\alpha_i$$

$$\beta = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}, \; \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

8 Electricity and Magnetism Equations

Maxwell's Equations in Vacuum (SI):

$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$, $\nabla \cdot \mathbf{B} = 0$

$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t}$

Maxwell's Equations in Matter (SI), and D and H:

$\nabla \cdot \mathbf{D} = \rho$, $\nabla \cdot \mathbf{B} = 0$

$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t}$

$\mathbf{D} = \epsilon_0\mathbf{E}$, $\mathbf{B} = \mu_0\mathbf{H}$

Continuity Equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$

Lorentz Force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Coulomb's Law (x2): $\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$, $\mathbf{F} = Q\mathbf{E}$

Gauss' Law: $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$

9 Miscellaneous Physics

Taylor Expansion: $f(\vec{x} + \vec{a}) = f(\vec{x}) + a_i \partial_i f(\vec{x}) + \mathcal{O}(\vec{a}^2)$

Gaussian Integral: $\int\limits_{-\infty}^{\infty} dx e^{-ax^2 + bx + c} = \sqrt{\frac{\pi}{a}} e^{b^2/(4a) + c}$

Value of fine structure constant: $\alpha \approx \frac{1}{137}$

Mass of electron in eV: $m_e c^2 = 0.511 eV$

Value of the Bohr radius: $a_0 = 0.529 \mathring{A}$