Modern Physics and Quantum Mechanics
Spring 2001 Proficiency

Useful constants:

- $e = 1.60 \times 10^{-19}$ C
- $hc = 1240$ eV $\cdot$ nm
- $c = 3.00 \times 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}$
- $m_e = 0.511 \ \frac{\mathrm{MeV}}{\mathrm{c}^2}$
- $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$
- $\epsilon_0 = 8.85 \times 10^{-12} \ \frac{\mathrm{C}^2}{\mathrm{Nm}^2}$

1. Consider a one-dimensional step potential of the form

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x \geq 0 \end{cases} ,$$

where $V_0 > 0$. A particle with total energy $E > V_0$ and mass m is incident on the step potential "from the left" (in other words: the particle starts at negative values of x and travels towards positive values of x).

   (a) Use the time-independent Schrödinger equation to determine the form of the particle's wave function in the two regions $x < 0$ and $x \geq 0$.

   (b) Derive expressions for the probabilities that the particle is
   (i) reflected (R), and
   (ii) transmitted (T).
   Hint: Recall that the probability density current is given by

$$j(x) = \mathrm{Re}\left(\Psi^* \frac{\hbar}{im} \frac{\partial \Psi}{\partial x}\right) ,$$

   and that R and T are ratios of probability density currents.

2. A monochromatic particle beam consists of particles whose total energy is 100 times their rest mass. The rest lifetime of the particles is 0.10 ns. In the laboratory, the distance between the point where the particles are generated and the detector is 6.0 m. What fraction of the generated particles reach the detector?

3. In a hydrogen atom the electron spin $\vec{S}_e$ and the proton spin $\vec{S}_p$ interact with each other via the hyperfine interaction and with an external magnetic field through the Zeeman effect. The magnetic field is applied along the z-direction, $\vec{B} = B\hat{z}$. The Hamiltonian of the system is given by

$$H = J\vec{S}_e \cdot \vec{S}_p + (\gamma_e \vec{S}_e + \gamma_p \vec{S}_p) \cdot \vec{B} ,$$

where J is the hyperfine coupling, and γ_e and γ_p are the gyromagnetic ratios of the electron and proton, respectively.

(a) State the energy eigenvalues and eigenstates for this Hamiltonian in the zero-field limit, $B = 0$.

(b) State the energy eigenvalues and eigenstates for the $J = 0$ case (no hyperfine interaction).

(c) Obtain the energy eigenvalues for the general case ($B \neq 0$ and $J \neq 0$).

4. Light of wavelength 300 nm strikes a metal plate, producing photoelectrons that move with a speed of 0.002 c.

   (a) What is the work function of the metal?

   (b) What is the critical wavelength for this metal, so that photoelectrons are produced?

   (c) What is the significance of the critical wavelength?