Notes on Siemens Ch. 6

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Interactions Beyond the Mean Field

- The mean field approximation gives us basic features of nuclei. But now we're going to move beyond the mean field approximation.
- The first thing we are going to do is look at the pairing term to the binding energy.
$$B_p = \frac{\left[(-1)^N + (-1)^Z\right]\delta}{A^{1/2}} \tag{1}$$
- This gives even-even nuclei a tighter binding energy. Also it turns out that the ground state of even-even nuclei have zero angular momentum.
- To explain these things we are going to go beyond the independent-particle motion (mean field).

Interactions Beyond the Mean Field

- Add a perturbation to the mean field Hamiltonian (H_R is called the residual interaction)

$$H = H_{MF} + H_R \tag{2}$$

  - The eigenstates of H_{MF} are Slater determinants (Correlated SD's?).
- One solution to this is to diagonalize H_R in the H_{MF} basis, but this requires large calculations.
- We are going to use other methods in this chapter. We will split (crudely) into long-range and short-range parts, and look at short-range parts here.

The δ-Force

- Look at degenerate states of H_{MF} because H_R with have a decisive influence.
- Start with H_{MF} in a full j state and two identical nucleons in the next j state.

$$\psi_{JM}^{nlj}(1,2) = \sum_{m_1 m_2} \langle jm_1 jm_2 |\, JM \rangle \, \mathcal{A}\left[\Phi_{nljm_1}(1)\Phi_{nljm_2}(2)\right] \tag{3}$$

$$\Phi_{nljm_1} = \frac{1}{2} u_n lj(r) \sum_{m,s} \left\langle lm \frac{1}{2} s \middle| jm_1 \right\rangle Y_l^m(\theta,\phi)\chi_s \tag{4}$$

- The shortest range for H_R is a δ-force.

$$H_R = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \tag{5}$$

The δ-Force

- This Hamiltonian gives an energy

$$E_R = V_0 \int \psi^*_{JM} \delta(\mathbf{r}_1 - \mathbf{r}_2) \psi_{JM} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \tag{6}$$

$$= \frac{V_0 \left[1 + (-1)^J\right] (2j+1)^2}{32\pi(2J+1)} \left| \left\langle j, \frac{1}{2}, j, -\frac{1}{2} \right| J, 0 \right\rangle \right|^2 \int\limits_0^\infty r^{-2} u^4_{nlj(r)dr} \tag{7}$$

- Note here that E_R vanishes for odd values of J. This means that two identical Fermi particles in the same j-shell can only be in even angular-momentum states.
- For an attractive force (V_0) the lowest energy has $J = 0$ and the first excited state is $J = 2$.

The δ-Force

- For all $j > \frac{3}{2}$ the difference in energies of these two states is

$$|(E_2 - E_0)/E_0| \approx \frac{3}{4} \tag{8}$$

  which is large as seen in figure 6.2 of the book.
- The two nucleons have their largest spatial overlap in this state.
- Thus an attractive δ-interaction decreases the energy.

The Degenerate Pairing Model

- A main feature of the δ-force that is maintained in the pairing force is that it only has non-zero matrix elements between time-reversed states. Also, they non-zero elements are all identital

$$\left\langle jm_1\overline{jm_1} \middle| jm_2\overline{jm_2} \right\rangle \equiv -G \tag{9}$$

- Let's use the basis states $j + \frac{1}{2} \equiv \Omega$. Now the Schrödinger equation becomes

$$-G \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & & & 1 \\ \vdots & \ddots & \vdots & \\ 1 & & \cdots & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_\Omega \end{pmatrix} = E \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_\Omega \end{pmatrix} \tag{10}$$

$$-G(x_1 + \cdots + x_\Omega) = Ex_1 = Ex_2 = \cdots = Ex_\Omega \tag{11}$$

The Degererate Pairing Model

$$-G(x_1 + \cdots + x_\Omega) = Ex_1 = Ex_2 = \cdots = Ex_\Omega \tag{12}$$

- This has solutions

$$E = -G\Omega, \qquad \vec{x} = \frac{1}{\sqrt{\Omega}}(1, 1, \cdots, 1) \tag{13}$$

and

$$E = 0, \qquad x_1 + x_2 + \cdots + x_\Omega = 0. \tag{14}$$