PHY531 Problem Set 1. Due January 27, 2015

Look on both sides!

1. a. Starting with retarded Coulomb's law for the Lorentz-gauge vector potential

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{1}{4\pi c} \int d^3 r' \frac{\boldsymbol{J}(\boldsymbol{r}', t - c^{-1}|\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|} \tag{1}$$

derive the Biot-Savart law for a static (i.e. time independent) current density

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{1}{4\pi c} \int d^3 r' \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \tag{2}$$

by calculating $B = \boldsymbol{\nabla} \times \boldsymbol{A}$. Compare your result with Jackson's Eq. 5.14.

   b. In the thin wire approximation, we assume that a wire's transverse dimensions (e.g. the diameter for a wire of circular cross section) are much smaller than any other length scale in the problem. For static currents, the integrated current over the wire cross section will be constant value I along the wire. Explain why, in this approximation, volume integrals over the current density can be replaced by a line integral over the path of the wire's axis multiplied by I.

   Use this result to show that the thin wire Biot-Savart result is

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{I}{4\pi c} \int_{\text{path of wire}} d\boldsymbol{r}' \times \frac{(\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \tag{3}$$

   Compare your result with Jackson's Eq. 5.4.

   c. A current I flows in a thin wire of length ℓ bent into the shape of an ellipse with area A. Show that the $\boldsymbol{B}$ field at the center of the ellipse has magnitude

$$|\boldsymbol{B}| = \frac{I\ell}{4Ac} \tag{4}$$

   Hint: Feel free to use any parameterization of a general ellipse, and then calculate its length, area, and $|\boldsymbol{B}|$. I found it was simplest for me to take the ellipse in the x-y plane with semiaxes a and b such that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. I performed the wire length calculation using the parameteriation $\boldsymbol{r}(\alpha) = a\cos\alpha\hat{\boldsymbol{x}} + b\sin\alpha\hat{\boldsymbol{y}}$. I performed the area and Biot-Savart calculations using the parameterization $\boldsymbol{r}(\phi) = r(\phi)\left[\cos\phi\hat{\boldsymbol{x}} + \sin\phi\hat{\boldsymbol{y}}\right]$, with $r(\phi)$ calculated to correctly give the ellipse.

2. Jackson problem 5.1

   Hint: First make sure you know what the solid angle calculation will look like so you know what to work toward.

3. Jackson problem 5.2

4. Jackson problem 5.3

5. Jackson problem 5.4

   Hint: Either Taylor series expand $\boldsymbol{B}$ *giving a power series in* ρ*, or equivalently show that* $\boldsymbol{B}$ *can be written as the negative gradient of a scalar potential which satisfies Laplace's equation, and expand it in a power series in* ρ*. Enforcing either Maxwell's equations or Laplace's equation will give you the desired result.*

6. Jackson problem 5.5