

Notes on Siemens Ch. 6

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Interactions Beyond the Mean Field

- The mean field approximation gives us basic features of nuclei. But now we're going to move beyond the mean field approximation.
- The first thing we are going to do is look at the pairing term to the binding energy (equation 4.3.2).

$$B_p = \frac{[(-1)^N + (-1)^Z] \delta}{A^{1/2}} \quad (1)$$

- This gives even-even nuclei a tighter binding energy. Also it turns out that the ground state of even-even nuclei have zero angular momentum.
- To explain these things we are going to go beyond the independent-particle motion (mean field).

Interactions Beyond the Mean Field

- Add a perturbation to the mean field Hamiltonian (H_R is called the residual interaction)

$$H = H_{MF} + H_R \quad (2)$$

- The eigenstates of H_{MF} are Slater determinants (Uncorrelated SD's?).
- One solution to this is to diagonalize H_R in the H_{MF} basis, but this requires large calculations.
- We are going to use other methods in this chapter. We will split (crudely) into long-range and short-range parts, and look at short-range parts here.

The δ -Force

- Look at degenerate states of H_{MF} because H_R will have a decisive influence.
- Start with H_{MF} in a full j state and two identical nucleons in the next j state.

$$\psi_{JM}^{nlj}(1, 2) = \sum_{m_1 m_2} \langle jm_1 jm_2 | JM \rangle \mathcal{A} [\Phi_{nljm_1}(1) \Phi_{nljm_2}(2)] \quad (3)$$

$$\Phi_{nljm_1} = \frac{1}{2} u_{nlj}(r) \sum_{m,s} \left\langle lm \frac{1}{2} s \middle| jm_1 \right\rangle Y_l^m(\theta, \phi) \chi_s \quad (4)$$

- The shortest range for H_R is a δ -force.

$$H_R = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (5)$$

The δ -Force

- This Hamiltonian gives an energy

$$E_R = V_0 \int \psi_{JM}^* \delta(\mathbf{r}_1 - \mathbf{r}_2) \psi_{JM} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \quad (6)$$

$$= \frac{V_0 [1 + (-1)^J] (2j + 1)^2}{32\pi(2J + 1)} \left| \left\langle j, \frac{1}{2}, j, -\frac{1}{2} \middle| J, 0 \right\rangle \right|^2 \int_0^\infty r^{-2} u_{nlj}^4(r) dr \quad (7)$$

- Note here that E_R vanishes for odd values of J . This means that two identical Fermi particles in the same j -shell can only be in even angular-momentum states.
- For an attractive force ($V_0 < 0$) the lowest energy has $J = 0$ and the first excited state is $J = 2$.

The δ -Force

- For all $j > \frac{3}{2}$ the difference in energies of these two states is

$$|(E_2 - E_0)/E_0| \approx \frac{3}{4} \quad (8)$$

which is large as seen in figure 6.2 of the book.

- The two nucleons have their largest spatial overlap in this state ($J = 0$).
- Thus an attractive δ -interaction decreases the energy.

The Degenerate Pairing Model

- A main feature of the δ -force that is maintained in the pairing force is that it only has non-zero matrix elements between time-reversed states. Also, they non-zero elements are all identical

$$\langle jm_1 \overline{jm_1} | V | jm_2 \overline{jm_2} \rangle \equiv -G \quad (9)$$

- Let's use the basis states $j + \frac{1}{2} \equiv \Omega$. Now the Schrödinger equation becomes

$$-G \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & & & 1 \\ \vdots & \ddots & \ddots & \\ 1 & & \cdots & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_\Omega \end{pmatrix} = E \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_\Omega \end{pmatrix} \quad (10)$$

$$-G(x_1 + \cdots + x_\Omega) = Ex_1 = Ex_2 = \cdots = Ex_\Omega \quad (11)$$

The Degenerate Pairing Model

$$-G(x_1 + \cdots + x_\Omega) = Ex_1 = Ex_2 = \cdots = Ex_\Omega \quad (12)$$

- This has solutions

$$E = -G\Omega, \quad \vec{x} = \frac{1}{\sqrt{\Omega}}(1, 1, \cdots, 1) \quad (13)$$

and

$$E = 0, \quad x_1 + x_2 + \cdots + x_\Omega = 0. \quad (14)$$

- Equation 13 refers to the $J = 0$ state and the degenerate $J > 0$ states have energy of equation 14.

The Degenerate Pairing Model

- Now assume we have n particles in the j -shell ($n \leq 2\Omega$), and p pairs of particles, i.e. $J = 0$ states.
- Using equation 13 and the fact that Ω is the number of possible pairs we get

$$E(n, p) = -Gp(\Omega - n + p + 1) \quad (15)$$

- Introduce *seniority*, $S = n - 2p$, the number of unpaired nucleons.

$$E(n, S) = -\frac{G}{4}(n - S)(2\Omega - n - S + 2) \quad (16)$$

$$E(n, 0) = -\frac{1}{4}Gn(2\Omega - n + 2) \quad (17)$$

$$E(2, 0) = -G\Omega \quad (18)$$

$$E(2\Omega, 0) = -Gn/2 \quad (19)$$

The Degenerate Pairing Model

- The pairing force creates as many pairs of particles as possible.
- Even-even have zero spin.
- Odd numbers of nuclei, spin is determined by unpaired nucleon.

$$E(2p + 1, p) - E(2p, p) = Gp \quad (20)$$

Which seems to agree with experiment

- For even-even nuclei, the lowest excited state is that of one broken pair, which has symmetric energy

$$E(2p, p - 1) - E(2p, p) = G\Omega \quad (21)$$