PHY6938 Mechanics Fall 2000 Diagnostic
April 2, 2002

1. Part 1:
Consider a reversible isothermal expansion of an ideal gas (the system) in contact with a heat reservoir at a temperature T, from an initial volume V_1 to a final volume V_2.

(a) What is the change of the internal energy of the system?

The internal energy of an ideal gas $U = C_V T = N c_V T$, where c_V is the specific heat per particle and C_V is the total specific heat, depends only on the number of gas molecules and the temperature of the system, so in an isothermal expansion where the temperature (and amount of gas) remains the same, the internal energy does not change, i.e. $\Delta U = 0$.

(b) Obtain the amount of work performed by the system.

The work done is

$$W = \int_{V_1}^{V_2} P dV,$$

and since for an ideal gas at a fixed temperature T we have $P = nRT/V$ we can write

$$W = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln\left(\frac{V_2}{V_1}\right).$$

(c) What is the amount of heat absorbed by the system?

Using the first law of thermodynamics, $\Delta Q = \Delta U + W$, and since $\Delta U = 0$ we have

$$\Delta Q = W = nRT \ln\left(\frac{V_2}{V_1}\right).$$

(d) What is the change of entropy of the system?

The change in entropy is

$$\Delta S = \int dS = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{\Delta Q}{T},$$

since the temperature T does not change, and so

$$\Delta S = nR \ln\left(\frac{V_2}{V_1}\right).$$

(e) What is the change of the entropy of the system plus the reservoir?

Since the temperature of the heat reservoir does not change by assumption, then since it loses heat ΔQ to the gas we know that its entropy change is just

$$\Delta S_{res} = -\Delta S = -nR \ln\left(\frac{V_2}{V_1}\right).$$

Note that this means that the total entropy change is zero, which is the definition of a reversible process.

Part 2: Next consider a free expansion of the above ideal gas from an initial volume V_1 to a final volume V_2.

(a) What is the change of temperature of the system?

In a free expansion, such as a vessel of volume V_1 separated from one of volume V_2 by a membrane which ruptures, the ideal (and so non-interacting) gas molecules have the same kinetic energy before and after the membrane ruptures, so that the temperature remains the same.

(b) Does the internal energy change?

No, since it depends only on the temperature and that has not changed. Alternatively, if you think of the motion of each of the molecules before and after the membrane ruptures, the kinetic energy of the molecules has not changed and so neither has U.

(c) Obtain the amount of work performed by the system.

When a membrane ruptures, even though V has changed, there is no work done by the system on its surroundings.

(d) Obtain the absorbed amount of heat.

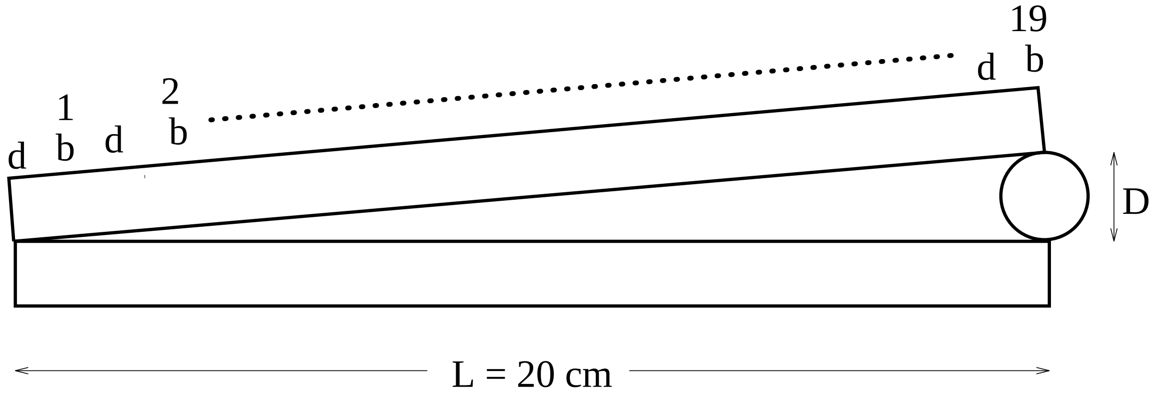
By the first law of thermodynamics, if there is no work done and no change in the internal energy there can be no heat absorbed.

(e) Obtain the change of entropy of the system and that of the reservoir.

If there is no heat absorbed from it, then there can be no change in the entropy of the reservoir. The gas, however, has now more available microstates (it has the same total energy but the energy levels are now closer together) and so has more entropy. Another way of saying this is that the entropy of the gas is a function of the state of the gas, and if it has the same state as after the isothermal expansion in Part 1 (d), then it has the same entropy, and so once again

$$\Delta S = nR \ln \left(\frac{V_2}{V_1} \right) .$$

Then the total entropy of the system has increased, as expected for an irreversible process.



2. The diameters of fine wires can be accurately measured using interference patterns. Two optically flat pieces of glass of length L are arranged with a wire between them as shown in the figure. The setup is illuminated with normally incident yellow light of wavelength $\lambda = 590$ nm. The length is $L = 20$ cm, and 19 bright fringes are seen along this length.

(a) Find the diameter of the wire.

Where the two plates touch, the light which reflects from the bottom of the top piece of glass and the light which reflects from the top of the bottom piece of glass travel have the same path length. However, the light from the second reflection has its phase inverted by π since it goes from a region of low density to one of high density, while the opposite is true for the second reflection so it is in phase. That means that these two reflected rays negatively interfere and there is a dark fringe, as shown. As we move along the top plate, the path length difference must increase by $\lambda/2$ (for a phase difference of π) by the time we get to the first bright fringe, and then to λ (for a phase difference of 2π) by the time we get to the second dark fringe, and $3\lambda/2$ for the second bright fringe, and so on. By the time we get to the nineteenth bright fringe, we must have a path length difference of $(18 + 1/2)\lambda = 37\lambda/2$ when we get to the 19^{th} bright fringe. Since the path difference is twice the separation of the plates and so twice the diameter of the wire (with the excellent approximation that the angular separation of the plates is very small so we have almost normal incidence), then

$$D = 37\lambda/4 = 5.47 \times 10^{-6} \text{ m}.$$

(b) If the 19^{th} fringe is not quite at the end, but there is no 20th fringe, give an estimate of the possible relative error in the measurement of the diameter.

The dark fringe spans a path length difference of $\lambda/2$, so we can make a relative error of a part in 37, or 2.7%.

3. The operation of a gasoline engine is approximately represented by the Otto cycle. This cycle consists of four different processes:

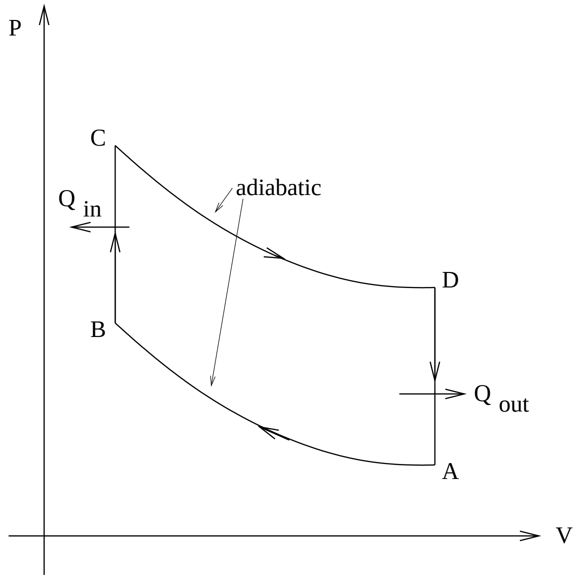
$A \to B$ **The gas is compressed adiabatically from volume V_A to volume V_B (compression stroke).**

$B \to C$ **The gas is heated at constant volume V_B (combustion of the gasoline mixture).**

$C \to D$ **The gas expands adiabatically from volume V_B to volume V_A (power stroke).**

$D \to A$ The gas is cooled at constant volume V_A (exhaust stroke).

(a) Compute the thermodynamic efficiency of the Otto cycle as a function of the compression ratio V_A/V_B and the heat capacity per particle at constant volume, c_V. Assume that the gas is ideal with temperature-independent heat capacities.



During the constant-volume heating and cooling, by the first law of thermodynamics (and since no work can be done by or on the gas) we have that

$$\Delta Q = \Delta U = \int_1^2 dU = C_V \int_{T_1}^{T_2} dT = C_V(T_2 - T_1),$$

so that we have

$$Q_{\text{out}} = C_V(T_D - T_A), \quad Q_{\text{in}} = C_V(T_C - T_B),$$

where these are the (positive) heats that are given up and put into the system, respectively. The thermodynamic efficiency of a cycle is defined to be the work done by the gas divided by the heat put in,

$$\epsilon = \frac{W}{Q_{\text{in}}},$$

and since the gas does not change its state over an entire cycle we know that its internal energy does not change. The first law of thermodynamics then tells us that the net heat added to the system is the work done by the system, so that $W = Q_{\text{in}} - Q_{\text{out}}$ and the efficiency is

$$\epsilon = \frac{W}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = \left(1 - \frac{T_D - T_A}{T_C - T_B}\right).$$

In an adiabatic expansion or compression we have the result (you should know how to prove this!) that

$$TV^{\gamma-1} = \text{constant},$$

where $\gamma = C_P/C_V$ is the ratio of the specific heat at constant pressure to that at constant volume. This will allow us to relate the (unknown) temperatures to the (known) volumes and so compute the efficiency.

Note that

$$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1},$$

so

$$T_C = T_D \left(\frac{V_D}{V_C}\right)^{\gamma-1}, \quad T_B = T_A \left(\frac{V_A}{V_B}\right)^{\gamma-1},$$

From the diagram we see that $V_D/V_C = V_A/V_B$ which is the compression ratio r. Then we have that

$$\epsilon = \left(1 - \frac{T_D - T_A}{T_C - T_B}\right) = \left(1 - \frac{T_D - T_A}{(T_D - T_A) r^{\gamma-1}}\right) = \left(1 - r^{1-\gamma}\right).$$

Now for an ideal gas we have that C_P and C_V differ by $nR = Nk$ (this can be derived from the first law of thermodynamics), so

$$\gamma - 1 = \frac{nR}{C_V} = \frac{Nk}{Nc_V} = \frac{k}{c_V},$$

since $R = N_A k$ and $nN_A = N$, and so, finally,

$$\epsilon = 1 - r^{-k/c_V}.$$

(b) Do higher or lower compression ratios give higher efficiency?

Obviously, the higher the compression ratio r the higher the efficiency.

4. A loop of wire is dipped into soapy water and forms a soap film when removed. The loop is held so the film is vertical. The index of refraction of the film is 1.4 and the light incident on the film has a wavelength of 560 nm in vacuum.

a. What is the wavelength of the light inside the film?

Recall that the the speed of light in a medium with refractive index n is c/n. Then $\nu\lambda = c/n$ and since ν can't change (the time taken for one oscillation cannot be altered by a medium or waves would pile up) then $\lambda = \lambda_{\text{vacuum}}/n = 400$ nm.

b. If the thickness of the film is $t = 10^{-6}$m, find the number of wavelengths contained in a thickness $2t$.

$200 \times 10^{-6}/(400 \times 10^{-9}) = 5.$

c. What happens to the phase of a light wave when it is reflected from
i) the front of the film, and
ii) the back of the film?

In going from a less dense to a more dense medium (the front of the film), the reflected light has its phase inverted (a phase difference of π) relative to the incident light. In going from a more dense medium to a less dense medium (the rear of the film) the reflected light is in phase with the incident light.

d. As time progresses, the film becomes thinner due to evaporation. The film then appears black in reflected light. Briefly explain why this occurs.

The phase difference between the light reflected from the rear of the film and the front of the film is

$$\delta = 2\pi \left(\frac{2t}{\lambda}\right) + \pi,$$

and as $t \to 0$ the two reflections destructively interfere.