Study guide for qualifying exams

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1 Classical Mechanics

1. Newtonian Mechanics
    (a) Newton's Laws/Kinematics
    (b) Energy
    (c) Momentum/Angular Momentum
2. Lagrangian Mechanics
    (a) Calculous of Variations
    (b) Principle of Least Action/Lagranges Equation
    (c) Generalized Coordinates
    (d) Holonomic/Non-Holonomic Constraints
    (e) Noether's Theorem
    (f) Rigid Body Motion
        i. Inertia Tensor
        ii. Euler's Equations
3. Hamiltonian Formalism
    (a) Legendre Transformation/Hamilton's Equations
    (b) Generalized Momenta/Cyclic Coordinates/Conserved Quantities
    (c) Liouville's Theorem
    (d) Poisson Brackets
    (e) Canonical Transformations

2 Statistical Mechanics

1. Thermodynamics Review
    (a) Laws of Thermodynamics
    (b) Intensive vs Extensive Variables
    (c) Thermodynamic Potentials and Ensembles
    (d) Maxwell's Relations
    (e) Various Definitions
        i. Compressibility
        ii. Heat Capacity etc.
2. Statistical Mechanics
    (a) Statistical Review
    (b) Partition Function/Trace
    (c) Thermodynamic Limit
    (d) Density Matrix
    (e) Ideal Gas
    (f) Ideal Bose Gas
    (g) Ideal Fermi Gas
    (h) Cluster Expansion

3 Quantum Mechanics

1. Shankar Math Review
2. Postulates
3. Free Particle
4. Particle in a Box
5. Harmonic Oscillator
6. Angular Momentum
7. Hydrogen Atom
8. Spin
9. Angular Momentum Addition
10. Time-Independent Perturbation Theory

11. Time-Dependent Perturbation Theory
    (a) Einstein A and B Coefficients
12. Scattering
13. WKB Formula
14. Dirac Equation

4 Electricity and Magnetism

1. Electrostatics
    (a) Coulomb's Law
    (b) Electrostatic Potentials
        i. Poisson/Laplace's Equations
    (c) Boundary Conditions
    (d) Method of Images
    (e) Multipole Expansion
    (f) Work and Energy
    (g) Electric Fields in Matter
2. Magnetostatics
    (a) Lorentz Force Law
    (b) Biot-Savart Law
    (c) Vector Potential
    (d) Magnetic Fields in Matter
3. Electrodynamics
    (a) Ohm's Law
    (b) Maxwell's Equations
    (c) Boundary Conditions to Maxwell's Equations
    (d) Continuity Equation
    (e) Poynting's Theorem
    (f) Maxwell Stress Tensor
    (g) Electromagnetic Waves
        i. The Wave Equation from Maxwell's Eq.
        ii. EM Waves in Matter

iii. Wave Guides

4. Scalar and Vector Potentials

5. Coulomb and Lorentz Gauge

6. Retarted Potentials

7. Lienard-Wiechert Potentials

8. Radiation

   (a) Electric/Magnetic Dipole Radiation

9. Helmholtz Theorem

10. Special Relativity

   (a) Einstein's Postulates
   (b) Lorentz Transformation
   (c) 4-Vectors
   (d) Field Tensor and Transformation
   (e) Relativistic Potentials

5 Classical Mechanics Equations

Newtonian Mechanics

Newton's Laws:

1. An object will maintain it's current motion unless acted upon by an external force.

2. $\vec{F} = m\vec{a}$

3. All forces occur in equal but directionally opposite pairs.

Second Law: $\vec{F} = m\vec{a} = \dot{\vec{p}}$
Angular Position/Velocity/Acceleration: $\theta = s/r$, $\omega = v/r$, $\alpha = a/r$
Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$
Torque: $\vec{\tau} = \vec{r} \times \vec{F} = \dot{\vec{L}}$
Centripital Acceleration: $a_c = v^2/r$
Centrifugal/Coriolis Forces: $\vec{F}_{cent} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$, $\vec{F}_{cor} = -2m\vec{\omega} \times \dot{\vec{r'}}$
Work to go from positions $\vec{a}$ to $\vec{b}$: $W_{ab} = \int\limits_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{s}$ **Conservative Force Field (2 eq):** W_{ab} is the same regardless of path so $\oint \vec{F} \cdot d\vec{s} = 0$, and thus we can write the force as $\vec{F} = -\nabla V(\vec{r})$.

Lagrangian Formalism

Functional Derivative: $\frac{\delta F}{\delta u}[x_0] = \lim_{\epsilon \to 0} \frac{F[x_0 + \epsilon u] - F[x_0]}{\epsilon} \to \frac{\delta F}{\delta x(t)}[x(t')] = \lim_{\epsilon \to 0} \frac{F[x(t') + \epsilon \delta(t'-t)] - F[x(t')]}{\epsilon}$
Principle of Least Action: $\delta S = 0$, where $S = \int_{t_i}^{t_f} L(\vec{q}, \dot{\vec{q}}, t) dt$
Lagranges Equation: $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = 0$
Holonomic Constraints: $f_\alpha(x^A, t) = 0,\ L' = L(x^A, \dot{x}^A) + \lambda_\alpha f_\alpha(x^A, t) \to \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = \lambda_\alpha \frac{\partial f_\alpha}{\partial x^A}$
Noether's Theorem: A continuous symmetry in the Action (and thus Lagrangian) result in a conserved quantity.
Moment of Inertia Tensor: $\vec{L} = \overleftrightarrow{I}\vec{\omega}$, $T = \frac{1}{2}\omega_a I_{ab} \omega_b$, $I_{ab} = \sum_i m_i((\vec{r}_i \cdot \vec{r}_i)\delta_{ab} - (\vec{r}_i)_a (\vec{r}_i)_b$
Euler's Equations: Only look at rotation, not translation. Conservation of Angular Momentum gives $I_i \dot{\omega}_i + \omega_j \omega_k (I_k - I_j) = 0$, for i,j,k being cyclic permutations of 1,2,3.

Hamiltonian Formalism

Generalized Momenta: $p_i = \frac{\partial L}{\partial \dot{q}_i}$, $\dot{p}_i = \frac{\partial L}{\partial q_i}$
Hamiltonian: $H(q_i, p_i, t) = \sum_{i=1}^{n} p_i \dot{q}_i - L(q_i, \dot{q}_i, t)$
Hamilton's Equations:

1. $\dot{p}_i = -\frac{\partial H}{\partial q_i}$
2. $\dot{q}_i = \frac{\partial H}{\partial p_i}$
3. $-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$

Cyclic/Ignorable Coordinates: q is ignorable if $\frac{\partial L}{\partial q} = 0$, i.e. if q does not appear in L. Thus $p = \frac{\partial L}{\partial \dot{q}}$ is conserved.
Liousille's Theorem: A volume of a region of phase space remains the same, even when the refion changes. $V = dq_1 \dots dq_n dp_1 \dots dp_n$.
Poisson Bracket: $f, g = \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}$.
Constant of Motion from Poisson Bracket: $\frac{df}{dt} = f, H + \frac{\partial f}{\partial t}$. If $I, H = 0$, then I is a constant of motion.
Canonical Transformation: Transformation $(q_i \to Q_i(q,p), p - I \to P_i(q,p))$ that leaves Hamilton's equations invariant.

6 Statistical Mechanics Equations

7 Quantum Mechanics Equations

8 Electricity and Magnetism Equations

9 Miscellaneous Physics

Taylor Expansion: $f(\vec{x}+\vec{a}) = f(\vec{x}) + a_i \partial_i f(\vec{x}) + \mathcal{O}(\vec{a}^2)$