Proficiency Exam and Diagnostic Exam, Fall 2001

Useful constants:

- $e = 1.60 \times 10^{-19}$ C
- $hc = 1240$ eV $\cdot$ nm
- $c = 3.00 \times 10^{8}$ $\frac{\text{m}}{\text{s}}$
- $m_e = 0.511$ $\frac{\text{MeV}}{\text{c}^2}$
- $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^{9}$ $\frac{\text{Nm}^2}{\text{C}^2}$
- $\epsilon_0 = 8.85 \times 10^{-12}$ $\frac{\text{C}^2}{\text{Nm}^2}$
- $\frac{\mu_0}{4\pi} = 10^{-7} \frac{\text{N}}{\text{A}^2}$
- $g = 9.81$ $\frac{\text{m}}{\text{s}^2}$

1. Four infinitely long wires parallel to the z-axis are arranged with $(x, y) = (0, d)$, $(0, -d)$, $(d, 0)$, and $(-d, 0)$, respectively. (Here $d > 0$.) The wires at $(0, d)$ and $(0, -d)$ carry a current of magnitude i in the positive z-direction, and the wires at $(d, 0)$ and $(-d, 0)$ carry a current of magnitude i in the negative z-direction.

   (a) What are the magnitude and direction of the magnetic field at any point along the x-axis?

   (b) What are the magnitude and direction of the magnetic field at any point along the y-axis?

2. A series LCR circuit is driven by an emf with angular frequency ω, so that the voltage across the resistor is $V_R(t) = V_0 \sin(\omega t)$. In answering the following questions, express everything in terms of the given quantities: L, C, R, ω, V_0, and ε_0.

   (a) Draw a phasor diagram for V_R, V_L, and V_C, and specify the angles between the phasors.

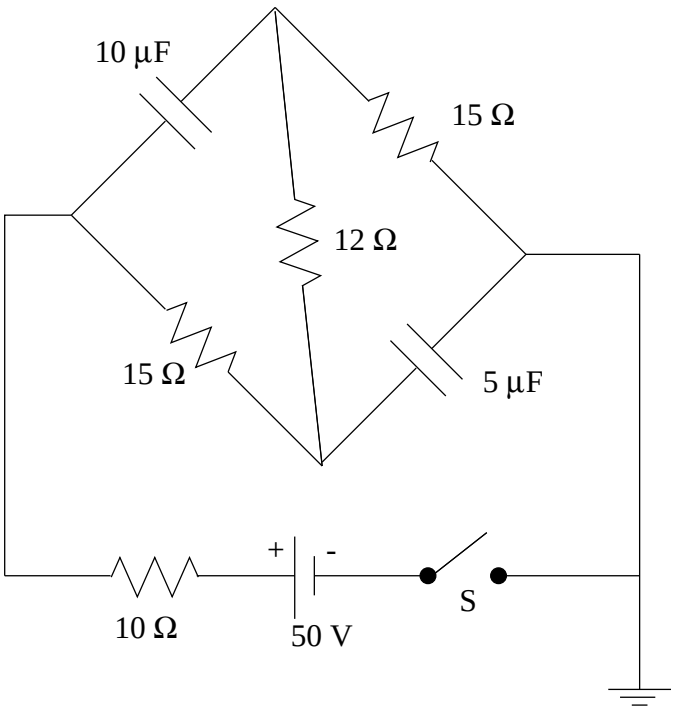
   (b) What is the current $I_L(t)$ through the inductance?

   (c) Find the voltage $V_L(t)$ across the inductance.

   (d) Find the voltage $V_C(t)$ across the capacitance.

   (e) Determine the angular frequency, ω_0, for which the voltage $V_R(t)$ across the resistance is the same as that across the emf source, i.e. $V_R(t) = \varepsilon(t)$.

3. A cloud is 200 m thick (200 m from the top to the bottom of the cloud). There is a positive point charge $+q$ at the top of the cloud and the same negative point charge $-q$ at the bottom. When the cloud is 300 m directly above a certain point on the ground (as measured from the bottom), the electric field at that point on the ground is measured to be 100 V/m. Assume that there are no other charges around in space, and that the ground is a perfect conductor.



(a) What is the magnitude of the point charges?

(b) What is the electric force on the cloud, due to the Earth?

4. The space between two metallic coaxial cylinders of length L and radii a and b is completely filled with a material of resistivity ρ.

   (a) What is the resistance between the two cylinders?

   (b) Find the current between the two cylinders if $\rho = 30\ \Omega\text{m}$, $a = 1.5$ cm, $b = 2.5$ cm, $L = 50$ cm, and a potential difference of 10 V is maintained between the two cylinders.

5. A proton traveling with a velocity of $\vec{v} = 1 \times 10^4\ \text{m/s}\ \hat{i} + 2 \times 10^4\ \text{m/s}\ \hat{j}$ is located at $x = 3$ m, $y = 4$ m at some time t. Find the magnetic field at time t at the following positions:

   (a) $x = 2$ m, $y = 2$ m

   (b) $x = 6$ m, $y = 4$ m

   (c) $x = 3$ m, $y = 6$ m

6. The capacitors in the circuit shown in the figure are initially uncharged.

   (a) What is the initial value of the current drawn from the battery when the switch S is closed?

   (b) What is the current drawn from the battery a long time after the switch is closed?

   (c) What are the final charges on the capacitors?