

1. A molecule consists of four identical atoms at the corner of a tetrahedron.

a) What is the number of (i) translational, (ii) rotational, and (iii) vibrational degrees of freedom for this molecule?

We have four particles which have three positions each, as they are moving in three dimensions. This gives a total of 12 degrees of freedom. Three of these are translations of the center of mass, and since the molecule has no continuous rotational symmetries (unlike a diatomic linear molecule, for example) then there are also three rotational degrees of freedom. This leaves six of the original twelve degrees of freedom which must be vibrational degrees of freedom. Note that we may not have this many *modes*, but the sum of the number of modes times the number of degrees of freedom within each mode will be six.

b) On the basis of the equipartition principle, what are the values of C_v and $\gamma = C_p/C_v$ for one mole of an ideal gas composed of these molecules?

The classical equipartition theorem states that for each independent quadratic variable in the energy (Hamiltonian) we have a mean energy of $k_B T/2$ when that system is at a temperature T . Note this only holds when the level spacing is small compared to $k_B T$. The kinetic energy

$$T = \sum_{i=1}^N \frac{m}{2} (v_x^2 + v_y^2 + v_z^2)$$

has three such independent quadratic degrees of freedom for each particle, so it has a mean of $3k_B T/2$ for each particle.

The same is true of the kinetic energy of rotation about the center of mass, since it looks like

$$T_{\text{rot}} = \sum_{i=1}^N \frac{1}{2} (I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2)$$

and so it also has a mean of $3k_B T/2$ for each particle.

The vibrational degrees of freedom enter the Hamiltonian with both a quadratic kinetic energy term and a quadratic potential energy term. Each vibrational degree of freedom within a normal mode behaves like a one-dimensional oscillator with energy

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2,$$

and so each vibrational degree of freedom contributes an average of $2 \times k_B T/2 = k_B T$ to the energy.

Putting these together, we have (recall there is one mole of the gas)

$$U = N_A \left(\frac{3}{2} k_B T + \frac{3}{2} k_B T + 6 k_B T \right) = 9 N_A k_B T = 9 R T$$

so that

$$C_v = 9R.$$

For an ideal gas we have that $C_p = C_v + nR$ so that for this mole of gas $C_p = C_v + R = 10R$, so that

$$\gamma = \frac{C_p}{C_v} = \frac{10}{9}.$$

c) Assuming that the vibrations of the molecule are quantized, what is C_v for temperatures such that $k_B T \ll \hbar \omega_v$? Here ω_v is the lowest frequency of vibration of the molecule.

At sufficiently low temperature the classical equipartition theorem no longer holds, because $k_B T$ can become comparable to the level spacing of the oscillator (or the rigid-rotator rotational spectrum). Since part d) implies that the rotational motion has much smaller frequencies than the vibrational motion, we know that for temperatures T so that $k_B T \ll \hbar \omega_v$ the vibrational oscillators are in their ground state, and so the internal energy U associated with them does not depend on T and we have

$$U = N_A \left(\frac{3}{2} k_B T + \frac{3}{2} k_B T \right),$$

and so $C_v = 3R$.

d) At very low T the rotations of the molecule are also quantized, i.e. $E_{\text{rot}} = (\hbar^2/2I)l(l+1)$, where $l = 0, 1, 2, \dots$ and I is the moment of inertia of the molecule. What is the value of C_v at very low temperatures ($k_B T \ll \hbar^2/2I$)? Assume the gas is still ideal at low T .

Now even the rotational degrees of freedom are in their ground state and so the energy associated with them does not depend on T , so all that is left are the translational degrees of freedom and

$$U = N_A \frac{3}{2} k_B T,$$

giving $C_v = 3R/2$.

2. Two slits of width $a = 0.015$ mm are separated by a distance $d = 0.06$ mm and illuminated by light of wavelength $\lambda = 650$ nm. How many bright fringes are seen in the central diffraction maximum?

The condition for a *minimum* of the diffraction pattern is

$$\frac{a}{2} \sin(\theta) = \frac{n\lambda}{2}, \quad n = 1, 2, 3, \dots,$$

where a is the slit width. The condition for a maximum of the interference pattern is

$$d \sin(\theta) = m\lambda, \quad m = 0, 1, 2, \dots,$$

where d is the slit spacing. Since in this case $d = 4a$, we have that the $m = 4$ interference maximum occurs at the position of the $n = 1$ diffraction minimum. That means that there are only three interference maxima on either side of the central diffraction, not counting the central ($m = 0$) maximum, since the fourth maximum will be wiped out by the diffraction

pattern. This gives seven bright interference fringes within the central diffraction maximum, regardless of the wavelength of the light.

3. Consider a real gas described by the van der Waals equation of state

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

where P is the pressure, T is the temperature, v is the volume per mole of the gas, R is the universal gas constant, and a and b are constants.

a) Briefly explain the physical origin and meaning of the constants a and b .

First examine the constant b . In the equation the term $v - b$ takes the place of the volume per mole v in the usual ideal gas law $Pv = RT$. This is because the gas molecules are not pointlike but have a finite size, and so we can interpret b as the volume taken up by a mole of these molecules if they are not moving.

The term involving a actually corrects the ideal gas law, which assumes that the molecules do not interact with each other, and comes from these interactions in a real gas. Since they go down like the volume per mole squared, they actually go like $1/r^6$ where r is the separation of the molecules, and so are of the van der Waals type. Note that the pressure is reduced by the term proportional to a , so each molecule is in fact attracted by the other molecules in the gas which results in no net force except near the walls. Near the walls the molecules feel a net force away from the wall and so the pressure is reduced.

b) The coefficient of thermal volume expansion is defined as $\alpha = v^{-1}(\partial v / \partial T)_P$. Show that for the van der Waals gas

$$\alpha = \frac{Rv^2(v - b)}{RTv^3 - 2a(v - b)^2}.$$

This is an exercise in finding total differentials. If we are keeping P fixed then the total differential of the gas law takes the form

$$RdT = -\frac{2a}{v^3}dv(v - b) + \left(P + \frac{a}{v^2}\right)dv.$$

Note that the expression we are to find does not involve P , so eliminate $P + a/v^2$ using the gas law

$$RdT = -\frac{2a}{v^3}dv(v - b) + \frac{RT}{v - b}dv = \frac{dv}{v^3(v - b)}(RTv^3 - 2a[v - b]^2),$$

so that

$$\alpha = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = \frac{v^2(v - b)}{RTv^3 - 2a(v - b)^2}.$$

c) Show that the isothermal compressibility, $\kappa_T = -v^{-1}(\partial v / \partial P)_T$, for the van der Waals gas is given by

$$\kappa_T = \frac{v^2(v - b)^2}{RTv^3 - 2a(v - b)^2}.$$

We now do the same again this time keeping T constant and letting P vary, so that

$$0 = dP(v - b) - \frac{2a}{v^3} dv (v - b) + \left(P + \frac{a}{v^2}\right) dv,$$

and using the same substitution we find

$$dP(v - b) = -\frac{dv}{v^3(v - b)} \left(RTv^3 - 2a[v - b]^2\right),$$

so that

$$\kappa_T = -\frac{1}{v} \left(\frac{\partial v}{\partial P}\right)_T = \frac{v^2(v - b)}{RTv^3 - 2a(v - b)^2}.$$

d) Obtain $(\partial P/\partial T)_v$ and verify that

$$\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_P \left(\frac{\partial v}{\partial P}\right)_T = -1.$$

If v is constant then the above collapses to

$$dP(v - b) = R dT,$$

so that

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v - b},$$

and

$$\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial T}{\partial v}\right)_P \left(\frac{\partial v}{\partial P}\right)_T = \frac{R}{v - b} \frac{1}{v\alpha} (-v\kappa_T) = -\frac{R}{v - b} \frac{\kappa_T}{\alpha} = -1.$$

4. An object is 15 cm to the left of a thin convex lens of focal length 10 cm. A concave mirror of radius 10 cm is 25 cm to the right of the lens.

a) Find the position of the final image formed by the mirror and the lens.

Careful use the lens equation, which also works for spherical mirrors, and that for the magnification of these optical systems will yield the results we need. The lens equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

works for lenses and spherical mirrors, where s is the distance of the object from the lens/mirror, s' is the distance of the image from the lens/mirror, and f is the focal length which is the image distance when $s = \infty$. These quantities are taken to be positive when the object, image, or center of curvature lies on the real side of the optical element. For lenses the real side is the incident side for objects and the transmission side for images and

centers of curvature. Positive s' means a real image, negative s' means a virtual image. The magnification is $m = -s'/s$, and if it is negative this means the image is inverted.

Applying this to the lens we have

$$\frac{1}{15 \text{ cm}} + \frac{1}{s'_1} = \frac{1}{10 \text{ cm}},$$

so that $s'_1 = 30 \text{ cm}$. Note that since s' is positive this image is real and inverted. Now this image would form 5 cm beyond the surface of the mirror, so it is not on the real side of the mirror. Note that for a spherical mirror the focal length is half the radius, so the equation for the mirror looks like

$$\frac{1}{-5 \text{ cm}} + \frac{1}{s'_2} = \frac{2}{10 \text{ cm}},$$

so that $s'_2 = 2.5 \text{ cm}$. Since this is positive we know that it occurs on the real side of the mirror inside the focal length, 2.5 cm to the left of the mirror, and so is a real image. Since for the mirror s'_2 is positive and s is negative the magnification is positive so the image remains inverted with respect to the object.

This real image will now behave like an object to the lens, so applying this equation once more we find

$$\frac{1}{22.5 \text{ cm}} + \frac{1}{s'_3} = \frac{1}{10 \text{ cm}},$$

so that $s'_3 = 18 \text{ cm}$.

b) Is the image real or virtual, upright or inverted?

This image is on the transmission side of the lens and so is a real image, and is inverted relative to the image from the mirror. The net result is an upright real image 18 cm to the left of the lens.

c) Show on a diagram where your eye must be to see this image.

Since light rays appear to emanate from this real image but are moving away from the lens, we will need to place our eye to the left of the real image to see it.

5. A 35 mm camera has a negative size of 24 mm by 36 mm. It is to be used to take a picture of a person 175 cm tall in which the image of the person just fills the height (24 mm) of the film. How far should the person stand from the camera if the focal length of the lens is 50 mm?

The lens equation gives

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f},$$

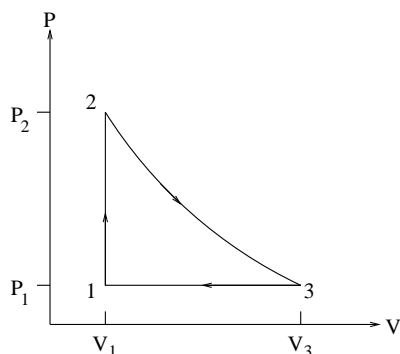
and the magnification is $m = -s'/s$. Using the desired magnification of $m = -2.4/175$ (note it will be negative for a converging lens) we can solve these two equations for s ,

$$\frac{1}{s} + \frac{1}{-ms} = \frac{1}{f}$$

$$\frac{1}{s} \left(\frac{-m+1}{-m} \right) = \frac{1}{f}$$

$$s = f \left(\frac{-m+1}{-m} \right) = 5 \text{ cm} \left(\frac{2.4/175 + 1}{2.4/175} \right) = 369.6 \text{ cm}.$$

6. An ideal gas ($\gamma = 1.5$) follows the cycle shown in the figure.



a) If $P_1 = 1 \text{ atm}$, $V_1 = 100 \text{ L}$, and $T_1 = 20^\circ \text{ C}$, find the amount of gas considered at point 1.

Use the ideal gas law and the value in the appropriate units $R = 0.0821 \text{ L} \cdot \text{atm}/(\text{mol} \cdot \text{K})$, and using $T_1 = 293 \text{ K}$, we have

$$n = \frac{P_1 V_1}{R T_1} = 4.16 \text{ mol}.$$

b) The gas is then heated at constant volume to point 2, where the pressure is $P_2 = 2 \text{ atm}$. What is the temperature of the gas there?

Using the ideal gas law again, we have that since V is constant

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$T_2 = \frac{P_2}{P_1} T_1 = 586 \text{ K}.$$

c) Next, the gas is isothermally expanded to point 3, such that $P_3 = P_1 = 1 \text{ atm}$. What is the work W_{23} done during the isothermal expansion?

Let's first find V_3 . Since the temperature is unchanged from T_2 , the volume is given by the ideal gas law

$$P_3 V_3 = P_2 V_2 = P_2 V_1$$

to be

$$V_3 = \frac{P_2}{P_3} V_1 = \frac{P_2}{P_1} V_1 = 2V_1 = 200 \text{ L}.$$

For an isothermal expansion we have

$$W_{2 \rightarrow 3} = \int_{V_1}^{V_3} P dV$$

and using the ideal gas law we have that $P = nRT_2/V$, so that

$$\begin{aligned} W_{2 \rightarrow 3} &= nRT_2 \int_{V_1}^{V_3} \frac{1}{V} dV = nRT_2 \ln(V_3/V_1) \\ &= (4.16 \text{ mol}) \cdot (0.0821 \text{ L} \cdot \text{atm}/(\text{mol} \cdot \text{K})) \cdot (586 \text{ K}) \ln(2) = 139 \text{ L} \cdot \text{atm}. \end{aligned}$$

d) How is the result modified if an adiabatic rather than isothermal expansion is considered between points 2 and 3?

For an adiabatic expansion we have no heat added to the system, and the expansion follows

$$PV^\gamma = \text{constant},$$

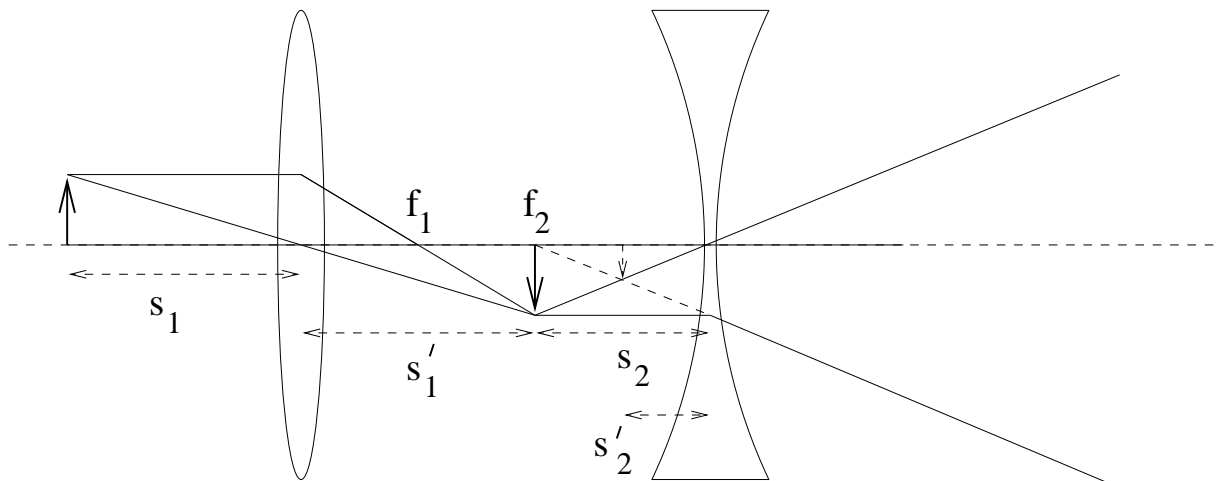
where $\gamma = C_p/C_v = 1.5$. The volume is now changed to

$$V'_3 = \left(\frac{P_2}{P_3} \right)^{\frac{1}{\gamma}} = 2^{2/3} V_1.$$

The work integral is modified to

$$\begin{aligned} W_{2 \rightarrow 3} &= P_2 V_2^\gamma \int_{V_1}^{V'_3} \frac{1}{V^{3/2}} dV = P_2 V_1^{3/2} \left[-2V^{-1/2} \right]_{V_1}^{V'_3} \\ &= 2P_2 V_1^{3/2} \left(V_1^{-1/2} - 2^{-1/3} \cdot V_1^{-1/2} \right) = 2P_2 V_1 \left(1 - 2^{-1/3} \right) \\ &= 2(2 \text{ atm}) \cdot (100 \text{ L}) \left(1 - 2^{-1/3} \right) = 82.5 \text{ L} \cdot \text{atm}. \end{aligned}$$

7. Two lenses are separated by 35 cm. An object is placed 20 cm to the left of the first lens, which is a converging lens of focal length 10 cm. The second lens is a diverging lens of focal length -15 cm. What is the position of the final image? Is the image real or virtual? Erect or inverted? What is the overall magnification of the image?



We can find everything we need from the lens formula(e)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \quad m = -\frac{s'}{s},$$

being careful to use a negative f for the diverging lens (since the focal point of light on the incident side is on the incident side and not the transmitted side, it is therefore negative), and use the figure to check our answers. For the first lens we have

$$\frac{1}{s'_1} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{10 \text{ cm}} - \frac{1}{20 \text{ cm}} = \frac{1}{20 \text{ cm}},$$

so that the first real image is 20 cm from the lens and has

$$m_1 = -\frac{s'_1}{s_1} = -1,$$

so that the image is the same size as the object and inverted.

For the second lens we have

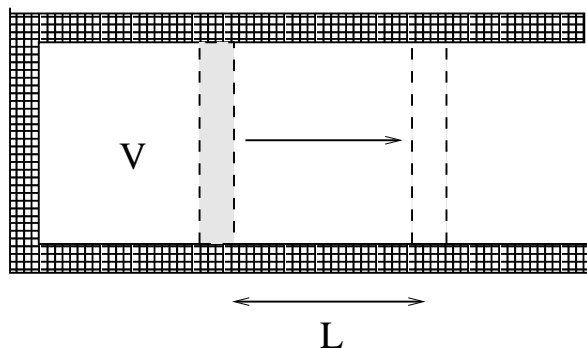
$$\frac{1}{s'_2} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{-15 \text{ cm}} - \frac{1}{15 \text{ cm}} = -\frac{2}{15 \text{ cm}},$$

so that the image is virtual (it is on the incident side, since $s'_2 < 0$) and 7.5 cm to the left of the second lens. The magnification of the second lens is

$$m_2 = -\frac{s'_2}{s_2} = -\frac{-7.5 \text{ cm}}{15 \text{ cm}} = \frac{1}{2},$$

so that the combined magnification is $m = m_1 m_2 = -0.5$ and the virtual image is inverted with an overall magnification of -0.5 .

8. The cylinder shown in the figure has a piston of mass M that can slide without friction. The area of the piston is S and the cylinder is filled with an ideal gas ($\gamma = 1.5$), with an initial volume V , and an initial pressure P . Assume that the outside pressure on the piston is zero (vacuum).



a) Determine the initial acceleration of the piston.

The initial force on the piston is given by the initial pressure P times the area of the piston to which this pressure is applied, $F = P \cdot A$. Then the initial acceleration is

$$a = \frac{F}{M} = \frac{PA}{M}.$$

b) Calculate the velocity of the piston after it has moved a distance L , assuming that the gas is thermally isolated (adiabatic).

Adiabatic expansions follow the rule $PV^\gamma = \text{constant}$, where γ is the ratio C_P/C_V . We can find the constant involved by using the initial condition, so that at some later time when the pressure is P' and the volume V' we have

$$P'V'^\gamma = PV^\gamma,$$

so that

$$P' = P \left(\frac{V}{V'} \right)^\gamma.$$

In order to find the velocity of the piston we will use conservation of energy, which states that the work done by the gas during the expansion is equal to the kinetic energy gained by the piston. The work done by the gas is

$$W = \int_V^{V_f} P' dV' = PV^{\frac{3}{2}} \int_V^{V_f} \frac{1}{V'^{\frac{3}{2}}} dV' = PV^{\frac{3}{2}} \left[\frac{-2}{V'^{\frac{1}{2}}} \right]_V^{V_f} = 2PV \left[1 - \left(\frac{V}{V_f} \right)^{\frac{1}{2}} \right].$$

The final volume is $V_f = V + LS$ so that

$$W = 2PV \left[1 - \left(\frac{V}{V + LS} \right)^{\frac{1}{2}} \right],$$

and using $W = Mv^2/2$ we have $v = (2W/M)^{1/2}$ and

$$v = \left(\frac{4PV}{M} \left[1 - \left(\frac{V}{V + LS} \right)^{\frac{1}{2}} \right] \right)^{\frac{1}{2}}.$$

c) What distance would the piston have to move for the temperature of the gas to drop to one half of the original value?

We will combine the ideal gas law with the adiabatic expansion law. Since the ideal gas law gives $P = nRT/V$ then we have that

$$PV^\gamma = nRTV^{\gamma-1} = nRT_f V_f^{\gamma-1},$$

so that

$$\frac{T_f}{T} = \left(\frac{V}{V_f} \right)^{\gamma-1} = \sqrt{\frac{V}{V_f}} = \sqrt{\frac{V}{V + LS}} = \frac{1}{2},$$

so that

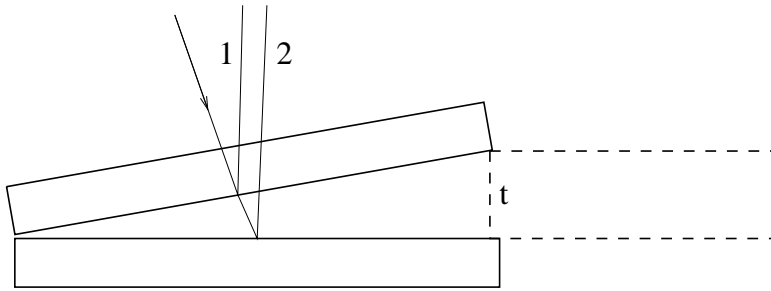
$$4V = V + LS$$

and

$$L = \frac{3V}{S}.$$

9. A wedge of air is formed between two glass plates held apart at one edge by a sheet of paper whose thickness is 4.1×10^{-5} m. Green light ($\lambda_{\text{vacuum}} = 552$ nm) strikes the glass plates nearly perpendicularly. Assume $n_{\text{glass}} = 1.52$, $n_{\text{air}} = 1.00$.

a) How many bright fringes occur between the place where the plates touch and the edge of the sheet of the paper?



The fringes occur because of interference between reflected rays 1 and 2 as illustrated in the diagram. Ray 1 is not inverted at the glass/air interface because it is going from a more dense to a less dense medium. Transmitted waves are always transmitted in phase. The reflected ray 2 is inverted at the air/glass boundary, so the two reflected rays are out of phase by π before we take into account the path difference. We will get bright fringes when the path difference δ is a half-integral number of wavelengths, so that

$$\delta = (m + \frac{1}{2})\lambda_{\text{air}}, \quad m = 0, 1, 2, \dots$$

At the right edge the path difference is just twice the thickness t of the paper, so there

$$2t = (m + \frac{1}{2})\lambda_{\text{air}},$$

and we see that the last fringe has

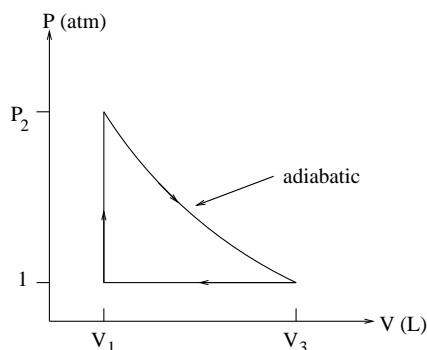
$$m = \frac{2t}{\lambda_{\text{air}}} - \frac{1}{2} = \frac{2 \cdot 4.1 \times 10^{-5}}{552 \times 10^{-9}} - \frac{1}{2} = 148,$$

and since we start counting from zero we see that we have 149 bright fringes.

b) Is there a dark fringe or a bright fringe where the plates touch? Why?

As we saw above, if there is no path difference the two reflected rays have a phase difference of π and destructively interfere. This means that close to the contact point we are in a dark fringe.

10. In the cycle shown in the figure, 1 mol of an ideal gas ($\gamma = 1.4$) is initially at a pressure of 1 atm and a temperature of 0°C . The gas is heated at constant volume to $T_2 = 150^\circ\text{C}$ and is then expanded adiabatically until its pressure is again 1 atm. It is then compressed at constant pressure back to its original state. Recall $R = 0.082 \text{ L} \cdot \text{atm}/(\text{mol} \cdot \text{K})$. Find:



a) the temperature T_3 after the adiabatic expansion

To find the temperature T_3 we will need the volume $V_2 = V_1$ and the pressure P_2 before the expansion, which we can find using the ideal gas law. We have

$$V_1 = \frac{nRT_1}{P_1} = \frac{1 \text{ mol} \cdot 0.082 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \cdot 273 \text{ K}}{1 \text{ atm}} = 22.4 \text{ L},$$

and so

$$P_2 = \frac{nRT_2}{V_1} = \frac{1 \text{ mol} \cdot 0.082 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \cdot 423 \text{ K}}{22.4 \text{ L}} = 1.55 \text{ atm}.$$

Now since the expansion is adiabatic to pressure P_1 we can find V_3 using the rule that $PV^\gamma = \text{constant}$, so that

$$V_3^\gamma = \frac{P_2}{P_1} V_1^\gamma,$$

and so

$$V_3 = \left(\frac{P_2}{P_1} \right)^{1/\gamma} V_1 = (1.55)^{1/1.4} \cdot 22.4 \text{ L} = 30.6 \text{ L}.$$

Now we can find T_3 using the ideal gas law,

$$T_3 = \frac{P_1 V_3}{nR} = \frac{1 \text{ atm} \cdot 30.6 \text{ L}}{1 \text{ mol} \cdot 0.082 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}} = 373 \text{ K}.$$

b) the heat entering or leaving the system during each process

Since there is no work done during $1 \rightarrow 2$ the heat entering the system is the change in the internal energy of the system, which is

$$Q_{1 \rightarrow 2} = C_V(T_2 - T_1).$$

We are not given the value of C_V , but since $\gamma = 1.4 = C_P/C_V = 1 + nR/C_V$ we know that $C_V = 5nR/2$, and so

$$Q_{1 \rightarrow 2} = \frac{5}{2} nR(150 \text{ K}) = 30.75 \text{ L} \cdot \text{atm}.$$

Since the expansion $2 \rightarrow 3$ is adiabatic, no heat enters or leaves the system. For the compression $3 \rightarrow 1$ at constant pressure P_1 we can use the definition of the heat capacity at constant pressure

$$Q_{3 \rightarrow 1} = C_P(T_3 - T_1) = \frac{7}{2}nR(-100 \text{ K}) = -28.7 \text{ L} \cdot \text{atm}.$$

c) the efficiency of this cycle

The efficiency of the cycle is defined to be the net work done during the cycle divided by the heat added during the phase when we add heat, which is $1 \rightarrow 2$, so that

$$\epsilon = \frac{W}{Q_{1 \rightarrow 2}}.$$

Note, however, that since the internal energy of the gas does not change after a complete cycle, we must have that $W = Q_{1 \rightarrow 2} + Q_{3 \rightarrow 1}$, so that

$$\epsilon = 1 + \frac{Q_{3 \rightarrow 1}}{Q_{1 \rightarrow 2}} = 1 - \frac{28.7}{30.75} = 6.7\%.$$

d) the efficiency of a Carnot cycle operating between the temperature extremes of this cycle.

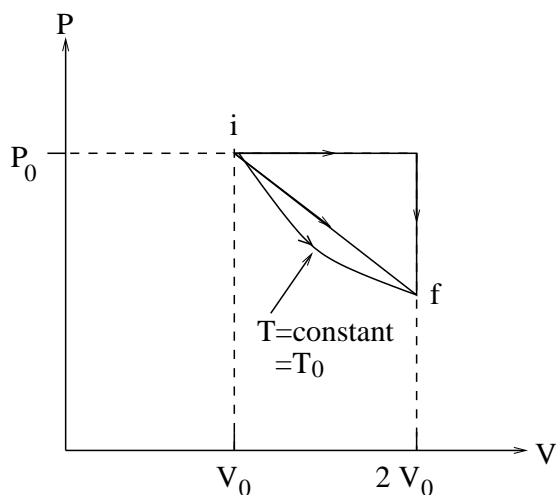
An ideal Carnot cycle which operates between the temperature $T_2 = T_{\text{hot}}$ and the temperature $T_3 = T_{\text{cold}}$ (the maximum temperature during the heating phase and the minimum temperature during the cooling phase, respectively) would have an isothermal expansion at the hot temperature T_{hot} and an isothermal compression at the low temperature T_{cold} , with the other two processes being an adiabatic compression (to T_{hot}) and an adiabatic expansion (to T_{cold}). Isothermal expansions and compressions involve heats Q_{hot} and Q_{cold} which are proportional to the temperature, through

$$Q = nRT \ln \left(\frac{V_f}{V_i} \right).$$

One can show using the rule for adiabatic expansions that the ratios of the volumes at the beginning and end of the adiabatic processes are equal, so that the ratio of the two heats is simply the ratio of the temperatures and the efficiency of the Carnot cycle is simply

$$\epsilon = 1 + \frac{Q_{\text{cold}}}{Q_{\text{hot}}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} = 1 - \frac{273}{423} = 35.5\%.$$

11. The figure shows three reversible processes joining initial and final states of n moles of a monatomic gas. For the three processes calculate:



a) the work done on the system

In what follows we will calculate the work done **by** the system (the usual convention) so that the answer to the question is always the negative of this.

Label the path from i to f which uses an expansion at constant pressure as path 1; it does work

$$W_1 = \int P dV = P_0 \int dV = P_0 V_0 = nRT_0.$$

Label the diagonal path as 2. To find the work done along this path we need to know P_f , which we can find from the ideal gas law, since we know from the third (curved) path that the final temperature has to be T_0 . Hence

$$P_f = \frac{nRT_0}{2V_0} = \frac{P_0 V_0}{2V_0} = \frac{P_0}{2},$$

so that (using geometry to find the area)

$$W_2 = V_0 \frac{P_0 + P_0/2}{2} = \frac{3}{4} P_0 V_0 = \frac{3}{4} nRT_0.$$

Finally, the third path is an isothermal expansion where $P = nRT_0/V$ so that

$$W_3 = \int_{V_0}^{2V_0} \frac{nRT_0}{V} dV = nRT_0 \ln(2) = nRT_0 \ln(2).$$

b) the heat interchanged with the environment

The heat added to the system is the work W done by the system plus the change in the internal energy of the system. For all paths the internal energy of the system does not change since the initial and final temperatures are the same, so in all cases the heat added to the system is simply the work done by the system.

c) the change in internal energy

Zero in all cases since the initial and final temperatures are equal.

d) the change in entropy. Express all results in terms of n and the initial temperature T_0 .

The entropy changes by $dS = dQ_{\text{rev}}/T$ when an amount of heat dQ_{rev} is added at a temperature T in a reversible process. For an ideal gas we have

$$dS = \frac{dU}{T} + \frac{dW}{T} = C_V \frac{dT}{T} + P \frac{dV}{T} = C_V \frac{dT}{T} + nR \frac{dV}{V},$$

so that

$$\Delta S = C_V \int_{T_i}^{T_f} \frac{dT}{T} + nR \int_{V_i}^{V_f} \frac{dV}{V}.$$

Since for all paths $T_i = T_f$ we have that

$$\Delta S = nR \int_{V_i}^{V_f} \frac{dV}{V} = nR \int_{V_0}^{2V_0} \frac{dV}{V} = nR \ln(2).$$

12. A mixture of red light ($\lambda_{\text{vacuum}} = 661 \text{ nm}$) and blue light ($\lambda_{\text{vacuum}} = 472 \text{ nm}$) shines perpendicularly on a thin layer of gasoline ($n_{\text{gas}} = 1.40$) lying on water ($n_{\text{water}} = 1.33$). The gasoline layer has a uniform thickness of $1.69 \times 10^{-7} \text{ m}$. What color is the light reflected from the gasoline? Neglect any effects(s) caused by the material under the water.

Because the light wave reflected from the air/gas interface is inverted, and that from the gas/water interface is not (all transmitted waves are transmitted in phase), we will get destructive interference when the path difference of the two waves in the gasoline is an integral number of wavelengths, and constructive interference when it is a half-integral number of wavelengths. The path difference is always twice the thickness of the gasoline layer for normal incidence. The wavelength of light in a medium is

$$\lambda = \frac{c/n}{\nu} = \frac{\lambda_{\text{vacuum}}}{n},$$

since the time frequency of a transmitted wave cannot change (think of a particle on the boundary responding to the incoming EM wave) and the speed of light in the medium is reduced by dividing by n . This means that the number of wavelengths of light in the path difference $\delta = 2t$ is

$$N = \frac{2tn}{\lambda_{\text{vacuum}}},$$

which is 0.716 for the red light and 1.00 for the blue light. This means that the interference for the blue light is destructive and we will see predominantly red light. Note that since the **intensities** of the two reflected blue waves will not exactly match because of the different ratios of the indices of refraction at the two boundaries, we will not get a perfect cancellation of the reflected blue light.