Notes on Siemens Ch. 3

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November 30, 2015

The Black Sphere

- Model neutron scattering from nuclei as a particle being absorbed by spherical object.
- Start by expanding an incident plane wave in terms of spherical harmonics.

$$e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{l} C_l Y_l^0(\theta) \tag{1}$$

$$e^{ikz} \approx \sum_{l} \frac{\sqrt{\pi}}{kr}\sqrt{2l+1}\, i^{l+1}\left(e^{-i(kr-\frac{l\pi}{2})} - e^{i(kr-\frac{l\pi}{2})}\right) Y_l^0(\theta) \tag{2}$$

- Here we have used the fact that $\mathbf{k}\cdot\mathbf{r}$ only depends on θ, and not on ϕ, thus $m = 0$. Also, we have used various identities and the orthonormality of spherical harmonics.

The Black Sphere

- Scattering only happens for short time

$$\phi(r \to \infty) = \sum_l \frac{\sqrt{\pi}}{kr}\sqrt{2l+1}\, i^{l+1}\left(e^{-i(kr-\frac{l\pi}{2})} - \eta_l e^{i(kr-\frac{l\pi}{2})}\right) Y_l^0(\theta) \tag{3}$$

- Scattered wave is just the total wave function minus the incident wave function, $\phi_{sct} = e^{ikz} - \phi(r \to \infty)$.

$$\phi(r \to \infty) = e^{ikz} + f(\theta)\frac{e^{ikr}}{r} \tag{4}$$

$$f(\theta) = \sum_l i\frac{\sqrt{\pi}}{k}\sqrt{2l+1}Y_l^0(\theta)(1-\eta_l) \tag{5}$$

- This looks like a scattering amplitude, $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$.

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- **Approximations:** Classical turning point is where $k^2 = l(l+1)/R^2 \approx (l+\frac{1}{2})^2/R^2$. If particle passes inside the range of force (R) you get absorption ($\eta_l = 0$), but if not you get none (η_l).

$$\frac{d\sigma}{d\Omega} = \frac{\pi}{k^2}\left|\sum_{l=0}^{kr-1/2}\sqrt{2l+1}Y_l^0(\theta)\right|^2 \tag{6}$$

- **More Approximations:** Here we approximate this for large and small angle scattering. I was not able to figure out the integrals so I'll just quote their answer here.

$$\frac{d\sigma}{d\Omega} \approx \begin{cases} \frac{2R}{\pi}k\theta^2 \sin\theta \cos^2\left(kR\theta + \frac{\pi}{4}\right), & \text{for } kR\theta \gg 1 \\ \frac{k^2R^4}{4}(1-(kR\theta/2)^2)^2, & \text{for } kR\theta \ll 1 \end{cases} \tag{7}$$

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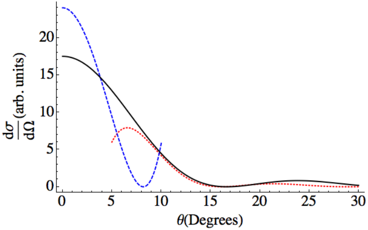

Figure: Rough reproduction of figure 3.2 in the book.

- To find the angle of minumum scattering I have taken the derivative of the high angle scattering and set it equal to zero to get.

$$\theta_{min} = \frac{\pi}{4kR}(2n - 1) \tag{8}$$

- Experiment must show that it's actually

$$\theta_{min} = \frac{5\pi}{4kR} \tag{9}$$

The Black Sphere

$$\theta_{min} = \frac{5\pi}{4kR} \tag{10}$$

- Now we can use this diffraction pattern to estimate the radius of nuclei. For Pb with $\epsilon = 84$ MeV we get $k = \sqrt{2m_N\epsilon/\hbar} \approx 2.0$ fm^{-1}. Now the graph above shows that $\theta_{min} \approx 15^\circ$. This gives us a radius of 7.5 fm.
- A quick google search gives Pb a radius of 7 fm.