

Notes on Siemens Ch. 3

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The Black Sphere

- Model neutron scattering from nuclei as a particle being absorbed by spherical object.
- Start by expanding an incident plane wave in terms of spherical harmonics.

$$e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_l C_l Y_l^0(\theta) \quad (1)$$

$$e^{ikz} \approx \sum_l \frac{\sqrt{\pi}}{kr} \sqrt{2l+1} i^{l+1} \left(e^{-i(kr - \frac{l\pi}{2})} - e^{i(kr - \frac{l\pi}{2})} \right) Y_l^0(\theta) \quad (2)$$

- Here we have used the fact that $\mathbf{k} \cdot \mathbf{r}$ only depends on θ , and not on ϕ , thus $m = 0$. Also, we have used various identities and the orthonormality of spherical harmonics.

The Black Sphere

- Scattering only happens for short time

$$\phi(r \rightarrow \infty) = \sum_l \frac{\sqrt{\pi}}{kr} \sqrt{2l+1} i^{l+1} \left(e^{-i(kr - \frac{l\pi}{2})} - \eta_l e^{i(kr - \frac{l\pi}{2})} \right) Y_l^0(\theta) \quad (3)$$

- Scattered wave is just the total wave function minus the incident wave function, $\phi_{sct} = e^{ikz} - \phi(r \rightarrow \infty)$.

$$\phi(r \rightarrow \infty) = e^{ikz} + f(\theta) \frac{e^{ikr}}{r} \quad (4)$$

$$f(\theta) = \sum_l i \frac{\sqrt{\pi}}{k} \sqrt{2l+1} Y_l^0(\theta) (1 - \eta_l) \quad (5)$$