PHY6938 Mechanics Fall and Spring 2001
January 7, 2002

1. A mass m is hung on the end of a light unstretchable cord of length L, forming a simple pendulum. The mass is pulled to one side so as to make an angle θ_1 with the vertical.

   (a) Find the potential energy of the system relative to its lowest position.

   (b) If the system is released at θ_1 from rest, find the velocity of the mass when it is at its lowest position.

   (c) Find the kinetic and potential energies at an angle $\theta_2 < \theta_1$ if the pendulum is released at θ_1 from rest.

2. A uniform spherical ball of mass M and radius R is set rotating about a horizontal axis with angular speed ω_0 and is placed gently on the floor. The initial center-of-mass velocity of the ball is zero. If the coefficient of sliding friction between the ball and the floor is μ, find the speed of the center of mass of the ball when it begins to roll without slipping.

   (The moment of inertia of the ball about an axis passing through its center of mass is $I = \frac{2}{5}MR^2$.)

3. Consider the Earth as a frame rotating about its axis with angular velocity $\vec{\omega}$. Particles moving relative to the Earth's surface are then subject to the Coriolis force given by $\vec{F} = -2m(\vec{\omega} \times \vec{v}_r)$, where $\vec{v}_r$ is the velocity of the particle relative to the rotating frame, and m is the mass of the particle. Choose the z–axis along the upwards vertical direction, the y–axis pointing North, and the x–axis to the East. Assume you are in the Northern hemisphere at a latitude such that the angle of the z–axis with $\vec{\omega}$ is θ.

   (a) A particle moves under the influence of gravitation and the Coriolis force. Write the equations of motion in the x, y, and z directions. Use the approximation that the component of the velocity in the z direction is much larger than the components in the x and y directions, i.e. $|v_z| \gg |v_x|, \; |v_y|$.

   (b) The particle is dropped at rest from a height h above the ground; it arrives at the ground with speed $v_0 = \sqrt{2gh}$. Find the magnitude and direction of the Coriolis deflection.

   (c) The particle is next thrown vertically upward with an initial speed v_0, so that it reaches the maximum height h [the same h as in part (b)], and then falls back to the ground. Find the magnitude and direction of the Coriolis deflection in this case.

   (d) Compare your results for parts (b) and (c).

4. A skydiver jumps from an airplane. We shall consider his "free" fall, before the parachute is opened. During the free fall, the magnitude of the frictional drag from the air is $F_\text{d} = Kv^2$, where v is the diver's speed.

   (a) Derive an expression for the terminal velocity of the skydiver, v_T. Express your answer in terms of the proportionality constant K, the acceleration of gravity, g, and the mass of the skydiver, m.

(b) Derive an expression for the speed of the skydiver as a function of time, t.

(c) Find v_{T} for a skydiver of mass $m = 80$ kg when $K = 0.25$ kg/m and $g = 9.8$ m/s^2.

5. A curve on a highway has a radius of curvature $r = 100$ m. The road in the curve is banked at an angle of $\theta = 30°$ with the horizontal. If the coefficient of static friction is $\mu = 0.3$, what is the maximum speed with which a car can go through the curve without skidding?

6. A satellite of mass m is in a circular orbit of radius R around the Earth. Assume m is much smaller than the mass of the Earth.

   (a) What are the forces acting on the satellite? State the equilibrium condition.

   (b) Obtain an expression for the period τ (the time it takes to make one complete orbit) as a function of the constants of the problem, e.g. the radius of the orbit.

   (c) Briefly state and discuss the conditions for a geosynchronous orbit.

   (d) What is the period of a geosynchronous orbit? At what radius R from the center of the Earth can a satellite be placed in a geosynchronous orbit?

   Useful constants:

   Mass of the Earth: $M_E = 5.98 \times 10^{24}$ kg

   Gravitational constant: $G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}sec^{-2}

7. A bucket of mass M (when empty) is initially at rest and has a mass m of water in it. The bucket is pulled straight upwards with a constant force F. The water in the bucket leaks out at a constant rate and with zero velocity relative to the bucket. At time T, the bucket becomes empty. Find the velocity of the bucket at time T.