E & M Proficiency Exam, Spring 2002

Useful constants:

- $e = 1.60 \times 10^{-19}$ C
- $m_e = 0.511 \ \frac{\text{MeV}}{\text{c}^2}$
- $c = 3.00 \times 10^8 \ \frac{\text{m}}{\text{s}}$
- $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \ \frac{\text{Nm}^2}{\text{C}^2}$
- $\epsilon_0 = 8.85 \times 10^{-12} \ \frac{\text{C}^2}{\text{Nm}^2}$

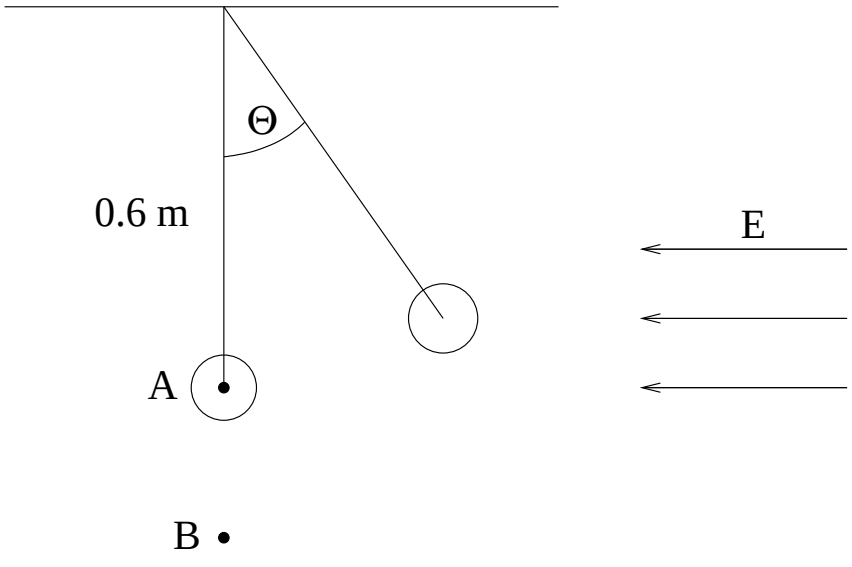
1. A parallel-plate capacitor is constructed using a dielectric whose constant varies with position. The plates have area A. The bottom plate is at $y = 0$ and the top plate is at $y = y_0$. The dielectric constant is given as a function of y according to $\kappa = 1 + \frac{3}{y_0} y$.

   (a) What is the capacitance?

   (b) Find the ratio σ_b/σ_f between the bound and free area charge densities on the the surfaces of the dielectric.

   (c) Use Gauss's law to find the induced volume charge density $\rho(y)$ within this dielectric.

   (d) Integrate the expression for the volume charge density found in (c) over the dielectric and show that the total induced bound charge, including that on the surfaces, is zero.



2. A non-conducting sphere of mass $m = 1.3\,\text{g}$ hangs vertically from a massless string of length $l = 0.6\,\text{m}$. The sphere carries a charge of $-3.4 \times 10^{-4}\,\text{C}$. A constant, uniform electric field of strength $E = 5.4$ V/m is oriented horizontally from right to left. (See the figure.)

(a) What is the equilibrium position (θ) of the sphere ?

(b) If the electric field is then removed, write an equation approximately describing the position of the sphere as a function of time [i.e., $\theta(t)$].

(c) When the sphere is at point A and swinging from left to right, what is the direction of the magnetic field at point B ?

(d) Qualitatively describe the motion of the sphere if, instead of removing the electric field, it is quickly reduced to half its original value.

3. An infinitely long cylinder of radius R has a uniform charge density σ deposited on its outer surface. The charges are fixed to the surface and do not move with respect to it. However, the cylinder itself is rotating at an angular speed ω about its axis. Find the magnetic field $\vec{B}$, as a function of the distance r from the cylinder axis, both inside and outside the cylinder.