2 December 2014, 1:30 pm.

P521: Classical Mechanics Final Exam

Due: 9 December 2014 (Tuesday), 1:30 pm, in GWC520.

Important:

- No collaboration is permitted on this exam. You may use any other resource but full citations should be included. The use of resources that are not cited will be treated as plagiarism.
- Questions about the exam should be emailed to me and should preferably also be posted on Blackboard for the benefit of the whole class.
- A hard copy of the exam must be turned in by the due deadline.

1. (20 points) Analyze the small excitations of an infinite, 1 dimensional, sequence of identical masses, m, that are connected by identical springs of spring constant k and natural length a. (System is depicted below.)
$$\ldots - \overset{k}{-} - m - \overset{k}{-} - m - \overset{k}{-} - m - \overset{k}{-} - m - \overset{k}{-} - \ldots \tag{1}$$
Find the energy eigenvalues and eigenmodes for small excitations in the limit $a \to 0$, $m/a \to \mu$, $ka \to Y$ where μ and Y are finite, non-zero constants. (This is a standard way to generalize classical mechanics to classical field theory and you can find the problem discussed in many texts. If you do use a text to learn about this topic, be sure to cite it.)

2. (10 points) Astronomers are/were closely watching comet ISON (mass $\sim 10^6$ kg) as it seems to have fallen into the Sun. Find the range of comet energy, E, and angular momentum, l, such that a comet will enter the Sun. Assume that only the gravitational force of the Sun is relevant (which is not true). Plot the region in the (l, E) plane in which there is a collision.

3. (10 points) a) The Hamiltonian for a system has the form
$$H = \frac{1}{2}\left(\frac{1}{q^2} + p^2 q^4\right) \tag{2}$$
Find the equation of motion for q.

   b) Find a canonical transformation that reduces H to the form of a harmonic oscillator. Show that the solution for the transformed variables is such that the equation of motion found in part (a) is satisfied.

4. (20 points) An automobile is started from rest with one of its doors initially at right angles. If the hinges of the door are toward the front of the car, the door will slam shut as the automobile picks up speed. Obtain a formula for the time needed for the door to close if the acceleration, f, is constant, the radius of gyration of the door about the axis of rotation is r_0, and the center of mass is at a distance a from the hinges. Find the time if $f = 2$ ft/sec^2 and the door is a uniform rectangle 50 inches wide.

5. (20 points) A particle in a 1 dimensional box bounces back and forth elastically between the ends while the size of the box, L, slowly decreases. By explicit solution, find the rate of change of the speed of the particle. Compare your result to the rate of change of speed obtained using an adiabatic invariant. Discuss what happens if L decreases rapidly?

6. (20 points) Consider the Sun, Earth, Moon system but in the situation where the Moon's orbit is initially in the plane whose normal points to the Sun. Set up the 3-body problem, make suitable simplifying assumptions, and discuss the Moon's orbit. What is the fate of the Moon if it started out in this way? (Either treat this problem analytically or else you can run the code you've developed over the semester to analyze these initial conditions.)