Study guide for qualifying exams

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1 Classical Mechanics Equations

Newtonian Mechanics

Newton's Laws:

1. An object will maintain it's current motion unless acted upon by an external force.
2. $\vec{F} = m\vec{a}$
3. All forces occur in equal but directionally opposite pairs.

Second Law: $\vec{F} = m\vec{a} = \dot{\vec{p}}$
Angular Position/Velocity/Acceleration: $\theta = s/r$, $\omega = v/r$, $\alpha = a/r$
Angular Momentum x2: $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$
Torque: $\vec{\tau} = \vec{r} \times \vec{F} = \dot{\vec{L}}$
Centripital Acceleration: $a_c = v^2/r$
Centrifugal/Coriolis Forces: $\vec{F}_{cent} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$, $\vec{F}_{cor} = -2m\vec{\omega} \times \dot{\vec{r'}}$
Work to go from positions $\vec{a}$ to $\vec{b}$: $W_{ab} = \int\limits_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{s}$
Conservative Force Field (2 eq): W_{ab} is the same regardless of path so $\oint \vec{F} \cdot d\vec{s} = 0$, and thus we can write the force as $\vec{F} = -\nabla V(\vec{r})$.

Lagrangian Formalism

Functional Derivative: $\frac{\delta F}{\delta u}[x_0] = \lim_{\epsilon \to 0} \frac{F[x_0+\epsilon u]-F[x_0]}{\epsilon} \to \frac{\delta F}{\delta x(t)}[x(t')] = \lim_{\epsilon \to 0} \frac{F[x(t')+\epsilon\delta(t'-t)]-F[x(t')]}{\epsilon}$
Principle of Least Action: $\delta S = 0$, where $S = \int_{t_i}^{t_f} L(\vec{q}, \dot{\vec{q}}, t) dt$
Lagranges Equation: $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = 0$
Holonomic Constraints: $f_\alpha(x^A, t) = 0$, $L' = L(x^A, \dot{x}^A) + \lambda_\alpha f_\alpha(x^A, t) \to \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = \lambda_\alpha \frac{\partial f_\alpha}{\partial x^A}$
Noether's Theorem: A continuous symmetry in the Action (and thus Lagrangian) result in a conserved quantity.
Moment of Inertia Tensor: $\vec{L} = \overleftrightarrow{I}\vec{\omega}$, $T = \frac{1}{2}\omega_a I_{ab} \omega_b$, $I_{ab} = \sum_i m_i (r_i^2 \delta_{\alpha\beta} - r_{i\alpha} r_{i\beta})$

Euler's Equations: Only look at rotation, not translation. Conservation of Angular Momentum gives $I_i\dot{\omega}_i + \omega_j\omega_k(I_k - I_j) = 0$, for i,j,k being cyclic permutations of 1,2,3.

Hamiltonian Formalism

Generalized Momenta: $p_i = \frac{\partial L}{\partial \dot{q}_i}$, $\dot{p}_i = \frac{\partial L}{\partial q_i}$

Hamiltonian: $H(q_i, p_i, t) = \sum_{i=1}^{n} p_i\dot{q}_i - L(q_i, \dot{q}_i, t)$

Hamilton's Equations:

1. $\dot{p}_i = -\frac{\partial H}{\partial q_i}$
2. $\dot{q}_i = \frac{\partial H}{\partial p_i}$
3. $-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$

Cyclic/Ignorable Coordinates: q is ignorable if $\frac{\partial L}{\partial q} = 0$, i.e. if q does not appear in L. Thus $p = \frac{\partial L}{\partial \dot{q}}$ is conserved.
Liousille's Theorem: A volume of a region of phase space remains the same, even when the refion changes. $V = dq_1 \dots dq_n dp_1 \dots dp_n$.
Poisson Bracket: $\{f, g\} = \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}$.
Constant of Motion from Poisson Bracket: $\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}$. If $I, H = 0$, then I is a constant of motion.
Canonical Transformation: Transformation $(q_i \to Q_i(q, p), p_i \to P_i(q, p))$ that leaves Hamilton's equations invariant.

2 Statistical Mechanics Equations

2.1 Thermodynamics

Laws of Thermodynamics:

1. Energy conservation. $dE = đQ - pdV$. $đQ$ just means that the heat is an inexact differential and the integral depends on the path.
2. $\Delta S \geq \int \frac{đQ}{T}$, where equality is for a process that is reversible (never leaves equilibrium).
3. Entropy at zero temperature is zero. In stat mech this means that the ground state is nondegenerate and $S \propto \ln(\mathrm{W})$, where W is the number of available states.

Intensive vs Extensive Variables: Intensive variables do NOT scale with system size (T, p, μ), while extensive do scale (E, S, V, N).
Thermodynamic Potentials:

- Internal Energy: $U(S, V, N)$

- Helmholtz Free Energy: $F(T,V,N) = U - TS$
- Enthalpy: $H(S,p,N) = U + pV$
- Gibbs Free Energy: $G(T,p,N) = U - TS + pV$
- Landau(Grand) Potential: $\Omega(T,V,\mu) = U - TS - \mu_i N_i$

Thermodynamic Ensembles:

1. Microcanonical: Does not exchange energy or particles with environment. Fixed E, N
2. Canonical: Does not exchange particles, but can exchange energy (heat bath). Fixed N, T
3. Grand canonical: Can exchange energy and particles with environment. Fixed T, μ.

Maxwell's Relations (4 main):

- $\frac{\partial^2 U}{\partial S \partial V} = -\left(\frac{\partial p}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$
- $\frac{\partial^2 F}{\partial T \partial V} = \left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$
- $\frac{\partial^2 H}{\partial S \partial p} = \left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial T}{\partial p}\right)_S$
- $\frac{\partial^2 G}{\partial T \partial p} = \left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$

Engine Efficience: $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{T_{out}}{T_{in}}$
Isobaric Thermal Expansion Coeffieicnt: $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$, How much the volume changes with a change in termperature.
Isothermal Compressibility: $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$, How much the volume changes when the pressure changes.
Isentropic(Adiabatic) Compressibility: $\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$, Same as above.
Specific Heat at Constant V: $C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$, Amount of heat per unit mass to raise the temp by 1 degree.
Specific Heat at Constant p: $C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial H}{\partial T}\right)_p$, Same as above.
Fermi Energy/Temperature: Chemical potential at $T = 0$. $\epsilon_F = \mu(T = 0)$

2.2 Statistical Mechanics

Number of microstates in a mactostate (ways to get n heads): $\Omega = \frac{N!}{\prod_i n_i!}$
Stirling's Approximation: $\ln n! = n \ln n - n$
How many order important ways to order n things: $n!$
How many order important waus to order n things r at a time: $\frac{n!}{(n-r)!}$
How many NOT order important ways to order n things r at a time: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
Microcanonical(Classical) Partition Function: $Z_m = \sum_s g_s e^{-\beta E_s}$

Canonical Partition Function: $Z_c = \mathrm{tr}\left(e^{-\beta \hat{H}}\right)$

Grand Canonical Partition Function: $Z_{gc} = \mathrm{tr}\left(e^{-\beta(\hat{H}-\mu\hat{N})}\right)$

Geometric Series: $\sum\limits_{n=0}^{\infty} x^n = \frac{1}{1-x}$

Classical limit of the trace of an operator: $\mathrm{tr}(\mathcal{O}) = \frac{1}{N!(2\pi\hbar)^{3N}} \int d^3r_1 \ldots d^3r_N \int d^3p_1 \ldots d^3p_N \mathcal{O}$, $N!$ is for identical particles.
Thermodynamic Limit: $T \to \infty, V \to \infty, N/V = const$
Expectation value for pure/mixed: $\langle \mathcal{O} \rangle_p = \langle \psi | \mathcal{O} | \psi \rangle, \langle \mathcal{O} \rangle_m = \sum_i P_i \langle \psi_i | \mathcal{O} | \psi_i \rangle$
Density Matrix (ex. Canonical Ensemble): $\rho = \sum_n P_n |\psi_n\rangle \langle \psi_n|, \rho_c = \frac{e^{-\beta\hat{H}}}{\mathrm{tr} e^{-\beta\hat{H}}}$
Expectation value with Density Matrix: $\langle \mathcal{O} \rangle = \mathrm{tr}(\mathcal{O}\rho)$
Trace of Density matrix: $\mathrm{tr}(\rho) = 1$
Time evolution of density matrix: $\frac{\partial}{\partial t}\rho = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right]$
Z_{gc} **for an ideal gas:** $Z_{gc} = \frac{V^N (2mT\pi)^{3N/2}}{N!(2\pi\hbar)^{3N}} e^{\beta\mu}$
Z_{gc} **for ideal fermi gas:** $Z_{gc} = \prod\limits_k \left(1 + e^{-\beta(\epsilon_k - \mu)}\right)$
Z_{gc} **for ideal bose gas:** $Z_{gc} = \prod\limits_k \frac{1}{\left(1-e^{-\beta(\epsilon_k-\mu)}\right)}$
Stuff here for black-body and phonons and bose condensates.
Explain Bose-Condensates with Bose statistics: $\lim\limits_{T->0} n(p) = \lim\limits_{\beta->\infty} \frac{1}{1-e^{\beta(\epsilon-\mu)}} \to 0$ unless $\epsilon \to \mu$, which happens at the ground state.
What is cluster expansion used for?: Systems of interacting particles.

3 Quantum Mechanics Equations

Properties of a vector space:

- Sum $|V\rangle + |W\rangle$
- Scalar product with properties
  1. closure: results in another vector in the space.
  2. distributive: $a(|V\rangle + |W\rangle = a|V\rangle + a|W\rangle)$, $(a+b)|V\rangle = a|V\rangle + b|V\rangle$
  3. associative: $a(b|V\rangle) = ab|V\rangle$, $|V\rangle + (|W\rangle + |Z\rangle) = (|V\rangle + |W\rangle) + |Z\rangle$
  4. commutative: $|V\rangle + |W\rangle = |W\rangle + |V\rangle$
  5. addative inverse: $|V\rangle + |-V\rangle = |0\rangle$
  6. null vector: $|V\rangle + |0\rangle = |V\rangle$

Hilbert space: Vector space with defined inner product.
Expand in orthonormal basis: $|V\rangle = \sum\limits_i vi\,|i\rangle$
Hermitian operator: $\mathcal{O}^\dagger = \mathcal{O}$
Anti-Hermitian operator: $\mathcal{O}^\dagger = \mathcal{O}$

Unitary operator: $UU^\dagger = \mathbb{I}$
Orthogonality: $\langle i|j\rangle = \delta_{ij}$
Completeness: $\sum_i i = \mathbb{I}$
Postulates of QM:

1. The state of a physical system, at some fixed time, is given by a normalized ray in a Hilbert space over the complex numbers. (ray is vector whose norm doesn't matter)

2. The ray evolves deterministically in time according to Schrödingers equation.

3. Observables correspond to self-adjoint (hermitian) operators.

4. If a particle is in the state $|\psi\rangle$ then a measurement of $\mathcal{O}$ will yield one of the eigenvalues of $\mathcal{O}$, ω. The state of the system changes to an eigenstate of $\mathcal{O}$, $|\omega\rangle$.

Schrödinger equation: $i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$
Free particle ψ_p and E_p: $\psi_p = Ae^{ikx} + Be^{-ikx}$, $k^2 = \frac{2mE_n}{\hbar^2}$, $E_p = \frac{p^2}{2m}$
Particle in a box ψ_n and E_n: $\psi_n = \sqrt{\frac{2}{L}}\sin(k_n x)$, $k_n = \frac{n\pi}{L}$, $E_n = \frac{\hbar^2\pi^2n^2}{2mL^2}$
Harmonic Oscillator $\hat{H}$, ψ_n and E_n: $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, $\psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{2^n n!}H_n(x)e^{-x^2/2}$, $E_n = (n+\frac{1}{2})\hbar\omega$
Raising and lowering operators and how to affect $|n\rangle$ (3-2):

- $a = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$, $a\,|n\rangle = \sqrt{n}\,|n-1\rangle$, $a\,|0\rangle = 0$

- $a^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$, $a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$\hat{H}$ in terms of a and $a^\dagger$: $\hat{H} = \hbar\omega(a^\dagger a + 1/2)$
Commutation relations for $\hat{H}$, a, $a^\dagger$:

- $[\hat{H}, a] = -a$

- $[\hat{H}, a^\dagger] = a^\dagger$

- $[a, a^\dagger] = 1$

$\mathbf{J}^2$ and J_z on the angular momentum state $|jm_j\rangle$:

- $\mathbf{J}^2\,|{=}\rangle\; j(j+1)\hbar^2\,|jm_j\rangle$

- $J_z\,|jm_j\rangle = m_j\hbar\,|jm_j\rangle$

Commutation relations for J_i and J_j and for $\mathbf{J}^2$ and J_i:

- $[J_i, J_j] = i\hbar J_k$

- $[\mathbf{J}^2, J_i] = 0$

J_z and $\mathbf{J}^2$ in position basis:

- $J_z = -i\hbar\frac{\partial}{\partial t}$
- $\mathbf{J}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$

Raising and Lowering Angular Momentum Operators on $|j,m\rangle$:
$J_\pm|j,m\rangle = \hbar[j(j+1) - m(m\pm 1)]^1/2\,|j,m\pm 1\rangle$
J_x and J_y in terms of J_+ and J_-: $J_x = \frac{1}{2}(J_+ + J_-)$, $J_y = \frac{1}{2i}(J_+ - J_i)$
Momentum eigenstate, $\langle x|p\rangle$: $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}$
Hydrogen Atom $V(r)$, ψ_n, E_n(x4): $V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$, $\psi_n = stuff{*}L^{2l+1}_{n-l-1}(\rho)Y^m_l(\theta,\phi)$(Laguerre)
$E_n = -\frac{1}{2n^2}\left(\frac{Ze^2}{4\pi\epsilon_0\hbar}\right)^2 m_e = -\frac{1}{2n^2}\alpha^2 m_e c^2 = -\frac{1}{n^2}13.6eV = -\frac{1}{2n^2}\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{a_0}\right)$,
$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$, $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$
Pauli matricies and commutation relations:
$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
$\sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$
$\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$
$[\sigma_a\sigma_b] = 2i\epsilon_{abc}\sigma_c$
Non-Deg Time-Ind Perturbation, $E_n^{(1)}$, $\left|n^{(1)}\right\rangle$, $E_n^{(2)}$:
$E_n^{(1)} = H'_{nn} = \left\langle n^{(0)}\right| H' \left|n^{(0)}\right\rangle$
$\left|n^{(1)}\right\rangle = \sum\limits_{m\neq n}\frac{\left\langle n^{(0)}\right|H'\left|n^{(0)}\right\rangle}{(E_n^{(0)} - E_m^{(0)})}\left|m^{(0)}\right\rangle$
$E_n^{(2)} = \sum\limits_{m\neq n}\frac{\left|\left\langle n^{(0)}\right|H'\left|n^{(0)}\right\rangle\right|^2}{(E_n^{(0)} - E_m^{(0)})}$

Deg Time-Ind Perturbation, $E_n^{(1)}$: Diagonalize the perturbation hamiltonian in the degenerate subspace.

Time-Dep Perturbation, $P_{i\to f}(t)$: $P_{i\to f}(t) = \frac{1}{\hbar^2}\left|\int_0^t dt'\,\langle f|\,H'(t')\,|i\rangle\, e^{i(E_f - E_i)t'/\hbar}\right|^2$

Fermi's golder rule, and $g(E_f) as \delta$: $R_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f|\,H'\,|i\rangle\right|^2 g(E_f)$, $g(E_f) \approx \delta(E_f^{(0)} - E_f^{(0)} - \hbar\omega)$
Einstein's Stimulated/Spontaneous emission coeffients:
Stimulated: $B_{if} = \frac{\pi e^2}{3\epsilon_0\hbar^2}\left|\langle f|\,\mathbf{r}\,|i\rangle\right|^2$
Spontaneous: $A_{if} = \frac{e^2\omega_{21}^3}{3\pi\epsilon_0\hbar c^3}\left|\langle f|\,\mathbf{r}\,|i\rangle\right|^2$
Total $\psi(\mathbf{r})$ in scattering problem:
$\psi(\mathbf{r}) = \psi_{inc}(\mathbf{r}) + \psi_s(\mathbf{r}) = \psi_{inc}(\mathbf{r}) + f(\theta,\phi)\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r}$
$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int d^3r' e^{-i\mathbf{k}'\cdot\mathbf{r}'}V(\mathbf{r}')\psi(\mathbf{r}')$
Differential Cross Section: $\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$
Born Approximation: In the above integral for $f(\theta,\phi)$ let $\psi \to \psi_{inc}$.
Dirac Equation:
$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$
$\gamma^0 = \beta$, $\gamma^i = \beta\alpha_i$

$\beta = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}$, $\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$

4 Electricity and Magnetism Equations

Maxwell's Equations in Vacuum (SI):
$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$, $\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$
Maxwell's Equations in Matter (SI), and D and H:
$\nabla \cdot \mathbf{D} = \rho$, $\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t}$
$\mathbf{D} = \epsilon_0 \mathbf{E}$, $\mathbf{B} = \mu_0 \mathbf{H}$
Continuity Equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$
Lorentz Force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
Coulomb's Law (x2): $\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$, $\mathbf{F} = Q\mathbf{E}$
Gauss' Law: $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$
Electrostatic Potential (x2): $\mathbf{E} = -\nabla \Phi$, $\Phi = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$
Laplace's Equation & General Solution(Spherical Coordinates, no ϕ): $\nabla^2 \Phi = 0$
$\Phi(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$
Poisson's Equation: $\nabla^2 \Phi = -\rho/\epsilon_0$
Explain the Method of Images: Because of the uniqueness theorem you can add charges OUTSIDE of the computational area to meet the same boundary conditions. A solution to this new configuration is also a solution to the initial configuration.
Method of Images (plane, sphere, hem boss):
plane: add one charge below plane.
sphere: 1 test charge inside sphere.
hem boss: 3 test charges.
Multipole Expansion of $\Phi(\mathbf{r})$: $\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \int d^3r' \frac{P_n(\cos\alpha)}{r^{n+1}} \rho(\mathbf{r}')$
Work and Energy in Electrostatics: The Energy of a system is the work it requires to assemble the system.
Atomic Polarizability (α): $\mathbf{p} = \alpha \mathbf{E}$
Polarization: Electric dipole moment per unit volume. $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$
Magnetization: Magnetic dipole moment per unit volume $\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$
Bound Charge: $\rho_b = -\nabla \cdot \mathbf{P}$
Bound Current: $\mathbf{J}_b = \nabla \times \mathbf{M}$
Linear Media x2: $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$, $\mathbf{M} = \chi_m \mathbf{H}$
Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}') \times |\mathbf{r}-\mathbf{r}'|}{|\mathbf{r}-\mathbf{r}'|}$
Ohm's Law: $\mathbf{J} = \sigma \mathbf{E}$, where σ is the conductivity
Resistivity: $\rho = 1/\sigma$
Boundary Conditions:
$D_1^\perp - D_2^\perp = \sigma_f$
$B_1^\perp - B_2^\perp = 0$

$\mathbf{E}_1^{\|} - \mathbf{E}_2^{\|} = \mathbf{0}$
$\mathbf{H}_1^{\|} - \mathbf{H}_2^{\|} = \mathbf{K}_f \times \hat{\mathbf{n}}$
Poynting's Theorem, units of S: $\mathbf{S} = \frac{energy}{time \cdot energy}$
$\frac{dW}{dt} = -\frac{d}{dt}\int d^3r \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\oint d\mathbf{a}\cdot(\mathbf{E}\times\mathbf{B}) = -\frac{d}{dt}(W_e + W_m) - \oint d\mathbf{a}\cdot\mathbf{S}$
Maxwell Stress Tensor and Static Force:
$T_{ij} = \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$
$\mathbf{F} = \oint d\mathbf{a}\cdot \overleftrightarrow{T}$
Index of Refraction: $n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$
What is a Waveguide: A waveguide is a conductor pipe such that $\mathbf{E}^{\|} = \mathbf{0}$ and $B^{\perp} = 0$ on the surface. Also the transverse components of the fields (x and y) can be determined from derivatives of the axial components (z).
Transverse electric/magnetic and TEM
TE: $E_z = 0$
TM: $B_z = 0$
TEM: both
E and B in terms of A and Φ: $\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$, $\mathbf{B} = \nabla\times\mathbf{A}$
Coulomb/Lorentz Gauge: $\nabla\cdot\mathbf{A} = 0$, $\nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial\Phi}{\partial t}$
Retarted Scalar and Vector Potentials: $\Phi = \frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}$, $\mathbf{A} = \frac{\mu_0}{4\pi}\int d^3r' \frac{\mathbf{J}(\mathbf{r}', t-|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}$
What are the Liénard-Wiechert Potentials?: Retarted potentials of a point charge with a specific trajectory.
Radiation Estimate $|\mathbf{r}-\mathbf{r}'|$ and $\frac{1}{|\mathbf{r}-\mathbf{r}'|}$: $|\mathbf{r}-\mathbf{r}'| \approx r - \frac{\mathbf{r}\cdot\mathbf{r}'}{r}$, $\frac{1}{|\mathbf{r}-\mathbf{r}'|} \approx \frac{1}{r} + \frac{\mathbf{r}\cdot\mathbf{r}'}{r^3}$
Radiation Dipole Approximation $e^{-i\omega\hat{\mathbf{r}}\cdot\mathbf{r}'/c} \approx 1$
Electric Dipole Moment: $\mathbf{p}(\mathbf{r}, t) = \int d^3r' \mathbf{r}' \rho(\mathbf{r}', t)$
Larmor Formula: $P = \frac{\mu_0}{6\pi c}q^2a^2$
Helmholtz Theorem: If you know the divergence (D) and the curl ($\mathbf{C}$) of a function $\mathbf{F}$ then $\mathbf{F} = -\nabla U + \nabla\times\mathbf{W}$ where
$U(\mathbf{r}) = \frac{1}{4\pi}\int d^3r' \frac{D(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$
$W(\mathbf{r}) = \frac{1}{4\pi}\int d^3r' \frac{\mathbf{C}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$
Einstein's Postulates of Special Relativity:
1. The laws of physics are the same in all inertial frames of reference.
2. The speed of light in free space has the same value c in all inertial frames of reference.

Boost in the x-direction in terms of x_i, γ and β:
$x_0' = \gamma(x_0 - \beta x_1)$
$x_1' = \gamma(x_1 - \beta x_0)$
$x_2' = x_2$
$x_3' = x_3$
$\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$

Boost in the x-direction in matrix form: $x'^{\mu} = \Lambda^{\mu}_{\nu}x^{\nu}$, $\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Covariant vs. Contravariant: covariant: a_μ, contravariant: a^μ, $(a_0, a_1, a_2, a_3) = (-a^0, a^1, a^2, a^3)$

Minkowski metric: $a_\mu = g_{\mu\nu}a^\nu$, $g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Four-(v, p, J, A):
$v^\mu = \frac{dx^\mu}{d\tau} = (\gamma c, \gamma \mathbf{v})$
$p^\mu = mv^\mu = (E/c, \mathbf{p})$
$J^\mu = (c\rho, \mathbf{J})$
$A^\mu = (\Phi/c, \mathbf{A})$
Relativistic Energy x2: $E = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}$
Field Tensor and Transformation: $F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$, $F'^{ij} = \Lambda^i_k \Lambda^j_l F^{kl} \rightarrow$
$F' = \Lambda F \Lambda^{\mathrm{T}}$
Maxwell's Equations with d'Alenbertian: $\Box^2 A^\mu = -\mu_0 J^\mu, \Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

5 Miscellaneous Physics

Taylor Expansion: $f(\vec{x} + \vec{a}) = f(\vec{x}) + a_i \partial_i f(\vec{x}) + \mathcal{O}(\vec{a}^2)$
Gaussian Integral: $\int\limits_{-\infty}^{\infty} dx e^{-ax^2+bx+c} = \sqrt{\frac{\pi}{a}} e^{b^2/(4a)+c}$
3 types of Boundary Conditions:
Dirichlet: $\Phi(\mathbf{a}) = const$
Neumann: $\frac{\partial \Phi(\mathbf{a})}{\partial \mathbf{n}} = \hat{\mathbf{n}} \cdot \nabla \Phi = const$
Robin: Linear combination of the first two
Value of fine structure constant: $\alpha \approx \frac{1}{137}$
Mass of electron in eV: $m_e c^2 = 0.511 eV$
Value of the Bohr radius: $a_0 = 0.529$ Å
Wave Equation: $\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$
Diffusion Equation: $\nabla^2 u - \frac{1}{D}\frac{\partial u}{\partial t} = 0$