PHY6938 Proficieny Exam Fall 2002
September 13, 2002
Modern Physics and Quantum Mechanics

1. Consider a one-dimensional step potential of the form

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x \geq 0 \end{cases}$$

where $V_0 > 0$. A particle with total energy $E > V_0$ and mass m is incident on the step potential "from the left" (in other words: the particle starts at negative values of x and travels toward positive values of x).

(a) Use the time-independent Schrödinger equation to determine the form of the particle's wave function in the two regions $x < 0$ and $x \geq 0$.

(b) Derive expressions for the probabilities that the particle is reflected (R) and transmitted (T).
Hint: Recall that the probability density current is given by

$$j(x) = \mathrm{Re}\left(\Psi^* \frac{\hbar}{im} \frac{\partial \Psi}{\partial x}\right),$$

and that R and T are ratios of probability density currents.

2. The nucleus ^{113}Cd captures a thermal neutron having negligible kinetic energy, producing ^{114}Cd in an excited state. The excited state of ^{114}Cd decays to the ground state by emitting a photon. Find the energy of the photon.

3. A current of quantum-mechanical particles of mass m can be written as

$$\vec{J} = -\frac{i\hbar}{2m}\left[\psi^* \nabla \psi - \psi \nabla \psi^*\right],$$

where ψ is the wave function. Assume that the particles move through a region of space where the potential is complex $V = V_r - iV_i$. Show that particles are being annihilated at a rate

$$R = \frac{2}{\hbar} V_i \, \psi^* \psi$$

per unit volume.
Hint: Use the time-dependent Schrödinger equation to obtain "the material derivative"

$$\frac{\partial}{\partial t}(\psi^* \psi) + \nabla \cdot \vec{J}.$$

4. Cosmic ray photons from space are bombarding your laboratory and smashing massive objects to pieces. Your detectors indicate that two fragments, each of mass m_0, depart such a collision moving at speed $0.6\,c$ at angles of 60° relative to the photon's original direction of motion.

(a) In terms of m_0 and c, what is the energy of the cosmic ray photon ?

(b) In terms of m_0, what is the mass M of the particle being struck (assumed to be initially stationary) ?