PHY6938 Proficieny Exam Fall 2002
September 13, 2002
Optics and Thermodynamics

1. Objects at finite temperature T emit electromagnetic radiation with a continuous spectrum, called Blackbody radiation. The radiated power per unit area and unit wavelength is given by the function

$$F(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1},$$

where λ is the wavelength, $c = 3.00 \times 10^8$m/s is the speed of light, $h = 6.26 \times 10^{-34}$ Js is Planck's constant, and $k_B = 1.381 \times 10^{-23}$J/K is Boltzmann's constant.

(a) Make a sketch of $F(\lambda)$ as a function of λ.

(b) The power density $F(\lambda)$ has a maximum at a particular wavelength, λ_{max}. Derive the relation

$$T\lambda_{\text{max}} \approx 2.90 \times 10^{-3} Km.$$

This result is known as Wien's Displacement law.
Hint: It may be useful to note that the two roots of the transcendental equation $5 - x = 5e^{-x}$ are 0 and approximately 4.965.

(c) Show that the total power radiated per unit area is given by the Stefan-Boltzmann law,

$$P = \sigma T^4,$$

with the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}$.
Hint: The following integral may be useful:

$$\int_{-\inf}^{\inf} \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$$

2. Two lenses are separated by 35 cm. An object is placed 20 cm to the left of the first lens, which is a converging lens of focal length 10 cm. The second lens if a diverging lens of focal length -15 cm. What is the position of the final image ? Is the image real or virtual ? Erect of inverted ? What is the overall magnification of the image?