18 October 2014, 12:00 pm

P521: Classical Mechanics Mid-Term Exam

Due: 20 October 2014 (Monday), 12:00 pm, in GWC520.

Important notice: No collaboration is permitted on this exam. Only the textbook by Goldstein, Tong's lecture notes, and class notes you may have taken, can be used. If you have technical questions about the exam, they should preferably be posted on Blackboard for the benefit of the whole class.

1. (20 points) Suppose that a particle moves in space subject to a conservative potential $V(\mathbf{r})$ but is constrained always to move on a surface whose equation is $\sigma(\mathbf{r}, t) = 0$. (The explicit dependence on t indicates that the surface may be moving.) The instantaneous force of constraint is taken as always perpendicular to the surface. Show analytically that the energy of the particle is not conserved if the particle moves in time. What physically is the reason for nonconservation of the energy under this circumstance? (The example of a bead on a moving rod may be helpful.)

2. (20 points) Consider the system of two identical pendulums of lengths $l_1 = l_2$ and mass $m_1 = m_2$. Accleration due to gravity is g. Suppose that the pendulums are connected by a weightless spring of spring constant k and whose natural length is equal to the distance between the points of suspension. Find the normal modes and frequencies of small oscillations.

3. (20 points) Consider a pendulum suspended at some location on the Earth. The length of the pendulum can be taken to be much shorter than the radius of the Earth. Approximate the Earth as a sphere. Find an equation for the angle, α, that the pendulum makes with the radial direction to the center of the Earth. Solve the equation in the static case (no oscillations) assuming that α is small. At what co-latitude is α maximum? What is the numerical value of the maximum α?

4. (20 points) Can the topmost, usually unstable, equilibrium position of a pendulum become stable if the point of suspension oscillates in the vertical direction? Justify your answer with calculations.