PHY6938 Proficieny Exam Spring 2003
March 28, 2003
Modern Physics and Quantum Mechanics

1. Light of wavelength 300 nm strikes a metal plate, producing photoelectrons that move with speed of $0.002c$.

   (a) What is the work function of the metal ?

   (b) What is the critical wavelength for this metal, so that photoelectrons are produced ?

   (c) What is the significance of the critical wavelength ?

2. A system consists of two distinguishable particles of mass m, bound in an infinite-strength one-dimensional square well potential of width a. They have no spin and do not interact. The one-particle states are given by

$$\Psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$$

   (a) Write the 2-body Hamiltonian in terms of the individual particle coordinates x_1 and x_2. Write down the wave-functions for all excited states.

   (b) Verify that they are eigenfunctions of the Hamiltonian and find their energy.

   (c) Consider now that both particles are identical bosons. Write down the excited state wave-functions. What changes with respect to part (b) ? You may ignore the normalization of the wavefunction.

   (d) What would change if the particles were identical spin 1/2 fermions ? Write down the ground state and first excited state wave-functions, in terms of eigenfunctions of the individual spins.

3. Consider an electron in a hydrogen atom that has the following wave function at a particular time, $t = 0$:

$$|\psi(0)\rangle = A\left(|100\rangle + 2i|210\rangle + 2|322\rangle\right).$$

   Here, each of the individual eigenvector terms are denoted by their quantum numbers N (principal), L (angular momentum), and M (angular momentum projection) in the following manner: $|NLM\rangle$.

   (a) Calculate the value of the normalization constant A.

   (b) Find the expectation value of the energy of this electron at $t = 0$. Express your answer in units of eV.

   (c) If a measurement of the z-projection of the electron's orbital angular momentum is made at $t = 0$, then with what probability are the results 0, $\hbar$, $2\hbar$, and $3\hbar$ obtained?

   (d) Write the expression for the wave function $|\Psi(t)\rangle$ at any time t after $t = 0$.

3. Cosmic ray photons from space are bombarding your laboratory and smashing massive objects to pieces! Your detectors indicate that two fragments, each of mass m_0, depart such a collision (between a photon and a particle of mass M) moving at speed 0.6 c at 60° to the photon's original direction of motion.

   (a) In terms of m_0 and c, what is the energy of the cosmic ray photon?

   (b) In terms of m_0, what is the mass M of the particle being struck (assumed originally stationary)?