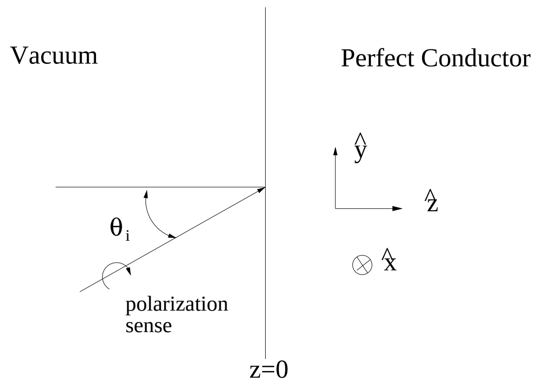
PHY531 Problem Set 5. Due March 26, 2015

1. The region $z < 0$ is vacuum, while the region $z > 0$ is filled with a perfect conductor. A circularly polarized plane electromagnetic wave of angular frequency ω is incident from negative z. The coordinates are chosen so that its wave vector is in the $y - z$ plane, and the angle of incidence is θ_i. The electric field rotates in the clockwise direction as viewed from the wave's source. The electric field can be written in the form Re $E_0\hat{\boldsymbol{\lambda}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t}$, with E_0 real. The geometry is shown below.



   a. Find expressions for $\hat{\boldsymbol{\lambda}}$ and $\boldsymbol{k}$ as functions of θ_i and ω.

   b. Use Maxwell's equations and Stokes and/or the divergence theorem to show how you derive the boundary conditions that $\boldsymbol{E}$ and $\boldsymbol{B}$ satisfy at $z = 0$.

   c. Calculate $\boldsymbol{E}$ and $\boldsymbol{B}$ everywhere.

   d. Use the Maxwell stress tensor to calculate the force per unit area on the plane.

2. Jackson problem 7.5. By $\sigma \gg \omega$, Jackson means that in the equation $\boldsymbol{\nabla} \times \boldsymbol{H}_\omega = \boldsymbol{J}_\omega - i\frac{\omega}{c}\epsilon(\omega)\boldsymbol{E}_\omega$ he writes $\boldsymbol{J}_\omega = \sigma\boldsymbol{E}_\omega$, and drops the $\omega\epsilon(\omega)$ term. Alternatively, you can view this as having $\boldsymbol{J}_\omega = 0$, and taking $\epsilon(\omega) = i\frac{\sigma}{\omega}$, since you really cannot separately measure the conduction and polarization contributions for $\omega > 0$.

   Your infinite thickness result for the reflection coefficient should agree with the result that Jackson gives for part b of problem 7.4. Penetration depth and skin depth are the same thing.

3. Jackson problem 7.8.

4. Show that if $E > v(x)$, that a nonrelativistic particle of mass m described by the 1-dimensional Schrödinger equation, scattering from the 1-dimensional potential $v(x)$ which is given by constant values V_j over a thicknesses t_j, can be described by the same mathematics as that in Jackson problem 7.8. If we write the wave function in each layer as

$$\psi_+ e^{ikx} + \psi_- e^{-ikx} \tag{1}$$

in each layer. Show that taking $\hbar\omega = E$, and $n_j = \sqrt{\frac{2mc^2(E-V_j)}{E^2}}$, the equations relating the $\psi_\pm$ coefficients are identical to those that relate $E_\pm$, so that this transfer matrix formalism can be used to calculate 1-d scattering.

Notice that larger index of refraction corresponds to more negative potentials.

5. Jackson problem 7.9. You may (or may not) find the result that you have no doubt learned in quantum mechanics, $\sigma_\ell \sigma_m = \delta_{\ell m} + i \sum_{k=1}^{3} \epsilon_{\ell m k} \sigma_k$, useful.