PHY6938 Modern and QM: Fall 99 to Spring 97

1. Consider an electron moving in one dimension in the presence of an attractive potential of the form:

$$V(x) = \begin{cases} -V_0, & |x| \leq a, \\ 0, & |x| > a. \end{cases}$$

a) Find the equation that determines the bound state energy eigenvalues. Namely, give an algebraic equation that, when solved, provides the energy as a function of V_0 and a.

b) Consider the limit of a very weak bound state, i.e. $|E|/V_0 \ll 1$. Find the ground state energy and wave function (apart from an overall multiplicative constant) in this limit. How can one determine this multiplicative constant?

2. Provide a brief qualitative description for each item listed below.

a) Heisenberg uncertainty principle

b) The fundamental forces

c) Michelson-Morley experiment

3. When light of wavelength 520 nm is incident on the surface of a metal, electrons are ejected with a maximum speed of 1.78×10^5 m/s. What wavelength is needed to give a maximum speed of 4.81×10^5 m/s?

4. A particle of mass m is confined in a one-dimensional infinite square well of width a.

a) Find the energy and the wave function of the n-th level.

b) A one-dimensional electron is confined between two impenetrable walls a distance 1 Å apart. What is the energy of the electron in the first excited state in electron volts? Note $m_e = 0.511$ MeV/c^2, and $\hbar c = 197.3$ MeV·fm.

c) What is the average force the electron applies on each wall in this state?

5. Provide a brief qualitative description for each item listed below.

a) Čerenkov radiation

b) Planck's constant

c) Rutherford scattering

6. Let $\mathbf{s}_1$ and $\mathbf{s}_2$ be the spin operators of two spin-1/2 particles.

a) Find the simultaneous eigenfunctions of the operators $\mathbf{s}^2$ and s_z, where $\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2$.

b) Assume the spin of these particles are coupled by an exchange interaction of the form $H = -J\mathbf{s}_1 \cdot \mathbf{s}_2$. What are the allowed energy states of the system?

c) Assuming $J > 0$, what is the degeneracy of the ground and the first excited state?

d) If an external magnetic field $\mathbf{H}$ is turned on, how are the allowed energy states modified? (*Hint:* the coupling of the field to the spins is of the Zeeman type, $H_Z = -g\mu_B \mathbf{s} \cdot \mathbf{H}$).

7 a). How fast does a muon have to travel to have the same energy as a charged pion at rest?

b) What is the de Broglie wavelength of the muon?

c) If you could measure the momentum of the muon with 10% accuracy, how well could you measure its position along its direction of motion?

8. Provide a brief qualitative description for each item listed below.

a) Compton effect

b) the difference between bosons and fermions

c) Rutherford scattering

9. Provide a brief qualitative description for each item listed below.

a) Hyperfine structure in a hydrogen atom

b) Rayleigh scattering

c) The four basic forces (describe each force)

10. Monochromatic blue light with a wavelength of 434.2 nm is incident on a sample of cesium. Electrons emitted from the cesium surface are observed to have velocities ranging up to 5.491×10^5 m/s. Note $m_e = 0.511$ MeV/c^2, $q_e = 1.602 \times 10^{-19}$ C, $hc = 1240$ eV·nm.

a) What is the work function for this sample of cesium?

b) Explain why there is a *range* of emitted electron velocities.

c) What is the wavelength of the fastest emitted electrons?

d) Now assume that the hydrogen discharge lamp produces 2.0 μW of power radiated in this particular blue Balmer series spectral line. If the lamp can be considered to be a point source and emits the light isotropically, estimate how many of these blue photons per second strike a circular cesium sample 7.5 cm in diameter and placed 10 cm from the lamp.

11. A photon with wavelength 24.8 fm strikes a proton at rest. The photon undergoes Compton scattering, and the scattered photon is seen by an observer in the lab to be emitted at 180° with respect to the direction of the incident photon. The mass of the proton is $M_p = 938$ MeV/c^2, and $hc = 1240$ MeV·fm.

a) What is the energy of the incident photon? What name would typically be given to classify this "type" of photon? Give a very brief explanation for your choice.

b) Using relativistic kinematics, find (i) the wavelength of the scattered photon and (ii) the de Broglie wavelength of the recoiling proton.

c) If we could observe this reaction occuring in the center-of-mass frame instead of the lab frame, what would we then see as the difference between the wavelengths of the incoming and scattered photons? Explain your answer.

12. Consider a one dimensional step potential of the form:

$$V(x) = \begin{cases} 0, & x < 0, \\ +V_0, & x \geq 0. \end{cases}$$

A particle with total energy E and mass m is incident on the step potential "from the left" (in other words, the particle starts at negative values of x and travels towards positive values of x). The particle's energy E is greater than V_0.

a) Use the time-independent Schrödinger equation to determine the form of the particle's wave function in the two regions $x < 0$ and $x \geq 0$.

b) Derive expressions for the probabilities that the particle is (i) reflected (R), and (ii) transmitted (T). (*Hint*: recall that the probability density current is given by

$$j(x) = \mathrm{Re}\left(\Psi^* \frac{\hbar}{im}\frac{\partial \Psi}{\partial x}\right),$$

and that R and T are ratios of probability density currents.)

13. A particle of mass m is confined to a one-dimensional, square potential well with infinitely high potential walls at $x = 0$ and L.

a) Find the ground state energy and wave function of the problem.

b) From arguments based on the uncertainty principle, estimate a lower bound for the smallest energy that a particle in this potential can have, and compare it with the result of part a).

c) Write the ground state wave function if two identical particles are introduced in the well. Suppose the particles carry spin 1/2 and do not interact with each other.

d) Now consider a square-well-like potential of depth $V_0 = 1000$ eV. Suppose it is given that the first four eigenstates of the problem have an energy $E_1 = 20$ eV, $E_2 = 70$ eV, $E_3 = 200$ eV, and $E_4 = 500$ eV, measured from the bottom of the well. Introduce eight electrons in the problem (they have spin). What is the minimum energy necessary to remove an electron from the ground state of the well and move it to an infinite distance? Assume the electrons do not interact among themselves.

14. Consider an electron in a hydrogen atom that has the following wave function at a particular time, $t = 0$:

$$|\psi(0)\rangle = A\left(|100\rangle + 2i|210\rangle + 2|322\rangle\right).$$

Here, each of the individual eigenvector terms are denoted by their quantum numbers N (principal), L (angular momentum), and M (angular momentum projection) in the following manner: $|NLM\rangle$.

a) Calculate the value of the normalization constant A.

b) Find the expectation value of the energy of this electron at $t = 0$. Express your answer in units of eV.

c) If a measurement of the z-projection of the electron's angular momentum is made at $t = 0$, then with what probability are the results 0, $h/2\pi$, h/π, and $3h/2\pi$ obtained?

d) Write the expression for the wave function $|\Psi(t)\rangle$ at any time t after $t = 0$.