

Study guide for qualifying exams

Cody L. Petrie

June 22, 2015

1 Classical Mechanics

1. Newtonian Mechanics

- (a) Newton's Laws/Kinematics
- (b) Energy
- (c) Momentum/Angular Momentum

2. Lagrangian Mechanics

- (a) Calculous of Variations
- (b) Principle of Least Action/Lagranges Equation
- (c) Generalized Coordinates
- (d) Holonomic/Non-Holonomic Constraints
- (e) Noether's Theorem
- (f) Rigid Body Motion
 - i. Inertia Tensor
 - ii. Euler's Equations

3. Hamiltonian Formalism

- (a) Legendre Transformation/Hamilton's Equations
- (b) Generalized Momenta/Cyclic Coordinates/Conserved Quantities
- (c) Liouville's Theorem
- (d) Poisson Brackets
- (e) Canonical Transformations

2 Statistical Mechanics

1. Thermodynamics Review
 - (a) Laws of Thermodynamics
 - (b) Intensive vs Extensive Variables
 - (c) Thermodynamic Potentials and Ensembles
 - (d) Maxwell's Relations
 - (e) Various Definitions
 - i. Compressibility
 - ii. Heat Capacity etc.
2. Statistical Mechanics
 - (a) Statistical Review
 - (b) Partition Function/Trace
 - (c) Thermodynamic Limit
 - (d) Density Matrix
 - (e) Ideal Gas
 - (f) Ideal Bose Gas
 - (g) Ideal Fermi Gas
 - (h) Photons (BB)
 - (i) Phonons
 - (j) Bose-Einstein Condensates
 - (k) Cluster Expansion

3 Quantum Mechanics

1. Shankar Math Review
2. Postulates
3. Free Particle
4. Particle in a Box
5. Harmonic Oscillator
6. Angular Momentum
7. Hydrogen Atom
8. Spin

- 9. Angular Momentum Addition
- 10. Time-Independent Perturbation Theory
- 11. Time-Dependent Perturbation Theory
 - (a) Einstein A and B Coefficients
- 12. Scattering
- 13. WKB Formula
- 14. Dirac Equation

4 Electricity and Magnetism

- 1. Electrostatics
 - (a) Coulomb's Law
 - (b) Electrostatic Potentials
 - i. Poisson/Laplace's Equations
 - (c) Boundary Conditions
 - (d) Method of Images
 - (e) Multipole Expansion
 - (f) Work and Energy
 - (g) Electric Fields in Matter
- 2. Magnetostatics
 - (a) Lorentz Force Law
 - (b) Biot-Savart Law
 - (c) Vector Potential
 - (d) Magnetic Fields in Matter
- 3. Electrodynamics
 - (a) Ohm's Law
 - (b) Maxwell's Equations
 - (c) Boundary Conditions to Maxwell's Equations
 - (d) Continuity Equation
 - (e) Poynting's Theorem
 - (f) Maxwell Stress Tensor

- (g) Electromagnetic Waves
 - i. The Wave Equation from Maxwell's Eq.
 - ii. EM Waves in Matter
 - iii. Wave Guides
- 4. Scalar and Vector Potentials
- 5. Coulomb and Lorentz Gauge
- 6. Retarded Potentials
- 7. Lienard-Wiechert Potentials
- 8. Radiation
 - (a) Electric/Magnetic Dipole Radiation
- 9. Helmholtz Theorem
- 10. Special Relativity
 - (a) Einstein's Postulates
 - (b) Lorentz Transformation
 - (c) 4-Vectors
 - (d) Field Tensor and Transformation
 - (e) Relativistic Potentials

5 Classical Mechanics Equations

Newtonian Mechanics

Newton's Laws:

1. An object will maintain it's current motion unless acted upon by an external force.
2. $\vec{F} = m\vec{a}$
3. All forces occur in equal but directionally opposite pairs.

Second Law: $\vec{F} = m\vec{a} = \dot{\vec{p}}$

Angular Position/Velocity/Acceleration: $\theta = s/r$, $\omega = v/r$, $\alpha = a/r$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F} = \dot{\vec{L}}$

Centripetal Acceleration: $a_c = v^2/r$

Centrifugal/Coriolis Forces: $\vec{F}_{cent} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})$, $\vec{F}_{cor} = -2m\vec{\omega} \times \dot{\vec{r}}$

Work to go from positions \vec{a} to \vec{b} : $W_{ab} = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{s}$

Conservative Force Field (2 eq): W_{ab} is the same regardless of path so $\oint \vec{F} \cdot d\vec{s} = 0$, and thus we can write the force as $\vec{F} = -\nabla V(\vec{r})$.

Lagrangian Formalism

Functional Derivative: $\frac{\delta F}{\delta u}[x_0] = \lim_{\epsilon \rightarrow 0} \frac{F[x_0 + \epsilon u] - F[x_0]}{\epsilon} \rightarrow \frac{\delta F}{\delta x(t)}[x(t')] = \lim_{\epsilon \rightarrow 0} \frac{F[x(t') + \epsilon \delta(t' - t)] - F[x(t')]}{\epsilon}$

Principle of Least Action: $\delta S = 0$, where $S = \int_{t_i}^{t_f} L(\vec{q}, \dot{\vec{q}}, t) dt$

Lagrange's Equation: $\frac{d}{dt} \frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = 0$

Holonomic Constraints: $f_\alpha(x^A, t) = 0$, $L' = L(x^A, \dot{x}^A) + \lambda_\alpha f_\alpha(x^A, t) \rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{x}^A} - \frac{\partial L}{\partial x^A} = \lambda_\alpha \frac{\partial f_\alpha}{\partial x^A}$

Noether's Theorem: A continuous symmetry in the Action (and thus Lagrangian) result in a conserved quantity.

Moment of Inertia Tensor: $\vec{L} = \overleftrightarrow{I} \vec{\omega}$, $T = \frac{1}{2} \omega_a I_{ab} \omega_b$, $I_{ab} = \sum_i m_i ((\vec{r}_i \cdot \vec{r}_i) \delta_{ab} - (\vec{r}_i)_a (\vec{r}_i)_b)$

Euler's Equations: Only look at rotation, not translation. Conservation of Angular Momentum gives $I_i \dot{\omega}_i + \omega_j \omega_k (I_k - I_j) = 0$, for i, j, k being cyclic permutations of 1, 2, 3.

Hamiltonian Formalism

Generalized Momenta: $p_i = \frac{\partial L}{\partial \dot{q}_i}$, $\dot{p}_i = \frac{\partial L}{\partial q_i}$

Hamiltonian: $H(q_i, p_i, t) = \sum_{i=1}^n p_i \dot{q}_i - L(q_i, \dot{q}_i, t)$

Hamilton's Equations:

1. $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

2. $\dot{q}_i = \frac{\partial H}{\partial p_i}$
3. $-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$

Cyclic/Ignorable Coordinates: q is ignorable if $\frac{\partial L}{\partial q} = 0$, i.e. if q does not appear in L .

Thus $p = \frac{\partial L}{\partial \dot{q}}$ is conserved.

Liouville's Theorem: A volume of a region of phase space remains the same, even when the region changes. $V = dq_1 \dots dq_n dp_1 \dots dp_n$.

Poisson Bracket: $f, g = \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}$.

Constant of Motion from Poisson Bracket: $\frac{df}{dt} = f, H + \frac{\partial f}{\partial t}$. If $I, H = 0$, then I is a constant of motion.

Canonical Transformation: Transformation $(q_i \rightarrow Q_i(q, p), p \rightarrow P_i(q, p))$ that leaves Hamilton's equations invariant.

6 Statistical Mechanics Equations

6.1 Thermodynamics

Laws of Thermodynamics:

1. Energy conservation. $dE = dQ - pdV$. dQ just means that the heat is an inexact differential and the integral depends on the path.
2. $\Delta S \geq \int \frac{dQ}{T}$, where equality is for a process that is reversible (never leaves equilibrium).
3. Entropy at zero temperature is zero. In stat mech this means that the ground state is nondegenerate and $S \propto \ln(W)$, where W is the number of available states.

Intensive vs Extensive Variables: Intensive variables do NOT scale with system size (T, p, μ) , while extensive do scale (E, S, V, N) .

Thermodynamic Potentials:

- Internal Energy: $U(S, V, N)$
- Helmholtz Free Energy: $F(T, V, N) = U - TS$
- Enthalpy: $H(S, p, N) = U + pV$
- Gibbs Free Energy: $G(T, p, N) = U - TS + pV$
- Landau(Grand) Potential: $\Omega(T, V, \mu) = U - TS - \mu_i N_i$

Thermodynamic Ensembles:

1. Microcanonical: Does not exchange energy or particles with environment. Fixed E, N
2. Canonical: Does not exchange particles, but can exchange energy (heat bath). Fixed N, T

3. Grand canonical: Can exchange energy and particles with environment. Fixed T, μ .

Maxwell's Relations (4 main):

- $\frac{\partial^2 U}{\partial S \partial V} = - \left(\frac{\partial p}{\partial S} \right)_V = \left(\frac{\partial T}{\partial V} \right)_S$
- $\frac{\partial^2 F}{\partial T \partial V} = \left(\frac{\partial p}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T$
- $\frac{\partial^2 H}{\partial S \partial p} = \left(\frac{\partial V}{\partial S} \right)_p = \left(\frac{\partial T}{\partial p} \right)_S$
- $\frac{\partial^2 G}{\partial T \partial p} = \left(\frac{\partial V}{\partial T} \right)_p = - \left(\frac{\partial S}{\partial p} \right)_T$

Engine Efficiency: $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{T_{out}}{T_{in}}$

Isobaric Thermal Expansion Coefficient: $\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$, How much the volume changes with a change in temperature.

Isothermal Compressibility: $\kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$, How much the volume changes when the pressure changes.

Isentropic(Adiabatic) Compressibility: $\kappa_S = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_S$, Same as above.

Specific Heat at Constant V: $C_V = \left(\frac{\partial Q}{\partial T} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V$, Amount of heat per unit mass to raise the temp by 1 degree.

Specific Heat at Constant p: $C_p = \left(\frac{\partial Q}{\partial T} \right)_p = \left(\frac{\partial H}{\partial T} \right)_p$, Same as above.

Fermi Energy/Temperature: Chemical potential at $T = 0$. $\epsilon_F = \mu(T = 0)$

6.2 Statistical Mechanics

Number of microstates in a macrostate (ways to get n heads): $\Omega = \frac{N!}{\prod_i n_i!}$

Stirling's Approximation: $\ln n! = n \ln n - n$

How many order important ways to order n things: $n!$

How many order important ways to order n things r at a time: $\frac{n!}{(n-r)!}$

How many NOT order important ways to order n things r at a time: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Microcanonical(Classical) Partition Function: $Z_m = \sum_s g_s e^{-\beta E_s}$

Canonical Partition Function: $Z_c = \text{tr} \left(e^{-\beta \hat{H}} \right)$

Grand Canonical Partition Function: $Z_{gc} = \text{tr} \left(e^{-\beta(\hat{H} - \mu \hat{N})} \right)$

Geometric Series: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

Classical limit of the trace of an operator: $\text{tr}(\mathcal{O}) = \frac{1}{N!(2\pi\hbar)^{3N}} \int d^3r_1 \dots d^3r_N \int d^3p_1 \dots d^3p_N \mathcal{O}$, $N!$ is for identical particles.

Thermodynamic Limit: $T \rightarrow \infty, V \rightarrow \infty, N/V = \text{const}$

Expectation value for pure/mixed: $\langle \mathcal{O} \rangle_p = \langle \psi | \mathcal{O} | \psi \rangle, \langle \mathcal{O} \rangle_m = \sum_i P_i \langle \psi_i | \mathcal{O} | \psi_i \rangle$

Density Matrix (ex. Canonical Ensemble): $\rho = \sum_n P_n |\psi_n\rangle \langle \psi_n|, \rho_c = \frac{e^{-\beta \hat{H}}}{\text{tr} e^{-\beta \hat{H}}}$

Expectation value with Density Matrix: $\langle \mathcal{O} \rangle = \text{tr}(\mathcal{O} \rho)$

Trace of Density matrix: $\text{tr}(\rho) = 1$

Time evolution of density matrix: $\frac{\partial}{\partial t} \rho = \frac{1}{i\hbar} [\hat{H}, \hat{\rho}]$

Z_{gc} for an ideal gas: $Z_{gc} = \frac{V^N (2mT\pi)^{3N/2}}{N! (2\pi\hbar)^{3N}} e^{\beta\mu}$

Z_{gc} for ideal fermi gas: $Z_{gc} = \prod_k (1 + e^{-\beta(\epsilon_k - \mu)})$

Z_{gc} for ideal bose gas: $Z_{gc} = \prod_k \frac{1}{(1 - e^{-\beta(\epsilon_k - \mu)})}$

Stuff here for black-body and phonons and bose condensates.

What is cluster expansion used for?: Systems of interacting particles.

7 Quantum Mechanics Equations

Postulates of QM:

1. The state of a physical system, at some fixed time, is given by a normalized ray in a Hilbert space over the complex numbers. (ray is vector whose norm doesn't matter)
2. The ray evolves deterministically in time according to Schrödingers equation.
3. Observables correspond to self-adjoint (hermitian) operators.
4. If a particle is in the state $|\psi\rangle$ then a measurement of \mathcal{O} will yield one of the eigenvalues of \mathcal{O} , ω . The state of the system changes to an eigenstate of \mathcal{O} , $|\omega\rangle$.

Schrödinger equation: $i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$

8 Electricity and Magnetism Equations

9 Miscellaneous Physics

Taylor Expansion: $f(\vec{x} + \vec{a}) = f(\vec{x}) + a_i \partial_i f(\vec{x}) + \mathcal{O}(\vec{a}^2)$