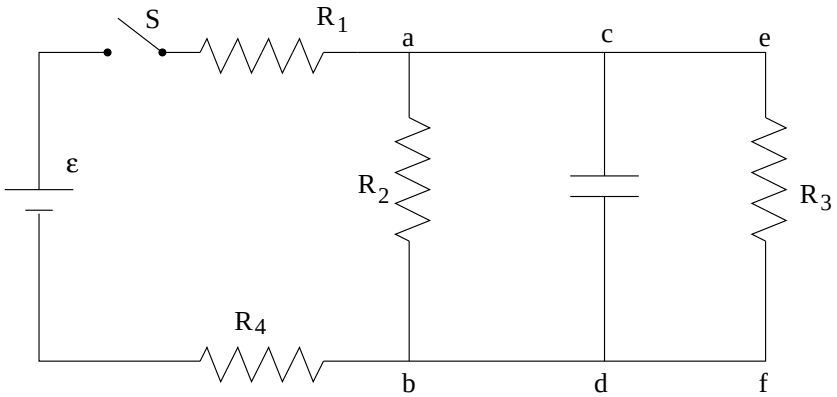
PHY6938 E & M Spring 99 and Fall 98

Note: hand in questions 1, 3, 5 and 6 as homework; we will work 2 and 4 in class on Tuesday February 27.

1. When switch S is closed, a current flows in the circuit which changes with time. Eventually, after a sufficiently long time, a steady state is reached in which all current is constant. Unless otherwise instructed, give all answers for this steady situation. For the circuit shown:



$\varepsilon = 10$ V
$R_1 = R_4 = 500\ \Omega$
$R_2 = 6$ kΩ
$R_3 = 1.2$ kΩ
$C = 6\ \mu$F

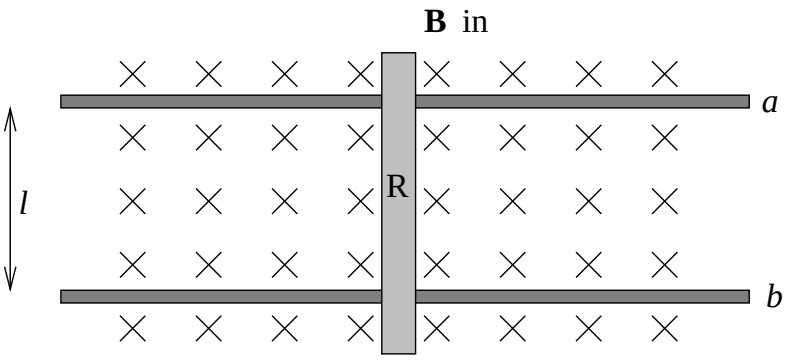
a) What is the battery current?

b) What is the potential difference between points a and b (i.e. $V_a - V_b$)?

c) What is the potential difference between points c and d? Between e and f?

d) If switch S is reopened after a long time, write the equation for the time dependence of the current in the section of the circuit containing the capacitor.

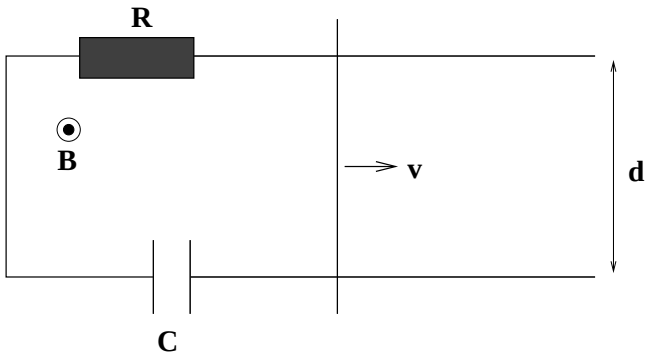
2. In the figure below, the rod has a resistance R and mass M and the rails have negligible resistance. A capacitor with charge Q_0 and capacitance C is connected between points a and b such that the current in the rod is downward. The rod is at rest at time $t = 0$.



a) Write the equation of motion for the rod on the rails.

b) Show that the terminal speed of the rod on the rails is related to the final charge on the capacitor.

3) A closed circuit loop consists of a resistor R, a capacitor C, two parallel conducting tracks a distance d apart, and a rigid wire that can slide along the tracks. A uniform magnetic field $\mathbf{B}$ is applied perpendicular to the loop. Initially there is no current in the loop and the wire is at rest. At $t = 0$ it starts to move to the right with constant speed v, and it comes to a sudden stop at time $t = t_0$. Determine the charge on the capacitor for $t > 0$.



4) A series AC circuit is operated at half its resonance frequency. The coil resistance is 40 Ω, its inductance is 80 mH, and the capacitance is 0.4 μF.

a) What is the resonance frequency of the circuit?

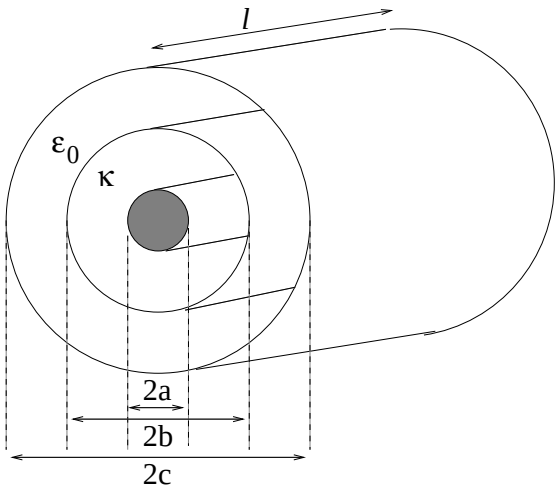
b) What is the impedance of the circuit at half the resonance frequency?

c) What is the phase angle between the voltage and the current at half the resonance frequency?

d) What should the impedance of the source feeding this cicruit be for maximum power transfer?

5) A coaxial capacitor of length l consists of an inner conductor of radius a, a cylindrical dielectric with dielectric constant k of inner radius a and outer radius b, and a shell of free

space of inner radius b and outer radius c. Beyond c is a conducting surface holding total charge q. The inner conductor carries charge $-q$.



a) Find the radial component of the electric field $\mathbf{E}$ in all regions of space.

b) Find the potential V in each region and the total potential across the capacitor (ignore all edge effects).

c) Find the capacitance.

d) Find the stored energy in the capacitor. Is the energy greater than or less than the energy stored in a similar capacitor with no dielectric?

6) What is the net electric force on charge q_1?