Notes on Siemens Ch. 6

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Interactions Beyond the Mean Field

- The mean field approximation gives us basic features of nuclei. But now we're going to move beyond the mean field approximation.
- The first thing we are going to do is look at the pairing term to the binding energy (equation 4.3.2).

$$B_p = \frac{\left[(-1)^N + (-1)^Z\right]\delta}{A^{1/2}} \tag{1}$$

- This gives even-even nuclei a tighter binding energy. Also it turns out that the ground state of even-even nuclei have zero angular momentum.
- To explain these things we are going to go beyond the independent-particle motion (mean field).

Interactions Beyond the Mean Field

- Add a perturbation to the mean field Hamiltonian (H_R is called the residual interaction)
$$H = H_{MF} + H_R \tag{2}$$
  - The eigenstates of H_{MF} are Slater determinants (Uncorrelated SD's?).
- One solution to this is to diagonalize H_R in the H_{MF} basis, but this requires large calculations.
- We are going to use other methods in this chapter. We will split (crudely) into long-range and short-range parts, and look at short-range parts here.

The δ-Force

- Look at degenerate states of H_{MF} because H_R with have a decisive influence.
- Start with H_{MF} in a full j state and two identical nucleons in the next j state.

$$\psi^{nlj}_{JM}(1,2) = \sum_{m_1 m_2} \langle j m_1 j m_2 | JM \rangle \, \mathcal{A} \left[\Phi_{nljm_1}(1) \Phi_{nljm_2}(2) \right] \tag{3}$$

$$\Phi_{nljm_1} = \frac{1}{2} u_{nlj}(r) \sum_{m,s} \left\langle lm \frac{1}{2} s \middle| j m_1 \right\rangle Y_l^m(\theta, \phi) \chi_s \tag{4}$$

- The shortest range for H_R is a δ-force.

$$H_R = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \tag{5}$$

The δ-Force

- This Hamiltonian gives an energy

$$E_R = V_0 \int \psi^*_{JM} \delta(\mathbf{r}_1 - \mathbf{r}_2) \psi_{JM} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \tag{6}$$

$$= \frac{V_0 \left[1 + (-1)^J\right] (2j+1)^2}{32\pi(2J+1)} \left| \left\langle j, \frac{1}{2}, j, -\frac{1}{2} \middle| J, 0 \right\rangle \right|^2 \int\limits_0^\infty r^{-2} u^4_{nlj(r)dr} \tag{7}$$

- Note here that E_R vanishes for odd values of J. This means that two identical Fermi particles in the same j-shell can only be in even angular-momentum states.
- For an attractive force $(V_0 < 0)$ the lowest energy has $J = 0$ and the first excited state is $J = 2$.

The δ-Force

- For all $j > \frac{3}{2}$ the difference in energies of these two states is

$$|(E_2 - E_0)/E_0| \approx \frac{3}{4} \tag{8}$$

which is large as seen in figure 6.2 of the book.

- The two nucleons have their largest spatial overlap in this state ($J = 0$).
- Thus an attractive δ-interaction decreases the energy.

The Degenerate Pairing Model

- A main feature of the δ-force that is maintained in the pairing force is that it only has non-zero matrix elements between time-reversed states. Also, they non-zero elements are all identital

$$\left\langle jm_1\overline{jm_1}\right| V \left|jm_2\overline{jm_2}\right\rangle \equiv -G \tag{9}$$

- Let's use the basis states $j+\frac{1}{2}\equiv\Omega$. Now the Schrödinger equation becomes

$$-G\begin{pmatrix}1 & 1 & \cdots & 1\\ 1 & & & 1\\ \vdots & \ddots & \vdots & \\ 1 & & \cdots & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_\Omega\end{pmatrix} = E\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_\Omega\end{pmatrix} \tag{10}$$

$$-G(x_1+\cdots+x_\Omega) = Ex_1 = Ex_2 = \cdots = Ex_\Omega \tag{11}$$

The Degenerate Pairing Model

$$-G(x_1 + \cdots + x_\Omega) = Ex_1 = Ex_2 = \cdots = Ex_\Omega \tag{12}$$

- This has solutions

$$E = -G\Omega, \qquad \vec{x} = \frac{1}{\sqrt{\Omega}}(1, 1, \cdots, 1) \tag{13}$$

and

$$E = 0, \qquad x_1 + x_2 + \cdots + x_\Omega = 0. \tag{14}$$

- Equation 13 refers to the $J = 0$ state and the degerarate $J > 0$ states have energy of equation 14.

The Degenerate Pairing Model

- Now assume we have n particles in the j-shell ($n \leq 2\Omega$), and p pairs of particles, i.e. $J = 0$ states.
- Using equation 13 and the fact that Ω is the number of possible pairs we get
$$E(n,p) = -Gp\left(\Omega - n + p + 1\right) \tag{15}$$
- Introduce *seniority*, $S = n - 2p$, the number of unpaired nucleons.

$$E(n,S) = -\frac{G}{4}(n-S)(2\Omega - n - S + 2) \tag{16}$$
$$E(n,0) = -\frac{1}{4}Gn(2\Omega - n + 2) \tag{17}$$
$$E(2,0) = -G\Omega \tag{18}$$
$$E(2\Omega,0) = -Gn/2 \tag{19}$$

The Degenerate Pairing Model

- The pairing force creates as many pairs of particles as possible.
- Even-even have zero spin.
- Odd numbers of nuclei, spin is determined by unpaired nucleon.
- Comparing odd-mass and even-mass nuclei we get,

$$E(2p+1,p) - E(2p,p) = Gp \tag{20}$$

  which seems to agree with experiment.
- For even-even nuclei, the lowest excited state is that of one broken pair, which has symmetric energy

$$E(2p,p-1) - E(2p,p) = G\Omega \tag{21}$$