Improved Trial Wave Functions for Quantum Monte Carlo Calculations of Nuclear Systems and Their Applications

Cody L. Petrie

February 22, 2019

Outline and Due Dates

Also add a list of figures???
Oliver will probably want to see pictures of scalings and timings. Scaling/processor etc. Include some nice pictures of these things.

1. Background/Motivation Tuesday Feb 19 Hardest section but really important. Make sure to explain why the stuff we're working on is interesting, this will lead into talking about other methods.

   (a) Other methods like HF and no-core shell model

   (b) General outline of the dissertation

2. Quantum Monte Carlo Friday Feb 22

   (a) VMC

   (b) DMC

   (c) AFDMC

      i. Mixed expectation values, operator breakup

   (d) Hamiltonian - Phenomenogolical + χEFT + some results with AFDMC/GFMC

3. Trial Wave Function Friday Feb 15

   (a) Slater Determinants

   (b) Pfaffian Explain what it's used for (superfluidity etc.) and why it's more efficient than the SD stuff (need many thousands of SD's to do the same thing).

   (c) Spin-Isospin Dependent Correlations

      i. Quadratic Correlations - include results
         A. Results

      ii. Exponential correlations . . . why they aren't working or how to do it, cluster decomposition If not working then don't say much about it. Explain that there are large variances that need to be taken care of etc.

1

Make sure to include stuff for each member of the committee.

1 Background and Motivation

Mention the recent paper Ground-state properties of doubly magic nuclei from the unitary-model-operator approach with the chiral two- and three-nucleon forces, they do calculations using the Unitary-Model-Operation Aproach (UMOA) on the same light doubly-magic systems that we do but using the χEFT NN and 3N potentials with the similarity renormalization group applied (to soften them?). Nuclear physics sheds light on the extremes. From the structure and processes of atomic nuclei and hypernuclei to the formation and structure of some of the largest objects in the universe, neutron stars. One of the largest obstacles to these regimes stems from our incomplete knowledge of the nuclear interation. Once we settle on a possible interaction, the next obstacle is to solve for properties of many-body nuclear systems using the selected, and often complicated, interaction. Currently the popular choices for 2- and 3-body nuclear interactions come in two flavors, phenomenological and those based in Chiral Effective Field Theory (χEFT). There are a large number of methods that have been developed to solve the many-body nuclear problem, though I will be using the Auxiliary Field Diffusion Monte Carlo (AFDMC) method. Other notable methods are the basis set methods such as no core shell model [1, 2], the coupled-cluster method [3], and the self-consistent Green's function method [4, 5]. For these methods the wave function of the nuclear system is written in terms of a truncated basis, often a harmonic oscillator basis. The momentum cutoff of the basis needs to be higher than the important momenta of the interaction that is being used, in order to do calculations in momentum space. This means however that calculations with sharp potentials (like local hard wall potentials) are difficult to do with basis set methods. They do employ techniques such as Similarity Renormalization Group [6] to soften these types of interaction. This allows them to decrease the number of basis functions used. One of the advantages of basis set methods is that they can use local and non-local, i.e. velocity dependent, potentials. The Quantum Monte Carlo (QMC) methods, which we are using in this work, complement these basis set methods. QMC methods are currently limited to mostly local potentials[1] [7], but can converge for a wide variety of local Hamiltonians. Also, Quantum Monte Carlo methods do not have the momentum cutoff limits or the poor scaling with basis set size of the basis set methods. This is from my comp so maybe work it over a bit

---

[1]Currently, interactions that are linear in the momentum can be used. Higher order terms are treated perturbatively.

One of the most accurate QMC methods is the Green's Function Monte Carlo (GFMC) method, which has had good success calculating properties of light nuclei and nuclear matter using 2- and 3-body potentials as well as electroweak currents [8]. GFMC has been used to calculate binding energies as well as excited states for nuclei up to ^{12}C as well as the nuclear equation of state (EOS) which has been used to study the structure of neutron stars. Nuclear calculations using the GFMC method are limited due to the explicit sum over spin states when calculating expectation values. In 1999 Schmidt and Fantoni [9] proposed the AFDMC method which is practically identical to GFMC in its Monte Carlo sampling of spatial integrals, however AFDMC uses Monte Carlo to sample the spin-isospin sums as well.

Add a bit about HF and how HF, GFMC, and AFDMC all use a similar form for the wave function

In this study, for simplicity, I have only used the AV6' phenom. potential...

Despite the difficulty, science has been making continuous steps toward that understanding. In 1935 Hideki Yukawa proposed the idea that the nuclear interaction, called the strong force, was governed by quanta or exchange particles called pions [10]. From this idea came the Yukawa potential, which is still used in modified form in many nuclear models today. The range of the force proposed by Yukawa was based on the mass of the exchange particle, and the strength was based only on the distance separating the particles. Today we often use potentials that depend on the separation distance between particles, but also their relative spins and isospins. These interactions can be quite complicated making a true understanding of the strong force difficult to achieve.

Currently it is believed that Quantum Chromodynamics (QCD) is the most correct theory to describe the strong force. However, due to asymptotic freedom, at low energies this theory becomes quite difficult to use and so other, approximate methods are often used to study the strong interaction. We use Quantum Monte Carlo methods to investigate different aspects of the strong interaction.

Many approximate methods exist to solve the nuclear many body problem. Some of these include ??????????

Continuing to better understand the interactions between nuclei will advance our understanding of many important processes in the universe.

2 Quantum Monte Carlo

2.1 Variational Monte Carlo

2.2 Diffusion Monte Carlo

2.3 Auxiliary Field Difusion Monte Carlo

Include a bit about GFMC here and show their good results, but also mention the limitations, which lead to needing AFDMC

3   Hamiltonian

Phenomenological plus χEFT potentials, and compare the two. What makes them different at NN (same operator structure)?

4   Trial Wave Function

An accurate trial wave function can drastically improve the accuracy of a variational QMC method such as VMC. Most highly accurate trial wave functions are entirely computationally intractable and are never implemented in QMC methods. In addition to being accurate and computationally tractable we need wave functions that satisfy known physical properties such as cluster decomposition as well as having an overall antisymmetry with respect to particle exchange due to nucleons obeying fermi statistics.

Cluster decomposition arises from the physical intuition that the wave function of two separate, non-interacting systems, A and B as in Figure 1, can be written as the product of their respective wave functions. Mathematically this can be represented as a product of
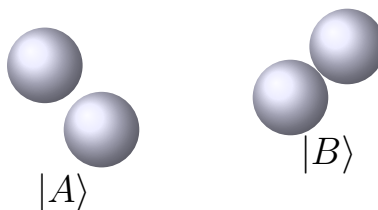

Figure 1: Two non interacting systems A and B, whose composite wave function is the product $|A + B\rangle = |A\rangle |B\rangle$.

n-body functions, where n is often 1 or 2 in our situation, though it could be higher. If a system is not cluster decomposable then unphysical correlations between non-interacting systems can occur.

The second property is that the wave function be antisymmetric overall. Since nucleons are fermions and the only degrees of freedom used in these calculations the product of different pieces of the wave function must be antisymmetric. Recent work in QMC has successfully included bosonic degrees of freedom such as pions [11], however that is not the case in this work.

4.1   Slater Determinant

One of the simplest wave functions that satisfies the two properties specified above is the Slater determinant. The Slater determinant has been the starting place for a variety of many-body calculations in nuclear and condensed matter physics alike. In condensed matter the many-body wave functions will often be written in terms of a sum of weighted Slater determinants, where some methods have been able to use a sum of up to 2 billion determinants

[12, 13]. In nuclear physics a single determinant is often used for closed shell calculations and a sum of a small number, $\mathcal{O}(10)$, of weighted determinants is used for open shell systems. A Slater determinant is an antisymmetrized product of single particle (non-interacting) wave functions

$$\Psi_{SD}(\mathbf{R}, S) = \mathcal{A}\left[\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)\dots\phi_A(\mathbf{r}_A)\right] = \begin{vmatrix} \phi_1(\mathbf{r}_1) & \phi_1(\mathbf{r}_2) & \dots & \phi_1(\mathbf{r}_A) \\ \phi_2(\mathbf{r}_1) & \phi_2(\mathbf{r}_2) & \dots & \phi_2(\mathbf{r}_A) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_K(\mathbf{r}_1) & \phi_K(\mathbf{r}_2) & \dots & \phi_K(\mathbf{r}_A) \end{vmatrix}, \quad (1)$$

where $\mathbf{R}$ and S are the spatial and spin coordinates of the walkers, $\mathcal{A}$ is the antisymitrization operator, and the $\phi_i(\mathbf{r}_j)$ are the overlap of the walker positions with the model single particle states. The single particle model states are made up of a radial and spin, iso-spin dependent parts,

$$\phi_k = \Phi_{nj}\left[C^j_{c_l,m_s}Y_{l,m_l}(\hat{r}_i)\chi_s(s_i)\right]_{j,m_j}, \quad (2)$$

where Φ_{nj} is the radial part and the rest contains the spherical harmonics $Y_{l,m_l}(\hat{r}_I)$ and spin and iso-spin states where the Clebsch-Gordan coefficients ensure the correct j and m_j quantum numbers, and the different states are given by the index k. To accurately describe the wave function of an open shell nuclei each state with the correct total angular momentum and parity J^π and isospin T is included as a seperate Slater determinant.

$$\langle \mathbf{R}S|\Phi\rangle_{J^\pi,T} = \sum_n c_n D\{\phi_k(\mathbf{r}_i, s_i)\} \quad (3)$$

Here the c_n coefficients are variational parameters used to minimize the energy given a set of possible state configurations. One of the simplest examples of an open shell nuclei would be ^{6}He whose ground state is a $J^\pi = 0^+$ state. The two protons and two of the neutrons could be in the full $(1S_{1/2})^2$ shell while the two remaining neutrons could be in the $(1P_{3/2})^2$ shell with their $m_j = \pm3/2, \pm1/2$ values being equal and opposite to ensure that $J = 0$. This state has two possible determinants. Other possible configurations for the two remaining neutrons would be $(1P_{1/2})^2$ with one possible determinant, $(1D_{5/2})^2$ with three possible determinants, $(2S_{1/2})^2$ with one possible determinant and $(1D_{3/2})^2$ with two possible determinants giving a total of nine possible determinants. Notice that the two neutrons could be in a combination of S and D shells but never an S and P or D and P to ensure the parity of the state is positive. The number of determinants used for open shell nuclei will control how accurate the trial wave function is but for closed shell nuclei such as ^{4}He or ^{16}O a single slater determinant describing the full shell configuration is sufficient.

The radial part Φ_{nj} of the single particle states are obtained as bound state solutions to the single particle Schrödinger equation with a Woods-Saxon potential wine-bottle potential.

$$v(r) = V_s\left[\frac{1}{1 + e^{(r-r_s)/a_s}} + \alpha_s e^{(-r/\rho_s)^2}\right] \quad (4)$$

Here the parameters, V_s, r_s, a_s, α_s and ρ_s are variational parameters used to shape the potential to obtain a minimum in energy.

5

The Slater determinant is a mean-field wave function and is often used with Jastrow type short range correlations.

$$\langle \mathbf{R}S|\psi_T\rangle = \langle \mathbf{R}S| \prod_{i<j} f(r_{ij}) |\phi\rangle_{SD} \tag{5}$$

These correlations are spin-isospin independent and depend only on the particle separation and improve upon the uncorrelated Slater determinant wave function significantly. To maintain the cluster decomposition the functions $f(r_{ij})$ must go to unity for large particle seperations. In this work we have used Slater determinant wave functions with a Jastrow factor and spin-isospin dependent correlations which will be discussed in a later section. ADD PLOT AND CITATION BACKING THIS CLAIM HERE.

4.2 Pfaffian Wave Function

Explain why it would be nice to have a pfaffian instead of a determinant and maybe mention some of the results related to superfluidity etc.

Another wave function that obeys these properties is the paired Pfaffian wave function. This wave function was developed to describe Cooper pairs which form when, at low temperature, paired fermions, such as electrons or liquid ^{3}He, are energetically favorable to free particles [14, 15]. This idea was then expanded and used to explain superconductivity as the condensation of these bosonic cooper pairs into the ground state [16, 17]. A Pfaffian wave function was then introduced to describe these paired systems by Bouchaud *et al.* in 1988 [18].

The BCS, or Pfaffian, pairing wave function can be written as an antisymmetrized product of pairing wave functions, thus keeping the antisymmetry of the constituent fermions explicitly. That is,

$$\Psi_{BCS}(\mathbf{R}S) = \mathcal{A}\left[\phi(\mathbf{r}_1, s_1, \mathbf{r}_2, s_2)\phi(\mathbf{r}_3, s_3, \mathbf{r}_4, s_4)\dots\phi(\mathbf{r}_{A-1}, s_{A-1}, \mathbf{r}_A, s_A)\right], \tag{6}$$

where $\mathcal{A}$ is the antisymmetrization operator, $\mathbf{r}_i$ and s_i are the walkers positions and spins, and ϕ are the pairing functions which can be separated into a spatial part, whose form is determined by the system, and a spin-isospin part, which are often written in terms of singlet and triplet states.

This wave function, like the Slater determinant, can be used with additional Jastrow-like correlations as in equation 5.

$$\langle \mathbf{R}S|\psi_T\rangle = \langle \mathbf{R}S| \prod_{i<j} f(r_{ij}) |\phi\rangle_{BCS} \tag{7}$$

For more information and a more detailed use and description of this wave function we refer the reader to [19].

4.3 Spin-Isospin Dependent Correlations

Explain what properties you need, etc. The nuclear Hamiltonian has a strong spin-isospin dependent part and to ensure good overlap with the trial wave function, the wave function must include spin-isospin dependent correlations. From here on I will be using the Slater

determinant for the long-range part of the wave function. To improve on the Jastrow correlations in equation 5, spin-isospin dependent correlations can be included that obey the properties of cluster decomposability and overall antisymmetry.

We have come up with two such correlations, the exponentially correlated,

$$\Psi_{\exp}(\mathbf{R}, S) = \langle \mathbf{R}S| \left[\prod_{i<j} f_c(r_{ij}) \right] e^{\sum\limits_{i<j} \sum\limits_{p} f_p(r_{ij})\mathcal{O}_{ij}^p} |\phi\rangle \tag{8}$$

and the symmetrized product wave functions,

$$\Psi_{\mathrm{SP}}(\mathbf{R}, S) = \langle \mathbf{R}S| \left[\prod_{i<j} f_c(r_{ij}) \right] \left[\mathcal{S} \prod_{i<j} \left(1 + \sum_{p} f_p(r_{ij})\mathcal{O}_{ij}^p \right) \right] |\phi\rangle . \tag{9}$$

where the S is the symmetrization operator, the $f_c(r_{ij}$ are the same Jastrow correlations as before, and the $\mathcal{O}_{ij}^p$ are the operators from the AV6' potential, $(1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}) \otimes (1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)$, where the tensor term is $S_{ij} = 3\boldsymbol{\sigma}_i \cdot \hat{r}_{ij} \boldsymbol{\sigma}_j \cdot \hat{r}_{ij} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$. The $f_p(r_{ij})$ functions contain variational parameters and the functional form is determined by solving a Schrödinger-type equation with the constraint that the wave function be continuous at the healing distance [20, 21].

The exponentially correlated wave function obeys cluster decomposition as long as the correlating functions, $f_p(r_{ij})$ go to zero as the particle seperation increases. This dampens out unphysical long-range particle correlations between physically separated systems. Also, due to the sum over particle pairs in the exponential, no explicit symmetrization is needed.

The symmetrized product wave function, introduced by Pandharipande and Wiringa in 1979 [20], requires an explicit symmetrization and is not obviously cluster decomposable. The addition of the 1 is need to ensure cluster decomposability because these $f_p(r_{ij}$ functions also approach zero as particle seperation increases.

When expanded to linear order these correlations are identical and can be written as

$$|\psi_T\rangle_{\mathrm{lin}} = \left[\prod_{i<j} f_c(r_{ij}) \right] \left(1 + \sum_{i<j} \sum_{p} f_p(r_{ij})\mathcal{O}_{ij}^p \right) |\phi\rangle . \tag{10}$$

These correlations are symmetric, allowing for the full wave function to be antisymmetric, however it has lost the cluster decomposability in the approximation. This is because the pair correlations are summed instead of beign inside a product. For higher orders expansions these two wave functions differ by commutation relations as well as the inclusion of additional correlation pairs. Until recently, only correlations up to linear order in the expansion were used for AFDMC calculations. Calculations for GFMC use a much better wave function, but have been limited to small nuclei up to ^{12}C. Calculations done with the AFDMC method have been slowely improving the trial wave function used, as a better wave function is surely needed to describe larger systems. In 2007 AFDMC binding energy calculations were done for ^{4}He, ^{16}O, and ^{40}Ca using only the Jastrow correlations [22]. These calculations were repeated in 2014 but with the addition of linear correlations [23] and I have plotted the respective results compared to current experimental values here for comparison. In figure 2 it is clear to see that the additional spin-isospin correlations are important for both systems larger than ^{4}He.
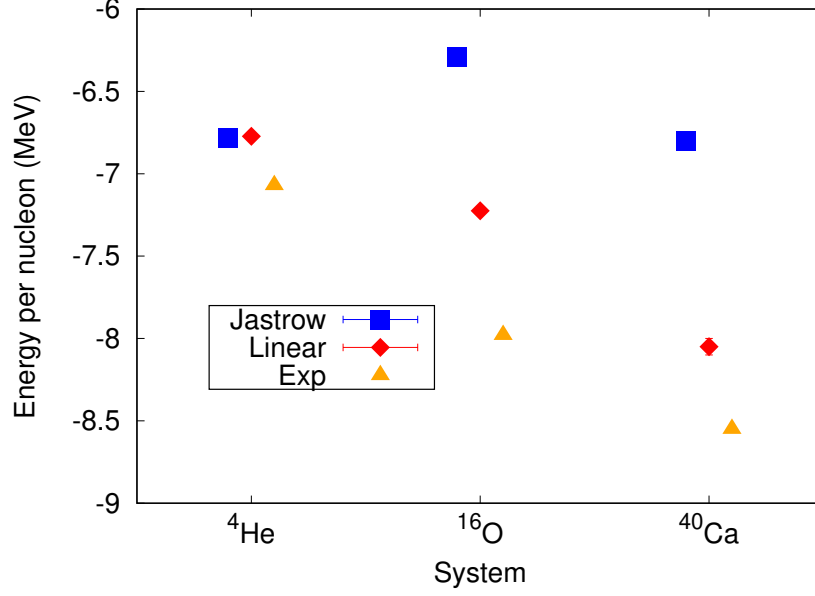
Figure 2: Binding energy calculations done with Jastrow correlations [22] compared to Jastrow plus linear spin-isospin dependent correlations [23] all compared to experimental results. All calculations were done with AFDMC and the AV6′ potential.

4.3.1 Quadratic Correlations

In this project we have included correlations up to quadratic order which includes up to 4 nucleons begin correlated at once. When expanded to quadratic order the symmetrized product wave function, equation 9, becomes

$$
|\psi_T\rangle_{\text{quad}} = \left[\prod_{i<j} f_c(r_{ij}) \right] \left[1 + \sum_{i<j} \sum_p f_p(r_{ij}) \mathcal{O}^p_{ij} \right.
$$
$$
\left. + \frac{1}{2} \sum_{i<j} \sum_p f_p(r_{ij}) \mathcal{O}^p_{ij} \sum_{\substack{k<l \\ ij \neq kl}} \sum_q f_q(r_{kl}) \mathcal{O}^q_{kl} \right] |\phi\rangle .
\tag{11}
$$

The subscripts on the sums which describe which correlations are allowed can be hard to visualize and so these correlation diagrams are used. All pair correlations are included in
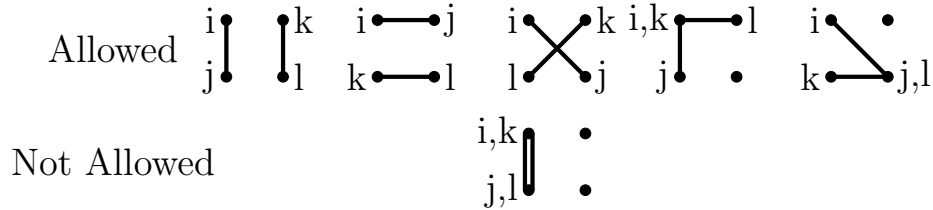

Figure 3: Diagrams used to visualize which correlations are included in the quadratic correlations in equation 11.

8

this wave function except pairs that are directly correlated with themselves, e.g. $\mathcal{O}_{23}\mathcal{O}_{23}$, where $\mathcal{O}_{ij}$ is a product of single particle operators such as $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ and $\mathcal{O}_{ij} = \mathcal{O}_{ji}$ due to the operators on different particles operating in different Hilbert spaces. For correlations where the same particle is included twice, the correlation operators do not commute and that term must be explicitly symmetrized. Currently in equation 11 and in the code all quadratic terms are symmetrized, e.g. the correlation $\mathcal{O}_{12}\mathcal{O}_{34}$ is symmetrized as $\frac{1}{2}(\mathcal{O}_{12}\mathcal{O}_{34} + \mathcal{O}_{34}\mathcal{O}_{12})$ even though only terms like $\mathcal{O}_{12}\mathcal{O}_{13}$ need this explicit symmetrization. This adds needless calculation time, but as I'll show, these terms don't seem to be important at all and can be omitted completely.

If the exponentially correlated wave function, equation 8, is expanded in the typical way, i.e. $\exp(A) = 1 + A + \frac{1}{2}A^2 \ldots$, then the quadratic correlations for the exponential wave function become

$$
|\psi_T\rangle_{\text{exp\_quad}} = \left[\prod_{i<j} f_c(r_{ij})\right] \left[1 + \sum_{i<j}\sum_p f_p(r_{ij})\mathcal{O}_{ij}^p \right.
$$
$$
\left. + \frac{1}{2}\sum_{i<j}\sum_p f_p(r_{ij})\mathcal{O}_{ij}^p \sum_{k<l}\sum_q f_q(r_{kl})\mathcal{O}_{kl}^q \right] |\phi\rangle .
\tag{12}
$$

Unlike the quadratic correlations derived from the symmetrized product this wave function includes all of the the terms in figure 3. There is not a large difference between these wave functions up to quadratic order and for here forward all references to quadratic correlations will refer to the expansion from the symmetrized product wave function.

Another way to include quadratic correlations is to only include terms that do not correlate the same particle twice leaving only independent pair correlations. This is the same whether you start from the exponentially correlated or the symmetrized product wave function and it can be written as

$$
|\psi_T\rangle_{\text{ip}} = \left[\prod_{i<j} f_c(r_{ij})\right] \left[1 + \sum_{i<j}\sum_p f_p(r_{ij})\mathcal{O}_{ij}^p \right.
$$
$$
\left. + \sum_{i<j}\sum_p f_p(r_{ij})\mathcal{O}_{ij}^p \sum_{k<l,\text{ip}}\sum_q f_q(r_{kl})\mathcal{O}_{kl}^q \right] |\phi\rangle
\tag{13}
$$

and the independent pair sum can be visualized as in figure 4. All terms where a parti-
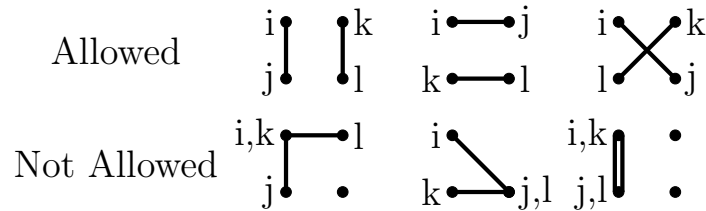

Figure 4: Diagrams used to visualize which correlations are included in the independent pair quadratic correlations in equation 13.

cle is included twice in the correlation are ignored, as a results, all correlation operators commute and the correlations are explicitly symmetric. Neither of these of these correlations maintains cluster decomposability, however an effort to build an antisymmetric and cluster decomposable wave function from the exponentially correlated wave function will be discussed.

The energy and it's uncertainty are used to judge the convergence of a propagated wave function in QMC and so a good wave function needs to be able to reproduce known binding energies. To this end I have done binding energy calculations with the quadratic correlations derived from the symmetrized product for ^{4}He, ^{16}O, ^{40}Ca, and symmetric nuclear matter (SNM) at saturation density, $\rho_0 = 0.16$ fm^{-3}, in a period box with 28 particles. The energy per particle for nuclei is plotted in figure 5 and the specific energies can be found in table 4.3.1. Talk about the energy data and then make sure to include scaling and add a
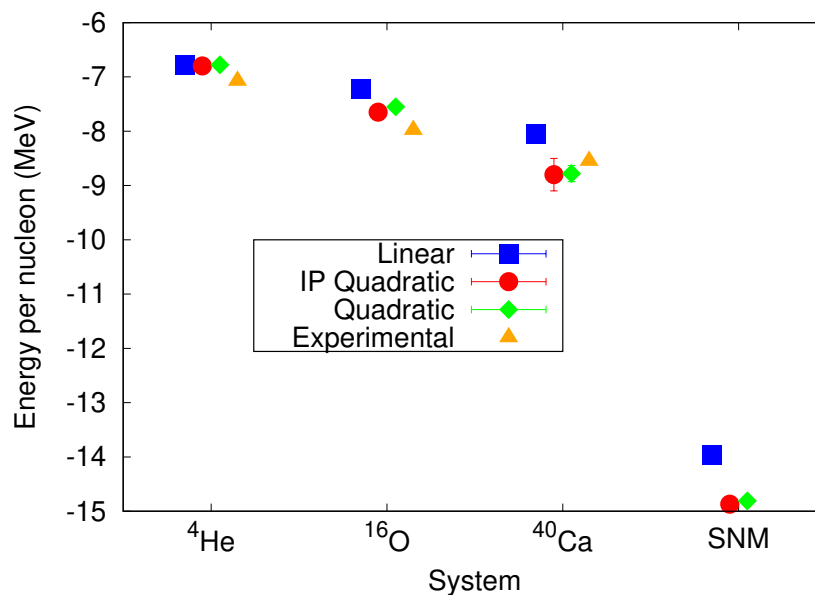

Figure 5: Energy per particle for small and medium closed shell nuclei with no Coulomb interaction with the AV6′ interaction. Each calculations was done with the linear, independent pair, and quadratic correlations. Energies are compared to experimental values where available and the statistical uncertainties are included.

brief discussion about scaling (maybe even an estimate about how much the scaling "would" be if quadratic correlations weren't fully symmetrized. ALSO ADD THE PLOT FOR THE FIT TO SATURATION DONE WHEN TALKING TO ACHIM
After showing results make sure to state and all further calculations will be with the IP correlation, even if you say "quadratic" correlations.

4.3.2 Exponential Correlations

Why aren't they working

System	Linear	Ind-Pair	Quadratic	Experimental
^{4}He	-6.785(10)	-6.798(8)	-6.778(8)	-7.074
^{16}O	-7.23(6)	-7.65(9)	-7.55(8)	-7.98
^{40}Ca	-8.05(8)	-8.8(3)	-8.78(15)	-8.55
SNM	-13.97(3)	-14.87(4)	-14.70(11)	

Table 1: Energy per particle calculated with no Coulomb interaction with the AV6′ interaction. Each calculations was done with the linear, independent pair, and quadratic correlations. Energies are compared to experimental values where available and the statistical uncertainties are included. THEN PUT THE PLOT ABOVE THIS!

4.3.3 Alessandro's correlations and T^2 fix to them - Maybe just do T^2 fix and apply it to exponential correlations

5 Alpha Particle Formation in Neutron Rich Matter

Include theory here and then talk about the results

6 Conclusion

6.1 Future Work

References

[1] Petr Navrátil, Sofia Quaglioni, Ionel Stetcu, and Bruce R Barrett. Recent developments in no-core shell-model calculations. *Journal of Physics G: Nuclear and Particle Physics*, 36(8):083101, 2009.

[2] Bruce R. Barrett, Petr Navrátil, and James P. Vary. *Ab initio* no core shell model. *Prog. Part. Nucl. Phys.*, 69:131–181, 2013.

[3] G Hagen, T Papenbrock, M Hjorth-Jensen, and D J Dean. Coupled-cluster computations of atomic nuclei. *Rep. Prog. Phys.*, 77(9):096302, 2014.

[4] W.H. Dickhoff and C. Barbieri. Self-consistent Green's function method for nuclei and nuclear matter. *Prog. Part. Nucl. Phys.*, 52(2):377 – 496, 2004.

[5] V. Somà, A. Cipollone, C. Barbieri, P. Navrátil, and T. Duguet. Chiral two- and three-nucleon forces along medium-mass isotope chains. *Phys. Rev. C*, 89:061301, Jun 2014.

[6] H. Hergert, S.K. Bogner, T.D. Morris, A. Schwenk, and K. Tsukiyama. The in-medium similarity renormalization group: A novel ab initio method for nuclei. *Phys. Rep.*, 621:165–222, 2016. Memorial Volume in Honor of Gerald E. Brown.

[7] J. E. Lynn and K. E. Schmidt. Real-space imaginary-time propagators for non-local nucleon-nucleon potentials. *Phys. Rev. C*, 86:014324, Jul 2012.

[8] J. Carlson, S. Gandolfi, F. Pederiva, Steven C. Pieper, R. Schiavilla, K.E. Schmidt, and R.B. Wiringa. Quantum Monte Carlo methods for nuclear physics. *Rev. Mod. Phys.*, 87:1067, 2015.

[9] K. E. Schmidt and S. Fantoni. A quantum Monte Carlo method for nucleon systems. *Phys. Lett. B*, 446:99–103, 1999.

[10] H. Yukawa. On the interaction of elementary particles. i. *Proc. Phys. Math. Soc. Japan.*, 17:48–57, 1935.

[11] Lucas Madeira, Alessandro Lovato, Francesco Pederiva, and Kevin E. Schmidt. Quantum monte carlo formalism for dynamical pions and nucleons. *Phys. Rev. C*, 98:034005, Sep 2018.

[12] B. Huron, J. P. Malrieu, and P. Rancurel. Iterative perturbation calculations of ground and excited state energies from multiconfigurational zerothorder wavefunctions. *The Journal of Chemical Physics*, 58(12):5745–5759, 1973.

[13] Junhao Li, Matthew Otten, Adam A. Holmes, Sandeep Sharma, and C. J. Umrigar. Fast semistochastic heat-bath configuration interaction. *The Journal of Chemical Physics*, 149(21):214110, 2018.

[14] Leon N. Cooper. Bound electron pairs in a degenerate fermi gas. *Phys. Rev.*, 104:1189–1190, Nov 1956.

[15] Anthony J. Leggett. A theoretical description of the new phases of liquid ^{3}He. *Rev. Mod. Phys.*, 47:331–414, Apr 1975.

[16] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Microscopic theory of superconductivity. *Phys. Rev.*, 106:162–164, Apr 1957.

[17] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Theory of superconductivity. *Phys. Rev.*, 108:1175–1204, Dec 1957.

[18] Bouchaud, J.P., Georges, A., and Lhuillier, C. Pair wave functions for strongly correlated fermions and their determinantal representation. *J. Phys. France*, 49(4):553–559, 1988.

[19] Lucas Madeira. *Quantum Monte Carlo Studies of Strongly Interacting Fermionic Systems*. PhD thesis, Arizona State University, June 2018.

[20] V. R. Pandharipande and R. B. Wiringa. Variations on a theme of nuclear matter. *Rev. Mod. Phys.*, 51:821–861, Oct 1979.

[21] V. R. Pandharipande and K. E. Schmidt. Variational calculations of simple bose systems. *Phys. Rev. A*, 15:2486–2495, Jun 1977.

[22] S. Gandolfi, F. Pederiva, S. Fantoni, and K.E. Schmidt. Auxiliary field diffusion Monte Carlo calculation of nuclei with $A \leq 40$ with tensor interactions. *Phys. Rev. Lett.*, 99:022507–1, 2007.

[23] S. Gandolfi, A. Lovato, J. Carlson, and Kevin E. Schmidt. From the lightest nuclei to the equation of state of symmetric nuclear matter with realistic nuclear interactions. *Phys. Rev. C*, 90, 2014.