Computational Nuclear Physics

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Nuclear Physics

- Pictures of things that nuclear physics is good for, like . . .
  - Nuclear power (on earth, spacecraft, etc.)
  - Medical imaging
  - Basic structure of nuclei
  - Neutron Stars

Integrals

- Ground state (lowest) energy

$$E_{gs} = \int \psi^*(\mathbf{R}) H \psi(\mathbf{R}) d\mathbf{r_1} d\mathbf{r_2} \ldots d\mathbf{r_N}$$

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- If you know what an integral is, go ahead and panic.
- If you don't know what an integral is, it's easy. Here I'll show you ...

What is an integral?

- Let's say you wanted to know the length of a line at disneyland. You can have 1 stick of known length. Does a smaller or larger stick make it faster? Which makes it more accurate?

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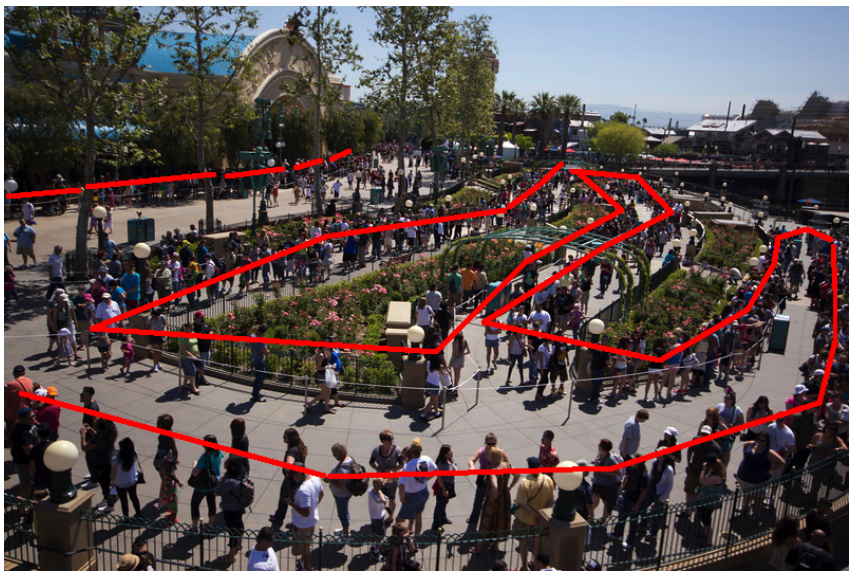
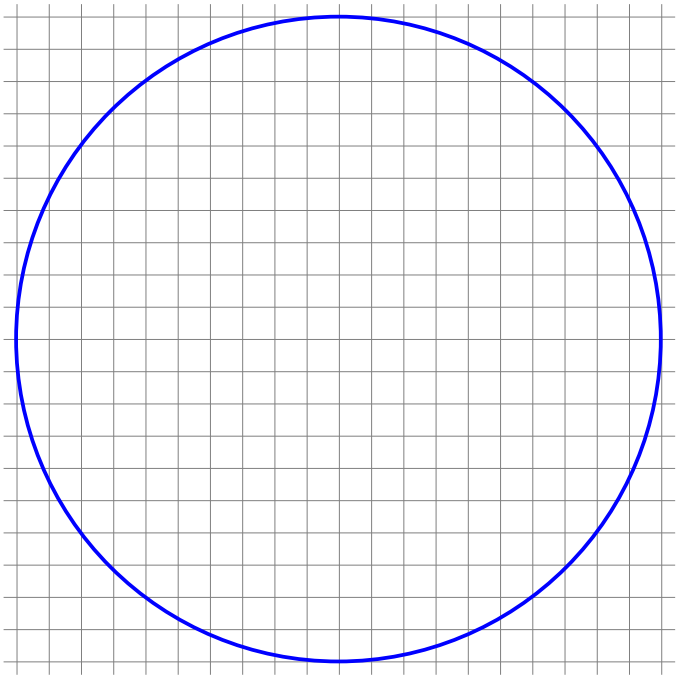


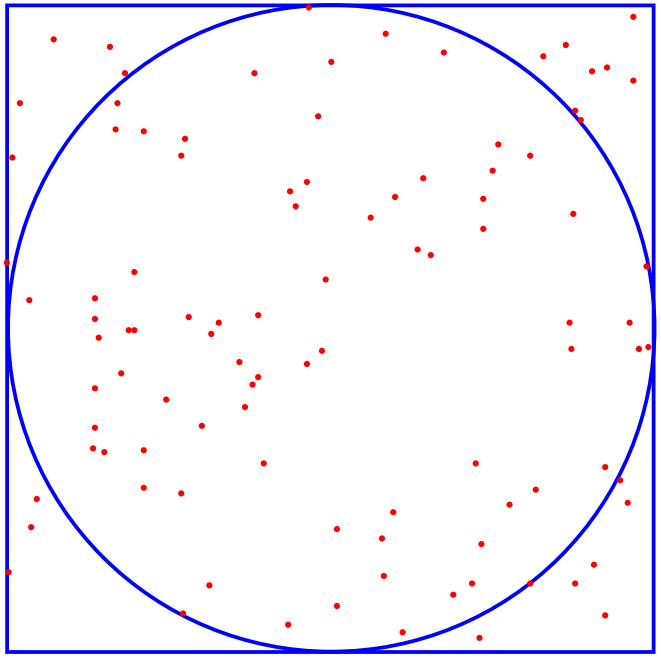
$$L = \sum_i l_i \xrightarrow{l_i \to 0} \int_{\text{line}} dl$$

Monte Carlo



$$A_{\text{circle}} = \sum_i \sum_j dx_i dy_j$$

Monte Carlo



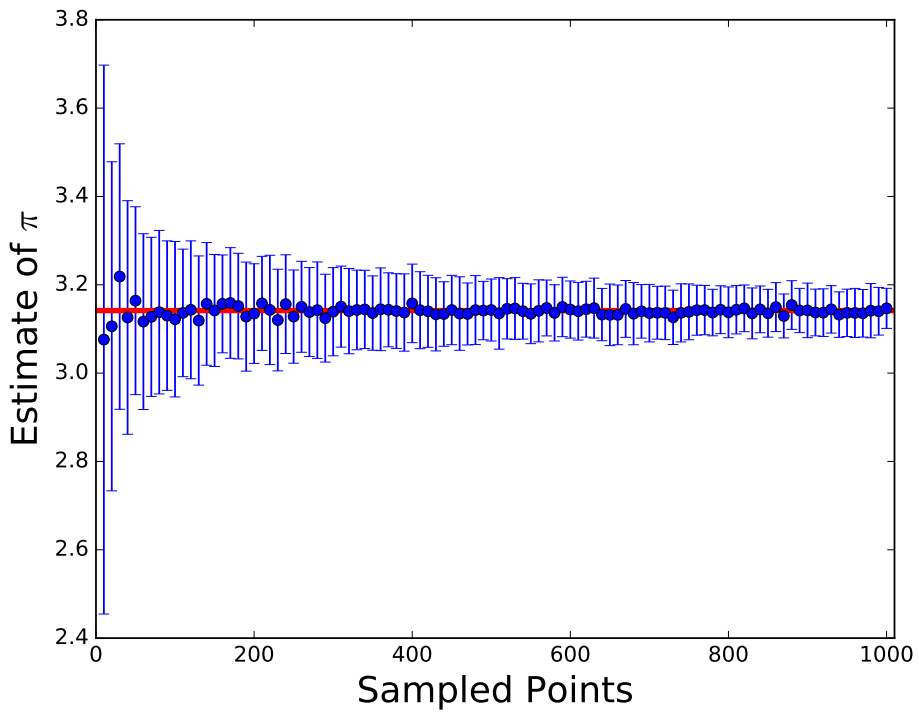
$$\frac{A_{\text{circle}}}{A_{\text{box}}} = \frac{\#\text{ points in the circle}}{\#\text{ points in the box (total)}}$$

Monte Carlo Example

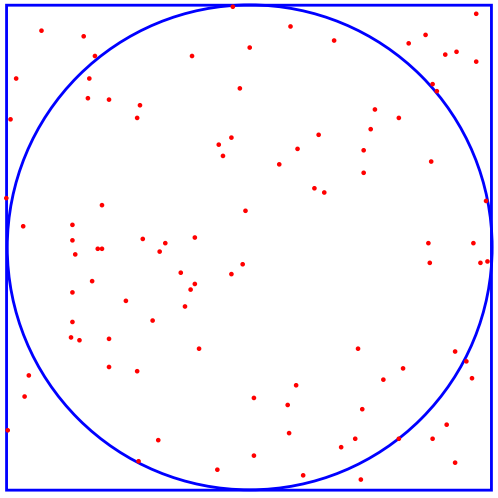
- You can estimate $\pi = 3.14159$ using the method above.

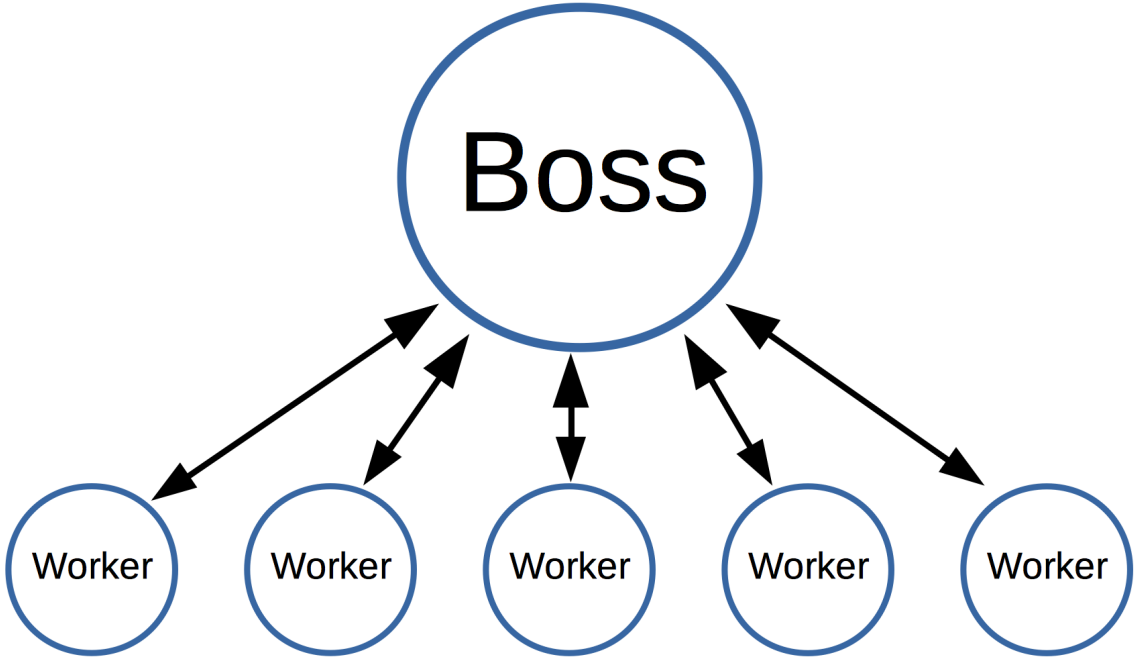
$$\frac{A_{\text{circle}}}{A_{\text{box}}} = \frac{\pi r^2}{(2r)(2r)} = \frac{\pi}{4} = \frac{\#\text{ points in the circle}}{\#\text{ points in the box (total)}}$$

$$\pi = 4\frac{\#\text{ points in the circle}}{\#\text{ points in the box (total)}}$$



Monte Carlo on a Supercomputer





$$E_{gs} = \int \psi^*(\mathbf{R}) H \psi(\mathbf{R}) d\mathbf{r_1} d\mathbf{r_2} \ldots d\mathbf{r_N}$$

$\psi_T^*(\mathbf{R}) H \psi_T(\mathbf{R})$ at each point

Monte Carlo in Nuclear Physics

$$E_{gs} = \int \psi^*(\mathbf{R}) H \psi(\mathbf{R}) d\mathbf{r_1} d\mathbf{r_2} \ldots d\mathbf{r_N}$$

- Guess Ψ_T
- Get a good guess for H from somebody else
- Put it on a supercomputer
- Change Ψ_T until you get the lowest energy you can (Variational Monte Carlo)

Better Ψ_T Results

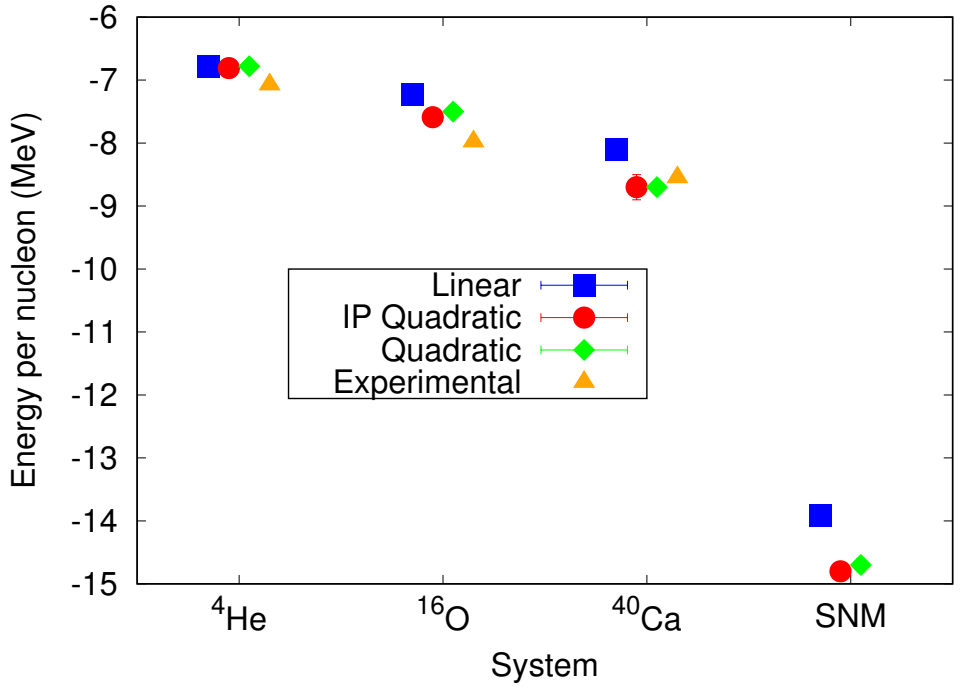
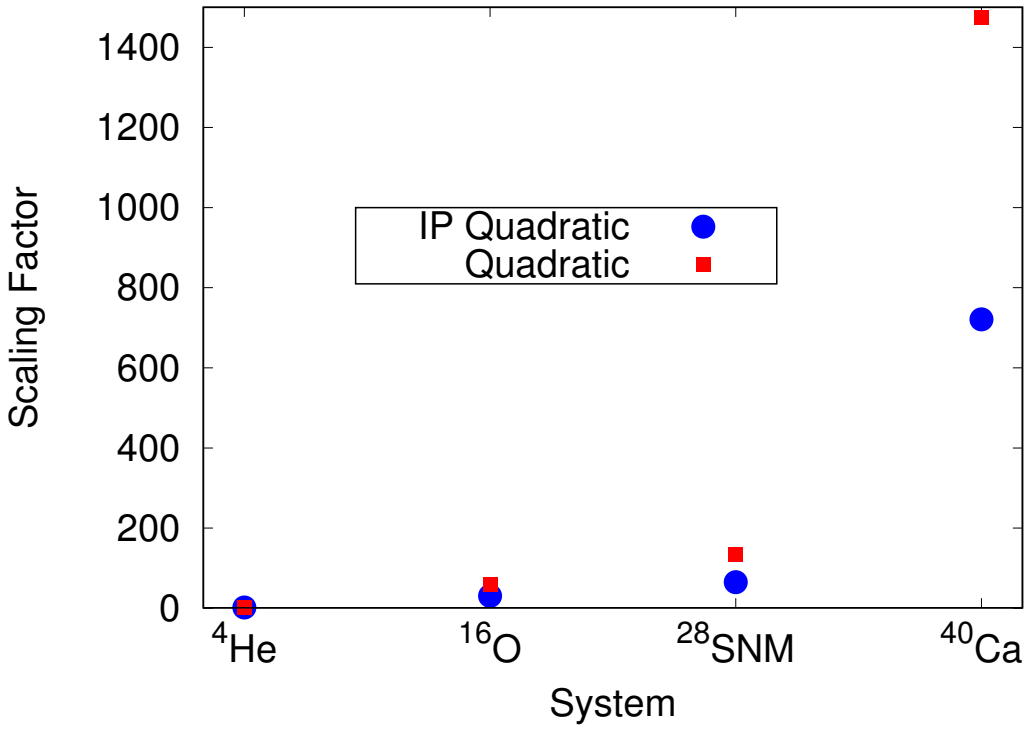


Table: Energy (*per nucleon) in MeV

System	Linear	IP Quadratic	Quadratic	Experimental
^{4}He	-27.14(4)	-27.22(3)	-27.11(3)	-28.295
^{16}O	-115.7(9)	-121.5(1.5)	-120.0(1.4)	-127.62
^{40}Ca	-324(3)	-347(8)	-349(5)	-342.1
SNM*	-13.92(6)	-14.80(7)	-14.70(11)	

D. Lonardoni et al. *Phys. Rev. C.*, **97**, 044318, 2018.

Better Ψ_T Cost

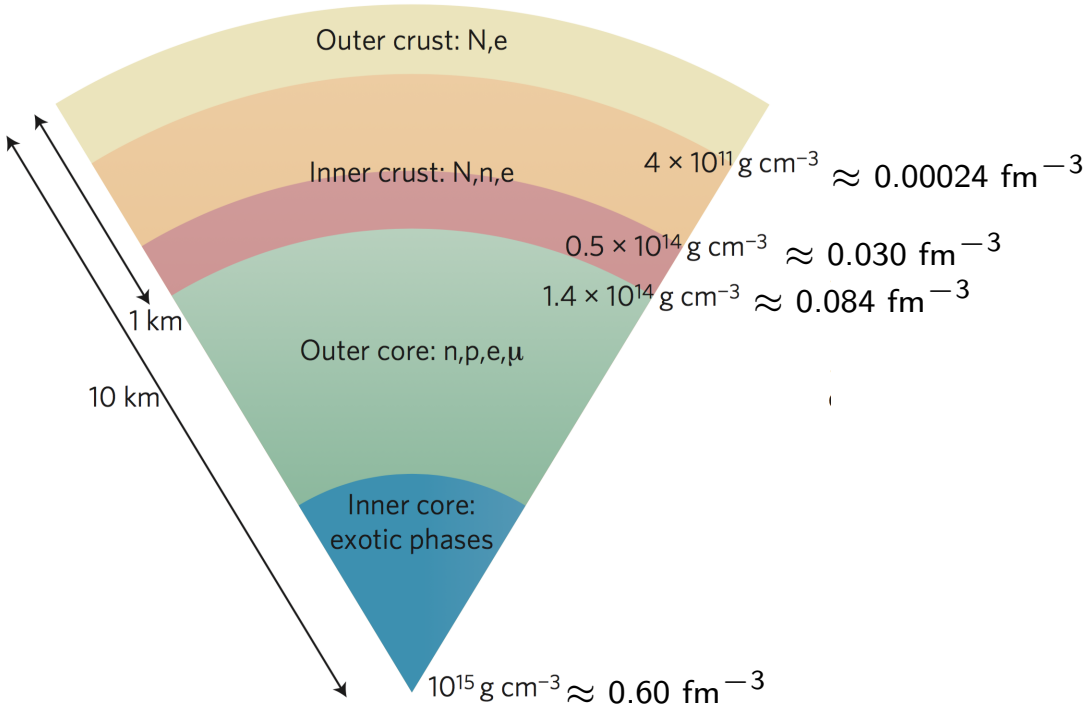


	^{4}He	^{16}O	SNM(28)	^{40}Ca
IP Quadratic	1.73	30.7	64.8	720.9
Quadratic	2.00	58.8	133.6	1473.9

^{4}He Nuclei Forming in Neutron Stars

- Use new wave function to study α formation in the inner crust of neutron stars.

$$E_\alpha = E_{\mathrm{Nn+2p}} - E_{\mathrm{(N\text{-}2)n}}$$



W. Newton *Nature Physics* **9**, 396-397 (2013)

Alpha Particle Clustering in Mostly Neutron Matter

- If alpha particles form in nearly neutron matter then we should be able to estimate their energy by

$$E_\alpha = E_{14n+2p} - E_{12n}$$

- Both energies decreased, but the combination did not always.

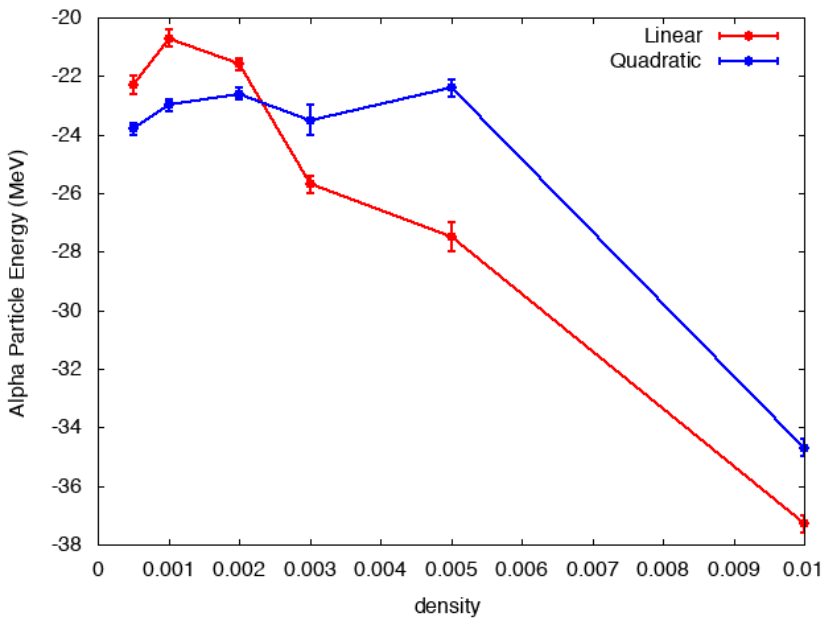


Table: Alpha energy in MeV

ρ (fm^{-3})	lin	ip
0.0005	-22.3(3)	-23.8(2)
0.001	-20.7(3)	-23.0(2)
0.002	-21.6(2)	-22.6(2)
0.003	-25.7(3)	-23.5(5)
0.005	-27.5(5)	-22.4(3)
0.01	-37.3(3)	-34.7(3)

Picture References

Ariel line on opening day (accessed 4 Aug 2018):
https://forums.wdwmagic.com/threads/omg-little-mermaid-is-sucha-failure.753509/page-2
Monte Carlo casino (accessed 6 Aug 2018):
http://www.montecarlosbm.com/luxury-casinos-monaco-3/monte-carlo-casino/