Dealing with Misspecification in Fixed-Confidence Linear Top-*m* Identification

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GitHub code

Model misspecification is pervasive in practical applications for recommendation systems, and while linear models are popular for their simplicity and the existence of efficient algorithms, they might be prone to errors when facing real-life data. **How to leverage feature information while preserving the correction of recommendations?**

A. Fixed-confidence misspecified Top-*m* identification

The goal is to identify the $m < K$ arms with the largest expected rewards, using fixed time-independent arm features. In a (possibly) ε-misspecified bandit model[1], the vector of expected rewards μ is a function of feature matrix $A \in \mathbb{R}^{K \times d}$

$$\mu \in M_\varepsilon := \{ A\theta+\eta : \theta \in \mathbb{R}^d, \eta \in \mathbb{R}^K, \|\eta\|_\infty \leq \varepsilon \}$$

Assume $\mu_1 \geq \ldots \geq \mu_m > \mu_{m+1} \geq \ldots \geq \mu_K$. A (m, δ)-PAC (Probably Approximately Correct) algorithm for fixed-confidence Top-*m* returns a subset of m arms $k_1, k_2, \ldots, k_m$ after τ samples with confidence 1-δ

$$\min_\tau \tau \text{ s.t. } \mathrm{Prob}(\{k_1, k_2, \ldots, k_m\} \subseteq [m]) \geq 1-\delta$$

B. Contributions

1. Tractable lower bound for Top-*m* identification

For model μ, using a change-of-measure argument,[2] expected minimum number of samples τ_δ needed to make a decision in the δ-Top-*m* problem is

$$E_\mu[\tau_\delta] \geq \log(1/2.4\delta)\, T^*(\mu, \varepsilon)^{-1} \ (\bigstar)$$
$$\text{where } T^*(\mu, \varepsilon) := \sup \{ \inf \{ \textstyle\sum_k \omega^k KL(\mu^k , \lambda^k) : \lambda \in \mathrm{Alt}(\mu,\varepsilon) \} : \omega \in \Delta_K \}$$

* The alternative set Alt(μ,ε) is a union of a polynomial number of halfspaces.

Lemma 1. $\mathrm{Alt}(\mu,\varepsilon) := \{ \lambda \in M_\varepsilon : [m] \text{ is not the set of } m\text{-best arms for } \lambda \}$
$= \bigcup_{i \in [m]} \bigcup_{j \notin [m]} \{ \lambda \in M_\varepsilon : \lambda_i < \lambda_j \}$

* Influence of ε on the lower bound on $E_\mu[\tau_\delta]$

Lemma 2. There is $\varepsilon(\mu) < +\infty$ such that when $\varepsilon > \varepsilon(\mu)$, the lower bound expression in (★) is equal to the lower bound for unstructured models.

* Adaptation to ε is impossible

Lemma 3. Consider ε' < ε. An algorithm cannot reach the optimal bound $\log(1/2.4\delta)T^*(\mu, \varepsilon')$ on subset $M_{\varepsilon'}$ while remaining δ-PAC on set M_ε.

Indeed, $M_{\varepsilon'} \subseteq M_\varepsilon$ implies that $T^*(\mu, \varepsilon) \leq T^*(\mu, \varepsilon')$.

2. MisLID algorithm for misspecified Top-*m* identification

Pull a barycentric spanner $\mathcal{S}$ of A s.t. $\mathrm{cst} \sum_{k \in \mathcal{S}} A_{k,\cdot} A_{k,\cdot}^T \succeq \mathrm{cst}\, I_d$
Compute emp. mean $\widehat{\mu}$, $\tilde{\mu} := \Pi_{\mathcal{M}_{\hat{\varepsilon}}}(\widehat{\mu})$
while $\inf_{\lambda \in \mathrm{Alt}(\tilde{\mu},\hat{\varepsilon})} \|\tilde{\mu} - \lambda\|^2_{N_{t-1}} \leq 2\beta_{t-1,\delta}$ **do**
  Obtain $\omega_t \in \Delta_K$ from $\mathcal{L}$
  Compute $\lambda_t \leftarrow \arg\min_{\lambda \in \mathrm{Alt}(\tilde{\mu},\hat{\varepsilon})} \|\tilde{\mu} - \lambda\|^2_{\omega_t}$
  Update $\mathcal{L}$ with an optimistic gain related to $\sum_{k \in [K]} \omega_t^k KL(\tilde{\mu}^k, \lambda_t^k)$
  Pull $k_t \sim \omega_t$
  Update $\widehat{\mu}_t$ and projection $\tilde{\mu}$
  $t \leftarrow t+1$
end while

Alg 1. MisLID algorithm for Gaussian bandits with variance 1

* Main ingredients

- A no-regret online learner L (any)

- Thresholds $(\beta_{t\delta})_t$ adapted to unstructured & linear models

$$\beta_{t\delta} \approx \log(1/\delta) + \min\{K \log t,\ d \log t + \varepsilon^2 t\}$$

- User-provided ε upper bound on misspecification

3. Analysis of the MisLID algorithm

Correctness.

Theorem 1. MisLID is (m, δ)-PAC on any model $\mu \in M_\varepsilon$.

Sample complexity. The sample complexity τ_δ on $\mu \in M_\varepsilon$ satisfies

$$E_\mu[\tau_\delta] \leq \tau + 2 \ (\bigstar\bigstar)$$
$$\text{with } \tau \leq T^*(\mu, \varepsilon)^{-1} [\log(1/\delta) + o(\log(1/\delta))\, C(\tau)]$$

where for ε ≈ 0 (linear models)

$$C(\tau) = O(\varepsilon^2\tau + \tau^{1/2}(\log K + d \log \tau)+(d \log \tau)^{1/2}) \ (1)$$

and for ε >> 0 (unstructured models):

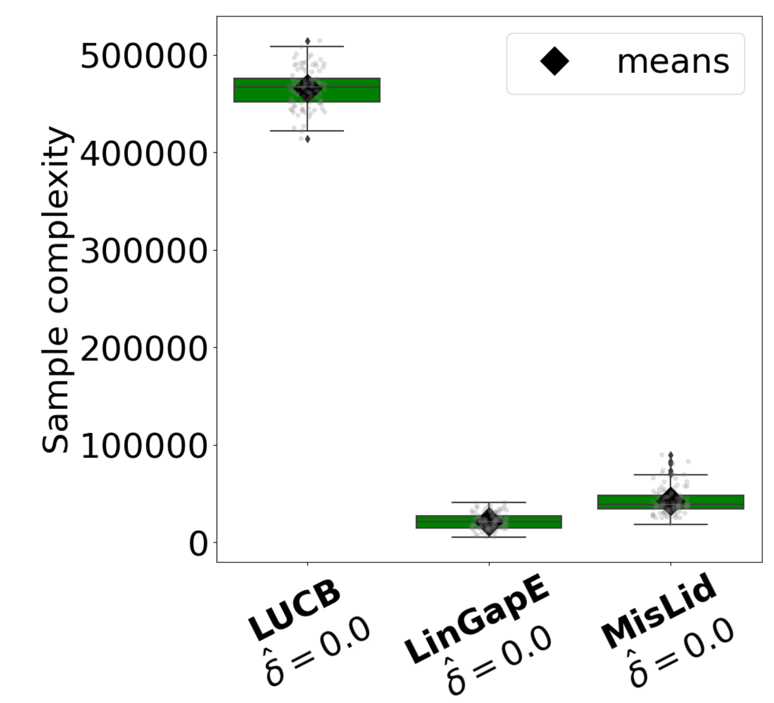
$$C(\tau) = O(\tau^{1/2}(\log K + K^{1/2} \log \tau)+(K \log \tau)^{1/2}) \ (2)$$

Theorem 2. MisLID
- is asymptotically optimal in the regime of δ → 0 (★★)
- doesn't have a polynomial dependence in K when ε = 0 (1)
- interpolates between an optimal linear bound and an optimal unstructured one as ε → +∞ (2).

C. Experimental study (across 100 runs)

Comparison to a linear (*m*-LinGapE[3]) and an unstructured (LUCB[4]) bandit algorithms for Top-*m* in two practical cases (δ=5%).

1. Application to drug repurposing (small misspecification ε)

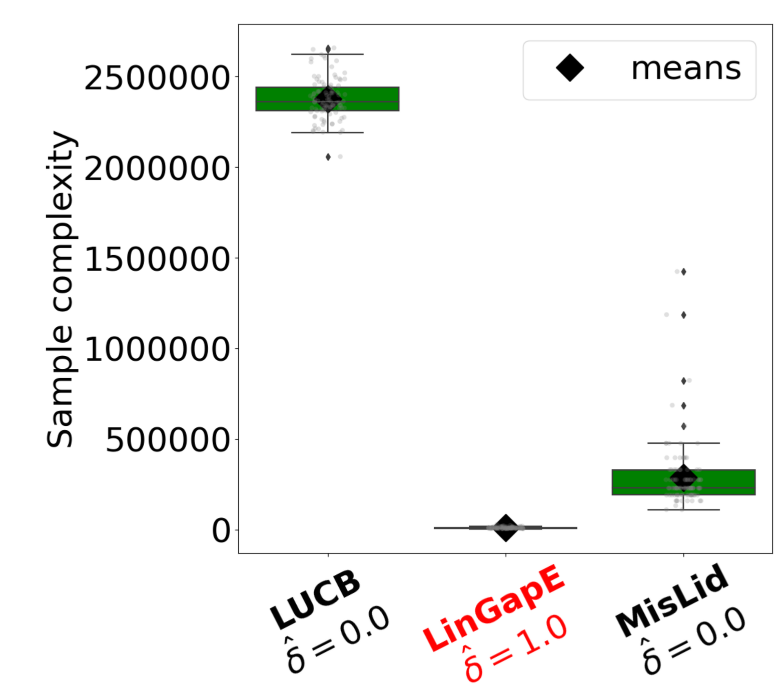


* All three algorithms are δ-correct.

* MisLID is competitive compared to LinGapE (multiplicative factor of 2 in sample complexity).

* Both a lot better than LUCB sample complexity-wise.

2. Application to online recommendation (large misspecification ε)



* LinGapE is consistently wrong, whereas both MisLID and LUCB are δ-correct.

* MisLID outperforms LUCB in terms of sample complexity by several orders of magnitude.

References

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[4] Kalyanakrishnan, Shivaram, et al. **"PAC subset selection in stochastic multi-armed bandits."** *ICML*. Vol. 12. 2012.