Regression Model

Experiment Description

实验基本要求

- 根据数据集 dataset_regression.csv， 求最小二乘解，画出回归曲线，给出训练误差。
- 将数据集 winequality-white.csv 按照4:1划分为训练集和测试集，构造线性回归模型，采用批量梯度下降或者随机梯度下降均可;输出训练集和测试集的均方误差(MSE)，画出MSE收敛曲线。

实验中级要求

尝试不同的学习率并进行MSE曲线展示，分析选择最佳的学习率。

Analysis

Least Square Fit

```
import numpy as np
import matplotlib.pyplot as plt
import csv
import operator
with open('dataset_regression.csv') as csvfile:
     reader = csv.reader(csvfile)
     dataset = [row for row in reader]
dataset.pop(0)
for i in dataset:
    for m in range(3):
        i[m] = float(i[m])
print(dataset)
n = len(dataset)
sum_xy = 0
sum_x = 0
sum_y = 0
sum_xx = 0
for i in range(n):
    sum_xy += dataset[i][1] * dataset[i][2]
    sum_x += dataset[i][1]
    sum_y += dataset[i][2]
    sum_xx += dataset[i][1] * dataset[i][1]
a1 = (sum_xy - (sum_x * sum_y) / n) / (sum_xx - n * (sum_x / n  * sum_x / n))
a0 = sum_y / n - a1 * sum_x / n
a1 = round(a1, 4)
a0 = round(a0, 1)
print("回归方程为：y=", a1 , "x+" , a0)
xt = []
yt = []
for i in dataset:
    xt.append(i[1])
    yt.append(i[2])
```

```
loss = 0
for i in range(n):
    loss += (yt[i] - a0 - a1 * xt[i]) ** 2
loss = loss / (2 * n)
print("loss的值为: ", loss)
fig = plt.figure(figsize=(4, 4))
ax = fig.add_subplot(1, 1, 1)
ax.scatter(xt, yt)
x = np.arange(-3, 4)
y = a1 * x + a0
plt.plot(x, y)
plt.show()
```

- Use batch gradient descent to construct the model for multi-dimentional data. Before using gradient descent, the data should be normalized first. Set learning rate at 0.5 0.3 0.1 0.01 0.001 and test.

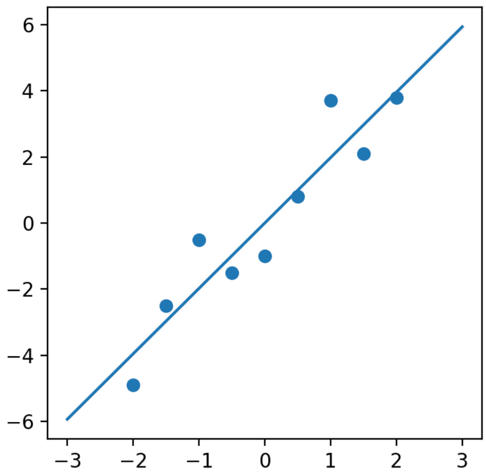
```
import numpy as np
import csv
import operator
from sklearn.model_selection import train_test_split
import matplotlib.pyplot as plt
with open('winequality-white.csv') as csvfile:
     reader = csv.reader(csvfile)
     dataset = [row for row in reader]
dataset.pop(0)
y = []
for i in dataset:
    for m in range(len(i)):
        i[m] = float(i[m])
    y.append(i[-1])
    i.pop(-1)
    i.insert(0, 1)
x_train, x_test, y_train, y_test = train_test_split(dataset, y, test_size = 0.2) #划分训练集
#归一化
def feature_scaling(x):
    for i in range(len(x[0])):
        max = -float('inf')
        min = float('inf')
        for m in range(len(x)):
            if x[m][i] > max:
                max = x[m][i]
            if x[m][i] < min:
                min = x[m][i]
        for m in range(len(x)):
            if max - min != 0:
                x[m][i] = (x[m][i] - min) / (max - min)
    return x
x_train = feature_scaling(x_train)
x_test = feature_scaling(x_test)
# theta = np.random.rand(len(x_train[0]))
```

```
# print(x_train[0])
# print(theta)
# print(theta * x_train)
# print(x_train)
# print(type(np.random.rand(len(x_train))))
def gradient_descent(x_train, y_train, x_test, y_test, learning_rate):
    loss = []
    theta = np.random.rand(len(x_train[0]))
    x_train = np.array(x_train)
    x_test = np.array(x_test)
    for index in range(1000):
        gradients = x_train.T.dot(x_train.dot(theta) - y_train) / len(x_train)
        theta = theta - learning_rate * gradients
        MSE = ((np.dot(x_test, theta) - y_test) ** 2).sum() / len(x_test)
        loss.append(MSE)
    return theta, loss
ls = []
for i in range(1000):
    ls.append(i)
x = np.array(ls)
theta0, loss0 = gradient_descent(x_train, y_train, x_train, y_train, learning_rate = 
0.5)
theta1, loss1 = gradient_descent(x_train, y_train, x_test, y_test, learning_rate = 0.5)
theta2, loss2 = gradient_descent(x_train, y_train, x_test, y_test, learning_rate = 0.3)
theta3, loss3 = gradient_descent(x_train, y_train, x_test, y_test, learning_rate = 0.1)
theta4, loss4 = gradient_descent(x_train, y_train, x_test, y_test, learning_rate = 0.01)
theta5, loss5 = gradient_descent(x_train, y_train, x_test, y_test, learning_rate = 
0.001)
# 画散点图
colors0 = '#000000'
colors1 = '#00CED1' #点的颜色
colors2 = '#DC143C'
colors3 = '#66CDAA'
colors4 = '#BEBEBE'
colors5 = '#00FA9A'
area = np.pi * 0.5**2  # 点面积
plt.scatter(x, loss0, s=area, c=colors0, alpha=0.4, label='train')
plt.scatter(x, loss1, s=area, c=colors1, alpha=0.4, label='learning_rate = 0.5')
plt.scatter(x, loss2, s=area, c=colors2, alpha=0.4, label='learning_rate = 0.3')
# plt.scatter(x, loss3, s=area, c=colors3, alpha=0.4, label='learning_rate = 0.1')
# plt.scatter(x, loss4, s=area, c=colors4, alpha=0.4, label='learning_rate = 0.01')
# plt.scatter(x, loss5, s=area, c=colors5, alpha=0.4, label='learning_rate = 0.001')
plt.legend()
plt.show()
```

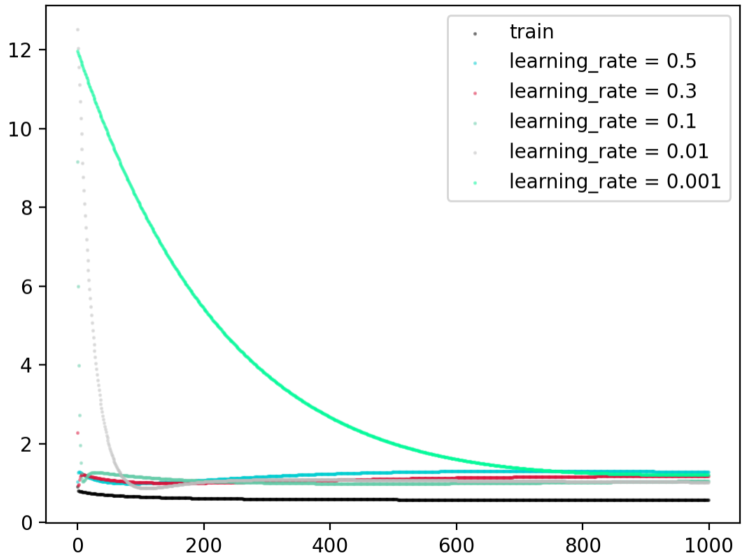
Experiment Result

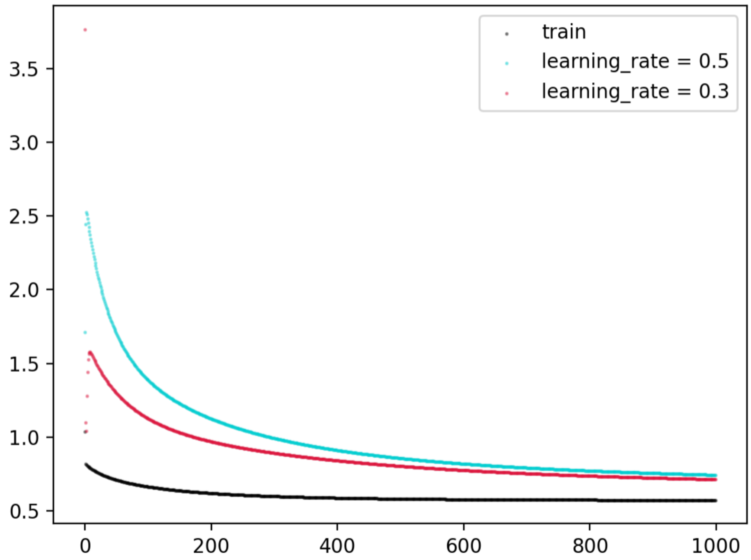
- Least Square Fit

```
[[1.0, -2.0, -4.9], [2.0, -1.5, -2.5], [3.0, -1.0, -0.5], [4.0, -0.5, -1.5], [5.0, 0.0, -1.0], [6.0, 0.5, 0.8], [7.0, 1.0, 3.7], [8.0, 1.5, 2.1], [9.0, 2.0, 3.8]]
回归方程为： y= 1.9767 x+ 0.0
loss的值为：  0.46287963055555564
```



- Gradient Descent





It can be concluded from the data that, extremely high or low learning rate can affect accuracy. Extremely high learning rate can lead to overfitting, and extremely low learning rate results in underfitting. The analysis shows that the optimal learning rate of this model is about 0.3.