The Hyper-Grid

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Continuation of Nugget 123:

$$N^2[((1)*\{N-1\},0)*N,((N)*N)*\{N-1\}]1$$

is a candidate LCFS-like formula, some sort of packed generator, of the first proposed LCFS code that, when unpacked, constructs a square grid of side N. Recap of the notation: the exponential, addition and subtraction operators work as normal, while the * symbol means: give me as many copies of the left handside of the operator as specified by the right handside.

I.e. (1,0)*3 = (1,0,1,0,1,0)

The formula should read as follows: There are N^2 nodes, a subsequence made of 1s repeated $N-1$ times, followed by a single 0, grouped and repeated N times, followed by another subsequence made of the value N repeated N times, grouped and repeated $N-1$ times. The final one means that all this should be repeated 1 time.

The exponent 2 clearly comes from the fact that we are building a 2d object. How would the formula look for any N^d with d any integer dimension? How would the formula look like for d=3? I suspect and hope that we can reuse some of the square grid formula. Luckily the 2x2x2 cubic graph has a LCF of [3,-3]4, feels promising. However I can't wrap my head around classic LCF codes and I think it may not exist for the 3x3 grid (or any grid actually).
Meanwhile the 3x3x3 cube has a more brutish LCFS, but at least it is easy to write down:

$$27[(1,1,0,1,1,0,1,1,0)*3,(3,3,3,3,3,3,0,0,0)*3,(9,9,9,9,9,9,9,9,9)*2]1$$

That can be folded into: [((1,1,0)*3)*3,(3,3,3,3,3,3,0,0,0)*3,((9)*9)*2]
I could continue folding these numbers but I think I can already hazard a formula for d=3 and any N:

$$N^3[((1)*\{N-1\},0)*N,(((N)*N)*\{N-1\},(0)*N),((N^2)*N^2)*\{N-1\}]$$

with the first two subsequences equal to the 2d grid case and just an added subsequence that takes care of the vertical edges. Nice. I think there is already a pattern here.

I'd say that we need a subsequence for each dimension and I'd hazard this formula:

$$N^d[((1)*\{N-1\}, 0)*N, (((N)*N)*\{N-1\}, (0)*N), ..., ((N^{d-1})*N^{d-1}))*\{N-1\}]$$

Behold the formula for a square grid of arbitrary dimensions d! I feel this could be expressed in a less convoluted manner. It looks like the product of some successor function, series or sequence of sequences. Maybe it is just a bog standard polynomial.

Each subsequence of the formula for each dimension i from 0 to d-1 may be produced by this more general formula:

$$(((N^i) * \{N^{i+1} - N^i\}, (0) * N^i) * N^{d-i}$$

More simply for each dimension i from 0 to d-1 of a grid of side N we need a subsequence equal to:

The value N^i repeated $N^{i+1} - N^i$ times, followed by N^i zeros, all repeated N^{d-i} times.

OR

from 1 to d:

The value N^{i-1} repeated $N^i - N^{i-1}$ times, followed by N^{i-1} 0s, all repeated N^{d-i+1} times.

Interestingly all hypercubes are just the special case of N=2 and arbitrary d.

As the * symbol is a bit confusing an alternative notation for the subsequences that would be more similar to the original LCF notation could be:

$$[[N^i]\{N^{i+1} - N^i\}, [0]N^i]N^{d-i}$$

In this notation we reuse the convention where the square brackets [] are followed by how many copies you want of the content within the brackets. Now we just need something to highlight that we want to substitute and unpack subsequences like that for each i from 0 to $d-1$. This could be done by some pseudo list comprehension.

Finally, here is the formula which includes the number of nodes required for any Hyper-Grid Graph of side N and d dimensions:

$$N^d[\{[[N^i]\{N^{i+1} - N^i\}, [0]N^i]N^{d-i} | i \in \mathbb{N}_0, i < d\}]1$$