Assignment 2 - Simulation of a Hamming-coded 16-QAM System

Digital Communications, EEEN3009J, Autumn 2020/21

You are required to write, in MATLAB, a time domain simulation of a communication system which uses *coded* 16-QAM modulation. The code used is a $(7,4)$ linear block code called a *Hamming code*. The system model is shown in Figure 1. The channel model uses symbol rate sampling and the only channel impairment is additive white Gaussian noise (AWGN)

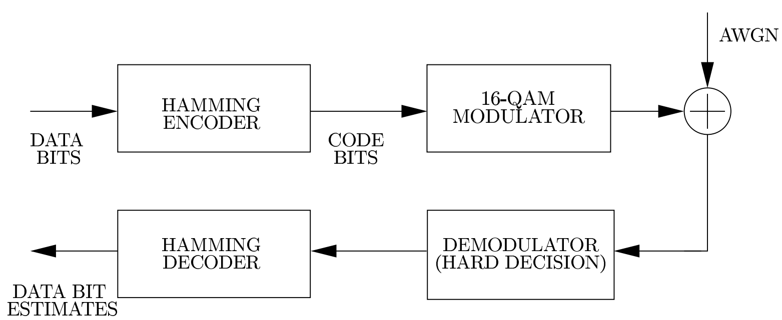

Figure 1: Block Diagram of the Hamming-coded 16-QAM system to be simulated.

The $(7,4)$ Hamming code has the following generator matrix:

$$\boldsymbol{G} = \left(\begin{array}{c|c} \boldsymbol{P} & \boldsymbol{I}_k \end{array} \right) = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{1}$$

Note that the matrix $\boldsymbol{G}$ is in systematic form. Therefore, from class notes, the parity-check matrix is given by $\boldsymbol{H} = \left(\begin{array}{c|c} \boldsymbol{I}_{n-k} & \boldsymbol{P}^T \end{array} \right)$. Given that the received vector may be written as $\boldsymbol{r} = \boldsymbol{c} + \boldsymbol{e}$, where $\boldsymbol{c}$ denotes the transmitted codeword and $\boldsymbol{e}$ the error vector, decoding of this code proceeds by using the received vector $\boldsymbol{r}$ to form the *syndrome* $\boldsymbol{s} = \boldsymbol{r}\boldsymbol{H}^T = \boldsymbol{e}\boldsymbol{H}^T$. The maximum-likelihood error vector $\boldsymbol{e}$ may then be identified via Table 1 below.

Table 1: Decoding table for the $(7, 4)$ Hamming code.

Syndrome $\boldsymbol{s}$	Error Vector $\boldsymbol{e}$
000	0000000
001	0010000
010	0100000
011	0000100
100	1000000
101	0000001
110	0001000
111	0000010

The following are the requirements:

- Use your simulation to plot the symbol error rate (SER) versus E_s/N_0 curve for the system. Plot SER on a log scale and E_s/N_0 in dB.
- Then, on the same graph, plot the *theoretical* SER curve for the system. To do this, you will need to derive an expression for the probability of symbol error for this system as a function of E_s/N_0.
- Also, on a separate plot show the simulated and theoretical bit error rate (BER) versus E_b/N_0 in dB.
- From the BER curves, estimate the value of E_b/N_0 above which the Hamming code offers improved performance over an uncoded system, i.e., find the values of BER and E_b/N_0 at which the two curves cross over each other.
- From the BER curves find the coding gain (in dB) at a BER of 10^{-4}.
- Your program should consist of a single m-file script, and should be appropriately annotated with comments. You should not use any procedures from the MATLAB communications toolbox.
- Your assignment should be submitted via Brightspace, and should contain two files:
  (a) Your MATLAB simulation m-file, and
  (b) A short report (**in PDF format**) containing the system performance graphs mentioned above. A brief commentary about the methods you used and the results you obtained should also be included in this report. The answers to the specific questions asked above should also be stated clearly in your report.
- The deadline is **11:30 pm (Dublin time) on Friday 20 November 2020**.

- **And most importantly:** The program you submit should be **your own work**. Programs will be scrutinized for evidence of copying. Programs in which copying is found will NOT be awarded a pass grade.