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On the GNSS Synchronization Performance Degradation under Interference Scenarios: Bias and Misspecified CRB

Lorenzo Ortega*[1,2] | Corentin Lubeigt[2,3] | Jordi Vilà-Valls[3] | Eric Chaumette[3]

[1]IPSA, 40, Boulevard de la Marquette, Toulouse, 31000, France
[2]TéSA, 7, Boulevard de la Gare, Toulouse, 31500, France
[3]ISAE-SUPAERO, 10, Avenue Edouard Belin, Toulouse, 31400, France

Correspondence
Lorenzo Ortega, Email: lorenzo.ortega@tesa.prd.fr

Abstract

Global navigation satellite systems (GNSS) are a key player in a plethora of applications, ranging from navigation and timing, to Earth observation or space weather characterization. For navigation purposes, interference scenarios are among the most challenging operation conditions, which clearly impact the maximum likelihood estimates (MLE) of the signal synchronization parameters. While several interference mitigation techniques exist, a theoretical analysis on the GNSS MLE performance degradation under interference, being fundamental for system/receiver design, is a missing tool. The main goal of this contribution is to provide such analysis, by deriving closed-form expressions of the misspecified Cramér-Rao (MCRB) bound and estimation bias, for a generic GNSS signal corrupted by an interference. The proposed bias and MCRB expressions are validated for a linear frequency modulation chirp signal interference.

KEYWORDS
GNSS synchronization, interference, maximum likelihood, misspecified CRB, bias analysis.

1 | INTRODUCTION

Global navigation satellite systems (GNSS) (Teunissen & Montenbruck, 2017) appear in a plethora of applications, ranging from navigation and timing, to Earth observation, attitude estimation or space weather characterization. Indeed, reliable position, navigation and timing information is fundamental in new application such as intelligent transportation systems or autonomous unmanned ground/air vehicles, for which GNSS have become the cornerstone source of positioning data, and this dependence can only but grow in the future. But GNSS were originally designed to operate in clear sky nominal conditions, and their performance clearly degrades under harsh environments. Among the non-nominal operation conditions, multipath, interferences (i.e., intentional (jamming) or unintentional) and spoofing are the most challenging ones, being a key issue in safety-critical scenarios (Amin et al., 2016). Interferences degrade GNSS performance, and can lead to denial of service or even counterfeit transmissions to control the receiver positioning solution. These effects have been reported in the state-of-the-art, and several interference mitigation countermeasures have been already proposed (Amin et al., 2017; Arribas et al., 2019; Borio & Gioia, 2021; Chien, 2015 2018; Fernández-Prades et al., 2016; Liu et al., 2022; Morales-Ferre et al., 2020; Pirayesh & Zeng, 2022).

It is well-known that an interference impacts the maximum likelihood estimator (MLE) of the signal synchronization parameters (i.e., delay, Doppler, phase), which is the key baseband signal processing in standard two-step GNSS receivers (Teunissen & Montenbruck, 2017). While several interference mitigation techniques exist (Morales-Ferre et al., 2020), as previously stated, a theoretical analysis on the GNSS MLE performance degradation induced by an interference (or a set of interferences) is a missing tool, being fundamental for system/receiver design. From an estimation perspective, and because the system of interest can be formulated as a Gaussian conditional signal model (CSM) under nominal conditions, it is sound to obtain the corresponding Cramér-Rao bound (CRB) (Trees & Bell, 2007). Indeed, the CRB gives an accurate estimation of the mean square error (MSE) of the MLE in the asymptotic region of operation, i.e., in the large sample and/or high signal-to-noise (SNR) regimes of the CSM (Renaux et al., 2006; Stoica & Nehorai, 1990). Even if CRBs for different GNSS receiver architectures under nominal conditions are available in the literature (see (Medina et al., 2020), (Medina et al., 2021), (McPhee et al., 2023a) and references therein), such performance bounds have not been studied for the interference case of interest in this contribution.

The main hypothesis is that the receiver is not aware that an interfence is present, and therefore, it assumes that the received signal is only corrupted by additive Gaussian noise as under nominal conditions. This implies that the signal model at the receiver input and the assumed signal model do not coincide, that is, there exists a model mismatch. In that case, the MLE is no longer unbiased, and the theoretical characterization implies to obtain closed-form expressions of: i) the estimation bias induced by the interference (this result has been first presented in Ortega et al. (2022)), and ii) the corresponding misspecified CRB (MCRB) (Richmond & Horowitz, 2015), (Fortunati et al., 2017), (Lubeigt et al., 2023), (McPhee et al., 2023b). The proposed bias and MCRB expressions are validated for a representative linear frequency modulation (LFM) chirp signal interference. Notice that once a compact MCRB form is derived, this can be used for: i) the derivation of metrics that allow to compare the robustness to interference of different GNSS signals, as well as for the design of new GNSS signals, and ii) the design of next-generation interference countermeasures.

2 | TRUE AND MISSPECIFIED SIGNAL MODELS

2.1 | Correctly Specified Signal Model

A GNSS band-limited signal $s(t)$, with bandwidth B, is transmitted over a carrier frequency f_c ($\lambda_c = c/f_c$, $\omega_c = 2\pi f_c$). The synchronization parameters to be estimated are the delay and Doppler shift, $\boldsymbol{\eta} = (\tau, b)^\top$. Under the narrowband assumption, the influence of the Doppler parameter on the baseband signal samples is negligible, $s((1-b)(t-\tau)) \approx s(t-\tau)$ (Dogandzic & Nehorai, 2001). For short observation times, a good approximation of the baseband output of the receiver's Hilbert filter (GNSS signal + interference) is (Skolnik, 1990),

$$x(t; \boldsymbol{\eta}) = \alpha s(t-\tau) e^{-j2\pi f_c (b(t-\tau))} + I(t) + n(t), \tag{1}$$

with $I(t)$ a bandlimited unknown interference (or set of interferences) within the frequency band of interest, $n(t)$ a complex white Gaussian noise with unknown variance σ_n^2 and $\alpha = \rho e^{j\Phi}$ a complex gain. The discrete vector signal model is built from $N = N_1 - N_2 + 1$ samples at $T_s = 1/F_s \leq 1/B$,

$$\mathbf{x} = \alpha \mathbf{a}(\boldsymbol{\eta}) + \mathbf{n} = \rho e^{j\Phi} \mathbf{a}(\boldsymbol{\eta}) + \mathbf{n} = \alpha \boldsymbol{\mu}(\boldsymbol{\eta}) + \mathbf{I} + \mathbf{n}, \tag{2}$$

with $\mathbf{x} = (\dots, x(kT_s), \dots)^\top$, $\mathbf{I} = (\dots, I(kT_s), \dots)^\top$, $\mathbf{n} = (\dots, n(kT_s), \dots)^\top$, $N_1 \leq k \leq N_2$ signal samples, and

$$\mathbf{a}(\boldsymbol{\eta}) = (\dots, s(kT_s - \tau) e^{-j2\pi f_c (b(kT_s - \tau)} + \frac{1}{\alpha} I(kT_s). \dots)^\top, \tag{3}$$

$$\boldsymbol{\mu}(\boldsymbol{\eta}) = (\dots, s(kT_s - \tau) e^{-j2\pi f_c (b(kT_s - \tau)}. \dots)^\top. \tag{4}$$

The unknown deterministic parameters can be gathered in vector $\boldsymbol{\epsilon}^\top = (\sigma_n^2, \rho, \Phi, \boldsymbol{\eta}^\top) = (\sigma_n^2, \boldsymbol{\theta}^\top)$, with $\rho \in \mathbb{R}^+, 0 \leq \Phi \leq 2\pi$. The correctly specified signal model is represented by a probability densitiy function (pdf) denoted as $p_\epsilon(\mathbf{x}; \boldsymbol{\epsilon})$, which follows a complex circular Gaussian distribution $\mathbf{x} \sim \mathcal{CN}(\alpha \mathbf{a}(\boldsymbol{\eta}), \sigma_n^2 \boldsymbol{I}_N)$.

2.2 | Misspecified Signal Model

The misspecified signal model represents the case where the interference is not considered, i.e., when a mismatched MLE (MMLE) is implemented at the receiver. This nominal case leads to the definition of the misspecified parameter vector $\boldsymbol{\eta}' = [\tau', b']^\top$, and the complete set of unknown parameters $\boldsymbol{\epsilon'}^\top = \left[\sigma'^2_n, \rho', \Phi', \boldsymbol{\eta'}^\top\right] = \left[\sigma'^2_n, \boldsymbol{\theta'}^\top\right]$, yielding the following signal model at the output of the Hilbert filter,

$$x'(t; \boldsymbol{\eta}') = \alpha' s(t-\tau') e^{-j2\pi f_c b'(t-\tau')} + n'(t), \tag{5}$$

with $n'(t)$ a complex white Gaussian noise with unknown variance σ'^2_n and $\alpha' = \rho' e^{j\Phi'}$. Again, we can build the discrete vector signal model from N samples at $T_s = 1/F_s$,

$$\mathbf{x}' = \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}') + \mathbf{n},\ \boldsymbol{\mu}(\boldsymbol{\eta}') = (\ldots, s(kT_s - \tau')e^{-j2\pi f_c b'(kT_s - \tau')}, \ldots)^\top. \tag{6}$$

The misspecified signal model is represented by a pdf denoted as $f_{\boldsymbol{\epsilon}'}(\mathbf{x}; \boldsymbol{\epsilon}')$ which follows a complex circular Gaussian distribution $\mathbf{x}' \sim \mathcal{CN}(\alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'), \sigma'^2_n \boldsymbol{I}_N)$, then

$$p_\epsilon(\mathbf{x}; \epsilon) = \frac{1}{\pi^N \sigma_n^{2N}} e^{\frac{-(\mathbf{x} - \alpha \boldsymbol{a}(\boldsymbol{\eta}))^H (\mathbf{x} - \alpha \boldsymbol{a}(\boldsymbol{\eta}))}{\sigma_n^2}}, \quad f_{\boldsymbol{\epsilon}'}(\mathbf{x}'; \boldsymbol{\epsilon}') = \frac{1}{\pi^N \sigma'^{2N}_n} e^{\frac{-(\mathbf{x} - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))^H (\mathbf{x} - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))}{\sigma'^2_n}} \tag{7}$$

Notice that considering the misspecified signal model induces a bias on the corresponding MMLE. These biased estimated parameters are commonly referred to as pseudotrue parameters, $\boldsymbol{\theta}_{pt}^\top = \left[\rho_{pt}, \Phi_{pt}, \tau_{pt}, b_{pt}\right]$. Note that for this particular contribution, we are not interested in the noise variance parameter.

3 | MMLE BIAS COMPUTATION VIA KULLBACK-LEIBLER DIVERGENCE

The pseudotrue parameters are simply those that give the minimum Kullback-Leibler Divergence (KLD) (Fortunati et al., 2017) $D(p_\epsilon || f_{\epsilon'}) = E_{p_\epsilon}\left[\ln p_\epsilon(\mathbf{x}; \epsilon) - \ln f_{\boldsymbol{\epsilon}'}(\mathbf{x}'; \boldsymbol{\epsilon}')\right]$, between the true and assumed models, where $E_{p_\epsilon}[\cdot]$ is the expectation with respect to (w.r.t.) the true model's pdf,

$$\boldsymbol{\theta}_{pt} = \arg\min_{\theta'} \left\{ D(p_\epsilon || f_{\epsilon'}) \right\} = \arg\min_{\theta'} \left\{ E_{p_\epsilon}\left[-\ln f_{\boldsymbol{\epsilon}'}(\mathbf{x}; \boldsymbol{\epsilon}')\right] \right\}, \tag{8}$$

$$E_{p_\epsilon}\left[-\ln f_{\epsilon'}\right] = -N \ln(\pi) - E_{p_\epsilon}\left[2N \ln(\sigma'_n)\right] + E_{p_\epsilon}\left[\frac{(\mathbf{x} - \alpha \mathbf{a}(\boldsymbol{\eta}) + \alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))^H (\mathbf{x} - \alpha \mathbf{a}(\boldsymbol{\eta}) + \alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))}{\sigma'^2_n}\right]. \tag{9}$$

We aim to compute the pseudotrue parameters $\boldsymbol{\theta}_{pt}^\top = \left[\rho_{pt}, \Phi_{pt}, \tau_{pt}, b_{pt}\right]$, then we have to minimize (8) w.r.t. the argument $\boldsymbol{\theta}'$, the equation can be simplified as follows,

$$\begin{aligned}
&\arg\min_{\theta'} \left\{ E_{p_\epsilon}\left[-\ln f_{\boldsymbol{\epsilon}'}(\mathbf{x}; \boldsymbol{\epsilon}')\right] \right\} \\
&= \arg\min_{\theta'} \left\{ E_{p_\epsilon} \left[\begin{array}{c} \frac{1}{\sigma'^2_n} \left[(\mathbf{x} - \alpha \mathbf{a}(\boldsymbol{\eta}))^H (\mathbf{x} - \alpha \mathbf{a}(\boldsymbol{\eta})) + (\mathbf{x} - \alpha \mathbf{a}(\boldsymbol{\eta}))^H (\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}')) \right. \\ +(\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))^H (\mathbf{x} - \alpha \mathbf{a}(\boldsymbol{\eta})) \\ \left. +(\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))^H (\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))\right] \end{array} \right] \right\} \\
&= \arg\min_{\theta'} \left\{ (\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}'))^H (\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}')) \right\} = \arg\min_{\theta'} \left\{ \left\| \alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}') \right\|^2 \right\}.
\end{aligned}$$

We define the orthogonal projector $\Pi_{\mathbf{A}}^\perp = \mathbf{I} - \Pi_{\mathbf{A}}$ with $\Pi_{\mathbf{A}} = \mathbf{A}\left(\mathbf{A}^H \mathbf{A}\right)^{-1} \mathbf{A}^H$, which leads to

$$\begin{aligned}
\|\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}')\|^2 &= \left\| \left(\Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')} + \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')}^\perp \right) \left(\alpha \mathbf{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}') \right) \right\|^2 \\
&= \left\| \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')} \left(\alpha \boldsymbol{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}') \right) \right\|^2 + \left\| \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')}^\perp \left(\alpha \boldsymbol{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}') \right) \right\|^2 \\
&= \left\| \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')} \alpha \boldsymbol{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}') \right\|^2 + \left\| \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')}^\perp \alpha \boldsymbol{a}(\boldsymbol{\eta}) \right\|^2 \\
&= \left\| \boldsymbol{\mu}(\boldsymbol{\eta}') \left(\frac{\boldsymbol{\mu}(\boldsymbol{\eta}')^H \alpha \boldsymbol{a}(\boldsymbol{\eta})}{\boldsymbol{\mu}(\boldsymbol{\eta}')^H \boldsymbol{\mu}(\boldsymbol{\eta}')} - \alpha' \right) \right\|^2 + \|\alpha \boldsymbol{a}(\boldsymbol{\eta})\|^2 - \left\| \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')} \alpha \boldsymbol{a}(\boldsymbol{\eta}) \right\|^2,
\end{aligned}$$

and then the parameters that minimize the KLD are,

$$\arg\min_{\theta'} \left\{ \|\alpha \boldsymbol{a}(\boldsymbol{\eta}) - \alpha' \boldsymbol{\mu}(\boldsymbol{\eta}')\|^2 \right\} \Leftrightarrow \begin{cases} \alpha_{pt} = \alpha \frac{\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})^H \boldsymbol{a}(\boldsymbol{\eta})}{\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})^H \boldsymbol{\mu}(\boldsymbol{\eta}_{pt})} \\ \boldsymbol{\eta}_{pt} = \arg\max_{\eta'} \left\{ \left\| \Pi_{\boldsymbol{\mu}(\boldsymbol{\eta}')} \alpha \boldsymbol{a}(\boldsymbol{\eta}) \right\|^2 \right\} \end{cases}$$

with $\alpha_{pt} = \rho_{pt} e^{j\Phi_{pt}}$ and $\boldsymbol{\eta}_{pt}^{\top} = \left[\tau_{pt}, b_{pt}\right]$. This may be connected with the asymptotic MMLE behavior (Fortunati et al., 2017),

$$\begin{cases} \widehat{\alpha}' = \frac{\mu(\widehat{\eta}')^H \mathbf{x}}{\mu(\widehat{\eta}')^H \mu(\widehat{\eta}')} \\ \widehat{\boldsymbol{\eta}}' = \arg\max_{\boldsymbol{\eta}'} \left\{ \left\| \Pi_{\mu(\boldsymbol{\eta}')} \mathbf{x} \right\|^2 \right\} \end{cases} \xrightarrow[SNR\to\infty]{} \begin{cases} \widehat{\alpha}' = \alpha \frac{\mu(\widehat{\eta}')^H a(\eta')}{\mu(\widehat{\eta}')^H \mu(\widehat{\eta}')} = \alpha_{pt} \\ \widehat{\boldsymbol{\eta}}' = \arg\max_{\boldsymbol{\eta}'} \left\{ \left\| \Pi_{\mu(\boldsymbol{\eta}')} \alpha \boldsymbol{a}(\boldsymbol{\eta}) \right\|^2 \right\} = \boldsymbol{\eta}_{pt} \end{cases}. \tag{10}$$

Because the pseudotrue parameters, obtained as the MMLE without noise, are those that give the minimum KLD between the true and assumed models, the bias is defined as $\Delta\alpha = \alpha_{pt} - \alpha$, $\boldsymbol{\Delta\eta} = \boldsymbol{\eta}_{pt} - \boldsymbol{\eta}$.

4 | CLOSED-FORM MCRB EXPRESSIONS FOR A BAND-LIMITED SIGNAL UNDER INTERFERENCE

In Richmond & Horowitz (2015), the MCRB was derived as an extension of the Slepian-Bangs formulas, a result that was later expressed as a combination of two information matrices in Fortunati et al. (2017): $\mathbf{A}(\boldsymbol{\theta}_{pt})$ and $\mathbf{B}(\boldsymbol{\theta}_{pt})$,

$$\mathbf{MCRB}(\theta_{pt}) = \mathbf{A}(\theta_{pt})^{-1}\mathbf{B}(\theta_{pt})\mathbf{A}(\theta_{pt})^{-1}, \tag{11}$$

where

$$\mathbf{A}(\theta_{pt}) = \frac{2}{\sigma_n^2}\Re\left\{(\delta\mathbf{m})^H\left(\frac{\partial^2\alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}\partial\boldsymbol{\theta}_{pt}^{\top}}\right)\right\} - \mathbf{B}(\theta_{pt}),$$
$$\mathbf{B}(\theta_{pt}) = \frac{2}{\sigma_n^2}\Re\left\{\left(\frac{\partial\alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}}\right)^H\left(\frac{\partial\alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}}\right)\right\},$$

$\delta\mathbf{m} \triangleq \alpha\mathbf{a}(\boldsymbol{\eta}) - \alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt}) = \alpha\boldsymbol{\mu}(\boldsymbol{\eta}) + \boldsymbol{I} - \alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})$ the mean difference between true and misspecified models.

4.1 | Single Source Fisher Information Matrix

In $\mathbf{B}(\theta_{pt})$ one can recognize the Fisher Information Matrix (FIM) of a single source CSM. A compact expression of this FIM, that depends only on the baseband signal samples, was recently derived in Medina et al. (2020). We recalled hereafter for completeness that

$$\mathbf{B}(\theta_{pt}) = \frac{2F_s}{\sigma_n^2}\Re\left\{\mathbf{QWQ}^H\right\} \tag{12}$$

with

$$\mathbf{W} = \begin{bmatrix} w_1 & w_2^* & w_3^* \\ w_2 & W_{2,2} & w_4^* \\ w_3 & w_4 & W_{3,3} \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} j\alpha_{pt}\omega_c b_{pt} & 0 & -\alpha_{pt} \\ 0 & -j\alpha_{pt}\omega_c & 0 \\ e^{j\Phi_{pt}} & 0 & 0 \\ \alpha_{pt} & 0 & 0 \end{bmatrix}, \tag{13}$$

where the elements of $\mathbf{W}$ can be expressed w.r.t. the baseband signal samples as,

$$w_1 = \frac{1}{F_s}\mathbf{s}^H\mathbf{s}, w_2 = \frac{1}{F_s^2}\mathbf{s}^H\mathbf{D}\mathbf{s}, w_3 = \frac{1}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,1}(0)\mathbf{s},$$
$$w_4 = \frac{1}{F_s}\mathbf{s}^H\mathbf{D}\mathbf{V}^{\Delta,1}(0)\mathbf{s}, W_{2,2} = \frac{1}{F_s^3}\mathbf{s}^H\mathbf{D}^2\mathbf{s}, W_{3,3} = F_s\mathbf{s}^H\mathbf{V}^{\Delta,2}(0)\mathbf{s}$$

with $\mathbf{s}$, the baseband samples vector, $\mathbf{D}$, $\mathbf{V}^{\Delta,1}(\cdot)$ and $\mathbf{V}^{\Delta,2}(\cdot)$ defined as,

$$\mathbf{s} = \left(\ldots, s(nT_s), \ldots\right)_{N_1\le n\le N_2}^{\top}, \tag{14a}$$
$$\mathbf{D} = \mathrm{diag}\left(\ldots, n, \ldots\right)_{N_1\le n\le N_2}, \tag{14b}$$
$$\left[\mathbf{V}^{\Delta,1}(q)\right]_{k,l} = \frac{1}{k-l-q}\left(\cos\left(\pi(k-l-q)\right) - \mathrm{sinc}\left(k-l-q\right)\right), \tag{14c}$$
$$\left[\mathbf{V}^{\Delta,2}(q)\right]_{k,l} = \pi^2\mathrm{sinc}\left(k-l-q\right) + 2\frac{cos\left(\pi(k-l-q)\right) - \mathrm{sinc}\left(k-l-q\right)}{(k-l-q)^2}. \tag{14d}$$

where the reader can refer to Appendix B for details on the closed-form expressions of $\mathbf{V}^{\Delta,1}(q)$ and $\mathbf{V}^{\Delta,2}(q)$.

4.2 | Model Mismatch Information Matrix

The matrix $\mathbf{A}(\boldsymbol{\theta}_{pt})$ accounts for the model misspecification. Its elements can also be expressed in a compact form as a function of the baseband signal and interference samples as,

$$\left[\mathbf{A}(\boldsymbol{\theta}_{pt})\right]_{p,q} = \frac{2F_s}{\sigma_n^2}\Re\left\{\left[\mathbf{Q}_q\right]_{p,.}\mathbf{W}^{\mathbf{A}}\tilde{\boldsymbol{\alpha}}^*\right\} - \left[\mathbf{B}(\boldsymbol{\theta}_{pt})\right]_{p,q},$$
$$\mathbf{W}^{\mathbf{A}} = [\mathbf{w}_1^{\mathbf{A}}\ \mathbf{w}_2^{\mathbf{A}}\ \mathbf{w}_3^{\mathbf{A}}],\ \tilde{\boldsymbol{\alpha}} = \left(\rho e^{j\Phi}, 1, -\rho_{pt}e^{j\Phi_{pt}}\right)^{\top}, \tag{15}$$

with $\mathbf{w}_1^{\mathbf{A}} = [\dots, w_{1,l}^{\mathbf{A}}, \dots]^{\top}$, $\mathbf{w}_2^{\mathbf{A}} = [\dots, w_{2,l}^{\mathbf{A}}, \dots]^{\top}$ and $\mathbf{w}_3^{\mathbf{A}} = [\dots, w_{3,l}^{\mathbf{A}}, \dots]^{\top}$ for $l \in (1, \cdots, 6)$, and where $\left[\mathbf{Q}_q\right]_{p,.}$ is the p-th row of the matrix $\mathbf{Q}_q$ (refer to Appendix A for $\mathbf{Q}_q$). With $\Delta\tau = \tau - \tau_{pt}$ and $\Delta b = b - b_{pt}$, $\mathbf{W}^{\mathbf{A}}$ is obtained from,

$$w_{1,1}^{\mathbf{A}}(\boldsymbol{\eta})^* = \frac{1}{F_s}\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\, e^{j\omega_c b\Delta\tau},, \tag{16a}$$

$$w_{1,2}^{\mathbf{A}}(\boldsymbol{\eta})^* = \frac{1}{F_s^2}\mathbf{s}^H\mathbf{D}\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\, e^{j\omega_c b\Delta\tau}, \tag{16b}$$

$$w_{1,3}^{\mathbf{A}}(\boldsymbol{\eta})^* = \frac{1}{F_s^3}\mathbf{s}^H\mathbf{D}^2\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\, e^{j\omega_c b\Delta\tau}, \tag{16c}$$

$$\begin{aligned}w_{1,4}^{\mathbf{A}}(\boldsymbol{\eta})^* = &\left(-\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,1}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s} + \right.\\ &\left.\frac{j\omega_c\Delta b}{F_s}\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\right).\, e^{j\omega_c b\Delta\tau},\end{aligned} \tag{16d}$$

$$\begin{aligned}w_{1,5}^{\mathbf{A}}(\boldsymbol{\eta})^* = &\left(-\frac{1}{F_s}\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s} - \frac{1}{F_s}\mathbf{s}^H\mathbf{D}\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,1}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\right.\\ &\left.+j\frac{\omega_c\Delta b}{F_s^2}\mathbf{s}^H\mathbf{D}\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\right)\ e^{j\omega_c b\Delta\tau},\end{aligned} \tag{16e}$$

$$\begin{aligned}w_{1,6}^{\mathbf{A}}(\boldsymbol{\eta})^* = &\left(-F_s\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,2}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s} - j2\omega_c\Delta b\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,1}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\right.\\ &\left.-\frac{(\omega_c\Delta b)^2}{F_s}\mathbf{s}^H\mathbf{U}\left(\frac{f_c\Delta b}{F_s}\right)\mathbf{V}^{\Delta,0}\left(\frac{\Delta\tau}{T_s}\right)\mathbf{s}\right)\ e^{j\omega_c b\Delta\tau},\end{aligned} \tag{16f}$$

$$w_{2,1}^{\mathbf{A}} = \frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}, \tag{16g}$$

$$w_{2,2}^{\mathbf{A}} = \frac{1}{F_s^2}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}, \tag{16h}$$

$$w_{2,3}^{\mathbf{A}} = \frac{1}{F_s^3}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}^2\mathbf{s}, \tag{16i}$$

$$w_{2,4}^{\mathbf{A}} = \left(\mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{j\omega_c b_{pt}}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}\right), \tag{16j}$$

$$\begin{aligned}w_{2,5}^{\mathbf{A}} = &\left(-\frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}\right.\\ &\left.+j\frac{\omega_c b_{pt}}{F_s^2}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}\right),\end{aligned} \tag{16k}$$

$$\begin{aligned}w_{2,6}^{\mathbf{A}} = &\left(-F_s\mathbf{I}^H\mathbf{V}^{\Delta,2}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + j2\omega_c b_{pt}\mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}\right.\\ &\left.-\frac{(\omega_c b_{pt})^2}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}\right),\end{aligned} \tag{16l}$$

$$w_{3,1}^{\mathbf{A}} = w_1,\ w_{3,2}^{\mathbf{A}} = w_2,\ w_{3,3}^{\mathbf{A}} = W_{2,2},\ w_{3,4}^{\mathbf{A}} = w_3,\ w_{3,5}^{\mathbf{A}} = w_4,\ w_{3,6}^{\mathbf{A}} = -W_{3,3}. \tag{16m}$$

with

$$\mathbf{U}(p) = \mathrm{diag}\left(\ldots, e^{-j2\pi pn}, \ldots\right)_{N_1 \leq n \leq N_2}, \tag{17}$$

$$\left[\mathbf{V}^{\Delta,0}(q)\right]_{k,l} = \mathrm{sinc}\,(k - l - q) \tag{18}$$

Proof. See Appendices A and B. □

4.3 | Implementation of the Bias and the Misspecified CRB expressions

In this section, we explain step by step how to perform the calculation of the bias and the MCRB of the synchronization parameters of the received signal:

$$x(t;\boldsymbol{\eta}) = \alpha s(t-\tau)\, e^{-j2\pi f_c(b(t-\tau))} + I(t) + n(t), \tag{19}$$

- First of all, it is necessary to calculate the parameters $\alpha_{pt} = \rho_{pt} e^{j\Phi_{pt}}$ and $\boldsymbol{\eta}_{pt}^{\top} = \left[\tau_{pt}, b_{pt}\right]$ from from equation (10).
- Then, we compute the bias of the synchronization parameters as $\Delta\alpha = \alpha_{pt} - \alpha$, $\boldsymbol{\Delta\eta} = \boldsymbol{\eta}_{pt} - \boldsymbol{\eta}$.
- To compute the MCRB, we start by computing the single source Fisher Information matrix $\mathbf{B}(\boldsymbol{\theta}_{pt})$. This process is described in Section 4.1.
- Then, we have to compute the model mismatch information matrix $\mathbf{A}(\boldsymbol{\theta}_{pt})$. To do this:
  - We compute $\tilde{\boldsymbol{\alpha}}$ from equation (15).
  - To compute $\mathbf{W}^{\mathbf{A}}$, we define $\Delta\tau = \tau - \tau_{pt}$ and $\Delta b = b - b_{pt}$. Then, we compute the elements of the matrix given by the equations (16a)-(16m).
  - Now, we compute the matrices $\mathbf{Q}_q$, with $q = \{1,2,3,4\}$. Those are included in Appendix A.
  - Finally, we compute $\left[\mathbf{A}(\boldsymbol{\theta}_{pt})\right]_{p,q} = \frac{2F_s}{\sigma_n^2}\Re\left\{\left[\mathbf{Q}_q\right]_{p,\cdot}\mathbf{W}^{\mathbf{A}}\tilde{\boldsymbol{\alpha}}^*\right\} - \left[\mathbf{B}(\boldsymbol{\theta}_{pt})\right]_{p,q}$.
- The MCRB is then computed $\mathbf{MCRB}(\boldsymbol{\theta}_{pt}) = \mathbf{A}(\boldsymbol{\theta}_{pt})^{-1}\mathbf{B}(\boldsymbol{\theta}_{pt})\mathbf{A}(\boldsymbol{\theta}_{pt})^{-1}$.

5 | VALIDATION

Let us consider the case where a GPS L1 C/A signal is interfered by a jammer that is generating a LFM chirp signal, which is defined as

$$I(t) = \Pi_T(t) \times e^{j\pi\alpha_c t^2 + j\phi},\ \Pi_T(t) = \begin{cases} A_i & \text{for } 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}, \tag{20}$$

with α_c the chirp rate, A_i the amplitude and $T = NT_s$ the waveform period. The instantaneous frequency is $f(t) = \frac{1}{2\pi}\frac{d}{dt}\left(\pi\alpha_c t^2\right) = \alpha_c t$, and therefore the waveform bandwidth is $B = \alpha_c T$. We consider the case where, after the Hilbert filter, the chirp is located at the baseband frequency, i.e., the central frequency of the chirp is $f_i = 0$. Then, the chirp equation can be rewritten as,

$$I(t) = \Pi_T(t) \times e^{j\pi\alpha_c (t-T/2)^2 + j\phi},\ \Pi_T(t) = \begin{cases} A_i & \text{for } 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}. \tag{21}$$

The MSE and bias results for the parameters of interest $\boldsymbol{\theta}^T = [\rho, \Phi, \boldsymbol{\eta}^T]$ are shown in Figures 1 -4 , w.r.t. the SNR at the output of the matched filter (i.e., SNR_{OUT}) and considering the following setup: a GNSS receiver with $F_s = 4$ MHz and a chirp bandwidth equal to 2 MHz, with initial phase $\phi = 0$ and amplitude $A_i = 10$. The number of Monte Carlo is set to 1000 iterations. In the results one can observe that i) the Root MSE ($\sqrt{MSE}$) of the true parameter converges to $\sqrt{MCRB + Bias^2}$, ii) the $\sqrt{MSE}$ of the pseudotrue parameter converges to the $\sqrt{MCRB}$, and iii) the value $\sqrt{MCRB}$ is always higher than $\sqrt{CRB}$ (refer to (Medina et al., 2020)), which represents the asymptotic estimation performance of the parameters without any source of interference. Such results validate and prove the exactness of the proposed MCRB and bias expressions. Finally, we would like to underline that the MCRB characterizes the misspecified MLE asymptotically and is therefore unable to evaluate what

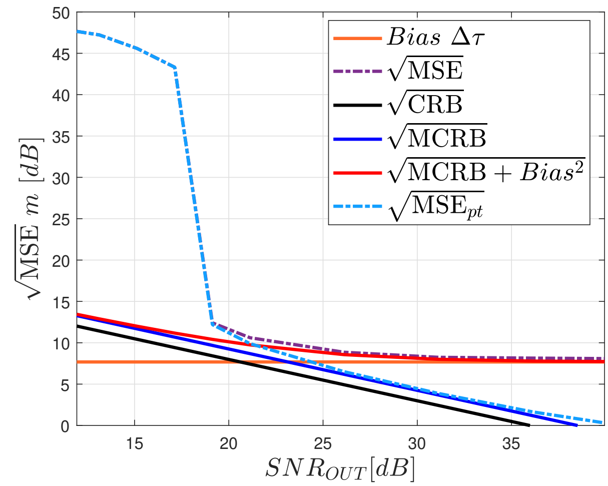

FIGURE 1 MML estimator root MSE for the time-delay τ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a chirp signal with $B = 2$ MHz, $A_i = 10$ and initial phase $\phi = 0$. Integration time set to 2 ms.

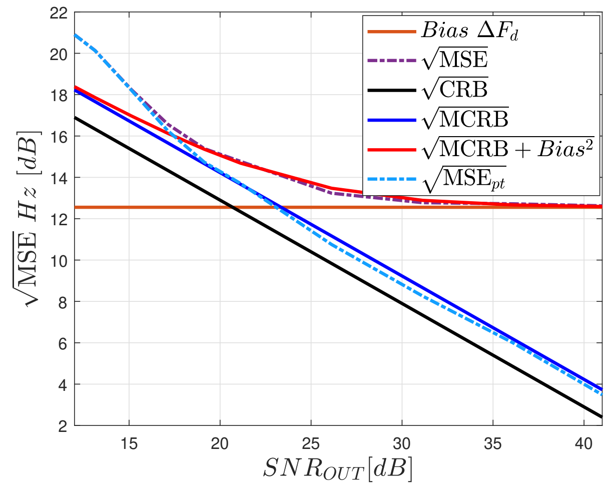

FIGURE 2 MML estimator root MSE for the Doppler F_d estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a chirp signal with $B = 2$ MHz, $A_i = 10$ and initial phase $\phi = 0$. Integration time set to 2 ms.

happens before the convergence region. Therefore the calculation of the MSE of the MMLE also indicates the threshold from which the MCRB theoretically characterizes the MSE of the MMLE.

In a second example, we evaluate the degradation caused by a single tone located at a frequency $f_i = 0.5$MHz. For this particular case, the interference samples are given by $\boldsymbol{I} = I(\cdots, A_i e^{j2\pi f_i kT_s + j\phi}, \cdots)$, which is a complex function, and can be rewritten as,

$$\boldsymbol{I} = I(\cdots, A_i e^{j2\pi f_i kT_s + j\phi}, \cdots) = I(\cdots, A_i \left(\cos(2\pi f_i kT_s + \phi) + j \sin(2\pi f_i kT_s + \phi)\right), \cdots) \tag{22}$$

with ϕ the initial phase of the tone and A_i the amplitude of the tone. For our particular scenario we set the initial phase to $\pi/2$ and $A_i = 10$. In Figures 5 -7 -9 -11 , we illustrate the MSE and bias results for the parameters of interest $\boldsymbol{\theta}^T = [\rho, \Phi, \boldsymbol{\eta}^T]$ as a function of the signal to noise ratio at the output of the match filter SNR_{OUT}. We set $F_s = 4$MHz and the integration time equal to 2ms. Note that the MSE converge to the theoretical result, re-validating the closed-form expressions. Moreover, in Figures 6 -8 -10 -12 , we also include one scenario where the integration time is set to 4 ms. Note that for this particular case, the bias is lower and Doppler estimation perfomance has been improved. Note that this can be proved theoretically thanks to the closed-form expressions of the Fisher information matrix which allows us to understand how the different design parameters affect the calculation of the MSE of the MLE. For this particular case, increasing the integration time increase the dimension of the matrices $\mathbf{D}$ and $\mathbf{D}^2$, which are related with the Fisher matrix parameters of the Doppler. The longer is the integration time, the better estimation performance is obtained.

6 | CONCLUSION

It is well documented in the literature that interference signals may have a huge impact on the GNSS receivers' performance, but to the best of the authors' knowledge, from an estimation perspective, the theoretical analysis of the impact of such intereferences on the first GNSS receiver stage (i.e., time-delay and Doppler estimation) is an important missing point. In practice, at the receiver there exists a model mismatch, and an interference induces both i) an estimation bias and ii) a variance degradation. In this contribution, we provided the theoretical closed-form expressions that characterize the MSE for the maximum likelihood estimates of the GNSS synchronization parameters, that is, bias and MCRB. Comparing these results to the standard CRB, associated to the unbiased maximum likelihood estimates without any interference, allows to theoretically characterize the performance degradation on the time-delay and Doppler estimation. The exactness of the proposed expressions was validated

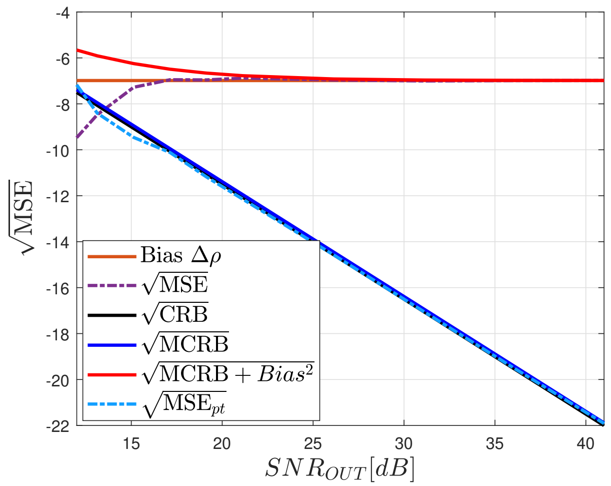

FIGURE 3 MML estimator root MSE for the amplitude ρ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a chirp signal with $B = 2$ MHz, $A_i = 10$ and initial phase $\phi = 0$. Integration time set to 2 ms.

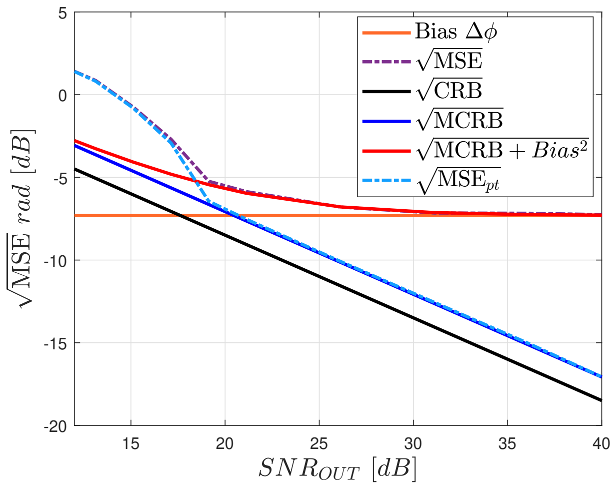

FIGURE 4 MML estimator root MSE for the phase Φ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a chirp signal with $B = 2$ MHz, $A_i = 10$ and initial phase $\phi = 0$. Integration time set to 2 ms.

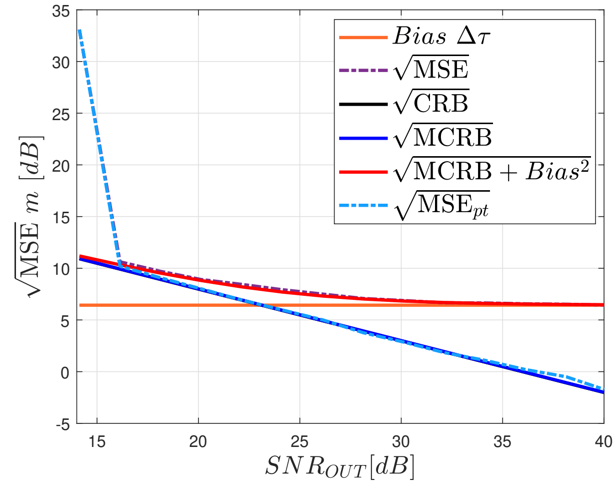

FIGURE 5 MML estimator root MSE for the time-delay τ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 2 ms.

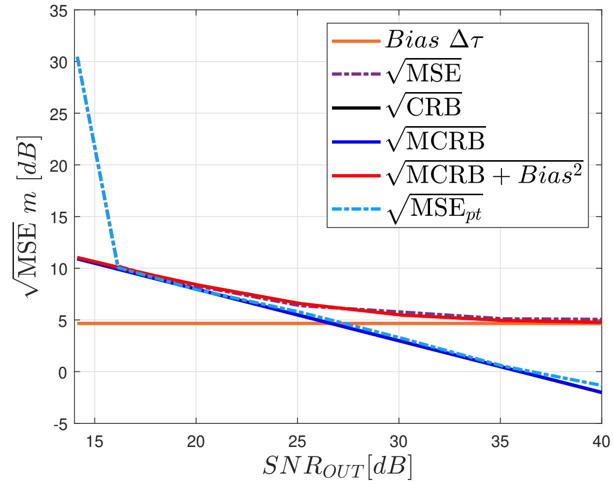

FIGURE 6 MML estimator root MSE for the time-delay τ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 4 ms.

for a representative case of a chirp interference jamming a GPS L1 C/A signal. Results were provided to show such validity and the impact on both time-delay and Doppler estimation. It is important to notice that such analysis may be the starting point for both the derivation of robustness metrics or new GNSS signals, and the design of interference countermeasures.

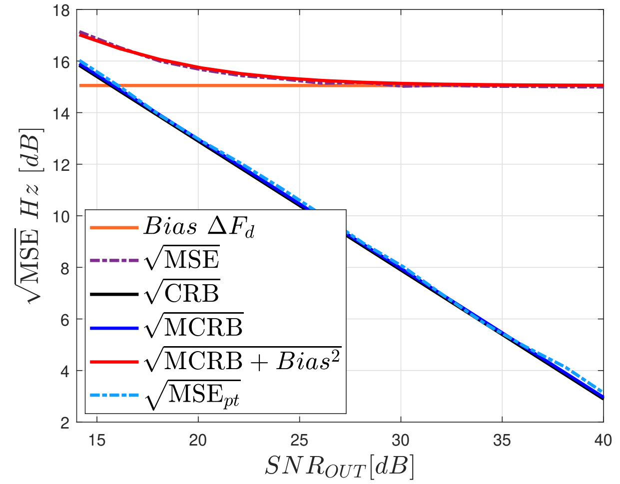

FIGURE 7 MML estimator root MSE for the Doppler estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 2 ms.

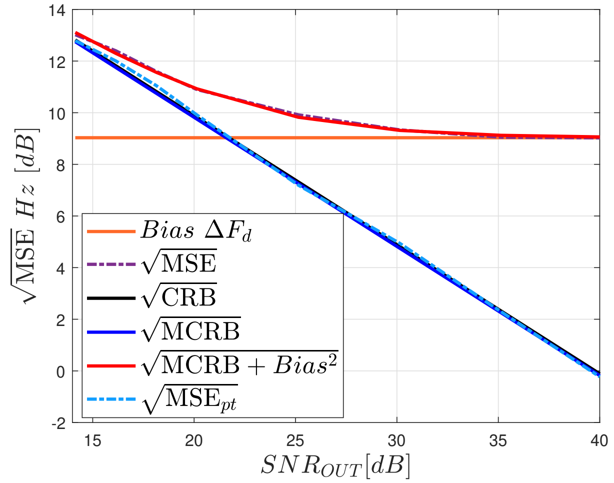

FIGURE 8 MML estimator root MSE for the Doppler estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 4 ms.

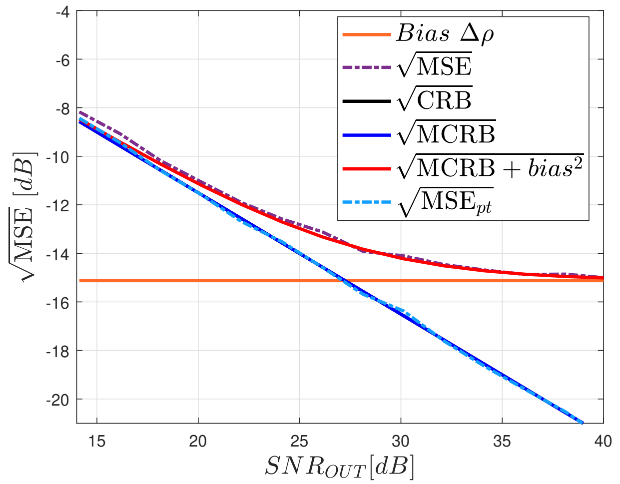

FIGURE 9 MML estimator root MSE for the amplitude ρ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 2 ms.

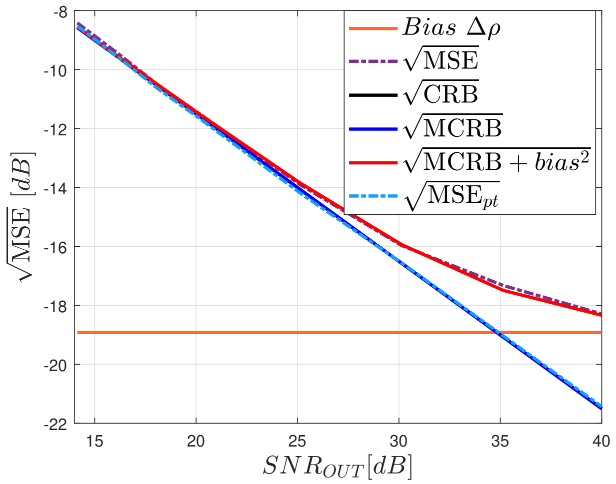

FIGURE 10 MML estimator root MSE for the amplitude ρ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 4 ms.

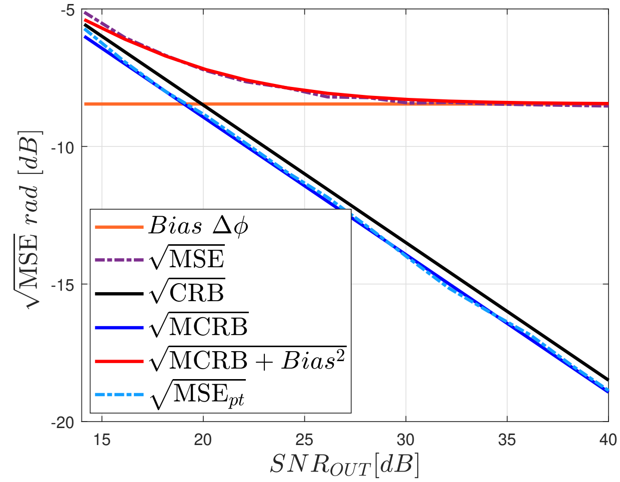

FIGURE 11 MML estimator root MSE for the phase Φ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 2 ms.

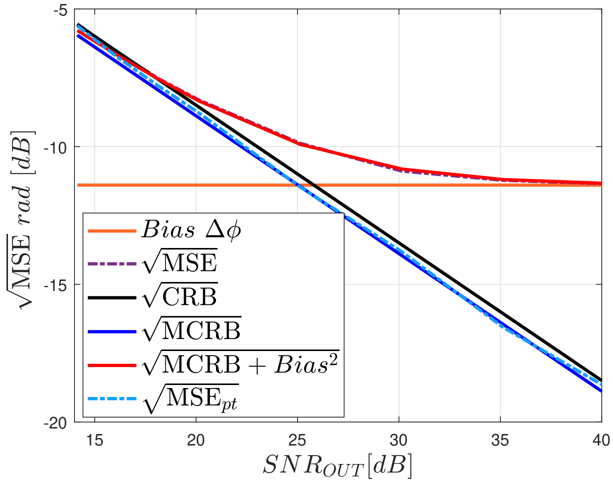

FIGURE 12 MML estimator root MSE for the phase Φ estimation w.r.t. the true and pseudotrue parameters, and the corresponding bounds. The interference is a tone signal with $f_i = 0.5$ MHz, $A_i = 10$ and initial phase $\phi = \pi/2$. Integration time set to 4 ms.

APPENDIX

A ON THE COMPUTATION OF $\mathbf{A}(\boldsymbol{\theta}_{PT})$

To compute $\mathbf{A}(\boldsymbol{\theta}_{pt})$, continuous time expressions are considered, $\mu(t;\boldsymbol{\eta}) = s(t-\tau)e^{-j\omega_c b(t-\tau)}$, $\delta m(t) = \alpha\mu(t;\boldsymbol{\eta}) + I(t) - \alpha_{pt}\mu(t;\boldsymbol{\eta}_{pt}) = \tilde{\mathbf{A}}(t)\tilde{\boldsymbol{\alpha}}$, with $\tilde{\mathbf{A}}(t) = \left[\mu(t;\boldsymbol{\eta}),\ I(t),\ \mu(t;\boldsymbol{\eta}_{pt})\right]$ and $\tilde{\boldsymbol{\alpha}} = \left(\rho e^{j\Phi}, 1, -\rho_{pt}e^{j\Phi_{pt}}\right)^{\top}$, which leads to the discrete expression $\delta\mathbf{m} = \tilde{\mathbf{A}}\tilde{\boldsymbol{\alpha}} = \left[\boldsymbol{\mu}(\boldsymbol{\eta}),\ \mathbf{I},\ \boldsymbol{\mu}(\boldsymbol{\eta}_{pt})\right]\tilde{\boldsymbol{\alpha}}$. The second derivative of interest can be written in a matrix form as:

$$\frac{\partial^2 \alpha_{pt}\boldsymbol{\mu}(t;\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}\partial\boldsymbol{\theta}_{pt}^{\top}} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 & \mathbf{Q}_3 & \mathbf{Q}_4 \end{bmatrix}\left(\mathcal{D}(t;\boldsymbol{\eta}_{pt}) \otimes \mathbf{I}_4\right) e^{-j\omega_c b_{pt}(t-\tau_{pt})},$$

with

$$\mathbf{Q}_1 = \begin{bmatrix} -\alpha_{pt}\omega_c^2 b_{pt}^2 & 0 & 0 & -j2\alpha_{pt}\omega_c b_{pt} & 0 & \alpha_{pt} \\ j\alpha_{pt}\omega_c & \alpha_{pt}\omega_c^2 b_{pt} & 0 & 0 & j\alpha_{pt}\omega_c & 0 \\ je^{j\Phi_{pt}}\omega_c b_{pt} & 0 & 0 & -e^{j\Phi_{pt}} & 0 & 0 \\ -\alpha_{pt}\omega_c b_{pt} & 0 & 0 & -j\alpha_{pt} & 0 & 0 \end{bmatrix}, \mathbf{Q}_2 = \begin{bmatrix} j\alpha_{pt}\omega_c & \alpha_{pt}\omega_c^2 b_{pt} & 0 & 0 & j\alpha_{pt}\omega_c & 0 \\ 0 & 0 & -\alpha_{pt}\omega_c^2 & 0 & 0 & 0 \\ 0 & -je^{j\Phi_{pt}}\omega_c & 0 & 0 & 0 & 0 \\ 0 & \alpha_{pt}\omega_c & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{Q}_3 = \begin{bmatrix} je^{j\Phi_{pt}}\omega_c b_{pt} & 0 & 0 & -e^{j\Phi_{pt}} & 0 & 0 \\ 0 & -je^{j\Phi_{pt}}\omega_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ je^{j\Phi_{pt}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{Q}_4 = \begin{bmatrix} -\alpha_{pt}\omega_c b_{pt} & 0 & 0 & -j\alpha_{pt} & 0 & 0 \\ 0 & \alpha_{pt}\omega_c & 0 & 0 & 0 & 0 \\ je^{j\Phi_{pt}} & 0 & 0 & 0 & 0 & 0 \\ -\alpha_{pt} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{D}(t;\tau_{pt}) = \begin{bmatrix} s(t-\tau_{pt}) \\ (t-\tau_{pt})s(t-\tau_{pt}) \\ (t-\tau_{pt})^2 s(t-\tau_{pt}) \\ s^{(1)}(t-\tau_{pt}) \\ (t-\tau_{pt})s^{(1)}(t-\tau_{pt}) \\ s^{(2)}(t-\tau_{pt}) \end{bmatrix} = \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \\ d_4(t) \\ d_5(t) \\ d_6(t) \end{bmatrix},$$

where $s^{(1)}(\cdot)$ and $s^{(2)}(\cdot)$ refer to the first and second time derivatives, respectively. A way to write the product of the mean difference term and the Hessian matrix, under its discrete form is,

$$\delta\mathbf{m}^H \left[\frac{\partial^2 \alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}\partial\boldsymbol{\theta}_{pt}^T}\right]_{p,q} = \left(\tilde{\mathbf{A}}\tilde{\boldsymbol{\alpha}}\right)^H \left(\left[\mathbf{Q}_q\right]_{p,.} \left[\dots, \mathcal{D}(kT_s;\boldsymbol{\eta}_{pt})e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})}, \dots\right]_{N_1\le k\le N_2}\right)^\top,$$

which can also be written as,

$$\delta\mathbf{m}^H \left[\frac{\partial^2 \alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}\partial\boldsymbol{\theta}_{pt}^T}\right]_{p,q} = \left[\mathbf{Q}_q\right]_{p,.} \sum_{k=N_1}^{N_2} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_6 \end{bmatrix},$$

with

$$\beta_l = \alpha^*\mu(kT_s;\boldsymbol{\eta})^* d_l(kT_s)e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})} + I(kT_s)^* d_l(kT_s)e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})} - \alpha_{pt}^*\mu(kT_s;\boldsymbol{\eta}_{pt})^* d_l(kT_s)e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})}$$

When the number of samples tends to infinity, each β_l is the sum of three integrals,

$$\lim_{(N_1,N_2)\to(-\infty,+\infty)} T_s \sum_{k=N_1}^{N_2} \mu(kT_s;\boldsymbol{\eta})^* d_l(kT_s)e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})} = \int_{\mathbb{R}} \mu(t;\boldsymbol{\eta})^* d_l(t)e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = w_{1,l}^{\mathbf{A}},$$

$$\lim_{(N_1,N_2)\to(-\infty,+\infty)} T_s \sum_{k=N_1}^{N_2} I(kT_s)^* d_l(kT_s)e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})} = \int_{\mathbb{R}} I(t)^* d_l(t)e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = w_{2,l}^{\mathbf{A}},$$

$$\lim_{(N_1,N_2)\to(-\infty,+\infty)} T_s \sum_{k=N_1}^{N_2} \mu(kT_s;\boldsymbol{\eta}_{pt})^* d_l(kT_s)e^{-j\omega_c b_{pt}(kT_s-\tau_{pt})} = \int_{\mathbb{R}} \mu(t;\boldsymbol{\eta}_{pt})^* d_l(t)e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = w_{3,l}^{\mathbf{A}},$$

which leads to the expression in (15),

$$\lim_{(N_1,N_2)\to(-\infty,+\infty)} \delta\mathbf{m}^H \left[\frac{\partial^2 \alpha_{pt}\boldsymbol{\mu}(\boldsymbol{\eta}_{pt})}{\partial\boldsymbol{\theta}_{pt}\partial\boldsymbol{\theta}_{pt}^T}\right]_{p,q} = F_s \left[\mathbf{Q}_q\right]_{p,.} \begin{bmatrix} w_{1,1}^{\mathbf{A}} & w_{2,1}^{\mathbf{A}} & w_{3,1}^{\mathbf{A}} \\ \vdots & \vdots & \vdots \\ w_{1,6}^{\mathbf{A}} & w_{2,6}^{\mathbf{A}} & w_{3,6}^{\mathbf{A}} \end{bmatrix} \tilde{\boldsymbol{\alpha}}^*$$

Then, the computation of $\mathbf{A}(\boldsymbol{\theta}_{pt})$ reduces to three sets of integrals. The first set of integrals is,

$$w_{1,1}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} s(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}dt, \quad w_{1,2}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} (t-\tau_{pt})s(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}dt,$$

$$w_{1,3}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} (t-\tau_{pt})^2 s(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}dt, \quad w_{1,4}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} s^{(1)}(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}dt,$$

$$w_{1,5}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} (t-\tau_{pt})s^{(1)}(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}dt, \quad w_{1,6}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} s^{(2)}(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}dt,$$

and the corresponding closed-form expressions are given in (16a)-(16f). The derivation of $w_{1,1}^{\mathbf{A}}(\boldsymbol{\eta})$, $w_{1,2}^{\mathbf{A}}(\boldsymbol{\eta})$ and $w_{1,4}^{\mathbf{A}}(\boldsymbol{\eta})$ can be found in Lubeigt et al. (2020) (equations (A.27), (A.28) and (A.29) respectively). The rest of the terms are derived in Appendix B. The second set of integrals is,

$$w_{2,1}^{\mathbf{A}} = \int_{\mathbb{R}} s(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt, \quad w_{2,2}^{\mathbf{A}} = \int_{\mathbb{R}} (t-\tau_{pt})s(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt,$$

$$w_{2,3}^{\mathbf{A}} = \int_{\mathbb{R}} (t-\tau_{pt})^2 s(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt, \quad w_{2,4}^{\mathbf{A}} = \int_{\mathbb{R}} s^{(1)}(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt,$$

$$w_{2,5}^{\mathbf{A}} = \int_{\mathbb{R}} (t-\tau_{pt})s^{(1)}(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt, \quad w_{2,6}^{\mathbf{A}} = \int_{\mathbb{R}} s^{(2)}(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt,$$

and the corresponding closed-form expressions are given in (16g)-(16l) The derivation of these terms is done in Appendix B. The last set is,

$$
\begin{aligned}
w^{\mathbf{A}}_{3,1} &= \int_{\mathbb{R}} s(t-\tau_{pt})s(t-\tau_{pt})^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_{pt}(t-\tau_{pt})\right)}dt = w_1,\\
w^{\mathbf{A}}_{3,2} &= \int_{\mathbb{R}} (t-\tau_{pt})s(t-\tau_{pt})s(t-\tau_{pt})^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_{pt}(t-\tau_{pt})\right)}dt = w_2,\\
w^{\mathbf{A}}_{3,3} &= \int_{\mathbb{R}} (t-\tau_{pt})^2 s(t-\tau_{pt})s(t-\tau_{pt})^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_{pt}(t-\tau_{pt})\right)}dt = W_{2,2},\\
w^{\mathbf{A}}_{3,4} &= \int_{\mathbb{R}} s^{(1)}(t-\tau_{pt})s(t-\tau_{pt})^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_{pt}(t-\tau_{pt})\right)}dt = w_3,\\
w^{\mathbf{A}}_{3,5} &= \int_{\mathbb{R}} (t-\tau_{pt})s^{(1)}(t-\tau_{pt})s(t-\tau_{pt})^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_{pt}(t-\tau_{pt})\right)}dt = w_4,\\
w^{\mathbf{A}}_{3,6} &= \int_{\mathbb{R}} s^{(2)}(t-\tau_{pt})s(t-\tau_{pt})^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_{pt}(t-\tau_{pt})\right)}dt = -W_{3,3}.
\end{aligned}
$$

B DERIVATION OF INTERFERENCE CONVOLUTION TERMS USING THE FOURIER TRANSFORM PROPERTIES

B.1 Prior Considerations

First the Fourier transform of a set of functions are to be evaluated. Remembering that the signal is band-limited of band $B \leq F_s$, one has:

$$s(t) \rightleftharpoons \mathrm{FT}\left\{s(t)\right\}(f) \triangleq S(f) = \left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi f nT_s}\right)1_{\left[-\frac{F_s}{2};\frac{F_s}{2}\right]} \tag{B1}$$

In order to tackle the issue that may come from the spectral shift due to Doppler effect. One simply needs to take F_s large enough so that $\frac{F_s}{2} \geq \frac{B}{2} + f_c \max\left\{|b|, |b_{pt}|, |b-b_{pt}|\right\}$.

A first expression is a simple application of the frequency shift relation when using the Fourier transform of a signal multiplied by a complex time-varying exponential.

$$s(t)e^{j2\pi f_c bt} \rightleftharpoons \mathrm{FT}\left\{s(t)e^{j2\pi f_c bt}\right\}(f) \triangleq S(f-f_c b) \tag{B2}$$

Then, let s_1 be defined by $s_1(t;b) = s(t)e^{j2\pi f_c bt}$, it is known that

$$ts_1(t;b) \leftrightharpoons \frac{j}{2\pi}\frac{\mathrm{d}}{\mathrm{d}f}\underbrace{\left(\mathrm{FT}\left\{s_1(t;b)\right\}(f)\right)}_{\text{(B2)}} \tag{B3}$$

therefore

$$ts(t)e^{j2\pi f_c bt} \leftrightharpoons \frac{j}{2\pi}\frac{\mathrm{d}}{\mathrm{d}f}\left(S(f-f_c b)\right) \tag{B4}$$

Similarly

$$t^2 s(t)e^{j2\pi f_c bt} \leftrightharpoons \left(\frac{j}{2\pi}\right)^2\frac{\mathrm{d}^2}{\mathrm{d}f^2}\left(S(f-f_c b)\right) \tag{B5}$$

Besides, with the superscript $^{(1)}$ referring to the first time derivative,

$$
\begin{aligned}
s_1^{(1)}(t;b) &\triangleq \frac{\mathrm{d}}{\mathrm{d}t}(s_1(t;b)) = s^{(1)}(t)e^{j2\pi f_c bt} + (j2\pi f_c b)s_1(t;b)\\
\Leftrightarrow\ s^{(1)}(t)e^{j2\pi f_c bt} &= s_1^{(1)}(t;b) - (j2\pi f_c b)s_1(t;b)
\end{aligned}
$$

Then, knowing the Fourier transform of the k-th time derivative of a function

$$\mathrm{FT}\left\{s^{(k)}(t)\right\}(f) \triangleq (j2\pi f)^k S(f) \tag{B6}$$

one directly gets

$$s^{(1)}(t)e^{j2\pi f_c bt} \rightleftharpoons j2\pi\left(f - f_c b\right) S(f - f_c b) \tag{B7}$$

Now, if s_2 is defined as $s_2(t; b) = ts(t)e^{j2\pi f_c bt}$,

$$\begin{aligned} s_2^{(1)}(t; b) &= s_1(t; b) + ts^{(1)}(t)e^{j2\pi f_c bt} + (j2\pi f_c b)s_2(t; b) \\ \Leftrightarrow\ ts^{(1)}(t)e^{j2\pi f_c bt} &= -\underbrace{s_1(t; b)}_{\text{(B2)}} + \underbrace{s_2^{(1)}(t; b)}_{\text{(B6)}} - (j2\pi f_c b)\underbrace{s_2(t; b)}_{\text{(B4)}} \end{aligned}$$

therefore,

$$ts^{(1)}(t)e^{j2\pi f_c bt} \leftrightharpoons -S(f - f_c b) - (f - f_c b)\frac{\mathrm{d}}{\mathrm{d}f}\left(S(f - f_c b)\right) \tag{B8}$$

Finally, by taking again s_1 as $s_1(t; b) = s(t)e^{j2\pi f_c bt}$,

$$\begin{aligned} s_1^{(2)}(t; b) &= s^{(2)}(t)e^{j2\pi f_c bt} + 2(j2\pi f_c b)s^{(1)}(t)e^{j2\pi f_c bt} + (j2\pi f_c b)^2 s_1(t; b) \\ \Leftrightarrow\ s^{(2)}(t)e^{j2\pi f_c bt} &= \underbrace{s_1^{(2)}(t; b)}_{\text{(B6)}} - (j4\pi f_c b)\underbrace{s^{(1)}(t)e^{j2\pi f_c bt}}_{\text{(B7)}} + 4\pi^2(f_c b)^2\underbrace{s_1(t; b)}_{\text{(B2)}} \end{aligned}$$

one obtains,

$$\begin{aligned} s^{(2)}(t)e^{j2\pi f_c bt} &\leftrightharpoons (j2\pi f)^2 S(f - f_c b) + 8\pi^2 f_c b(f - f_c b)S(f - f_c)) + 4\pi^2(f_c b)^2 S(f - f_c) \\ &\leftrightharpoons -4\pi^2\left(f - f_c b\right)^2 S(f - f_c b) \end{aligned} \tag{B9}$$

B.2 Evaluation of the Integrals

B.2.1 Derivation of Integral $w_{1,3}^{\mathbf{A}}(\boldsymbol{\eta})$

$$w_{1,3}^{\mathbf{A}}(\boldsymbol{\eta}) = \int_{\mathbb{R}} (t - \tau_{pt})^2 s(t - \tau_{pt})s(t - \tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt}) - b(t-\tau)\right)}dt = e^{-j\omega_c b\Delta\tau}\int_{\mathbb{R}} u^2 s(u)s(u - \Delta\tau)^* e^{j\omega_c \Delta bu}\mathrm{d}u$$

Then, using the Fourier transform properties over the hermitian product,

$$\begin{aligned} w_{1,3}^{\mathbf{A}}(\boldsymbol{\eta})e^{j\omega_c b\Delta\tau} &= \int_{\mathbb{R}} \underbrace{u^2 s(u)e^{j\omega_c \Delta bu}}_{\text{(B5)}}(s(u - \Delta\tau))^*\,\mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(\left(\frac{j}{2\pi}\right)^2 \frac{\mathrm{d}^2}{\mathrm{d}f^2}\left(S(f - f_c\Delta b)\right)\right)\left(S(f)e^{-j2\pi f\Delta\tau}\right)^*\mathrm{d}f \\ &= \frac{1}{F_s^3}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\mathbf{s}^T\mathbf{D}^2\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\boldsymbol{v}(f)^*\right)e^{j2\pi f\frac{\Delta\tau}{T_s}}\left(\mathbf{s}^H\boldsymbol{v}(f)\right)\mathrm{d}f = \frac{1}{F_s^3}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{D}^2\mathbf{s} \end{aligned}$$

Hence

$$\boxed{w_{1,3}^{\mathbf{A}}(\boldsymbol{\eta}) = \frac{1}{F_s^3}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{D}^2\mathbf{s}\,e^{-j\omega_c b\Delta\tau},} \tag{B10}$$

with $\mathbf{U}(p)$ defined in (17) and $\mathbf{V}^{\Delta,0}(q)$ defined in (18). Note that:

$$\boldsymbol{v}(f) = \begin{pmatrix} \dots & e^{j2\pi fn} & \dots \end{pmatrix}^T_{N_1 \le n \le N_2} \tag{B11}$$

$$\mathbf{V}^{\Delta,0}(q) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{v}(f)\boldsymbol{v}^H(f)e^{-j2\pi fq}\mathrm{d}f \tag{B12}$$

and

$$\left[\mathbf{V}^{\Delta,0}(q)\right]_{k,l} = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j2\pi f(k-l-q)}\mathrm{d}f = \left[\frac{e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{\sin(\pi(k-l-q))}{\pi(k-l-q)} = \operatorname{sinc}(k-l-q). \tag{B13}$$

B.2.2 Derivation of Integral $w^{\mathbf{A}}_{1,5}$

$$w^{\mathbf{A}}_{1,5}(\boldsymbol{\eta}) = \int_{\mathbb{R}} (t-\tau_{pt})s^{(1)}(t-\tau_{pt})s(t-\tau)^* e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b_k(t-\tau)\right)}\mathrm{d}t = e^{-j\omega_c b\Delta\tau}\int_{\mathbb{R}} us^{(1)}(u)s(u-\Delta\tau)^* e^{j\omega_c\Delta bu}\mathrm{d}u$$

Therefore,

$$w^{\mathbf{A}}_{1,5}(\boldsymbol{\eta})e^{j\omega_c b\Delta\tau} = \int_{\mathbb{R}} \underbrace{us^{(1)}(u)e^{j\omega_c\Delta bu}}_{(B8)}(s(u-\Delta\tau))^*\,\mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}}\left(-S(f-f_c\Delta b)-(f-f_c\Delta b)\frac{\mathrm{d}}{\mathrm{d}f}\left(S(f-f_c\Delta b)\right)\right)\left(S(f)e^{-j2\pi f\Delta\tau}\right)^*\mathrm{d}f$$

$$= -\frac{1}{F_s}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\mathbf{s}^T\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\boldsymbol{\nu}(f)^*\right)e^{j2\pi f\frac{\Delta\tau}{T_s}}\left(\mathbf{s}^H\boldsymbol{\nu}(f)\right)\mathrm{d}f + \frac{1}{F_s}\int_{-\frac{1}{2}}^{\frac{1}{2}} j2\pi f\left(\mathbf{s}^T\mathbf{D}\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\boldsymbol{\nu}(f)^*\right)e^{j2\pi f\frac{\Delta\tau}{T_s}}\left(\mathbf{s}^H\boldsymbol{\nu}(f)\right)\mathrm{d}f$$

$$- j2\pi\frac{f_c\Delta b}{F_s^2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\mathbf{s}^T\mathbf{D}\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\boldsymbol{\nu}(f)^*\right)e^{j2\pi f\frac{\Delta\tau}{T_s}}\left(\mathbf{s}^H\boldsymbol{\nu}(f)\right)\mathrm{d}f$$

$$= -\frac{1}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s} + \frac{1}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,1}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{D}\mathbf{s} - j2\pi\frac{f_c\Delta b}{F_s^2}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{D}\mathbf{s}$$

Hence

$$\boxed{\begin{aligned} w^{\mathbf{A}}_{1,5}(\boldsymbol{\eta}) = \Bigg(&-\frac{1}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s} + \frac{1}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,1}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{D}\mathbf{s} \\ &-j\frac{\omega_c\Delta b}{F_s^2}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{D}\mathbf{s}\Bigg)\, e^{-j\omega_c b\Delta\tau}\end{aligned}} \tag{B14}$$

with $\mathbf{U}$, $\mathbf{V}^{\Delta,0}$ and $\mathbf{V}^{\Delta,1}(q)$ defined in (17), (18) and (14c). Note that

$$\mathbf{V}^{\Delta,1}(q) = j2\pi\int_{-\frac{1}{2}}^{\frac{1}{2}} f\boldsymbol{\nu}(f)\boldsymbol{\nu}^H(f)e^{-j2\pi fq}\mathrm{d}f \tag{B15}$$

and

$$\begin{aligned}\left[\mathbf{V}^{\Delta,1}(q)\right]_{k,l} &= j2\pi\int_{-\frac{1}{2}}^{\frac{1}{2}} f e^{j2\pi f(k-l-q)}\mathrm{d}f = j2\pi\left(\left[\frac{f e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\right]_{-\frac{1}{2}}^{\frac{1}{2}} - \int_{-\frac{1}{2}}^{+\frac{1}{2}}\frac{e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\mathrm{d}f\right) \\ &= \frac{j2\pi}{j2\pi(k-l-q)}\left(\left[\frac{1}{2}e^{j\pi(k-l-q)} - \left(-\frac{1}{2}\right)e^{-j\pi(k-l-q)}\right] - \left[\frac{e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\right]_{-\frac{1}{2}}^{\frac{1}{2}}\right) \\ &= \frac{1}{k-l-q}\left(\cos(\pi(k-l-q)) - \operatorname{sinc}(k-l-q)\right)\end{aligned} \tag{B16}$$

B.2.3 Derivation of Integral $w^{\mathbf{A}}_{1,6}$

$$w^{\mathbf{A}}_{1,6}(\boldsymbol{\eta}) = \int_{\mathbb{R}} s^{(2)}(t-\tau_{pt})s(t-\tau)^{*}e^{-j\omega_c\left(b_{pt}(t-\tau_{pt})-b(t-\tau)\right)}\mathrm{d}t = e^{-j\omega_c b\Delta\tau}\int_{\mathbb{R}} s^{(2)}(u)s(u-\Delta\tau)^{*}e^{j\omega_c\Delta bu}\mathrm{d}u$$

Therefore,

$$w^{\mathbf{A}}_{1,6}(\boldsymbol{\eta})e^{j\omega_c b\Delta\tau} = \int_{\mathbb{R}} \underbrace{s^{(2)}(u)e^{j\omega_c\Delta bu}}_{\text{(B9)}}(s(u-\Delta\tau))^{*}\,\mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(-4\pi^2(f-f_c\Delta b)^2 S(f-f_c\Delta b)\right)\left(S(f)e^{-j2\pi f\Delta\tau}\right)^{*}\mathrm{d}f$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\left(-F_s(4\pi^2 f^2) - j4\pi f_c\Delta b(j2\pi f) - 4\pi^2\frac{(f_c\Delta b)^2}{F_s}\right)\left(\mathbf{s}^T\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\boldsymbol{\nu}(f)^{*}\right)\right)\times e^{j2\pi f\frac{\Delta\tau}{T_s}}\left(\mathbf{s}^H\boldsymbol{\nu}(f)\right)\mathrm{d}f$$

$$= -F_s\mathbf{s}^H\mathbf{V}^{\Delta,2}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s} - j4\pi f_c\Delta b\mathbf{s}^H\mathbf{V}^{\Delta,1}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s}$$
$$- 4\pi^2\frac{(f_c\Delta b)^2}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s}$$

Hence

$$\boxed{\begin{aligned} w^{\mathbf{A}}_{6}(\boldsymbol{\eta}) = \Big(&-F_s\mathbf{s}^H\mathbf{V}^{\Delta,2}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s} - j2\omega_c\Delta b\mathbf{s}^H\mathbf{V}^{\Delta,1}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s} \\ &-\frac{(\omega_c\Delta b)^2}{F_s}\mathbf{s}^H\mathbf{V}^{\Delta,0}\left(-\frac{\Delta\tau}{T_s}\right)\mathbf{U}\left(-\frac{f_c\Delta b}{F_s}\right)\mathbf{s}\Big)\, e^{-j\omega_c b\Delta\tau}\end{aligned}} \tag{B17}$$

with $\mathbf{U}(\cdot)$ defined in (17), $\mathbf{V}^{\Delta,0}(\cdot)$ defined in (18), $\mathbf{V}^{\Delta,1}$ defined in (14c) and $\mathbf{V}^{\Delta,2}(\cdot)$ defined in (14d). Note that

$$\mathbf{V}^{\Delta,2}(q) = 4\pi^2\int_{-\frac{1}{2}}^{\frac{1}{2}} f^2\boldsymbol{\nu}(f)\boldsymbol{\nu}^H(f)e^{-j2\pi fq}\mathrm{d}f \tag{B18}$$

and

$$\left[\mathbf{V}^{\Delta,2}(q)\right]_{k,l} = 4\pi^2\int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 e^{j2\pi f(k-l-q)}\mathrm{d}f = 4\pi^2\left(\left[\frac{f^2 e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\right]_{-\frac{1}{2}}^{\frac{1}{2}} - \int_{-\frac{1}{2}}^{+\frac{1}{2}}\frac{2fe^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\mathrm{d}f\right)$$

$$= \frac{4\pi^2}{j2\pi(k-l-q)}\frac{1}{4}\left[e^{j\pi(k-l-q)} - e^{-j\pi(k-l-q)}\right] - \frac{8\pi^2}{j2\pi(k-l-q)}\left(\left[\frac{fe^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\right]_{-\frac{1}{2}}^{\frac{1}{2}} - \int_{-\frac{1}{2}}^{+\frac{1}{2}}\frac{e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\mathrm{d}f\right)$$

$$= \pi^2\mathrm{sinc}\,(k-l-q) - \frac{8\pi^2}{(j2\pi(k-l-q))^2}\times\left(\left[\frac{1}{2}e^{j\pi(k-l-q)} - \left(-\frac{1}{2}\right)e^{-j\pi(k-l-q)}\right] - \left[\frac{e^{j2\pi f(k-l-q)}}{j2\pi(k-l-q)}\right]_{-\frac{1}{2}}^{\frac{1}{2}}\right)$$

$$= \pi^2\mathrm{sinc}\,(k-l-q) + \frac{8\pi^2}{4\pi^2(k-l-q)^2}\times(\cos(\pi(k-l-q)) - \mathrm{sinc}\,(k-l-q))$$

$$= \pi^2\mathrm{sinc}\,(k-l-q) + 2\frac{cos\,(\pi(k-l-q)) - \mathrm{sinc}\,(k-l-q)}{(k-l-q)^2} \tag{B19}$$

B.2.4 Derivation of Integral $w^{\mathbf{A}}_{2,1}$

$$w^{\mathbf{A}}_{2,1} = \int_{\mathbb{R}} s(t-\tau_{pt})I(t)^{*}e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = \int_{\mathbb{R}} s(u)I(u+\tau_{pt})^{*}e^{-j\omega_c b_{pt}u}\mathrm{d}u$$

Then, using the Fourier transform properties over the hermitian product,

$$
\begin{aligned}
w_{2,1}^{\mathbf{A}} &= \int_{\mathbb{R}} \underbrace{s(u)e^{-j\omega_c b_{pt} u}}_{\text{(B2)}} \left(I(u+\tau_{pt})\right)^* \mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} S(f+f_c b_{pt}) \left(I(f)e^{j2\pi f \tau_{pt}}\right)^* \mathrm{d}f \\
&= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi(f+f_c b_{pt})nT_s}\right) e^{-j2\pi f \tau_{pt}} \left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f nT_s}\right)^* \mathrm{d}f \\
&= \frac{1}{F_s}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}} \left(\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f n}\right)^* \mathrm{d}f \\
&= \frac{1}{F_s}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\mathbf{s}^T\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{v}(f)^*\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}} \left(\mathbf{I}^H\boldsymbol{v}(f)\right)\mathrm{d}f \\
&= \frac{1}{F_s}\mathbf{I}^H\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{v}(f)\boldsymbol{v}^H(f)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} = \frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}
\end{aligned}
$$

Hence

$$
\boxed{w_{2,1}^{\mathbf{A}} = \frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}} \tag{B20}
$$

B.2.5 Derivation of Integral $w_{2,2}^{\mathbf{A}}$

$$
w_{2,2}^{\mathbf{A}} = \int_{\mathbb{R}} (t-\tau_{pt})s(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = \int_{\mathbb{R}} us(u)I(u+\tau_{pt})^* e^{-j\omega_c b_{pt} u}\mathrm{d}u
$$

Therefore,

$$
\begin{aligned}
w_{2,2}^{\mathbf{A}} &= \int_{\mathbb{R}} \underbrace{us(u)e^{-j\omega_c b_{pt} u}}_{\text{(B4)}} \left(I(u+\tau_{pt})\right)^* \mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(\frac{j}{2\pi}\frac{\mathrm{d}}{\mathrm{d}f}\left(S(f+f_c b_{pt})\right)\right) \left(I(f)e^{j2\pi f \tau_{pt}}\right)^* \mathrm{d}f \\
&= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(\frac{1}{F_s}\frac{j}{2\pi}(-j2\pi T_s)\sum_{n=N_1}^{N_2} s(nT_s)ne^{-j2\pi(f+f_c b_{pt})nT_s}\right) e^{-j2\pi f \tau_{pt}} \times \left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f nT_s}\right)^* \mathrm{d}f \\
&= \frac{1}{F_s^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\sum_{n=N_1}^{N_2} s(nT_s)ne^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}} \left(\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f n}\right)^* \mathrm{d}f \\
&= \frac{1}{F_s^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\mathbf{s}^T\mathbf{D}\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{v}(f)^*\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}} \left(\mathbf{I}^H\boldsymbol{v}(f)\right)\mathrm{d}f \\
&= \frac{1}{F_s^2}\mathbf{I}^H\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{v}(f)\boldsymbol{v}^H(f)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s} = \frac{1}{F_s^2}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}
\end{aligned}
$$

Hence

$$\boxed{w^{\mathbf{A}}_{2,2} = \frac{1}{F_s^2}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{Ds}} \tag{B21}$$

with $\mathbf{U}$ and $\mathbf{V}^{\Delta,0}$, defined in (17) and (18), respectively, and $\mathbf{D}$ defined in (14b).

B.2.6 Derivation of Integral $w^{\mathbf{A}}_{2,3}$

$$w^{\mathbf{A}}_{2,3} = \int_{\mathbb{R}} (t-\tau_{pt})^2 s(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = \int_{\mathbb{R}} u^2 s(u)I(u+\tau_{pt})^* e^{-j\omega_c b_{pt}u}\mathrm{d}u$$

Therefore,

$$w^{\mathbf{A}}_{2,3} = \int_{\mathbb{R}} \underbrace{u^2 s(u)e^{-j\omega_c b_{pt}u}}_{\text{(B5)}}\left(I(u+\tau_{pt})\right)^*\mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}}\left(\left(\frac{j}{2\pi}\right)^2\frac{\mathrm{d}^2}{\mathrm{d}f^2}\left(S(f+f_c b_{pt})\right)\right)\left(I(f)e^{j2\pi f\tau_{pt}}\right)^*\mathrm{d}f$$

$$= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}}\left(\frac{1}{F_s}\left(\frac{j}{2\pi}\right)^2(-j2\pi T_s)^2\sum_{n=N_1}^{N_2}s(nT_s)n^2e^{-j2\pi(f+f_c b_{pt})nT_s}\right)e^{-j2\pi f\tau_{pt}}\left(\frac{1}{F_s}\sum_{n=N_1}^{N_2}I(nT_s)e^{-j2\pi f nT_s}\right)^*\mathrm{d}f$$

$$= \frac{1}{F_s^3}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\sum_{n=N_1}^{N_2}s(nT_s)n^2e^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\sum_{n=N_1}^{N_2}I(nT_s)e^{-j2\pi f n}\right)^*\mathrm{d}f$$

$$= \frac{1}{F_s^3}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\mathbf{s}^T\mathbf{D}^2\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{v}(f)^*\right)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\mathbf{I}^H\boldsymbol{v}(f)\right)\mathrm{d}f$$

$$= \frac{1}{F_s^3}\mathbf{I}^H\left(\int_{-\frac{1}{2}}^{\frac{1}{2}}\boldsymbol{v}(f)\boldsymbol{v}^H(f)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}^2\mathbf{s} = \frac{1}{F_s^3}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}^2\mathbf{s}$$

Hence

$$\boxed{w^{\mathbf{A}}_{2,3} = \frac{1}{F_s^3}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}^2\mathbf{s}} \tag{B22}$$

with $\mathbf{U}$, $\mathbf{V}^{\Delta,0}$ and $\mathbf{D}$ defined in (17), (18) and (14b) respectively.

B.2.7 Derivation of Integral $w^{\mathbf{A}}_{2,4}$

$$w^{\mathbf{A}}_{2,4} = \int_{\mathbb{R}} s^{(1)}(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = \int_{\mathbb{R}} s^{(1)}(u)I(u+\tau_{pt})^* e^{-j\omega_c b_{pt}u}\mathrm{d}u$$

Therefore,

$$w^{\mathbf{A}}_{2,4} = \int_{\mathbb{R}} \underbrace{s^{(1)}(u)e^{-j\omega_c b_{pt} u}}_{\text{(B7)}} \left(I(u+\tau_{pt})\right)^* \mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(j2\pi(f+f_c b_{pt})S(f+f_c b_{pt})\right)\left(I(f)e^{j2\pi f \tau_{pt}}\right)^* \mathrm{d}f$$

$$= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(j2\pi(f+f_c b_{pt})\frac{1}{F_s}\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi(f+f_c b_{pt})nT_s}\right) e^{-j2\pi f \tau_{pt}} \left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f nT_s}\right)^* \mathrm{d}f$$

$$= \frac{1}{F_s}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(j2\pi(fF_s+f_c b_{pt})\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}} \left(\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f n}\right)^* \mathrm{d}f$$

$$= \frac{1}{F_s}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(j2\pi(fF_s+f_c b_{pt})\mathbf{s}^T\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{\nu}(f)^*\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}} \left(\mathbf{I}^H\boldsymbol{\nu}(f)\right)\mathrm{d}f$$

$$= \mathbf{I}^H\left(j2\pi\int_{-\frac{1}{2}}^{\frac{1}{2}} f\boldsymbol{\nu}(f)\boldsymbol{\nu}^H(f)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{j2\pi f_c b_{pt}}{F_s}\mathbf{I}^H\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{\nu}(f)\boldsymbol{\nu}^H(f)e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}$$

$$= \mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{j2\pi f_c b_{pt}}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}$$

Hence

$$\boxed{w^{\mathbf{A}}_{2,4} = \mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{j\omega_c b_{pt}}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s}} \tag{B23}$$

with $\mathbf{U}$, $\mathbf{V}^{\Delta,0}$ and $\mathbf{V}^{\Delta,1}$ defined in (17), (18) and (14c).

B.2.8 Derivation of Integral $w^{\mathbf{A}}_{2,5}$

$$w^{\mathbf{A}}_{2,5} = \int_{\mathbb{R}} (t-\tau_{pt})s^{(1)}(t-\tau_{pt})I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})}dt = \int_{\mathbb{R}} us^{(1)}(u)I(u+\tau_{pt})^* e^{-j\omega_c b_{pt} u}\mathrm{d}u$$

Therefore,

$$w^{\mathbf{A}}_{2,5} = \int\limits_{\mathbb{R}} \underbrace{us^{(1)}(u)e^{-j\omega_c b_{pt} u}}_{\text{(B8)}} \left(I(u+\tau_{pt})\right)^* \mathrm{d}u = \int\limits_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(-S(f+f_c b_{pt}) - (f+f_c b_{pt})\frac{\mathrm{d}}{\mathrm{d}f}\left(S(f+f_c b_{pt})\right)\right)\left(I(f)e^{j2\pi f\tau_{pt}}\right)^* \mathrm{d}f$$

$$= \int\limits_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(-\left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi(f+f_c b_{pt})nT_s}\right) - (f+f_c b_{pt})\left(\frac{1}{F_s}(-j2\pi T_s)\sum_{n=N_1}^{N_2} s(nT_s)ne^{-j2\pi(f+f_c b_{pt})nT_s}\right)\right)$$

$$\times e^{-j2\pi f\tau_{pt}}\left(\frac{1}{F_s}\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f nT_s}\right)^* \mathrm{d}f$$

$$= -\frac{1}{F_s}\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \left(\sum_{n=N_1}^{N_2} s(nT_s)e^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f n}\right)^* \mathrm{d}f$$

$$+\frac{1}{F_s}\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} j2\pi f\left(\sum_{n=N_1}^{N_2} s(nT_s)ne^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f n}\right)^* \mathrm{d}f$$

$$+ j2\pi\frac{f_c b_{pt}}{F_s^2}\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \left(\sum_{n=N_1}^{N_2} s(nT_s)ne^{-j2\pi f n}e^{-j2\pi\frac{f_c b_{pt}}{F_s}n}\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\sum_{n=N_1}^{N_2} I(nT_s)e^{-j2\pi f n}\right)^* \mathrm{d}f$$

$$= -\frac{1}{F_s}\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \left(\mathbf{s}^T\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{v}(f)^*\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\mathbf{I}^H\boldsymbol{v}(f)\right)\mathrm{d}f + \frac{1}{F_s}\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} j2\pi f\left(\mathbf{s}^T\mathbf{D}\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{v}(f)^*\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\mathbf{I}^H\boldsymbol{v}(f)\right)\mathrm{d}f$$

$$+ j2\pi\frac{f_c b_{pt}}{F_s^2}\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \left(\mathbf{s}^T\mathbf{D}\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\boldsymbol{v}(f)^*\right) e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\left(\mathbf{I}^H\boldsymbol{v}(f)\right)\mathrm{d}f$$

$$= -\frac{1}{F_s}\mathbf{I}^H\left(\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{v}(f)\boldsymbol{v}(f)^H e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{1}{F_s}\mathbf{I}^H\left(j2\pi\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} f\boldsymbol{v}(f)\boldsymbol{v}(f)^H e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}$$

$$+ j2\pi\frac{f_c b_{pt}}{F_s^2}\mathbf{I}^H\left(\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{v}(f)\boldsymbol{v}(f)^H e^{-j2\pi f\frac{\tau_{pt}}{T_s}}\mathrm{d}f\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}$$

$$= -\frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s} + j2\pi\frac{f_c b_{pt}}{F_s^2}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}$$

Hence

$$\boxed{w^{\mathbf{A}}_{2,5} = -\frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{s} + \frac{1}{F_s}\mathbf{I}^H\mathbf{V}^{\Delta,1}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s} + j\frac{\omega_c b_{pt}}{F_s^2}\mathbf{I}^H\mathbf{V}^{\Delta,0}\left(\frac{\tau_{pt}}{T_s}\right)\mathbf{U}\left(\frac{f_c b_{pt}}{F_s}\right)\mathbf{D}\mathbf{s}} \tag{B24}$$

with $\mathbf{U}$, $\mathbf{V}^{\Delta,0}$, $\mathbf{V}^{\Delta,1}$ and $\mathbf{D}$ defined in (17), (18), (14c) and (14b).

B.2.9 Derivation of Integral $w^{\mathbf{A}}_{2,6}$

$$w^{\mathbf{A}}_{2,6} = \int_{\mathbb{R}} s^{(2)}(t-\tau_{pt}) I(t)^* e^{-j\omega_c b_{pt}(t-\tau_{pt})} dt = \int_{\mathbb{R}} s^{(2)}(u) I(u+\tau_{pt})^* e^{-j\omega_c b_{pt} u} \mathrm{d}u$$

Therefore,

$$w^{\mathbf{A}}_{2,6} = \int_{\mathbb{R}} \underbrace{s^{(2)}(u) e^{-j\omega_c b_{pt} u}}_{(B9)} \left(I(u+\tau_{pt}) \right)^* \mathrm{d}u = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(-4\pi^2 (f + f_c b_{pt})^2 S(f + f_c b_{pt}) \right) \left(I(f) e^{j2\pi f \tau_{pt}} \right)^* \mathrm{d}f$$

$$= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \left(\left(-4\pi^2 f^2 - 8\pi^2 f f_c b_{pt} - 4\pi^2 (f_c b_{pt})^2 \right) \left(\frac{1}{F_s} \sum_{n=N_1}^{N_2} s(nT_s) e^{-j2\pi (f + f_c b_{pt}) nT_s} \right) \right) \times e^{j2\pi f \tau_{pt}} \left(\frac{1}{F_s} \sum_{n=N_1}^{N_2} I(nT_s) e^{-j2\pi f nT_s} \right)^* \mathrm{d}f$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\left(-4\pi^2 (f F_s)^2 - 8\pi^2 (f F_s) f_c b_{pt} - 4\pi^2 (f_c b_{pt})^2 \right) \left(\frac{1}{F_s} \sum_{n=N_1}^{N_2} s(nT_s) e^{-j2\pi f n} e^{-j2\pi \frac{f_c b_{pt}}{F_s} n} \right) \right)$$

$$\times e^{-j2\pi f \frac{\tau_{pt}}{T_s}} \left(\frac{1}{F_s} \sum_{n=N_1}^{N_2} I(nT_s) e^{-j2\pi f n} \right)^* \mathrm{d}f \, F_s$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\left(-F_s (4\pi^2 f^2) + j4\pi f_c b_{pt} (j2\pi f) - 4\pi^2 \frac{(f_c b_{pt})^2}{F_s} \right) \left(\mathbf{s}^T \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \boldsymbol{v}(f)^* \right) \right) e^{-j2\pi f \frac{\tau_{pt}}{T_s}} \left(\mathbf{I}^H \boldsymbol{v}(f) \right) \mathrm{d}f$$

$$= -F_s \mathbf{I}^H \left(4\pi^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} f^2 \boldsymbol{v}(f) \boldsymbol{v}(f)^H e^{-j2\pi f \frac{\tau_{pt}}{T_s}} \mathrm{d}f \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s} + j4\pi f_c b_{pt} \mathbf{I}^H \left(j2\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} f \boldsymbol{v}(f) \boldsymbol{v}(f)^H e^{-j2\pi f \frac{\tau_{pt}}{T_s}} \mathrm{d}f \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s}$$

$$- 4\pi^2 \frac{(f_c b_{pt})^2}{F_s} \mathbf{I}^H \left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \boldsymbol{v}(f) \boldsymbol{v}(f)^H e^{-j2\pi f \frac{\tau_{pt}}{T_s}} \mathrm{d}f \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s}$$

$$= -F_s \mathbf{I}^H \mathbf{V}^{\Delta,2} \left(\frac{\tau_{pt}}{T_s} \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s} + j4\pi f_c b_{pt} \mathbf{I}^H \mathbf{V}^{\Delta,1} \left(\frac{\tau_{pt}}{T_s} \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s} - 4\pi^2 \frac{(f_c b_{pt})^2}{F_s} \mathbf{I}^H \mathbf{V}^{\Delta,0} \left(\frac{\tau_{pt}}{T_s} \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s}$$

Hence

$$\boxed{w^{\mathbf{A}}_{2,6} = -F_s \mathbf{I}^H \mathbf{V}^{\Delta,2} \left(\frac{\tau_{pt}}{T_s} \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s} + j2\omega_c b_{pt} \mathbf{I}^H \mathbf{V}^{\Delta,1} \left(\frac{\tau_{pt}}{T_s} \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s} - \frac{(\omega_c b_{pt})^2}{F_s} \mathbf{I}^H \mathbf{V}^{\Delta,0} \left(\frac{\tau_{pt}}{T_s} \right) \mathbf{U} \left(\frac{f_c b_{pt}}{F_s} \right) \mathbf{s}} \tag{B25}$$

with $\mathbf{U}$, $\mathbf{V}^{\Delta,0}$ and $\mathbf{V}^{\Delta,1}$ defined in (17), (18) and (14c).

B.3 Matrix Properties

Based on the definitions of matrices $\mathbf{V}^{\Delta,0}$, $\mathbf{V}^{\Delta,1}$, $\mathbf{V}^{\Delta,2}$ and $\mathbf{U}$, one can do the following remarks:

- $\left(\mathbf{V}^{\Delta,0}(q) \right)^H = \mathbf{V}^{\Delta,0}(-q)$,
- $\left(\mathbf{V}^{\Delta,1}(q) \right)^H = -\mathbf{V}^{\Delta,1}(-q)$,
- $\left(\mathbf{V}^{\Delta,2}(q) \right)^H = \mathbf{V}^{\Delta,2}(-q)$,
- $(\mathbf{U}(p))^H = \mathbf{U}(-p)$.

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CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

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