IBDP Physics Internal Assessment

Candidate Number: gzk284

Research Topic: What is the relationship between electric current and the stopping time of eddy current brakes (ECBs) in high speed trains?

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1 Foreword

I have always been obsessed with trains throughout the entirety of my childhood; the streamlined aesthetics and speediness of a train simply fascinated me. But when I was small, I was panicky with the high pitched screeching noises of a train as well, which occurs frequently when travelling in Hong Kong's underground MTR (Mass Transit Railway). I later knew that the screeching noises were probably a result of the locomotive's brakes.

Having visited Japan 3 times in my childhood, the Shinkansen has once again captivated my enthusiasm for trains. I was in love with the Shinkansen 700 series due to its unique aesthetic, its "duck bill" nose in particular; incidentally, when I was doing some research regarding the 700 series (Figure 1), I realised one key feature that differentiated itself from most of the trains in Hong Kong: it had an eddy current braking system as opposed to the conventional pneumatic brake (air brake), which allows it to decelerate elegantly and quietly.

In class, I learnt that by Lenz's law, eddy currents, which are a result of electromagnetic induction, are created to oppose the change in magnetic field. With this in mind, in this investigation, I have taken the opportunity to conduct a quantitative research on eddy current brakes (ECBs).

Figure 1 Shinkansen 700 series (Wikipedia Commons, 2009)

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2 Introduction to eddy current brakes (ECB)

Brakes are used by railway locomotives to decelerate, to control acceleration in downhill situations or to keep a locomotive standing still. Some conventional railway brakes are (1) band brakes which consist of a band that tightens around a rotating cylindrical object, (2) clasp brakes which literally clasp the tyres to decelerate, (3) hydraulic brakes which apply fluid pressure for deceleration, and (4) friction brakes which relies on the friction between two surfaces pressed against each other.

An ECB, however, relies on the drag force provided by the electromagnetic force between an electromagnet and a metallic conductor. When there is relative motion between the conductor and the magnet, by Faraday's Law of induction, this will induce an electromotive force that is proportional to the rate of change of magnetic flux linkage and loops of electrical current within the conductor known as eddy currents. By Lenz's law, the eddy currents will create a counter magnetic field that opposes the magnetic field of the magnet.

As a result, in a case where the magnet is stationary and the conductor is in motion, the conductor will encounter a drag force from the magnet which is proportional to the velocity of the conductor; this will cause it to retard.

Since the eddy currents have to flow through the resistance of the conductor, the kinetic energy of the moving conductor will eventually be released as heat, which is a phenomenon known as joule heating (Thangaraju, 2015). For this reason, ECBs will not cause the screeching noises traditionally found in brakes relying on friction, and they are not prone to wearing out since the magnet is never in contact with the conductor.

With respect to the postulations and principles above, a study by Sokolov has proposed the model derived from D.Schieber valid at low speeds. The model is:

$$F_e = \frac{1}{4}\frac{\pi}{\rho}D^2 d B^2 c v \quad \cdots (1)$$

$$c = \frac{1}{2} \times \left(1 - \frac{1}{4} \frac{1}{\left(1 + \frac{R}{A}\right)^2 \left(\frac{A-R}{D}\right)^2} \right)$$

F_e Braking force of ECB

ρ $Resistivity\ of\ disc\ material$

B $Strength\ of\ Magnetic\ Field$

d $Thickness\ of\ disc\ (m)$

D $Diameter\ of\ Soft\ Iron\ pole\ (m)$

c $Proportionality\ factor, ratio\ of\ total\ disc\ resistance\ to\ resistance\ of\ part\ under\ pole$

v $Tangential\ Speed\ of\ Rotating\ disc$

R $Distance\ of\ centre\ of\ pole\ from\ centre\ of\ disc$

A $Total\ radius\ of\ disc$

Thus, an ECB's braking force is proportional to the tangential velocity of the moving conductor, or in other words:

$$F_e \propto v$$

Therefore, a disadvantage of ECBs is, since there must be a relative motion between the conductor and magnet to cause a rate of change of magnetic flux, it has no "holding force" for a locomotive that is standing still.

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3 Preliminary work for experiment

For real ECBs in the 700 Series Shinkansen, a conductive yet non-ferromagnetic metal disc (known as a rotor) is attached to the vehicle's wheel, with electromagnets of opposite poles located on both sides of the disc. Hence, when the driver steps on the brake pedal, a certain magnitude of current will be created, which will create a magnetic field; this will brake the rotor and the entire axle (Sokolov, 2016).

For the purposes of my experiment, I will only be creating a setup with a rotor (motor with disc rotating with it) and 2 electromagnets, and I will like to investigate the quantitative relationship between the magnetic field strength of the electromagnet and the stopping time of the rotating disc. Yet, the electromagnets have to be closely positioned to the disc, and there is no available lab equipment for measuring the magnetic field in between two closely positioned electromagnets.

By ampere's law, in a direct magnetisation setup where the current is passed directly through the disk, the magnetic field around that wire is directly proportional to its current (Deziel, 2018). In other words, the excitation (magnetic field generated by means of an electric current) current of the electromagnet is proportional to its magnetic field strength. Therefore, the relationship between excitation current and stopping time of the aluminium disc will be investigated.

Variables

Independent Variable

Excitation current of electromagnet: This is achieved by adjusting the knob on the rheostat.

Dependent Variable

Stopping time of rotating disc: This is collected by the light gate.

Justifications for Controlled Variables and selected apparatus

Controlled Variable	Justification
Air gap of 5mm on both sides of the disc and position of electromagnet	Magnetic field strength is weaker as distance increases from the electromagnet, so the position of electromagnet relative to disc must be kept constant
Power Supply for rotor of 4V	Braking force of an ECB is proportional to the tangential velocity of the moving conductor, so initial velocity must be maintained constant. During my preliminary experiment, it was discovered that using a lower voltage like 2V has resulted in stopping times too short to be measured, while using a higher voltage like 6V has melted the glue sticking the disc to the shaft and made the rotation wobbly; 4V has been chosen as the best compromise.
Initial position of disc: slit of disc at the top	Since the light gate calculates the speed of the rotating disc by counting the frequency for light passing through the slit, a same initial position will ensure that the initial velocity is calculated correctly.
Duration of turning on motor	A 115 second duration ensures that the rotor can reach its maximum speed.
Other apparatus kept constant	Justification
Aluminium disc	While past studies have proven a wide range of metals like zinc suitable for similar occasions, aluminium was proven the most effective for similar experiments (Baharom et al. , 2011) Furthermore, I was able to obtain pieces of scrap aluminium for free in shops selling aluminium framed windows.
Light gate	While other options like the rotary motion sensor was available, the light gate is proven the best option since it does not add additional friction to the rotor. It only required a slit cut on the disc.
Transformers with 1000 coils	In this experiment, transformers act as electromagnets, and since more turns of coils result in a greater magnetic field strength, I chose the 1000-coil transformers, which are the ones with the most turns of coils in our school lab. This will allow me to test with a wider range of magnetic field strengths.

Table 1 Justifications for Controlled Variables and selected apparatus

Experiment setup

Figure 2 is an overview of my experiment setup whereas Figure 3 is a circuit diagram for my setup.

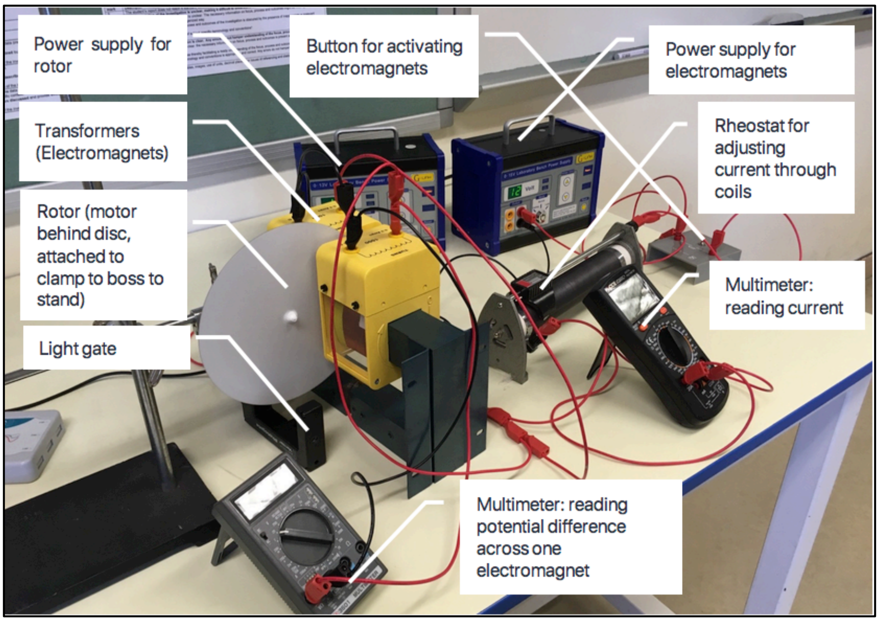

Figure 2 Photo of setup

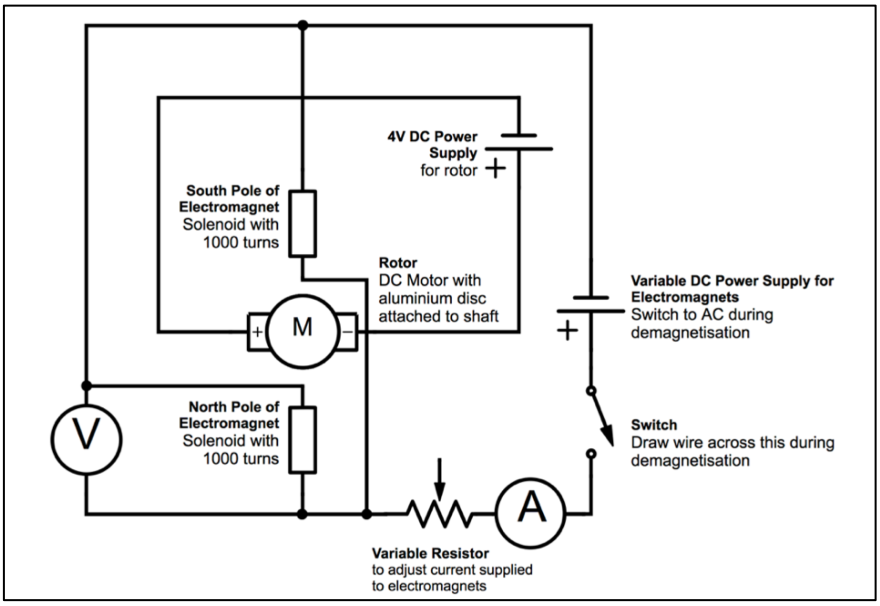

Figure 3 Circuit diagram for setup

<u>Procedures</u>

1) Setup the experiment according to the figure 2.
2) Adjust the rheostat to adjust the current to the desired value; take down the values of current of entire circuit and potential difference across one electromagnet.
3) Ensure slit is at the top position of the disc. This is maintained for consistency.
4) Click “Start” for collecting data on computer and start the stopwatch. In 5 seconds, switch on the motor; To minimise the time to switch on supply to 4V, set the power supply to 4V and plug in one cable; plug in the another cable to reach 4V immediately
5) After 120 seconds or 2 minutes on the stopwatch, switch off power supply for rotor and immediately press the switch for the electromagnet; Hold on to the switch until rotor has completely stopped moving.
6) Before the next trial, **the electromagnets must be demagnetised.** To demagnetise the magnet, change the supply of the electromagnets to AC current; Starting at 10V A.C., drop the voltage slowly, 1V every 3 seconds. Remember placing a cable across the switch before doing so and removing it after doing so.

4 Experiment Results and Analysis of Data

Raw Data Graphs

Velocity-Time Graphs for all trials

Since the light gate collects the gate state whenever the light is unblocked, over 70,000 cells of data has been generated, which is too large to be fit into this document. As a compromise and for environmental concerns, only the averaged data and data for electromagnet with supply 0.3A has been offered in the appendices. The raw data graph of velocity against time for all experiments (each with 3 trials) is all shown in figure 4. Note that the error bars are too small to be seen; the absolute uncertainties for velocity and time are is $\pm 0.001\ ms^{-1}$ s and $\pm 0.000001\ s$ respectively, which is the resolution of the equipment.

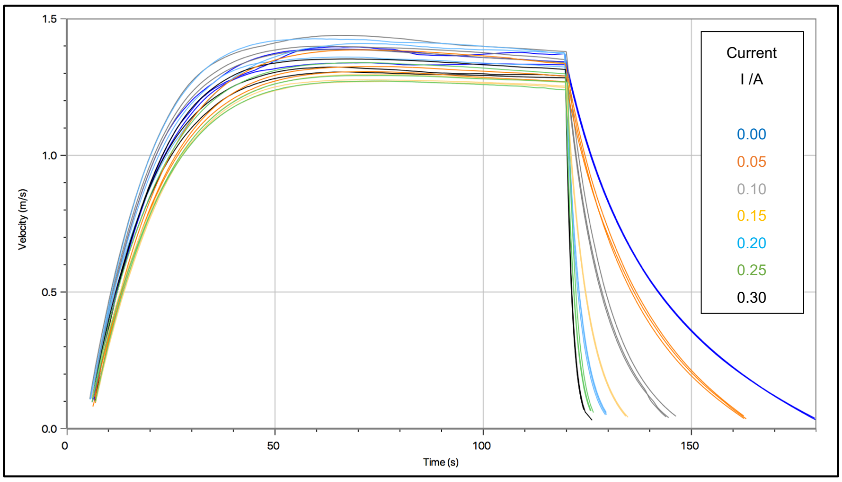

Figure 4 Raw Data: Velocity of against time, legend in graph.

Processed Data Graphs

Velocity-Time Graph for each Independent Variable (Average of 3 trials)

To gain a more concise perspective for how the disc decelerates with each electromagnet, the data from 3 trials of each experiment will be collated together. This involves taking data from the maximum deceleration onwards for each trial, and averaging the duration from last data point and the velocity for the 3 corresponding data set (which are now in sync). This results in one line for every electromagnet configuration (7 lines for 7 configurations). For example, table 2 is the first few data points of electromagnet with current 0A (for which the acceleration is at a minimum), which will be utilised in later calculations.

Run 1 (Trial 1)			Run 2 (Trial 2)			Run 3 (Trial 3)			Average Velocity	Average Time
Time	Velocity	Acc	Time	Velocity	Acc	Time	Velocity	Acc		
119.605761	1.367	-0.07	119.812572	1.340	-0.07	119.944956	1.328	-0.07	1.345	0.000000
119.605987			119.812803			119.945190				0.000230
119.642374	1.364	-0.07	119.849927	1.337	-0.07	119.982640	1.325	-0.07	1.342	0.037217
119.642601			119.850158			119.982874				0.037448
119.679058	1.362	-0.07	119.887354	1.335	-0.07	120.020400	1.323	-0.07	1.340	0.074508
119.679285			119.887586			120.020633				0.074738

Table 2 First 6 data points for experiment using electromagnet with current 0A, explanations for table headings in table 3.

Note that there is a blank cell for every other cell since the light gate works by calculating the frequency the light can pass through the slit of the disc. Hence, it collects data when the gate state is 0 or when it is unblocked. In addition, since the table headings might be too small; table 3 outlines some details for each heading:

Time	Velocity	Acc	Average Velocity	Average Time

t/s $\Delta t = \pm 0.000001s$	v/ms^{-1} $\Delta v = \pm 0.001ms^{-1}$	a/ms^{-2} $\Delta a = \pm 0.01ms^{-2}$	v/ms^{-1} $\Delta v = \pm 0.001ms^{-1}$	t/s $\Delta t = \pm 0.000001s$
Timestamp from start of experiment (cumulative). The first 120s include 5s of buffering and 115s of motor switched on, which will allow the rotor to reach its maximum velocity. Only data points that come at and after the maximum deceleration are used for processing as there are too many data points before 120s.	Instantaneous velocity	Instantaneous Acceleration	Average velocity with respect to average time	The average time for the first row is zero since it is the first data set. For the next row onwards, it is the average duration from the last point added to the previous time (time is cumulative in this case).
Absolute uncertainty is listed with respect to the smallest unit given by data from digital photo gate (Vernier, 2018).			Absolute uncertainties remain the same as the absolute uncertainties of the corresponding columns.	

Table 3 Details for table headings in table 2, justifications for absolute errors given

The average velocity is evaluated by averaging the velocity across every row. The average time for the first row is zero since it is the first data set. For the next row onwards, it is the average of the duration from the last point added to the previous time (time is cumulative in this case). For example, for row 2,

$$Avg\ Time = \frac{(119.605987 - 119.605761) + (119.812803 - 119.812572) + (119.945190 - 119.944956)}{3} + 0$$

$$= 0.00023033 \approx 0.000230\ seconds$$

These calculations is repeated for the 7 electromagnet configurations (each consisting of 3 trials). The average times and average velocities for every experiment will then be plotted against each other on the following graph. Figure 5 shows the graph of data fitted with exponential regression models:

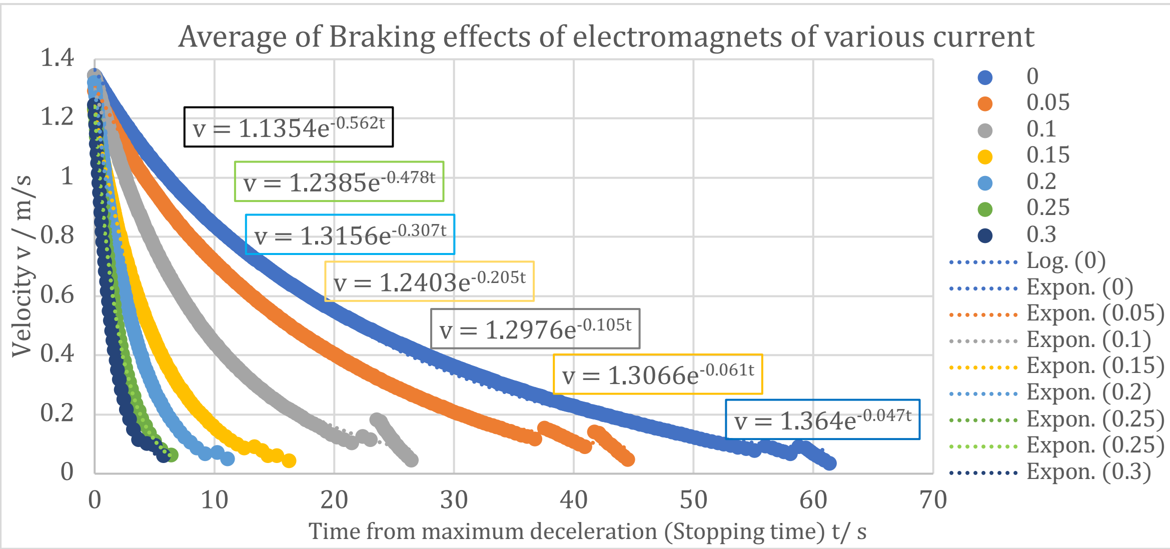

Figure 5 Processed data: Average velocity against time, legend in graph

While the graph shows a clear trend of proportionality, several observations can be made. For the above natural exponential graphs with general equation $v = ce^{bt}$, "c" represents the v-intercepts, whereas "b" indicates the growth factor of the variable "v". Hence, in theory, "c", which is the initial velocity, must remain constant, since the motor has been given a potential difference of 4V throughout the experiment. The fact that "c" varies between 1.1354 and 1.364 ms^{-1} indicates that the various imperfections in the rotor.

On the other hand, although "b" does not have a physical meaning, differentiating velocity gives deceleration $a = cbe^{bt}$. While there is a rate of change of acceleration or jerk, initial or maximum deceleration where t=0 is obtainable, which gives $a_{max} = cb$. Since growth factor b is increasing in magnitude when a greater current is applied to the electromagnet, for example: from 0.047 to 0.562 when current is increased from 0 to 0.3A, it can be deduced that maximum deceleration a_{max} increases with current.

Summary Data Table for Raw Data and Processed Graphs

However, the light gate not only offered data for instantaneous velocity. Acceleration data is given as well. In table 4, raw data for each configuration of the electromagnet (at a respective current and potential difference) is given. Note that the stopping time is processed data, and it is the time from maximum deceleration (minimum acceleration) to the moment of completion, where the rotor has stopped spinning. There are several assumptions:

1) Although the ideal voltmeter is supposed to have infinite resistance, it is placed in parallel with a high-resistance electromagnet (transformer), so there may be a substantial error in measuring the potential difference across it. Assume voltmeter is ideal.
2) Assume the magnetic field generated by the induced eddy currents is negligible.
3) Assume initial velocity for all trials are constant, even though it varies due to imperfections in the rotor; nonetheless, this assumption enables calculations relating to average braking force to be performed.

Note that error bars for potential difference across electromagnet and time intervals are invisible on graph; their absolute uncertainties are stated in the following table.

Excitation Current of electromagnets I/A $\Delta I = \pm 0.01A$	Potential Difference across each electromagnet $P.D./V$ $\Delta P.D. = \pm 0.01V$	Trial	Maximum Deceleration a_{max}/ms^{-2} $\Delta a_{max} = \pm 0.01ms^{-2}$	Stopping Time t/s $\Delta t = \pm 0.000001s$	Average Maximum Deceleration $\overline{a_{max}}/ms^{-2}$ $\Delta\overline{a_{max}} = \pm 0.01ms^{-2}$	Average Stopping Time $\bar{t}/s$ $\Delta\bar{t} = \pm 0.000001s$
Absolute uncertainty is listed with respect to the smallest unit (resolution) from digital multimeter.			Absolute uncertainty the smallest unit given by data from digital photo gate (Vernier, 2018).		Absolute uncertainties remain the same as the absolute uncertainties of the corresponding columns.	
		1	-0.07	62.256258		
0.00	0.00	2	-0.07	60.828349	-0.07	61.455534
		3	-0.07	61.281995		
		1	-0.08	43.990678		
0.05	2.31	2	-0.08	44.427722	-0.08	44.563511
		3	-0.08	45.272132		
		1	-0.16	27.572531		
0.10	4.46	2	-0.15	26.399369	-0.16	26.502517
		3	-0.16	25.535650		
		1	-0.25	16.578486		
0.15	9.00	2	-0.25	15.701608	-0.25	15.991377
		3	-0.25	15.694036		
		1	-0.42	11.050418		
0.20	8.96	2	-0.40	11.238213	-0.41	10.999577
		3	-0.40	10.710099		

0.25	10.00	1	-0.59	7.039282	-0.57	7.208168
		2	-0.55	7.106932		
		3	-0.56	7.478290		
0.30	13.46	1	-0.80	7.852687	-0.81	6.147882
		2	-0.80	5.189445		
		3	-0.82	5.401515		

Table 4 Summary table for raw and processed experimental data.

In the figures 6 and 7, excitation current of the electromagnets.is plotted against average stopping time and average maximum deceleration (minimum acceleration is maximum deceleration) respectively.

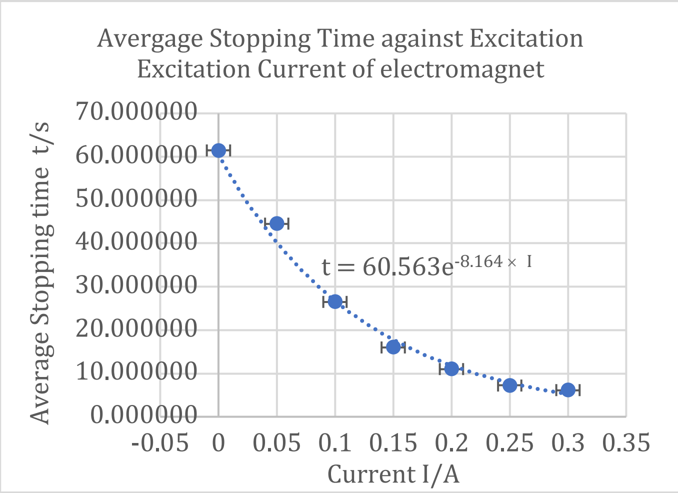

Figure 6 Average stopping time against excitation current of electromagnet

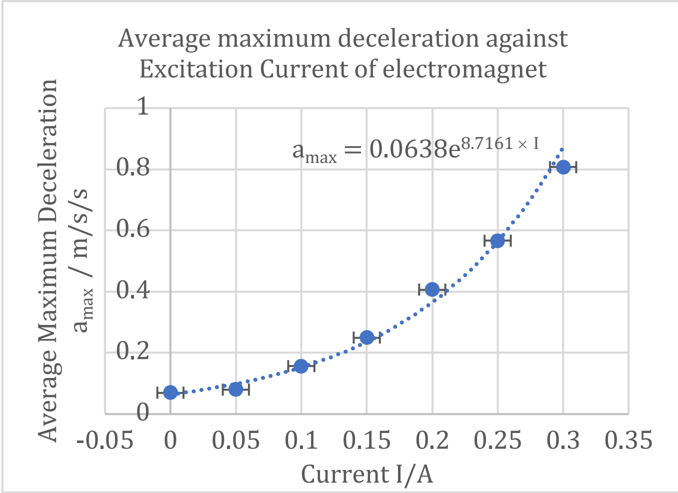

Figure 7 Average maximum deceleration against excitation current of electromagnet.

Once again, all the data has been fitted with exponential regression models, and the fact that the lines fit within every error bar shows that they are accurate models. On a side note, while a_{max} is not $0ms^{-2}$ when no current is applied, it is not a random error; instead, this constant reveals the existence of friction between the shaft and the motor. Yet, for the purposes of this experiment, friction has not been eliminated in the calculations.

Since an exponential graph is not useful for analysis, therefore, linearization will be performed.

Linearization of Graphs

a) The D.Schieber's Model (1) proposes braking force F_e is proportional to the square of magnetic field strength applied, that is:

$$F_e \propto B^2$$

b) The change in momentum for every experiment is assumed to be the same since 1) the same disk is used which means the mass is constant; 2) the final velocity of the rotating disk is zero, and; 3) the initial velocity is assumed to be constant. Therefore, by Newton's second law of motion, average braking force F_e is proportional to the rate of change of momentum of the rotating disk as outlined, and hence it is also inversely proportional to stopping time of the disk:

$$F_e \propto \frac{\Delta p}{t} \; or \; F_e \propto \frac{1}{t} \; ;$$

c) Hence, the squared of the magnetic field strength should be inversely proportional to the stopping time of a rotating disk, or in notation:

$$B^2 \propto \frac{1}{t}$$

d) However, since excitation current is proportional to the magnetic field strength of a direct magnetisation component as outlined by Ampere's Law (Deziel, 2018), or in notation:

$$I \propto B$$

e) Therefore, in theory:

$$I^2 \propto \frac{1}{t} \cdots (2)$$

f) On the other hand, since the initial average braking force is also directly proportional to the maximum deceleration of the disk with respect to Newton's second law of motion, or in notation:

$$F_e \propto a_{max}$$

g) Therefore, with respect to the principle outlined in (a), (c) and (d),

$$I^2 \propto a_{max} \cdots (3)$$

In other words, with respect to (2) and (3), a graph of $I^2 \propto \frac{1}{t}$ and $I^2 \propto a_{max}$ will be plotted such that they will yield a linear regression line. The maximum and minimum slope lines have been plotted with respect to the absolute uncertainties of $I^2, \frac{1}{t}$ and a_{max} as well. The results are as shown in figure 8 and 10 respectively:

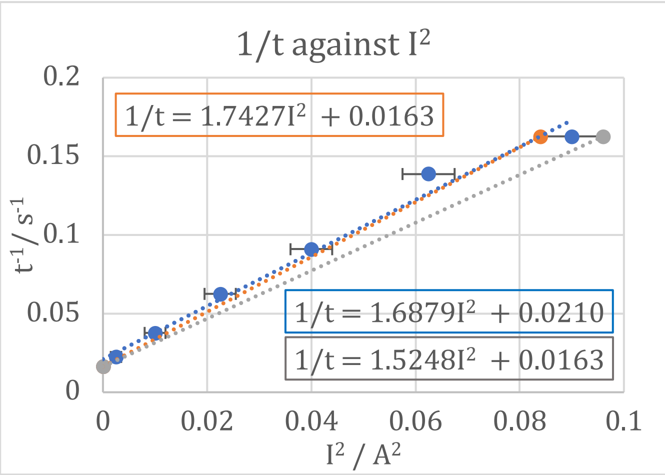

Figure 8 Graph of 1/t against I²

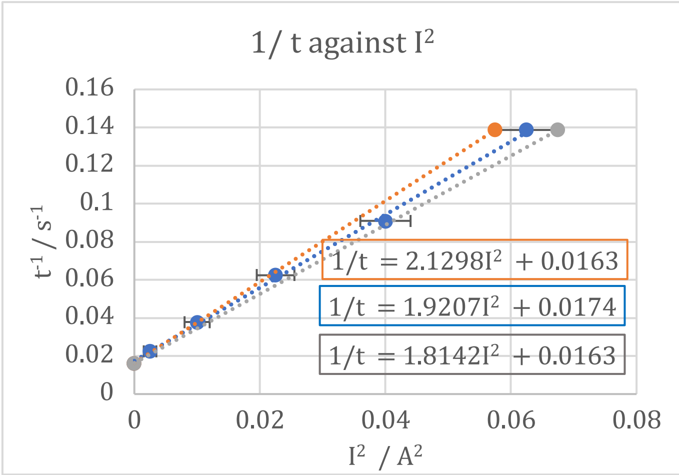

Figure 9 Figure 8 with 6th point (anomaly) removed

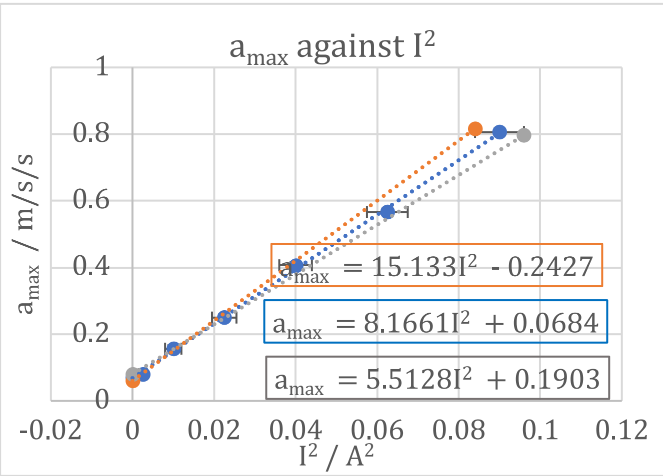

Figure 10 Maximum deceleration against current squared.

Initially, the trend did not appear promising since it did not pass through the error bars of the 6th data point. Yet, removing the 7th point allowed for a linear regression that passed through all error bars as shown in figure 9. This anomaly indeed suggests a major systematic error in the last experiment.

Note that the error bars and absolute uncertainties of I^2 are now ±0, 0.001, 0.002, 0.003, 0.004, 0.005 and 0.006A for the 7 points respectively, which is a result of doubling the percentage uncertainty of excitation current (I) when squaring itself. Since the absolute uncertainty of time is $\Delta\bar{t} = \pm 0.000001s$, the percentage uncertainty and absolute uncertainty of $\frac{1}{t}$ is an infinitesimal number that is invisible on graph.

Indeed, the fact that the best fit linear regression models $1/t = 1.9207I^2 + 0.0174$ and $a_{max} = 8.1661I^2 + 0.0684$ lie within the error bars indicates that they are good model for both $I^2 \propto \frac{1}{t}$ and $I^2 \propto a_{max}$. Indeed, the deduced relationships of the trendlines are all consistent with the model proposed by D.Schieber.

5 Conclusion

After the experimental data has been collected and processed, it can be concluded that the results support D.Schieber's model to a great extent. This can be proven by the fact that all linear and exponential regression models for the raw and processed data graphs lie within the error bars of the variables; this verifies the relationships for both $I^2 \propto \frac{1}{t}$ and $I^2 \propto a_{max}$ and in return supports the claims made in D.Schieber's model, and its premises made with respect to ampere's Law and newton's second law of motion.

One can notice the relatively high precision in data obtained from the light gate, which includes the time interval in between blocked and unblocked gate states, instantaneous velocity and instantaneous acceleration. The absolute uncertainties for these variables were ±0.000001s, ±0.001ms^{-1} and 0.01ms^{-2} respectively, which is quite negligible since they are invisible on the graphs. The absolute uncertainties for excitation current through electromagnet and potential difference across electromagnet are 0.01A and 0.01V respectively, which are considerably larger but still relatively small for a school lab experiment. Yet, the regression line for all graphs, with the exception of one data point in the graph for $I^2 against \frac{1}{t}$, managed to pass through all error bars, which means that the regression models are all statistically significant.

All in all, the experimental data has a relatively high precision as demonstrated by small absolute uncertainties, and a high accuracy as shown by the regression lines lying within the error bars. In other words, the systematic and random errors were considerably small. Therefore, I hereby declare $1/t = 1.9207 \times I^2 + 0.0174$ and $a_{max} = 8.1661 \times I^2 + 0.0684$ successful regression models for the relationship $I^2 \propto \frac{1}{t}$ and $I^2 \propto a_{max}$ respectively, and I hereby this experiment a successful investigation.

6 Evaluation

As a whole, the processed data does not reveal evident errors, which means that many errors in the experiment are of low significance. However, the weaknesses and limitations of the data collection process and methodology is apparent in the raw data section. The following outlines the possible sources of systematic and random errors in the experiment, possible limitations in the methodology and the improvements that may be made.

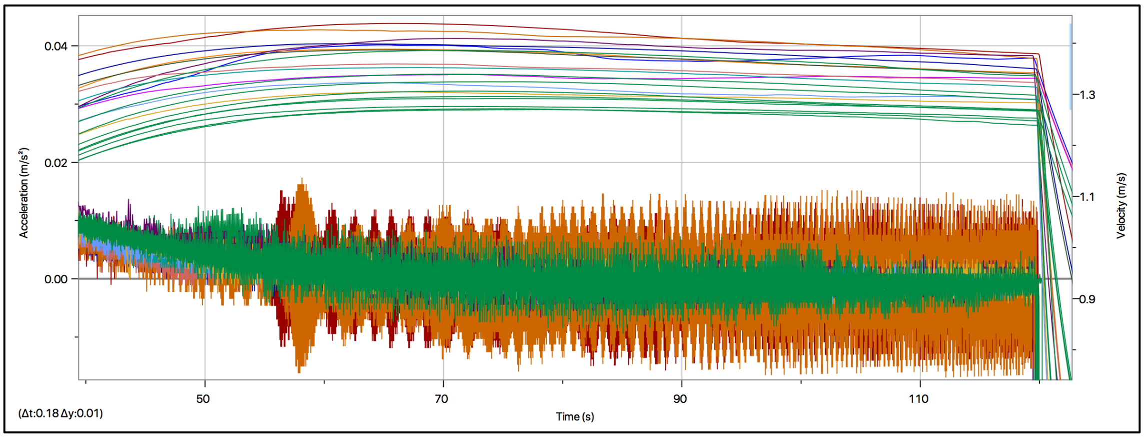

Figure 11 Velocity (right y-axis) and acceleration (left y-axis) of rotor against time

1) Figure 11 is the raw data of velocity and acceleration of rotor against time, where the top cluster of lines are for velocity (right y-axis) and lower cluster of lines are for acceleration (left y-axis). In the experiment, a 2 minute buffer was given to allow the motor to reach its top speed before the electromagnetic brake was applied. However, as outlined by the graph, the top speed of the rotor is inconsistent across various trials, with the range being nearly 0.02ms^{-1}, and for many trials, the maximum speed dropped after 70

seconds. This was likely due to the glue between the shaft and aluminium disk melting, leading to increased wobbling of the disk, as shown by the large variation in acceleration of disk between 50 and 120 seconds. This random error has resulted in unprecise detection by the light; to improve, a more heat resistant glue can be used and a motor with higher power rating should be utilised, such that there will be less heat dissipated during high power usage.

2) There is a time delay for starting the motor at 5 seconds and switching the motor off and switching on the electromagnet at 120 seconds. To reduce the error during data collection, I made sure:
    a. The power supply was at 4V such that I could directly plug in the cable at 5 seconds and allow the motor supply to reach 4V immediately (without pressing the buttons to reach 4V from 0V).
    b. Switch entire power supply of rotor off immediately at 120 seconds to reduce latency.
    c. Rheostat was configured beforehand. I also demagnetised the electromagnet by slowly dropping the current of the AC supply from a high voltage. Before I started the experiment, I removed the wire across the switch (such that the current would only pass if the switch was closed). This allowed me to close the switch at 120 seconds and output the desired current immediately.

   Moreover, in an attempt to eliminate the time delay entirely from the processed data, the processed data only analyses data after the maximum deceleration.

   These measures, however, did not make these random errors less conceivable in the raw data graphs. One way to reduce the error is to automate the switch opening and closing process with a microcontroller kit, though it would be time consuming to program the sequence and to setup the hardware. To a certain extent, if the program is not executed in a smooth manner, the reliability might be even lower.

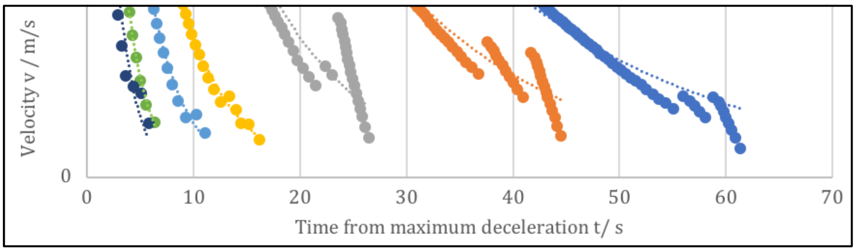

Figure 12 average velocity against time from maximum deceleration of rotor (Cropped from figure 5)

3) Figure 12 shows the average velocity against time from maximum deceleration of rotor, and in this segment it appears that the velocity has sudden "spikes" before it completely stops. Due to small random errors, most likely the time delay in activating the electromagnet, the stopping time for the same configuration varies across 3 trials. As a result of averaging the velocity of the 3 trials from maximum deceleration, when the trial with the shortest stopping time is complete, only the velocity of the 2 remaining trials is averaged, which is higher since the disk continues rotating. The same goes on for when only the velocity of 1 remaining trial is plotted on the graph. One way to solve the problem is to remove the data after the trial with the shortest stopping time is completed. However, this would mean that the data from all 3 trials will not be fully utilised, which undermines the purpose of repeating the experiment.

4) The y-intercepts in the regression equations $a_{max} = 8.1661 \times I^2 + 0.0684$ and $a_{max} = 0.0638e^{8.7161 \times I}$, namely 0.0684 and 0.0638ms^{-2} represent the deceleration when there is no current applied to the electromagnet. In theory, there should be no deceleration in this scenario, and for this reason, this number represents the friction existent in the motor component. In ideal situations, using a motor with infinitesimal friction would have yielded a result that perfectly fits the relationship in D.Schieber's model, which would allow the researcher to take into account the other factors in the model.

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7 Limitations

While the relationship between magnetic field strength and stopping time is distinctly noticeable in the experiment, there are various real-life implications that has prevented me to answer the question with respect to real ECBs in high speed trains and to extrapolate or relate the conclusions of this experiment to ECBs in high speed trains. The following are several real-life problems that have not been accounted for in this experiment:

1) In a typical Shinkansen, the disk (similar to the one utilised in the experiment) is attached to the middle of a shaft, which has the wheels positioned at the ends of the shafts. In some cases, the shafts are attached to other rotors which indirectly rotate the wheels. As a result of this complex setup, there is more friction loss in the mechanisms, further magnifying the effect of braking.
2) In this experiment, the mass of the aluminium disk is very low and the rate of change of momentum as it is rotating is relatively low as well. The motion of a high speed locomotive combined with the substantial mass of the shafts, wheels and other components of the brake will result in a momentum of considerable size. Since the average braking force of the brake has to be proportional to the rate of change of momentum of the combined mass of these components, the average braking force has to be larger.
3) In ECBs, the eddy currents are dissipated as heat. Hence, while increasing the magnetic field will decelerate the locomotive quicker, it will result in more heat, further compromising on the longevity of the train components. Hence, ECBs are likely not utilised to their full potential on high speed trains.
4) The confidential details of an ECB on a 700 series Shinkansen, including the material of the disk and width of air gap between electromagnets and disk, is not disclosed by Kawasaki, which makes it impossible to replicate the real setup of the brake. These details are key factors that have to be accounted for the D.Schieber's model. Other environmental factors like air resistance and dynamic friction between wheels and rails, and friction in rotor has not been accounted as well.

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Appendix 1: Averaged data Overview

Variable	Current	Potential Difference	Acceleration	Velocity	Time
Units	A	V	m/s/s	m/s	s

Current	**0.00**		**0.05**		**0.10**		**0.15**		**0.20**		**0.25**		**0.30**	
Potential Difference across Electromagnet	0.00		2.31		4.46		9.00		8.96		10.00		13.46	
Average Maximum Deceleration	0.07		0.08		0.16		0.25		0.41		0.57		0.81	
Average Retardaton Time	61.455534		44.56351067		26.50251667		15.99137667		10.99957667		7.208168		6.147882333	
	Average Velocity	Average Time	Average Velocity	Average Time	Average Velocity	Average Time	Average Velocity	Average Time	Average Velocity	Average Time	Average Velocity	Average Time	Average Velocity	Average Time
	1.345	0.000000	1.292	0.000000	1.345	0.000000	1.228	0.000000	1.321	0.000000	1.231	0.000000	1.245	0.000000
		0.000230		0.000243		0.000230		0.000255		0.000236		0.000254		0.000251
	1.342	0.037217	1.288	0.038773	1.339	0.037273	1.218	0.040876	1.305	0.038107	1.208	0.041010	1.212	0.040711
		0.037448		0.039017		0.037505		0.041134		0.038345		0.041269		0.040969
	1.340	0.074508	1.285	0.077642	1.333	0.074706	1.208	0.082094	1.290	0.076662	1.185	0.082824	1.179	0.082541
		0.074738		0.077886		0.074939		0.082353		0.076904		0.083088		0.082807
	1.337	0.111870	1.282	0.116607	1.327	0.112305	1.198	0.123661	1.275	0.115683	1.161	0.125479	1.147	0.125558
		0.112101		0.116852		0.112538		0.123922		0.115927		0.125750		0.125831
	1.334	0.149305	1.279	0.155670	1.322	0.150066	1.188	0.165583	1.259	0.155176	1.137	0.169013	1.114	0.169830
		0.149537		0.155915		0.150301		0.165846		0.155424		0.169289		0.170111
	1.332	0.186812	1.276	0.194830	1.316	0.187997	1.178	0.207866	1.243	0.195159	1.114	0.213463	1.081	0.215434
		0.187045		0.195076		0.188233		0.208132		0.195410		0.213744		0.215723
	1.329	0.224394	1.273	0.234088	1.310	0.226095	1.168	0.250517	1.228	0.235645	1.090	0.258868	1.048	0.262454
		0.224626		0.234334		0.226331		0.250785		0.235898		0.259157		0.262753
	1.327	0.262047	1.270	0.273443	1.304	0.264364	1.158	0.293542	1.212	0.276642	1.066	0.305272	1.015	0.310982
		0.262281		0.273690		0.264602		0.293812		0.276899		0.305566		0.311290
	1.324	0.299774	1.266	0.312896	1.298	0.302804	1.147	0.336947	1.197	0.318168	1.042	0.352720	0.982	0.361118
		0.300009		0.313144		0.303043		0.337220		0.318428		0.353021		0.361437
	1.322	0.337576	1.263	0.352446	1.292	0.341417	1.137	0.380740	1.182	0.360235	1.019	0.401259	0.949	0.412975
		0.337810		0.352694		0.341656		0.381015		0.360499		0.401567		0.413305
	1.319	0.375450	1.260	0.392093	1.286	0.380204	1.127	0.424927	1.166	0.402858	0.995	0.450942	0.916	0.466675
		0.375685		0.392342		0.380445		0.425205		0.403125		0.451257		0.467017
	1.316	0.413399	1.257	0.431839	1.281	0.419168	1.117	0.469517	1.151	0.446052	0.971	0.501822	0.883	0.522357
		0.413635		0.432088		0.419410		0.469797		0.446323		0.502145		0.522712
	1.314	0.451423	1.254	0.471684	1.275	0.458310	1.106	0.514514	1.135	0.489834	0.948	0.553960	0.849	0.580175
		0.451658		0.471934		0.458553		0.514797		0.490108		0.554292		0.580544
	1.311	0.489520	1.251	0.511628	1.269	0.497632	1.096	0.559926	1.120	0.534219	0.924	0.607419	0.816	0.640305
		0.489756		0.511879		0.497876		0.560212		0.534497		0.607759		0.640689
	1.309	0.527693	1.248	0.551673	1.263	0.537135	1.086	0.605760	1.104	0.579226	0.900	0.662266	0.783	0.702943
		0.527930		0.551925		0.537380		0.606049		0.579508		0.662615		0.703343
	1.306	0.565940	1.244	0.591819	1.257	0.576820	1.076	0.652023	1.088	0.624871	0.877	0.718577	0.750	0.768314
		0.566177		0.592071		0.577067		0.652314		0.625158		0.718935		0.768732
	1.304	0.604263	1.241	0.632065	1.251	0.616691	1.066	0.698722	1.073	0.671174	0.853	0.776431	0.716	0.836675
		0.604500		0.632318		0.616938		0.699016		0.671465		0.776800		0.837113
	1.301	0.642661	1.238	0.672413	1.246	0.656746	1.056	0.745867	1.057	0.718153	0.829	0.835913	0.683	0.908321
		0.642899		0.672667		0.656995		0.746163		0.718448		0.836292		0.908781
	1.299	0.681134	1.235	0.712862	1.240	0.696989	1.046	0.793465	1.042	0.765824	0.806	0.897120	0.649	0.983591
		0.681373		0.713117		0.697239		0.793765		0.766123		0.897510		0.984075
	1.296	0.719684	1.232	0.753414	1.234	0.737422	1.036	0.841526	1.026	0.814210	0.782	0.960146	0.615	1.062877
		0.719923		0.753669		0.737673		0.841828		0.814514		0.960547		1.063388
	1.294	0.758309	1.229	0.794068	1.228	0.778045	1.025	0.890061	1.011	0.863334	0.758	1.025114	0.582	1.146635
		0.758549		0.794324		0.778298		0.890366		0.863643		1.025528		1.147176
	1.291	0.797012	1.226	0.834825	1.222	0.818861	1.015	0.939080	0.995	0.913219	0.735	1.092126	0.548	1.235393
		0.797251		0.835081		0.819114		0.939388		0.913532		1.092554		1.235967
	1.288	0.835790	1.223	0.875686	1.217	0.859871	1.005	0.988590	0.980	0.963887	0.711	1.161337	0.515	1.329778
		0.836031		0.875942		0.860126		0.988901		0.964206		1.161779		1.330390
	1.286	0.874646	1.220	0.916650	1.211	0.901077	0.995	1.038600	0.964	1.015364	0.688	1.232877	0.482	1.430531
		0.874887		0.916907		0.901333		1.038915		1.015687		1.233335		1.431186
	1.283	0.913579	1.217	0.957717	1.205	0.942481	0.985	1.089121	0.949	1.067675	0.664	1.306909	0.449	1.538549
		0.913820		0.957976		0.942738		1.089440		1.068004		1.307382		1.539253
	1.281	0.952589	1.214	0.998890	1.199	0.984083	0.975	1.140163	0.933	1.120846	0.641	1.383622	0.416	1.654937
		0.952831		0.999149		0.984342		1.140484		1.121181		1.384113		1.655697
	1.278	0.991676	1.211	1.040166	1.193	1.025886	0.965	1.191733	0.918	1.174905	0.617	1.463195	0.383	1.781106
		0.991919		1.040425		1.026146		1.192059		1.175245		1.463705		1.781933
	1.276	1.030842	1.208	1.081546	1.188	1.067892	0.955	1.243844	0.903	1.229881	0.594	1.545875	0.350	1.918893
		1.031085		1.081806		1.068153		1.244173		1.230227		1.546406		1.919799
	1.273	1.070085	1.204	1.123029	1.182	1.110103	0.945	1.296506	0.887	1.285805	0.570	1.631889	0.317	2.070728
		1.070329		1.123290		1.110365		1.296838		1.286157		1.632442		2.071730
	1.271	1.109407	1.201	1.164618	1.176	1.152519	0.935	1.349727	0.872	1.342713	0.547	1.721545	0.284	2.239913
		1.109651		1.164880		1.152783		1.350062		1.343071		1.722122		2.241037
	1.268	1.148808	1.198	1.206313	1.170	1.195143	0.925	1.403516	0.856	1.400642	0.523	1.815157	0.250	2.431183
		1.149052		1.206576		1.195408		1.403856		1.401006		1.815761		2.432465
	1.265	1.188287	1.195	1.248116	1.165	1.237979	0.915	1.457887	0.841	1.459625	0.500	1.913083	0.217	2.651620
		1.188532		1.248379		1.238245		1.458230		1.459997		1.913715		2.653117
	1.263	1.227846	1.192	1.290028	1.159	1.281025	0.905	1.512853	0.825	1.519703	0.476	2.015761	0.183	2.912518
		1.228091		1.290291		1.281293		1.513200		1.520081		2.016424		2.914321
	1.260	1.267484	1.189	1.332049	1.154	1.324286	0.895	1.568429	0.810	1.580920	0.452	2.123669	0.149	3.234093
		1.267729		1.332313		1.324555		1.568780		1.581306		2.124367		3.236382
	1.258	1.307201	1.186	1.374180	1.148	1.367762	0.885	1.624629	0.794	1.643323	0.429	2.237391	0.114	3.660728
		1.307447		1.374444		1.368032		1.624984		1.643717		2.238129		3.663965
	1.255	1.346999	1.183	1.416421	1.142	1.411454	0.875	1.681463	0.779	1.706956	0.405	2.357619	0.102	4.357060
		1.347245		1.416687		1.411726		1.681822		1.707358		2.358400		4.364328
	1.253	1.386876	1.180	1.458773	1.136	1.455366	0.865	1.738948	0.763	1.771869	0.381	2.485181	0.095	5.084563
		1.387123		1.459039		1.455639		1.739310		1.772278		2.486011		5.089126
	1.250	1.426834	1.176	1.501236	1.131	1.499500	0.855	1.797092	0.748	1.838108	0.357	2.621073	0.061	5.763386
		1.427081		1.501504		1.499774		1.797458		1.838526		2.621960		5.765268
	1.248	1.466872	1.173	1.543812	1.125	1.543858	0.846	1.855905	0.733	1.905727	0.333	2.766515	0.031	6.314564
		1.467120		1.544080		1.544134		1.856276		1.906153		2.767467		
	1.245	1.506991	1.170	1.586500	1.119	1.588442	0.836	1.915384	0.717	1.974770	0.308	2.922978		
		1.507240		1.586769		1.588720		1.915760		1.975205		2.924003		
	1.243	1.547192	1.167	1.629301	1.113	1.633255	0.826	1.975564	0.702	2.045296	0.284	3.092292		
		1.547441		1.629571		1.633534		1.975945		2.045741		3.093407		
	1.240	1.587473	1.164	1.672216	1.108	1.678299	0.816	2.036466	0.687	2.117366	0.260	3.276810		
		1.587723		1.672487		1.678579		2.036851		2.117821		3.278030		
	1.237	1.627836	1.161	1.715246	1.102	1.723576	0.807	2.098100	0.672	2.191047	0.235	3.479628		
		1.628087		1.715517		1.723857		2.098489		2.191513		3.480977		
	1.235	1.668281	1.158	1.758390	1.096	1.769088	0.797	2.160494	0.657	2.266412	0.211	3.704990		
		1.668532		1.758661		1.769370		2.160888		2.266888		3.706497		
	1.232	1.708808	1.155	1.801650	1.090	1.814837	0.787	2.223670	0.641	2.343540	0.186	3.958898		
		1.709059		1.801922		1.815122		2.224069		2.344028		3.960610		
	1.230	1.749417	1.151	1.845026	1.085	1.860827	0.777	2.287657	0.626	2.422522	0.161	4.250192		
		1.749669		1.845299		1.861112		2.288061		2.423022		4.252180		
	1.227	1.790108	1.148	1.888519	1.079	1.907058	0.767	2.352467	0.611	2.503464	0.135	4.592782		
		1.790361		1.888792		1.907346		2.352877		2.503977		4.595162		
	1.225	1.830883	1.145	1.932128	1.074	1.953535	0.757	2.418118	0.596	2.586468	0.109	5.010931		

	1.831136		1.932403		1.953824		2.418533		2.586994		5.013915
1.222	1.871740	1.142	1.975855	1.068	2.000259	0.747	2.484642	0.580	2.671645	0.082	5.555045
	1.871994		1.976130		2.000550		2.485062		2.672185		5.559145
1.220	1.912681	1.139	2.019700	1.062	2.047233	0.737	2.552040	0.564	2.759114	0.062	6.378240
	1.912935		2.019976		2.047525		2.552467		2.759669		
1.217	1.953705	1.136	2.063662	1.057	2.094459	0.728	2.620352	0.550	2.848999		
	1.953960		2.063939		2.094753		2.620784		2.849570		
1.215	1.994814	1.133	2.107743	1.051	2.141941	0.718	2.689588	0.534	2.941437		
	1.995068		2.108020		2.142236		2.690026		2.942023		
1.213	2.036006	1.130	2.151943	1.045	2.189679	0.708	2.759780	0.519	3.036567		
	2.036261		2.152221		2.189976		2.760224		3.037171		
1.210	2.077282	1.127	2.196262	1.039	2.237678	0.698	2.830959	0.503	3.134541		
	2.077538		2.196541		2.237977		2.831409		3.135163		
1.208	2.118644	1.124	2.240702	1.034	2.285939	0.688	2.903151	0.488	3.235521		
	2.118900		2.240981		2.286240		2.903608		3.236163		
1.205	2.160090	1.121	2.285263	1.028	2.334467	0.678	2.976389	0.473	3.339703		
	2.160347		2.285543		2.334769		2.976853		3.340365		
1.203	2.201621	1.118	2.329945	1.022	2.383262	0.668	3.050706	0.458	3.447288		
	2.201879		2.330226		2.383566		3.051176		3.447972		
1.200	2.243239	1.115	2.374749	1.017	2.432328	0.658	3.126139	0.443	3.558527		
	2.243497		2.375031		2.432634		3.126617		3.559235		
1.198	2.284942	1.112	2.419677	1.011	2.481667	0.648	3.202718	0.427	3.673688		
	2.285201		2.419959		2.481975		3.203204		3.674421		
1.195	2.326732	1.109	2.464728	1.006	2.531284	0.638	3.280488	0.412	3.793060		
	2.326991		2.465011		2.531593		3.280981		3.793819		
1.193	2.368609	1.106	2.509902	1.000	2.581180	0.628	3.359469	0.397	3.916982		
	2.368869		2.510187		2.581490		3.359970		3.917772		
1.190	2.410572	1.103	2.555202	0.994	2.631358	0.619	3.439727	0.381	4.045876		
	2.410832		2.555487		2.631670		3.440236		4.046697		
1.188	2.452623	1.100	2.600628	0.989	2.681821	0.609	3.521282	0.365	4.180179		
	2.452883		2.600914		2.682135		3.521799		4.181036		
1.185	2.494761	1.097	2.646180	0.983	2.732572	0.599	3.604192	0.350	4.320389		
	2.495022		2.646466		2.732888		3.604718		4.321285		
1.183	2.536987	1.094	2.691859	0.978	2.783615	0.589	3.688498	0.334	4.467065		
	2.537249		2.692147		2.783933		3.689033		4.468002		
1.180	2.579301	1.091	2.737666	0.972	2.834951	0.579	3.774252	0.318	4.620846		
	2.579563		2.737955		2.835271		3.774797		4.621831		
1.178	2.621703	1.088	2.783602	0.966	2.886586	0.569	3.861508	0.302	4.782485		
	2.621966		2.783891		2.886907		3.862061		4.783522		
1.176	2.664194	1.085	2.829667	0.961	2.938520	0.558	3.950326	0.286	4.952866		
	2.664457		2.829957		2.938844		3.950890		4.953961		
1.173	2.706774	1.081	2.875861	0.955	2.990759	0.548	4.040770	0.271	5.133029		
	2.707038		2.876151		2.991084		4.041344		5.134188		
1.171	2.749444	1.078	2.922185	0.950	3.043305	0.538	4.132906	0.255	5.324205		
	2.749709		2.922477		3.043632		4.133491		5.325437		
1.168	2.792203	1.075	2.968641	0.944	3.096162	0.528	4.226788	0.239	5.527905		
	2.792468		2.968933		3.096491		4.227385		5.529222		
1.166	2.835052	1.072	3.015228	0.938	3.149335	0.518	4.322495	0.222	5.745939		
	2.835318		3.015521		3.149666		4.323103		5.747353		
1.163	2.877991	1.069	3.061947	0.933	3.202826	0.508	4.420095	0.206	5.980552		
	2.878258		3.062241		3.203159		4.420716		5.982077		
1.161	2.921021	1.066	3.108800	0.927	3.256639	0.498	4.519666	0.190	6.234648		
	2.921288		3.109095		3.256974		4.520299		6.236307		
1.159	2.964141	1.063	3.155786	0.922	3.310778	0.488	4.621293	0.174	6.512020		
	2.964409		3.156082		3.311115		4.621939		6.513840		
1.156	3.007353	1.060	3.202907	0.916	3.365247	0.477	4.725063	0.157	6.817757		
	3.007621		3.203204		3.365587		4.725723		6.819775		
1.154	3.050656	1.057	3.250163	0.910	3.420051	0.467	4.831072	0.140	7.159012		
	3.050924		3.250460		3.420392		4.831747		7.161283		
1.151	3.094051	1.054	3.297555	0.905	3.475192	0.457	4.939423	0.123	7.546230		
	3.094320		3.297853		3.475536		4.940112		7.548834		
1.149	3.137538	1.051	3.345082	0.899	3.530676	0.447	5.050221	0.105	7.995711		
	3.137808		3.345382		3.531021		5.050927		7.998785		
1.146	3.181118	1.048	3.392747	0.894	3.586505	0.437	5.163589	0.086	8.535908		
	3.181388		3.393047		3.586853		5.164311		8.539704		
1.144	3.224790	1.045	3.440549	0.888	3.642685	0.427	5.279655	0.067	9.225221		
	3.225061		3.440850		3.643035		5.280395		9.230322		
1.142	3.268555	1.042	3.488490	0.883	3.699219	0.416	5.398550	0.071	10.237627		
	3.268826		3.488791		3.699571		5.399308		10.246677		
1.139	3.312414	1.039	3.536569	0.877	3.756112	0.406	5.520418	0.050	11.085588		
	3.312686		3.536871		3.756466		5.521195				
1.137	3.356366	1.036	3.584788	0.871	3.813367	0.396	5.645409				
	3.356639		3.585092		3.813723		5.646206				
1.134	3.400413	1.033	3.633147	0.866	3.870988	0.386	5.773692				
	3.400686		3.633452		3.871347		5.774511				
1.132	3.444553	1.030	3.681648	0.861	3.928980	0.375	5.905450				
	3.444827		3.681953		3.929342		5.906292				
1.129	3.488788	1.027	3.730290	0.855	3.987348	0.365	6.040905				
	3.489063		3.730596		3.987712		6.041771				
1.127	3.533118	1.024	3.779075	0.849	4.046095	0.355	6.180284				
	3.533392		3.779382		4.046461		6.181175				
1.124	3.577542	1.021	3.828003	0.844	4.105227	0.344	6.323828				
	3.577818		3.828310		4.105595		6.324747				
1.122	3.622061	1.018	3.877075	0.839	4.164747	0.334	6.471798				
	3.622337		3.877384		4.165117		6.472746				
1.120	3.666676	1.015	3.926292	0.833	4.224660	0.324	6.624480				
	3.666952		3.926602		4.225033		6.625458				
1.117	3.711386	1.012	3.975655	0.827	4.284973	0.313	6.782187				
	3.711663		3.975966		4.285349		6.783197				
1.115	3.756192	1.009	4.025163	0.822	4.345691	0.303	6.945288				
	3.756470		4.025475		4.346070		6.946333				
1.112	3.801095	1.006	4.074820	0.816	4.406819	0.292	7.114181				
	3.801373		4.075132		4.407200		7.115264				
1.110	3.846094	1.003	4.124623	0.811	4.468363	0.282	7.289322				
	3.846373		4.124936		4.468747		7.290447				
1.108	3.891191	1.000	4.174576	0.805	4.530328	0.271	7.471222				
	3.891470		4.174891		4.530714		7.472390				
1.105	3.936384	0.997	4.224678	0.800	4.592719	0.261	7.660467				
	3.936664		4.224994		4.593108		7.661684				
1.103	3.981676	0.994	4.274931	0.794	4.655544	0.250	7.857710				
	3.981957		4.275248		4.655935		7.858980				
1.100	4.027065	0.991	4.325335	0.789	4.718806	0.239	8.063740				
	4.027347		4.325653		4.719200		8.065069				
1.098	4.072553	0.988	4.375892	0.784	4.782514	0.228	8.279457				
	4.072835		4.376211		4.782910		8.280850				
1.096	4.118140	0.985	4.426602	0.778	4.846672	0.218	8.505888				
	4.118423		4.426921		4.847072		8.507352				
1.093	4.163826	0.982	4.477466	0.773	4.911287	0.207	8.744258				
	4.164109		4.477786		4.911690		8.745803				
1.091	4.209611	0.979	4.528484	0.767	4.976366	0.195	8.996075				
	4.209895		4.528806		4.976771		8.997711				
1.089	4.255496	0.976	4.579658	0.762	5.041914	0.184	9.263202				
	4.255780		4.579981		5.042322		9.264942				
1.086	4.301481	0.973	4.630989	0.756	5.107939	0.173	9.547914				
	4.301766		4.631312		5.108351		9.549773				
1.084	4.347566	0.970	4.682475	0.750	5.174448	0.162	9.853082				

	4.347852		4.682800		5.174862		9.855084
1.081	4.393753	0.967	4.734120	0.745	5.241447	0.150	10.182460
	4.394039		4.734446		5.241865		10.184630
1.079	4.440040	0.964	4.785923	0.740	5.308945	0.138	10.541159
	4.440328		4.786250		5.309367		10.543536
1.077	4.486429	0.961	4.837885	0.734	5.376949	0.125	10.936426
	4.486718		4.838212		5.377373		10.939068
1.074	4.532921	0.958	4.890005	0.729	5.445466	0.113	11.379301
	4.533209		4.890333		5.445893		11.382301
1.072	4.579514	0.956	4.942286	0.723	5.514502	0.099	11.888554
	4.579803		4.942616		5.514933		11.892082
1.070	4.626210	0.953	4.994727	0.718	5.584067	0.085	12.502323
	4.626500		4.995057		5.584501		12.506789
1.067	4.673010	0.950	5.047333	0.712	5.654169	0.091	13.356739
	4.673299		5.047664		5.654607		13.364999
1.065	4.719912	0.947	5.100102	0.707	5.724816	0.077	13.972435
	4.720203		5.100435		5.725257		13.975309
1.063	4.766918	0.944	5.153040	0.701	5.796016	0.061	14.480349
	4.767210		5.153374		5.796460		14.484145
1.060	4.814028	0.941	5.206143	0.696	5.867777	0.060	15.227713
	4.814321		5.206477		5.868226		15.235320
1.058	4.861243	0.938	5.259416	0.690	5.940110	0.042	16.224483
	4.861536		5.259752		5.940561		
1.056	4.908562	0.935	5.312856	0.685	6.013022		
	4.908856		5.313193		6.013477		
1.053	4.955987	0.932	5.366468	0.679	6.086524		
	4.956281		5.366806		6.086982		
1.051	5.003517	0.929	5.420250	0.674	6.160623		
	5.003812		5.420590		6.161085		
1.049	5.051154	0.926	5.474207	0.669	6.235330		
	5.051449		5.474547		6.235796		
1.046	5.098896	0.923	5.528336	0.663	6.310653		
	5.099192		5.528676		6.311124		
1.044	5.146745	0.920	5.582640	0.658	6.386605		
	5.147042		5.582982		6.387079		
1.042	5.194701	0.917	5.637118	0.652	6.463195		
	5.194999		5.637462		6.463673		
1.039	5.242764	0.914	5.691775	0.647	6.540436		
	5.243063		5.692120		6.540918		
1.037	5.290936	0.911	5.746608	0.641	6.618338		
	5.291235		5.746954		6.618824		
1.035	5.339216	0.908	5.801620	0.636	6.696913		
	5.339516		5.801967		6.697404		
1.032	5.387604	0.905	5.856811	0.630	6.776172		
	5.387905		5.857160		6.776667		
1.030	5.436102	0.902	5.912184	0.625	6.856127		
	5.436403		5.912532		6.856626		
1.028	5.484710	0.899	5.967736	0.620	6.936788		
	5.485011		5.968086		6.937292		
1.025	5.533427	0.896	6.023471	0.614	7.018166		
	5.533729		6.023822		7.018674		
1.023	5.582255	0.893	6.079387	0.609	7.100282		
	5.582558		6.079740		7.100795		
1.021	5.631193	0.891	6.135487	0.603	7.183150		
	5.631497		6.135841		7.183668		
1.019	5.680243	0.888	6.191769	0.598	7.266785		
	5.680547		6.192124		7.267308		
1.016	5.729404	0.885	6.248239	0.592	7.351206		
	5.729709		6.248595		7.351734		
1.014	5.778678	0.882	6.304894	0.587	7.436430		
	5.778984		6.305251		7.436963		
1.012	5.828064	0.879	6.361737	0.581	7.522471		
	5.828371		6.362096		7.523009		
1.009	5.877564	0.876	6.418769	0.576	7.609344		
	5.877871		6.419128		7.609888		
1.007	5.927176	0.873	6.475989	0.570	7.697066		
	5.927484		6.476350		7.697614		
1.005	5.976903	0.870	6.533400	0.565	7.785649		
	5.977211		6.533762		7.786203		
1.002	6.026743	0.867	6.591002	0.559	7.875117		
	6.027053		6.591365		7.875677		
1.000	6.076699	0.864	6.648798	0.554	7.965487		
	6.077009		6.649162		7.966052		
0.998	6.126770	0.861	6.706787	0.548	8.056776		
	6.127081		6.707153		8.057347		
0.996	6.176957	0.859	6.764971	0.542	8.149005		
	6.177268		6.765338		8.149581		
0.993	6.227259	0.856	6.823353	0.537	8.242193		
	6.227571		6.823722		8.242776		
0.991	6.277678	0.853	6.881934	0.531	8.336363		
	6.277991		6.882304		8.336952		
0.989	6.328215	0.850	6.940714	0.526	8.431536		
	6.328528		6.941086		8.432131		
0.986	6.378868	0.847	6.999696	0.520	8.527735		
	6.379183		7.000068		8.528336		
0.984	6.429639	0.844	7.058879	0.515	8.624983		
	6.429955		7.059252		8.625592		
0.982	6.480529	0.841	7.118264	0.509	8.723304		
	6.480845		7.118639		8.723919		
0.980	6.531537	0.838	7.177853	0.504	8.822724		
	6.531854		7.178229		8.823346		
0.977	6.582665	0.835	7.237646	0.498	8.923267		
	6.582982		7.238023		8.923896		
0.975	6.633912	0.832	7.297645	0.492	9.024959		
	6.634230		7.298024		9.025595		
0.973	6.685279	0.829	7.357852	0.487	9.127827		
	6.685598		7.358232		9.128470		
0.970	6.736767	0.827	7.418267	0.481	9.231894		
	6.737086		7.418649		9.232546		
0.968	6.788375	0.824	7.478892	0.475	9.337197		
	6.788696		7.479275		9.337856		
0.966	6.840106	0.821	7.539728	0.470	9.443765		
	6.840427		7.540112		9.444432		
0.964	6.891958	0.818	7.600775	0.464	9.551629		
	6.892280		7.601160		9.552303		
0.961	6.943933	0.816	7.662035	0.459	9.660821		
	6.944256		7.662422		9.661504		
0.959	6.996031	0.813	7.723510	0.453	9.771373		
	6.996355		7.723898		9.772065		
0.957	7.048253	0.810	7.785200	0.448	9.883323		
	7.048577		7.785588		9.884023		
0.954	7.100599	0.807	7.847106	0.442	9.996705		
	7.100924		7.847497		9.997415		
0.952	7.153068	0.804	7.909230	0.436	10.111561		
	7.153394		7.909623		10.112279		
0.950	7.205664	0.801	7.971575	0.431	10.227928		

	7.205990		7.971968		10.228657
0.948	7.258384	0.798	8.034140	0.425	10.345852
	7.258711		8.034536		10.346590
0.945	7.311230	0.796	8.096928	0.420	10.465375
	7.311558		8.097324		10.466123
0.943	7.364203	0.793	8.159938	0.414	10.586544
	7.364532		8.160336		10.587303
0.941	7.417302	0.790	8.223173	0.409	10.709410
	7.417632		8.223572		10.710179
0.938	7.470530	0.787	8.286634	0.403	10.834026
	7.470860		8.287035		10.834806
0.936	7.523884	0.784	8.350323	0.397	10.960446
	7.524215		8.350725		10.961237
0.934	7.577368	0.782	8.414240	0.391	11.088730
	7.577699		8.414644		11.089533
0.932	7.630979	0.779	8.478389	0.386	11.218938
	7.631313		8.478794		11.219754
0.929	7.684721	0.776	8.542769	0.380	11.351136
	7.685055		8.543176		11.351963
0.927	7.738592	0.773	8.607383	0.374	11.485387
	7.738927		8.607791		11.486228
0.925	7.792594	0.770	8.672233	0.369	11.621762
	7.792930		8.672643		11.622615
0.923	7.846727	0.768	8.737320	0.363	11.760333
	7.847063		8.737731		11.761201
0.920	7.900992	0.765	8.802645	0.357	11.901180
	7.901329		8.803057		11.902062
0.918	7.955389	0.762	8.868210	0.351	12.044389
	7.955726		8.868624		12.045287
0.916	8.009917	0.759	8.934018	0.346	12.190049
	8.010256		8.934433		12.190962
0.914	8.064579	0.756	9.000069	0.340	12.338252
	8.064919		9.000486		12.339181
0.911	8.119375	0.753	9.066366	0.334	12.489096
	8.119715		9.066784		12.490042
0.909	8.174305	0.751	9.132909	0.328	12.642691
	8.174646		9.133330		12.643654
0.907	8.229370	0.748	9.199702	0.323	12.799150
	8.229712		9.200124		12.800131
0.905	8.284570	0.745	9.266744	0.317	12.958589
	8.284913		9.267167		12.959589
0.902	8.339907	0.743	9.334037	0.311	13.121133
	8.340250		9.334462		13.122153
0.900	8.395379	0.740	9.401584	0.305	13.286915
	8.395724		9.402011		13.287956
0.898	8.450990	0.737	9.469386	0.299	13.456082
	8.451335		9.469815		13.457144
0.896	8.506737	0.734	9.537446	0.293	13.628788
	8.507083		9.537876		13.629873
0.894	8.562624	0.731	9.605766	0.287	13.805206
	8.562971		9.606198		13.806314
0.891	8.618649	0.729	9.674348	0.282	13.985517
	8.618997		9.674782		13.986650
0.889	8.674814	0.726	9.743193	0.276	14.169917
	8.675163		9.743628		14.171075
0.887	8.731118	0.723	9.812304	0.269	14.358593
	8.731468		9.812741		14.359780
0.885	8.787564	0.720	9.881684	0.264	14.551805
	8.787914		9.882122		14.553021
0.883	8.844150	0.718	9.951336	0.258	14.749808
	8.844502		9.951776		14.751054
0.880	8.900878	0.715	10.021262	0.252	14.952878
	8.901231		10.021704		14.954157
0.878	8.957749	0.712	10.091465	0.246	15.161319
	8.958102		10.091908		15.162633
0.876	9.014763	0.709	10.161944	0.239	15.375469
	9.015118		10.162390		15.376820
0.874	9.071922	0.706	10.232704	0.233	15.595705
	9.072277		10.233151		15.597094
0.872	9.129224	0.703	10.303745	0.227	15.822452
	9.129580		10.304194		15.823885
0.869	9.186672	0.701	10.375072	0.221	16.056183
	9.187028		10.375522		16.057660
0.867	9.244265	0.698	10.446684	0.215	16.297419
	9.244622		10.447137		16.298945
0.865	9.302004	0.695	10.518586	0.208	16.546751
	9.302363		10.519041		16.548329
0.863	9.359891	0.692	10.590782	0.202	16.804880
	9.360251		10.591238		16.806516
0.861	9.417926	0.689	10.663276	0.195	17.072556
	9.418286		10.663734		17.074255
0.858	9.476109	0.686	10.736071	0.189	17.350676
	9.476470		10.736531		17.352443
0.856	9.534442	0.684	10.809170	0.182	17.640247
	9.534804		10.809633		17.642089
0.854	9.592924	0.681	10.882574	0.175	17.942526
	9.593288		10.883038		17.944452
0.852	9.651557	0.678	10.956287	0.169	18.259059
	9.651921		10.956753		18.261080
0.850	9.710341	0.675	11.030309	0.161	18.592051
	9.710707		11.030777		18.594184
0.847	9.769278	0.672	11.104646	0.153	18.946203
	9.769643		11.105116		18.948489
0.845	9.828366	0.670	11.179298	0.142	19.331935
	9.828733		11.179770		19.334449
0.843	9.887607	0.667	11.254271	0.132	19.767168
	9.887975		11.254746		19.770064
0.841	9.947003	0.664	11.329561	0.123	20.256448
	9.947371		11.330037		20.259701
0.839	10.006552	0.661	11.405181	0.114	20.807363
	10.006922		11.405659		20.811087
0.837	10.066257	0.658	11.481120	0.103	21.468061
	10.066628		11.481600		21.472747
0.834	10.126119	0.655	11.557398	0.125	22.355316
	10.126491		11.557881		22.362856
0.832	10.186137	0.652	11.634000	0.115	22.995438
	10.186510		11.634485		22.998465
0.830	10.246313	0.649	11.710944	0.180	23.572291
	10.246687		11.711431		23.577178
0.828	10.306647	0.647	11.788220	0.174	23.857819
	10.307022		11.788709		23.858416
0.826	10.367141	0.644	11.865840	0.167	23.955516
	10.367516		11.866331		23.956137
0.823	10.427793	0.641	11.943797	0.161	24.057122
	10.428170		11.944290		24.057768
0.821	10.488607	0.639	12.022104	0.154	24.163029

	10.488985		12.022600		24.163703
0.819	10.549582	0.636	12.100758	0.147	24.273693
	10.549961		12.101256		24.274399
0.817	10.610718	0.633	12.179768	0.140	24.389644
	10.611097		12.180268		24.390385
0.815	10.672016	0.630	12.259128	0.133	24.511531
	10.672397		12.259631		24.512312
0.813	10.733478	0.627	12.338854	0.126	24.640156
	10.733860		12.339359		24.640981
0.810	10.795103	0.624	12.418936	0.119	24.776467
	10.795486		12.419443		24.777345
0.808	10.856894	0.621	12.499390	0.111	24.921693
	10.857278		12.499900		24.922631
0.806	10.918850	0.619	12.580213	0.103	25.077411
	10.919235		12.580725		25.078421
0.804	10.980973	0.616	12.661414	0.095	25.245703
	10.981359		12.661928		25.246801
0.802	11.043263	0.613	12.742989	0.086	25.429510
	11.043650		12.743506		25.430719
0.800	11.105721	0.610	12.824952	0.077	25.633074
	11.106109		12.825470		25.634426
0.798	11.168349	0.607	12.907293	0.068	25.862797
	11.168738		12.907814		25.864345
0.795	11.231146	0.604	12.990028	0.057	26.129705
	11.231536		12.990552		26.131544
0.793	11.294114	0.601	13.073152	0.045	26.455292
	11.294506		13.073679		
0.791	11.357255	0.599	13.156673		
	11.357647		13.157202		
0.789	11.420568	0.596	13.240596		
	11.420962		13.241127		
0.787	11.484056	0.593	13.324923		
	11.484450		13.325457		
0.785	11.547717	0.590	13.409659		
	11.548113		13.410196		
0.782	11.611555	0.587	13.494809		
	11.611952		13.495349		
0.780	11.675568	0.584	13.580381		
	11.675966		13.580924		
0.778	11.739758	0.581	13.666375		
	11.740157		13.666920		
0.776	11.804125	0.578	13.752794		
	11.804525		13.753342		
0.774	11.868670	0.576	13.839645		
	11.869071		13.840196		
0.772	11.933394	0.573	13.926935		
	11.933797		13.927488		
0.770	11.998299	0.570	14.014667		
	11.998702		14.015223		
0.767	12.063384	0.567	14.102846		
	12.063789		14.103406		
0.765	12.128652	0.564	14.191478		
	12.129057		14.192040		
0.763	12.194101	0.561	14.280567		
	12.194508		14.281132		
0.761	12.259735	0.559	14.370117		
	12.260143		14.370685		
0.759	12.325554	0.556	14.460135		
	12.325962		14.460705		
0.757	12.391558	0.553	14.550627		
	12.391968		14.551200		
0.755	12.457748	0.550	14.641601		
	12.458160		14.642177		
0.753	12.524127	0.547	14.733063		
	12.524539		14.733643		
0.750	12.590693	0.544	14.825019		
	12.591107		14.825602		
0.748	12.657449	0.541	14.917473		
	12.657864		14.918059		
0.746	12.724395	0.538	15.010430		
	12.724811		15.011019		
0.744	12.791532	0.535	15.103894		
	12.791950		15.104487		
0.742	12.858863	0.533	15.197871		
	12.859281		15.198467		
0.740	12.926385	0.530	15.292368		
	12.926805		15.292967		
0.738	12.994103	0.527	15.387389		
	12.994524		15.387992		
0.736	13.062017	0.524	15.482941		
	13.062439		15.483548		
0.733	13.130128	0.521	15.579031		
	13.130550		15.579641		
0.731	13.198436	0.518	15.675664		
	13.198861		15.676277		
0.729	13.266943	0.515	15.772847		
	13.267369		15.773463		
0.727	13.335651	0.512	15.870588		
	13.336078		15.871208		
0.725	13.404560	0.509	15.968893		
	13.404988		15.969516		
0.723	13.473671	0.506	16.067766		
	13.474101		16.068393		
0.721	13.542986	0.503	16.167212		
	13.543417		16.167843		
0.719	13.612505	0.500	16.267240		
	13.612937		16.267874		
0.717	13.682229	0.497	16.367855		
	13.682663		16.368493		
0.714	13.752160	0.494	16.469067		
	13.752595		16.469708		
0.712	13.822300	0.491	16.570883		
	13.822735		16.571528		
0.710	13.892648	0.489	16.673311		
	13.893085		16.673960		
0.708	13.963205	0.486	16.776357		
	13.963644		16.777010		
0.706	14.033975	0.483	16.880029		
	14.034415		16.880686		
0.704	14.104956	0.480	16.984335		
	14.105398		16.984997		
0.702	14.176152	0.477	17.089284		
	14.176595		17.089950		
0.699	14.247564	0.474	17.194883		
	14.248008		17.195553		
0.697	14.319192	0.471	17.301142		

	14.319637		17.301816
0.695	14.391038	0.468	17.408071
	14.391484		17.408749
0.693	14.463102	0.465	17.515679
	14.463550		17.516361
0.691	14.535387	0.462	17.623974
	14.535836		17.624662
0.689	14.607893	0.459	17.732970
	14.608344		17.733661
0.687	14.680621	0.457	17.842667
	14.681073		17.843363
0.684	14.753573	0.454	17.953075
	14.754026		17.953776
0.682	14.826750	0.451	18.064208
	14.827205		18.064914
0.680	14.900153	0.448	18.176079
	14.900610		18.176788
0.678	14.973785	0.445	18.288698
	14.974243		18.289413
0.676	15.047645	0.442	18.402076
	15.048104		18.402795
0.674	15.121736	0.439	18.516222
	15.122196		18.516946
0.672	15.196058	0.436	18.631149
	15.196520		18.631879
0.670	15.270614	0.433	18.746868
	15.271077		18.747602
0.668	15.345403	0.430	18.863388
	15.345868		18.864128
0.665	15.420428	0.427	18.980725
	15.420895		18.981471
0.663	15.495690	0.424	19.098889
	15.496158		19.099639
0.661	15.571191	0.422	19.217893
	15.571660		19.218648
0.659	15.646931	0.419	19.337753
	15.647402		19.338513
0.657	15.722913	0.415	19.458486
	15.723386		19.459252
0.655	15.799138	0.412	19.580106
	15.799612		19.580878
0.653	15.875607	0.409	19.702624
	15.876083		19.703401
0.651	15.952322	0.407	19.826055
	15.952799		19.826838
0.649	16.029284	0.404	19.950414
	16.029763		19.951203
0.647	16.106495	0.401	20.075717
	16.106975		20.076512
0.644	16.183956	0.398	20.201980
	16.184437		20.202782
0.642	16.261670	0.394	20.329224
	16.262153		20.330031
0.640	16.339638	0.391	20.457469
	16.340123		20.458283
0.638	16.417863	0.388	20.586736
	16.418349		20.587556
0.636	16.496345	0.385	20.717042
	16.496833		20.717869
0.634	16.575087	0.382	20.848405
	16.575576		20.849240
0.632	16.654089	0.379	20.980845
	16.654580		20.981686
0.630	16.733354	0.376	21.114373
	16.733847		21.115222
0.628	16.812882	0.373	21.249016
	16.813377		21.249871
0.626	16.892677	0.370	21.384796
	16.893173		21.385658
0.624	16.972739	0.367	21.521736
	16.973237		21.522606
0.621	17.053071	0.364	21.659862
	17.053571		21.660739
0.619	17.133674	0.361	21.799197
	17.134176		21.800082
0.617	17.214551	0.357	21.939771
	17.215054		21.940664
0.615	17.295703	0.354	22.081610
	17.296207		22.082512
0.613	17.377131	0.351	22.224741
	17.377638		22.225651
0.611	17.458838	0.348	22.369192
	17.459347		22.370110
0.609	17.540827	0.345	22.514990
	17.541337		22.515916
0.607	17.623097	0.342	22.662165
	17.623610		22.663101
0.605	17.705653	0.339	22.810753
	17.706167		22.811698
0.603	17.788495	0.335	22.960788
	17.789011		22.961742
0.601	17.871626	0.332	23.112306
	17.872144		23.113269
0.599	17.955048	0.329	23.265336
	17.955567		23.266309
0.596	18.038762	0.326	23.419912
	18.039283		23.420895
0.594	18.122771	0.323	23.576070
	18.123294		23.577062
0.592	18.207077	0.320	23.733849
	18.207601		23.734851
0.590	18.291681	0.316	23.893290
	18.292208		23.894303
0.588	18.376586	0.313	24.054435
	18.377115		24.055461
0.586	18.461795	0.310	24.217332
	18.462325		24.218369
0.584	18.547308	0.307	24.382028
	18.547840		24.383077
0.582	18.633128	0.303	24.548580
	18.633662		24.549639
0.580	18.719258	0.300	24.717032
	18.719794		24.718103
0.578	18.805699	0.296	24.887432
	18.806238		24.888516
0.576	18.892455	0.293	25.059832

	18.892995		25.060930
0.574	18.979528	0.290	25.234293
	18.980070		25.235405
0.571	19.066919	0.287	25.410881
	19.067463		25.412006
0.569	19.154631	0.283	25.589659
	19.155177		25.590798
0.567	19.242663	0.280	25.770691
	19.243211		25.771844
0.565	19.331017	0.276	25.954044
	19.331568		25.955212
0.563	19.419701	0.273	26.139793
	19.420253		26.140977
0.561	19.508712	0.270	26.328014
	19.509266		26.329214
0.559	19.598053	0.266	26.518787
	19.598609		26.520003
0.557	19.687725	0.262	26.712209
	19.688284		26.713443
0.555	19.777730	0.259	26.908380
	19.778290		26.909631
0.553	19.868071	0.256	27.107399
	19.868634		27.108669
0.551	19.958752	0.252	27.309375
	19.959317		27.310664
0.549	20.049767	0.248	27.514415
	20.050334		27.515724
0.547	20.141123	0.245	27.722643
	20.141692		27.723972
0.545	20.232829	0.241	27.934187
	20.233400		27.935538
0.543	20.324879	0.238	28.149184
	20.325453		28.150557
0.541	20.417280	0.234	28.367784
	20.417855		28.369181
0.538	20.510036	0.230	28.590139
	20.510613		28.591561
0.536	20.603143	0.227	28.816414
	20.603722		28.817861
0.534	20.696610	0.223	29.046796
	20.697192		29.048269
0.532	20.790435	0.219	29.281504
	20.791019		29.283007
0.530	20.884624	0.215	29.520773
	20.885210		29.522306
0.528	20.979177	0.211	29.764851
	20.979766		29.766414
0.526	21.074098	0.208	30.013993
	21.074689		30.015592
0.524	21.169389	0.204	30.268496
	21.169983		30.270128
0.522	21.265055	0.200	30.528684
	21.265651		30.530354
0.520	21.361096	0.196	30.794945
	21.361694		30.796654
0.518	21.457517	0.192	31.067687
	21.458118		31.069439
0.516	21.554320	0.188	31.347402
	21.554923		31.349200
0.514	21.651509	0.184	31.634629
	21.652114		31.636477
0.512	21.749086	0.179	31.929991
	21.749694		31.931895
0.510	21.847055	0.175	32.234188
	21.847665		32.236151
0.508	21.945418	0.170	32.548077
	21.946031		32.550106
0.506	22.044180	0.166	32.872719
	22.044796		32.874819
0.504	22.143343	0.161	33.209355
	22.143961		33.211539
0.502	22.242912	0.157	33.559536
	22.243532		33.561812
0.500	22.342889	0.152	33.925210
	22.343512		33.927595
0.498	22.443281	0.147	34.309335
	22.443906		34.311855
0.495	22.544089	0.142	34.715847
	22.544716		34.718526
0.493	22.645316	0.136	35.149524
	22.645946		35.152403
0.491	22.746967	0.130	35.618693
	22.747599		35.621850
0.489	22.849044	0.124	36.138582
	22.849680		36.142168
0.487	22.951553	0.117	36.743459
	22.952191		36.747887
0.485	23.054497	0.152	37.580103
	23.055138		37.589485
0.483	23.157880	0.147	37.942751
	23.158524		37.944288
0.481	23.261706	0.143	38.189413
	23.262353		38.191016
0.479	23.365979	0.138	38.447165
	23.366629		38.448846
0.477	23.470703	0.132	38.717704
	23.471355		38.719475
0.475	23.575880	0.126	39.005434
	23.576536		39.007347
0.473	23.681517	0.120	39.315201
	23.682175		39.317255
0.471	23.787618	0.114	39.649790
	23.788279		39.652035
0.468	23.894186	0.107	40.017960
	23.894850		40.020472
0.466	24.001227	0.100	40.435400
	24.001894		40.438335
0.464	24.108744	0.091	40.938824
	24.109414		40.942643
0.462	24.216742	0.140	41.710286
	24.217415		41.721585
0.460	24.325226	0.135	42.082380
	24.325902		42.083164
0.458	24.434201	0.130	42.207956
	24.434880		42.208771
0.456	24.543671	0.125	42.338557

	24.544354		42.339406
0.454	24.653645	0.120	42.474764
	24.654329		42.475651
0.452	24.764123	0.114	42.617325
	24.764811		42.618255
0.450	24.875112	0.108	42.767154
	24.875804		42.768135
0.448	24.986617	0.102	42.925307
	24.987311		42.926346
0.446	25.098641	0.096	43.093237
	25.099339		43.094345
0.443	25.211192	0.090	43.272757
	25.211892		43.273948
0.441	25.324272	0.083	43.466455
	25.324976		43.467750
0.439	25.437886	0.075	43.677981
	25.438593		43.679409
0.437	25.552041	0.067	43.912990
	25.552752		43.914598
0.435	25.666742	0.058	44.181049
	25.667457		44.182929
0.433	25.781996	0.047	44.501137
	25.782714		
0.431	25.897812		
	25.898533		
0.429	26.014193		
	26.014918		
0.427	26.131147		
	26.131876		
0.425	26.248680		
	26.249412		
0.423	26.366799		
	26.367534		
0.421	26.485508		
	26.486247		
0.419	26.604814		
	26.605557		
0.416	26.724721		
	26.725468		
0.414	26.845235		
	26.845985		
0.412	26.966363		
	26.967117		
0.410	27.088111		
	27.088869		
0.408	27.210486		
	27.211248		
0.406	27.333498		
	27.334264		
0.404	27.457153		
	27.457923		
0.402	27.581460		
	27.582234		
0.400	27.706424		
	27.707203		
0.398	27.832054		
	27.832836		
0.395	27.958355		
	27.959142		
0.393	28.085337		
	28.086128		
0.391	28.213006		
	28.213802		
0.389	28.341371		
	28.342171		
0.387	28.470441		
	28.471246		
0.385	28.600226		
	28.601034		
0.383	28.730733		
	28.731546		
0.380	28.861971		
	28.862788		
0.378	28.993951		
	28.994772		
0.376	29.126681		
	29.127507		
0.374	29.260173		
	29.261004		
0.372	29.394436		
	29.395272		
0.370	29.529480		
	29.530321		
0.367	29.665316		
	29.666162		
0.365	29.801956		
	29.802806		
0.363	29.939412		
	29.940268		
0.361	30.077695		
	30.078556		
0.359	30.216818		
	30.217684		
0.357	30.356789		
	30.357661		
0.354	30.497623		
	30.498499		
0.352	30.639329		
	30.640212		
0.350	30.781923		
	30.782811		
0.348	30.925415		
	30.926309		
0.346	31.069821		
	31.070720		
0.344	31.215152		
	31.216057		
0.341	31.361425		
	31.362336		
0.339	31.508654		
	31.509571		
0.337	31.656855		
	31.657778		
0.335	31.806042		
	31.806971		
0.333	31.956231		

	31.957166
0.330	32.107436
	32.108378
0.328	32.259674
	32.260623
0.326	32.412962
	32.413918
0.324	32.567316
	32.568278
0.321	32.722752
	32.723721
0.319	32.879289
	32.880265
0.317	33.036944
	33.037928
0.314	33.195741
	33.196731
0.312	33.355696
	33.356693
0.310	33.516828
	33.517833
0.307	33.679160
	33.680172
0.305	33.842712
	33.843732
0.303	34.007511
	34.008539
0.301	34.173579
	34.174616
0.298	34.340940
	34.341984
0.296	34.509611
	34.510663
0.294	34.679620
	34.680681
0.291	34.850992
	34.852061
0.289	35.023750
	35.024828
0.287	35.197918
	35.199005
0.284	35.373525
	35.374620
0.282	35.550593
	35.551698
0.280	35.729154
	35.730269
0.277	35.909244
	35.910368
0.275	36.090894
	36.092027
0.272	36.274131
	36.275275
0.270	36.458990
	36.460143
0.268	36.645504
	36.646668
0.265	36.833710
	36.834885
0.263	37.023640
	37.024826
0.260	37.215335
	37.216532
0.258	37.408836
	37.410044
0.255	37.604189
	37.605408
0.253	37.801437
	37.802670
0.250	38.000629
	38.001874
0.248	38.201812
	38.203069
0.246	38.405030
	38.406299
0.243	38.610332
	38.611615
0.241	38.817775
	38.819071
0.238	39.027414
	39.028725
0.236	39.239314
	39.240638
0.233	39.453535
	39.454874
0.231	39.670128
	39.671482
0.228	39.889163
	39.890532
0.226	40.110718
	40.112103
0.223	40.334874
	40.336276
0.220	40.561723
	40.563142
0.218	40.791346
	40.792783
0.215	41.023831
	41.025285
0.212	41.259267
	41.260741
0.210	41.497756
	41.499249
0.207	41.739402
	41.740914
0.204	41.984308
	41.985841
0.202	42.232597
	42.234151
0.199	42.484391
	42.485967
0.196	42.739824
	42.741423
0.193	42.999027
	43.000651
0.190	43.262154

	43.263803
0.187	43.529375
	43.531051
0.184	43.800857
	43.802559
0.181	44.076782
	44.078513
0.178	44.357352
	44.359113
0.176	44.642784
	44.644575
0.172	44.933304
	44.935128
0.169	45.229171
	45.231031
0.166	45.530656
	45.532550
0.163	45.838055
	45.839987
0.160	46.151708
	46.153680
0.157	46.471962
	46.473977
0.154	46.799238
	46.801298
0.150	47.134021
	47.136130
0.147	47.476826
	47.478988
0.143	47.828205
	47.830422
0.140	48.188820
	48.191097
0.136	48.559407
	48.561749
0.133	48.940793
	48.943206
0.129	49.333945
	49.336434
0.125	49.740035
	49.742610
0.121	50.160428
	50.163100
0.117	50.596729
	50.599506
0.113	51.050911
	51.053809
0.108	51.525482
	51.528517
0.104	52.023607
	52.026803
0.099	52.549505
	52.552893
0.094	53.108972
	53.112599
0.089	53.710661
	53.714597
0.084	54.368658
	54.373031
0.077	55.109572
	55.114649
0.092	56.000666
	56.007392
0.086	56.604970
	56.607581
0.080	57.040276
	57.043144
0.074	57.522943
	57.526184
0.067	58.078666
	58.082552
0.090	58.777414
	58.782962
0.085	59.350508
	59.351733
0.080	59.553047
	59.554357
0.074	59.770454
	59.771870
0.067	60.006488
	60.008041
0.061	60.266940
	60.268673
0.053	60.560968
	60.562964
0.044	60.905296
	60.907716
0.033	61.338632

Appendix 2: Sample Raw Data: Electromagnet on 0.30A Supply

Data obtained from Vernier Photogate, extracted via Labquest Interface to Logger Pro.
Only data coming at and after maximum deceleration has been extracted for this appendix.

Variable	Acceleration	Velocity	Time	Distance
Units	m/s/s	m/s	s	m

Run 10 (Trial 1)					Run 11 (Trial 2)					Run 12 (Trial 3)					Processed	
Time	Gate State	Distance	Velocity	Acc	Time	Gate State	Distance	Velocity	Acc	Time	Gate State	Distance	Velocity	Acc	Average Velocity	Average Time
119.867484	0	137.05	1.271	-0.80	119.828962	0	134.00	1.240	-0.80	119.945255	0	132.10	1.224	-0.82	1.245	0.000000
119.867730	1				119.829213	1				119.945510	1					0.000251
119.907314	0	137.10	1.239	-0.80	119.869827	0	134.05	1.207	-0.79	119.986692	0	132.15	1.190	-0.80	1.212	0.040711
119.907566	1				119.870086	1				119.986955	1					0.040969
119.948190	0	137.15	1.207	-0.78	119.911820	0	134.10	1.174	-0.77	120.029315	0	132.20	1.156	-0.78	1.179	0.082541
119.948449	1				119.912086	1				120.029586	1					0.082807
119.990173	0	137.20	1.175	-0.76	119.955005	0	134.15	1.142	-0.75	120.073197	0	132.25	1.123	-0.75	1.147	0.125558
119.990439	1				119.955279	1				120.073475	1					0.125831
120.033325	0	137.25	1.143	-0.74	119.999455	0	134.20	1.109	-0.73	120.118411	0	132.30	1.089	-0.73	1.114	0.169830
120.033598	1				119.999737	1				120.118698	1					0.170111
120.077716	0	137.30	1.110	-0.72	120.045243	0	134.25	1.076	-0.71	120.165043	0	132.35	1.056	-0.71	1.081	0.215434
120.077998	1				120.045534	1				120.165339	1					0.215723
120.123423	0	137.35	1.078	-0.70	120.092455	0	134.30	1.043	-0.69	120.213186	0	132.40	1.022	-0.69	1.048	0.262454
120.123713	1				120.092755	1				120.213492	1					0.262753
120.170525	0	137.40	1.046	-0.68	120.141184	0	134.35	1.010	-0.67	120.262938	0	132.45	0.988	-0.66	1.015	0.310982
120.170824	1				120.141494	1				120.263254	1					0.311290
120.219113	0	137.45	1.013	-0.66	120.191532	0	134.40	0.977	-0.64	120.314411	0	132.50	0.955	-0.64	0.982	0.361118
120.219422	1				120.191852	1				120.314739	1					0.361437
120.269282	0	137.50	0.981	-0.64	120.243612	0	134.45	0.944	-0.62	120.367731	0	132.55	0.921	-0.62	0.949	0.412975
120.269601	1				120.243944	1				120.368071	1					0.413305
120.321140	0	137.55	0.948	-0.62	120.297553	0	134.50	0.911	-0.60	120.423033	0	132.60	0.888	-0.60	0.916	0.466675
120.321470	1				120.297896	1				120.423386	1					0.467017
120.374806	0	137.60	0.916	-0.60	120.353493	0	134.55	0.878	-0.58	120.480472	0	132.65	0.854	-0.57	0.883	0.522357
120.375148	1				120.353850	1				120.480839	1					0.522712
120.430415	0	137.65	0.883	-0.58	120.411590	0	134.60	0.844	-0.56	120.540222	0	132.70	0.820	-0.55	0.849	0.580175
120.430769	1				120.411960	1				120.540603	1					0.580544
120.488120	0	137.70	0.850	-0.56	120.472017	0	134.65	0.811	-0.54	120.602478	0	132.75	0.787	-0.53	0.816	0.640305
120.488488	1				120.472403	1				120.602876	1					0.640689
120.548093	0	137.75	0.818	-0.54	120.534973	0	134.70	0.778	-0.52	120.667464	0	132.80	0.753	-0.51	0.783	0.702943
120.548475	1				120.535376	1				120.667880	1					0.703343
120.610523	0	137.80	0.785	-0.52	120.600684	0	134.75	0.745	-0.50	120.735435	0	132.85	0.719	-0.49	0.750	0.768314
120.610922	1				120.601105	1				120.735871	1					0.768732
120.675629	0	137.85	0.752	-0.50	120.669410	0	134.80	0.711	-0.47	120.806687	0	132.90	0.685	-0.46	0.716	0.836675
120.676046	1				120.669850	1				120.807145	1					0.837113
120.743662	0	137.90	0.719	-0.48	120.741443	0	134.85	0.678	-0.45	120.881560	0	132.95	0.651	-0.44	0.683	0.908321
120.744097	1				120.741905	1				120.882042	1					0.908781
120.814905	0	137.95	0.686	-0.45	120.817121	0	134.90	0.644	-0.43	120.960449	0	133.00	0.617	-0.42	0.649	0.983591
120.815362	1				120.817608	1				120.960957	1					0.984075
120.889682	0	138.00	0.652	-0.43	120.896832	0	134.95	0.611	-0.41	121.043818	0	133.05	0.583	-0.40	0.615	1.062877
120.890163	1				120.897345	1				121.044356	1					1.063388
120.968366	0	138.05	0.619	-0.41	120.981040	0	135.00	0.578	-0.39	121.132199	0	133.10	0.549	-0.37	0.582	1.146635
120.968873	1				120.981584	1				121.132771	1					1.147176
121.051384	0	138.10	0.586	-0.39	121.070276	0	135.05	0.544	-0.36	121.226219	0	133.15	0.515	-0.35	0.548	1.235393
121.051920	1				121.070853	1				121.226829	1					1.235967
121.139231	0	138.15	0.553	-0.36	121.165151	0	135.10	0.511	-0.34	121.326654	0	133.20	0.482	-0.32	0.515	1.329778
121.139799	1				121.165766	1				121.327307	1					1.330390
121.232453	0	138.20	0.520	-0.34	121.266434	0	135.15	0.478	-0.32	121.434408	0	133.25	0.448	-0.30	0.482	1.430531
121.233057	1				121.267092	1				121.435110	1					1.431186
121.331737	0	138.25	0.488	-0.32	121.374990	0	135.20	0.445	-0.29	121.550622	0	133.30	0.414	-0.28	0.449	1.538549
121.332381	1				121.375697	1				121.551382	1					1.539253
121.437848	0	138.30	0.456	-0.29	121.491924	0	135.25	0.412	-0.27	121.676740	0	133.35	0.380	-0.26	0.416	1.654937
121.438538	1				121.492688	1				121.677566	1					1.655697
121.551752	0	138.35	0.423	-0.27	121.618618	0	135.30	0.379	-0.25	121.814649	0	133.40	0.346	-0.23	0.383	1.781106
121.552495	1				121.619448	1				121.815556	1					1.781933
121.674665	0	138.40	0.391	-0.25	121.756863	0	135.35	0.346	-0.23	121.966853	0	133.45	0.312	-0.21	0.350	1.918893
121.675467	1				121.757771	1				121.967860	1					1.919799
121.808128	0	138.45	0.359	-0.23	121.909033	0	135.40	0.313	-0.21	122.136723	0	133.50	0.278	-0.19	0.317	2.070728
121.809002	1				121.910037	1				122.137852	1					2.071730
121.954153	0	138.50	0.327	-0.21	122.078311	0	135.45	0.280	-0.19	122.328976	0	133.55	0.244	-0.17	0.284	2.239913
121.955112	1				122.079435	1				122.330264	1					2.241037
122.115419	0	138.55	0.295	-0.19	122.269186	0	135.50	0.246	-0.16	122.550646	0	133.60	0.210	-0.14	0.250	2.431183
122.116485	1				122.270462	1				122.552150	1					2.432465
122.295640	0	138.60	0.262	-0.17	122.488336	0	135.55	0.213	-0.14	122.812586	0	133.65	0.175	-0.12	0.217	2.651620
122.296838	1				122.489820	1				122.814395	1					2.653117
122.500238	0	138.65	0.229	-0.15	122.745785	0	135.60	0.179	-0.12	123.133233	0	133.70	0.140	-0.10	0.183	2.912518
122.501612	1				122.747554	1				123.135499	1					2.914321
122.737034	0	138.70	0.196	-0.13	123.058044	0	135.65	0.145	-0.10	123.548902	0	133.75	0.105	-0.07	0.149	3.234093
122.738645	1				123.060237	1				123.551964	1					3.236382
123.018105	0	138.75	0.163	-0.11	123.456913	0	135.70	0.110	-0.08	124.148867	0	133.80	0.070	-0.05	0.114	3.660728
123.020050	1				123.459821	1				124.153724	1					3.663965
123.364859	0	138.80	0.129	-0.09	124.016188	0	135.75	0.075	-0.05	125.331834	0	133.85			0.102	4.357060
123.367322	1				124.020592	1				125.346770	1					4.364328
123.820274	0	138.85	0.095	-0.06	125.008111	0	135.80								0.095	5.084563
123.823666	1				125.018407	1										5.089126
124.497926	0	138.90	0.061	-0.04											0.061	5.763386
124.503572	1															5.765268
126.151461	0	138.95	0.031	0.00											0.031	6.314564
126.198714	1															
127.720171	1	139.00														
	0															