

any cell  $c$  whose address is given by a couple of integers,  $(c.x, c.y)$ ,  $c.x, c.y \in \mathbb{Z}$ , the Moore neighbourhood is the set  $N_c$  of cells  $d$  for which  $|d.x - c.x|, |d.y - c.y| \leq 1$ . The local transition function  $f_A$  is a mapping from  $Q_A^9$  into  $Q_A$ , where  $Q_A$  is the finite set of states of  $A$ , which rules the change of state in the cell at each tic of the clock:

$$\eta_A(c, t+1) = f_A(\{\eta_A(d, t) \mid d \in N_c\}),$$

where  $\eta_A(c, t)$  is the state of the automaton at the cell  $c$ , at time  $t$ .

The global transition function  $G_A$ , is defined as a mapping from  $Q_A^{\mathbb{Z}}$  into itself defined as follows. If  $\xi \in Q_A^{\mathbb{Z}}$  is a configuration,  $G_A(\xi)$  is the configuration at the next time and it is defined by:  $G_A(\xi)(c) = f_A(\{\xi(d)\} \mid d \in N_c)$ .

The theorems established by Moore and Myhill, see [20,21] state that for a cellular automaton  $A$  in the plane with Moore neighbourhood,  $G_A$  is surjective if and only if it is injective on the set of finite configurations. This property was used by Jarkko KARI to establish that the injectivity, the surjectivity and the reversibility of cellular automata in the plane are undecidable, see [4].

In the nineties and very recently, a few works appeared on the question whether the question of the existence of Garden of Eden is connected with the growth function of the group associated to a Cayley graphs. This function of  $n$  gives the number of elements of the group which can be represented by a product of at most  $n$  generators of the group and their inverses. In [6,2,1], the question is considered in different settings of Cayley groups. In [6], a counter-example is given when the group has an exponential growth: then, there is an injective global function which is not surjective and there is a surjective global function which is not injective.

However, the techniques of Cayley graphs do not apply to infinitely many tessellations of the hyperbolic plane. They apply only when the generating polygon is of the form  $\{2p, 2q\}$ , which means that its number of sides is even and that the number of polygons around a vertex in the tiling is also even, see [3]. In the other cases, especially for polygons with an odd number of sides and with an odd number of copies around a vertex, these techniques do not apply. However, for all the mentioned tessellations, whatever the parities, the technique indicated in [7] always apply: there, tools are developed which allow to locate the cells in the hyperbolic plane, facilitating the implementation of cellular automata in these contexts.

Cellular automata in the hyperbolic plane appeared for the first time in [16,17]. After the publication of [7], this study received a new impulse, see for instance [11] for references. In [10], I have shown that Hedlund's characterization of cellular automata in the plane can be transported to cellular automata in the hyperbolic plane at the price of an additional property on the cellular automaton. I also have noticed that the proof of Moore and Myhill in the Euclidean case does not pass to the hyperbolic plane. In the light of the mentioned paper in the case of Cayley graphs, it seems plausible that the answer is that there are also counter-examples. In fact, as shown in Section 3, there are examples of an injective cellular automaton which is not surjective and of a surjective cellular automaton which is not injective, and the local transition function which we give in these examples are much more