argument shows that it is not injective: whatever the number of cells, we have a choice. Also, we already have a choice for $c_0(x_0)$. □

Proof of Theorem 4.2. The states are again 0 or 1 and we denote by $N(x,t,1)$ the number of neighbours y of x, with $y \neq x$ for which $s(y,t) = 1$, where, again, $s(y,t)$ is the state at y and at time t. We define the transition function in such a way that:

$$s(x,t+1) = \begin{cases} 0 & \text{if } N(x,t,1) \in \{0,1,4,7\} \\ 1 & \text{if } N(x,t,1) \in \{2,3,5,6\} \end{cases},$$

It is again clear that this transition function can be defined by rotation invariant rules.

The surjectivity comes from the fact that if we have two cells at our disposal among the neighbours of a cell x, this is enough to fix the value at x according to this rule. This can be checked by Table 1, below.

In the table, $N^*(x,t,1)$ is the number of 1's on the already fixed neighbours of x. Each entry tells us how many cells to put to 1. If the entry says 0, this means that both free cells are put to 0.

Note that if we have more free cells at our disposal, we can use the same table to fix the values: two cells a and b among the free ones are fixed according to the table and the others are fixed to 0 or to 1 in order to obtain the value $N^*(x,t,1)$ of the table for the cells which are distinct from a and b.

Table 1 *The values of $N^*(x,t,1)$ for the example of Theorem 4.2.*

$N^*(x,t,1)$	0	1	2	3	4	5
if 0 needed	0	0	2	2	0	2
if 1 needed	2	2	0	0	2	0

Now, the configuration c_0 where every cell has the state 0 is transformed into c_0 and the configuration c_1 where every cell has the state 1 is also transformed into c_0. Accordingly, the global function is not injective. Now, it is also not injective on finite configurations.

Indeed, let us fix a cell x_0. The configuration $c(x_0)$ defined by 1 at x_0 and 0 everywhere else is also transformed into c_0. Similarly, if we have scattered 1's at a distance at least 4 from each other, then such a configuration is also transformed into c_0. Indeed, the requirement on the distance entails that the neighbourhoods of the 1's are disjoint. And so, we can take only finitely many of them if required. □

5 Conclusion

We have proved that injectivity and surjectivity are independent in the case of cellular automata in the hyperbolic plane. In the case of rotation invariant cellular automata, the problem of finding an example of an injective cellular automaton