what it was defined before the change. This informal definition can be made more precise, by fixing a way to number the neighbours of a cell or, which is equivalent, to number the sides of the polygon which supports the automaton. Let us say that for the central cell, side 1 is fixed once and for all. The other sides are numbered increasingly while counter-clockwise turning around the cell starting from side 1. For the other cells, fix side 1 to be the side shared by the cell with its father and, similarly number the other sides by counter-clockwise turning around the cell. Then, a rule of the automaton can be displayed in the following format, see [12,19,18] :

$$\eta_0, \eta_1, ..., \eta_\alpha \to \eta_0^1,$$

where η_0 is the current state of the cell, η_i, with $i \in \{1..\alpha\}$, where $\alpha = 5$ or $\alpha = 7$, is the current state of neighbour i and η_0^1 is the new state of the cell.

We say that a cellular automaton on the pentagrid or on the heptagrid is rotation invariant if and only if for each rule as above, the rule

$$\eta_0, \eta_{\pi(1)}, ..., \eta_{\pi(\alpha)} \to \eta_0^1,$$

also belongs to the set of rules for any circular permutation π on $\{1..\alpha\}$.

In what follows, $s(x,t)$ denotes the state at x and at time t. We have the following two properties :

Theorem 4.1 *(Margenstern-Kari) There is a rotation invariant cellular automaton on the pentagrid or on the heptagrid which is surjective but not injective.*

Theorem 4.2 *(Margenstern-Kari) There is a rotation invariant cellular automaton on the pentagrid or on the heptagrid which is surjective but not injective even on finite configurations.*

Proof of Theorem 4.1. It will be enough that $s(x,t)$ takes its values in $\{0,1\}$. We define the transition rules in such a way that:

$$s(x,t+1) = \begin{cases} 0 & \text{if } \#\{s(y,t) = 0\} \text{ is even} \\ 1 & \text{if } \#\{s(y,t) = 1\} \text{ is even} \end{cases},$$

where, in this formula, y denotes a neighbour of x which is not x and $\#\{P(s(y,t))\}$ is the number of y's around x such that $s(y,t)$ satisfy P. This is clearly a rotation invariant function.

Consider a configuration c and again, denote the central cell by x_0. It is not difficult to see that $c_0(x_0)$ can be 0 or 1 indifferently. Once it is fixed, we can fix the level 1 around x_0, setting two cells at the value $c(x_0)$ and the others at $1-c(x_0)$. Then, by induction, on each level, we have that the number of neighbours to be fixed is 3 or 2. In fact, the number is 4 for the first tile which we consider at level 1. For the others it is 3, the last one being excepted, for which it is 2. Now, having 2 or 3 cells at our disposal is enough to fix the parity of the required value to be odd. If the number of the required value is odd it is enough to give this value to one new cell and the other value to the others. If it is even, we have the choice: even to give the required value to an even number of cells, 0 or 2, we have the choice, and the other value to the other cells.

This proves that the global function is surjective. Moreover, the just above