possible to locate cells and which plays a key role in the proofs of the theorem of Section 3.

The important property indicated by Fig. 2 is that in the case of the hyperbolic plane, the penta- and the heptagrid are generated by a **tree**. The tree structure is underlined by the parts of the tiling which are detached and placed around the central tile. In the case of the pentagrid, five such regions, each one spanned by the same tree, are placed around the central tile, in a rotation symmetric way. In the case of the ternary heptagrid, we have seven regions. A remarkable property is that the generating tree is the same for the pentagrid and for the ternary heptagrid, see [11]. This tree is called **Fibonacci tree** as the number of its cells on a level k is f_{2k+1}, where $\{f_k\}_{k\in I\!N}$ is the Fibonacci sequence where $f_0 = f_1 = 1$, see [16]. From this, coordinates can be computed to locate the cells of a cellular automaton, see [7,11].

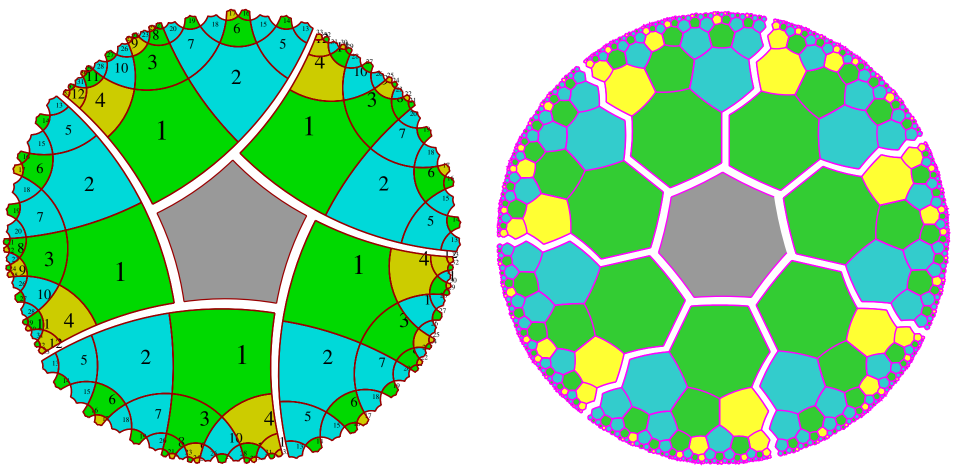

Fig. 2. On the left: the pentagrid, on the right: the underlying tree which spans the tiling.

As a consequence, except the cell which is placed at the central tile, we call it the **central cell**, each cell of the cellular automaton, has a father: its father as a node of the tree in the region it falls in.

This is an important point which will be used in Section 3.

The existence of a father for all cells, except the central one, plays the role of a direction in the hyperbolic plane, in the same way as the four traditional directions play a key role in the Euclidean plane.

In most formal presentations of cellular automata in the Euclidean plane − people usually say CA in the plane − the set of cells is identified with $Z\!\!\!Z\ 2$. This identification is so evident that it requires some effort to realize that it connects two different things and how it performs the connection.

In the Euclidean case, the above identification consists in three steps. First, we