simple than in [6].

In Section 2, we remind a few notions about cellular automata in the hyperbolic plane, also see [11].

2 Cellular automata in the hyperbolic plane

It is not needed to be very familiar with hyperbolic geometry to have a good representation of what happens in the hyperbolic plane. Fortunately, there are good models for that. In this paper, we make use of Poincaré's disc model. The hyperbolic plane is the set of points inside a fixed open disc U of the plane and the points of ∂U, the border of U, are called the points at infinity. In this model, lines are the trace in U of diametral lines or of circles which are orthogonal to ∂U, see Fig. 1, below. The interest of this model is that the angles between lines in the model are the true angles in the hyperbolic plane.

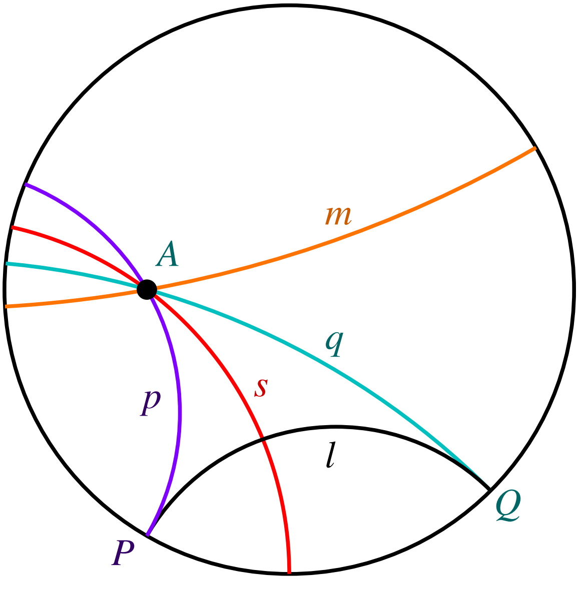

Fig. 1. The Poincaré's disc model. Remark the two lines p and q. They both pass through the point A and they are both parallel to the line ℓ. The line m passes through A and does not cut the line ℓ, even at infinity.

Cellular automata live on regular grids. There are infinitely many ones in the hyperbolic plane. Here, we choose the simplest ones in some sense: the pentagrid and the ternary heptagrid, see [11] for a detailed study of these grids. The pentagrid is constructed on the replication of a copy of the regular pentagon with right angles in its sides and, recursively, of the images in their sides. The ternary heptagrid is constructed in a similar way starting from the regular heptagon with $\dfrac{2\pi}{3}$ as its angles.

A view of these grids are given in Fig. 2 in a way which indicates how it is