

# About the Garden of Eden Theorems for Cellular Automata in the Hyperbolic Plane

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## Abstract

The Garden of Eden theorems are well known theorems established by Moore and Myhill in the early sixties connecting injectivity and surjectivity for the global function of cellular automata in the (Euclidean) plane. In this paper, it is shown that the properties established by Moore and Myhill are no more true for cellular automata in the hyperbolic plane.

*Keywords:* Cellular automata, hyperbolic plane, global function of transition.

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## 1 Introduction

Cellular automata are a well known model of computation, studied from already fifty years. This model is a very powerful tool for simulation of various phenomenas, from gas flows up to the circulation of crowds of pedestrians in railway stations and other physical, chemical or sociological problems of everyday life. At the same time, they are very simple dynamical systems which raise a lot of difficult theoretical questions which are far from being solved. Cellular automata are also a model of computations to which the theory of computability applies. There are a lot of results about the complexity of computations with cellular automata. There are also a lot of results on the Turing complete power of computation of this model.

In the early fifties, Moore and Myhill established an interesting property of the global function of cellular automata in the plane, in fact, the Euclidean plane. Remember that a cellular automaton  $A$  in the plane is defined by the definition of a neighbourhood, here the Moore neighbourhood, and a local transition function. For

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