which would not be surjective is still open.

From these results, it appears that a new argument has to be found to prove the undecidability of the surjectivity or the reversibility of cellular automata in the hyperbolic plane. For the injectivity, the problem can be proved undecidable by transferring to the hyperbolic plane the way opened by Jarkko Kari, although it is not all that easy, see [13]. It seems reasonable to conjecture that the surjectivity and the reversibility of the global function of a cellular automaton are undecidable. An argument, in favour of this conjecture is that the tiling problem for the hyperbolic plane is also undecidable, as it was proved by Maurice Margenstern and by Jarkko Kari, independently and by very different methods, see [5,8,14].

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