is known by the cell from $f(x)$. For x_0, as the sectors spanned by a Fibonacci tree are numbered, we define $\sigma_\ell(x_0)$ as the root of the tree which received the smallest number.

Now, we define B as follows:

$$\eta_B(x,t{+}1) = \mathrm{xor}(\eta_B(x,t), \eta_B(\sigma_\ell(x),t)),$$

where $\eta_B(x,t)$ is the state of the cell x at time t under B.

It is not very difficult to see that G_B is not injective. If we define c_0 by assigning the state 0 to all cells and c_1 by assigning the state 1 to each cell, it is not difficult to see that $G_B(c_0) = G_B(c_1) = c_0$.

Now, let us check that G_B is surjective. Indeed, fix a configuration c_1 and we have to define a configuration c_0 such that $G_B(c_0) = c_1$.

Consider x_0. Define $c_0(x_0) = 0$. Then, applying the definition, we have that $c_1(x_0) = \mathrm{xor}(0, c_0(\sigma_\ell(x_0))) = c_0(\sigma_\ell(x_0))$. And so, this defines c_0 at $\sigma_\ell(x_0)$. Define $c_0(x) = 0$ for all the other sons of x_0 than $\sigma_\ell(x_0)$.

By induction, assume that we have defined the level $n{+}1$ and that the surjectivity holds for all cells up to the level n, this level being included. From what we have seen, this is the case for $n = 0$.

On the level $n{+}2$, define all white nodes y by $c_0(y) = 0$. Now, consider a node x of the level $n{+}1$. As $c_0(x) = a$ is defined, we have, by definition, $G_B(c_0)(x) = \mathrm{xor}(a, c_0(\sigma_\ell(x)))$. This always defines c_0 at $\sigma_\ell(x)$. Indeed, $c_0(\sigma_\ell(x)) = c_1(x)$ if $a = 0$ and $c_0(\sigma_\ell(x)) = 1{-}c_1(x)$ if $a = 1$. And so, considering all nodes x of the level $n{+}1$, this defines c_0 for all black nodes of the level $n{+}2$. Moreover, now $G_B(c_0)(x) = c_1(x)$ for all cells x of the level $n{+}1$ too And the definition of c_0 at the level $n{+}2$ is complete.

And so, by induction, we proved that G_B is surjective.

The second variant requires to know the sons of a cell. This is easy to define from $f(x)$ for any cell x with $x \neq x_0$. For x_0, we consider that all its neighbours are its sons. Now, we define G_B as follows:

$$\eta_B(x,t{+}1) = \min\{\mathrm{xor}(\eta_B(x,t), \eta_B(y,t)) \,|\, y \in S_x\},$$

where S_x is the set of the sons of x.

The argument is the same as in the first variant.

Note that, in the proof of the surjectivity, we can easily see that B cannot be injective, as long as the state of many cells can be fixed arbitrarily. □

4 The case of rotation invariant cellular automata

It was proved in [15] that an analog of Hedlund's for cellular automata hold for the hyperbolic plane provided that an additional property is satisfied by the automaton, namely that the set of its rules is **rotation invariant**.

Intuitively, this means that if the neighbourhood of a cell is changed by a rotation of the neighbourhood around the cell, then the new state of the cell is the same as