Is the Rent Too High? Land Ownership and Monopoly Power

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Abstract

We investigate the sources and scope of landowner market power. We model pricing power arising from two sources: horizontal differentiation among buildings and sorting, a novel source. Zoning regulations can interact with market power to increase markups and we derive conditions under which restricting landownership concentration reduces rents. Estimating the model using a new building-level dataset of Manhattan rents, we find markups are on average roughly 21% of rents. Counterfactual rezoning policies resulting in 500 new units reduce markups on unconstrained units by \$3.61. Finally, we find no empirical scope for concentration restrictions to improve welfare in our data.

1 Introduction

Property rights grant landowners exclusive use over parcels of land. Since Adam Smith, economists have considered whether these rights endow landowners with monopoly powers and how those powers might affect spatial equilibria.[1] Property rights alone need not generate monopoly power, and standard assumptions in urban models generate perfectly competitive real estate markets.[2]

This paper investigates the underlying causes and the possible economic impact of market power due to land rights. We build a quantifiable model that nests both old and new theoretical mechanisms that can generate monopoly pricing power for landowners and use the model to answer two questions: is this power economically meaningful, and how should this alter our understanding of urban land use policies? Using data on Manhattan multi-unit residential rental prices, we find monopoly markups are on average about 21% of price. We show how monopoly markups may interact in unexpected ways with urban policies, including zoning regulations, and examine the possibility of a novel form of policy aimed at restricting concentration of land ownership. However, we find no scope for such policy in our data.

Our model nests two theoretical channels through which property rights generate monopoly markups for landowners: horizontal (idiosyncratic taste) differentiation of parcels and a novel channel where complementarity between vertically (quality) differentiated parcels and renters generates landowner pricing power. In Section 2, we present the full model then isolate each channel to demonstrate how each contributes to landowner pricing power. The first channel, first introduced by Chamberlain,[3] shows how horizontal differentiation among land parcels makes the rental market monopo-

[1] For Smith, the fact that the landlords could rent land without expenditures on improvements lead him to believe that rent was a "monopoly price" (Smith, 1776). Ricardo (1817) considered land to be a differentiated factor of production, so that prices reflected differentials in marginal product. Marx argued that monopoly land rents came from three sources: quality differences, markups designed to limit access to land, and extraction of rents from producers selling at markups (Evans, 1991).

[2] See Brueckner (1987) for a unified, formal Alnso-Muth-Mills (AMM) model and Glaeser (2007) for standard modelling in urban economic models.

[3] Chamberlain (1933) argued differences in consumer access to urban retail space create monopoly power for retailers, translating into landowner rents, even when land is in itself homogeneous. This variation generates downward-sloping demand for space at each location among a discrete set of parcels in a city.

listically competitive. Exogenous differences between locations in a city interact with individual taste heterogeneity in a discrete-choice framework. As is standard, this generates downward-sloping demand for units at each plot.

In the second channel, vertical differentiation among both parcels and renters creates sorting which generate monopoly power for landowners. Each renter values different parcels according to her own marginal benefits, but increasing complementarity between renter income and parcel qualities means that the marginal difference in utility for two given plots is increasing in renter income. This generates positive assortative matching and increasing surplus in types and a downward sloping demand for each parcel.

In Section 3, we discuss the theoretical policy implications of monopoly markups. We show how the presence of monopoly markups has two implications for the study of zoning policy. First, policy makers with information on costs hoping to achieve increased quantities and lower prices will be unhappily surprised by the effects of eliminating zoning regulations, which reduce prices and increase quantities to monopolistic rather than efficient levels. Second, and less obvious, in a city which is only partially constrained by zoning, zoning regulations have the potential to impact prices at unconstrained or unzoned locations through increased markups. We show that in the horizontal case, when the cost function is nondecreasing, zoning constraints always raise prices at unzoned plots.

We then explore the potential for municipalities to generate efficiencies by limiting the concentration of landownership. While, to our knowledge, no such policy is in place, restrictions on concentration have been recently proposed by Berlin housing activists.[4] Intuitively, landowners with multiple lots can potentially internalize the impact of one plot's pricing decision on their other plots. When cost-related substitution effects between plots are sufficiently small, this can lead to higher prices and markups. We show that in the horizontal case with non-decreasing cost, landowners with higher concentration always raise markups, and higher concentration increases prices at all other plots as well.

[4]See Stone (2019).

While these theoretical channels may exist, a separate question is whether they are empirically relevant. The extent to which landowner market power affect rents will depend on the strength of complementarities between the two sides of the market, as well as the degree to which consumers see housing at similar plots as substitutes. In Section 4, we translate our theoretical framework into an econometric model that can be used to answer this question.

To estimate our model, we generate a new dataset of rental income for multi-unit residential buildings in Manhattan using tax assessment data and formulas for generating tax assessments from rental income obtained from the NYC Department of Finance.[5] By applying these formulas to the tax assessment rolls in reverse, we can back out total rental income for 9,330 buildings in Manhattan. Section 5 details this construction.

One important aspect of our empirical environment is the ubiquity of constraints on prices and quantities in the form of rent control restrictions and zoning regulations. We use all residential structures to help estimate the demand parameters in our model, but markups are only valid for policy-unconstrained buildings.[6] Using our data, we back out the nature of constraints on each structure in Manhattan and calculate markups only on unconstrained buildings.

In Section 6, we report our model's findings. We find prices include an average markup over marginal costs of \$325 per month.[7] We find similar results for the set of recently constructed buildings.

[5]We use the 2010 New York city Primary Land Use Tax Lot Output (PLUTO) dataset on all buildings in the city in combination with the Fiscal Year 2012 Final Assessment Roll (FAR) dataset that includes market value information based on the reported building level income from 2010.

[6]We use details from the PLUTO dataset in combination with zoning regulations and, separately, we merge the 2012 NYC Department of Homes and Community Building Registration File that includes all rent controlled and stabilized buildings to document whether each building in our sample is effectively price and/or quantity constrained. We find that 92% of rental buildings are either zoning constrained or rent controlled / stabilized in our sample.

[7]Our estimation method is based on differentiated product demand estimation developed by Berry, Levinsohn and Pakes (1995) and the resulting literature. Within urban structural estimation, our method is most closely related to Bayer, McMillan and Rueben (2004), who model a discrete choice of housing types in the San Francisco metro area where buyers of different types have any horizontal preference for housing and neighborhood features, and use instruments based on the characteristics of houses beyond a certain distance from a neighborhood. See Kuminoff, Smith and Timmins (2013) for a review of the literature.

Finally, in Section 7 we use our results to assess the quantitative impact of our two policies: up-zoning and the implementation of constraints on ownership concentration. Consistent with our theory, the ubiquity of zoning constraints appears to have an appreciable impact on prices at unconstrained lots. We use the cross-price elasticities generated by our estimates to assess the impact of relaxing zoning constraints on unconstrained plots. On average, a policy change resulting in the construction of roughly 500 additional units at zoning-constrained plots reduces markups by at least $3.31 per unit at unconstrained, monopoly-priced building.

We then use the ownership structure available in our data to investigate the empirical impact of ownership concentration on pricing. Contradicting the theoretical result in Section 3, we find that landowners with larger market shares have lower markups, which may imply they benefit from scale economies in renting. This is true even when we control for model-derived measure of unobservable plot quality. We then generate a Herfindahl-Hirschman Index (HHI) for each census tract, and examine the relationship between estimated markups and the HHI index. Tracts with higher degrees of concentration exhibit higher prices and markups, implying localized concentration may matter. However, when we account for model-derived estimates of plot unobservables, the relationship disappears. We conclude that the initial positive realtionship is most likely due to unobserved differences in quality between tracts, and in our context, there appears to be no scope for reducing landowner monopoly power through restricting ownership concentration below current levels.

Our paper makes two theoretical and two empirical contributions. Our first theoretical contribution is the vertical channel described by our model. The increasing complements at the source of this mechanism is related to a literature on vertical differentiation and sorting, and in particular to a literature measuring the value of amenities and the degree of sorting. Following Tiebout (1956), this literature generally considers sorting on public goods between communities, as opposed to plots, and does not generally consider the effect of this sorting on pricing.[8] To our knowledge, the idea that vertical differentiation can lead to monopoly pricing in land markets has not been explored

[8] Dowding, John and Biggs (1994) and Boadway and Tremblay (2012) review the Tiebout model and Barseghyan and Coate (2016) create a dynamic extension of the model.

since the early political economists discussed the issue.[9]

By contrast, several papers, especially Arnott (1989), Arnott and Igarashi (2000), and Basu and Emerson (2003) use this horizontal differentiation to theoretically analyze the effects of rent controls in the context of landowner monopoly power. In these models, rent controls may aid renters because they create greater housing variety and limit landlord pricing power. Similarly, costly search increases the willingness to pay for initial housing matches which increases pricing power.[10] While this literature has opened the door to the possibility of monopoly power in land ownership, it is largely silent on its true empirical scope and relevance. Thus, our main empirical contribution is to generate an estimate of the size of monopoly markups.

Our second theoretical and empirical contributions lie in our investigation of the interactions between monopoly markups and urban policy. Here, we apply new results from the growing literature on multi-product oligopoly to the setting of landownership, where parcels are differentiated products, including Affeldt, Filistrucchi and Klein (2013), Jaffe and Weyl (2013), and Nocke and Schutz (2018b). We especially apply the results of Nocke and Schutz (2018a) to the impact of zoning on monopoly markups at unzoned plots and and extend their results to find conditions under which concentrated landownership also generates increases at prices at all plots. We are also tied to an empirical literature on the price and quantity effects of urban constraints, including Diamond, McQuade and Qian (2019) and Hsieh and Moretti (2019).[11]

[9] Shaked and Sutton (1982) formalize a vertical differentiation model with positive markups (and constant marginal cost); however, the authors implicitly also assume complementarities between plot quality and consumer type. Our contribution highlights that this complementarity is a necessary factor; else price differences represent quality differences as in Ricardo (1817).

[10] Basu and Emerson (2003) shows that a tenancy rent control scheme – where rent can freely adjust between tenants – may not be distinguishable from standard price ceilings due to a combination of inflation, private information about renter's optimal term of residence, and rent induced selection. A similar problem is approached empirically by Barker (2003) but with little discussion of monopoly power.

[11] See also: McMillen and McDonald (1999, 2002); Glaeser, Gyourko and Saks (2005); Ihlanfeldt (2007); Glaeser and Ward (2009); Paciorek (2013); Turner, Haughwout and Van Der Klaauw (2014); Gyourko and Molloy (2015); Hilber and Vermeulen (2015); Jackson (2016); Albouy and Ehrlich (2018).

2 Model

In this section, we lay out the model. There are two sets of agents in the model: renters endowed with income and in need (inelastically) of a location of residence and a finite set of landowners endowed with a location. The landowner chooses the rental rate at her location and provides, at some marginal cost, space to renters.

We first set up the optimization routines for each agent, and then define and solve the equilibrium in two cases: first, without vertical differentiation in location quality, and then without horizontal differentiation. We show how, in each case, the model delivers pricing power to each landowner.

2.1 Setup

2.1.1 Plots and Landowners

The space, a city, is comprised of a set of discrete locations or plots, A, which differ according to their underlying quality a, drawn without replacement from any distribution $G_1(a)$. Higher values of a have higher amenity value to renters, and we refer to a as "location quality" and differences in a as vertical differentiation in plots. A location's realized quality a will also be used henceforth to index each location in the set A.[12]

Each plot has a unique landowner who maximizes her profits by choosing the rent level at their location. Landowners provide a mass of renters housing at a positive, differentiable marginal cost $c_a(q)$, where q is the mass of renters the landowner accommodates at her location a in equilibrium. Total revenue is rent r collected times q. A given landowner a's profits are therefore

$$\pi_a = r \cdot q - c_a(q).$$

Because landowners will price above marginal cost, an interior solution will be found whether marginal cost is increasing, constant, or decreasing and can vary arbitrarily between plots. This will be important in our estimation, which does not rely on observing the cost function.

[12] Because the number of plots is finite, we can assume that no two plots will have identical values of a.

This setup assumes a single agent determines the constructed quantity and rental price of units, and our cost function includes the cost of both of these activities. Appendix B shows that equilibrium prices and quantities are unchanged when we separate the development and rental price problems. However, one implication of having a joint development-pricing problem is that when we estimate the model, markups will be over a "shadow" marginal cost of construction and operation of a marginal unit, as if the building were being developed for rent from scratch, and we cannot differentiate between maintanance-related costs and development-related costs. In the divided model of Appendix B, this markup is capitalized into the price of the building.

2.1.2 Renters

There is a mass, M, of heterogeneous renters, indexed by $i \in N$, where each renter's type y is drawn from some distribution $G_2(y)$. Renters draw random utilities $\epsilon_{i,a}$ for each location $a \in A$ These idiosyncratic random utility draws, drawn from a type-one extreme value distribution $G_3(\epsilon)$, constitute what we refer to as horizontal differentiation among locations. Renters' utility is derived from consumption and location choice. We allow utility to vary independently by agents' income as well:

$$U_{y,a} = F(a, y, I_y - r(a)) + \epsilon_{i,a}, \tag{1}$$

where consumption is equivalent to I_y, income of renter type y, minus rent. Renters choose among all locations a and an outside-option location $a_{out} \in A$ to maximize utility.[13]

2.2 Equilibrium

An equilibrium will be defined by a schedule of rents and quantities $\{(r_a, q_a)\}_{a \in A}$ that maximize landowner profits, assign renters to locations a such that no renter can improve welfare by choosing to pay rents at any other plot, and clear the real estate market. Thus, for each type y, the original density of types y is accounted for across all their

[13]We include the outside option to match our empirical model.

chosen locations a and the outside option, $g(y) = \sum_A q_a(y) + q_0(y)$.[14]

In each of the following subsections, we make additional assumptions on the renter's payoff function F and the distributions of types to isolate each source of monopoly power for landowners.

2.2.1 Equilibrium Under Horizontal Differentiation

In this subsection, we solve the model under the further assumptions that the distribution of types a and y are degenerate. We normalize the utility of the outside option $a_{out} = 0$, so that all urban utilities are relative to this baseline. Without the vertical channel in operation, the renters' location decision becomes a discrete choice problem where renters have multinomial logit errors.

Assuming that the random utility ϵ_{ia} are drawn from a type-one extreme value distribution with scale parameter σ_ϵ, a renter's demand is the probability that the renter will choose a particular plot, which yields the familiar multinomial logit choice probabilities:

$$\text{Individual} \quad d_a = \frac{e^{F(a,y_i,I_{y_i}-r(a))/\sigma_\epsilon}}{1 + \sum\limits_{a' \in A} \{e^{F(a',y_i,I_{y_i}-r(a'))/\sigma_\epsilon}\}} \tag{2}$$

$$\text{Market} \quad D_a = \int [d_a(y)]\, \mathsf{d}G_2(y) = M \cdot d_a. \tag{3}$$

This distribution provides intuitive closed form solutions for individual demand, which is integrated over individual heterogeneity for market level demand.[15] We can immediately see two important features of the market. First, demand for a given plot depends on the amenity and rents of all plots in the market, and $\sum_a d_a = 1$ which represents the fact that every individual makes a choice. Second, all products are substitutes for each other, and the substitution patterns are functions of the marginal utility of consumption, rents, and demand shares.[16]

[14]We do not consider combinations of $G_2(y)$, cost functions, and $G_1(a)$ which result in the full mass of renters choosing the outside option.

[15]Note, as we assume for this case that $G_2(y)$ is degenerate, all renters have the same expected demand, so market demand is the mass of renters times any individual's demand.

[16]Note, substitution patterns from plot a to a' for an individual renter only depend on price and demand share of the plot a'; this is a consequence of the implicit Independence of Irrelevant Alternative

In equilibrium, all landlords of type a must have the same price.[17] The equilibrium will therefore be symmetric and satisfies the usual monopoly pricing rule, a percentage markup of marginal cost:

$$r(a) = mc(D_a) - \frac{D_a}{\partial D_a/\partial r} = \left[\frac{\varepsilon_a}{\varepsilon_a + 1}\right] mc(D_a), \tag{4}$$

where ε_a is the own price elasticity.

The equilibrium rent at each plot equals marginal cost plus a markup related to the curvature of demand, which is a function of the marginal utility of consumption, the scale of the idiosyncratic tastes, and substitution behavior of renters.[18] Equation 4 yields an implicit equilibrium rent function, $r^\star = r(J; a, mc(), F(), \sigma_\epsilon)$. The solution implies strictly positive markups in rents that is bounded above zero.[19] An economic consequence of the markup is that some renters will not enter the market even though they would if plots were priced a marginal cost; i.e., $D_a(r^M) < D_a(mc(D_a))$. Thus, there exist potential renters with a willingness to pay for space greater than their impact on marginal cost, but are nevertheless priced out of the market.

To close the model, we apply the market clearing condition that the total number of renters in and out of the city equal the total number of renters:

$$\int \left(\sum_{a' \in A} d_{a'}(r^\star, y)\right) \mathsf{d}G(y)dy = \int g(y)\mathsf{d}y = G(y). \tag{5}$$

2.2.2 Equilibrium Under Vertical Differentiation

In this subsection, we solve the model under the alternative assumptions that the renters' payoff function F are log supermodular in renter and location type and that utility draws for renters are zero $\epsilon_{i,a} = 0\, \forall i, a$ (i.e., no horizontal differentiation).

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assumption, also called the 'Red Bus / Blue Bus' problem; see Train (2009) for a textbook discussion.

[17] Given the degenerate distribution of amenities, a symmetric solution to the landlord's problem can be reasoned verbally. Suppose all landlords with amenity value a set rent at some equilibrium $r^\star(a)$. Any individual deviation to a higher price leads to less demand since amenities are equivalent, but any deviation to a lower price would lead to greater demand.

[18] Caplin and Nalebuff (1991) and Perloff and Salop (1985) show that such an always equilibrium exists.

[19] See Bajari and Benkard (2003) for more implications from the horizontal discrete choice model.

Here, the source of monopoly power is more subtle. Without the assumed complementarity between the vertical differentiation on both sides of the market, landowners price at marginal cost. However, with increasing match surplus, not only does utility (gross of rents) increase in location quality, as it does in the standard framework, but, because of the complement between types, the relative utility – net of utility at other locations and equilibrium differences in rent – is variable among renter types, generating downward sloping demand for landowners.

Utility is log-supermodular in renter and parcel type:

$$F(a, c, y) = F_1(a, y) \cdot F_2(I_y - r), \tag{6}$$

where function F_1 is log-supermodular in a and y, and F_2 is some function of consumption $I_y - r$. This is an assumption about complementarity between resident's type and location quality, which in turn guarantees, in equilibrium, sorting between the two dimensions of vertical differentiation, along with increasing complements and surpluses.[20] These increasing complements will, in equilibrium, generate the downward sloping demand function for each landowner and guarantee rent gradients are steeper than the gradients in the underlying location quality.

The equilibrium in this market will be characterized by a set of rents r_a for each location $a \in A$, an assignment of types y to locations a, y_a, and quantities q_a, such that given rents at all other locations, no renter wishes to locate elsewhere and all renters are accommodated.

Note that given a rent schedule $\{r_a\}_{a \in A}$, the willingness to pay (WTP) of type y for location a will be given by:

$$WTP(y, a) = \min_{\forall b \in A \backslash a} F_1(a, y) \cdot F_2(I_y - r_a) - F_1(b, y) \cdot F_2(I_y - r_b). \tag{7}$$

From the equation above, it should be clear that the assumption of increasing complements generates differential willingness to pay for any given plot. This is the basis for Propositions 1 and 2, which characterize the equilibrium and describe willingness of

[20]See Topkis (1998).

each type y at their assigned location a.

Proposition 1. *A sorting equilibrium exists with a rent schedule R such that rent r_a is increasing in a for all occupied plots, and cutoffs $\{y_1, \ldots, y_{a-1}\}$ on the support of $G_2(y)$, such that at each cutoff y_a, individuals to the left of the cutoff are assigned to location a, while those to the right are assigned to location $a+1$.*

Proposition 1 describes the equilibrium of the model in the case of vertical differentiation. The equilibrium is given by as set of rents and assignments, or cutoffs. A plot is unoccupied if $y_a = y_{a-1}$. See Appendix A for a proof.

Next, we turn a description of willingness to pay at a given plot for each type y at their equilibrium assignment.

Proposition 2. *In any sorting equilibrium, between any cutoff y_{a-1} and y_a, the willingness to pay of individuals assigned to location a is heterogeneous and single-peaked in type y at some $y_{apeak} \in [y_{a-1}, y_a]$.*

The intuition behind this result is that increasing complementarity acts within assignments of continuous types to the discrete number of plots. Those types on the margin of the assignment must be indifferent, while those in the interior are remote from either outside option and therefore have the highest willingness to pay for their assigned location.

The landowner pricing rule chooses q, and effectively y_{a-1} and y_a such that

$$r_a - mc_a(q) = -\frac{G_2(y_a) - G_2(y_{a-1})}{g_2(y_a)\frac{dy_a}{dr_a} - g_2(y_{a-1})\frac{dy_{a-1}}{dr_a}}. \tag{8}$$

Note that $\frac{dy_a}{dr_a} < 0$, $\frac{dy_{a-1}}{dr_a} > 0$, and therefore markups are positive. As landowners adjust price r_a upwards, they lose renters on two margins, the lowest-type assigned to their plot, y_a, who flee to the cheaper next-best option $a-1$, and those near the top of the distribution at their location that spend more for the option $a+1$.

To close the model, the housing market clears. Note that cutoffs are continuous, and for any y_1, if y_1 chooses a plot in the city all $y > y_1$ do as well. If WTP is negative for the lowest type $\underline{y}$ at the lowest location $\underline{a}$, some mass of types will choose the outside option.

3 Policy Implications of Theory

In this section, we assess the effects of several policies in the context of monopoly markups. First, we discuss the impact of zoning. We show that in the horizontal case, zoning raises rents of plots that are *not* constrained by zoning, even when marginal costs are constant. Second, we discuss how, under non-decreasing marginal costs, concentration of land ownership raises markups and rents at all plots. We conclude by discussing the scope for analysis of of monopoly power in several other urban policies.

3.1 Old Policies, New Implications

One implication of the above model is that, even in the absence of any positive or negative spillovers, a laissez-faire policy of no zoning will not necessarily be first-best. Because a monopolist landlord restricts quantity, the quantity difference between zoning-restricted and an identical, unrestricted plot with a monopolist landlord is less than the difference between zoning-restricted plots and a competitively priced plot. Indeed, reverse height restrictions, forcing more than the market provided density, could be welfare improving.

In reality, zoning constraints are not binding everywhere. To the extent that zoning constrains bind at a particular plot, the quantity must be restricted beyond the monopoly-optimal quantity, and rents as a result must be higher. However, in a city where only some plots are constrained by zoning rules, those constraints also impact rents at unzoned plots by affecting equilibrium monopoly power at unconstrained plots.

In both the vertical and horizontal case, the rent at a given plot is inversely proportional to rents at other plots. However, making an assertion regarding the overall effect of zoning on rents and markups at other plots is more difficult because of the potential for shifts in equilibrium markups through shifts in demand. When we restrict ourselves to just the horizontal case, we can state the following:

Proposition 3. *In the logit case with non-decreasing marginal cost, the imposition of binding zoning constraints on a given plot impacts the rent and markups at all other plots, including unzoned plots and plots where zoning constraints do not bind.*

By raising rents at competing locations, binding zoning constrains have spillover effects at other locations. Likewise, relaxing zoning constraints at one plot brings down rents everywhere. Of course, even when priced competitively, if marginal costs are increasing, by limiting supply at one location, zoning can impact rents and quantities at other locations. But Proposition 3 points out that monopoly power exacerbates the price effects by changing optimal markups. In other words, even in a world of constant marginal costs, zoning constraints at one plot would raise rents at all other plots in the city by exacerbating monopoly markups. This effect operates through the cross-price elasticities, which, in the multinomial logit case can be signed and compared across any equilibria.

3.2 New Policies, New Implications

Under monopoly pricing, higher rents can generate a positive pecuniary externality on other landowners, and, by increasing demand and affecting elasticity, monopoly markups at one plot may positively impact markups, rents, and profits at other plots. When landowners own multiple parcels, they internalize these externalities, which may result in higher markups and rents overall. Intuitively, monopolists with greater share of a market may reduce quantity to a greater extent in order to maximize total profits.

In general, however, the impact of changes in land ownership concentration is analogous to mergers in the multi-product oligopoly setting. As in that setting, we cannot make statements on the effects of concentration on the equilibrium without additional assumptions. We extend Nocke and Schutz (2018b) to generate the following proposition:

Proposition 4. *In the logit case with non-decreasing marginal cost, all else equal, landowners with higher market share have higher markups and rents; an increase in the ownership share of one landowner will generate increases in markups and rents at all the landowner's plots, and increases in rents at all other plots.*

Because we cannot assume marginal cost is constant, we introduce an even more flexible cost function than those found in Nocke and Schutz (2018b,a). That in turn requires an extension to the result on the relationship between own share and others' share on markup and rent. Appendix A proves these extensions.

Note that Proposition 4 is only guaranteed to hold when we can exclude the possibilities of scale economies and when there are no systematic variations in individual valuations by individual characteristics; i.e., no sorting. Intuitively, if landowners can raise profits by forcing more individuals into one plot, generating scale, or if they can affect the sorting equilibrium through manipulations to the rents of multiple plots, they may find it optimal to reduce, rather than increase rents and/or markups.

An important implication of this result is that manipulating the ownership structure of plots, in particular by prohibiting concentration of ownership, may affect rents and welfare. In Section 7, we look for evidence of scope for such policies in our New York city dataset.

3.3 Other regulations: rent control, inclusionary zoning, and use laws

Finally, we briefly discuss the interaction of monopoly pricing with rent controls, inclusionary zoning, and use laws.

Where previously introduced into the housing literature, the concept of monopoly power among landowners has been used to advocate for rent controls. The intuition is that reducing rents in the presence of monopoly markups can achieve the efficient equilibrium. Although we are not aware that it has been similarly studied previously, inclusionary zoning policies, which mandate affordable housing be included in new developments can be considered as a policy which moves monopoly quantities to efficient levels.

Diamond et al. (2019) show that rent controls generate an extensive margin impact. While it is beyond the scope of this paper to discuss exit and entry, Appendix B shows how monopoly markups are capitalized into land rents and could impact such decisions. In this context, inclusionary zoning can be thought of as having similar ideal quantity effects to rent controls without reducing monopoly profit and therefore without affecting entry decisions.

Finally, we point out that zoning use laws may also operate on monopoly margins.

While we only consider markups in a residential market, if demand elasticities vary between residential and commercial markets, use laws may reduce markups by constraining landowners to build in less profitable markets with more elastic demand.

4 Estimating Elasticities and Markups

To empirically assess the monopoly forces we describe, we estimate the building level demand elasticity for Manhattan rental buildings in 2010. We follow the literature of empirically estimating differentiated product models with consumer heterogeneity based Berry et al. (1995) (BLP) and the citing literature.[21] Below, we describe our empirical model and identification strategy.

4.1 Renter Demand Econometric Model

As in our theoretical model, the urban rental market made up of all individuals who will choose to live in a rental property.[22] We differentiate the choice set geographically, such that we consider all rental properties in Manhattan as 'inside' goods and all rental properties in the other boroughs as part of an 'outside' good.[23]

We assume that renter i utility for unit j is linear in idiosyncratic parameters and tastes:

$$U_{ij} = \delta_j + m_i(r_j) + X_j\beta_i + \epsilon_{ij} \tag{9}$$

$$= \delta_j + \frac{\alpha}{y_i} r_j + \left(X_j\beta + \sum_h \{\gamma_h v_{ih} x_{jh}\} \right) + \epsilon_{ij}. \tag{10}$$

where y is renter income, X a vector of observed unit covariates, δ a scalar unit unobservable amenity, v_h an individual preference shifter for covariate x_h, and ϵ is a individual taste shifter for the product. We assume that $\beta_{h,i} \sim \mathsf{N}(\beta_h, \gamma_h)$, so that we only need to

[21] Some examples for applications: Nevo (2001); Davis (2006); Armantier and Richard (2008); Berry and Jia (2010); Chu (2010); for methodological studies: Nevo (2000); Ackerberg and Rysman (2005); Dubé et al. (2012); Knittel and Metaxoglou (2014); Gandhi and Houde (2018).

[22] Our market definition may be better stated as *large* rental properties as we only consider rental properties with four or more units.

[23] This is analogous of comparing utility from a Manhattan property to the average non-Manhattan property for each individual renter.

estimate the mean and variance parameters of the random coefficient distribution. We follow the convention of allowing the distribution of rent to follow the empirical income distribution by using $m_i(r_j) = \frac{\alpha}{y_i} r_j$ and estimating only α (Berry et al., 1999; Nevo, 2001).

Under the assumption that ϵ is distributed Type 1 Extreme Value, then the model connects to the data in the following way:

$$D_j = \int_i \Pr(a_i = j|\chi, \theta) \mathsf{d}F(\theta) = \int_i \frac{\mathsf{e}^{(\delta_j + \frac{\alpha}{y_i} r_j + X_j \beta_i)}}{1 + \sum_{k \in \{J\}} \mathsf{e}^{(\delta_k + \frac{\alpha}{y_i} r_k + X_k \beta_i)}} \mathsf{d}F(\theta), \tag{11}$$

where the ϵ's have been integrated out and the outside option utility, U_{i0}, is normalized to zero.[24]

4.2 Estimation Routine

We estimate the econometric model using market level variables on building choice shares, rents, and characteristics, $\{s_j, r_j, X_j\}$. Using the structure of the econometric model, we simulate R renters by drawing $(y_i, \vec{v}_i)$[25] to calculate the individual demands, and then use Monte Carlo integration for market demand. We allow for the possibility that building rents are correlated with unobservable amenities and identify the parameters using instrumental variable techniques.

Estimation generally has four steps, which are iterated until parameter convergence.[26] `First`, a non-linear inversion step to find mean product utility, μ, given an initial set of parameters, θ.[27] `Second`, use linear GMM to estimate mean utility parameters, β, which identifies the unobserved mean utility characteristic, δ. `Third`, use a non-linear IVGMM search routine to estimate the non-linear parameters, (α, γ), using the moment condi-

[24]Essentially, renters belong to 'types' based on their preferences, θ_i, and we integrate over the probability of observing that 'type' of renter.

[25]We use Halton sequences to approximate uniform random draws. Income is simulated by using a log normal distribution with mean and variance based on the ACS 2010 file.

[26]Formally, we use a Iterated Efficient Simulated IVGMM estimation routine with a nested fixed point. The only difference from BLP is that we iterate the GMM procedure until parameter convergence rather than the usual 2-Step because we find that the non-linear parameters are quite sensitive to the initial weighting matrix in our problem. While the 2-Step approach is consistent, in finite samples the 2-Step parameters depend on the initial weight matrix and can be subject to greater misspecification errors, leading us to use an Iterated GMM approach (Hansen and Lee, 2019).

[27]We use a tolerance of $\|\mu_j^{r+1} - \mu_j^r\|_\infty < 10^{-12}$. For more details, see appendix C.

tion $\mathsf{E}[Z' \cdot \delta]$. `Fourth`, update the weight matrix using the residuals from step 3, and repeat until the parameter vector converges, $\|\theta^{s+1} - \theta^s\|_2 \approx 0$.

4.3 Identification

One way of thinking about the model is that there are two endogenous variables for every observation: rent and market share.[28,29] Thus, we need to instrument in order to estimate substitution patterns while accounting for potential unobserved correlations with rent, $\mathsf{Exp}[r \cdot \delta] \neq 0$. As in the literature, we identify the substitution patterns by specifying them as a function of the building characteristics depending on renter heterogeneity. That is, the random coefficients, γ, specify substitution patterns and we require instruments that are relevant to substitution between products, $Z^{(x)}$. As rents are likely correlated with building level unobserved amenities, δ_j, we need an instrument that shifts price but is unrelated to the building level amenity, $Z^{(r)}$.

With a set of instruments, $Z = (X, Z^{(x)}, Z^{(r)})$, the identifying moment condition is

$$\mathsf{E}[\delta(X, r, s; \theta) \mid Z] = 0, \tag{12}$$

which leads to our use of $\mathsf{E}[Z'\delta]$ as the empirical moment we wish to minimize. We describe the instruments we use in more detail after we introduce the data.

4.4 Elasticities and Markup Calculations

Once we estimate our model parameters, θ, we can calculate building level elasticities and markups that will inform our understanding of monopoly power in the Manhattan market. We calculate the building level demand elasticities using the analytical derivatives of the logit demand functions, and we calculate the percent markup assum-

[28] For a rigorous discussion of identification, see Berry and Haile (2014, 2016).
[29] We assume that the building level variables are exogenous and can additionally serve as instruments.

ing landlords solve a Bertrand price competition game:

$$\varepsilon_j = \frac{\partial D_j}{\partial r_j}\frac{r_j}{D_j} = \left[\int_i \left(\frac{\alpha}{y_i}\right) \cdot d_{ij} \cdot (1 - d_{ij}) \mathsf{d}F(\theta)\right] \frac{r_j}{D_j} \tag{13}$$

$$\mathsf{Lerner}_j = \frac{r_j^\star - mc_j}{r_j^\star} = \left(\frac{-1}{\varepsilon_j}\right) \tag{14}$$

We calculate the 'aggregate elasticity' which provides the responsiveness of renters to a 1% increase in rent for all 'inside' buildings (Berry and Jia, 2010; Conlon and Gortmaker, 2019):

$$\varepsilon^{\text{Agg}} = \sum_{k \in \{J\}} \frac{D_j(\{r_k + \Delta r_k\}_{k \in J}) - D_j}{\Delta}\Bigg|_{\Delta = 1\%}. \tag{15}$$

This parameter is closest in spirit to the parameter that most housing demand literature estimates, which has typically found inelastic values seemingly incompatible with monopoly pricing (Albouy et al., 2016). We reconcile this fact by showing that the relevant pricing elasticity for housing prices is based on the own price elasticity, ε_j, rather than the aggregate elasticity, ε^{Agg}.

5 Data

The first data items we use come from public administrative building level data from the New York City Department of Planning and Department of Finance. The 2010 New York city Primary Land Use Tax Lot Output (PLUTO) dataset on all buildings in the city[30] provides us with building characteristics, ownership, address, zoning, tax assessment, and building level rental income information. We combine this with the Fiscal Year 2012 Final Assessment Roll (FAR) dataset that includes market value information based on the reported building level income from 2010. In addition we merge the 2012 NYC Department of Homes and Community Building Registration File that includes all rent controlled and stabilized buildings.[31]

The PLUTO dataset does not account for vacancies, so we use the 2010 Decennial

[30]The 2010 PLUTO file does not include 2010 Census Block or Tract identifiers but the 2011 does, so we merge the two files to include the 2010 boundaries.

[31]We use the 2012 file because we were unable to access the 2010 file.

Census to allocate rental households to buildings.[32] We use the 2010 American Community Survey to find the mean and variance of the income distribution for Manhattan. To determine the size of the rental market, we use the total number of renter households in the five NYC boroughs that are in buildings with four or more units.[33]

5.1 Sample

For estimation, we use all private buildings classified as multi-family rental buildings in Manhattan in 2010 with four plus units, where all units are residential units and there is no missing data. When we construct the instruments based on exogenous rival building characteristics, we expand the sample to include mixed-use buildings. We exclude mixed-use buildings in the estimation because we cannot separate building income due to residential sources versus commercial tenant sources.[34] For more details, see Appendix D.[35]

5.2 Variables

The building level variables that affect mean-utility are a constant, building age, distance to the CBD, distance to nearest subway station, years since the building was last updated, average unit square-feet, whether the building has an elevator, the number of residential buildings, office buildings, retail buildings, and open parks in the census block.[36] We also include census tract fixed effects. For random coefficients, we only include the first five variables (plus constant) and omit the fixed effect and the block level counts.

In addition, building level average rent has a random coefficient as described earlier. Average rent is indirectly reported by the DOF as *market value* and recovered using the

[32]We calculate the block level vacancy rate for rental buildings and multiply this times the number of units in a building. This method assumes that vacancy rates are uniform within census blocks, but may vary across census blocks.

[33]The 2010 Census reports the number of renter households but not by number of units, so we scale the 2010 Census value by the ratio of renters in four+ unit buildings to all renters from the 2010 ACS.

[34]The is data is observed by NYC DOF but is not released publicly.

[35]Note that we do *not* push mixed use buildings to the outside good; instead, we simply do not include them in the estimation. Under the model assumptions, this causes an efficiency loss but does not bias our results.

[36]Geodesic distance measures are in log miles and using latitude and longitude coordinates.

following formula:

$$\text{MarketValue}_j = \text{GIM}_j \cdot \mathsf{Avg}\left(\text{Annual Rent}\right)_j \cdot \text{units}_j, \tag{16}$$

where the Gross Income Multiplier is determined by the DOF based on the building's income per square foot.[37] For more details, see appendix E.

5.3 Instruments

As stated before, we use two sets of instruments to estimate random coefficients. The first is to deal with correlation between rents and unobserved amenities, $Z^{(r)}$, and the second is used to identify substitution patterns between products, $Z^{(x)}$.

The $Z^{(r)}$ is the land value of the building plot, which captures the market value of vacant land where the building is located. Land value is constructed by the NYC DOF using a database of building transactions and rental building incomes and costs. We interpret this measure as capturing the opportunity cost of the landlord for renting the space out versus some other purpose, and is the value which must be covered by renters in order for the landlord to participate.[38] This instrument is constructed using sales of similar plots which may not necessarily be spatially near a given building. Thus, the value should not be systematically correlated with local building level unobservable residential amenities.[39]

The $Z^{(x)}$ is constructed using functions of rival building characteristics.[40] Our preferred specifications use "Quadratic Differentiation Instruments" based on Gandhi and Houde (2018). These are a finite order basis function approximation of the optimal in-

[37]These calculations are based on FY 2010 Assessment Roll Guidelines, online at the NYC DOF; see the "NYC DOF Property Tax Guide for Tax Class 2" for a user friendly guide. The AR guidelines appear to change over the years, becoming more complex over time, and so it may not be possible to use the guidelines to impute building level income for a given year.

[38]Technically, land value is a based on a sales concept, and so needs to be properly discounted in order to be comparable with unit rents.

[39]That is, the exclusion restriction is only violated if, conditional on building observables and tract FEs, constructed land value from sales around the city are correlated with unobservable amenities at the building level.

[40]When creating the rival set $K(j)$, we exclude rivals within a 1km radius of a given building, based on Bayer et al. (2004) and Bayer et al. (2007) who use rings of 5 and 3 miles, respectively for their instrument construction using homes in the San Francisco bay area.

struments in the sense of Amemiya (1977) and Chamberlain (1987). For a given covariate h for building j with rivals $K(j)$, each instrument is defined as:[41]

$$Z_{hj}^{\mathrm{DQ}} = \sum_{k \in \{K(j)\}} (x_{hk} - x_{hj})^2. \tag{17}$$

For complete details, see appendix F where we describe our instrument construction at length.[42]

5.4 Summary Statistics

Table 1 presents summary statistics for our Manhattan data. Each column represents a cut of the data that we use. As explained above, the first is used for calculating our instruments, the second is used in our estimation, and the third is the set of unconstrained buildings which we can calculate counterfactual price effects.

Manhattan rental buildings account for about 24% of the total large-building rental market in the city. While roughly half of all buildings are mixed use, mixed use buildings account for slightly more than half of households due to differences in building size, 24.8 versus 20.5. Unconstrained buildings have noticeably different characteristics. These buildings are in areas with higher vacancy rates, are more expensive, are newer, and are larger. Figure 1 plots the total number of households and mean unit rents by census tract.[43]

6 Estimation Results

Table 2 presents our main empirical results.[44] We estimate utility parameters based on our empirical model, then calculate building level elasticities and percent-markups over marginal cost. We present the traditional two-step GMM and our preferred IGMM results using "Quadratic Differentiation Instruments", as described in section 5.3. Of our

[41] We also create a differentiation instrument for average rent, accounting for potential endogeneity as prescribed in Gandhi and Houde (2018).

[42] Additionally, consult appendix G for other details in implementing our estimation procedure.

[43] In appendix D we plot additional spatial distributions, such as zoning constraints and rent control.

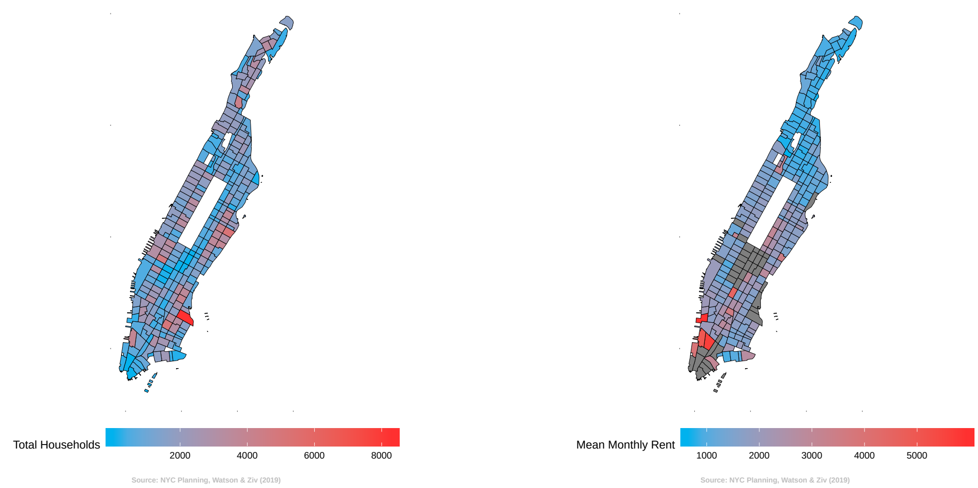
[44] We present robustness results in appendix H using alternative instruments and solvers.

Table 1: Summary Stats:
Manhattan Rental Buildings

	IV Sample	Estimation Sample	Unconstrained Sample
Total Market Share	23.9%	10.2%	0.6%
Res.Units per Building	24.8	20.5	21.1
Households per Building	22.7	18.8	19.0
Vacancy Rate	10%	10%	11%
Percent Mixed-Use	48%	0%	0%
Percent Rent Controlled	52%	54%	0%
Percent Zoning Constrained	80%	82%	0%
Median Monthly Rent*	–	$1,243.75	$1,927.08
Median Rent by Median Income*	–	30%	46%
Median Monthly Land Value per Unit	$162.00	$154.84	$460.00
Years Since Construction	94.33	94.71	87.12
Years Since Update	49.11	48.79	36.32
log(Distance CBD)	1.34	1.59	1.50
log(Distance Subway)	-1.94	-1.89	-1.97
Avg Unit Sqft	776.48	746.38	1166.11
Buildings	18,052	9,330	553

Building data from PLUTO 2010 file, rent data based on FAR FY2012 file. Households allocated based on building units and 2010 Decennial Census and American Community Survey. Vacancy rate is one minus the total households in building divided by total building units. Median income in 2010 for NYC is $ 50,000. A building is mixed-use if the building has positive commercial area. A building is considered rent controlled if it is on the 2012 Rental Controlled and Stablized list. A building is zoning constrained if the building would not be allowed to create an additional unit based on building floor-area-ratios and minimum unit area requirements. Geodesic distances are in miles based on building (lat,lon) coordinates. 'Updates' are based on building renovation projects since original construction. Monthly land value per unit is [Land Value / (12 x Residential Units)].
(*) – Rent data is only available for single use buildings

Figure 1: Distribution of Manhattan Quantities & Rents



Left: total census tract households. Right: mean tract unit rents; missing values due to mixed-use buildings.

estimated parameters, we only present our estimate of α, with its heteroskedasticy robust standard error.[45] Using the equation 13 we calculate the own price elasticity, equation 14 the percent markup, and equation 15 the aggregate elasticity.

In both specifications, our rent coefficients are statistically significantly different from zero and either all or the vast majority of buildings are priced in the elastic region of demand, as predicted by monopoly pricing. When we iterate the estimation routine until parameter convergence, we estimate more elastic demands implying smaller markups.[46] The 2-step GMM results also have larger standard errors on the rent coefficient and an objective function value a order of magnitude greater. We interpret this as a finite sample issue which is alleviated as we iterate the procedure, improving our estimate of the optimal weighting matrix.

We estimate median own price elasticities around -4.3 for the entire sample and -4.8 for the unconstrained sample. We expect these unconstrained landlords have the

[45]This parameter can be roughly interpreted as the marginal utility of non-housing consumption.

[46]Our MATLAB implementation of the IGMM 'QD+P' model did not converge using our criterion of $\|\theta^{s+1} - \theta^{s}\|_2 < 0.01$; rather, parameters oscillated repeatedly between two sets of estimates with a norm difference of 0.068. In section H table 11, our alternative solver `pyblp` results largely agree with both parameters sets, so we present the estimates with the slightly smaller GMM objective function value.

most control over their rents compared to landlords dealing with rent controlled units or pressing against zoning constraints. A larger magnitude elasticity typically implies being 'further up' the demand curve due to more limited quantity supplied. We find that the median percent mark-ups is between $(21\%, 23\%)$ of total rent. In other words, were units priced at the marginal cost reflective of the production and maintanance of buildings, we would expect rents to be about 80% of their current levels. Figure 2 plots the mean tract Lerner index by Census tract.

Without other studies as benchmarks, we cannot be sure how different our results would be in another context. One the one hand, the close proximity of substitute units in Manhattan should reduce monopoly power. On the other hand, as discussed in Section 3, the ubiquity of policy constraints may artificially raise markups in Manhattan beyond what they would be in less constrained environments.

Note that our results differ from the literature on the elasticity of housing demand. Our elasticity of interest is conceptually different than that targeted by that literature, which seeks to measure the substitution between quantity of housing and consumption. In that literature, housing demand typically found to be inelastic.

When we estimate the aggregate elasticity in our data, which is more akin to the parameter estimated in the prior housing demand literature, we find similarly inealstic demand with an elasticity is -0.146. This estimate is slightly lower than the consensus range in the prior literature: $(-0.64, -0.3)$ (Albouy et al., 2016).[47] This may be due to a differences in sample than most estimates or the different methodology, as our outside good includes other housing choices in NYC rather than pure consumption.

One issue with our results may be the age of the Manhattan housing stock. While Appendix B discusses the case of seperated developer and landlord quantity and price decisions, the analysis hinges on the ability of developers to correctly anticipate the demand faced by landlords. However, the mean apartment building construction date was roughly one century before the current residential demand was realized, and one may worry that this assumption is not tenable.

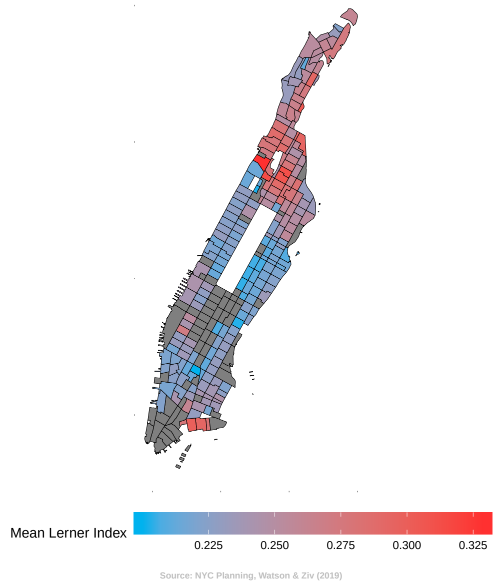
[47] Using a hedonic approaches with building level data, Gyourko and Voith (2000) and Chen et al. (2011) find elasticities that are compatible with monopoly pricing; however, only the latter notes the connection with monopolistic landlords.

Table 2: Main Estimation Results

	2-Step GMM	IGMM
α	-34.32	-77.30
SE	(23.80)	(18.10)
$\mathsf{Med}(\varepsilon_j)$	-2.61	-4.30
$\mathsf{Med}(\varepsilon_j \mid \text{Unconstrained})$	-2.92	-4.84
$\mathsf{Med}(\mathsf{Lerner}_j)$	38%	23%
$\mathsf{Med}(\mathsf{Lerner}_j \mid \text{Unconstrained})$	34%	21%
$\%\{\lvert\varepsilon_j\rvert > 1\}$	99%	100%
ε^{Agg}	-0.135	-0.146
GMM Obj	390.95	34.82

$N = 9,330$ for all models; standard errors robust to heteroskedasticity; census tract fixed effects and building characteristics included in all models. Both models use "Quadratic Differentiation Instruments" based on Gandhi and Houde (2018), as described in section 5.3. The own price elasticity is ε_j and the aggregate price elasticity, ε^{Agg}, is based on Berry and Jia (2010). Buildings are 'unconstrained' if *not* rent-stabilized and *not* zoning-constrained; $N_{\text{Uncons.}} = 553$.

Figure 2: Distribution of Lerner Index for Manhattan



Lerner index results use estimates form IGMM model with DQ+P instruments.

In table 12 in appendix H, we display descriptive statistics and results for the 50 unconstrained buildings that were built in the last 10 years, which are more likely to have correctly anticipated contemporary demand. However, we find that these new buildings are actually slightly more elastic with median elasticity of -4.91 and a median markup that is 20% of rent. Thus, while the concern about building age is valid, it does not appear to have a first-order effect on our estimates.

7 Empirical Policy Implications

In this section, we use our data and the results of our model to quantify the potential effects of two policies: zoning restrictions and concentration controls. First, we use the model-implied elasticities to examine the effect of a 1% across-the-board reduction in zoning quantity constraints. Because our empirical approach takes no stand on costs, our results can be treated as an upper bound for the spillover impact of zoning. We find modest but appreciable spillovers.

Second, we explore the potential for concentration to impact welfare. We test two implications of our proposition: that landowners with higher shares raise markups and rents, and that higher levels of concentration increase rents overall. We show that in our data, these conclusions can be rejected, implying the landowners with higher concentrations may benefit from scale economies. While there may be some counterfactual concentration in NYC under which policies aimed at reductions in concentration could raise welfare, that is not the case in the current equilibrium.

7.1 Up-Zoning Effects on Unconstrained Plots

We want to consider the price effect on unconstrained buildings of reducing zoning constraints on the constrained buildings. The price effect that we estimate is the partial equilibrium change in monopoly mark-ups for unconstrained buildings given a reduction in zoning constraints in the set of 4,291 zoning-constrained, non-rent controlled residential buildings in our sample.[48,49]

Without further assumptions on the curvature of marginal costs, we cannot estimate the partial equilibrium change in building rents. If marginal cost is assumed increasing in quantity supplied (yielding an upward sloping supply curve), then the partial equilibrium decrease in demand should decrease marginal costs, which decreases rent. *Only if* one assumes that marginal costs are (locally) constant, then our estimates reflect partial equilibrium rent changes. As such, our change in the markup is the *lower-bound* of the magnitude of the zoning price effect.

To implement this, we `first` use the estimated building level demand elasticity to calculate the rent change required to increase building market share by 1% from its equilibrium level for all zoning-constrained buildings, $\{r_k^{\text{cf}}\}_{k \in \mathcal{Z}}$.[50] `Second`, we totally differentiate the monopoly pricing rule with respect to all zoning-constrained build-

[48]Because we do not empirically model the supply side of the market, especially in the context of zoning regulation, we do not solve for full counterfactual rents with looser zoning constraints, which we would interpret as the general equilibrium change.

[49]Additionally, our marginal effect is *not* the same as considering if each zoning constrained building added an additional household, which would require solving the full monopoly pricing problem. For example a one unit change for 5-unit buildings is a 20% change rather than a marginal change.

[50]Since a 1% increase in market share is not the same as adding an additional unit, we only interpret this as the rent-equivalent to a marginal zoning change rather than a true change in zoning policy.

ing rents and solve for a given unconstrained building's rent change.[51] `Third`, we plug our estimated parameters and the counterfactual rent change vector of the constrained buildings into the total differential formula to find our rent changes for each unconstrained building, $\{\mathsf{d}r_j^{\mathsf{cf}}\}_{j\in\mathcal{U}}$. We calculate this as:

$$\mathsf{d}r_j^{\mathsf{cf}} = \sum_{k\in\{\mathcal{Z}\}} \left\{ \left(\frac{D_j \frac{\partial m_j}{\partial r_k} - m_j \frac{\partial D_j}{\partial r_k}}{2(m_j)^2 - D_j \frac{\partial m_j}{\partial r_j}} \right) \mathsf{d}r_k^{\mathsf{cf}} \right\}., \tag{18}$$

where $m_j = \partial D_j / \partial r_j$. In addition, we calculate the partial equilibrium market share effect for unconstrained buildings from the zoning shock:

$$\mathsf{d}D_j^{\mathsf{cf}} = \sum_{k\in\{\mathcal{Z}\}} \frac{\partial D_j}{\partial r_k} \mathsf{d}r_k^{\mathsf{cf}}. \tag{19}$$

For more details, see appendix I.

Table 3 presents our results. Using our preferred estimates, we find that the rent-equivalent of a 1% exogenous increase in market share for zoning-constrained buildings leads to a mean mark-up *decrease* of $ 3.31 per unit or $ 65.36 per building. This implies a very small -0.060 elasticity for each building with respect to all zoning-constrained buildings. We find that the mean percent change in unconstrained building market share is -0.013%, so that loosening zoning would cause at least some residents to substitute to the previously constrained buildings.

However, as there are more zoning-constrained buildings, the market share increase in these buildings dominates the decreased demand for the unconstrained buildings. We calculate that the total market share of non-mixed-use rental buildings increases by ≈ 0.03 percentage points, from 10.20% to 10.23%, which is about 490 additional households from the four boroughs outside of Manhattan.[52]

Altogether, we interpret these two partial equilibrium facts as additional rationales for easing residential zoning restrictions. Without monopoly power, only changes in

[51] As this is a partial equilibrium analysis, for each unconstrained building we hold fixed all *other* unconstrained buildings' rents.

[52] Since non-mixed-use rental buildings are roughly half of the rental buildings for both constrained and unconstrained buildings, a back-of-the-envelope estimate for the total market effect is roughly one thousand additional households.

marginal cost would affect rent. The price effect we calculate represents *additional* downward pressure on rents that arises purely through the monopoly forces in the model. In addition, these results imply that at least part of the large equilibrium markups on unconstrainted plots we find in our estimation may be a result of spillovers from (the numerous) zoning-constrained plots.

Table 3: Zoning Effects for Manhattan Single Use Buildings

	2-Step	IGMM
Direct Price Effect of Looser Zoning		
$\frac{\mathsf{d}D_k^{\mathsf{cf}}}{D_k}$	1%	1%
$\mathsf{d}r_k^{\mathsf{cf}}$	-\$66.27	-\$39.65
Spillover Markup Effect of Looser Zoning		
$\frac{\mathsf{d}D_j^{\mathsf{cf}}}{D_j}$	-0.015%	-0.013%
$\mathsf{d}r_j^{\mathsf{cf}}\big\vert_{\mathsf{d}mc_j=0}$	-\$3.76	-\$3.31
Implied Spillover Zoning Elasticity		
$\frac{\mathsf{d}r_j^{\mathsf{cf}}}{r_j} \Big/ \frac{\mathsf{d}D_k^{\mathsf{cf}}}{D_k}$	-0.042	-0.060
Net Manhattan HHs		
$\left(\sum_{j\in\mathcal{J}} \mathsf{d}D_j^{\mathsf{cf}}\right) M_{\text{NYC}}$	491	489

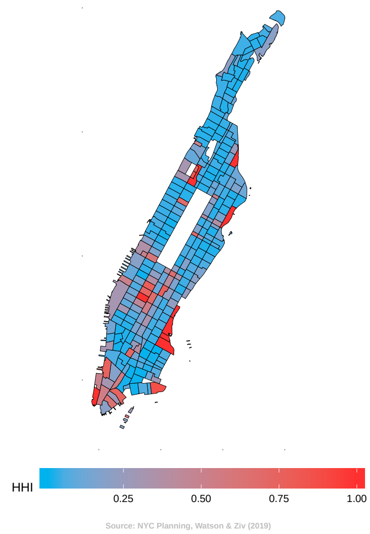
Both columns use "Quadratic Differentiation IVs" based on Gandhi and Houde (2018), as described in section 5.3. $\mathcal{U}$ are 553 non-zoning constrained, non-rent controlled buildings, $\mathcal{Z}$ are 3,738 zoning constrained but non-rent controlled buildings. Buildings are 'unconstrained' if *not* zoning-constrained For more details, see appendix I.

7.2 Concentration in New York City

Finally, we examine the relationship between rents, markups, and concentration in our data. Proposition 4 predicts landowners in the cross section with identical costs and amenities and higher shares of the market will have higher markups and rents, and that as concentration increases, rents and markups at all plots increase.

For each landlord f with buildings j in location t, we calculate the landlord market

Figure 3: Distribution of Ownership Concentration in Manhattan



HHI index of within tract ownership concentration.

share of t as

$$s_f^t = \frac{\left(\sum_{j \in f_t} D_j\right)}{\sum_{f' \in F_t} \left(\sum_{j \in f'_t} D_j\right)}$$

where F_t is the set of landowners with buildings in t.

We then test the basic prediction that rent and markups increase in shares. Our first specification estimates

$$\ln[r_p] = \alpha_0 + \alpha_1 \cdot \ln[s_f^t] + \alpha \cdot X + \epsilon_p, \tag{20}$$

where s_f^t is described above and X is a set of controls including all observables described in section 5 and potentially an estimate of the plot unobservable quality derived in section 6.

Columns 1 and 7 of Table 4 estimate the model in the above equation for all unconstrained plots, where $s_f^t = s_f^{\text{MN}}$, the landowners share of Manhattan. We find a negative relationship inconsistent with the predictions of Proposition 4. Furthermore, Columns 2 and 8 repeat the previous exercise but add the model's estimate of the unobserved amenity value at each plot. The negative relationship is smaller but still significant. One potential interpretation of these results is that landowners with multiple plots achieve scale economies that reduce costs. Another possibility is that concentrated landowners select plots with lower costs or that the unobservable included does not fully account for unobserved variation.

Next, we look at concentration at the tract level using $s_f^t = s_f^{\text{tract}}$. While local concentration is beyond the scope of our model, we hope to use variation in concentration across areas in the city to test the second piece of Proposition 4, that concentration raises costs at all plots. First, we repeat the above analysis at the tract level. Columns 3 and 4 regress rents on local landowner concentration and observable plot amenities, without and with controls for model-derived unobservable, respectively. Columns 9 and 10 repeat the same for markups. Findings are similar to those in the full-city sample.

Second, to understand the effect of localized landowner concentration on other landowners, we estimate

$$\ln[r_p] = \alpha_0 + \alpha_1 \cdot \ln[\mathsf{HHI}_f] + \alpha \cdot X + \epsilon_p, \tag{21}$$

where $\mathsf{HHI}_f = \sum_{f' \in F \setminus f} \left(s_{f'}^t\right)^2$ is the concentration ratio in tract t measures, for each landowner f in tract t the concentration of all other landowners in the tract. Figure **??**, plots tract-level HHI measures for Manhattan using the PLUTO data.

Columns 5 and 11 report a positive but insignificant effect of localized concentration on landowners' plots. Columns 6 and 12 show that these effects are eliminated when controlling for model-derived unobservable. While these appears to be some scope for the potential for localized concentration to impact rents, much of this appears due to unobserved tract differences.

Here again, without an instrument for costs we cannot assess whether results are affected by selection. In addition, concentration may work globally through the city rather than locally. Overall, however, we conclude that our results do not appear consistent with the hypothesis that policies directed at reducing the concentration of ownership under the current ownership structure in Manhattan cannot generate efficiency gains. Of course, this may not be the case in a counterfactual, more highly concentrated ownership structure or in a different setting.

Finally, a caveat of this analysis here is that the true ownership in Manhattan may be more concentrated than given by the PLUTO data, where individual plots may be reported to be owned by subsidiaries of larger organizations whose name is not reported in our data.

Table 4: Rents and Markups on Concentration

	Log Rent					
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$\ln[s_f^{\text{MN}}]$	-0.061*	-0.058**				
	(0.025)	(0.012)				
$\ln[s_f^{\text{tract}}]$			-0.047*	-0.069**		
			(0.022)	(0.015)		
$\ln[\text{HHI}_f]$					0.065	0.009
					(0.039)	(0.026)
Unobservable		0.203**		0.206**		0.203**
		(0.006)		(0.006)		(0.006)
Observations	553	553	553	553	553	553
R-squared	0.418	0.827	0.411	0.829	0.409	0.814
	Log Markup					
	(7)	(8)	(9)	(10)	(11)	(12)
$\ln[s_f^{\text{MN}}]$	-0.046*	-0.043**				
	(0.019)	(0.010)				
$\ln[s_f^{\text{tract}}]$			-0.036*	-0.053**		
			(0.017)	(0.012)		
$\ln[\text{HHI}_f]$					0.047	0.002
					(0.031)	(0.021)
Unobservable		0.162**		0.165**		0.163**
		(0.005)		(0.005)		(0.005)
Observations	553	553	553	553	553	553
R-squared	0.449	0.838	0.443	0.841	0.441	0.827

All regressions include full set of controls used in model estimation. Robust standard errors in parentheses.
$^{**}p < 0.01, ^{*}p < 0.05$.

8 Conclusion

To investigate the role and impact of monopoly power over land, we built and estimated a model where monopolist landowners set rents to maximize profits. We show how markups are generated from two separate sources: horizontal plot differentiation, and two-sided vertical differentiation with sorting.

To estimate the model, we generate a new building-level dataset of rents of multi-unit buildings in New York. We combine assessment data and NYC Department of Finance documents to find each building's total rent. We use this set of buildings to estimate the demand parameters of the model. Using lists of rent-controlled buildings and zoning information, we separate a set of unconstrained buildings where these parameters can be used to back out markups. We find average markups are above 20% of rent. While our model may misspecify markups for buildings where developers had incomplete information on demand, and this may be especially true for older buildings, we find similar percentage markups on the newest buildings in our model.

We then investigate the policy implications of these markups. Theoretically, monopoly control means that, under at least some circumstances, zoning regulations have spillover effects on unregulated plots through effects on markups. We estimate that this effect to be such that a policy relaxing zoning quantity constraints by 1% overall reduces markups on each unzoned plot by about $3 per unit.

The model also predicts, under similar conditions, concentration can raise markups further. Using our data, we can directly test these predictions, but find limited support for the conclusion that at present levels of concentration, policies targeting the ownership structure of plots in New York could improve welfare.

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A Propositions

A.1 Proposition 1

First note that in any equilibrium, rents must be strictly increasing in a. Because plots are increasing in their quality for all agents, any other arrangement would mean a renter at a lower-a plot could pay weakly less in rent and move to a strictly higher a plot. But because utility is increasing in a holding rent constant, this renter would not be optimizing at the proposed equilibrium.

Note also that the assumption of log-supermodularity guarantees a sorting equilibrium as in Topkis (1998). For an intuition as to why, note that the utility difference between any two plot is increasing in renter type. In particular, note that this sorting generates the set of cutoffs $\{y_1, \ldots, y_{a-1}\}$ which act as assignments for types y to locations. Any violation of this assignment would mean an individual can improve utility, it must be the case that either the higher-type individual can improve utility by moving to a higher-a plot or the lower-type individual can improve utility by moving down to the lower-a plot.

A.2 Proposition 2

Note that the willingness to pay is smooth in y, and therefore that types on the two margins of plot a, y_{a-1} (and y_a), must be indifferent between a and either $a-1$ (or $a+1$).

Although the rents at all all locations may impact an individual's willingness to pay, following Mirrlees (1971), only each agent's top outside option must be considered, and in a sorting equilibrium, this relevant outside option for any agent at y' must be either $a-1$ or $a+1$. Note that if any agent y''s top outside option is $a-1$, all $y_{a-1} < y'' < y$ relevant outside option must also be $a-1$.

Now note from equation **??** that the complementarity between type y and outside option $a+1$ (and $a-1$) is increasing (and decreasing, respectively) in y. Finally, note that it is impossible for the highest outside option to be $a+1$ for type y_1 and $a-1$ for y_2 when $y_1 < y_2$. It must therefore be the case that WTP is increasing monotonically from y_{a-1} to $y_a peak$.

A.3 Propositions 3 and 4

Here we prove that when an landlord's parcel ownership concentration increases, the landlord increases the prices at all properties. We apply the framework of Nocke and Schutz (2018b) and Nocke and Schutz (2018a) to calculate the price effect by utilizing the ι-markup of the landlord. The authors use a nested-logit model, but we simplify the result removing the nesting structure.[53]

Lemma: In the logit case with non-decreasing marginal cost, $\frac{\partial r_j}{\partial s_f} > 0,\ \forall\, j \in f$. Below, we show this in the two product and general case.

A.3.1 Oligopolist Pricing Equation

Let each landlord solves the following joint-profit equation:

$$\max_{\{r_j\}_{j\in f}} \sum_{j\in f} r_j D_j - C_j(D_j). \tag{22}$$

Following the insight from Nocke and Schutz (2018b), the first order for each property satisfies:

$$\left(r_j - \frac{\partial C_j}{\partial D_j}\right) = \frac{-1}{\alpha} + \pi_f = \frac{-1}{\alpha(1-s_f)}. \tag{23}$$

We can rearrange 23 to solve for rent:

$$r_j = \frac{\partial C_j}{\partial D_j} - \frac{1}{\alpha(1-s_f)} > 0, \tag{24}$$

where marginal cost is positive to yield an upward sloping supply curve. Denote marginal cost as $\frac{\partial C_j}{\partial D_j} = c_j$. We will assume that its derivative is positive: $\tilde{c}_j := \frac{\partial c_\ell}{\partial D_\ell} \geq 0,\ \forall \ell \in J$.[54]

A.3.2 Two Product Case

Recall that under logit demand:

[53] These results also remove individual heterogeneity in renter preferences in order to take advantage of the IIA property.

[54] A micro-foundation is that the residential space production function is concave in inputs which implies that the cost function in convex in quantity; hence, marginal cost is non-decreasing in quantity.

$$\frac{\partial D_j}{\partial r_j} = \alpha D_j(1 - D_j) < 0 \tag{25}$$
$$\frac{\partial D_k}{\partial r_j} = -\alpha D_j D_k > 0 \tag{26}$$

Price Effects:

$$r_j = \frac{-1}{\alpha(1 - s_f)} + c_j(D_j) \tag{27}$$
$$\implies \frac{\partial r_j}{\partial s_f} = \frac{-1}{\alpha(1 - s_f)^2} + \frac{\partial c_j}{\partial D_j}\left(\frac{\partial D_j}{\partial r_j}\frac{\partial r_j}{\partial s_f} + \frac{\partial D_j}{\partial r_j}\frac{\partial r_k}{\partial s_f}\right) \tag{28}$$

by symmetry

$$\frac{\partial r_j}{\partial s_f} = \frac{\frac{-1}{\alpha(1-s_f)^2} + \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_k}\left[\frac{\frac{-1}{\alpha(1-s_f)^2} + \frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_j}\frac{\partial r_j}{\partial s_f}}{\left(1 - \frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_k}\right)}\right]}{\left(1 - \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_j}\right)} \tag{29}$$
$$= \frac{-1}{\alpha(1 - s_f)^2}\left[\frac{1 - \frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_k} + \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_k}}{\left(1 - \frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_k}\right)\left(1 - \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_j}\right) - \left(\frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_k}\right)\left(\frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_j}\right)}\right] \tag{30}$$

imposing Logit

$$= \frac{-1}{\alpha(1 - s_f)^2}\left[\frac{1 - \frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_k} + \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_k}}{1 - \frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_k} - \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial r_j} - \frac{\partial c_j}{\partial D_j}\frac{\partial c_k}{\partial D_k}\frac{\partial D_k}{\partial r_j}\alpha(1 - s_f)}\right] > 0 \tag{31}$$

A.3.3 General Product Case

Note that we have the following:

$$[r_i] = [\Gamma(s_f) \cdot 1_f] + [c_i(D_i)] \tag{32}$$

$$\mathsf{D}_{s_f} r = [\Gamma'(s_f) \cdot 1_f] + \mathsf{D}_D c \cdot \mathsf{D}_r D \cdot \mathsf{D}_{s_f} r \tag{33}$$

$$\implies \mathsf{D}_{s_f} r \cdot [\mathbb{I} - \mathsf{D}_D c \cdot \mathsf{D}_r D] = [\Gamma'(s_f) \cdot 1_f] \tag{34}$$

$$\implies \mathsf{D}_{s_f} r = [\mathbb{I} - \mathsf{D}_D c \cdot \mathsf{D}_r D]^{-1} \cdot [\Gamma'(s_f) \cdot 1_f] \tag{35}$$

A.4 Definitions and Lemmas

Definition A.1. Strictly (Row) Diagonally Dominant : for every row, i, the element along the diagonal, a_{ii}, is greater in magnitude than the sum of the magnitudes of each non-diagonal element in the row $a_{i,j},\ j \neq i$. That is,

$$|a_{i,i}| > \sum_{j \neq i} |a_{i,j}|$$

Definition A.2. `Z`-matrix : a matrix whose off-diagonal entries are less than or equal to zero.

Definition A.3. `M`-matrix : a `Z`-matrix where every real eigenvalue of A is positive.

Lemma 1. *If A is a `Z`-matrix that is strictly diagonally dominant, then A is an `M`-matrix by* Gershgorin Circle Theorem.

Lemma 2. *If A is an `M`-matrix with positive diagonals and negative off diagonals, then $B = A^{-1}$ is monotone positive; i.e., $b_{ij} > 0,\ \forall\, i, j$; proof in Fan 1958.*

A.4.1 General Case Proof

We need to show that the lemma holds and that the vector $B \cdot \Gamma'(s)$ is a monotone positive vector. Let $[\mathbb{I} - \mathsf{D}_D c \cdot \mathsf{D}_r D] = A$.

First, see that A is (a) a Z-matrix that is (b) Strictly (Row) Diagonally Dominant :

(a) for each row, using logit demand, we have

$$a_{i,i} = 1 - \tilde{c}_i \alpha D_i (1 - D_i) > 0 \tag{36}$$

$$a_{i,j} = \tilde{c}_i \alpha D_i D_j < 0 \tag{37}$$

(b) plug into definition of (row) diagonally dominant

$$\Longrightarrow 1 + \tilde{c}_i |\alpha| D_i (1 - D_i) > \sum_{j \in f \setminus i} \tilde{c}_i |\alpha| D_i D_j = \tilde{c}_i |\alpha| D_i \sum_{j \in f \setminus i} D_j \tag{38}$$

$$\Longrightarrow 1 + \tilde{c}_i |\alpha| D_i > \tilde{c}_i |\alpha| D_i \cdot s_f. \tag{39}$$

Thus A satisfies lemma 2, so B is a monotone positive matrix.
Second, $\Gamma'(s_f) = \frac{\mathsf{d}}{\mathsf{d}s_f} \frac{-1}{\alpha(1-s_f)} = \frac{-1}{\alpha(1-s_f)^2} > 0$.
Thus as $B \cdot \Gamma'(s_f)$ is a series of multiplication and addition of positive numbers, so $\mathsf{D}_{s_f} r$ must be a monotone positive vector.

B Separate Developer and Landlord Decisions

The standard assumption in the urban literature is that a competitive construction sector purchases land to produce urban space that is then put on the rental market (or sold to initial owners). We have modeled the choice environment as landowners producing the urban space they provide to the rental market. In this section, we show that under the assumption of competitive construction and the existence of owners of differentiated land that our model leads to the same allocation. This implies that the standard assumptions imply that urban space is constrained. We show this in the horizontal sorting case.

Consider a developer who as *already* purchased land from a land-owner and must now decide how much urban space to provide to the rental market. The construction firms are price takers in factors and space, but can make a quantity choice. We consider the dual builder's problem of maximizing location conditional profit or minimizing costs subject to a level of demand by choosing labor and capital:

$$\max_{k,h}\{r \cdot q_j(k,h) - ik - wh\} \iff \min_{k,h}\{ik + wh \text{ s.t. } q_j(k,h) = d_j(r)\}$$

Given that these are dual problems, they each yield the same solution. Let's consider the cost minimization problem's solution of a building cost $B_j(r, d_j(r))$. With free entry, $\pi_j = r \cdot d_j(r) - B_j(r, d_j(r)) \geq 0$. This provides the builder's solution if the builder buys the right to develop location $j \in J$. The builder will develop a plan for each $j \in J$ and seeks to purchase land from land-owners.

Now, we must consider how land-owners set the price of land, r_j. Clearly, $r_j = \pi_j$, else another developer would bid up the price. This creates an open bid auction for each location, so the land price must also be bid up to the highest potential location profit, which is the monopoly location profit. Suppose a builder decides to set rent at cost and provide enough space to clear the market, then this builder must bid $\pi^{ce} = 0$. Another builder decides to reduce space and increase rent to clear the market, and so bids $\pi^m > 0$. The land-owner will choose the second bidder.

Here, free entry into the construction sector creates the incentives to engage in monopolistic behavior in the rental market when there is downward sloping demand. If urban space was viewed as homogeneous by renters, then developers would not be able to adjust market rents and space and make profits since all renters would have the same willingness to pay.

C BLP Inversion Step

For intuition, if we omit the random coefficients, then the model becomes a standard logit specification using grouped data. Berry (1994) shows that the mean utility can be solved for in closed form as:

$$\ln[s_j] - \ln[s_0] = \delta_j + X_j\beta + \alpha r_j. \tag{40}$$

One can use a linear 2SLS specification to estimate $\{\alpha, \beta\}$.

With random coefficients, the above does not work. However, BLP show that the following is a contraction mapping algorithm guaranteed to converge:

$$\mu_j^{r+1} = \mu_j^r + \left(\ln[s_j] - \ln[D_j(\mu_j^r;\theta)]\right) \ ,\forall j. \tag{41}$$

When $\|\mu_j^{r+1} - \mu_j^r\|_\infty \approx 0$ the algorithm has converged.[55] Once μ is recovered, then we can use the model's moment conditions to estimate $\{\beta, \alpha, \gamma\}$.

D Detailed Construction of Sample

We begin with all buildings in Manhattan according to the 2010 PLUTO file: 43,663. Next, we drop plots that are

1. missing location information, plots that are under construction, vacant, or are parks: 41,718
2. residential area is zero, there are zero residential units, or market values equal zero : 30,708
3. plots where the building is not classified as a private rental building (i.e., we drop owner occupied single family residences, condominium and cooperative buildings, 100% publicly owned buildings, any remaining commercially classified buildings, buildings designated as land-marks): 18,062
4. missing building characteristic information: 18,052

The sample of 18,052 buildings are used to create the "Differentiation Instruments" that are functions of the exogenous building characteristics of rivals outside of a 1km radius of a given building.

To arrive at the estimation sample, we drop buildings where

1. there is positive commercial building area: 9,396
2. the census tracts has fewer than 5 remaining buildings: 9,330

These 9,330 buildings constitute the estimation sample on which we estimate the model.

We drop buildings with commercial area – mixed use buildings – because we cannot be sure that we area measuring average residential rents as we cannot separate commercial and tenant income sources. As noted earlier, this is not the same as treating these

[55] We use a tolerance of 10^{-12}, and we always start the algorithm with the linear specification mean value.

buildings as outside goods for the model. Utility parameters are identified under the assumption that the parameters do not depend on whether the building has commercial space.[56]

We drop buildings in tracts with fewer than 5 buildings to ensure that tracts FEs are estimated with a somewhat reasonable number of buildings. A more practical issue is that the model can become numerically unstable when we include high dimensional FEs with few observations per group, so we arbitrarily include a threshold of 5 buildings per tract.

D.1 Summary Stats by Neighborhood Tabulation Area

We present the IV and Estimation samples separately since both are used. The IV sample is closer to the true market, but is not used to to mismeasurement of rents for mixed-use buildings. We present the results by Neighborhood Tabulation Areas, which is statistical area used by city planners, which is a collection of census tracts.[57]

In table 5 and 6, we present initial summary statistics by NTA for the buildings used in the IV Sample and the Estimation Sample. These tables feature total market share for the NTA, percent of rental buildings that are mixed use, percent that are zoning constrained, percent that are rent controlled or stabilized, the mean and median number of households per building, the average unit monthly rent, the building level land value per unit per month (to put on same scale as monthly rent), and the annual rent as a percentage of median income. A building is considered mixed-use if the building has positive commercial area. A building is considered zoning constrained if, under current regulations, the building would not be allowed to create an additional unit based on building floor-area-ratios and minimum unit area requirements. A building is considered rent controlled if it is on the 2012 Rental Controlled and Stablized list, as described earlier.

In table 7, we present the summary statistics by Neighborhood Tabulation Areas for building characteristics of the Estimation Sample. All of these variables (plus a constant

[56]Unreported monte carlo tests show that under the assumptions of the model, parameters remain unbiased. At worst, we believe the model is less efficiently estimated due to smaller samples.

[57]There are 27 NTAs in the sample and 245 census tracts; an NTA will have between 3 and 15 census tracts.

Table 5: Summary Stats:
IV Sample

NTA	Total Market Share	% Mixed Use	Vacancy Rate	% Zoning Constrained	% Rent Controlled	Mean HHs	Median HHs	Month Rent ($)	Annual Rent by Median Income
Battery Park City-Lower Manhattan	0.001	0.92	0.13	0.14	0.92	35	4.1	3696.48	0.89
Central Harlem North-Polo Grounds	0.013	0.28	0.15	0.48	0.68	16.4	8.7	999.21	0.24
Central Harlem South	0.007	0.24	0.14	0.47	0.72	12.4	8.2	1094.47	0.26
Chinatown	0.006	0.9	0.07	0.42	0.81	14.4	14.6	1451.29	0.35
Clinton	0.012	0.57	0.1	0.6	0.99	27.2	11.8	1907.21	0.46
East Harlem North	0.006	0.52	0.11	0.45	0.82	13.8	7.4	1199.81	0.29
East Harlem South	0.006	0.52	0.09	0.56	0.78	18.2	8.8	1311.1	0.31
East Village	0.01	0.63	0.06	0.41	0.82	16.1	13.1	1987.39	0.48
Gramercy	0.005	0.59	0.08	0.5	0.87	24.8	9.5	2277.49	0.55
Hamilton Heights	0.008	0.24	0.1	0.58	0.71	18	9.6	949.13	0.23
Hudson Yards-Chelsea-Flat Iron-Union Square	0.012	0.5	0.09	0.51	0.86	27.5	11	2227.84	0.53
Lenox Hill-Roosevelt Island	0.013	0.61	0.11	0.66	0.92	30.5	15.5	2189.02	0.53
Lincoln Square	0.007	0.31	0.11	0.35	0.94	24.7	9	2255.01	0.54
Lower East Side	0.006	0.58	0.07	0.16	0.79	20.5	11.7	1735.99	0.42
Manhattanville	0.002	0.27	0.07	0.84	0.71	22.4	18.9	990.24	0.24
Marble Hill-Inwood	0.008	0.26	0.05	0.79	0.59	40.6	37.2	993.24	0.24
Midtown-Midtown South	0.005	0.96	0.21	0.57	0.89	39.9	6.9	2816.86	0.68
Morningside Heights	0.006	0.28	0.07	0.55	0.8	27	17.2	1548.14	0.37
Murray Hill-Kips Bay	0.009	0.61	0.09	0.57	0.92	28.6	9.3	2196.98	0.53
SoHo-TriBeCa-Civic Center-Little Italy	0.008	0.85	0.09	0.35	0.87	21.1	8.7	2788.82	0.67
Turtle Bay-East Midtown	0.005	0.71	0.15	0.31	0.9	27.1	7.9	2484.78	0.6
Upper East Side-Carnegie Hill	0.006	0.61	0.17	0.53	0.84	14.6	7.4	3125.6	0.75
Upper West Side	0.018	0.22	0.08	0.46	0.87	19	9.3	1846.55	0.44
Washington Heights North	0.011	0.34	0.05	0.88	0.39	38.2	33	1032.19	0.25
Washington Heights South	0.013	0.35	0.06	0.85	0.57	29.9	26.4	1058.22	0.25
West Village	0.011	0.58	0.09	0.35	0.8	16.6	8.5	2428.29	0.58
Yorkville	0.015	0.46	0.09	0.78	0.95	25.4	14.5	2015.93	0.48
Total	0.227	0.48	0.1	0.52	0.8	21.7	10.9	1787.83	0.43

Building Data from PLUTO 2010 file, Rent Data based on FAR FY-2012 file. Household allocated based on Building Units and 2010 Decennial Census and American Community Survey.
Vacancy rate is one minus the total households in building divided by total building units. Median income in 2010 is $ 50,000.

Table 6: Summary Stats:
Estimation Sample

NTA	Total Market Share	Vacancy Rate	% Zoning Constrained	% Rent Controlled	Mean HHs	Median HHs	Month Rent ($)	Land Value per Unit per Month ($)	Annual Rent by Median Income
Battery Park City-Lower Manhattan	0	0.09	0.33	1	8.1	7.6	3070.3	2888.1	0.74
Central Harlem North-Polo Grounds	0.009	0.15	0.43	0.74	16.2	8.6	943.98	854.57	0.23
Central Harlem South	0.005	0.14	0.43	0.76	11.7	8.1	1028.54	912.38	0.25
Chinatown	0.001	0.08	0.35	0.85	15.9	15.5	1427.5	1279.58	0.34
Clinton	0.005	0.09	0.69	0.99	26.4	17.3	1630.49	826.97	0.39
East Harlem North	0.003	0.11	0.43	0.82	15	7.8	1126.2	984.12	0.27
East Harlem South	0.002	0.09	0.59	0.85	16.5	9.7	1226.68	532.58	0.29
East Village	0.004	0.07	0.47	0.87	17.2	15	1722.1	1436.47	0.41
Gramercy	0.002	0.08	0.49	0.91	18.9	11.3	1935.48	1455.95	0.46
Hamilton Heights	0.006	0.1	0.54	0.73	16.9	9.1	880.04	715.25	0.21
Hudson Yards-Chelsea-Flat Iron-Union Square	0.005	0.09	0.55	0.92	21.6	13.9	1725.75	2019.08	0.41
Lenox Hill-Roosevelt Island	0.004	0.1	0.7	0.94	25.2	17.9	1787.22	681.73	0.43
Lincoln Square	0.002	0.11	0.41	0.96	12.7	9	1876.48	1888.49	0.45
Lower East Side	0.002	0.06	0.21	0.81	21.4	14.3	1734.56	1115.78	0.42
Manhattanville	0.001	0.06	0.8	0.77	22.4	18.8	873.19	310.42	0.21
Marble Hill-Inwood	0.005	0.05	0.78	0.65	37.5	30.3	938.55	139.16	0.23
Midtown-Midtown South	0	0.17	0.62	0.88	21	8.6	1649.15	659.58	0.4
Morningside Heights	0.004	0.06	0.52	0.8	25.2	18.7	1473.33	322.43	0.35
Murray Hill-Kips Bay	0.002	0.09	0.63	0.92	19.6	15.2	1858.13	1983.71	0.45
SoHo-TriBeCa-Civic Center-Little Italy	0.001	0.07	0.4	0.91	14.4	9.5	2207.85	1792.42	0.53
Turtle Bay-East Midtown	0.001	0.17	0.31	0.94	16.1	7.9	2040.02	3039.19	0.49
Upper East Side-Carnegie Hill	0.002	0.17	0.54	0.82	11.4	8.2	2424.89	3252.75	0.58
Upper West Side	0.011	0.08	0.44	0.87	15	9.1	1697	2103.21	0.41
Washington Heights North	0.007	0.05	0.88	0.43	36.6	29.5	936.92	100.82	0.22
Washington Heights South	0.008	0.06	0.85	0.61	27.7	22.9	941.6	178.46	0.23
West Village	0.004	0.09	0.34	0.86	15.6	8.9	2065.7	3263.61	0.5
Yorkville	0.006	0.09	0.83	0.96	19.3	17.3	1782.79	641.78	0.43
Total	0.103	0.1	0.54	0.82	18.9	10.8	1453.26	1269.22	0.35

Building Data from PLUTO 2010 file, Rent Data based on FAR FY-2012 file. Household allocated based on Building Units and 2010 Decennial Census and American Community Survey.
Vacancy rate is one minus the total households in building divided by total building units. Land Value per Unit per Month = Land Value / (Units * 12). Median income in 2010 is $ 50,000.

and census tract FEs) have linear parameters, but only the first five have random coefficients.

Table 7: Summary Stats:
Estimation Sample

NTA	Age	$\ln[\text{Dist}_{\text{CBD}}]$	$\ln[\text{Dist}_{\text{Subw}}]$	Years Since Update	Avg Unit Area	% Elevator	# Retail & Theaters	# Parks	# Offices	# Residential
Battery Park City-Lower Manhattan	110	-0.94	-1.29	77.67	1176.13	0.33	0.33	0.33	0.33	11
Central Harlem North-Polo Grounds	93.65	2.05	-1.82	46.46	741.63	0.1	0.78	0.2	0.2	37.84
Central Harlem South	98.51	1.94	-2.02	49.36	778.57	0.11	0.5	0.02	0.12	37.58
Chinatown	89.38	-0.15	-1.88	57.31	769.12	0.24	2.93	0.38	0.32	21
Clinton	91.01	1.27	-1.61	41.02	651.42	0.16	1.69	0.12	0.44	32.91
East Harlem North	94.56	1.94	-1.52	41.97	794.64	0.16	1.94	0.09	0.49	28.42
East Harlem South	92.74	1.83	-2.12	41.3	765.21	0.13	2.34	0.17	0.11	20.35
East Village	99.77	0.4	-1.72	52.7	660.17	0.1	1.51	0.04	0.34	34.42
Gramercy	89.84	0.69	-1.78	51.03	722.95	0.27	0.82	0.06	0.72	23.96
Hamilton Heights	92.26	2.13	-2.09	48.16	763.92	0.2	0.6	0.03	0.12	32.29
Hudson Yards-Chelsea-Flat Iron-Union Square	96.8	0.8	-2.22	52.11	638.46	0.15	1.36	0.15	0.63	36.86
Lenox Hill-Roosevelt Island	90.13	1.54	-1.17	47.61	617.18	0.23	0.56	0.01	0.34	24.65
Lincoln Square	100.5	1.53	-2.16	53.3	713.74	0.14	0.58	0.14	0.47	39.1
Lower East Side	84.85	0.31	-1.3	42.29	760.03	0.21	1.06	0.24	0.24	24.72
Manhattanville	98.17	2.04	-2.46	33.94	779.97	0.18	0.24	0	0.09	19.96
Marble Hill-Inwood	80.98	2.45	-2.17	36.92	796.15	0.2	1.21	0.07	0.2	13.46
Midtown-Midtown South	76.12	0.93	-2.4	58.38	831.85	0.38	2	0	3.5	16
Morningside Heights	94.61	1.9	-2.07	52.95	904.16	0.38	0.17	0.16	0.26	19.06
Murray Hill-Kips Bay	90.38	0.95	-1.55	51.58	666.56	0.27	0.85	0.03	1.39	25.61
SoHo-TriBeCa-Civic Center-Little Italy	96.78	-0.16	-2.46	60.35	914.04	0.17	1.42	0.16	0.53	18.57
Turtle Bay-East Midtown	88.15	1.28	-1.58	51.42	765.64	0.25	1.56	0.1	1.46	26.71
Upper East Side-Carnegie Hill	92.16	1.59	-1.95	58.75	875.78	0.3	1.29	0	0.96	21.04
Upper West Side	100.98	1.7	-2.07	50.47	773.07	0.16	0.27	0.02	0.09	36.32
Washington Heights North	87.26	2.36	-2.16	47.29	809.39	0.28	1.07	0.05	0.05	9.99
Washington Heights South	94.51	2.25	-2.13	48.89	846.85	0.26	0.51	0.03	0.08	15.34
West Village	101.26	0.4	-2.14	59.96	716.52	0.12	1.29	0.03	0.29	28.25
Yorkville	89.01	1.67	-1.04	42.74	622.18	0.16	0.55	0	0.23	32.02
Total	94.59	1.58	-1.89	48.77	748.05	0.18	0.89	0.08	0.3	29.65

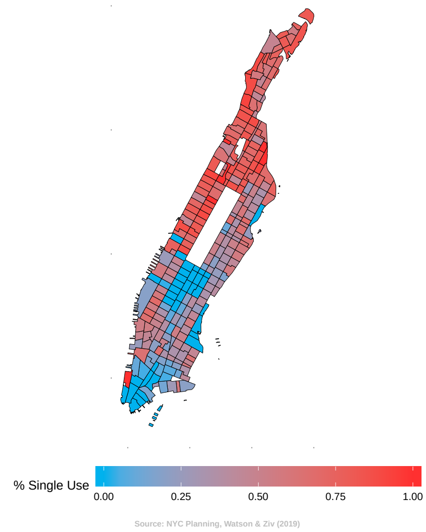
Building Data from PLUTO 2010 file, Rent Data based on FAR FY-2012 file. Household allocated based on Building Units and 2010 Decennial Census and American Community Survey.
Counts (#) are number of buildings in a census block classified as a category. Distances are in log geodesic miles.

D.2 Spatial Distribution of Single Use, Zoning Constrained, & Rent Control

In figures 4, 5, and 6, we plot the spatial distribution of building use status, zoning constrained status, and rent control status. We define a building as being mixed use if we observe positive commercial space in the building; else, single use. Commercial space includes retail space, office space, or (for a minority of buildings) industrial space. For mixed use buildings, we cannot differentiate commercial versus residential sources of building income.

For figure 5, a building is considered zoning constrained if the landlord could not legally add another unit at the minimum legally allowed area without affecting existing building units. Within our data we able to observe that whether a building's Floor Area

Figure 4: Distribution of Building Use in Manhattan



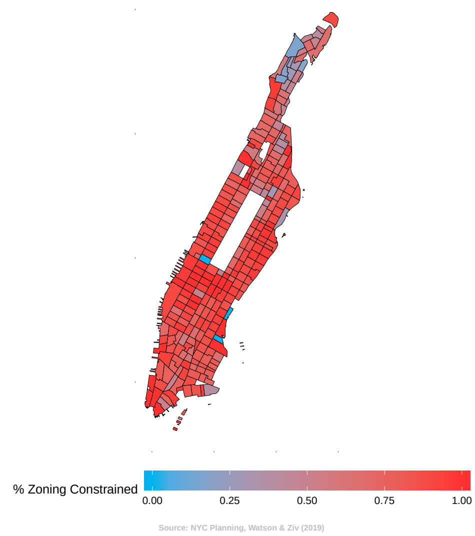
Mixed-use based on whether there is positive commercial building space.

Ration (FAR_j) is below its maximum allowable FAR (MaxFAR_j). A building can be below its MaxFAR but still zoning constrained if $(\text{MaxFAR}_j) - \text{FAR}_j)$ is less than the minimum allowable unit FAR, meaning a landlord cannot legally add an additional unit. Thus, a building is zoning constrained if (1) $(\text{MaxFAR}_j) - \text{FAR}_j) \leq 0$ or (2) $(\text{MaxFAR}_j) - \text{FAR}_j) \leq$ (Legal Min Unit FAR). We find that while 80% of rental buildings are zoning constrained only 30% are constrained due to (1).[58] This potentially implies that developers incorporate zoning constraints, which if binding would limit revenues, by building larger units that may attract higher income renters.

Finally, in figure 6, we plot the spatial distribution of rent controlled buildings. We define rent controlled status by whether a building is on the 2012 NYC Department of Homes and Community Building Registration File. A building is on this list if the building has at least one unit that is rent controlled or rent stabilized. Being rent controlled implies that a landlord is not in complete control of unit pricing, so to some extent the landlord is constrained.

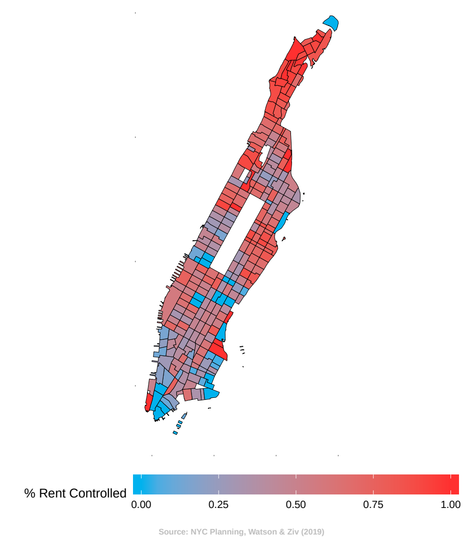
[58] For single-use buildings this is 81.7% and 34.2% and for mixed-use buildings this is 79.2% and 25.8%, respectively.

Figure 5: Distribution of Zoning Constraints in Manhattan



Zoning-constrained is defined by the inability to legally add an addition unit without affecting existing units.

Figure 6: Distribution of Rent Control in Manhattan



Rent control status defined by presence on 2012 NYC Department of Homes and Community Building Registration File.

E Detailed Construction of Average Building Rent

Steps

1. Merge FY2012 FAR to 2010 PLUTO
   - This puts Market Value, Bldg Square Feet, and Building Class in same dataset
2. Calculate MV / SqFt = m
3. Use the NYC DOF FY2012 Assessment Roll Guidelines to learn the Income to Market Value Mapping
   - This document is available online at the DOF website
   - This document describes the mapping of building income – by building class, location, and number of units – to building market value
4. Invert the mapping to recover building incomes.

This data is ultimately sourced from the Real Property Income and Expense (RPIE) statements that all income generating property owners are required to file annually and face financial penalties for not filing. Nevertheless, not all property owners will file this report. If an owner does not file, the DOF has the right to assign a market value based on its best judgement. In addition, the DOF documentation says that they will adjust report amounts that seem extreme; e.g., a building reporting high costs and no income in an area where other buildings are report incomes above costs. Without access to the RPIE statements, it is not possible to determine which properties have been adjusted.

E.1 The Mapping

As it turns out, the mapping uses a non-linear multiplier, the 'gross income multiplier' (GIM_j), which is a function of $(Y_j/SqFt_j) = y_j$. Specifically, the mapping is:

$$m_j = GIM_j \cdot y_j$$

The building level data does not contain GIM_j. However, the DOF Assessment Guidelines show that this value can be directly inferred using m_j. In the table below, we describe the DOF mapping that goes from observed income to market value: $G\colon Y \times SqFt \to M$.

Table 8: Example Mapping of Market Value to Income

y	GIM_{Low}	GIM_{High}	m	Y_j
$[y_1, y_2]$	$\frac{m_1}{y_1}$	$\frac{m_2}{y_2}$	$[m_1, m_2]$	$= MV_j \cdot \frac{y_2}{m_2}$
$[y_2, y_3]$	-	$\frac{m_3}{y_3}$	$[m_2, m_3]$	$= MV_j \cdot \frac{y_3}{m_3}$
$[y_3, y_4]$	-	$\frac{m_4}{y_4}$	$[m_3, m_4]$	$= MV_j \cdot \frac{y_4}{m_4}$

Known:

- MV_j – Observed Market Value
- m_j – Observed Market Value by SqFt
- $\tilde{G} \colon M \times SqFt \to Y$ – Inverse GIM Mapping from (MV, SqFt) to Income

Unknown: Y – Unobserved Building Income

Thus, we calculate building income as

$$\implies Y_j = SqFt_j \cdot (m_j/GIM_j)$$

E.1.1 Robustness of Calculations

We have two ways of checking the robustness of our calculations. First, the NYC DOF publishes the Notice of Property Value, which includes the income, market value, and GIM. Second, the NYC DOF publishes some rental building incomes and market values in an ancillary dataset.

The Notice of Property Value are accessible using an online tool for property owners to view forms they are sent regarding their property taxes. We used this tool to ensure that our assignments are correct. We found that our income values generally matched the reported values with the exception of some new buildings (which did not have reliable income information). At this time, we are not able scan all of these documents to recover building incomes, which would be the most thorough way of recovering this information.

Alternatively, we observe building incomes from an ancillary dataset, the Condo/Coop Comparable Rental Income for Manhattan. By law, condominium buildings must be valued for tax purposes as rental buildings. To accomplish this, the DOF matches condominiums with rental properties and calculates and expected, market value and income of the condominiums. They publish these comparisons and include the rental building income and market value used in the comparisons. Thus, we are able to check our results for the matched buildings. Our values are nearly identical except for inconsistent rounding behavior on the part of the NYC DOF, typically in the owner's favor.[59]

E.2 Average Building Rents

Once we have building incomes, we want to know the average building rent; however, building level income may contain non-residential income if the building is mixed use.[60] Property owners with income generating properties must file an annual income and expense report with the sources of income specified. With publicly available data, there is no way to distinguish the different sources of income.

We deal with this by dropping buildings with positive commercial area from the estimation. We do not drop these buildings from calculating market share or the instruments which use exogenous rival building information. Alternatively, one could try to estimate a commercial space premium over residential income per square foot, assuming a competitive price at some geographic level, and then scale the building income by the appropriate ratio. However, we do not know of any available source at any reasonable geography.

Once we drop the buildings with commercial space, we then calculate average building rent as

$$\mathsf{Avg}(r)_j = Y_j/\text{units}_j.$$

This value represents the average annual residential income for an existing unit. If we divide this by twelve, then this is the average monthly rent for a unit in a fully residential building.

[59]We are able to check against 1,883 rental buildings, and we find 83 buildings where the absolute difference between our assigned GIM and the empirical ratio of market value to income is greater than 0.1; this represents an error rate around 4% of buildings. Again, these errors are due to inconsistent behavior by the NYC DOF.

[60]Another source is if the building is able to lease any roof or wall space for advertisements.

F Instrument Construction

We primarily use "Quadratic Differentiation Instruments," based on Gandhi and Houde (2018), with a spatial radius, as in Bayer et al. (2004, 2007). These instruments are meant to be *an* approximation to the optimal instruments in the sense of Amemiya (1977) and Chamberlain (1987).[61]

The 'true' optimal instruments are based on the partial derivative of the structural error term:

$$Z^{\mathsf{opt}} = \mathsf{Var}(\delta_j)^{-1} \cdot \mathsf{E}\left[\frac{\partial \delta_j}{\partial \beta} \quad \frac{\partial \delta_j}{\partial \alpha} \quad \frac{\partial \delta_j}{\partial \sigma} \middle| Z \right]. \tag{42}$$

This has exactly as many moments as parameters, so is exactly identified and no iterative weighting matrix is necessary.

To calculate this object, one must take a stand on the conditional distribution of the structural error, solve the Bertrand pricing problem, back out model-implied structural errors, and then calculate the derivatives. In a major methodological advancement, Conlon and Gortmaker (2019) describe how, given an initial set of estimates, one can calculate this object relatively quickly for most problems. Their `pyblp` software automates most of these steps with various options; however, this is not possible in our problem. Because we do not accurately observe prices for mixed-use buildings, which is roughly half of the choice set, we cannot credibly solve the Bertrand pricing problem.[62] Even conditional on obtaining the true parameter vector, our implied substitution between buildings will be biased up or down based on whether commercial rents are greater or less than residential rents in those buildings, which will bias the calculated 'optimal instrument.'

Nevertheless, Gandhi and Houde (2018) show that the optimal instruments can be approximated, in any dataset, by symmetric functions of the differences in building level covariates without needing to solve the Bertrand pricing problem. Their results formalize the intuition of the more traditional "BLP Instruments" that mark-ups are shifted

[61] Somewhat more formally they are a finite-order basis-function approximation to the optimal instruments.

[62] In addition, with rent control and zoning constraints, we would need to solve a constrained Bertrand pricing problem, which is not coded in `pyblp`.

by utilizing the 'product-space-distance' between products, where more isolated products as more immune to price shocks. However, there are still many choices of potential finite basis functions that can be used.

The authors suggest two 'flavors' for practitioners. First, they propose "Quadratic Differentiation Instruments":

$$Z_{hj}^{\mathrm{DQ}} = \sum_{k \in \{K(j)\}} (x_{hk} - x_{hj})^2, \tag{43}$$

where $K(j)$ is a set of rivals for plot j.

Second, they propose "Local Differentiation Instruments":

$$Z_{hj}^{\mathrm{DL}} = \sum_{k \in \{K(j)\}} 1\left[\, |x_{hk} - x_{hj}| < \mathsf{sd}(X_h)\,\right], \tag{44}$$

where $\mathsf{sd}(X_h)$ is the empirical standard deviation of variable X_h. However, their results indicate that there are infinite possible approximations to the optimal instruments.

To deal with endogeneity of prices (or any covariate), the authors recommend using a predicted price using plausibly exogenous variation, such as the following additional example:

$$Z_{r,j}^{\mathrm{DQ}} = \sum_{k \in \{K(j)\}} \left(\mathsf{E}[r_k \mid X, W] - \mathsf{E}[r_j \mid X, W]\right)^2, \tag{45}$$

where $\mathsf{E}[r_k \mid X = x_k, W = w_k]$ is from a first stage regression on all exogenous information, (X, W), where W are any variables excluded from the utility function.[63]

G Additional Estimation Details

To aid our estimation, we follow most modern practices in estimating demand parameters. Many of these are based on advice found in Nevo (2000) (*N*), Knittel and Metaxoglou (2014) (*KM*), and Conlon and Gortmaker (2019) (*CG*).

[63]Note, Gandhi and Houde (2018) specify W as any already available instrument, which Conlon and Gortmaker (2019) interpret to include $\{Z_{hj}^{\mathrm{DQ}}\}_{h \in H}$ for the building $X's$. Currently, we do not use $\{Z_{hj}^{\mathrm{DQ}}\}_{h \in H}$ as part of W, so that X are building characteristics in the utility function and W is land value from the NYC DOF.

`First`, we scale all $Z = (X, Z^{(x)}, Z^{(r)})$ variables by their empirical standard deviations to put their variances on the same order of magnitude. As in Brunner et al. (2017), we find this alleviates most model convergence issues.

`Second`, we use an 'overflow safe' method of calculating market shares which gives some protection when a solver inadvertently uses a parameter vector that is far from the true vector, as described in section 3.4 of *CG*.

`Third`, for the inversion step we always use the Berry (1994) logit inversion as the starting value, we use an accelerated fixed point algorithm, called `SQUAREM`, as described in section 3.2 of *CG*, and we use a fixed tolerance of $\|\mu^{s+1} - \mu^s\|_\infty < 10^{-12}$. *KM* show a loose or variable tolerance can cause catastrophic error propagation from the inversion step to the GMM estimates to the gradient, which can veer the optimization algorithm far off course.

`Fourth`, we use supply the analytical gradients of the GMM objective function using a gradient based solver, as described in *N* and benchmarked by *KM* and *CG*. This not only speeds up computation relative to gradient-free or approximated gradients but is also more reliable.

`Finally`, for technical and theoretical reasons we do *not* include a supply side for the model in estimation. Our main theoretical reasons are that we do not know enough about the marginal cost function for rental buildings nor do we wish to fully model the zoning and rent control constraints for a landlord. Brushing theoretical concerns aside, the analytical derivative of the supply moments effectively requires storing a $J \times J \times J$ three-dimensional matrix (where $J = 9,330$) in computer memory, which is not feasible using even for many super computers. We believe the primary empirical benefit of a supply moment would be to increase precision and ensure elastic demand. However, as the majority of our results do not suffer from either problem – see appendix H – we do not think the supply side is necessary for the model's estimation.

H Additional Estimation Results

We estimate demand parameters using a logit model, which assumes no heterogeneity in utility parameters, and a random coefficient logit model, which does allow for heterogeneity. For the former, we present the OLS, the two-stage least squares, the 2-Step Efficient GMM, and the Iterative Efficient GMM results; for the latter, we only present the 2-Step and Iterative Efficient Simulated GMM. The iterative GMM methods update the optimal weighting matrix until the parameter estimates converge.

In 10 we use "Quadratic Differentiation Instruments" ('DQ') and in table 9 we add an additional differentiation instrument based on predicted rents ('DQ+P'), both sets based on Gandhi and Houde (2018). For details on the instrument construction, see appendix F.

We present results that omit the 'Predicted Rent' DQ instrument for two reasons. First, this instrument depends on a first stage regression which is subject to our arbitrary linear functional form. Second, we use the entire IV sample to estimate predicted rents, including mixed use buildings, which we are unsure of their rents.[64] We find that omitting the 'Predicted Rent' DQ instrument leads to a slightly smaller estimate of α, but still in the same range as when we include the instrument. We interpret this as losing some identifying power for substitution patterns, so we use the 'DQ+P' as our main results.[65]

Of our estimated parameters, we only present our estimate of α, with its heteroskedasticy robust standard error.[66] For the linear models, this is the coefficient on average building rent divided by median income for Manhattan, $r_j/\,\mathsf{Med}(y_i)$. For the non-linear models, this is the coefficient on average building rent divided by the income of the simulated market participants, r_j/y_i. We find that the OLS estimate is severely positively biased due to price endogeneity with unobserved building amenities. We find that the parameter becomes larger in magnitude after we employ instrumental variable methods, larger as we iterate the optimal GMM weighting matrix, and larger again when we incorporate more preference heterogeneity using the non-linear models.[67] Our IV esti-

[64] While we cannot separately identify the residential rents, reinterpreting the regression as predicting building income per unit leads to no conceptual issue.

[65] In practice, using the 'DQ+P' rather than the 'DQ' results gives us more conservative estimates of the monopoly forces.

[66] This parameter can be roughly interpreted as the marginal utility of non-housing consumption.

[67] Our MATLAB implementation of the non-linear IGMM 'QD+P' model did not converge using our criterion of $\|\theta^{s+1} - \theta^{s}\|_2 < 0.01$; rather, parameters oscillated repeatedly between two sets of estimates with a norm difference of 0.068. Given that our `pyblp` results in table 11 largely agree with both parameters sets, we present the estimates with the slightly smaller GMM objective function value.

mates are all statistically significant.

We next present the own price elasticity, ε_j, calculated using the analytic formula based on the logit shares. Similar to our estimates of α, the elasticities become more elastic across models. For the OLS and 2SLS estimates, the vast majority of the elasticities are actually *inelastic*, which is incompatible with monopoly pricing. However, the linear and non-linear GMM estimates have the majority of elasticities complying with monopoly pricing. Only for the Iterated Simulated GMM model are all elasticities actually elastic. We also find that the building that are neither rent constrained nor zoning constrained have more elastic demand.

Overall, we find quite large mark-ups. While there is little to benchmark these estimates from the literature, the calculated mark-ups for the linear models seem implausibly large. Only the non-linear IGMM mark-ups seem to be approaching a sensible value at between $32-39\%$ for the 'DQ' and between $26-30\%$ for the 'DQ+P' models.

Finally, we present the aggregate elasticity, ε^{Agg}. We find the aggregate elasticity is always *inelastic*, with little variation across most IV models. Most estimated housing demand elasticities are inelastic, which is inconsistent with monopoly pricing, but this is reconciled by the fact that most papers appear to estimate an aggregate elasticity rather than a the elasticity that actually governs the landlord's pricing problem, where the landlord is more concerned with competition between buildings rather than the market as a whole.

In addition to our main results, table 11 shows additional results from alternative differentiation-instruments varieties as well as alternative software. In the first column pair for each are our main specification using "Quadratic + Predicted Rent", the second pair use "Local", and the third pair use "Local + Predicted Rent". We also compare our results to alternative software by using the `pyblp`, coded in Python, from Conlon and Gortmaker (2019). We use the same simulation draws, tolerances, and variables as our MATLAB implementation.

We estimate nearly identical estimates between our MATLAB and `pyblp` implementation for our "DQ" without predicted price instruments, so we omit the comparison columns to save space. We find that there are some noticeable differences between im-

Table 9: Results for 'DQ+P' IV

	Logit				RC Logit	
	OLS	2SLS	GMM-2step	IGMM	GMM-2step	IGMM
α	-0.063	-1.709	-3.034	-7.281	-38.169	-77.220
SE	(0.010)	(0.164)	(0.114)	(0.487)	(23.799)	(18.104)
$\mathsf{Med}(\varepsilon_j)$	-0.135	-0.510	-0.906	-2.173	-2.534	-4.304
$\mathsf{Med}(\varepsilon_j \mid \text{Unconstrained})$	-0.209	-0.791	-1.403	-3.367	-2.834	-4.827
$\mathsf{Med}(\mathsf{Lerner}_j)$	–	84%	71%	44%	39%	23%
$\mathsf{Med}(\mathsf{Lerner}_j \mid \text{Unconstrained})$	–	74%	57%	29%	35%	21%
$\%\{\lvert\varepsilon_j\rvert > 1\}$	0%	11%	42%	93%	99%	100%
ε^{Agg}	-0.012	-0.044	-0.077	-0.184	-0.135	-0.146
GMM Obj	–	428.307	140.957	114.440	390.948	34.810

$N = 9,330$ for all models; standard errors robust to heteroskedasticity; census tract fixed effects and building characteristics included in all models
All models use "Quadratic Differentiation Instruments + Predicted Price" based on Gandhi and Houde (2018)
ε_j is own price elasticity and ε^{Agg} is aggregate price elasticity, based on Berry and Jia (2010)
Buildings are 'unconstrained' if *not* rent-stabilized and *not* zoning-constrained; $N_{\text{Uncons.}} = 553$

plementations for the "Quad + Predicted-Rent" and especially the "Local" results, but almost no difference between "Local + Predicted-Rent". As noted, the MATLAB implementation of the 'DQ+P' model fails to neatly converge but the `pyblp` implementation successfully converges rather quickly nearly identical results. The "Local" differs between the implementations for both the 2-Step and the IGMM and the elasticities are noticeably smaller (implying implausibly large mark-ups). The "Local+PR" are quite similar between implementations and estimation procedures, and the IGMM is only marginally more elastic than the traditional 2-Step estimates.

Finally, in unreported estimates utilizing "BLP" instruments, we find that both implementations fail to converge and the 2-Step results lead to mostly inelastic demands. We interpret this as the BLP instruments not having 'enough' predictive power to identify the model.

In table 12, we show summary statistics and partial results for the subsample of unconstrained buildings that are less than ten years old.

Table 10: Results for 'DQ' IV

	Logit				RC Logit	
	OLS	2SLS	GMM-2step	IGMM	GMM-2step	IGMM
α	-0.063	-2.49	-5.165	-6.604	-38.169	-63.371
SE	(0.010)	(0.464)	(0.207)	(0.460)	(21.293)	(14.306)
$\mathsf{Med}(\varepsilon_j)$	-0.135	-0.743	-1.536	-1.967	-2.473	-3.584
$\mathsf{Med}(\varepsilon_j \mid \text{Unconstrained})$	-0.209	-1.151	-2.380	-3.047	-2.863	-4.116
$\mathsf{Med}(\mathsf{Lerner}_j)$	–	74%	58%	48%	40%	28%
$\mathsf{Med}(\mathsf{Lerner}_j \mid \text{Unconstrained})$	–	63%	40%	32%	35%	24%
$\%\{\lvert\varepsilon_j\rvert > 1\}$	0%	28%	82%	91%	99%	100%
ε^{Agg}	-0.012	-0.064	-0.131	-0.167	-0.143	-0.151
GMM Obj	–	222.696	140.344	50.413	326.902	41.482

$N = 9,330$ for all models; standard errors robust to heteroskedasticity; census tract fixed effects and building characteristics included in all models
All models use "Quadratic Differentiation Instruments" based on Gandhi and Houde (2018)
ε_j is own price elasticity and ε^{Agg} is aggregate price elasticity, based on Berry and Jia (2010)
Buildings are 'unconstrained' if *not* rent-stabilized and *not* zoning-constrained; $N_{\text{Uncons.}} = 553$

Table 11: Alternative Instruments and Solvers for RC Logit Model

	Matlab						PyBLP					
	Quad + Predicted-Rent		Local		Local + Predicted-Rent		Quad + Predicted-Rent		Local		Local + Predicted-Rent	
	GMM-2step	IGMM	GMM-2step	IGMM	GMM-2step	IGMM	GMM-2step	IGMM	GMM-2step	IGMM	GMM-2step	IGMM
α	38.169	-77.220	-5.731	-8.081	-32.634	-33.006	-36.22	-77.295	-3.301	-10.555	-32.949	-33.307
SE	(23.799)	(18.104)	(6.687)	(6.554)	(7.933)	(10.581)	(19.4784)	(10.239)	(27.300)	(17.100)	(7.970)	(10.800)
$\mathsf{Med}(\varepsilon_j)$	-2.534	-4.304	-1.1248	-1.3824	-2.606	-2.6196	-2.608	-4.297	-0.76	-1.567	-2.614	-2.626
$\mathsf{Med}(\varepsilon_j \mid \text{Unconstrained})$	-2.834	-4.827	-1.3339	-1.5781	-2.8407	-2.8561	-2.916	-4.835	-0.967	-1.762	-2.851	-2.866
$\mathsf{Med}(\mathsf{Lerner}_j)$	39%	23%	84%	72%	38%	38%	38%	23%	91%	64%	38%	38%
$\mathsf{Med}(\mathsf{Lerner}_j \mid \text{Unconstrained})$	34%	21%	75%	63%	35%	35%	34%	21%	87%	57%	35%	35%
$\%\{\lvert\varepsilon_j\rvert > 1\}$	99%	100%	76%	94%	100%	100%	100%	100%	13%	98%	100%	100%
ε^{Agg}	-0.135	-0.146	-0.058	-0.069	-0.13	-0.13	-0.137	-0.147	-0.046	-0.079	-0.13	-0.13

$N = 9,330$ for all models; standard errors robust to heteroskedasticity; census tract fixed effects and building characteristics included in all models
Instruments are based on Gandhi and Houde (2018) , "Predicted Rent" incorporates an additional differentiation-instrument using $\mathsf{E}[r_j \mid x_j, z_j^{(r)}]$
PyBLP results use `pyblp` software developed by Conlon and Gortmaker (2019) , but use same draws and instruments as Matlab results
ε_j is own price elasticity and ε^{Agg} is aggregate price elasticity, based on Berry and Jia (2010)
Buildings are 'unconstrained' if *not* rent-stabilized and *not* zoning-constrained; $N_{\text{Uncons.}} = 553$

Table 12: Summary Stats:
Manhattan Rental Buildings

	New Unconstrained Sample
Total Market Share	0.1%
Res.Units per Building	39.2
Households per Building	33.4
Vacancy Rate	11%
Percent Mixed-Use	0%
Percent Rent Controlled	0%
Percent Zoning Constrained	0%
Median Monthly Rent*	$2,362.97
Median Rent by Median Income*	57%
Median Monthly Land Value per Unit	$180.65
Years Since Construction	4.72
Years Since Update	4.72
log(Distance CBD)	1.29
log(Distance Subway)	-1.78
Avg Unit Sqft	1541.53
$\text{Med}(\varepsilon_j)$	-4.91
$\text{Med}(\text{Lerner}_j)$	20%
Buildings	50

Building data from PLUTO 2010 file, rent data based on FAR FY2012 file. Households allocated based on building units and 2010 Decennial Census and American Community Survey. Vacancy rate is one minus the total households in building divided by total building units. Median income in 2010 for NYC is $ 50,000. A building is mixed-use if the building has positive commercial area. A building is considered rent controlled if it is on the 2012 Rental Controlled and Stablized list. A building is zoning constrained if the building would not be allowed to create an additional unit based on building floor-area-ratios and minimum unit area requirements. Geodesic distances are in miles based on building (lat,lon) coordinates. 'Updates' are based on building renovation projects since original construction. Monthly land value per unit is [Land Value / (12 x Residential Units)].
(*) – Rent data is only available for single use buildings

I Total Derivative of Monopoly Pricing Rule

The monopoly pricing rule is

$$r_j^\star = mc_j - \frac{D(r_j^\star; \{r_k^\star\}_k)}{m(r_j^\star; \{r_k^\star\}_k))}, \tag{46}$$

where $m_j = \frac{\partial D_j}{\partial r_j}$.

Totally differentiating this function, we get

$$\begin{aligned} \mathsf{d}r_j &= \mathsf{d}mc_j - \left[\frac{\frac{\partial D_j}{\partial r_j}\mathsf{d}r_j + \sum_{k\in\mathcal{Z}} \frac{\partial D_j}{\partial r_k}\mathsf{d}r_k}{m_j} - \frac{\frac{\partial m_j}{\partial r_j}\mathsf{d}r_j + \sum_{k\in\mathcal{Z}} \frac{\partial m_j}{\partial r_k}\mathsf{d}r_k}{(m_j)^2} D_j \right] && (47) \\ &= \mathsf{d}mc_j - \left(1 - \frac{\frac{\partial m_j}{\partial r_j}D_j}{(m_j)^2}\right)\mathsf{d}r_j - \sum_{k\in\{\mathcal{Z}\}} \left\{ \left(\frac{\frac{\partial D_j}{\partial r_k}}{m_j} - \frac{\frac{\partial m_j}{\partial r_k}D_j}{(m_j)^2} \right) \mathsf{d}r_k \right\} && (48) \end{aligned}$$

where

$$\begin{aligned} \frac{\frac{\partial m_j}{\partial r_j}}{(m_j)^2} &= \frac{\int_i \alpha_i^2 D_{ij}(1-D_{ij})(1-2D_{ij})\mathsf{d}F(i)}{\left(\int_i \alpha_i^2 D_{ij}(1-D_{ij})\mathsf{d}F(i)\right)^2} && (49) \\ \frac{\frac{\partial D_j}{\partial r_k}}{m_j} &= \frac{-\int_i \alpha_i D_{ij} D_{ik}\mathsf{d}F(i)}{\int_i \alpha_i^2 D_{ij}(1-D_{ij})\mathsf{d}F(i)} && (50) \\ \frac{\frac{\partial m_j}{\partial r_k}}{(m_j)^2} &= \frac{-\int_i \alpha_i^2 D_{ij} D_{ik}(1-2D_{ij})\mathsf{d}F(i)}{\left(\int_i \alpha_i^2 D_{ij}(1-D_{ij})\mathsf{d}F(i)\right)^2} && (51) \end{aligned}$$

Solving equation 48 for $\mathsf{d}r_j^{\mathsf{cf}}$ when $\mathsf{d}mc_j = 0$ yields equation 18.

With preference heterogeneity – i.e., random coefficients – then the expression has no closed form solution, but is easily calculated with our estimated parameters and Monte Carlo integration. For intuition, if there were no individual agent heterogeneity in preferences, then

$$\begin{aligned} \mathsf{d}r_j &= (1-D_j)\mathsf{d}mc_j + \frac{D_j}{(1-D_j)} \sum_{k\in\mathcal{Z}} \{D_k \mathsf{d}r_k\} && (52) \\ &= (1-D_j)\mathsf{d}mc_j + \frac{D_j}{(1-D_j)} \mathsf{Avg}_D(\mathsf{d}r_k). && (53) \end{aligned}$$

Without a full model of building costs, we cannot calculate $\mathrm{d}mc_j$, so we cannot calculate the true partial equilibrium change in unconstrained prices. Under the assumption of (locally) constant marginal costs, then our measure *equals* the partial equilibrium change in rental prices. Under the assumption of strictly increasing marginal costs, then $\mathrm{d}mc_j < 0$, so our measure would be the lower bound of the *magnitude* of the rent change. Without additional assumptions, our measure calculates the partial equilibrium change in the monopoly mark-up of unconstrained buildings due to a zoning-shock.