Comment: Estimating Both Supply and Demand Elasticities Using Variation in a Single Tax Rate

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Abstract

Zoutman, Gavrilova, & Hopland (Econometrica, 2018) show that by knowing on 'which side of the market' an 'exogenous' tax is levied one can use a single tax instrument to estimate both a supply and a demand elasticity. This seemingly goes against the intuition that one needs two instruments for two parameters; i.e., a 'supply' and a 'demand' instrument.

I show that the result is only true with partial equilibrium assumptions. Without further assumptions, tax reform induced general equilibrium price spillover effects imply that the tax rates are correlated with the unobserved structural errors. Thus, tax rates on their own are invalid instruments for at least one of the parameters. However, I show that if one can calculate a measure of spillovers, then one can still estimate the two elasticities using one tax reform, but with the spillover measure as an additional instrument.

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1 Introduction

Zoutman, Gavrilova, & Hopland (2018) [ZGH] show that knowing 'which side of the market' a tax is levied allows one to identify both a supply and a demand elasticity using a single "exogenous" tax instrument, and advocate estimating consumption, housing, and labor elasticities as applications of the method. Citing ZGH, Bibler, Teltser and Tremblay (2018) estimate Airbnb demand and tax evasion and Fajgelbaum, Goldberg, Kennedy and Khandelwal (2020) estimate tariff rate pass through from the 2018 US 'trade war.'

In this comment, I suggest that researchers consider whether the tax variation they use has the potential to create 'spillover effects' that cause the single tax rate 2SLS estimator to be inconsistent. Specifically, I describe how the assumption of "exogenous tax variation" implies partial equilibrium tax incidence, how heterogeneous spillovers cause tax rates to be invalid instruments, and how one can use spillovers, if quantifiable, as identifying variation.[1] Thus, I show one can use a single tax *reform* to identify two elasticities, but not necessarily a single tax rate.

Consider a tax reform that directly affects a treated market but not other untreated markets. The partial equilibrium [PE] treatment effect only considers the quantity and price responses in the treated market, holding other endogenous variables fixed; whereas, the general equilibrium [GE] allows all market quantities and prices to adjust. I denote 'spillovers' as the additional adjustment to quantities and prices beyond the PE effects. I assert that that these spillover effects will be correlated with the tax treatment but heterogeneous across markets, which can be microfounded as in Agrawal and Hoyt (2019) and Watson (2020) for the goods and factor market, respectively.

GE spillovers induce correlation of the tax treatment with unobservable demand or supply shocks causing the instrumental variable approach of ZGH to be inconsistent for the elasticity on the side with the spillovers. The key point is that in a Simultaneous Equation Model [SEM], the first stage error term is composed of the structural equations' errors, so exogeneity of tax treatment in the structural equation implies selection on observables in the first stage. Exogeneity implies spillovers must be uncorrelated with treatment which contradicts the assertion. Thus the tax rates are now invalid instruments. However, if the researcher can form a measure of spillover effects, then this can provide a second instrument to identify two parameters.

[1] ZGH discuss salience, tax avoidance, and lack of pass-through as empirical challenges but not spillover based confounding given their implicit partial equilibrium assumption.

2 Background: ZGH 2018

ZGH use a general price and quantity relationship, but I specify a labor market setting to ground ideas.[2] Begin with the following simultaneous equations model:

$$l_{it}^D = \alpha_0 + \alpha_1 w_{it} + u_{it}^D \qquad l_{it}^S = \beta_0 + \beta_1 w_{it} + \beta_1 \tau_{it} + u_{it}^S \qquad l_{it}^S = l_{it}^D, \tag{1}$$

where all variables are in logs and $\ln[(1+\tau)] \approx \tau$. Labor demand depends on gross wages while supply depends on net wages, which satisfies what ZGH call the 'Ramsey Exclusion Restriction.'

Proposition 1. *ZGH (2018)*
If τ is exogenous with the above SEM, then $\frac{\widehat{\mathsf{Cov}}(l,\tau)}{\widehat{\mathsf{Cov}}(w,\tau)} \to_p \alpha_1$ and $\frac{\widehat{\mathsf{Cov}}(l,\tau)}{\widehat{\mathsf{Cov}}(w+\tau,\tau)} \to_p \beta_1$, where 'exogenous' means that $\mathsf{Cov}(\tau, u^D) = 0$ and $\mathsf{Cov}(\tau, u^S) = 0$.

Proof:
The argument can be seen using the 'first stage'[FS] and 'reduced form' [RF] of the models. The FS is found by equating (1) and (2) and then solving for w, and the RF by substituting the FS into either (1) or (2):

$$w_{it} = \frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} + \frac{-\beta_1}{\beta_1 - \alpha_1}\tau_{it} + \frac{u^D - u^S}{\beta_1 - \alpha_1} := \pi_0 + \pi_1 \tau_{it} + v_{it}^w, \tag{2}$$

$$l_{it} = \frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\beta_1 - \alpha 1} + \frac{-\alpha_1\beta_1}{\beta_1 - \alpha_1}\tau_{it} + \frac{\beta_1 u^D - \alpha_1 u^S}{\beta_1 - \alpha_1} := \mu_0 + \mu_1 \tau_{it} + v_{it}^L. \tag{3}$$

The Wald / 2SLS estimator is based on the following covariances:

$$\frac{\mathsf{Cov}(l,\tau)}{\mathsf{Cov}(w,\tau)} = \frac{\mathsf{Cov}(\mu_0 + \mu_1\tau + v^L, \tau)}{\mathsf{Cov}(\pi_0 + \pi_1\tau + v^w, \tau)} = \frac{\mu_1 \mathsf{Var}(\tau) + \mathsf{Cov}(v^L, \tau)}{\pi_1 \mathsf{Var}(\tau) + \mathsf{Cov}(v^w, \tau)}, \tag{4}$$

$$\frac{\mathsf{Cov}(l,\tau)}{\mathsf{Cov}([w+\tau],\tau)} = \frac{\mathsf{Cov}(\mu_0 + \mu_1\tau + v^L, \tau)}{\mathsf{Cov}(\pi_0 + (\pi_1+1)\tau + v^w, \tau)} = \frac{\mu_1 \mathsf{Var}(\tau) + \mathsf{Cov}(v^L, \tau)}{(\pi_1+1) \mathsf{Var}(\tau) + \mathsf{Cov}(v^w, \tau)}. \tag{5}$$

A sufficient condition for the Wald / 2SLS estimators to consistently estimate both structural parameters is that $\mathsf{Cov}(v^L, \tau) = 0$ and $\mathsf{Cov}(v^w, \tau) = 0$ which implies the necessary exogeneity condition in the proposition.[3]

[2]A consumption good example is two goods that are complements/substitutes, such as coffee and cream/tea.

[3]ZGH allow for control variables, which I suppress, and show in an appendix that their result extends to multi-product markets as long as each good's tax rate has *some* independent variation.

3 Implication of Exogeneity

As in the SEM, consider a labor market where wages determine labor quantities for different skilled workers that are imperfectly substitutable in the production function.[4] Suppose a tax reform for a labor skill group induces a shift in that skill's labor supply. If firms are able to readjust their labor bundles in response to changes in workers' marginal products, then, depending on the reform's direct incentive effects, firms will demand more or less labor from every skill type relative to PE allocation. Call this demand change a spillover effect that is heterogeneous for each type of worker – see Watson (2020) for further discussion and application to the Earned Income Tax Credit.[5] Thus, one would get the following 'incidence equation':

$$\underbrace{\mathsf{d}w_{it}}_{\text{Wage Change in Data}} = \underbrace{\gamma_1 \mathsf{d}\tau_{it} + \gamma_2 \mathsf{d}Z_{it}}_{\text{Incidence Induced Change}} + \underbrace{\gamma_0 + e_{it}}_{\text{Unobs Wage Change}} \tag{6}$$

where $\mathsf{d}Z$ is a theoretical measurement of the GE spillover effect. I assume that $\mathsf{Cov}(e, \tau) = \mathsf{Cov}(e, Z) = 0$, but I assert that $\mathsf{Cov}(\tau, Z) \neq 0$.

3.1 Reconciling the First Stage and Incidence Equations

To reconcile the two equations, the following equivalence must hold:

$$\underbrace{\gamma_0 + e + \gamma_1 \mathsf{d}\tau + \gamma_2 \mathsf{d}Z}_{\text{Incidence + Unobs}} = \underbrace{\mathsf{d}w}_{\text{Data}} = \underbrace{\frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} + \frac{-\beta_1}{\beta_1 - \alpha_1}\mathsf{d}\tau_{it} + \frac{\mathsf{d}u^D - \mathsf{d}u^S}{\beta_1 - \alpha_1}}_{\text{SEM}} \tag{7}$$

One obvious way to reconcile the two equations is the following:

$$e = \frac{-1}{\beta_1 - \alpha 1}\mathsf{d}u^S \qquad \gamma_2 \mathsf{d}Z = \frac{1}{\beta_1 - \alpha 1}\mathsf{d}u^D \qquad \gamma_0 = \frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} \qquad \gamma_1 = \frac{-\beta_1}{\beta_1 - \alpha_1}, \tag{8}$$

where $\gamma_2 \mathsf{d}Z$ reflects the demand schedule shift — a demand 'shock.' One can allow for additional demand unobservable wage variation as long as the tax change and spillover measures are uncorrelated; e.g., $e = a_1 \mathsf{d}u^S + a_2 \mathsf{d}u_x^D$ with $\mathsf{Cov}(u_x^D, \tau) = \mathsf{Cov}(u_x^D, Z) = 0$.

[4] If the goods are perfect complements/substitutes, then the tax incidence is not shared, so the first stage is unidentified (as ZGH note) and there are no spillovers.

[5] In Watson (2020), an EITC expansion increases low wage labor supply → increases the marginal product of untreated high wage workers → shifts high wage demand → increases the marginal product of the low skill group; this feedback eventually settles at a new GE allocation.

Proposition 2.
If there exist demand spillovers Z with $u^D = f(Z)$ and treatment is correlated with spillovers $\mathsf{Cov}(\tau, Z) \neq 0$*, then* $\mathsf{Cov}(\tau, u^D) \neq 0$*, so the ZGH method is inconsistent as tax changes are invalid instruments.*

Remark. The analysis in ZGH is correct, but the equivalence of "exogeneity" and partial equilibrium is not made clear. Further ZGH mention that exogeneity may only be conditional on a vector of covariates, x. Unless one is willing to assume that the spillover effects are captured by included control variables, the ZGH method will be inconsistent for reforms that generate GE responses with unspecified heterogeneous spillover effects. The solution is to either find groups that face the same spillovers but different treatment assignment or the same assignment but different spillover effects.

Remark. Interestingly, despite the fact that τ is correlated with both v^L and v^W making τ an invalid instrument in the technical sense, in this linear example the Wald / 2SLS estimator for the supply elasticity is consistent: $\frac{\mathsf{Cov}(l,\tau)}{\mathsf{Cov}(w+\tau,\tau)} = \frac{-\alpha_1\beta_1 \mathsf{Var}(\tau)+\beta_1 \mathsf{Cov}(\tau,u^D)}{-\alpha_1 \mathsf{Var}(\tau)+\mathsf{Cov}(\tau,u^D)} = \beta_1$. With non-linear structural equations, which can be interpreted as treatment intensity effects of wages, spillovers bias the average marginal LATE coefficient from the PE case (Mogstad and Wiswall, 2010; Lochner and Moretti, 2015).

4 Using Spillovers to Estimate Elasticities

Using the RF equation, note the following:

$$\frac{\partial l}{\partial Z} = \frac{\beta_1}{\beta_1 - \alpha_1}\frac{\partial u^D}{\partial Z} \quad \& \quad \frac{\partial w}{\partial Z} = \frac{1}{\beta_1 - \alpha_1}\frac{\partial u^D}{\partial Z} \quad \Longrightarrow \frac{\partial l/\partial Z}{\partial w/\partial Z} = \beta_1. \tag{9}$$

It is straight-forward to show: $\frac{\partial w}{\partial Z} = \frac{\partial [w+\tau]}{\partial Z}$ and $\frac{\partial l/\partial u^S}{\partial w/\partial u^S} = \alpha_1$. As per intuition, one needs 'demand instruments' for supply parameters and needs 'supply instruments' for demand parameters.

In the context of the labor market SEM, one can use the tax reform treatment as a supply shifter and a measure of spillovers as a demand shifter. Let $\dot{y}_x$ be the residual from from a regression of y on x.

Proposition 3.
If τ is exogenous with the above SEM, then $\frac{\widehat{\mathsf{Cov}}(\dot{l}_\tau,\dot{Z}_\tau)}{\widehat{\mathsf{Cov}}(\dot{w}_\tau,\dot{Z}_\tau)} \to_p \beta_1$ and $\frac{\widehat{\mathsf{Cov}}(\dot{l}_Z,\dot{\tau}_Z)}{\widehat{\mathsf{Cov}}(\dot{w}_Z,\dot{\tau}_Z)} \to_p \alpha_1$, where 'exogenous' means that $\mathsf{Cov}(\tau, u^S) = 0$.

5 Conclusion

Zoutman, Gavrilova and Hopland (2018) discuss identification of supply and demand elasticities using a single tax instrument with two assumptions: knowing which side of the market faces and responds to taxes and that taxes are exogenous. I discuss how exogeneity implies an absence of spillovers – i.e., partial equilibrium – and provide an example where spillovers cause the Wald / 2SLS estimate to be inconsistent for at least one of the elasticities. I additionally show that having a measure of spillovers can provide an additional instrument which allows for identification of both elasticities.

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