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Lecture 06a
Introduction to Root Locus

Lecture is on YouTube

The YouTube video entitled 'Introduction to Root Locus' that covers this lecture is located at https://youtu.be/V3iveC_fa04

Outline

-Root Locus Introduction

Root Locus Introduction

One of the most common engineering problems is understanding how varying a certain parameter in a system affects the system response or performance. These are sometimes referred to as parameter variation studies or sensitivity studies.

Examples

-How does varying the wing sweep angle on an F-14 affect the maximum range of the aircraft?
-How does varying the wheel size of a ground robot affect lateral stability in a turn?
-How does varying the aggressive of a controller affect the system response?

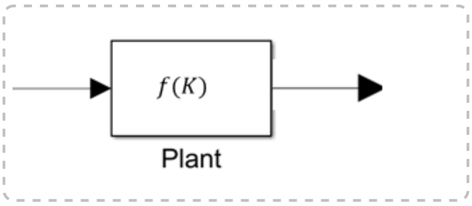
For linear dynamic systems, we often are interested in how the poles of the system respond to variations in system parameters.

The **root locus method** is a way to examine how the roots of a system change with variation of a certain system parameter. As the name suggests, the root locus is the location of the closed loop poles as a function of the varying parameter.

Another name for this might be 'Pole Position' :)

Furthermore, note that the definition of the root locus does not specify a particular feedback structure. This means that we can define a root locus for any type of system by simply asking "how do the closed loop pole locations vary as we change a certain system parameter"? Traditionally this tunable system parameter is called K. Some examples of systems are shown below

Example System 1



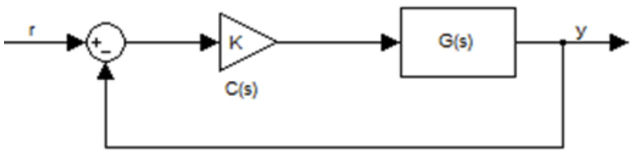
In this case, the plant is a function of the tunable parameter K. Depending on where K enters into the equations of motion (and therefore into the transfer function), this will affect the location of the poles of the system. For example, if the parameter K is in the numerator of the transfer function, then the poles will not be affected by if K is in the denominator then it will change the poles.

Example System 2

In this case, we would like to see where the closed loop poles (the poles of $T(s) = Y(s)/R(s)$) go as the gain parameter, K, is varied. We can see that this is a trivial case because the poles of $T(s)$ are not a function of the tunable system parameter, K.

Example System 3: Traditional/Classical System

The tradition root locus problem (and the one that we will study) asks, "How to the poles of the closed loop transfer function vary with a proportional gain controller with negative unity feedback?". In other words, the specific feedback control structure that we will study and generate a root locus for is shown below



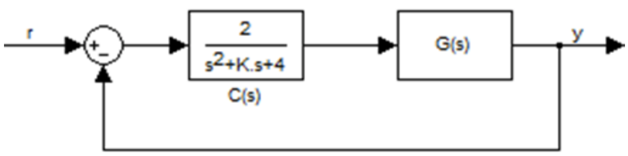
In this case, we can compute the closed loop transfer function (see video on Block Diagram Algebra)

$$T(s) = \frac{K\,G(s)}{1+K\,G(s)}$$

char eq = $1 + K\,G(s)$

The traditional root locus considers the case where the controller is a simple proportional gain, K, and asks the question, how do the roots of $T(s) = Y(s)/R(s)$ change as the parameter K is varied from 0 to ∞.

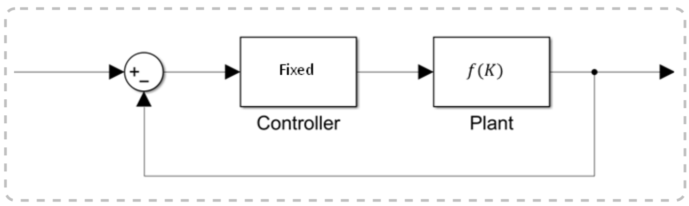
Example System 4



Note that the gain K does not need to simply be a constant proportional gain, it can be part of the controller.

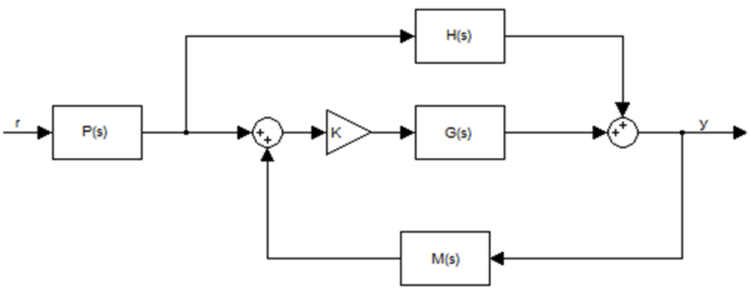
In this example, the tunable system parameter is not a static gain, but rather it is a coefficient of the transfer function $C(s)$. Once again, the root locus is still defined as the locations of the poles of $T(s) = Y(s)/R(s)$ as the parameter K is varied. In this case, K is not a simple gain but nevertheless, we see that changing K will in some fashion change the poles of $T(s)$

Example System 5



In this example, we seek to understand how the overall closed loop poles vary as a parameter of the plant is varied.

Example System 6



In this example, we seek to find out how to the closed loop poles of $T(s) = Y(s)/R(s)$ vary as the gain parameter, K, is varied. We see that this is a more complicated problem because the closed loop system poles a definitely a function of K but how they change as a function of K is not clear.

This lays the foundation for the root locus method which we further in several future videos

Understanding and Sketching the Root Locus (2:57)
Using ‘rlocus’ in Matlab to Plot the Root Locus (0:20)
Using Root Locus to Meet Performance Requirements (0.29)
Designing a PID Controller Using the Root Locus Method (1:04)