Christopher Lum
lum@uw.edu

Lecture 10b
Geodetic Coordinates: Computing Latitude and Longitude

Lecture is on YouTube

The YouTube video entitled ‘Geodetic Coordinates: Computing Latitude and Longitude’ that covers this lecture is located at https://youtu.be/4BJ-GpYbZlU.

Outline

-Geodetic Coordinates

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References:
-Krakiwsky, Edward J., and Thomson, Donald B. . “Geodetic Position Computations No. NB-DSE-TR-39. Department of Surveying Engineering, University of New Brunswick, 1974.

-Pg. 37 of Stevens and Lewis

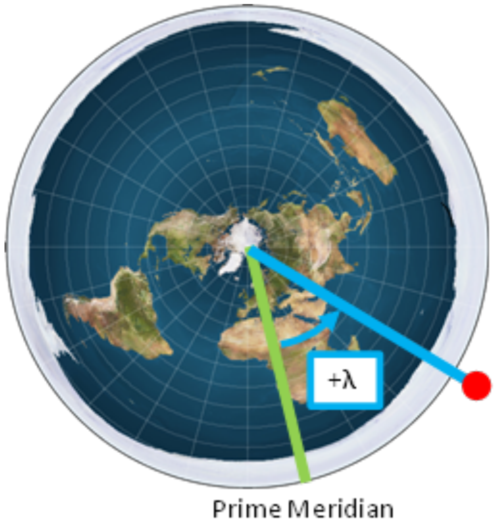
Recall from ‘Lecture 02a - Various Frames of Reference’ where we discussed the WGS-84 spheroid model of the earth. We now continue our discussion on this topic. In particular, we would like to relate how velocities in the NED frame relate to changes in latitude, longitude, and altitude over the surface of the Earth. We need to first be specific about what latitude, longitude, and altitude are. In this case, we would like to calculate the geodetic position over the surface of the Earth. This is determined by using a normal to the spheroid model.

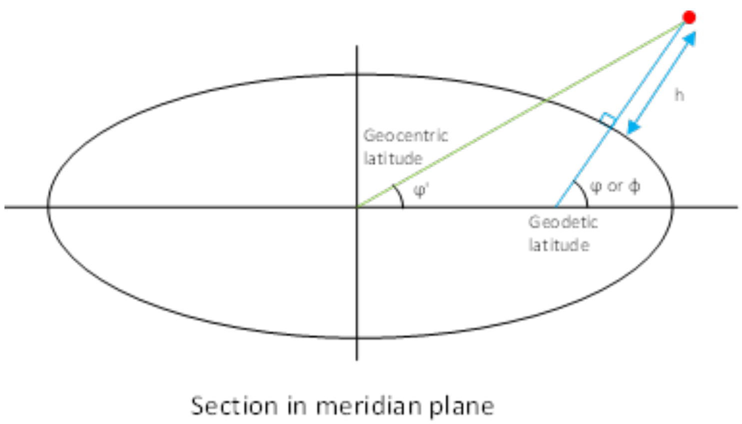
Geodetic altitude, h, is the height above the spheroid, along the normal as shown below.

Geodetic latitude, ϕ, is the angle that the normal makes with the geodetic equatorial plane, and is positive in the Northern Hemisphere.

Terrestrial longitude, λ, angle from the prime meridian and measured in the equatorial plane. This starts at 0^o at the Prime Meridian and runs + eastward and - westward.

Note that this notation is slightly different from Stevens and Lewis. The notation of λ for terrestrial longitude is more consistent with other texts and sources.





Therefore, the position of the vehicle in this coordinate system is given by the triple

$$\begin{pmatrix} \phi \\ \lambda \\ h \end{pmatrix} = \text{geodetic position of vehicle}$$

From Krakiwsky and Thomson, section 1.2

"On the surface of an ellipsoid, an infinite number of planes can be drawn through a point on the surface which contains the normal at this point. These planes are known as <u>normal planes</u>. The curves of intersection of the normal planes and the surface of the ellipsoid are called <u>normal sections</u>. At each point, there are two mutually perpendicular normal sections whose curvatures are maximum and minimum, which are called the <u>principal normal sections</u>. These principal sections are the <u>meridian</u> and <u>prime vertical</u> normal sections, and their radii of curvature are denoted by M and N respectively (Figure 2 and 3)."

Let us first consider the *meridian radius of curvature*, *M*, which is the radius of curvature in the meridian plane, that is the radius of curvature of the generating ellipse.

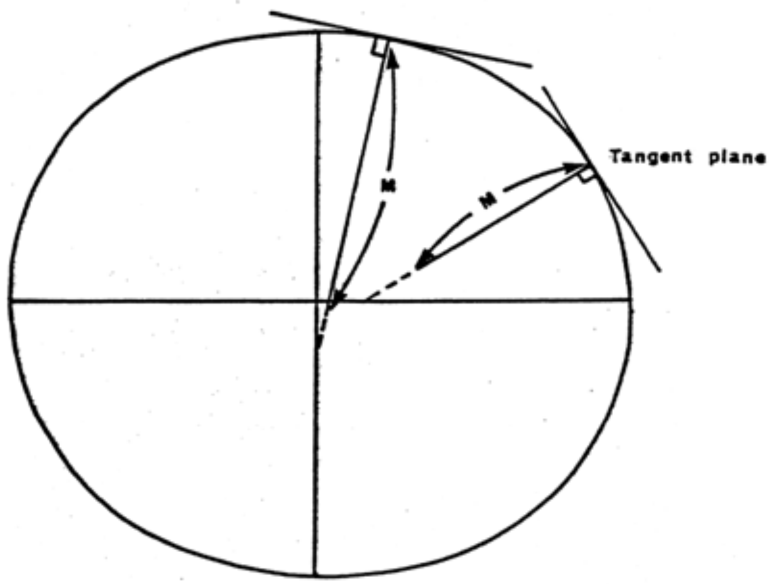


Figure 2
MERIDIAN NORMAL SECTION SHOWING THE MERIDIAN RADIUS OF CURVATURE (M)

This is given by

$$M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin(\phi)^2\right)^{3/2}}$$

The related *prime vertical radius of curvature*, *N*, is the radius of curvature in the plane containing the spheroid normal and a normal to the meridian plane. It is equal to the distance along the normal, from the spheroid surface to the semi-minor axis

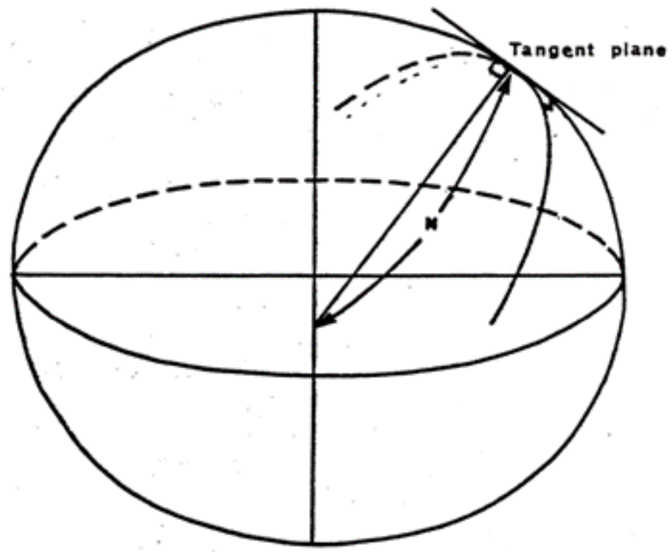


Figure 3
PRIME VERTICAL NORMAL SECTION SHOWING THE PRIME VERTICAL RADIUS OF CURVATURE (N)

This is given by

$$N = \frac{a}{\left(1 - e^2 \sin(\phi)^2\right)^{1/2}}$$

At this point, it may be useful to create a Simulink library block that calculates these for the WGS-84 model

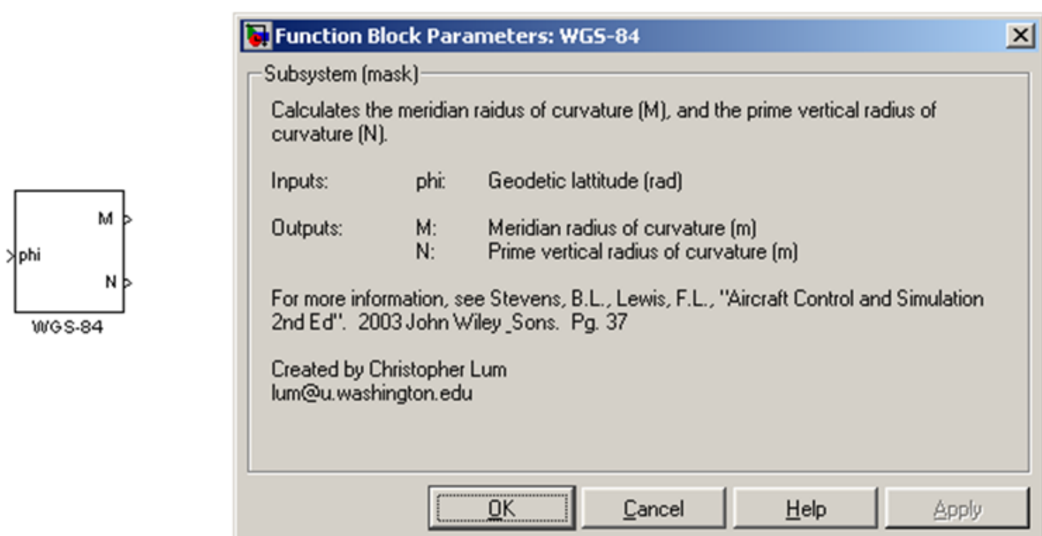


Figure 1: WGS-84

This block computes the meridian radius of curvature, M, and the prime vertical radius of curvature, N, for a given geodetic latitude (ϕ). This uses an ellipsoid with semi-major axis and first eccentricity defined by the WGS-84 world model. These parameters are

$a = 6\,378\,137.0\,m$ (semi-major axis)
$e = 0.081819190842622$ (first eccentricity)

M and N are calculated using equations above

We can use a spherical to Cartesian mapping to transform the position from geodetic to Earth centered Earth fixed coordinates.

$$\overline{p}^{\text{ECEF}} = \begin{pmatrix} (N+h)\cos(\phi)\cos(\lambda) \\ (N+h)\cos(\phi)\sin(\lambda) \\ \left[N\left(1-e^2\right)+h\right]\sin(\phi) \end{pmatrix} \qquad \textbf{\textit{(Eq.1.4-8)}}$$

Finally, the most useful relationship we can establish is how the velocities in the local North, East, down frame relate to $\dot{\phi}$, $\dot{\lambda}$, and $\dot{h}$.

In the northerly direction, the meridian radius of curvature becomes immediately useful as we can write

$$V_N = (M+h)\,\dot{\phi}$$

Or in a more useful form of

$$\dot{\phi} = \frac{V_N}{(M+h)}$$

A similar, slightly more complex calculation can be done in the easterly direction

$$\dot{\lambda} = \frac{V_E}{(N+h)\cos(\phi)}$$

Because the local NED frame is aligned with the local normal, we have

$$\dot{h} = -V_D$$

These make the dynamic equations to integrate in order to keep track of geodetic position if given inputs of the vehicle velocity in the NED frame.

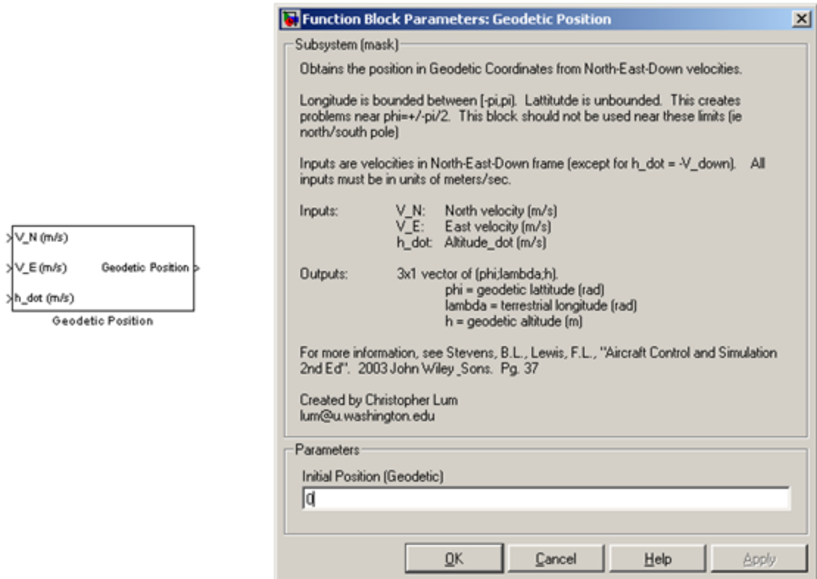


Figure 3: Geodetic Position

-Initial Position (Geodetic): Initial geodetic position $\begin{pmatrix} \phi(0) \\ \lambda(0) \\ h(0) \end{pmatrix} \quad \begin{matrix} (radians) \\ (radians) \\ (m) \end{matrix}$ [3x1 vector value]

Inputs/Output

Input: –V_N: Velocity north *(m/s)* [1x1 scalar value]

–V_E: Velocity east *(m/s)* [1x1 scalar value]

–h_dot: Geodetic altitude rate of change *(m/s)* [1x1 scalar value]

Output: –Geodetic Position : $\begin{pmatrix} \phi \\ \lambda \\ h \end{pmatrix} \begin{matrix} (radians) \\ (radians) \\ (m) \end{matrix}$ [3x1 vector value]

We can implement the block as (while being careful of the interface difference between $\dot{h}$ and $\dot{V}_D$).