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Radius of a Circle Inscribed in a Triangle

Lecture is on YouTube

The YouTube video entitled ‘Radius of a Circle Inscribed in a Triangle’ that covers this lecture is located at https://youtu.be/AlTYdT7kF38

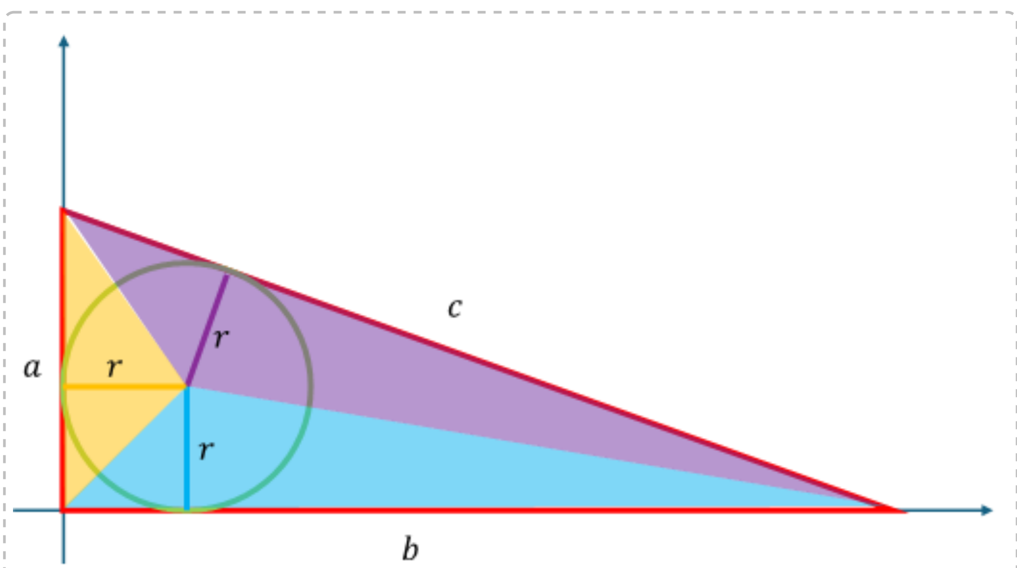
Outline

-Simulink 3D Animation

Notes

https://en.wikipedia.org/wiki/Incircle_and_excircles#:~:text=In%20geometry%2C%20the%20incircle%20or,center%20called%20the%20triangle’s%20incenter

Right Triangle



Area of the triangle is given as

$$A_T = \frac{1}{2}\, a\, b$$

In[ ]:=
```
AT = 1/2 a b;
```

The area of each small triangle is given as

$$A_b = \frac{1}{2}\, a\, r \qquad \text{(blue)}$$

$$A_o = \frac{1}{2}\, b\, r \qquad \text{(orange)}$$

$$A_p = \frac{1}{2}\, c\, r \qquad \text{(purple)}$$

In[ ]:=
```
Ab = 1/2 a r;
Ao = 1/2 b r;
Ap = 1/2 c r;
```

So the total area is also given as

$$A_T = A_b + A_o + A_p \qquad \textbf{\textit{(Eq.1)}}$$

$$\frac{1}{2}\, a\, b = \frac{1}{2}\, a\, r + \frac{1}{2}\, b\, r + \frac{1}{2}\, c\, r$$

$$a\, b = (a + b + c)\, r$$

$$\frac{a\, b}{a + b + c} = r \qquad \text{note: } c = \text{sqrt}\left(a^2 + b^2\right)$$

In[ ]:=
```
r = (a b)/(a + b + c);
```

Example

In[ ]:=
```
aGiven = 3;
bGiven = 4;
cGiven = (aGiven^2 + bGiven^2)^(1/2)
```

Out[ ]= 5

In[ ]:=
```
rGiven = r /. {a → aGiven, b → bGiven, c → cGiven}
```

Out[ ]= 1

Non-Right Triangle

If the triangle is not a right triangle, we can still apply Eq.1

$$A_T = A_b + A_o + A_p \qquad \textbf{\textit{(Eq.1)}}$$

But total area of the triangle is given by Heron’s Formula https://en.wikipedia.org/wiki/Heron%27s_formula

$$A_T = \sqrt{s(s-a)(s-b)(s-c)}$$

where $\quad s = \frac{a+b+c}{2}$ (semi perimeter)

So Eq.1 becomes

$$\sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2} a r + \frac{1}{2} b r + \frac{1}{2} c r$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2} r(a+b+c)$$

$$r = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a+b+c}$$

Example

```
In[ ]:= aGiven = 3;
bGiven = 4;
cGiven = 6.3582; (*corresponds to thetaA = 130 deg*)
```

```
In[ ]:= sGiven = (aGiven + bGiven + cGiven)/2
Out[ ]= 6.6791
```

```
In[ ]:= ATGiven = Sqrt[sGiven (sGiven - aGiven) (sGiven - bGiven) (sGiven - cGiven)]
Out[ ]= 4.59631
```

```
In[ ]:= rGiven = (2 ATGiven)/(aGiven + bGiven + cGiven)
Out[ ]= 0.688163
```