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Lecture05b
Wind Tunnel Corrections and Data Reduction

Lecture is on YouTube

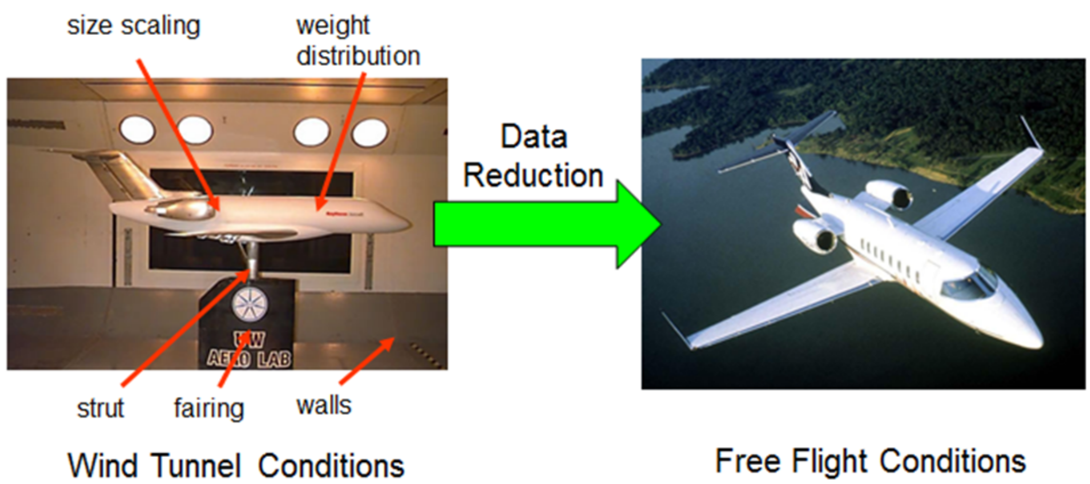
The YouTube video entitled ‘Wind Tunnel Corrections and Data Reduction’ that covers this lecture is located at https://youtu.be/6FZ6sIZSYrc.

Outline

-Data Reduction
- -A. Indicated to Actual q
- -B. Balance Interactions
- -C. Tares : Strut/Fork, Tare and Interference
- -D. Weight Tares
- -E. Moment Transfers
- -F. Blockage Corrections
- -G. Initial Coefficients
- -H. Flow Angularity (Crossflow and Upflow)
- -I. Wall Corrections
- -J. Final Coefficients / Corrected Angle of Attack
- -K. Axis Transfers

Data Reduction

Once we have acquired the data, it may be tempting to directly use this with our dynamic model. This would be a mistake because the data needs to be corrected to free flight conditions. This takes into account the fact that the test conditions in the wind tunnel are not the same as free flight.

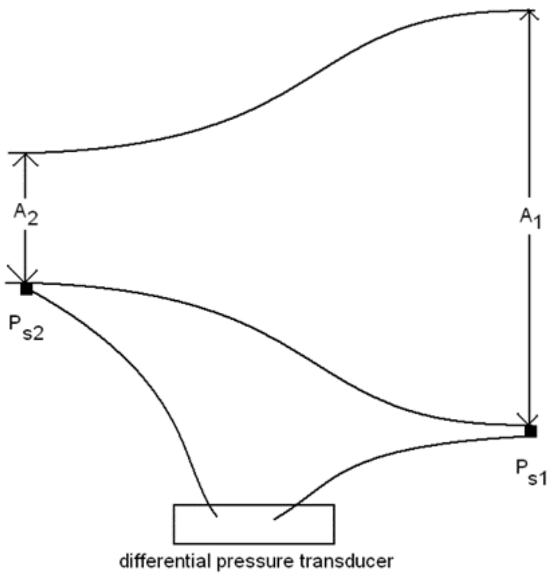


Need to do data reduction to correct the data to free flight conditions. Several corrections that are needed

A. Indicated to actual q
B. Balance Interactions
C. Tares: Strut/fork, tare and interference
D. Weight Tares
E. Moment transfers
F. Blockage corrections
G. Initial Coefficients
H. Flow angularity (cross flow and upflow)
I. Wall corrections
J. Final Coefficients / Corrected Angle of Attack
K. Axis Transfers

A. Indicated to Actual q

Wind tunnel measures dynamic pressure using two sets of static pressure strips.



Continuity equation (mass flow conservation)

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For incompressible flow, $\rho_1 = \rho_2$

$$A_1 V_1 = A_2 V_2$$

$$V_1 = V_2 \frac{A_2}{A_1} \qquad \textbf{\textit{(Eq.A.1)}}$$

Total pressure is also conserved,

$$p_{o1} = p_{o2}$$

$$p_{s1} + \frac{1}{2} \rho_1 V_1^2 = p_{s2} + \frac{1}{2} \rho_2 V_2^2$$

Solving for V_2

$$\frac{1}{2} \rho_2 V_2^2 = p_{s1} - p_{s2} + \frac{1}{2} \rho_1 V_1^2 \qquad \text{note: } V_1 = V_2 \frac{A_2}{A_1}$$

$$\frac{1}{2} \rho_2 V_2^2 = p_{s1} - p_{s2} + \frac{1}{2} \rho_1 \left(V_2 \frac{A_2}{A_1}\right)^2$$

$$\rho_2 V_2^2 = 2 (p_{s1} - p_{s2}) + \rho_1 V_2^2 \frac{A_2^2}{A_1^2}$$

$$\rho_2 V_2^2 - \rho_1 V_2^2 \frac{A_2^2}{A_1^2} = 2(p_{s1} - p_{s2})$$

$$\left(\rho_2 - \rho_1 \frac{A_2^2}{A_1^2}\right) V_2^2 = 2(p_{s1} - p_{s2}) \qquad \text{assuming incompressible:} \qquad \rho_1 = \rho_2$$

$$\rho\left(1 - \frac{A_2^2}{A_1^2}\right) V_2^2 = 2(p_{s1} - p_{s2})$$

$$V_2^2 = \frac{2(p_{s1}-p_{s2})}{\rho\left(1-(A_2/A_1)^2\right)}$$

$$V_2 = \sqrt{\frac{2(p_{s1}-p_{s2})}{\rho\left(1-(A_2/A_1)^2\right)}}$$

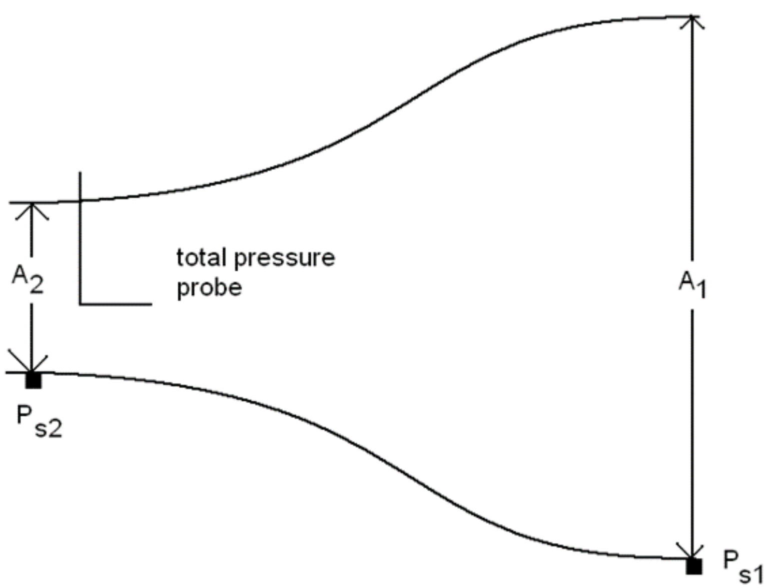
The differential static pressure is measured using two static pressure strips on the floor of the test section.

So the dynamic pressure at the test section is given by

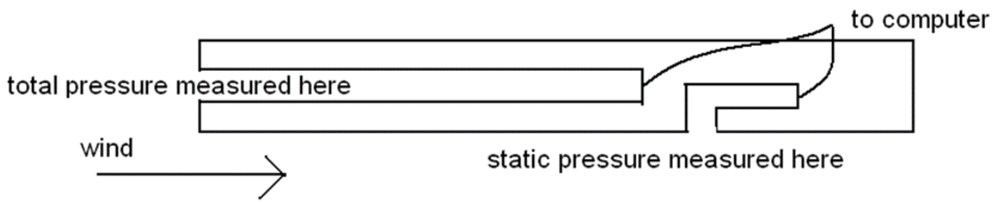
$$q_2 = \frac{1}{2}\rho V_2^2$$

$$= \frac{(p_{s1}-p_{s2})}{\left(1-(A_2/A_1)^2\right)} \qquad \textbf{\textit{(Eq.A.2)}}$$

Alternatively, could also obtain dynamic pressure using a pitot-static pressure probe.



The construction is as shown below



So the dynamic pressure is simply

$$q = p_o - p_s \qquad \textbf{(Eq.A.3)}$$

This is more accurate than the two static pressure strips but we can't have a pressure rake in the tunnel with the model simultaneously. Therefore, we generate a correction factor of

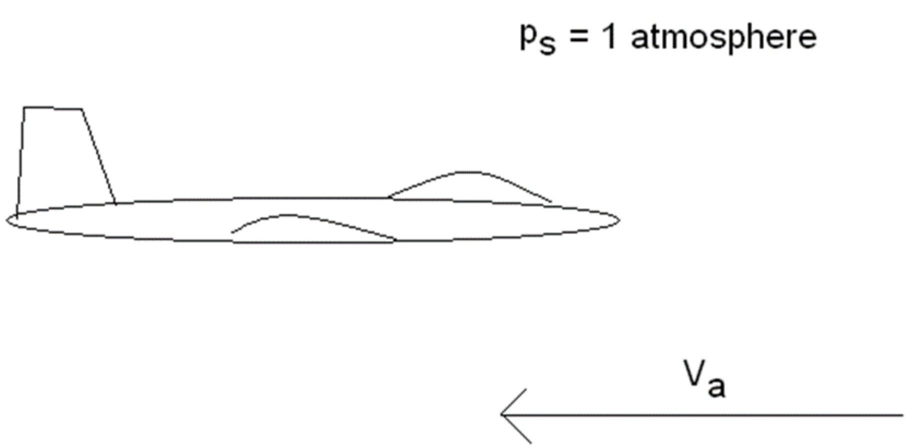
$$q_a = \frac{q_a}{q_i}\, q_i \qquad \textbf{(Eq.A.4)}$$

where q_i = dynamic pressure calculated using two static pressure strips

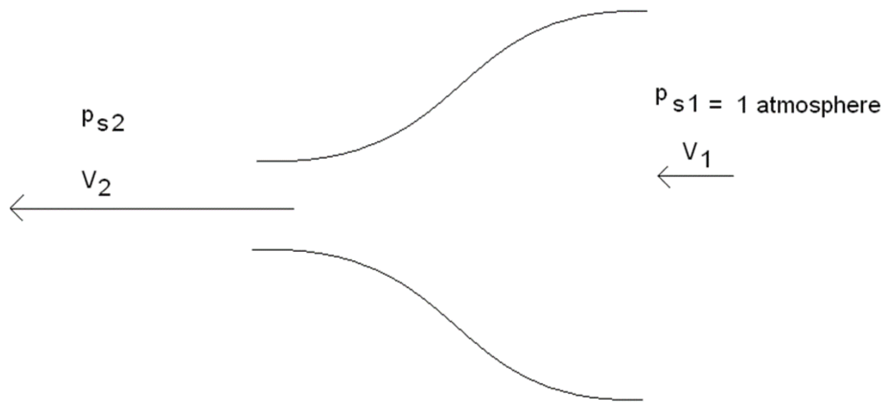
q_a / q_i = correction factor (usually near 1)

q_a =actual dynamic pressure

While we're talking about pressures, what is the static pressure of an aircraft in flight? It should be p_s = 1 atmosphere.



However, what about in a nozzle (ie wind tunnel bellmouth to test section)



We already know that total pressure is conserved

$$p_o = q_1 + p_{s1} = q_2 + p_{s2}$$

Assuming that V_1 is small, then $p_{s1} \approx 1$ atm and

$$p_o = \frac{1}{2}\rho V_1^2 + p_{s1} \qquad \text{recall: } V_1 = V_2 \frac{A_2}{A_1}$$

$$= \frac{1}{2}\rho \left(V_2 \frac{A_2}{A_1}\right)^2 + p_{s1}$$

$$= \frac{1}{2}\rho V_2^2 \left(\frac{A_2}{A_1}\right)^2 + p_{s1} \qquad \text{note:} \quad \frac{1}{2}\rho V_2^2 = q_2$$

$$p_o = q_2\left(\frac{A_2}{A_1}\right)^2 + p_{s1}$$

$$p_{s1} = p_o - q_2\left(\frac{A_2}{A_1}\right)^2 \qquad \text{recall:} \quad p_o = q_2 + p_{s2}$$

$$p_{s1} = q_2 + p_{s2} - q_2\left(\frac{A_2}{A_1}\right)^2$$

$$p_{s1} = \left(1 - \left(\frac{A_2}{A_1}\right)^2\right) q_2 + p_{s2}$$

Since $A_2 < A_1$, the first term is positive and therefore, we see that

$$p_{s2} \leq p_{s1}$$

If $p_{s1} \approx 1$ atmosphere, then static pressure in the test section is less that desired.

To fix this problem, UWAL uses an approach of venting the test section to the atmosphere (ie cutting a hole in the side of the test section). This forces $p_{s2} \approx 1$ atmosphere

B. Balance Interactions

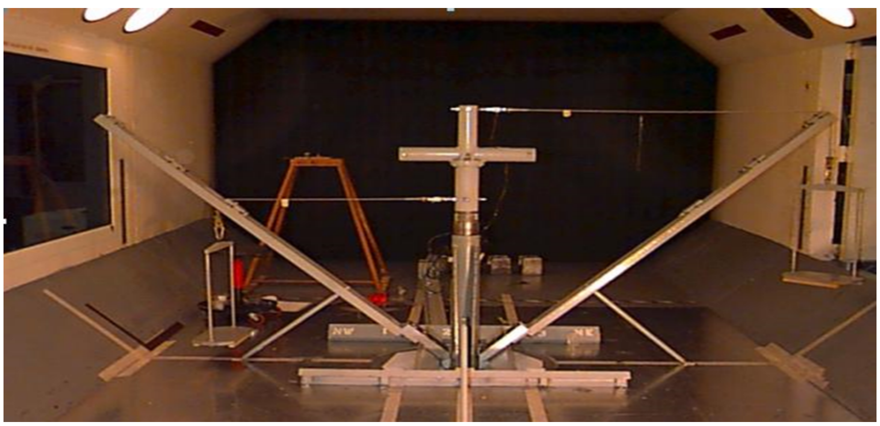
The balance forces and moment interact with each other. In other words, applying a pure lift force does not yield only a lift reading from the balance, it also affects the other channels. This is accounted for with a second order correction

$$\begin{pmatrix} L_{\text{BU}} \\ D_{\text{BU}} \\ \text{SF}_{\text{BU}} \\ \text{PM}_{\text{BU}} \\ \text{RM}_{\text{BU}} \\ \text{YM}_{\text{BU}} \end{pmatrix} = A_1 \begin{pmatrix} L_R \\ D_R \\ \text{SF}_R \\ \text{PM}_R \\ \text{RM}_R \\ \text{YM}_R \end{pmatrix} + A_2 \begin{pmatrix} {L_R}^2 \\ {D_R}^2 \\ {\text{SF}_R}^2 \\ {\text{PM}_R}^2 \\ {\text{RM}_R}^2 \\ {\text{YM}_R}^2 \end{pmatrix}$$

(last time I checked, it was only second order non-linear in yaw and side force)

where A_i = 6 x6 interaction matrix

How to obtain the matrix A_1 and A_2? Use a complex rig which can apply forces and moments exactly in tunnel axis. This applies a pure rolling moment which pushes the right wingtip down which is a positive rolling moment according to KWT sign convention.



Consider a balance which only measured 3 components (lift, drag, and side force). Then consider applying a known force to the balance and measuring what the balance reads.

$$\overline{x}_{\text{BU}} = \begin{pmatrix} L_{\text{BU}} \\ D_{\text{BU}} \\ S_{\text{BU}} \end{pmatrix} = \begin{pmatrix} \text{known lift quantity applied to balance} \\ \text{known drag quantity applied to balance} \\ \text{known sideforce quantitify applied to balance} \end{pmatrix}$$

$$\overline{x}_R = \begin{pmatrix} L_R \\ D_R \\ S_R \end{pmatrix} = \begin{pmatrix} \text{raw lift value read by balance} \\ \text{raw drag value read by balance} \\ \text{raw sideforce value read by balance} \end{pmatrix}$$

Before we tackle a second order correction, let us first consider generating a first order correction of the form

$$L_{BU} = a_{11} L_R + a_{12} D_R + a_{13} S_R$$
$$D_{BU} = a_{21} L_R + a_{22} D_R + a_{23} S_R$$
$$S_{BU} = a_{31} L_R + a_{32} D_R + a_{33} S_R$$

Or $\overline{x}_{BU} = A \overline{x}_R$ ***(Eq.B.1)***

where $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

Goal is to now find A such that $\overline{x}_{BU} = A \overline{x}_R$. In this case, we effectively have 3 equations and 9 unknowns so we have infinite choices for the A matrix.

Method 1

What if we have more than one test point? Rewrite Eq.B.1 as

$$A \overline{x}_R - \overline{x}_{BU} = \overline{0}$$

$A \overline{x}_R{}^1 - \overline{x}_{BU}{}^1 = \overline{0}$ (test point 1)

$A \overline{x}_R{}^2 - \overline{x}_{BU}{}^2 = \overline{0}$ (test point 2)

$\vdots$

$A \overline{x}_R{}^n - \overline{x}_{BU}{}^n = \overline{0}$ (test point n)

We can collect the known quantities in a vector as

$$\overline{x} = \begin{pmatrix} \overline{x}_R{}^1 \\ \overline{x}_{BU}{}^1 \\ \overline{x}_R{}^2 \\ \overline{x}_{BU}{}^2 \\ \vdots \\ \overline{x}_R{}^n \\ \overline{x}_{BU}{}^n \end{pmatrix}$$

This becomes

$\tilde{A} \overline{x} = \overline{0}$ ***(Eq.B.2)***

where $\tilde{A} = \begin{pmatrix} A & -I & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & A & -I & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \ldots & \ldots & A & -I \end{pmatrix}$

So from this we see that there may not be a solution (since we now have $3n$ equations and only 9 unknowns)

We can formulate this as an optimization problem

$$\underset{A \in \mathbb{R}^{3\times 3}}{\text{minimize}}\, f_o(A) \equiv \frac{1}{2} \| \tilde{A}\, \overline{x} \|^2$$

Method 2

Some of you probably noticed that this is a somewhat standard regression problem. To make this more clear, let's rewrite in a more understandable form

$$L_{\text{BU}} = a_{11} L_R + a_{12} D_R + a_{13} S_R$$
$$D_{\text{BU}} = a_{21} L_R + a_{22} D_R + a_{23} S_R$$
$$S_{\text{BU}} = a_{31} L_R + a_{32} D_R + a_{33} S_R$$

Or alternatively

$$\overline{x}_{\text{BU}} = B\, \overline{a} \qquad \textbf{\textit{(Eq.B.3)}}$$

where $B = \begin{pmatrix} L_R & D_R & S_R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_R & D_R & S_R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_R & D_R & S_R \end{pmatrix}$

$$\overline{a} = (\, a_{11} \quad a_{12} \quad a_{13} \quad a_{21} \quad a_{22} \quad a_{23} \quad a_{31} \quad a_{32} \quad a_{33}\,)^T$$

We see that this is only possible if $\overline{x}_{\text{BU}} \in R(B)$. This should be true if any component L_R, D_R, or S_R is not equal to 0.

What if we have multiple data points? We can stack all the applied forces and the measured forces in the form

$$\overline{X}_{\text{BU}} = \begin{pmatrix} \overline{x}_{\text{BU}}{}^1 \\ \overline{x}_{\text{BU}}{}^2 \\ \vdots \\ \overline{x}_{\text{BU}}{}^n \end{pmatrix} \quad \tilde{B} = \begin{pmatrix} B^1 \\ B^2 \\ \vdots \\ B^n \end{pmatrix}$$

So we can write all the data points as

$$\overline{X}_{\text{BU}} = \tilde{B}\, \overline{a} \qquad \textbf{\textit{(Eq.B.4)}}$$

This is of the form, $\overline{b} = A\, \overline{x}$. This is setup as an optimization problem of

$$\underset{\overline{a} \in \mathbb{R}^9}{\text{minimize}}\, \frac{1}{2} \| \tilde{B}\, \overline{a} - \overline{X}_{\text{BU}} \|^2 \qquad \textbf{\textit{(Eq.B.5)}}$$

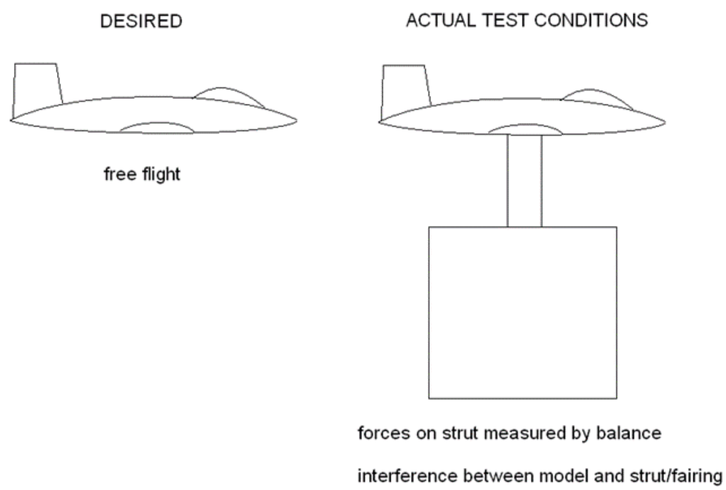
The solution is the well known pseudoinverse

$$\bar{a}^* = \left(\tilde{B}^T \tilde{B}\right)^{-1} \tilde{B}^T \bar{X}_{BU} \qquad \textbf{\textit{(Eq.B.6)}}$$

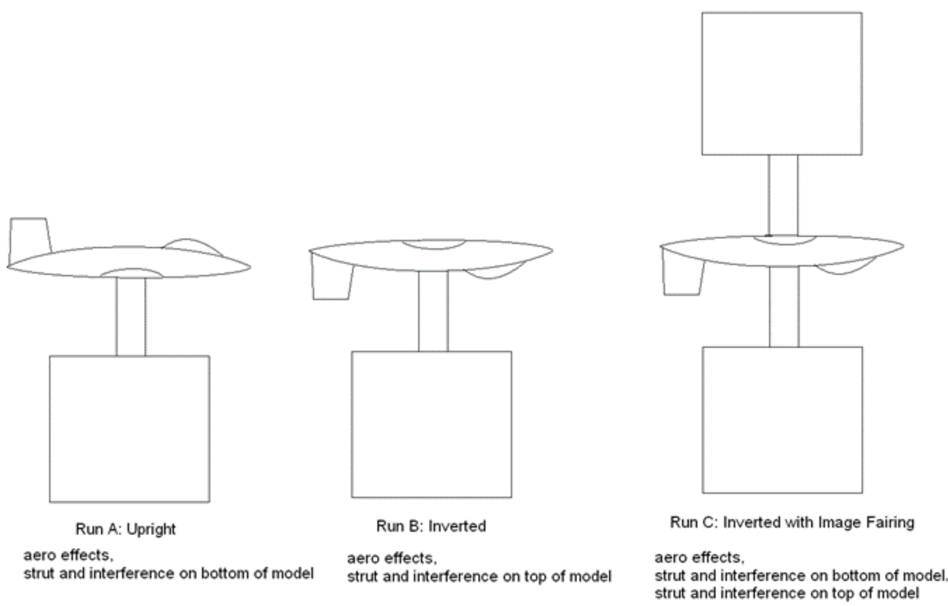
We can extend this idea to generate a second order correction as well.

C. Tares : Strut/Fork, Tare and Interference

We want the wind tunnel to measure the aerodynamic forces/moments on a free aircraft. What are we actually measuring?



To find the effects of the strut and fairing, can invert the model



So if we now take

$$D \equiv C - B$$

$$D = \begin{pmatrix} \text{aero effects} \\ \text{strut and interference on bottom} \\ \text{stut and inteference on top} \end{pmatrix} - \begin{pmatrix} \text{aero effects} \\ \text{strut and interference on top} \end{pmatrix} =$$

$$(\text{strut and intererence on bottom})$$

If we now normalize this by the dynamic pressure at which the data was acquired, we obtain the strut tare

$$T = \frac{D}{q_a} \qquad \text{(strut tare)} \qquad \textbf{\textit{(Eq.C.1)}}$$

We can then apply strut tare to upright aero data

$$A - T\, q_A = \begin{pmatrix} \text{aero effects} \\ \text{strut and interference on bottom} \end{pmatrix} - (\text{strut and interference on bottom}) = (\text{aero effects})$$

So the forces/moments which are now corrected for strut and interference is given by

$$\begin{aligned} L_{\mathrm{TI}} &= L_{\mathrm{BU}} - \mathrm{NLIFT} \cdot q_A \qquad \textbf{\textit{(Eq.C.2)}} \\ D_{\mathrm{TI}} &= D_{\mathrm{BU}} - \mathrm{NDRAG} \cdot q_A \\ &\vdots \\ R_{\mathrm{TI}} &= R_{\mathrm{BU}} - \mathrm{NRM} \cdot q_A \end{aligned}$$

where NLIFT = normalized strut and interference tare for lift

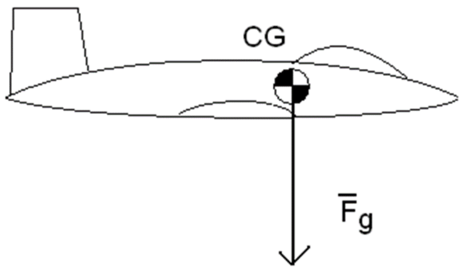
X_{TI} = channel corrected for both balance interactions and tare and interference

This is time consuming, requiring three runs for each configuration. To make this simpler, a single strut tare is sometimes applied to multiple, similar configuration runs.

It is worth mentioning that in order to calculate these tares, the model needs to have the ability to be inverted and also accommodate an image fairing. As a test engineer, you will need to ensure that your wind tunnel model is built with these capabilities.

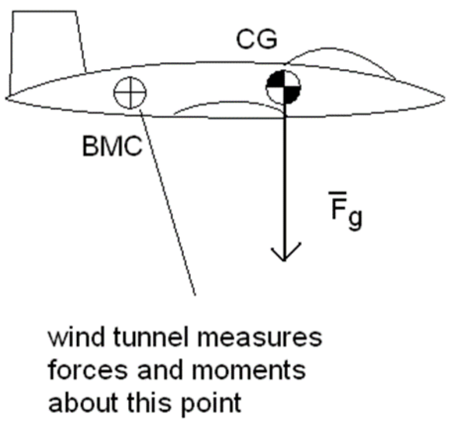
D. Weight Tares

Once again, we only want the aerodynamic forces and moments. For actual, full size aircraft, when working with data related the moments, we typically want the moments about the center of gravity of the aircraft. Note that in a real aircraft, is there is no moment about the CG due to gravity (in other words, gravity does not affect the pitching, yawing, or rolling moments).



In a wind tunnel, we need to take into account the fact that the CG of the model and the balance moment center (BMC) are not coincident.

Perform demonstration where student holds model aircraft near the back (to emulate the balance) and show that they experience a moment because they are not at the CG of the aircraft.



The balance therefore measures both aerodynamic moments on the model and moments due to the center of gravity of the model not being located at the BMC. To correct for this, we need to determine the location of the CG with respect to the BMC for each model configuration. This is done one of two ways

1. Full Table Tare
    a. Take a WOZ at $\alpha \approx \psi \approx 0$
    b. Pitch and yaw model to specific α and ψ value
    c. Take data with wind off
    d. Repeat at different α, ψ

This gives enough information to generate a look-up table of moments due to gravity which can be remove at each test point

2. Three Point Weight Tare
    a. Take a WOZ at $\alpha \approx \psi \approx 0$ (point 1)
    b. Pitch model to negative α, negative ψ and take data with wind off (point 2)

c. Pitch model to positive α, positive ψ, and take data with wind off (point 3)
d. Calculate center of gravity location from these points.

So the correction becomes

$$P_B = P_{\text{TI}} - \text{PMWT} \qquad \textbf{(Eq.D.1)}$$
$$R_B = R_{\text{TI}} - \text{RMWT}$$

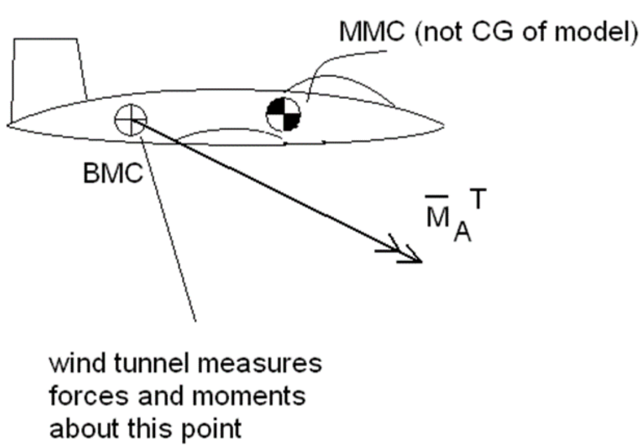
where P_B, R_B = pitch and roll corrected for everything previously discussed as well as weight tare

Note that you do not need to correct for yaw since tunnel axis is perpendicular to gravity vector so the model's offset CG will never create a yawing moment about the BMC.

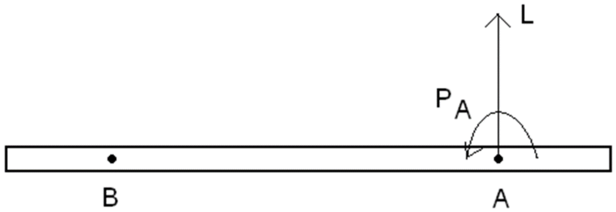
It is also worth mentioning that the weight tare will take into account the moments due to the offset CG of the model, not due to the CG of the real aircraft. Furthermore, since we are only gathering aerodynamic forces/moments, the CG of the model does not need to be anywhere near the CG of the real aircraft. In fact, many wind tunnel models are constructed from solid steel or aluminum where the CG is wildly different from the CG of the actual, full scale aircraft. This leads into a discussion of moment transfers.

E. Moment Transfers

The CG of the model is not the CG of the real aircraft. The customer will often request that the moments be transferred to several locations. These are where they anticipated the CG of the actual aircraft will lie.



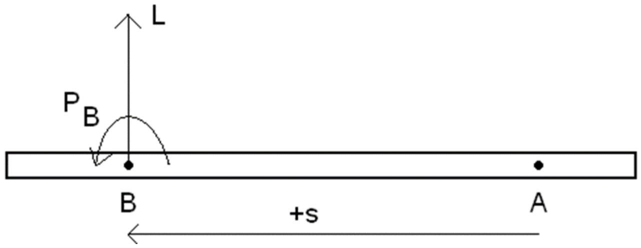
Recall the simplified example from last lecture. For example, in the longitudinal direction, we can look at a one dimensional picture of pitch



In this situation, the moment about point A is simply

moment about point A = P_A

However if the moment are taken about point B? In this case the picture is



In this situation, the moment about point A is given by

moment about point A = $P_B - s\,L$

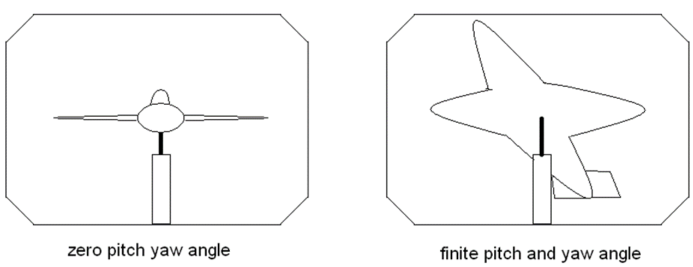
Equating these two equations yields

$$P_A = P_B - s\,L$$

There is a homework assignment dealing with a full 3D situation with all forces/moments.

F. Blockage Corrections

Consider the model at $\alpha \approx \psi \approx 0$ and then at a finite pitch and yaw angle.



As model pitches and yaws, the cross sectional area is reduced.

Recall that the pressure strips in the floor of the tunnel measure q_i. We then correct this to what we call q_a using the relation $q_a = \frac{q_a}{q_i}\, q_i$.

We need to further increase q_a to account for blockage (nozzle effect) due to the blockage of the model (and potentially the wake as well)

$q_c = q_a(1+\varepsilon)^2$ ***(Eq.E.1)***

There are two methods for calculating ε

Method 1. Shindo's Simplified Tunnel Correction Method

$$\varepsilon = \frac{S_w}{C_A}\left[C_{\text{DU}} - C_{\text{LU}}{}^2\left(\frac{1}{\pi\,\text{AR}} - S_w\,\frac{C_w}{C_A}\right)\right]$$

where C_A = cross sectional area of empty test section

Method 2. Maskall's Blockage Correction

$$\varepsilon_{\text{SB}} = \frac{K_A V_A + K_B V_B}{C_A{}^{3/2}}$$

$$\varepsilon_{\text{WB}} = \frac{S_W}{4\,C_A}\,C_{D_o} + \frac{S_w K_w}{4\,C_A}\,(C_{D_u} - C_{D_i} - C_{D_o})$$

$$\varepsilon = \varepsilon_{\text{SB}} + \varepsilon_{\text{WB}}$$

Notice that Maskall's correction explicitly takes into account the blockage due to wake (ε_{WB}).

G. Initial Coefficients

At this point, have enough information to generate some initial aerodynamic coefficients

$$C_{L_{\text{INI}}} = \frac{L_B}{q_c\,S_W} \qquad C_{M_{\text{INIxx}}} = \frac{M_{\text{MMC}}}{q_c\,S_W\,\bar{c}}$$

$$C_{D_{\text{INI}}} = \frac{D_B}{q_c\,S_W} \qquad C_{R_{\text{INIxx}}} = \frac{R_{\text{MMC}}}{q_c\,S_w\,b_{\text{ref}}}$$

$$C_{Y_{\text{INI}}} = \frac{S_B}{q_c\,S_W} \qquad C_{N_{\text{INIxx}}} = \frac{Y_{\text{MMC}}}{q_c\,S_w\,b_{\text{ref}}}$$

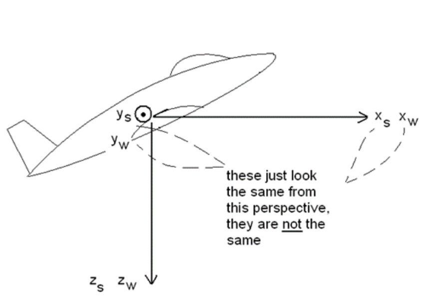
Notice that pitch uses $\bar{c}$ as characteristic length whereas roll and yaw use b_{ref}. This is important and we will revisit this later.

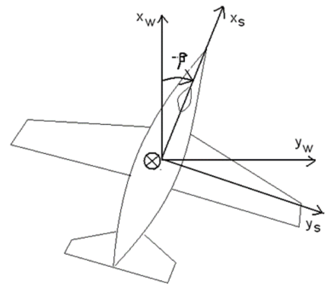
H. Flow Angularity (Crossflow and Upflow)

We want to look at this using the stability and wind axes transformations. Recall with this system used with these rotation matrices, the axis of the body system were defined as

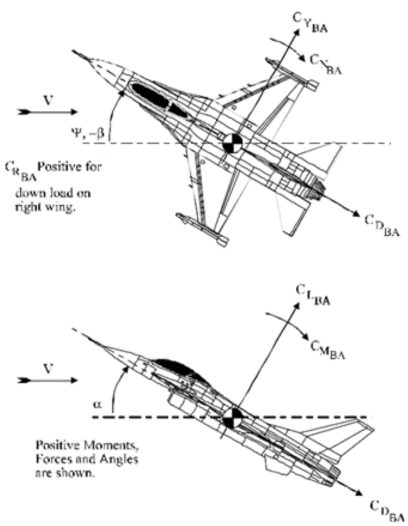
x_b = positive forward ***(Eq.K.1)***
y_b = positive out right wingtip
z_b = positive down

The wind axis is setup in the same direction as this but rotated by the angles α and β.





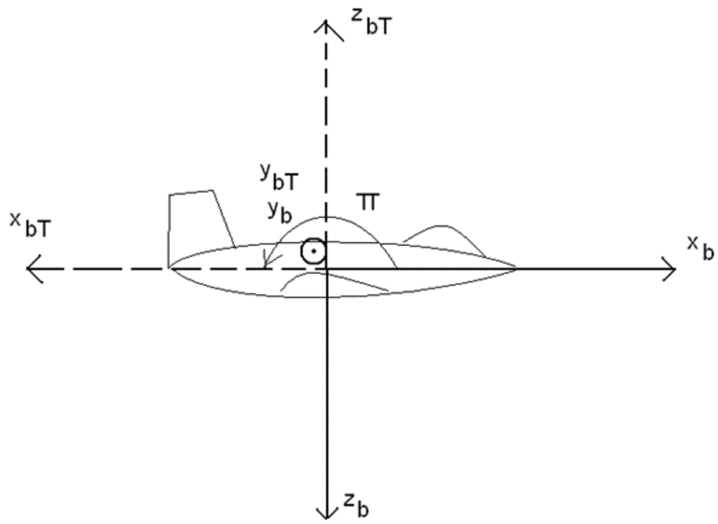
One issue is that the wind tunnel defines the body axis slightly differently. Let us examine a picture from the Kirsten Wind Tunnel showing body frame.



Let us denote the wind tunnel body frame as F_{b_T}. From the picture, we see that it is defined as follows

x_{b_T} = positive backwards **(Eq.K.2)**
y_{b_T} = positive out right wingtip
z_{b_T} = positive up

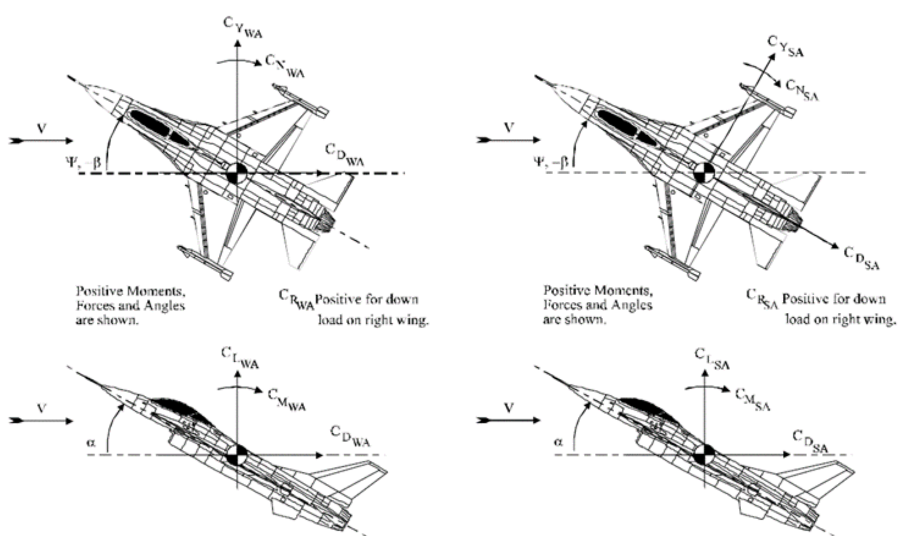
Drawing the two frames on top of each other we have



We see that the rotation between these two frames is about the y axis through and angle of π

$$C_{b_T/b} = \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix} \qquad \text{(rotate from body axis to wind tunnel body axis)}$$

In a similar fashion, the stability and wind frame in the wind tunnel are different from our previous definitions.



The wind and stability axis are likewise defined with the angles α and β so we also have

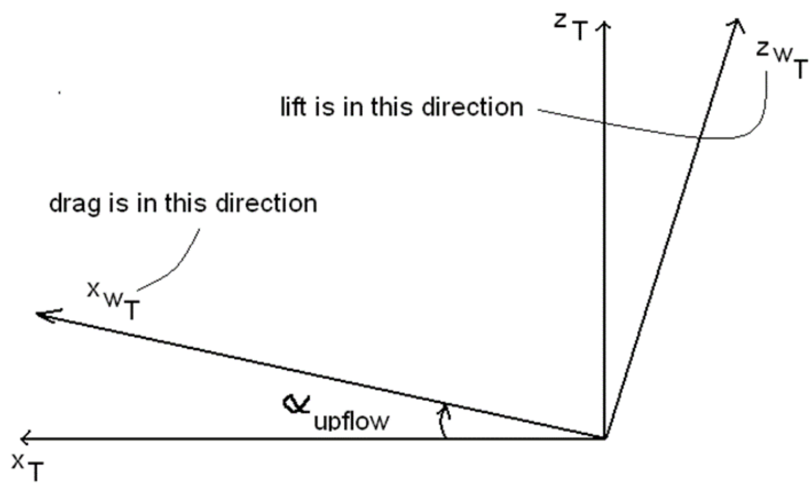
$$C_{S_T/S} = \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix} \qquad \text{(rotate from stability axis to wind tunnel stability axis)}$$

$$C_{W_T/W} = \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix} \qquad \text{(rotate from wind axis to wind tunnel wind axis)}$$

What frame does the balance express forces and moments in? It expresses values in the tunnel frame, F_T. This frame is defined as

x_T = positive backwards parallel with test section floor
y_T = positive towards right wall parallel with test section floor
z_T = positive up normal to test section floor

What if airstream is not aligned with tunnel axis? For example, what if there is a non-zero α_{upflow} (note: this is actually the wind frame used in the tunnel, F_{W_T}, shown in the picture)



By definition, drag is parallel to the wind, not to the tunnel axis. Similarly, lift is perpendicular to the wind, not to the tunnel axis.

This is very similar to the stability to body frame rotation we talked about previously. We can think of the tunnel frame as the body frame. So to rotate forces/moments from the tunnel to the wind frame, we simply use

$$C_{S_T/W_T}(\beta_{\text{cross}}) = \begin{pmatrix} \cos(\beta_{\text{cross}}) & -\sin(\beta_{\text{cross}}) & 0 \\ \sin(\beta_{\text{cross}}) & \cos(\beta_{\text{cross}}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_{T/S_T}(\alpha_{\text{upflow}}) = \begin{pmatrix} \cos(\alpha_{\text{upflow}}) & 0 & -\sin(\alpha_{\text{upflow}}) \\ 0 & 1 & 0 \\ \sin(\alpha_{\text{upflow}}) & 0 & \cos(\alpha_{\text{upflow}}) \end{pmatrix}$$

So the total rotation is

$$C_{T/w_T}(\alpha_{\text{upflow}}, \beta_{\text{cross}}) = C_{T/s_T}(\alpha_{\text{upflow}})\, C_{s_T/w_T}(\beta_{\text{cross}})$$

```
Off[General::"spell1", General::"spell"]
```

$$\texttt{CsTwT[}\beta\texttt{cross\_] = } \begin{pmatrix} \texttt{Cos[}\beta\texttt{cross]} & \texttt{-Sin[}\beta\texttt{cross]} & 0 \\ \texttt{Sin[}\beta\texttt{cross]} & \texttt{Cos[}\beta\texttt{cross]} & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$\texttt{CTsT[}\alpha\texttt{upflow\_] = } \begin{pmatrix} \texttt{Cos[}\alpha\texttt{upflow]} & 0 & \texttt{-Sin[}\alpha\texttt{upflow]} \\ 0 & 1 & 0 \\ \texttt{Sin[}\alpha\texttt{upflow]} & 0 & \texttt{Cos[}\alpha\texttt{upflow]} \end{pmatrix};$$

```
CTwT[αupflow_, βcross_] = CTsT[αupflow].CsTwT[βcross];
CTwT[αupflow, βcross] // MatrixForm
```

$$\begin{pmatrix} \texttt{Cos[}\alpha\texttt{upflow] Cos[}\beta\texttt{cross]} & \texttt{-Cos[}\alpha\texttt{upflow] Sin[}\beta\texttt{cross]} & \texttt{-Sin[}\alpha\texttt{upflow]} \\ \texttt{Sin[}\beta\texttt{cross]} & \texttt{Cos[}\beta\texttt{cross]} & 0 \\ \texttt{Cos[}\beta\texttt{cross] Sin[}\alpha\texttt{upflow]} & \texttt{-Sin[}\alpha\texttt{upflow] Sin[}\beta\texttt{cross]} & \texttt{Cos[}\alpha\texttt{upflow]} \end{pmatrix}$$

So if the tunnel measures forces in the tunnel axis, we have

$$\overline{F}_A^{\,T} = \begin{pmatrix} \text{force in } x_T \text{ axis (we've been calling this } C_{D_{\text{INI}}}\, q_c\, S_w) \\ \text{force in } y_T \text{ axis (we've been calling this } C_{Y_{\text{INI}}}\, q_c\, S_w) \\ \text{force in } z_T \text{ axis (we've been calling this } C_{L_{\text{INI}}}\, q_c\, S_w) \end{pmatrix}$$

So the true forces in the wind frame are

$$\overline{F}_A^{\,w_T} = C_{w_T/T}(\alpha_{\text{upflow}}, \beta_{\text{cross}})\, \overline{F}_A^{\,T}$$

We can normalize to obtain the coefficients in the wind frame.

```
CwTT[αupflow_, βcross_] = Transpose[CTwT[αupflow, βcross]];
```

$$\texttt{CwTT[}\alpha\texttt{upflow, }\beta\texttt{cross].}\begin{pmatrix} \texttt{CDini} \\ \texttt{CYini} \\ \texttt{CLini} \end{pmatrix} \texttt{ // MatrixForm}$$

$$\begin{pmatrix} \texttt{CDini Cos[}\alpha\texttt{upflow] Cos[}\beta\texttt{cross] + CLini Cos[}\beta\texttt{cross] Sin[}\alpha\texttt{upflow] + CYini Sin[}\beta\texttt{cross]} \\ \texttt{CYini Cos[}\beta\texttt{cross] - CDini Cos[}\alpha\texttt{upflow] Sin[}\beta\texttt{cross] - CLini Sin[}\alpha\texttt{upflow] Sin[}\beta\texttt{cross]} \\ \texttt{CLini Cos[}\alpha\texttt{upflow] - CDini Sin[}\alpha\texttt{upflow]} \end{pmatrix}$$

Let us look at the situation at the Kirsten wind tunnel.

<u>Show slide about bellmouth turning vanes and how each is tuned to try and minimize upflow and cross flow.</u>

In the case of the Kirsten Wind Tunnel, $\alpha_{\text{upflow}} = -0.012^o$, $\beta_{\text{cross}} = 0^o$

$$\texttt{CwTT[}\alpha\texttt{upflow, 0].}\begin{pmatrix} \texttt{CDini} \\ \texttt{CYini} \\ \texttt{CLini} \end{pmatrix} \texttt{ // MatrixForm}$$

$$\begin{pmatrix} \texttt{CDini Cos[}\alpha\texttt{upflow] + CLini Sin[}\alpha\texttt{upflow]} \\ \texttt{CYini} \\ \texttt{CLini Cos[}\alpha\texttt{upflow] - CDini Sin[}\alpha\texttt{upflow]} \end{pmatrix}$$

So we have

$$\overline{F}_A^{\,w_T} = \begin{pmatrix} C_{D_{\text{INI}}} \cos(\alpha_{\text{upflow}}) + C_{L_{\text{INI}}} \sin(\alpha_{\text{upflow}}) \\ C_{Y_{\text{INI}}} \\ C_{L_{\text{INI}}} \cos(\alpha_{\text{upflow}}) - C_{D_{\text{INI}}} \sin(\alpha_{\text{upflow}}) \end{pmatrix} q_c\, S_w$$

Note that since α_{upflow} is so small, we can apply the small angle approximation with little error

$$\overline{F}_A^{\,w_T} \approx \begin{pmatrix} C_{D_{\text{INI}}} + C_{L_{\text{INI}}}\, \alpha_{\text{upflow}} \\ C_{Y_{\text{INI}}} \\ C_{L_{\text{INI}}} - C_{D_{\text{INI}}}\, \alpha_{\text{upflow}} \end{pmatrix} q_c\, S_w$$

We see that the effect of upflow on lift is of magnitude $-C_{D_{\text{INI}}}\, \alpha_{\text{upflow}}$. In most cases, both $C_{D_{\text{INI}}}$ and α_{upflow} are small so their product is even smaller. Combining this with the fact that lift is typically much larger than drag, we can assume that $-C_{D_{\text{INI}}}\, \alpha_{\text{upflow}}$ is negligible and we have

$$\overline{F}_A^{\,w_T} \approx \begin{pmatrix} C_{D_{\text{INI}}} + C_{L_{\text{INI}}}\, \alpha_{\text{upflow}} \\ C_{Y_{\text{INI}}} \\ C_{L_{\text{INI}}} \end{pmatrix} q_c\, S_w$$

So the upflow correction is only applied to drag and the magnitude of the coefficient correction is given by

$$\Delta C_{D_{\text{upflow}}} = \alpha_{\text{upflow}}\, C_{L_{\text{INI}}}$$

We can perform a similar analysis on the moment coefficients but it will turn out to be a similar small magnitude corrections so it is not applied. Furthermore, we should defer the formal analysis of this until after we have discussed the final axis transfer as there are nuances associated with applying rotation matrices to moment coefficient data.

I. Wall Corrections

In free flight, the aircraft produces downwash. Can think of this as a balance of momentum where air is expelled with a downward force to suspend the aircraft in the air.

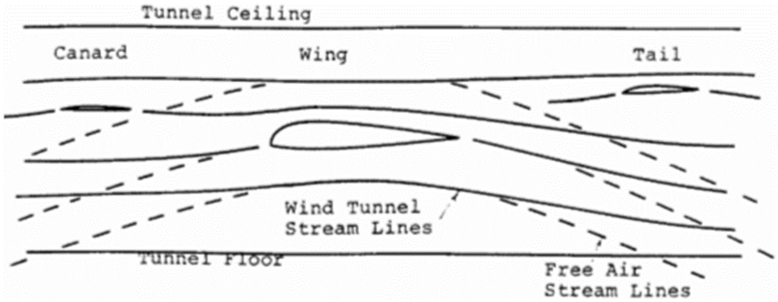


Fig. 4: The change in angle of attack at the wing, tail and canard due to the wind tunnel walls

So the effective angle of attack of the model is not what it would be in free flight. The angle of attack in the wind tunnel is actually lower than what it should be.

Angle of attack correction is given by

$$\Delta\alpha_{\mathrm{WC}} = \delta_W\left(\frac{S_W}{C}\right) C_{L_W}$$

where δ_W = correction parameter

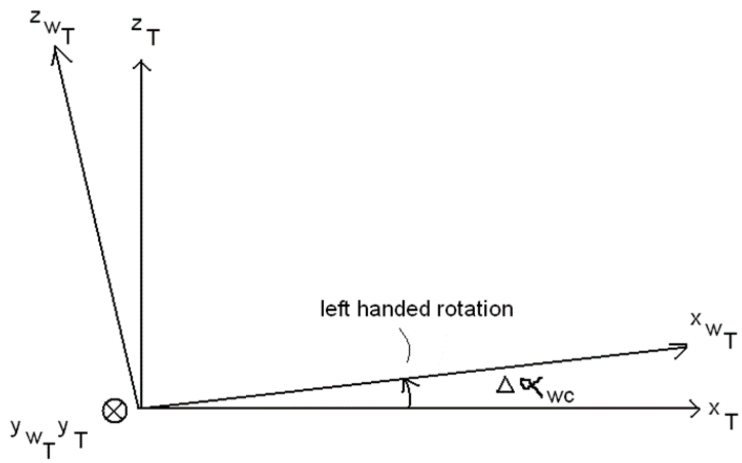
C = test section cross sectional area, not accounting for the fillets (96 ft^2)

Typically, $\delta_W > 0$ and since all other parameters are positive, the $\Delta\alpha_{\mathrm{WC}}$ correction is positive.

Notice that the magnitude of the correction is proportional to the lift of the wing (C_{L_W}). This makes sense from a geometric perspective. In free flight, in order to generate more lift, the streamlines must be deflected downwards more (like a rocket engine expelling more momentum downwards). So in the tunnel, the more lift you are generating, the more the streamlines are unnaturally bent due to the walls and therefore, the more the angle of attack must be corrected.

Notice that this requires computing the lift of the wing only. In many cases it is impractical to test the lift of the wing only as this would require removing the fuselage, empennage, etc. In many cases, the only other surface which generates significant lift is the horizontal tail. Therefore, a reasonable approximation is to simply remove the tail and perform a "tail-off" run in order to measure the lift generated by the wing/body combination for a given configuration. This tail-off run for a given configuration is known as a "lift-set". Obtaining lift-sets for every configuration is time consuming so often, lift sets are just obtained for crucial configurations. For other non-crucial runs, a lift-set with a similar configuration is used. Similar runs are determined by runs where the lift from the wing is most likely not changing much (ie a changes in aileron angles probably will not change the lift of the wing but a change in flap setting will probably have significant impact on the lift of the wing).

The other consequence of this angle of attack correction is the rotation of the lift and drag forces. This is similar to the upflow correction.



Since this is a left handed rotation

$$C_{w_T/T}(\Delta\alpha_{wc}) = \begin{pmatrix} \cos(-\Delta\alpha_{wc}) & 0 & -\sin(-\Delta\alpha_{wc}) \\ 0 & 1 & 0 \\ \sin(-\Delta\alpha_{wc}) & 0 & \cos(-\Delta\alpha_{wc}) \end{pmatrix}$$

```
CwTT[Δαwc_] = ( Cos[-Δαwc]  0  -Sin[-Δαwc] )
              (     0       1       0      ) ;
              ( Sin[-Δαwc]  0   Cos[-Δαwc] )
```

```
CwTT[Δαwc].( D )
           ( Y ) // MatrixForm
           ( L )
```

```
( D Cos[Δαwc] + L Sin[Δαwc] )
( Y                         )
( L Cos[Δαwc] - D Sin[Δαwc] )
```

So we have the corrected values of lift, drag, and sideforce as

$$\overline{F}_A^{\,w_T} = \begin{pmatrix} D\cos(\Delta\alpha_{wc}) + L\sin(\Delta\alpha_{wc}) \\ Y \\ L\cos(\Delta\alpha_{wc}) - D\sin(\Delta\alpha_{wc}) \end{pmatrix}$$

Once again, using the same reasoning as in the flow angularity correction section (ie small $\Delta\alpha_{wc}$ and $L >> D$), we have

$$\overline{F}_A^{\,w_T} = \begin{pmatrix} D + L\,\Delta\alpha_{wc} \\ Y \\ L \end{pmatrix}$$

So we see that we can roll this wall correction factor up into an additive drag correction of the form

$$\Delta C_{D_{wc}} = \Delta\alpha_{wc}\, C_{L_{INI}}$$

Wall corrections also affect the angle of attack of the tail

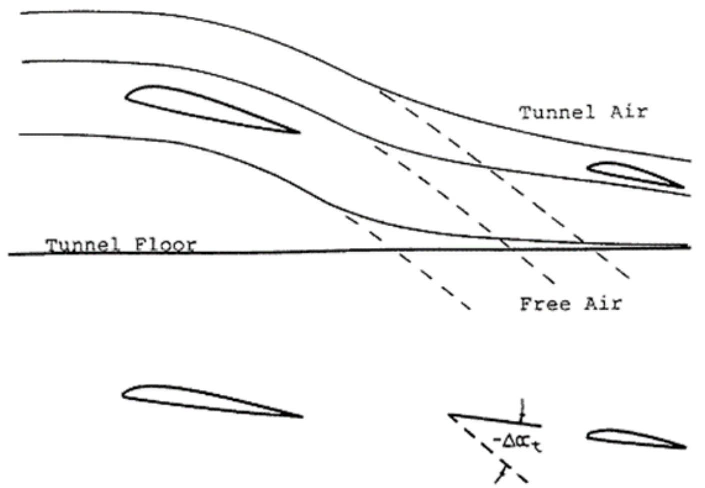
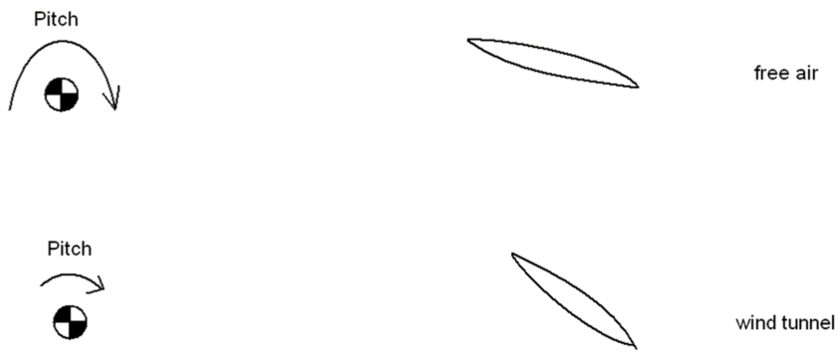


Fig. 3: Effect of tunnel walls on flow of air past wing and tail

So in the wind tunnel, the angle of attack of the tail is actually higher than it should be. This may effect both the lift and pitching moment from the tail. The effect of lift of the tail is considered insignificant, but the effect of this $\Delta\alpha_t$ on the pitching moment is not insignificant and must be taken into account.



To correct for this phenomenon we apply a pitching moment correction of the form

$$\Delta C_{M_{WC}} = -\delta_{A_S} \frac{\partial C_M}{\partial \delta_s} C_{L_W} \frac{S_W}{C}$$

In most cases, $\delta_{A_S} > 0$. Note that increasing the stabilizer angle actually decreases overall pitching moment. Therefore, for most cases, $\frac{\partial C_M}{\partial \delta_s} < 0$. Since C_{L_W}, S_W, and C are all positive, we see that the overall correction is positive. Since the pitching moment measured in the wind tunnel is actually too low, $\Delta C_{M_{WC}}$ should be positive.

The parameter $\partial C_M / \partial \delta_s$ is typically obtained during testing. This measures how the pitching moment changes with a change in tail stabilizer angle. This means having a tail which be positioned at various angles of incidence and then performing various runs. To simplify testing, typically one value of $\partial C_M / \partial \delta_s$ for all configurations. This may need to be changed if you have two different tails or are interested in very accurate pitching moment corrections.

Note that in some cases, we may not want to apply wall corrections.

<u>Show picture of ground plane</u>

In these configurations, we want the effects of the ground (AKA the walls) to be included in the measurements. These effects are typically called "ground effect".

J. Final Coefficients / Corrected Angle of Attack

So with all these corrections, the final coefficients are given by

$$C_{L_{\text{WA}}} = C_{L_{\text{INI}}}$$
$$C_{D_{\text{WA}}} = C_{D_{\text{INI}}} + \Delta C_{D_{\text{wc}}} + \Delta C_{D_{\text{upflow}}}$$
$$C_{Y_{\text{WA}}} = C_{Y_{\text{INI}}}$$

$$C_{M_{\text{WA}}} = C_{M_{\text{INI}}} + \Delta C_{M_{\text{wc}}}$$
$$C_{R_{\text{WA}}} = C_{R_{\text{INI}}}$$
$$C_{N_{\text{WA}}} = C_{N_{\text{INI}}}$$

The angle of attack is corrected to

$$\alpha_c = \alpha_i + \alpha_{\text{upflow}} + \Delta\alpha_{\text{wc}}$$

For completeness, recall that the corrected dynamic pressure is given by

$$q_c = q_a (1 + \varepsilon)^2$$

K. Axis Transfers

Finally, coefficients can be transferred from wind axis to stability

$$C_{L_{\text{SA}}} = C_{L_{\text{WA}}}$$
$$C_{D_{\text{SA}}} = C_{D_{\text{WA}}} \cos(\psi) - C_{Y_{\text{WA}}} \sin(\psi)$$
$$C_{Y_{\text{SA}}} = C_{Y_{\text{WA}}} \cos(\psi) + C_{D_{\text{WA}}} \sin(\psi)$$

$$C_{M_{\text{SAxx}}} = C_{M_{\text{WAxx}}} \cos(\psi) - \frac{b_{\text{ref}}}{\bar{c}} C_{R_{\text{WAxx}}} \sin(\psi)$$
$$C_{R_{\text{SAxx}}} = C_{R_{\text{WAxx}}} \cos(\psi) + \frac{\bar{c}}{b_{\text{ref}}} C_{M_{\text{WAxx}}} \sin(\psi)$$
$$C_{N_{\text{SAxx}}} = C_{N_{\text{WAxx}}}$$

And to the body axis using

$$C_{L_{\text{BA}}} = C_{L_{\text{WA}}} \cos(\alpha) + C_{D_{\text{WA}}} \cos(\psi) \sin(\alpha) - C_{Y_{\text{WA}}} \sin(\psi) \sin(\alpha)$$
$$C_{D_{\text{BA}}} = C_{D_{\text{WA}}} \cos(\psi) \cos(\alpha) - C_{L_{\text{WA}}} \sin(\alpha) - C_{Y_{\text{WA}}} \sin(\psi) \cos(\alpha)$$
$$C_{Y_{\text{BA}}} = C_{Y_{\text{WA}}} \cos(\psi) + C_{D_{\text{WA}}} \sin(\psi)$$

$$C_{M_{\text{BAxx}}} = C_{M_{\text{WAxx}}} \cos(\psi) - \frac{b_{\text{ref}}}{\bar{c}} C_{R_{\text{WAxx}}} \sin(\psi)$$
$$C_{R_{\text{BAxx}}} = C_{R_{\text{WAxx}}} \cos(\psi) \cos(\alpha) + \frac{\bar{c}}{b_{\text{ref}}} C_{M_{\text{WAxx}}} \sin(\psi) \cos(\alpha) - C_{N_{\text{WAxx}}} \sin(\alpha)$$
$$C_{N_{\text{BAxx}}} = C_{N_{\text{WAxx}}} \cos(\alpha) + C_{R_{\text{WAxx}}} \cos(\psi) \sin(\alpha) + \frac{\bar{c}}{b_{\text{ref}}} C_{M_{\text{WAxx}}} \sin(\psi) \sin(\alpha)$$

These could be rewritten in more convenient form using matrix notation using the rotations between stability and wind axis (not the tunnel stability and wind axes).

$$C_{b/s}(\alpha) = \begin{pmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix}$$

$$C_{s/w}(\beta) = \begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Cbs[α_] = ( Cos[α]  0  -Sin[α] )
          (   0     1     0    ) ;
          ( Sin[α]  0   Cos[α] )
Csw[β_] = ( Cos[β]  -Sin[β]  0 )
          ( Sin[β]   Cos[β]  0 ) ;
          (   0        0     1 )
```

Recall with this system used with these rotation matrices, the axis of the body system were defined as

x_b = positive forward **(Eq.K.1)**
y_b = positive out right wingtip
z_b = positive down

Recall that the rotation between the tunnel body and the body frame (and similarly for the tunnel stability to stability and tunnel wind to wind axes) were constant rotations about the y axis through and angle of π

$$C_{b_T/b} = \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix}$$ (rotate from body axis to wind tunnel body axis)

$$C_{s_T/s} = \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix}$$ (rotate from stability axis to wind tunnel stability axis)

$$C_{w_T/w} = \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix}$$ (rotate from wind axis to wind tunnel wind axis)

Furthermore,

Lift as positive up
Drag as positive back
Sideforce as positive out right wingtip

Pitch as positive nose up
Yaw as positive nose right
Roll as positive right wingtip down

Given this information, we can write

$$\overline{F}_A^{\,w_T} = \begin{pmatrix} C_{D_{\text{WA}}} \\ C_{Y_{\text{WA}}} \\ C_{L_{\text{WA}}} \end{pmatrix} q_c\, S_W$$

To rotate this to the wind tunnel stability axis, we use

$$\overline{F}_A^{\,s_T} = C_{s_{T/s}} \cdot C_{s/w}(\beta) \cdot C_{w/w_T}\, \overline{F}_A^{\,w_T}$$

$$= \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix} \begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix}^T \begin{pmatrix} C_{D_{\text{WA}}} \\ C_{Y_{\text{WA}}} \\ C_{L_{\text{WA}}} \end{pmatrix} q_c\, S_W$$

```
FAsT = {{Cos[π], 0, -Sin[π]}, {0, 1, 0}, {Sin[π], 0, Cos[π]}}.Csw[β].Transpose[{{Cos[π], 0, -Sin[π]}, {0, 1, 0}, {Sin[π], 0, Cos[π]}}].{CDwa, CYwa, CLwa};
FAsT // MatrixForm
```

$$\begin{pmatrix} \text{CDwa}\,\text{Cos}[\beta] + \text{CYwa}\,\text{Sin}[\beta] \\ \text{CYwa}\,\text{Cos}[\beta] - \text{CDwa}\,\text{Sin}[\beta] \\ \text{CLwa} \end{pmatrix}$$

Finally, recall that $\beta = -\psi$

```
FAsT /. {β → -ψ} // MatrixForm
```

$$\begin{pmatrix} \text{CDwa}\,\text{Cos}[\psi] - \text{CYwa}\,\text{Sin}[\psi] \\ \text{CYwa}\,\text{Cos}[\psi] + \text{CDwa}\,\text{Sin}[\psi] \\ \text{CLwa} \end{pmatrix}$$

Similarly for the body axis equations, we have

$$\overline{F}_A^{\,b_T} = C_{b_{T/b}} \cdot C_{b/s}(\alpha) \cdot C_{s/w}(\beta) \cdot C_{w/w_T}\, \overline{F}_A^{\,w_T}$$

$$= \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix} \cdot$$

$$\begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\pi) & 0 & -\sin(\pi) \\ 0 & 1 & 0 \\ \sin(\pi) & 0 & \cos(\pi) \end{pmatrix}^T \begin{pmatrix} C_{D_{\text{WA}}} \\ C_{Y_{\text{WA}}} \\ C_{L_{\text{WA}}} \end{pmatrix} q_c S_W$$

```
FAbT = ( Cos[π]  0  -Sin[π]
         0       1   0
         Sin[π]  0   Cos[π] ).Cbs[α].Csw[β].Transpose[( Cos[π]  0  -Sin[π]
                                                         0       1   0
                                                         Sin[π]  0   Cos[π] )].( CDwa
                                                                                 CYwa
                                                                                 CLwa );
```

And making the substitution that $\beta = -\psi$

```
FAbT /. {β → -ψ} // MatrixForm
```

$$\begin{pmatrix} \text{CDwa} \cos[\alpha] \cos[\psi] - \text{CLwa} \sin[\alpha] - \text{CYwa} \cos[\alpha] \sin[\psi] \\ \text{CYwa} \cos[\psi] + \text{CDwa} \sin[\psi] \\ \text{CLwa} \cos[\alpha] + \text{CDwa} \cos[\psi] \sin[\alpha] - \text{CYwa} \sin[\alpha] \sin[\psi] \end{pmatrix}$$

It may be temping to perform a similar procedure with the moment coefficient. However you need to be very careful. If the different moment coefficients are normalized by different characteristics lengths, then these coefficients must be re-normalized to an actual engineering moment (not a coefficient) before performing a rotation.