Christopher Lum
lum@uw.edu

Lecture 04a
Introduction to Bode Plots

Lecture is on YouTube

The YouTube video entitled ‘Introduction to Bode Plots’ that covers this lecture is located at https://youtu.be/KX7GNqy3k7w.

Outline

-Bode Plots

Bode Plots

So we see that when the input is a sinusoid, the steady state output of the system is equal to the input sinusoid amplified by $|G(j\omega)|$ and shifted in phase by $\theta = \angle G(j\omega)$

$$y_{ss}(t) = A\,|G(j\omega)|\,\sin(\omega t + \theta) \qquad \textbf{(Eq.1)}$$

where $\quad |G(j\omega)| = \sqrt{(\mathrm{Re}[G(j\omega)])^2 + (\mathrm{Im}[G(j\omega)])^2}$

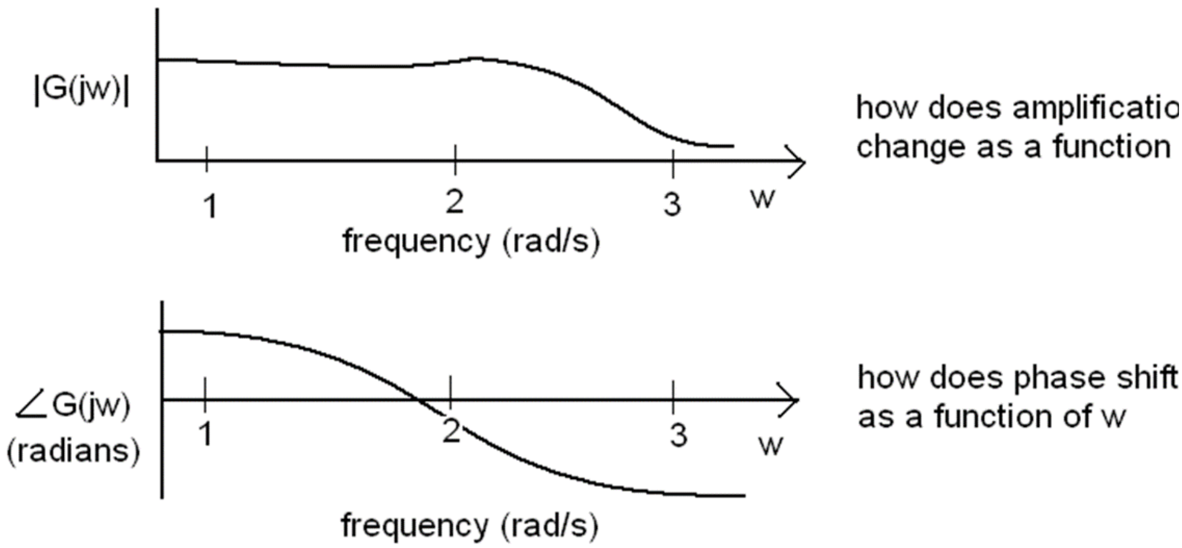
$$\theta = \angle G(j\omega) = \mathrm{atan2}(\mathrm{Im}[G(j\omega)], \mathrm{Re}[G(j\omega)]) = \mathrm{atan2}(y, x)$$

In the previous lecture, we examined the response at a single frequency.

The plot of magnitude and phase as a function of ω is known as a **Bode plot**. This was developed in the 1930s by American engineer Hendrik Wade Bode. He has Dutch ancestry and his last name may have been pronounced “bow-duh” but most people say “bow-dee”

Perform demo/example of a mass, spring, damper. Show how the amplitude and phase varies with frequency.

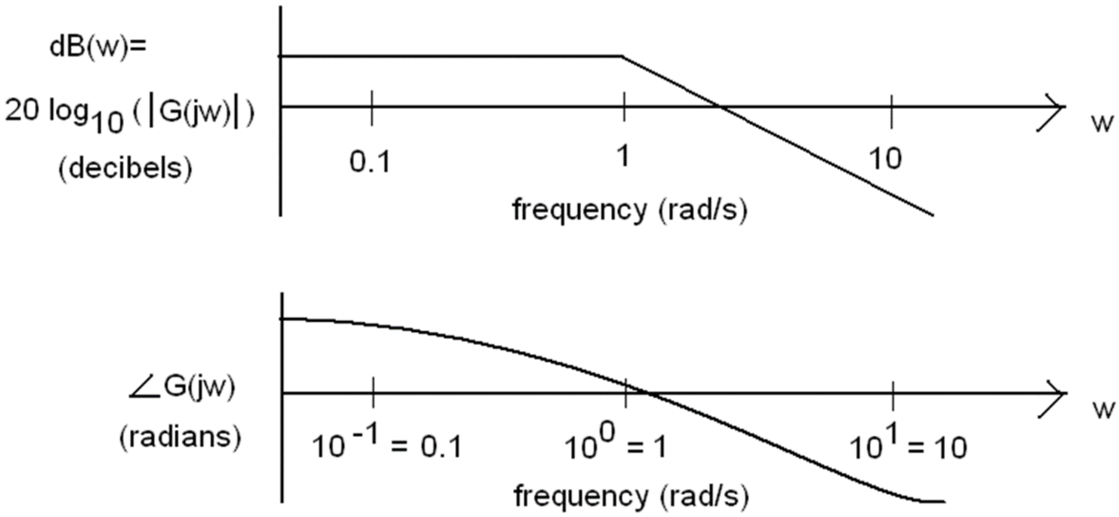
It becomes useful to plot the amplification and phase shift as a function of ω.



It turns out that is more useful to look at semi-logarithmic plots. In other words, we plot the x-axis in a $\log_{10}$ scale. Note: in Matlab, you can use the function ‘semilogx’ to draw the x-axis in a log scale. Warning: in Matlab and Mathematica, ‘Log’ denotes the natural logarithm, not the base 10 logarithm.

Furthermore, the amplification can be measured in terms of decibels which is defined as

$20 \log_{10}(\,|\,G(j\,\omega)\,|\,)$ = amplification measured in decibels



Let's look at what a unity amplification corresponds to in decibels.

$20 \log_{10}(1) = 0$

So we see that if the decibels greater than 0 imply an amplification of the signal whereas the decibel value of less than zero implies an attenuation of the signal

dB positive $\Longleftrightarrow$ signal amplified

dB negative $\Longleftrightarrow$ signal attenuated

Example: Mass/Spring/Damper

Let's use the same mass/spring/damper system we just looked at. Recall that the transfer function was

$$G(s) = \frac{Z(s)}{U(s)} = \frac{3}{s^2+\frac{1}{2}\,s+4} \qquad \text{(recall: } m = 1/3,\ c = 1/6,\ k = 4/3\text{)}$$

We showed in the previous lecture that we can write $G(j\,\omega)$ as

$$G(j\,\omega) = \alpha + \beta\,j$$

where $\alpha = \mathrm{Re}[G(j\,\omega)] = \frac{12-3\,\omega^2}{16-\frac{31\,\omega^2}{4}+\omega^4}$

$\beta = \mathrm{Im}[G(j\,\omega)] = \frac{-\frac{3\,\omega}{2}}{16-\frac{31\,\omega^2}{4}+\omega^4}$

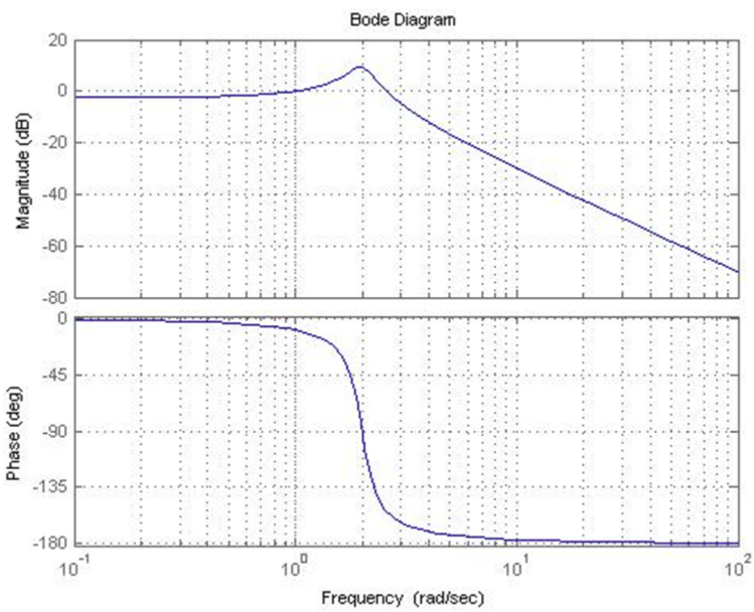
We could now calculate the bode plot using

$$20\log_{10}(\,|\,G(j\,\omega)\,|\,) = 20\log_{10}\left(\sqrt{\alpha^2+\beta^2}\right)$$
$$\angle G(j\,\omega) = \tan^{-1}(\beta/\alpha)$$

We can numerically calculate the amplification and phase shift for various frequencies. To do this

1. Generate a list of frequencies where you would like to evaluate the bode plot at.
2. Compute the amplification and phase shift at each of these frequencies.
3. Convert the amplification to dB
4. Plot on a $\log_{10}$ x-axis.

Matlab can do this for us using the 'bode' command. The result is shown below



Let's see if this confirms our previous result. By reading the plot at $\omega = 3$, we see that

$$20 \log_{10}(\mid G(3\,j) \mid) \approx -4.76$$

$$\log_{10}(\mid G(3\,j) \mid) \approx -\frac{4.76}{20}$$

$$\left| G(3\,j) \right| \approx 10^{-\frac{4.76}{20}}$$

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10^(-4.76/20)
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0.578096
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So we have

$$\mid G(3\,j) \mid \approx 0.57809$$

The phase angle can be read directly off the plot

$$\angle G(3\,j) \approx -163\,\frac{\pi}{180}$$

These are the same results that we obtained previously

More information about resonant frequency is in a dedicated lecture.