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Lecture 06c
Using ‘rlocus’ in Matlab to Plot the Root Locus

Lecture is on YouTube

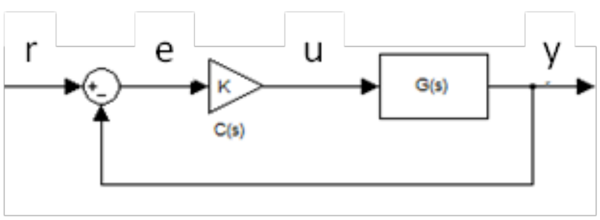
The YouTube video entitled ‘Using ‘rlocus’ in Matlab to Plot the Root Locus’ that covers this lecture is located at https://youtu.be/im19KuzjWwo.

Outline

-Classic Root Locus
-rlocus

Classic Root Locus

Recall from the previous lecture that the root locus architecture we were considering was



This had a closed loop transfer function of

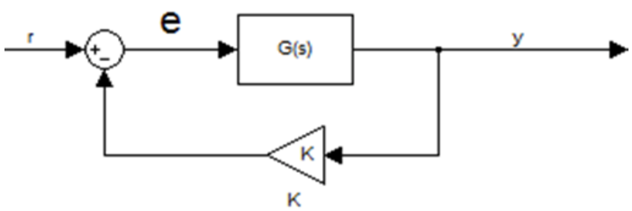
$$T(s) = \frac{K\,G(s)}{1+K\,G(s)}$$

Therefore the closed loop characteristic equation we previously considered was given by

$$\Delta(s) = 1 + K\,G(s) = 0$$

rlocus

Matlab provides the function ‘rlocus’ to help sketch root locus plots. However, reading the documentation shows that this actually computes the root locus for the following system.



We can compute the closed loop transfer function for this system. We do this by first computing the output signal, $Y(s)$.

$$Y(s) = G(s)\,E(s)$$

$$Y(s) = G(s)\,(R(s) - K\,Y(s))$$

$$Y(s) = G(s)\,R(s) - K\,G(s)\,Y(s)$$

$$Y(s) + K\,G(s)\,Y(s) = G(s)\,R(s)$$

$$(1 + K\,G(s))\,Y(s) = G(s)\,R(s)$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+K\,G(s)} \qquad \textbf{\textit{(Eq.A.1)}}$$

Comparing Eq.A.1 with Eq.1, we see that the characteristic equation of the closed loop system for both the system we want to analyze (Eq.1) and the system analyzed by ‘rlocus’ is the same. Therefore, we can safely use ‘rlocus’ to sketch the root locus for our system (Eq.10).

Previous Example 1

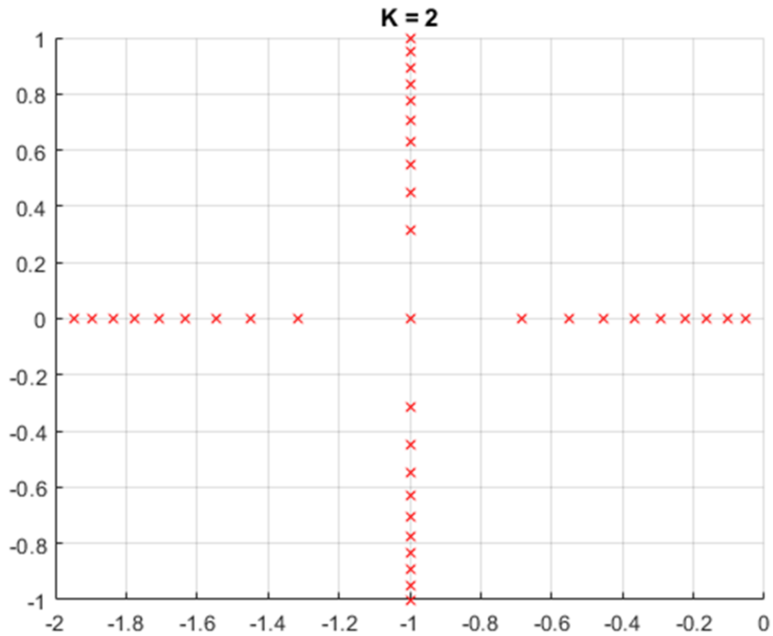
Recall the example we had of

$$G_1(s) = \frac{1}{s(s+2)} = \frac{1}{s^2+2\,s}$$

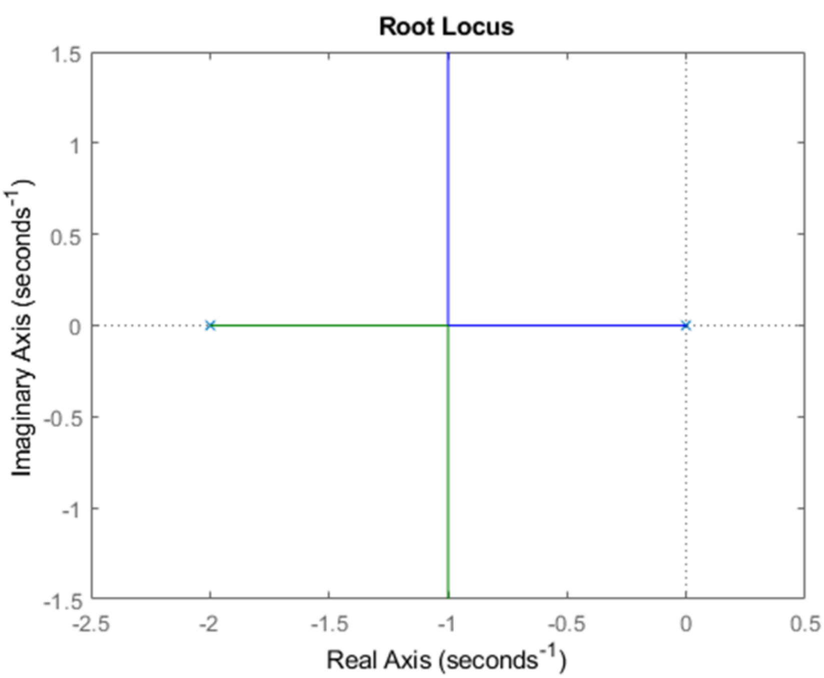
We showed that the closed loop characteristic equation under the classical feedback architecture was

$$\Delta(s) = s^2 + 2\,s + K$$

We can perform a manual, numerical calculation of the root locus by iteratively solve $\Delta(s) = 0$ for various K values



Based on our previous discussion, we know that rlocus will compute the root locus for a slightly different architecture, but the resulting closed loop characteristic equation is the same.

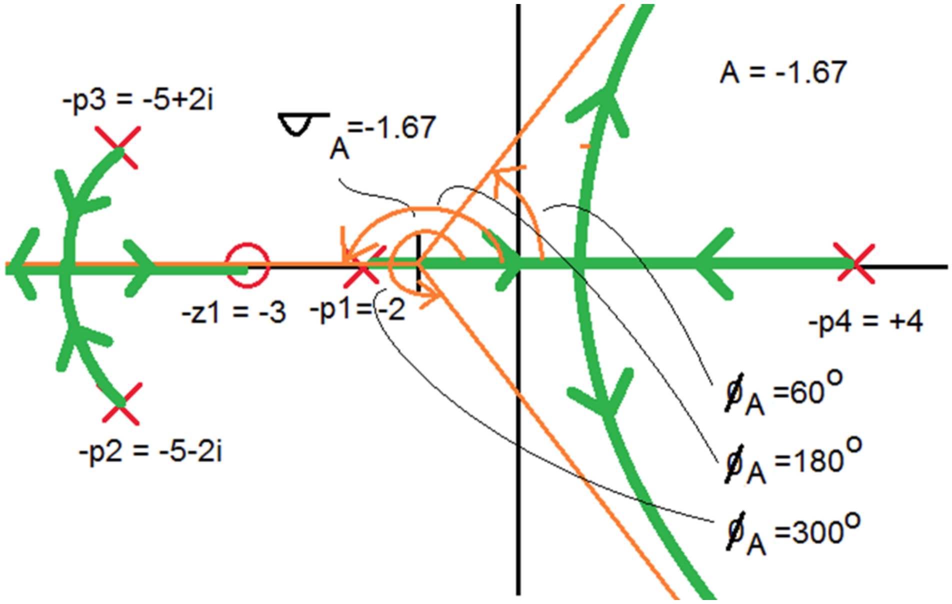


Previous Example 2

Recall the example we had of

$$G_6(s) = \frac{s+3}{s^4+8\,s^3+s^2-138\,s-232} = \frac{(s+3)}{(s+2)\,(s+5+2\,i)\,(s+5-2\,i)\,(s-4)}$$

Our approximate sketch of the root locus is shown below

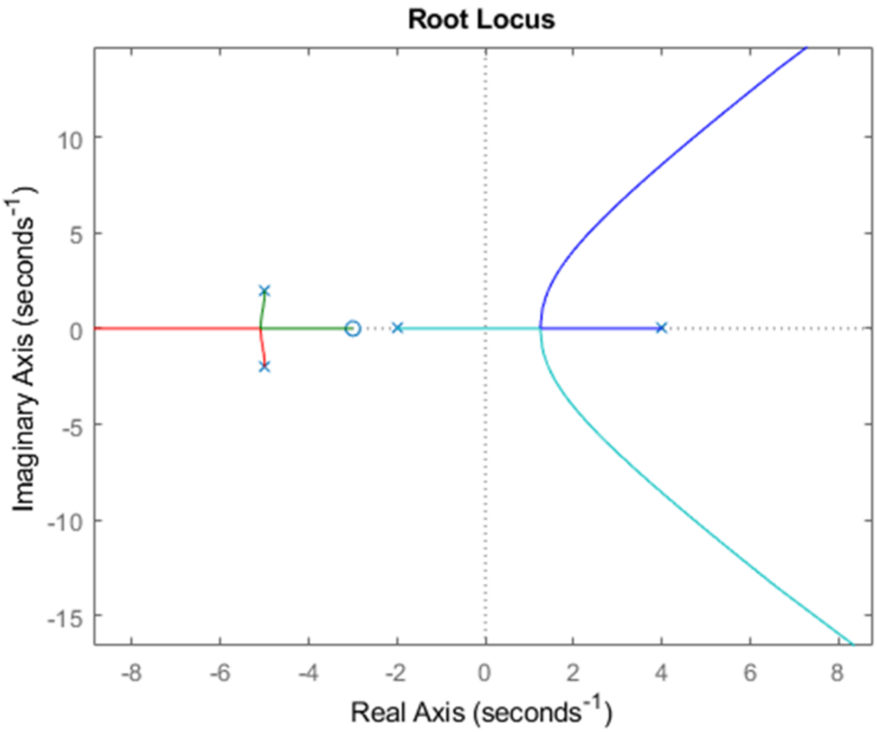


We can verify this with the 'rlocus' command. The appropriate Matlab commands are shown below

```
G6_num = [1 3];
G6_den = [1 8 1 -138 -232];
roots(G6_den)            %check open loop roots
G6 = tf(G6_num, G6_den)

figure
rlocus(G6)
```

The resulting figure is shown below



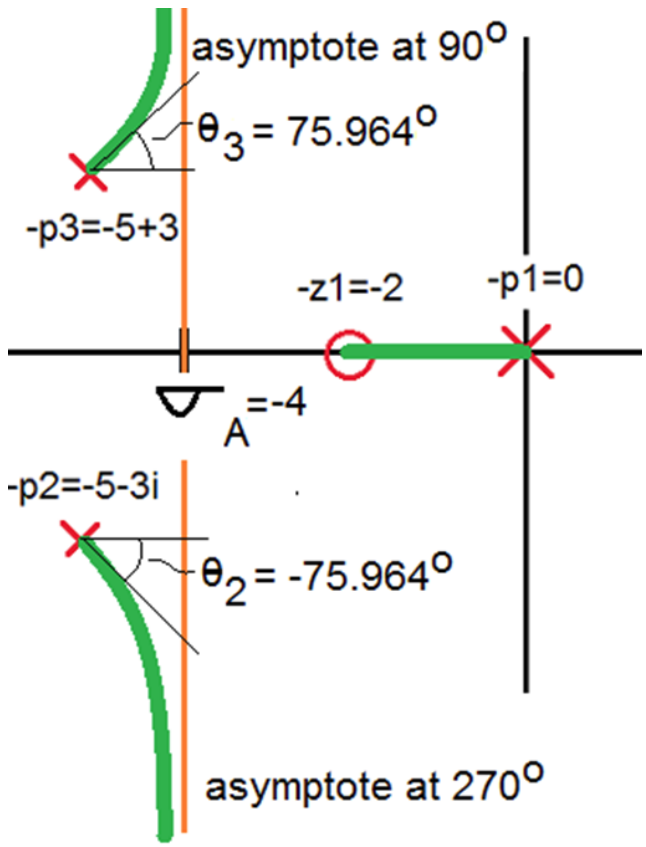
As can be seen, these match up quite well.

Previous Example 2

Recall the example we had of

$$G_7(s) = \frac{s+2}{s^3+10\,s^2+34\,s} = \frac{(s+2)}{s(s+5+3\,i)\,(s+5-3\,i)}$$

Our approximate sketch of the root locus is shown below



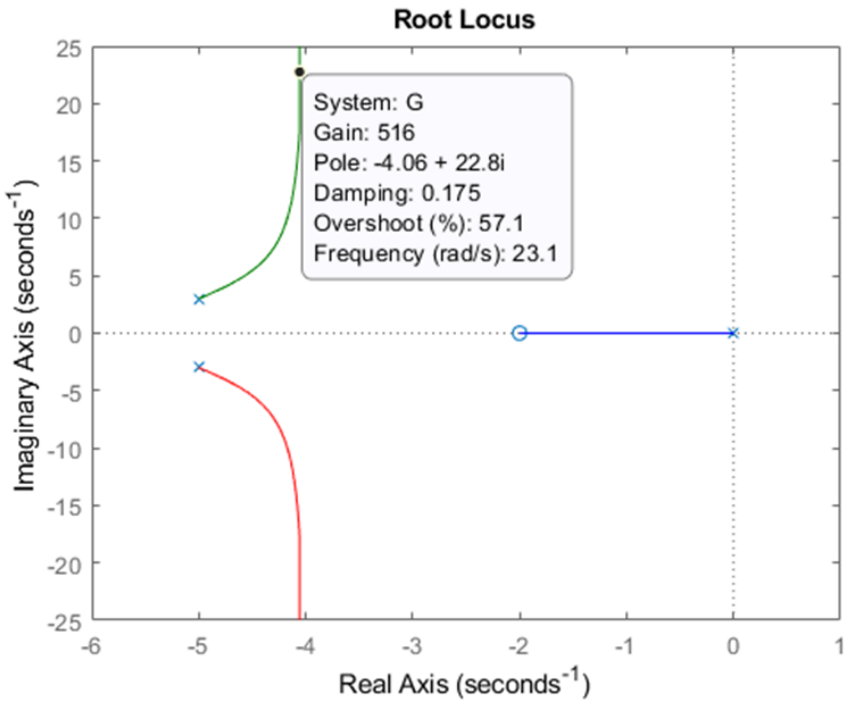
We can verify this with the 'rlocus' command. The appropriate Matlab commands are shown below (note we use 'zpk' instead of 'tf' simply to obtain practice with this other function)

```
z7 = [-2];
p7 = [0 -5-3*i -5+3*i];
k7 = 1;

G7 = zpk(z7,p7,k7)

figure
rlocus(G7)
```

The resulting figure is shown below



As can be seen, these match up quite well. Furthermore, note that you can click on areas of the root locus plot and see what the associated gain *K* is.