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Lecture 05d
Manipulating Aerodynamic Coefficients

The YouTube video entitled ‘Manipulating Aerodynamic Coefficients’ that covers this lecture is located at https://youtu.be/Mv6aUQkK59s

Outline

-Review of Aerodynamic Coefficients
-Adding/Subtracting Aerodynamic Coefficients
-Rotating Aerodynamic Coefficients

Review of Aerodynamic Coefficients

Recall that the dimensionless aerodynamic coefficients for the 3 forces (lift, drag, and sideforce) are defined as

$$C_L = \frac{L}{q\,S}$$

$$C_D = \frac{D}{q\,S}$$

$$C_Y = \frac{\text{SF}}{q\,S}$$

where q = dynamic pressure (N/m^2)
S = reference area, typically wing planform area (m^2)
L, D, SF = lift, drag, sideforce respectively (N)

In a similar fashion, the dimensionless aerodynamic coefficients for the 3 moments (roll, pitch, yaw) are defined as

$$C_l = \frac{\text{RM}}{q\,S\,b_{\text{ref}}}$$

$$C_m = \frac{\text{PM}}{q\,S\,\bar{c}}$$

$$C_n = \frac{\text{YM}}{q\,S\,b_{\text{ref}}}$$

where b_{ref} = reference length, typically wingspan (m)
$\bar{c}$ = reference length, typically mean aerodynamic chord (m)
RM, PM, YM = rolling, pitching, and yawing moment respectively ($N\,m$)

Adding/Subtracting Aerodynamic Coefficients

Example: Properly Adding Coefficients

A UAV is being tested in a wind tunnel. The development team is split up into two groups. Team A is in charge of developing the wing body combination whereas Team B is in charge of the tail.

Team A determines that UAV should have a wing area of $S = 2\,m^2$. At a dynamic pressure of $q = 15\,N/m^2$ this wing generates $L = 24\,N$ of lift. Team A therefore computes the coefficient of lift of the wing as

$$C_{L_{\text{wb}}} = \frac{L}{q\,S} = \frac{24}{15*2} = 0.8$$

Team B is in charge of designing the tail and chooses an shape with $S_t = 0.1\,m^2$. At this same dynamic pressure, it generates $L_t = 0.87\,N$ of lift. This team normalizes the coefficient using the area of the tail and compute the lift coefficient of the tail as

$$C_{L_t} = \frac{L_t}{q\,S_t} = \frac{0.87}{15*0.1} = 0.58$$

At this point the two teams decide to predict the performance of the combined wing/body/tail combination. In this case, the INCORRECT approach would be to directly add the lift coefficients

$$C_L = C_{L_{\text{wb}}} + C_{L_t} = 0.8 + 0.58 = 1.38 \qquad \text{(INCORRECT)}$$

This corresponds to a lift force of

$$L = C_L\,q\,S = 1.38*15*2 = 41.4\,N \quad \text{(INCORRECT)}$$

This grossly over-predicts the actual performance of the aircraft. We see that we should add the two lift forces together $L = 24\,N + 0.87\,N = 24.87\,N$. We can obtain this if we properly renormalize the coefficient of lift of the tail using the area of the main wing

$$C_L = C_{L_{\text{wb}}} + C_{L_t}\left(\frac{S_t}{S}\right) = 0.8 + 0.58 \times \left(\frac{0.1}{2}\right) = 0.829 \qquad \text{(CORRECT)}$$

We see that this correctly corresponds to a total lift of $L = 24\,N + 0.87\,N = 24.87\,N$

$$L = C_L\, q\, S = 0.829 * 15 * 2 = 24.87\, N \quad \text{(CORRECT)}$$

Rotating Aerodynamic Coefficients

The moment coefficients can be written in a vector

$$\overline{C}_M = \begin{pmatrix} C_l \\ C_m \\ C_n \end{pmatrix}$$

Suppose that these coefficients were obtained in a wind tunnel, and therefore reported in the wind axis. It may be tempting to add notation to denote which frame these are expressed in

$$\overline{C}_M{}^w$$

The problem now becomes if you attempt to rotate this to another axis using a rotation matrix. In other words

$$\overline{C}_M{}^b \neq C_{b/w}(\alpha,\, \beta)\, \overline{C}_M{}^w$$

The reason for this is that $\overline{C}_M$ is not really a vector that is expressed in a frame. This is due to the inconsistent way that the coefficients are normalized (some use $\overline{c}$ and some use b_{ref}).

It is safer to first dimensionalize the vector to obtain the dimensional moment vector expressed in the wind frame first

$$\overline{M}^w = \begin{pmatrix} C_l\, q\, S\, b_{\text{ref}} \\ C_m\, q\, S\, \overline{c} \\ C_n\, q\, S\, b_{\text{ref}} \end{pmatrix}^w$$

Since this is an actual vector, this can safely be rotated to any frame using any rotation matrix

$$\overline{M}^b = C_{b/w}(\alpha,\, \beta)\, \overline{M}^w$$

Now, if desired, you can then renormalize to a coefficient

$$\overline{C}_M{}^b = \begin{pmatrix} \frac{\overline{M}^b(1)}{q\, S\, b_{\text{ref}}} \\ \frac{\overline{M}^b(2)}{q\, S\, \overline{c}} \\ \frac{\overline{M}^b(3)}{q\, S\, b_{\text{ref}}} \end{pmatrix}$$