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Lecture 02f
Roots of a Polynomial

Lecture is on YouTube

The YouTube video entitled 'Finding Roots of a Polynomial Using Matlab, Mathematica, and a TI-83' that covers this lecture is located at https://youtu.be/J8il5eB_VS8.

Outline

-Roots of a Polynomial

Finding Roots

At this point, one might ask, how do we find the poles of a complex function? If you recall, poles are defined as values of s which make the function go to ∞. In other words, these are roots of the denominator polynomial. We can use the Matlab or Mathematica to help solve for the roots.

Example: Matlab and Mathematica to Find Roots of a Polynomial

Consider the polynomial

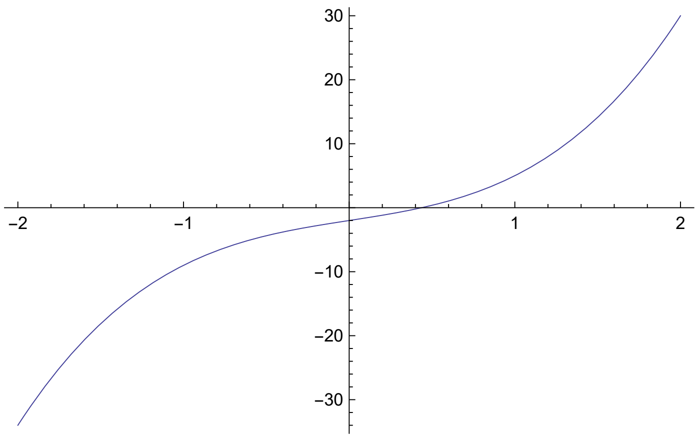
$$p(s) = 3s^3 + 4s - 2$$

We are interested in finding the roots of this polynomial, so we are interested in solving

$$p(s) = 0$$

We can plot $p(s)$ to see if we can graphically find the roots

```
p[s_] = 3 s^3 + 4 s - 2;
Plot[p[s], {s, -2, 2}]
```



Graphically, we see that there is a real root at approximately 0.4, but we need to account for the possibility of imaginary roots. We can use Matlab's 'roots' function to do this.

To do this, we need to define the polynomial within Matlab. To do this, we can first write this out explicitly in descending powers of *s*. In other words

$$p(s) = 3\,s^3 + 0\,s^2 + 4\,s - 2$$

We now define this polynomial in Matlab as an array of coefficients of *s* in descending order

```
%Define the polynomial by listing the coefficients in descending order
%(notice the entry of 0)
C = [3 0 4 -2];

%find the roots
roots(C)
```

This generates the output of

```
ans =

 [illegible]

 [illegible]

 [illegible]
```

Alternatively, we can directly do this in Mathematica using the 'Roots'

```
Roots[p[s] == 0, s]
```

$$s == \frac{1}{3}\left(-\frac{4}{\left(9+\sqrt{145}\right)^{1/3}}+\left(9+\sqrt{145}\right)^{1/3}\right) \;||\; s == \frac{2\times\left(1+i\sqrt{3}\right)}{3\left(9+\sqrt{145}\right)^{1/3}}-\frac{1}{6}\times\left(1-i\sqrt{3}\right)\left(9+\sqrt{145}\right)^{1/3} \;||$$

$$s == \frac{2\times\left(1-i\sqrt{3}\right)}{3\left(9+\sqrt{145}\right)^{1/3}}-\frac{1}{6}\times\left(1+i\sqrt{3}\right)\left(9+\sqrt{145}\right)^{1/3}$$

We can also use the more general 'Solve' function.

```
roots = Solve[p[s] == 0, s] // Simplify
```

$$\left\{\left\{s \to \frac{-4 + \left(9 + \sqrt{145}\right)^{2/3}}{3 \left(9 + \sqrt{145}\right)^{1/3}}\right\}, \left\{s \to \frac{4 + 4\,i\,\sqrt{3} + i\,\left(i + \sqrt{3}\right)\,\left(9 + \sqrt{145}\right)^{2/3}}{6 \left(9 + \sqrt{145}\right)^{1/3}}\right\}, \left\{s \to \frac{4 - 4\,i\,\sqrt{3} + \left(-1 - i\,\sqrt{3}\right)\,\left(9 + \sqrt{145}\right)^{2/3}}{6 \left(9 + \sqrt{145}\right)^{1/3}}\right\}\right\}$$

And we can ask for the roots in numerical form so we can compare with Matlab

```
roots // N
```

$\{\{s \to 0.437287\}, \{s \to -0.218643 + 1.21522\,i\}, \{s \to -0.218643 - 1.21522\,i\}\}$

We see that we obtain the same result

```
Clear[p, roots]
```