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Lecture 07a
Bandwidth of a Dynamic System

Lecture is on YouTube

The YouTube video entitled 'Bandwidth of a Dynamic System' that covers this lecture is located at https://youtu.be/evVi_D7C6mA.

Outline

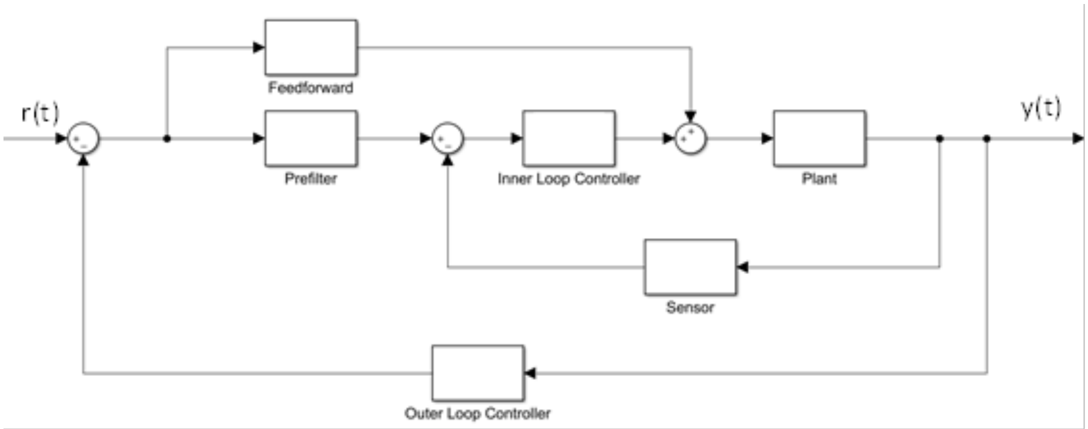
-System Bandwidth

System Bandwidth

Another performance metric that is system bandwidth. While this has several definitions, the one that is most commonly associated with controlled dynamic systems is the frequency range where the closed loop system is able to track before the output starts to become attenuated/degraded. Mathematically, we can define the bandwidth of the system as the frequency where the response drops 3 dB from the DC gain

ω_{bw} = frequency where system response drops 3 dB from the DC gain

For example, consider the complicated system shown below. Note that the blocks do not need to be linear, the aforementioned definition of bandwidth is applicable to even a nonlinear systems.



We can ask the question, "How fast can we vary $r(t)$ and expect $y(t)$ to track this input signal?" If $r(t)$ is slowly varying, we would expect that $y(t)$ should be able to follow the input signal. A good test for this would be to use $r(t)$ as a sinusoid with a low angular frequency. However, as we make the inputs change more quickly, it is likely that the system will not be able to respond as quickly.

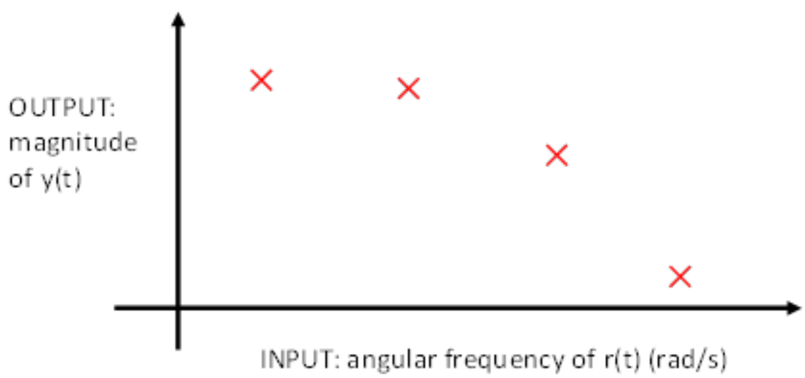
Demo 1: Student trying to run between two lines

See Matlab code to generate animation to perform demo.

Demo 2: Dog tracking laser pointer

See YouTube video

A very crude plot of what a typical response might look is shown below



This is very much similar to the magnitude plot of a Bode plot.

As such, a common way to measure bandwidth is to generate a bode plot of the <u>closed loop</u> system (AKA the transfer function of $y(t)$ in response to $r(t)$).

Example: Linear System

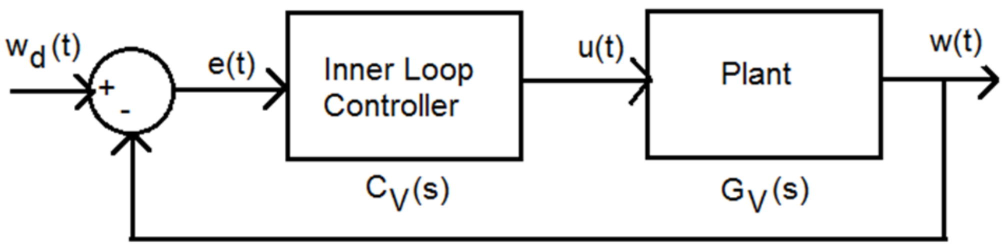
If the system is linear, we can predict how the bandwidth will change with system parameters.

Recall that previously, we showed that the transfer function for a DC motor (between velocity and armature voltage) was given by

$$G_V(s) = \frac{\dot{\theta}(s)}{V_a(s)} = \frac{\frac{K_T}{J_m L_a}}{s^2+\left(\frac{c L_a+J_m R+J_m R_m}{J_m L_a}\right)s+\left(\frac{K_T K_V+c R+c R_m}{J_m L_a}\right)} = \frac{46163}{s^2+1021 s+4845}$$

```
GV[s_] = 46163 / (s^2 + 1021 s + 4845);
```

Consider a closed loop DC motor system where we attempt to control velocity.



Let us start with a simple proportional controller

$$C_V(s) = K$$

```
CV[s_] = K;
```

The closed loop transfer function of the system is

$$T_V(s) = \frac{\omega(s)}{\omega_d(s)} = \frac{C_V(s)\,G_V(s)}{1+C_V(s)\,G_V(s)}$$

```
TV[s_, K_] = CV[s] × GV[s] / (1 + CV[s] × GV[s]) // Simplify
```

$$\frac{46163\,K}{4845 + 46163\,K + 1021\,s + s^2}$$

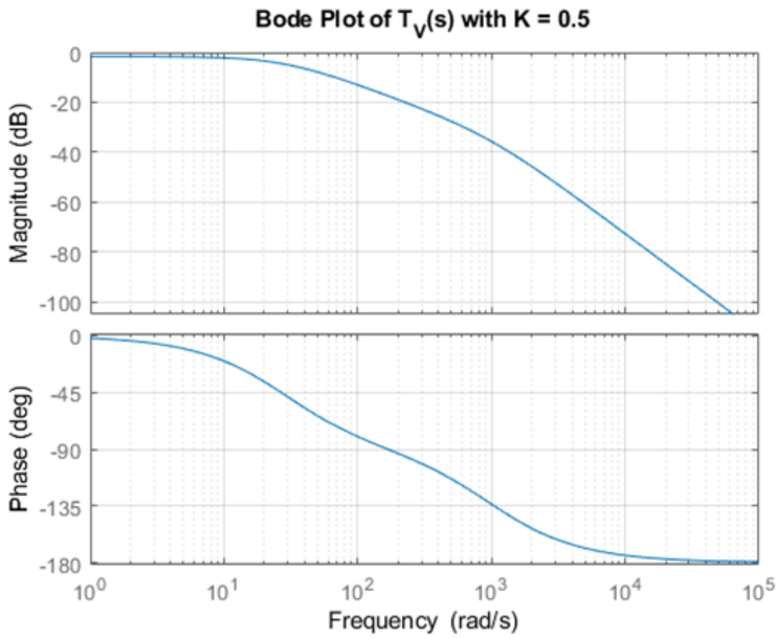
$K = 0.5$

If we start with $K = 0.5$, the closed loop system becomes

```
KLow = 1 / 2;
TV[s, KLow] // Simplify
```

$$\frac{46163}{55853 + 2042\,s + 2\,s^2}$$

We can plot the closed bode plot of $T_V(s)$



This measures how well, the system is able to respond to a commanded velocity, $\omega_d(s)$.

We can compute the DC gain of $T_V(s)$

$$\text{DC gain} = T_V(0)$$

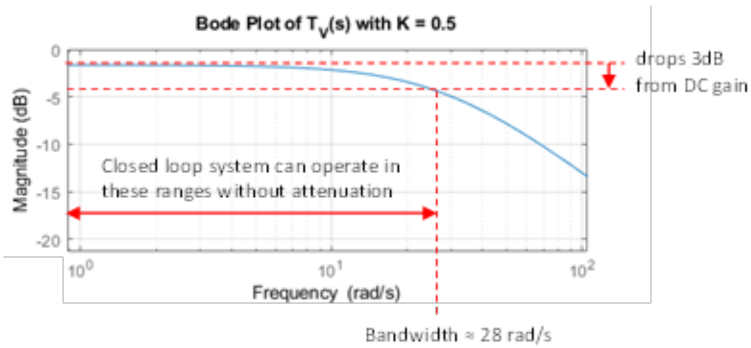
```
(*Compute gain*)
DCGain = TV[0, KLow];
DCGain // N

(*in dB*)
DCGainDB = 20 Log10[DCGain];
DCGainDB // N
```

```
0.826509

-1.65505
```

So at low frequencies, the system is able to track to 82.7% of the commanded input velocity (equivalent to -1.66 dB). We seek the point where the system drops 3 dB from this value, in other words we are looking for the -4.66 dB point. Focusing on only the magnitude plot and zooming into the lower frequencies, we can graphically find this point.



So we see that in this case, with $K = 0.5$ we have

$$\omega_{bw} \approx 28 \text{ rad/s}$$

***K* = 5.5**

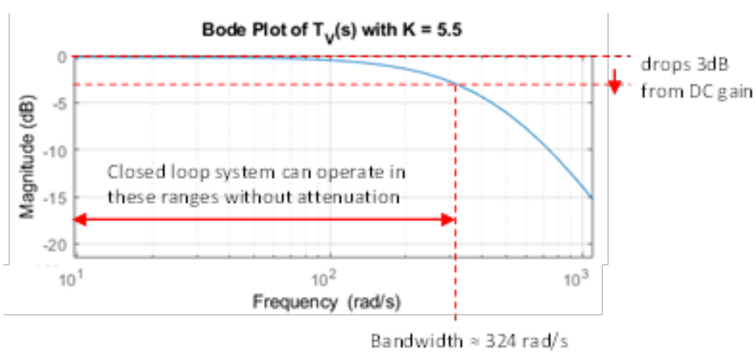
Note that the system bandwidth is a function of the entire system. For example, if we change the plant or controller, the bandwidth will change. For example, if we use a more aggressive controller of $K = C(s) = 5.5$, the DC gain becomes

```
KHigh = 5.5;
DCGain2 = TV[0, KHigh]
DCGain2DB = 20 Log10[DCGain2]
```

```
0.981275
```

```
-0.164188
```

The closed loop bode plot becomes



So we see that in this case, with $K = 5.5$ we have

$$\omega_{bw} \approx 324 \text{ rad/s}$$

To summarize, the bandwidth of the system measures the highest frequency that the system can operate at before it starts attenuating (ie unable to track in the desired input signal).

In this example of controlling the speed of a DC motor with $K = 0.5$, if we use a slowly varying desired set-point of $\omega_d(t) = 20 \sin(3\,t)$, we expect that the motor will be able to track this slowly varying set-point and we expect that $\omega(t) \approx (0.826509) * 20 \sin(3\,t)$ at steady state. The amplitude would be approximately unchanged as long as the input frequency was below the bandwidth of 28 rad/s.

However, if we increase the frequency of the input to say $\omega_d(t) = 20 \sin(200\,t)$, we expect that the system may not be able to track this and the steady state output will be severely attenuated. To combat this, we nee to increase the controller gain K to 5.5 to increase the system bandwidth beyond the input frequency.