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Lecture09d
Translating Inputs, Outputs, and Initial Conditions Between Linear and Nonlinear Dynamic Systems

Lecture is on YouTube

The YouTube video entitled 'Translating Inputs, Outputs, and Initial Conditions Between Linear and Nonlinear Dynamic Systems' that covers this lecture is located at https://youtu.be/FDK0bOmOjZo.

Outline

-Comparing Linear and Nonlinear Models
- -Scenario 1: Response to Inputs
- -Scenario 2: Response to Initial Conditions
- -Scenario 3: Linear Controller

-Example: RCAM Aircraft Model

Comparing Linear and Nonlinear Models

We can now compare the linear and non-linear models.

Recall the nonlinear model was given as

$$\dot{\bar{x}}(t) = \bar{f}(\bar{x}(t), \bar{u}(t))$$
$$\bar{y}(t) = \bar{g}(\bar{x}(t), \bar{u}(t))$$

The linear model was obtained by trimming about an operating point $\bar{x}_o$, $\bar{u}_o$. Note that this trim point also defines a trim output as

$$\bar{y}_o = \bar{g}(\bar{x}_o, \bar{u}_o)$$

We then define perturbations from the trim point as

$$\Delta\,\bar{x}(t) = \bar{x}(t) - \bar{x}_o$$

$\Delta\,\overline{u}(t) = \overline{u}(t) - \overline{u}_o$
$\Delta\,\overline{y}(t) = \overline{y}(t) - \overline{y}_o$

We can then linearize about this point to obtain a linear model as

$\Delta\,\dot{\overline{x}}(t) = A\,\Delta\,\overline{x}(t) + B\,\Delta\,\overline{u}(t)$
$\Delta\,\overline{y}(t) = C\,\Delta\,\overline{x}(t) + D\,\Delta\,\overline{u}(t)$

Most of our design and analysis will be using the linear model. Afterwards, we will want to use the linear controller on the non-linear model.

As such, we need to be careful when:

1. Computing/using inputs $\overline{u}(t)$
2. Computing/using outputs $\overline{y}(t)$
3. Computing/using initial conditions $\overline{x}(0)$

Scenario 1: Response to Inputs

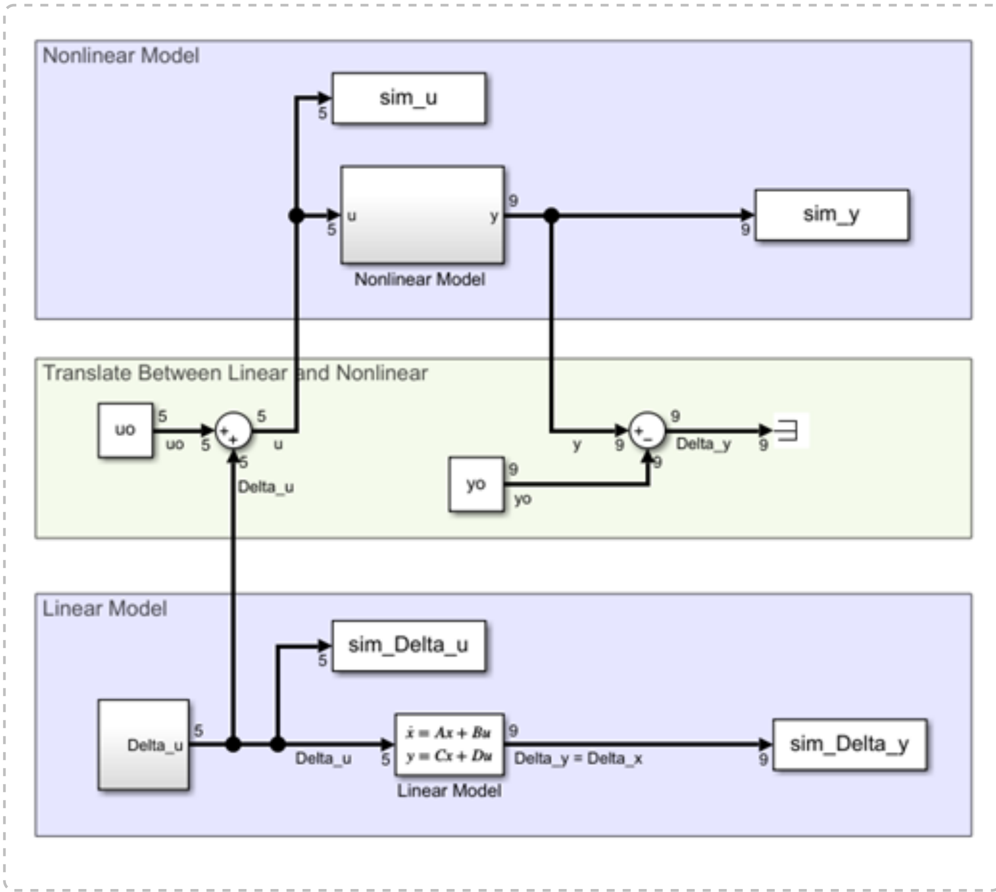
Discuss how to apply inputs to both models. Focus on plant model.

Scenario 2: Response to Initial Conditions

Discuss how to apply equivalent initial conditions to both models. Focus on plant model.

Scenario 3: Linear Controller

Discuss how to apply a linear controller to a nonlinear plant.



Example: RCAM Aircraft Model

Consider the linear model of the system at straight and . Recall that the linearization point when deriving our longitudinal model was

$u_o = 84.9905\ m/s$
$v_o = 0\ m/s$
$w_o = 1.2713\ m/s$
$p_o = 0\ \text{rad}/s$
$q_o = 0\ \text{rad}/s$
$r_o = 0\ \text{rad}/s$
$\phi_o = 0\ \text{rad}$
$\theta_o = 0.015\ \text{rad} = 0.859\ \text{deg}$
$\psi = 0\ \text{rad}$

$u_{1o} = 0\ \text{rad}$
$u_{2o} = -0.1780\ \text{rad} = -10.2\ \text{deg}$
$u_{3o} = 0\ \text{rad}$

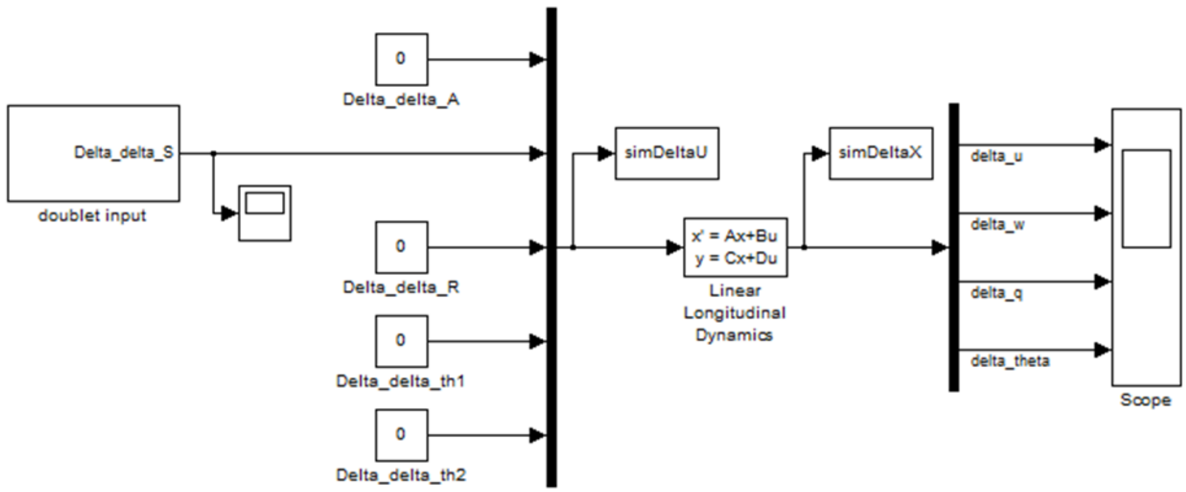
$u_{4o} = 0.0821$

$u_{5o} = 0.0821$

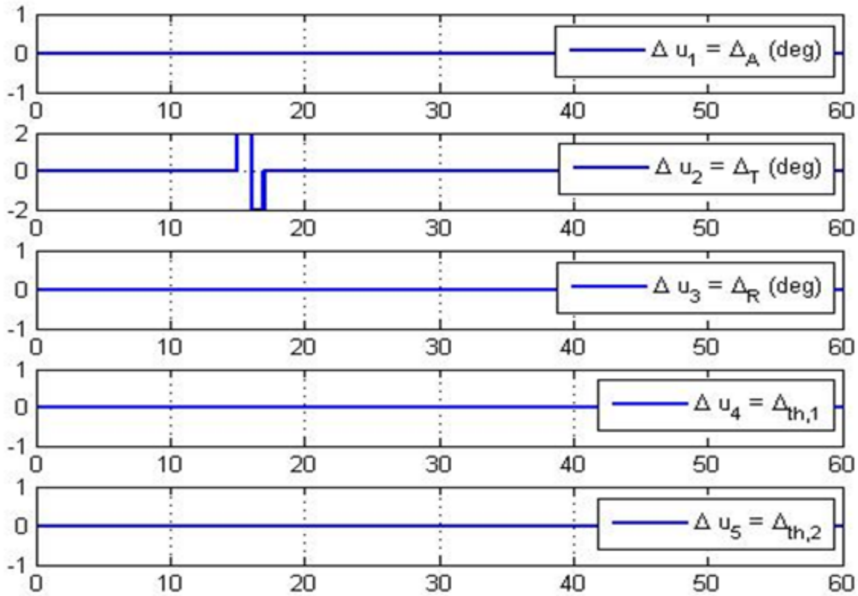
Suppose we apply an initial condition to the linear system of

$$\Delta\,\overline{x}(0) = \begin{pmatrix} \Delta\,u(0) \\ \Delta\,w(0) \\ \Delta\,q(0) \\ \Delta\,\theta(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 3\,\frac{\pi}{180} \end{pmatrix}$$

Furthermore, suppose that we introduce a doublet input into the 2nd control input. The appropriate Simulink model is shown below

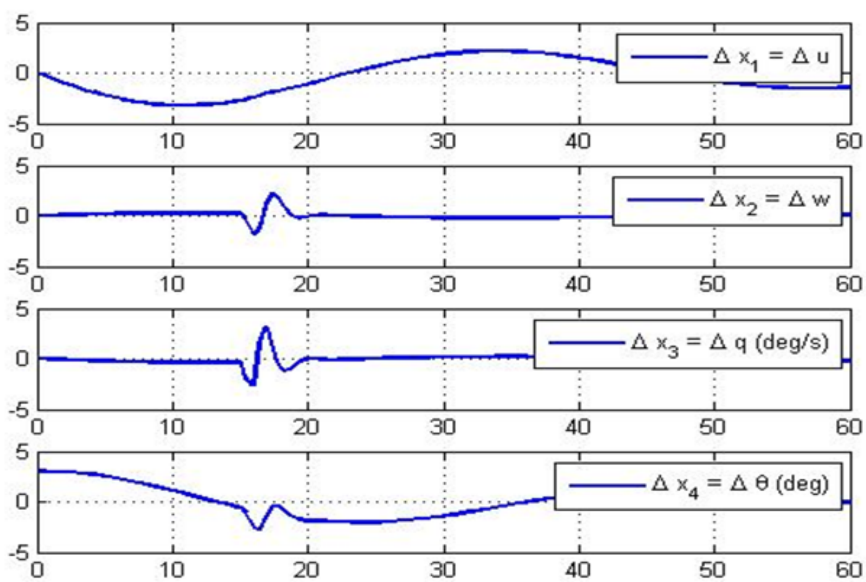


The input is shown below



The important thing to realize is that although all these signals are near 0, this corresponds to the non-linear model having control surfaces deflected from the position $\overline{u}_o$. Similarly, when the doublet of $\Delta u_2 = \pm 2^o$, this actually corresponds to the tail on the nonlinear model being deflected first to $2^o + (-10.2^o) = -8.2^o$, then down to $-2^o + (-10.2^o) = -12.2^o$, and then finally back to $0^o + (-10.2^o) = -10.2^o$.

This initial condition and input generates the following response



In a similar fashion, we see that these signals are near 0, but this does not mean that the non-linear aircraft has u, w, q, θ near zero. The response of the nonlinear system will follow $\overline{x}_{\text{long}}(t) = \Delta\,\overline{x}_{\text{long}}(t) + \overline{x}_{\text{long},o}$

We can simulate the non-linear model using similar conditions (see previous lecture on how to compare linear and non-linear systems). We leave this as an exercise to the reader as part of a homework assignment.