Christopher Lum
lum@uw.edu

Lecture 06b
Building a Matlab/Simulink Model of an Aircraft: The Research Civil Aircraft Model (RCAM)

The YouTube video entitled ‘Building a Matlab/Simulink Model of an Aircraft: The Research Civil Aircraft Model (RCAM)’ that covers this lecture is located at https://youtu.be/m5sEln5bWuM.

Outline

- -Matlab Function: RCAM_model.m
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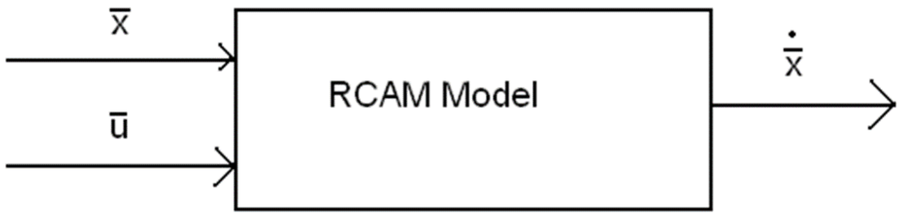
Matlab Function: RCAM_model.m

The goal is to now create a function that computes these state derivatives.



General skeleton code flow RCAM_model.m is

A. Unpack State and Control Vector
B. Define Constants
1. Control Limits/Saturation
2. Intermediate Variables
3. Nondimensional Aerodynamic Force Coefficients in F_s
4. Aerodynamic Force in F_b
5. Nondimensional Aerodynamic Moment Coefficient About Aerodynamic Center in F_b
6. Aerodynamic Moment About Aerodynamic Center in F_b
7. Aerodynamic Moment About Center of Gravity in F_b
8. Propulsion Effects
9. Gravity Effects
10. Explicit First Order Form

A . Unpack State and Control Inputs

See RCAM_model.m

B . Define Constants

From pg. 9 of the RCAM document.   Note that we're using $Z_{cg} = 0.10\,\bar{c}$, the values can be different depending on whether Table 2.4 or Table 2.5 is consulted.

Symbol	Alphanumeric		Name	Default	Unit
Mass Parameters					
m	MASS	=	aircraft total mass	120 000	kg
Aerodynamic Parameters					
$\bar{c}$	CBAR	=	mean aerodynamic chord	6.6	m
l_t	LTAIL	=	distance between AC of the wing-body (AC_{wb}), and AC of the tail (AC_t)	24.8	m
S	S	=	wing planform area	260.0	m^2
S_t	STAIL	=	tail planform area	64.0	m^2
X_{cg}	XCG	=	x position of the CoG in F_M	0.23 $\bar{c}$	m
Y_{cg}	YCG	=	y position of the CoG in F_M	0	m
Z_{cg}	ZCG	=	z position of the CoG in F_M	$\boxed{0}$	m
Engine Parameters					
X_{APT1}	XAPT1	=	x position of application point of thrust of engine 1 in F_M	0.0	m
Y_{APT1}	YAPT1	=	y position of application point of thrust of engine 1 in F_M	−7.94	m
Z_{APT1}	ZAPT1	=	z position of application point of thrust of engine 1 in F_M	−1.9	m
X_{APT2}	XAPT2	=	x position of application point of thrust of engine 2 in F_M	0.0	m
Y_{APT2}	YAPT2	=	y position of application point of thrust of engine 2 in F_M	7.94	m
Z_{APT2}	ZAPT2	=	z position of application point of thrust of engine 2 in F_M	−1.9	m

Table 2.4 Parameters definitions

Inconsistent

Parameters		Bounds					Nominal
m	MASS	100 000 kg	<	m	<	150 000 kg	120 000 kg
X_{cg}	XCG	0.15 $\bar{c}$	<	X_{cg}	<	0.31 $\bar{c}$	0.23 $\bar{c}$
Y_{cg}	YCG	−0.03 $\bar{c}$	<	Y_{cg}	<	0.03 $\bar{c}$	0.0 $\bar{c}$
Z_{cg}	ZCG	0.00 $\bar{c}$	<	Z_{cg}	<	0.21 $\bar{c}$	$\boxed{0.10\ \bar{c}}$

Table 2.5 Possible parameter choices in RCAM, see also section 3.2.3.

F_M is a measurement reference frame. The origin is located at the leading edge of the mean aerodynamic chord. x_M is positive pointing backwards, y_M is positive to the right, and z_M is positive pointing up.

1. Control Limits/Saturation

For now, we implement this control saturation inside the RCAM_model.m function. Later, we will move this outside the function.

2. Intermediate Variables

See RCAM_model.m

3. Nondimensional Aerodynamic Force Coefficients in F_s

Recall from previous lecture that these were modeled as

$$C_{L_{wb}} = n(\alpha - \alpha_{L=0}) \qquad \text{if } \alpha \le 14.5 \tfrac{\pi}{180} \text{ rad} \qquad \textbf{\textit{(Eq.2.22)}}$$
$$= a_3\,\alpha^3 + a_2\,\alpha^2 + a_1\,\alpha + a_o \qquad \text{if } \alpha \ge 14.5 \tfrac{\pi}{180}$$

where $\alpha_{L=0} = -11.5 \frac{\pi}{180}$ rad (α at zero lift)

$n = 5.5$

$a_3 = -768.5 \qquad a_2 = 609.2 \, a_1 = -155.2 \qquad a_o = 15.2$

One option would be to shuffle this to another function.

However, in the interest of keeping all the code together in one file, it may be more convenient to hard code this into the same RCAM_model.m file. It is recommended that you keep all code in a single file at this point.

4. Aerodynamic Force in F_b

See RCAM_model.m

5. Nondimensional Aerodynamic Moment Coefficient About Aerodynamic Center in F_b

See RCAM_model.m

6. Aerodynamic Moment About Aerodynamic Center in F_b

See RCAM_model.m

7. Aerodynamic Moment About Center of Gravity in F_b

Recall that the RCAM document has a typo.

See RCAM_model.m

8. Propulsion Effects

See RCAM_model.m

9. Gravity Effects

See RCAM_model.m

10. Explicit First Order Form

We have now accounted for all the forces and moments. Let us form an explicit expression for $\dot{x}_1$, $\dot{x}_2$, and $\dot{x}_3$. Recall that this is given by Eq.2.1 which is repeated here for convenience.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix}_b = \frac{1}{m}\overline{F}^b - \overline{\omega}_{b/e}{}^b \times \overline{V}^b \qquad \overline{M}_{cg}{}^b = \left(\overline{M}_{A_{cg}}{}^b + \overline{M}_{E_{cg}}{}^b\right)$$

where $\overline{F}^b = \overline{F}_g{}^b + \overline{F}_E{}^b + \overline{F}_A{}^b$

Next, the expression for $\dot{x}_4$, $\dot{x}_5$, and $\dot{x}_6$. Recall that these are given by Eq.6, which is repeated here for convenience.

$$\begin{pmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix}_b = I_b^{-1}\left(\overline{M}_{cg}{}^b - \overline{\omega}_{b/e}{}^b \times I_b\, \overline{\omega}_{b/e}{}^b\right) \qquad \textbf{(Eq.6)}$$

where $\quad \overline{M}_{cg}{}^b = \overline{M}_{A_{cg}}{}^b + \overline{M}_{E_{cg}}{}^b$

Finally, the derivatives of the Euler angles are given by Eq.3.7, which is repeated here for convenience.

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}^b \qquad \textbf{(Eq.3.7)}$$

Note: don't want to use inv(Ib). Instead, calculate this offline to improve efficiency. Also note that the RCAM document is inconsistent in its definition of I_b. Be sure to use the version on pg. 12, not pg. 91.

$$\texttt{Ib = m} \begin{pmatrix} 40.07 & 0 & -2.0923 \\ 0 & 64 & 0 \\ -2.0923 & 0 & 99.92 \end{pmatrix};$$

```
invIb = Inverse[Ib];
invIb // MatrixForm
```

$$\begin{pmatrix} \frac{0.0249836}{m} & 0. & \frac{0.000523151}{m} \\ 0. & \frac{0.015625}{m} & 0. \\ \frac{0.000523151}{m} & 0. & \frac{0.010019}{m} \end{pmatrix}$$

$$I = \begin{bmatrix} I_x & 0 & I_{xz} \\ 0 & I_y & 0 \\ I_{xz} & 0 & I_z \end{bmatrix} = m \begin{bmatrix} 40.07 & 0 & -2.0923 \\ 0 & 64 & 0 \\ -2.0923 & 0 & 99.92 \end{bmatrix}$$

Pg. 12

Inconsistent in the thousandths and ten-thousandths places

```
parameter   mass = 120000.0 { mass / kg },
            Ix = 40.07 * mass      { x-moment of inertia / kg*m^2 },
            Ixy =  0               { xy-product of inertia / kg*m^2 },
            Ixz = -2.09323 * mass  { xy-product of inertia / kg*m^2 },
            Iy = 64 * mass         { y-moment of inertia / kg*m^2 },
            Iyz =  0               { yz-product of inertia / kg*m^2 },
            Iz = 99.92 * mass      { z-moment of inertia / kg*m^2 }
```

Should be xz

Pg. 91

Simulink Implementation

I recommend that you use an ‘Interpreted Matlab Function’ instead of a ‘MATLAB Fcn’ block.

See YouTube video

Move Control Limit Saturation outside of RCAM.m

See YouTube video