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Lecture 08b
Designing a PID Controller Using the Ziegler Nichols Method

Lecture is on YouTube

YouTube video entitled ‘Designing a PID Controller Using the Ziegler-Nichols Method’ covering this is located at https://youtu.be/n829SwSUZ_c.

Outline

-Introduction
-Ziegler Nichols Method

Introduction

We would now like to design a PID controller which involves choosing the gains K_p, K_I, and K_D. There are several ways to do this. One method is to use the Ziegler Nichols method (https://en.wikipedia.org/wiki/Ziegler% E2 %80 %93 Nichols_method).

Ziegler Nichols Method

This procedure can be used for stable plants in order to determine a set of gains which function desirably.

Several version of the method provide a procedure and algorithm to obtain the parameters K_P, T_i, and T_d which can be used to implement a control law of

$$u(t) = K_P\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)\,d\tau + T_d\frac{d\,e(t)}{d\,t}\right) \qquad \textbf{(Eq.1)}$$

Comparing this with the traditional PID control law of

$$u(t) = K_P\,e(t) + K_I\int_0^t e(\tau)\,d\tau + K_D\frac{d\,e(t)}{d\,t} \qquad \textbf{(Eq.2)}$$

We see that K_I and K_D are given by

$K_I = \frac{K_P}{T_i}$ **(Eq.3)**

$K_D = T_d K_P$ **(Eq.4)**

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KI[KP_, Ti_] = KP / Ti;
KD[KP_, Td_] = Td KP;
```

The procedure to obtain these parameters involves using only a proportional controller and slowly increasing the gain K_P until persistent oscillations occur.

Procedure

1. Start with the proportional gain with a small value (K_P =small). Set other gains to zero ($K_I = K_D = 0$)
2. Increase gain K_P until system becomes neutrally stable (consistent oscillations of constant amplitude).
3. Record the critical gain ($K_U = K_P$ at oscillation) and the oscillation period T_U, in seconds.
4. Look up the values of K_P, T_i , and T_d based on the following table

Control Type	K_P	T_i	T_d		$K_I = K_P/T_i$	$K_D = T_d K_P$
PID (classic)	$0.6\ K_U$	$T_U/2$	$T_U/8$		$1.2K_U/T_U$	$0.075\ K_U T_U$
P	$0.5\ K_U$	-	-		-	-
PI	$0.45\ K_U$	$T_U/1.2$	-		$0.54K_U/T_U$	-
PD	$0.8\ K_U$	-	$T_U/8$		-	$0.1K_U T_U$
Pessen Integration Rule	$0.7\ K_U$	$2T_U/5$	$3\ T_U/20$		$1.75K_U/T_U$	$0.105\ K_U T_U$
Some Overshoot	$K_U/3$	$T_U/2$	$T_U/3$		$(2/3)K_U/T_U$	$(1/9)K_U/T_U$
No Overshoot	$0.2\ K_U$	$T_U/2$	$T_U/3$		$(2/5)K_U/T_U$	$(1/15)K_U/T_U$

5. Compute the gains K_I and K_P using Eq.3 and Eq.4

For example, to compute a classical PID

```
(*Parameters from Ziegler Nichols for Classical PID*)
KPGiven = 0.6 KU;
TiGiven = TU / 2;
TdGiven = TU / 8;

(*What are the derived KI and KD?*)
KI[KPGiven, TiGiven]
KD[KPGiven, TdGiven]
```

$\frac{1.2\ KU}{TU}$

0.075 KU TU

Note that I created an implementation of this algorithm as a Matlab function located at https://github.com/clum/UWMatlab/blob/master/UWMatlab/Controls_Functions/ZieglerNichols.m.

There are several advantages and disadvantages of this method

Advantages

-Does not require advanced understanding of dynamic system/controls.

-Does not require a mathematical model/simulation of the system.

-Often produces good initial estimate of parameters.

Disadvantages

-Requires a stable system.

-Requires conducting experiments on the real system (this can be costly)

-System must be able to be driven unstable by increasing gain (for example a system of $\frac{1}{s+1}$ will never display persistent oscillations and therefore K_U is undefined).

Example: Transfer Function

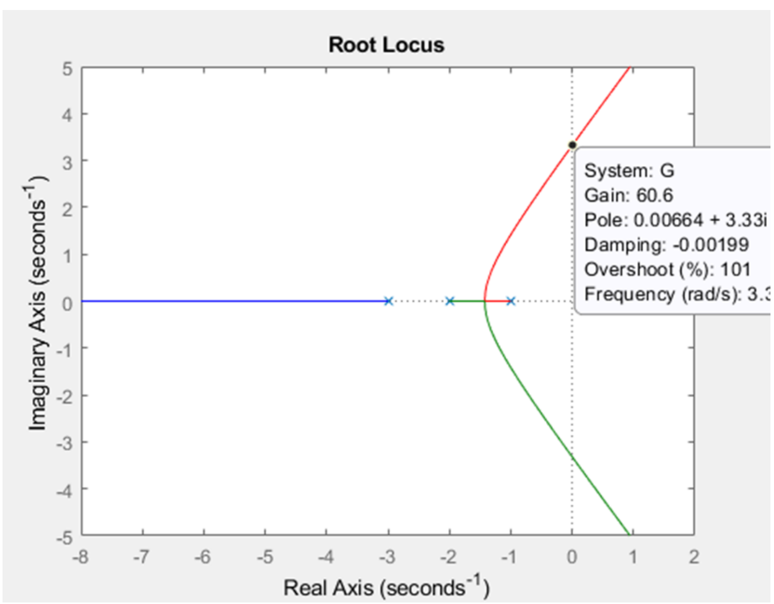
Consider the example transfer function that we examined during our discussion of Routh-Hurwitz stability

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

```
G[s_] = 1/((s + 1) (s + 2) (s + 3));
Expand[Denominator[G[s]]]
```

$6 + 11\,s + 6\,s^2 + s^3$

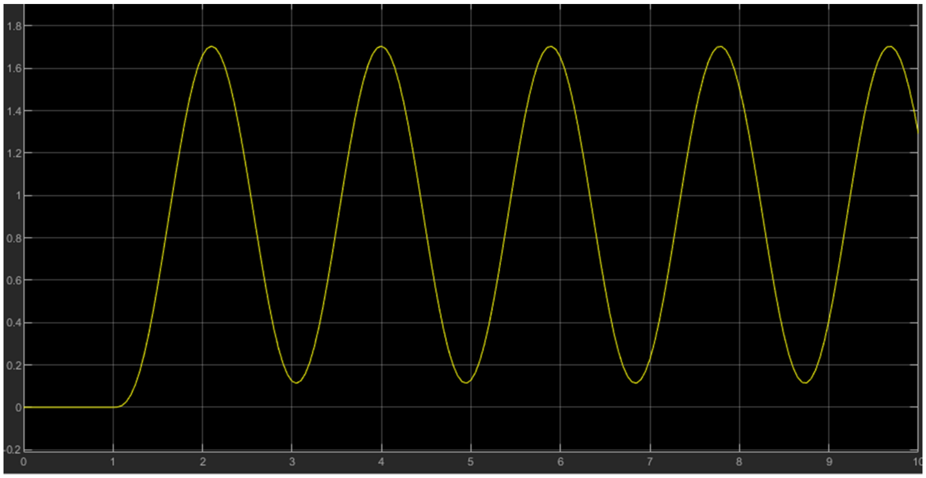
Recall the root locus of this system was as shown below



We used Routh-Hurwitz to show that a proportional gain of $K = 60$ yields a neutrally stable system, therefore

$$K_U = 60$$

We can simulate the system with this gain to obtain T_U



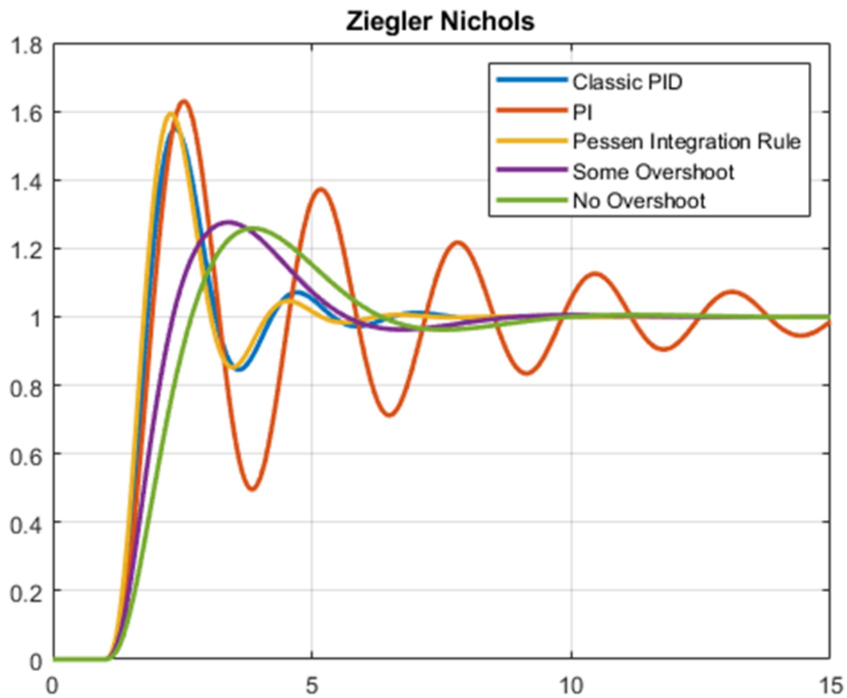
We measure

$$T_U \approx 1.9 \text{ seconds}$$

Computing gains using various techniques

	Classic PID	PI	Pessen Integration Rule	Some Overshoot	No Overshoot
K_P	36	27	42	20	12
K_I	37.9	17.1	55.3	21.1	12.6
K_D	8.6	0	12	12.7	7.6

The responses are shown below

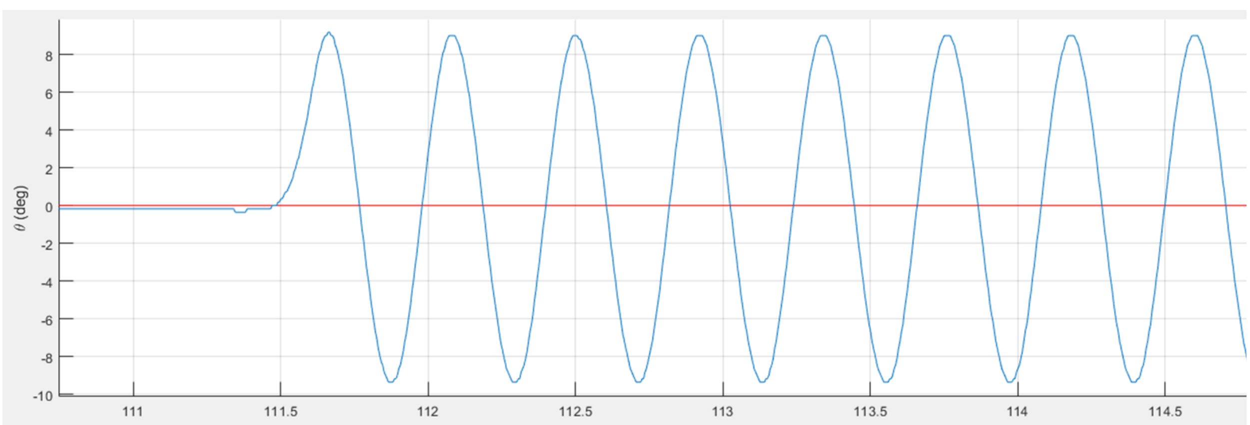


As can be seen, these all yield reasonable responses.

Example: DC Motor Position Control

The real power of the Ziegler-Nichols method is that it can be applied to systems where we do not have a mathematical model or simulation of the system. It can be applied directly to a real system to find controller gains.

Consider the DC motor position control system. In this case, we have added weights to the flywheel in order to increase the moment of inertia. Experimental data with K = 45 is shown below



We find that

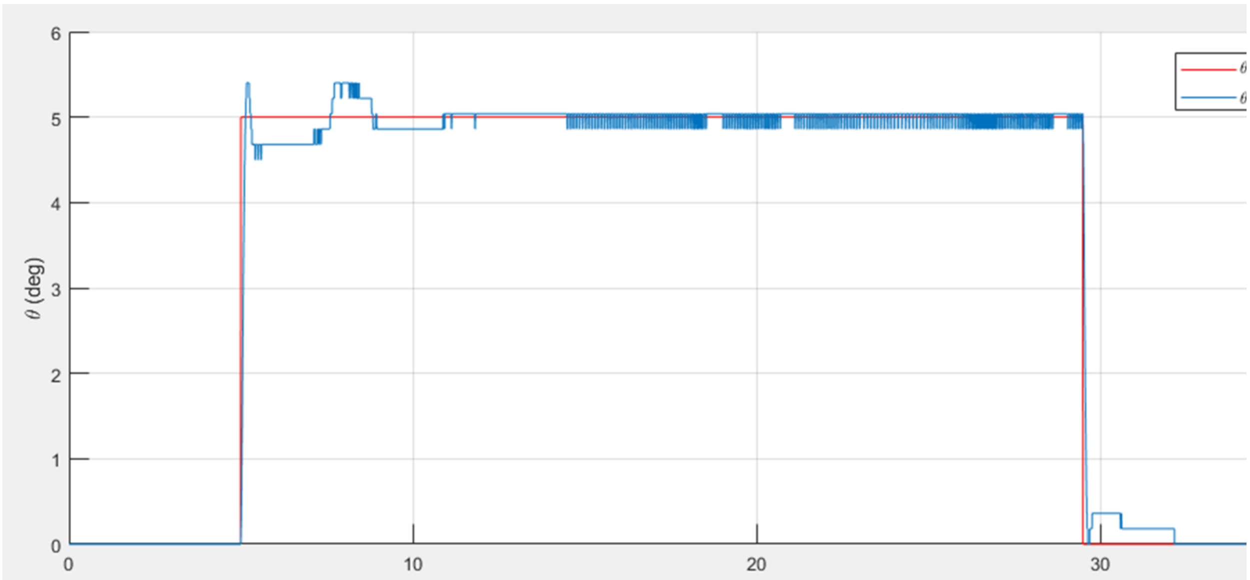
$K_U = 45$

$T_U = 0.96$ seconds

Computing using the Classical PID Ziegler-Nichols yields

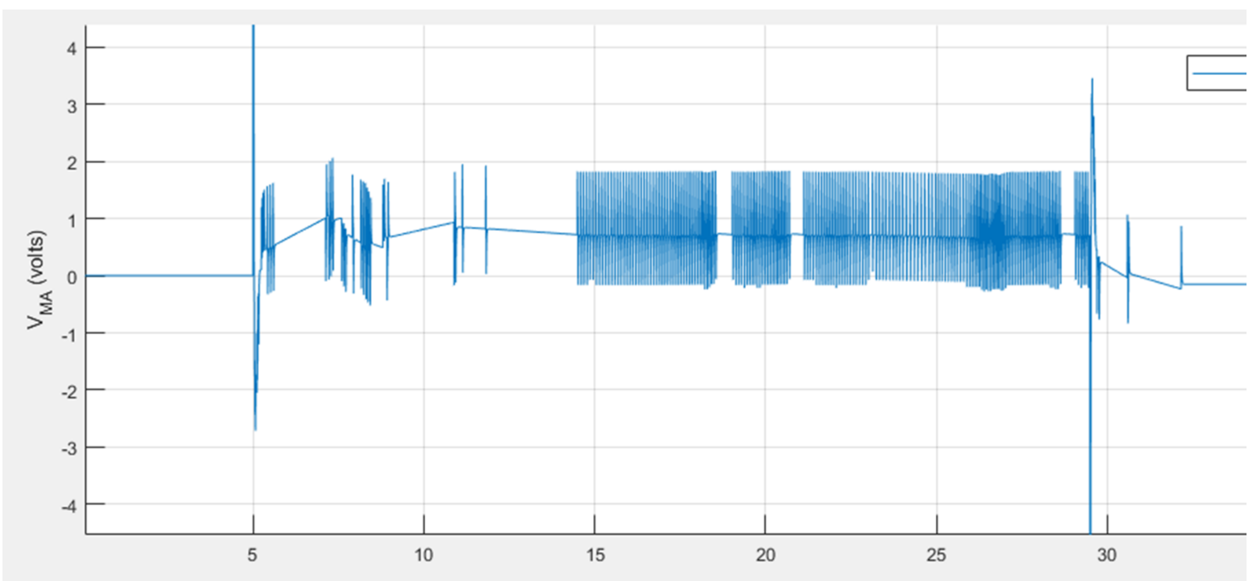
$K_P = 27$
$K_I = 56.25$
$K_D = 3.24$

The experimental result to a step of 5 degrees and then back to 0 degrees is shown below



As can be seen, the system performs well.

Perhaps just as importantly, with this real system, we should investigate the control signal to ensure it is reasonable. The control signal during this time is shown below



As can be seen, the control is mostly small. There are some spikes that are as large as ±30 volts that saturate the controller when the step change is introduced but they are quickly reduced to the acceptable regions.