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Lecture03g
Analytically Solving Systems of Linear Ordinary Differential Equations

Lecture is on YouTube

The YouTube video entitled ‘Analytically Solving Systems of Linear Ordinary Differential Equations’ that covers this lecture is located at https://youtu.be/i2QkjxtXKos .

Outline

-Solutions of State Space Systems
    -Example
-System Stability

Solution of State Space Systems

As we have seen, it is possible to write an system of arbitrarily high order ODEs as a system of first order ODEs in a form called a state space representation (see YouTube video on state space systems). Writing the system in state space form can be consider the modeling aspect of the problem (in other words the state space representation could be considered the mathematical model of how a system behaves). It is now natural to ask, what is the solution to this state space system?

Assumptions: A is a constant matrix (time-invariant)

Given the linear system of the form

$$\dot{\overline{x}}(t) = A\,\overline{x}(t) + B\,\overline{u}(t) \qquad \textbf{\textit{(Eq.1.1)}}$$

with $\overline{x}(t_o) = \overline{x}_o$

The question now becomes, can we find an explicit analytical solution for $\overline{x}(t)$?

Auxiliary Matrix Function, $K(t)$

Consider an auxiliary matrix function as (we will choose this function later)

$$K(t) = \text{auxiliary matrix function}$$

To make the analysis easier, we note that if we have the quantity $K(t)\,\overline{x}(t)$, we can differentiate (using the product rule) this to obtain

$$\frac{d}{dt}[K(t)\,\overline{x}(t)] = \dot{K}(t)\,\overline{x}(t) + K(t)\,\dot{\overline{x}}(t) \qquad \textbf{(Eq.1.2)}$$

We can verify this quickly

```
K[t_] = ( Sin[θ[t]]    t^3
          3 Exp[3 t]   t^2 + t^4 );
x[t_] = ( Tan[t]
          t^3 + 1/t );
D[K[t].x[t], t] == (D[K[t], t].x[t] + K[t].D[x[t], t])
Clear[K, x]
```

Out[ ]= True

Let us rewrite Eq.1.1 as

$$\dot{\overline{x}}(t) - A\,\overline{x}(t) = B\,\overline{u}(t)$$

Pre-multiplying by the auxiliary matrix function on the left, we have

$$K(t)\,\dot{\overline{x}}(t) - K(t)\,A\,\overline{x}(t) = K(t)\,B\,\overline{u}(t) \qquad \textbf{(Eq.1.3)}$$

We choose $K(t)$ to be of the following form

$$K(t) = K(t_o)\,e^{-A(t-t_o)} \qquad \textbf{(Eq.1.5)}$$

We note that $K(t_o)$ is a constant matrix and therefore we can take the derivative of this expression and apply our previous result (see video entitled ‘Introduction to the Matrix Exponential’ https://y-outu.be/e_guF0dwwA4) that $\frac{d}{dt}\left[e^{B\,t}\right] = B\,e^{B\,t} = e^{B\,t}\,B$.

$$\frac{d}{dt}[K(t)] = \frac{d}{dt}\left[K(t_o)\,e^{-A(t-t_o)}\right]$$

$$= K(t_o)\,\frac{d}{dt}\left[e^{-A(t-t_o)}\right]$$

$$= K(t_o)\,\frac{d}{dt}\left[e^{-A\,t}\,e^{A\,t_o}\right]$$

$$= K(t_o)\,\frac{d}{dt}\left[e^{-A\,t}\right]e^{A\,t_o} \qquad \text{recall: } \frac{d}{dt}\left[e^{-A\,t}\right] = -A\,e^{-A\,t} = e^{-A\,t}\,A.$$

$$= K(t_o)\,e^{-A\,t}(-A)\,e^{A\,t_o}$$

$$= -K(t_o)\,e^{-A\,t}\left(A\,e^{A\,t_o}\right) \qquad \text{recall: } \frac{d}{dt_o}\left[e^{A\,t_o}\right] = A\,e^{A\,t_o} = e^{A\,t_o}\,A$$

$$= -K(t_o)\, e^{-A\,t}\left(e^{A\,t_o}\, A\right)$$

$$= -K(t_o)\, e^{-A\,t}\, e^{A\,t_o}\, A$$

$$= -K(t_o)\, e^{-A(t-t_o)}\, A$$

$$= -\left[K(t_o)\, e^{-A(t-t_o)}\right] A$$ recall from Eq.1.5: $K(t) = K(t_o)\, e^{-A(\,t-t_o)}$

$$\dot{K}(t) = -K(t)\, A \qquad \textbf{\textit{(Eq.1.6)}}$$

Example: Verify $\dot{K}(t) = -K(t)\,A$

Let us check this quickly with a concrete example.

```
In[]:= A = ( -2  1
             -1  2 );
K[t_] = MatrixExp[-A (t - to)] // Simplify;
Kd[t_] = D[K[t], t] // Simplify;
Kd[t] == -A.K[t] // Simplify
Kd[t] == -K[t].A // Simplify
Clear[A, K, Kd]
```

```
Out[]= True
Out[]= True
```

Substitute $K(t)$ into Original ODE

Therefore, by multiplying the original differential equation by $K(t)$ we have

$K(t)\,\dot{\overline{x}}(t) - K(t)\,A\,\overline{x}(t) = K(t)\,B\,\overline{u}(t)$ note: since $\dot{K}(t) = -K(t)\,A$ from Eq.1.6

$K(t)\,\dot{\overline{x}}(t) + \dot{K}(t)\,\overline{x}(t) = K(t)\,B\,\overline{u}(t)$ note: $K(t)\,\dot{\overline{x}}(t) + \dot{K}(t)\,\overline{x}(t) = \frac{d}{dt}[K(t)\,\overline{x}(t)]$ from Eq.1.2 (product rule)

$$\frac{d}{dt}[K(t)\,\overline{x}(t)] = K(t)\,B\,\overline{u}(t) \qquad \textbf{\textit{(Eq.1.9)}}$$

We can now integrate both sides between t_o and t

$$\int_{t_o}^{t} \frac{d}{dt}[K(t)\,\overline{x}(t)]\,dt = \int_{t_o}^{t} K(t)\,B\,\overline{u}(t)\,dt$$

$$K(t)\,\overline{x}(t) - K(t_o)\,\overline{x}(t_o) = \int_{t_o}^{t} K(t)\,B\,\overline{u}(t)\,dt$$

Let us change the variable of integration on the right side from t to τ to avoid confusion

$$K(t)\,\overline{x}(t) - K(t_o)\,\overline{x}(t_o) = \int_{t_o}^{t} K(\tau)\,B\,\overline{u}(\tau)\,d\tau$$

Furthermore, we can substitute in Eq.1.5 for $K(t)$ to obtain

$$K(t_o)\, e^{-A(t-t_o)}\, \overline{x}(t) - K(t_o)\, e^{-A(t_o-t_o)}\, \overline{x}(t_o) = \int_{t_o}^{t} K(t_o)\, e^{-A(\tau-t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

$$K(t_o)\left(e^{-A(t-t_o)}\, \overline{x}(t) - \overline{x}(t_o)\right) = K(t_o) \int_{t_o}^{t} e^{-A(\tau-t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

We note that $K(t_0)$ is constant and non-singular (matrix exponentials are always an invertible matrix, see Wikipedia entry) so we can left multiply both sides by $K(t_o)^{-1}$ to obtain

$$e^{-A(t-t_o)}\, \overline{x}(t) - \overline{x}(t_o) = \int_{t_o}^{t} e^{-A(\tau-t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

We can now premultiply by $e^{A(t-t_o)}$ on the left

$$e^{A(t-t_o)}\, e^{-A(t-t_o)}\, \overline{x}(t) - e^{A(t-t_o)}\, \overline{x}(t_o) = e^{A(t-t_o)} \int_{t_o}^{t} e^{-A(\tau-t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

$$\overline{x}(t) - e^{A(t-t_o)}\, \overline{x}(t_o) = e^{A(t-t_o)} \int_{t_o}^{t} e^{-A(\tau-t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

Notice that $e^{A(t-t_o)}$ can be moved inside the integral on the right side (since it is not a function of τ).

$$\overline{x}(t) - e^{A(t-t_o)}\, \overline{x}(t_o) = \int_{t_o}^{t} e^{A(t-t_o)}\, e^{-A(\tau-t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

$$\overline{x}(t) - e^{A(t-t_o)}\, \overline{x}(t_o) = \int_{t_o}^{t} e^{(A t - A t_o - A \tau + A t_o)}\, B\, \overline{u}(\tau)\, d\tau$$

$$\overline{x}(t) - e^{A(t-t_o)}\, \overline{x}(t_o) = \int_{t_o}^{t} e^{(A t - A \tau)}\, B\, \overline{u}(\tau)\, d\tau$$

$$\overline{x}(t) - e^{A(t-t_o)}\, \overline{x}(t_o) = \int_{t_o}^{t} e^{A(t-\tau)}\, B\, \overline{u}(\tau)\, d\tau$$

Finally, solving for $\overline{x}(t)$ yields

$$\overline{x}(t) = e^{A(t-t_o)}\, \overline{x}(t_o) + \int_{t_o}^{t} e^{A(t-\tau)}\, B\, \overline{u}(\tau)\, d\tau \qquad \textbf{(Eq.1.10)}$$

Which is the solution to a linear ordinary differential equation (AKA a state space representation).

Note: a very common mistake is to substitute $\overline{u}(t)$ instead of $\overline{u}(\tau)$ in Eq.1.10.

The integral is sometimes referred to as the convolution integral and in a loose sense is related to the particular solution (it involves the external input $\overline{u}(t)$). The term $e^{A(t-t_o)}\, \overline{x}(t_o)$ is loosely referred to as the homogeneous or autonomous response of the system as it describes how the system responds to initial conditions by itself (see YouTube videos entitled 'Homogeneous Linear Ordinary Differential Equations' https://youtu.be/3Kox-3APznI and 'Nonhomogeneous Linear Ordinary Differential Equations' https://youtu.be/t98ILS2YdrU). Computing the term $e^{A(t-t_o)}$ is investigated in the next section.

A large portion of this problem involves computing the matrix exponential $e^{A(t-t_o)}$. This is sometimes referred to as the **state transition matrix**

$$\phi(t, t_o) = e^{A(t-t_o)}$$

To simplify the analysis, let us assume that $t_o = 0$ (ie we start time at $t = 0$). We note that this does not reduce the generalizability of the analysis process.

$$\phi(t) = e^{A t} \qquad \textbf{\textit{(Eq.3)}}$$

There are there are several ways to compute the transition matrix

1. Infinite Series
2. Laplace Method
3. Modal Transformation (Diagonalization)
4. Cayley-Hamilton

The infinite series method will yield an infinite expression, which is often of little use. Therefore we will instead concentrate on the Laplace, Modal Transformation, and Cayley-Hamilton techniques. These techniques are documented in separate videos.

Example

Consider the state space representation of

$$\dot{\overline{x}}(t) = A\,\overline{x}(t) + B\,\overline{u}(t)$$

where $\quad A = \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

$$\overline{u}(t) = \begin{pmatrix} 2\sin(10\,t) \\ 20\,t \end{pmatrix}$$

In[ ]:=
```
A = ( -2  1
      -1  2 );
B = ( 3  -2
      1   1 );
u[t_] = ( 2 Sin[10 t]
            20 t     );
```

We can consider initial conditions of

$$\overline{x}(0) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

In[ ]:=
```
x0 = {{3}, {-2}};
```

We can chose $t_o = 0$ and apply Eq.2 to write

$$\overline{x}(t) = e^{A(t-t_o)}\,\overline{x}(t_o) + \int_{t_o}^{t} e^{A(t-\tau)}\,B\,\overline{u}(\tau)\,d\tau$$

$$= e^{A\,t}\,\overline{x}(0) + \int_{0}^{t} e^{A(t-\tau)}\,B\,\overline{u}(\tau)\,d\tau$$

$$= e^{A\,t}\,\overline{x}(0) + \int_{0}^{t} e^{A\,t-A\,\tau}\,B\,\overline{u}(\tau)\,d\tau$$

$$= e^{A\,t}\,\overline{x}(0) + \int_{0}^{t} e^{A\,t}\,e^{-A\,\tau}\,B\,\overline{u}(\tau)\,d\tau \qquad \text{note: } e^{A\,t} \text{ is not a function of } \tau$$

$$= e^{A\,t}\,\overline{x}(0) + e^{A\,t}\int_{0}^{t} e^{-A\,\tau}\,B\,\overline{u}(\tau)\,d\tau$$

$$\overline{x}(t) = \overline{x}_{\text{homogeneous}}(t) + \overline{x}_{\text{particular}}(t)$$

where $\overline{x}_{\text{homogeneous}}(t) = e^{A\,t}\,\overline{x}(0)$

$\overline{x}_{\text{particular}}(t) = e^{A\,t}\int_{0}^{t} e^{-A\,\tau}\,B\,\overline{u}(\tau)\,d\tau$

We have to compute the matrix exponential of $e^{A\,t}$ and $e^{-A\,\tau}$. Note that $e^{-A\,\tau} \neq -e^{A\,\tau}$ so unfortunately we have to calculate $e^{-A\,\tau}$

We can compute the various matrix exponentials

In[ ]:=
```
eAt = MatrixExp[A t];
eMinusAτ = MatrixExp[-A τ];

eAt // MatrixForm
eMinusAτ // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2}\,e^{-\sqrt{3}\,t} + \frac{e^{-\sqrt{3}\,t}}{\sqrt{3}} + \frac{e^{\sqrt{3}\,t}}{2} - \frac{e^{\sqrt{3}\,t}}{\sqrt{3}} & -\frac{e^{-\sqrt{3}\,t}}{2\,\sqrt{3}} + \frac{e^{\sqrt{3}\,t}}{2\,\sqrt{3}} \\ \frac{e^{-\sqrt{3}\,t}}{2\,\sqrt{3}} - \frac{e^{\sqrt{3}\,t}}{2\,\sqrt{3}} & \frac{1}{2}\,e^{-\sqrt{3}\,t} - \frac{e^{-\sqrt{3}\,t}}{\sqrt{3}} + \frac{e^{\sqrt{3}\,t}}{2} + \frac{e^{\sqrt{3}\,t}}{\sqrt{3}} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2}\,e^{-\sqrt{3}\,\tau} - \frac{e^{-\sqrt{3}\,\tau}}{\sqrt{3}} + \frac{e^{\sqrt{3}\,\tau}}{2} + \frac{e^{\sqrt{3}\,\tau}}{\sqrt{3}} & \frac{e^{-\sqrt{3}\,\tau}}{2\,\sqrt{3}} - \frac{e^{\sqrt{3}\,\tau}}{2\,\sqrt{3}} \\ -\frac{e^{-\sqrt{3}\,\tau}}{2\,\sqrt{3}} + \frac{e^{\sqrt{3}\,\tau}}{2\,\sqrt{3}} & \frac{1}{2}\,e^{-\sqrt{3}\,\tau} + \frac{e^{-\sqrt{3}\,\tau}}{\sqrt{3}} + \frac{e^{\sqrt{3}\,\tau}}{2} - \frac{e^{\sqrt{3}\,\tau}}{\sqrt{3}} \end{pmatrix}$$

We can compute the autonomous/homogeneous response

In[∘]:=
```
xHomogeneous[t_] = eAt.x0 // Simplify;
xHomogeneous[t] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{6} e^{-\sqrt{3} t} \left(9 + 8 \sqrt{3} + \left(9 - 8 \sqrt{3}\right) e^{2 \sqrt{3} t}\right) \\ \frac{1}{6} e^{-\sqrt{3} t} \left(-6 + 7 \sqrt{3} - \left(6 + 7 \sqrt{3}\right) e^{2 \sqrt{3} t}\right) \end{pmatrix}$$

We can compute the particular response. We can first compute the integrand (again, be careful to use $\overline{u}(\tau)$, not $\overline{u}(t)$ in this step).

In[∘]:=
```
integrand[τ_] = eMinusAτ.B.u[τ] // Simplify;
integrand[τ] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{3} e^{-\sqrt{3} \tau} \left(-10 \times \left(6 - 5 \sqrt{3} + \left(6 + 5 \sqrt{3}\right) e^{2 \sqrt{3} \tau}\right) \tau + \left(9 - 5 \sqrt{3} + \left(9 + 5 \sqrt{3}\right) e^{2 \sqrt{3} \tau}\right) \text{Sin}[10 \tau]\right) \\ \frac{1}{3} e^{-\sqrt{3} \tau} \left(-10 \times \left(-3 - 4 \sqrt{3} + \left(-3 + 4 \sqrt{3}\right) e^{2 \sqrt{3} \tau}\right) \tau + \left(3 - \sqrt{3} + \left(3 + \sqrt{3}\right) e^{2 \sqrt{3} \tau}\right) \text{Sin}[10 \tau]\right) \end{pmatrix}$$

We can integrate this between 0 and t

In[∘]:=
```
xParticular[t_] = eAt.Integrate[integrand[τ], {τ, 0, t}] // Simplify;
xParticular[t] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{10}{927} \times \left(1236 - 591 e^{-\sqrt{3} t} - 500 \sqrt{3} e^{-\sqrt{3} t} - 591 e^{\sqrt{3} t} + 500 \sqrt{3} e^{\sqrt{3} t} - 3090 t - 54 \text{Cos}[10 t] + 9 \text{Sin}[10 t \\ \frac{2}{927} \times \left(5 e^{-\sqrt{3} t} \left(318 - 409 \sqrt{3} + \left(318 + 409 \sqrt{3}\right) e^{2 \sqrt{3} t} - 618 e^{\sqrt{3} t} (1 + 4 t)\right) - 90 \text{Cos}[10 t] + 9 \text{Sin}[10 t \end{pmatrix}$$

So the total response is $\overline{x}(t) = \overline{x}_{\text{homogeneous}}(t) + \overline{x}_{\text{particular}}(t)$

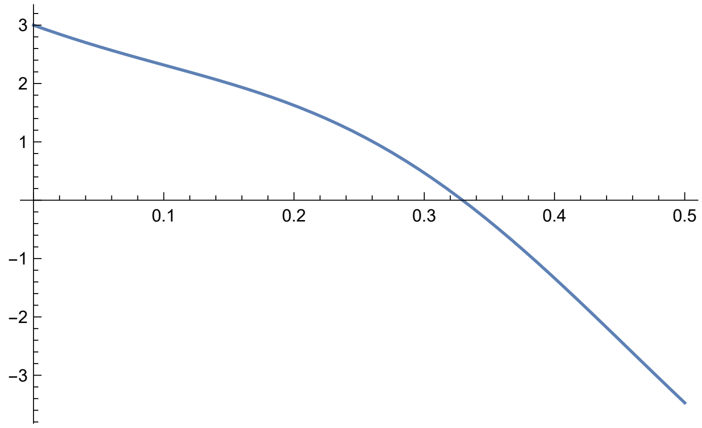
In[∘]:=
```
x[t_] = xHomogeneous[t] + xParticular[t] // Expand;
x[t] // MatrixForm
```

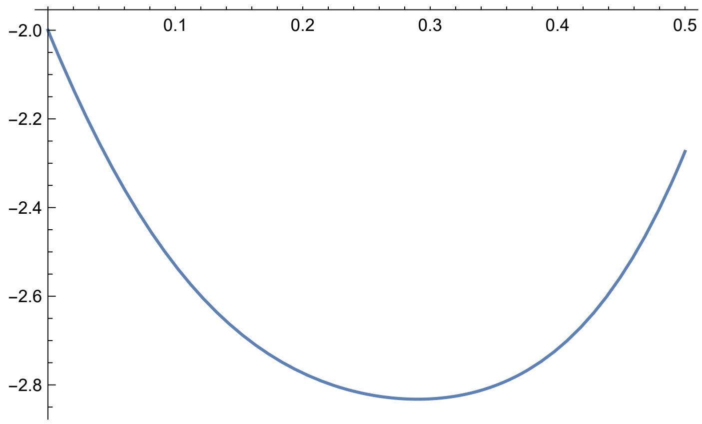
Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{40}{3} - \frac{3013}{618} e^{-\sqrt{3} t} - \frac{3764 e^{-\sqrt{3} t}}{309 \sqrt{3}} - \frac{3013 e^{\sqrt{3} t}}{618} + \frac{3764 e^{\sqrt{3} t}}{309 \sqrt{3}} - \frac{100 t}{3} - \frac{60}{103} \text{Cos}[10 t] + \frac{10}{103} \text{Sin}[10 t] \\ -\frac{20}{3} + \frac{751}{309} e^{-\sqrt{3} t} - \frac{6017 e^{-\sqrt{3} t}}{618 \sqrt{3}} + \frac{751 e^{\sqrt{3} t}}{309} + \frac{6017 e^{\sqrt{3} t}}{618 \sqrt{3}} - \frac{80 t}{3} - \frac{20}{103} \text{Cos}[10 t] + \frac{2}{103} \text{Sin}[10 t] \end{pmatrix}$$

We can plot this to see the response over a given period of time $t \in [0, 0.5]$

In[ ]:=
```
Plot[x[t][[1, 1]], {t, 0, 0.5}]
Plot[x[t][[2, 1]], {t, 0, 0.5}]
```

Out[ ]=


Out[ ]=


We can compare this with a numerical (AKA Matlab/Simulink) solution (go to Matlab)

System Stability

We are now in a position to ask one of the most popular questions of ODEs “Is this system stable”? Recall that the general state space system takes the form of

$$\dot{\overline{x}}(t) = A\,\overline{x}(t) + B\,\overline{u}(t) \qquad \textbf{(Eq.1)}$$

Informally, the concept of stability is really asking, does any state become unbounded as time increases (assuming no external inputs) for some initial condition? In other words

$$\text{stable} \Rightarrow x_i \to 0 \ \text{ for all } i = 1, 2, \ldots, n \ \text{ for any initial condition} \qquad \textbf{(Eq.2)}$$

$$\text{unstable} \Rightarrow x_i \to \pm\infty \text{ for some } i \in \{1, 2, \ldots, n\} \text{ for some initial condition} \qquad \textbf{(Eq.3)}$$

Recall that the solution to this system of differential equations is given by

$$\overline{x}(t) = e^{A(t-t_o)}\,\overline{x}(t_o) + \int_{t_o}^{t} e^{A(t-\tau)}\,B\,\overline{u}(\tau)\,d\tau$$

Recall that we are interested in the situation of no external inputs. Furthermore, we can consider $t_o = 0$ to make notation easier. So we have

$$\overline{x}(t) = e^{A\,t}\,\overline{x}(0)$$

Note that we can perform a similarity transformation on the matrix A where $A = T\,\tilde{A}\,T^{-1}$

$$\overline{x}(t) = e^{\left(T\,\tilde{A}\,T^{-1}\right)t}\,\overline{x}(0)$$

$$= T\,e^{\tilde{A}\,t}\,T^{-1}\,\overline{x}(0)$$

If we choose T to be the matrix of eigenvectors of A (which is non-singular), we see that $\tilde{A}$ is diagonal with the eigenvalues along the diagonals

$$= T\,e^{\begin{pmatrix} \lambda_1\,t & 0 & \ldots & 0 \\ 0 & \lambda_2\,t & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_n\,t \end{pmatrix}}\,T^{-1}\,\overline{x}(0)$$

Because the matrix is diagonal, we can easily compute its matrix exponential

$$\overline{x}(t) = T \begin{pmatrix} e^{\lambda_1\,t} & 0 & \ldots & 0 \\ 0 & e^{\lambda_2\,t} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & e^{\lambda_n\,t} \end{pmatrix} T^{-1}\,\overline{x}(0) \qquad \textbf{\textit{(Eq.4)}}$$

where λ_i = eigenvalue of A

Eq.4 is an extremely helpful expression of $\overline{x}(t)$. Since T, T^{-1}, and $\overline{x}(0)$ are just constants, it shows that the solution $\overline{x}(t)$ is simply linear combinations of the terms $e^{\lambda_i\,t}$ for $i = 1, 2, \ldots, n$. Therefore, the eigenvalues tell everything we need to know in terms of the system response, $\overline{x}(t)$.

From our previous experience, we know that for the expression

$$y(t) = e^{s\,t}$$

If $\text{Re}(s) > 0$, then $y(t)$ grows exponentially. Conversely, if $\text{Re}(s) < 0$, then $y(t)$ decays exponentially. Extending this analysis for our current situation, we therefore see that if even one of the eigenvalues has positive real part, then one or more of the states x_i will become unbounded.

$$\text{Re}(\lambda_i) > 0 \Rightarrow \quad e^{\lambda_i\,t} \to \infty \quad \Rightarrow \quad \text{one or more } x_i(t) \to \pm\infty$$

So we see that the criterion for stability is

All eigenvalues have real part strictly less than 0 ⇒ stable

Any eigenvalue has real part strictly greater than 0 ⇒ unstable

Math Joke:

Question: An international flight from Warsaw, Poland to Los Angeles was flying over the Grand Canyon. Being an international flight from Poland to the USA, the flight was filled with approximately half Polish citizens and half US citizens. For some reason, the two groups naturally segregated themselves with the Polish citizens siting on the port side of the aircraft and the Americans sitting on the starboard side. The flight was uneventful until it was flying over the Arizona when the pilot announced to the cabin that “if you look out the starboard windows you can see the Grand Canyon”. One of the Polish citizens moved to that side of the plane and the aircraft instantly became exponentially unstable and crashed. Why?

Answer: Because you had a single pole in the right half plane.