Christopher Lum
lum@uw.edu

Lecture 02b
4 Quadrant Inverse Tangent

The YouTube video entitled ‘The 4 Quadrant Inverse Tangent (atan2) and Other Inverse Trigonometric Functions’ that covers this lecture is located at https://youtu.be/UWrkh_N1bfE.

Outline

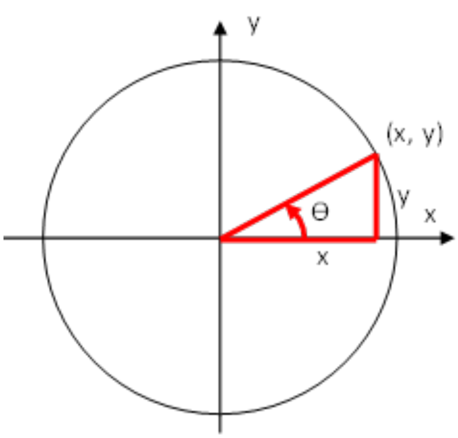
-atan2

Inverse Tangent

Recall the definition of tangent

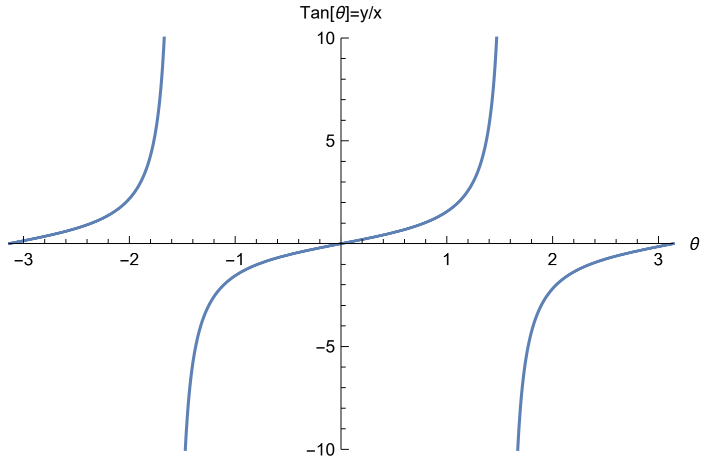
$\tan(\theta) = y / x$

The graphic that goes along with this definition is shown below



We can plot the tangent function

```
θMin = -π;
θMax = π;
Plot[Tan[θ], {θ, θMin, θMax},
 AxesLabel → {"θ", "Tan[θ]=y/x"}, PlotRange → {{θMin, θMax}, {-10, 10}}]
```



We see that that this is not a 1-to-1 function. In other words, if we consider the inverse function

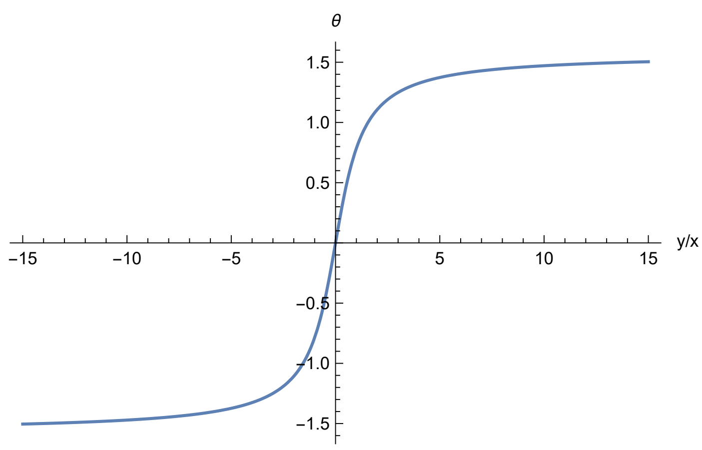
$$\theta = \tan^{-1}(y/x)$$

$$\theta = \tan^{-1}(\alpha)$$

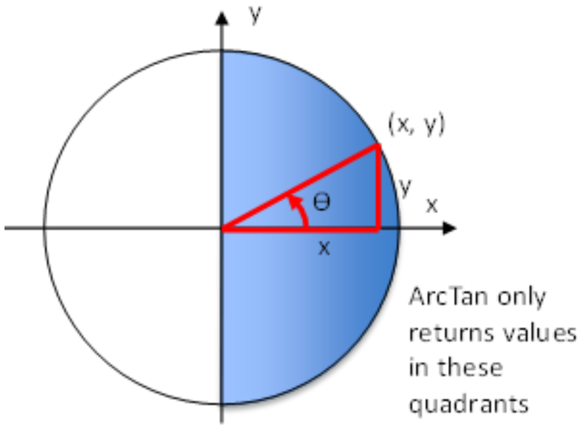
where $\alpha = y/x$

Then for any given value of $\alpha = y/x$, we must chose which θ value to return. When calling the inverse tangent function, most software implementations chose to return a value in the range of $[-\pi/2, \pi/2]$ or $[-90^o, 90^o]$.

```
Plot[ArcTan[α], {α, -15, 15}, AxesLabel → {"y/x", "θ"}]
```



In effect, this only tells half the story because ArcTan only returns values within the two quadrants shown below



This can be a problem because in some cases, it returns the “incorrect” value. For example, consider your aircraft has a velocity of $\begin{pmatrix} x_1 & y_1 \end{pmatrix}^T = \begin{pmatrix} 10 & 4 \end{pmatrix}^T$ m/s. If we want to understand the angle that the aircraft velocity makes with the $+x$ axis, we can simply use

```
x = 10;
y = 4;
θ1 = ArcTan[y / x] 180/π // N
```

```
21.8014
```

However, what if the velocity was $\begin{pmatrix} x_2 & y_2 \end{pmatrix}^T = \begin{pmatrix} -10 & -4 \end{pmatrix}^T$ m/s. From geometry, we clearly see that the correct answer is $\theta_2 = 180 + 21.8 = 201.8^o = -158.2^o$. However, if we use the $\tan^{-1}$ = ArcTan function, we obtain

```
x = -10;
y = -4;
θ2 = ArcTan[y / x] 180/π // N
```

```
21.8014
```

Which is clearly incorrect. The problem is that the negative signs cancel out and by the time you pass the argument to the $\tan^{-1}$ = ArcTan function, the information about which quadrant the answer should reside in is lost.

To combat this problem, we can use the atan2 function which is sometimes referred to a a “2-argument arctangent” or “4 quadrant inverse tangent”.

$$\theta = \text{atan2}(y, x)$$

This effectively implements

$$\operatorname{atan2}(y,x)=\begin{cases}\operatorname{atan}(y/x) & \text{if } x>0\\ \operatorname{atan}(y/x)+\pi & \text{if } x<0 \text{ and } y\geq 0\\ \operatorname{atan}(y/x)-\pi & \text{if } x<0 \text{ and } y<0\\ \pi/2 & \text{if } x=0 \text{ and } y>0\\ -\pi/2 & \text{if } x=0 \text{ and } y<0\\ \text{undefined} & \text{if } x=0 \text{ and } y=0\end{cases}$$

Example

Consider the aircraft example above

```
x = -10;
y = -4;
θ2Correct = ArcTan[x, y] 180/π // N
  (*Be careful and note that Mathematica requires x as the first input*)
```

```
-158.199
```

We see that this returns the proper result

Warning

Be careful of the order of inputs when using Matlab’s atan2 vs. Mathematica’s ArcTan

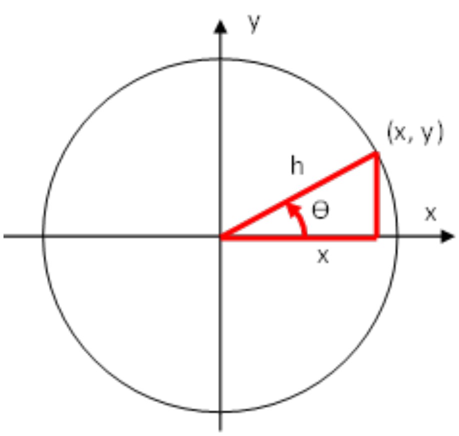
Matlab:	atan2(y,x)
Mathematica:	ArcTan[x,y]

Inverse Cos/Sin

The same analysis can be applied to cos and sin.

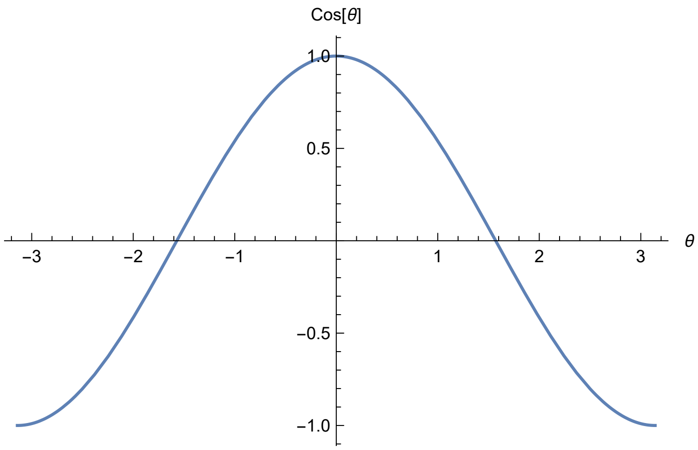
$\cos(\theta) = x/h$

The graphic that goes along with this definition is shown below



We can plot the cosine function

```
θMin = -π;
θMax = π;
Plot[Cos[θ], {θ, θMin, θMax}, AxesLabel → {"θ", "Cos[θ]"}]
```



Again, we see that that this is not a 1-to-1 function. In other words, if we consider the inverse function

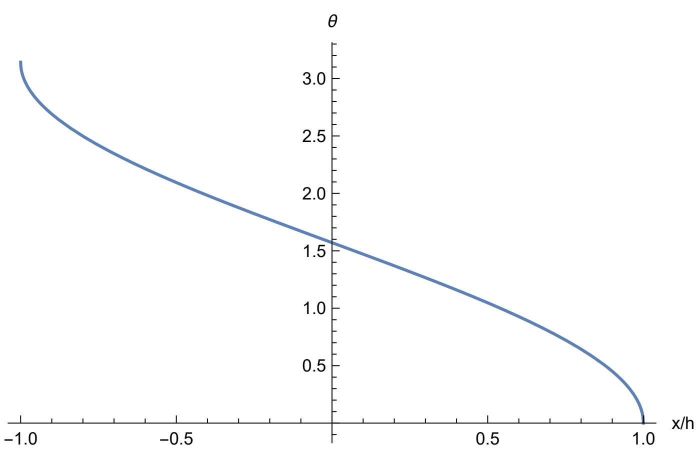
$$\theta = \cos^{-1}(x/h)$$

$$\theta = \cos^{-1}(\beta)$$

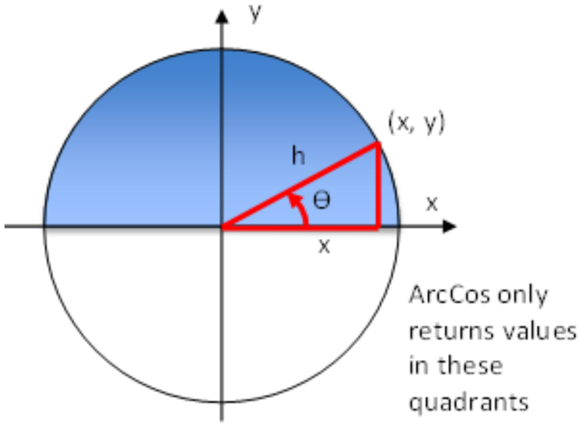
where $\beta = x/h$

Then for any given value of x/h, we must chose which θ value to return. Most software implementations chose to return a value in the range of $[0, \pi]$ or $[0^o, 180^o]$.

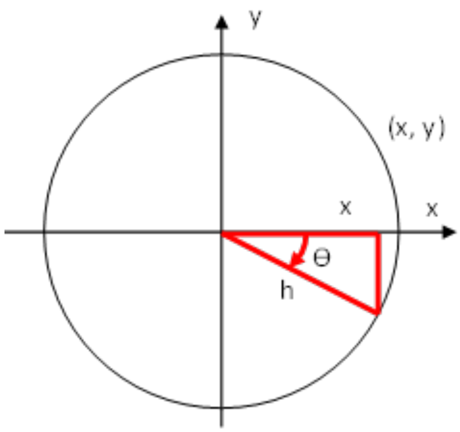
```
Plot[ArcCos[β], {β, -1, 1}, AxesLabel → {"x/h", "θ"}]
```



Again, in effect, this only tells half the story because it only returns values within the two quadrants shown below



This can be a problem because in some cases, it returns the "incorrect" value. For example, if you wanted a value as show below



In this case, since the hypotenuse does not have a sign and is always positive, we need to pass in a second argument to denote that we want solutions from the bottom two quadrants.

$$\theta = \text{acos2}(x/h, \text{quadrant}) = \begin{cases} \text{acos}(x/h) & \text{if quadrant == 'TopQuadrants'} \\ -\text{acos}(x/h) & \text{if quadrant == 'BottomQuadrants'} \end{cases}$$

Example

Consider...