Hypothesis Testing Notes

General Procedure:
Let μ be the mean __(in context of question)_
Let X be _____(if not defined by question)____

For unknown population variance, $s^2 = \frac{n}{n-1}(sample\ variance)$ for unbiased estimate (Sampling)

Test $H_o: \mu =$ __
against $H_1: \mu \ </\neq/>$ __
__ tail test at ____ level of significance

Sample statistics: If you need to calculate due to population mean/ population variance being unknown, be careful to note the negative sign in calculating unbiased estimate of population variance, $s^2 = \frac{1}{n-1}[\sum x^2 - \frac{(\sum x)^2}{n}]$.

Test statistic:
Under H_o, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ approximately since n is large
$Z = \frac{\bar{X}-\mu_o}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ approximately

Could be s instead of σ if population variance unknown.

For known population variance,
Test statistic: $Z = \frac{\bar{X}-\mu_o}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$
For unknown population variance,
Test statistic: $Z = \frac{\bar{X}-\mu_o}{\frac{s}{\sqrt{n}}} \sim N(0,1)$
Where in both cases, X follows a normal distribution OR follows a normal distribution approximately when n is large

Directly input values.
Take standard distribution by convention

Method 1:
Critical region: Reject H_o, if $z \ </>$ _____
Using GC, $z_{calculated} =$ _______ $</>$ _____
Since $z_{calculated}$ lies within critical region, we reject H_o.

Method 2:
p-value= _____ $</>$ ___(level of significance)__
There is sufficient/ insufficient evidence at ___% level of significance that _____________.

p-value can be calculated directly with GC
Take note that when you key into GC "Z-test", under σ you key in sample/ population variance **without the "sample size, n".**
However, take note that when you key into GC e.g. normal cdf to find probability of sample means exceeding a particular value, under σ you key in sample/ population variance **with the "sample size, n".**

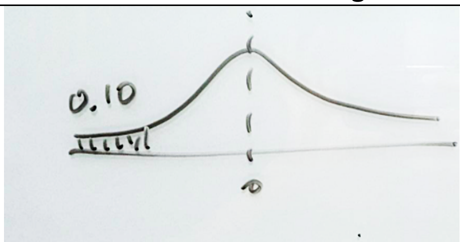
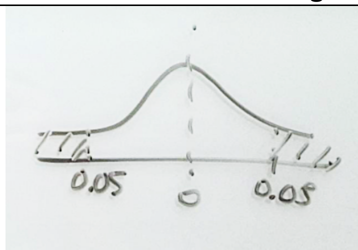
Pointers:

1. H_o must be attributed with equal sign.
   Example: A wholesaler claimed that the mass of his watermelons has a mean mass of at least 3kg. The mean mass of a sample of 50 watermelons from the wholesaler was measured and the mass of watermelons in the sample was found to have standard deviation of 0.6kg.
   → Unknown distribution (require CLT), unknown variance (require unbiased estimator)
   → Interpret as Test $H_o: \mu = 3$ against $H_1: \mu < 3$

2. Concepts of p-value and level of significance

We reject H_o if p-value < level of significance		
Test $H_o: \mu = \mu_o$ against $H_1: \mu > \mu_o$	p-value = P $(Z \geq z_{calculated})$	one tail test (upper tail test)
Test $H_o: \mu = \mu_o$ against $H_1: \mu < \mu_o$	p-value = P $(Z \leq z_{calculated})$	one tail test (lower tail test)
Test $H_o: \mu = \mu_o$ against $H_1: \mu \neq \mu_o$	p-value = P $(\|Z\| \geq \|z_{calculated}\|)$	two tail test

Note that if you are using method 1 (using $z_{calculated}$), need to be careful to ensure that the benchmark z value is calculated with the **right probability values** for **two tail test** in particular.

Example	
One tail test at 10% level of significance	Two tail test at 10% level of significance
	

N.B. If later parts of question requires you to relate p-value from 1-tail test to p-value of 2 tail test, need to multiply p-value by 2, before comparing to the benchmark p-value (inferred from level of significance).

3. Take note of reverse hypothesis questions
   - What is the conclusion?
   - Should the p-value approach be used or should the $z_{calculated}$ and critical range approach be used?
   - Possible questions: (1) Find the range of values of sample mean (2) Find the range of values for population mean (3) Find the least value for sample size

4. Other sub questions you might encounter
   - State what it means for a sample to be random in this context.
     Example: Every biscuit bar has an equal probability of being selected for the sample, and the selection of biscuit bars are made independently.

   - Do we need to assume that variable (e.g. X) follows a normal distribution?
     Example: There is no need to assume that X is normally distributed as $n = 150 > 30$ is large, hence $\bar{X}$ follows a normal distribution approximately.
     N.B. Don't cite CLT. It is used when population variance is known in RV's case.

   - Explain, in this context, the meaning of 'at 2% significance level''
     Example: At 2% level of significance means that there is a probability of 0.02 that the test will wrongly indicate that the mean mass of strawberry jam is less than 200 grams when in fact, it is 200 grams.

   - Explain the meaning of the calculated p value in this context.
     Example: 0.0301 is the probability of obtaining a sample mean as extreme or more extreme than 121 g, assuming the population mean mass of bananas is 125 g.
     Example: The p-value is the smallest significance level to conclude that the population mean time has changed from 30min.

N.B. May need to assume that standard deviation of e.g. coating of a computer device remains unchanged, if question provides you with the **actual value of population variance**. If further asked on why the actual value of population variance may not be used: standard deviation may have changed due to wear out of mechanical parts.