Electric Field

$$U = q(\Delta V)$$
$$E = -dV/dr$$
$$F\_E = -dU/dr$$

Point charges:

$$F_E = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2} \text{ (Unit: N, Vector)}$$ [1]

$$E = \frac{F}{q} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2} \text{ (Unit; } NC^{-1} \text{ } or \text{ } Vm^{-1}\text{, Vector)}$$

$$V = \frac{U}{q} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r} \text{ (Unit: } V \text{ } or \text{ } JC^{-1}\text{, Scalar)}$$

$$U = \frac{1}{4\pi\varepsilon_0} \times \frac{Qq}{r} \text{ (Unit: J, Scalar)}$$

Parallel Plates:

$$F = qE = \frac{qV}{d}\text{, hence } a = \frac{F}{m} = \frac{qV}{dm}$$ [2]

Note: Must include signs of Q/q when calculating V and U. For F and E, can omit if you are only concerned with magnitude. That being said, should be keenly aware of the vector nature of F and E and know what you are doing. Also, do not mix up point charges and plates concepts.

Coulomb's Law (action reaction pair)
The magnitude of the electric force between two-point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation.

Electric Field
A region of space in which a (stationary) electric charge experiences an electric force

Electric Field Strength
The electric field strength at a point is defined as the electric force per unit **positive** charge placed at that point.

Electric Potential
The electric potential V at a point in an electric field is defined as the work done (against the electric force) per unit **positive** charge by an **external** agent in bringing a **small** test charge from infinity to that point, without any change in net KE of the point charge.

Electric Potential Energy
The electric potential energy U of a point charge in an electric field is defined as the work done (against the electric force) by an **external** agent in bringing a charge from infinity to that point, without any change in net KE of the point charge.

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[1] $\varepsilon_0 = 8.85 \times 10^{-12} m^{-3} kg^{-1} s^{-4} A^{-2}$, and $\frac{1}{4\pi\varepsilon_0} = 8.99 \, x 10^9$ **only in vacuum/ air.** r is the centre-to-centre distance between charges Usually ignore F_G in calculation as negligible compared to F_E

[2] Constant potential gradient except at the edges. May be used in conjunction with kinematics (uniform acceleration)

Notes:
Could be important for electric charge to be **stationary** in Efield definition, so that <u>no magnetic field is set up and the charge experiences no magnetic force</u>.
Could be important that **small** positive test charge be used in Efield strength definition, so that <u>electric field lines in electric field being measured is not distorted</u>.

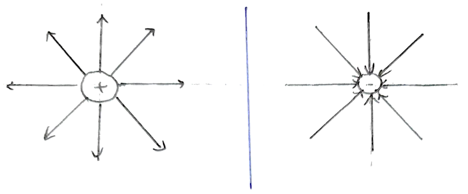
- If solving for net electric force/ electric field strength for a system of point charges, **calculate the magnitude of vector quantity before resoling as vectors**.

- If solving for electric potential/ electric potential energy for a system of point charges, **calculate out the magnitude of scalar quantity, before summing as scalar**.

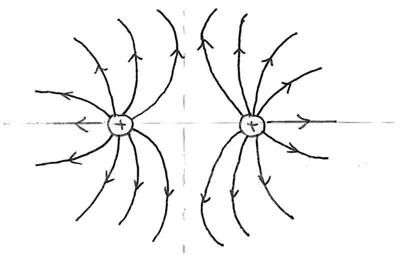
- To analyse changes in PE/ KE of moving charges, need to consider (1) **direction of Efield,** (2) **direction of motion**, and (3) **whether the charge is +ve/ -vely charged.**
- For example, +ve charge moving in direction of field loses PE but gains KE. KE gain = WD by electric force.

Properties of conductors in electrostatic equilibrium:
- Excess charges reside entirely on surface of conductor
- Efield in conductor is 0
- Conductor is at same potential

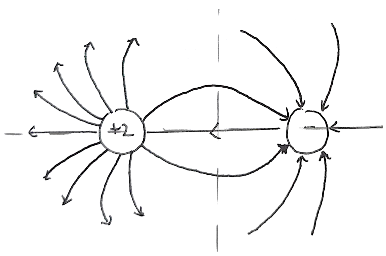
Sketching:



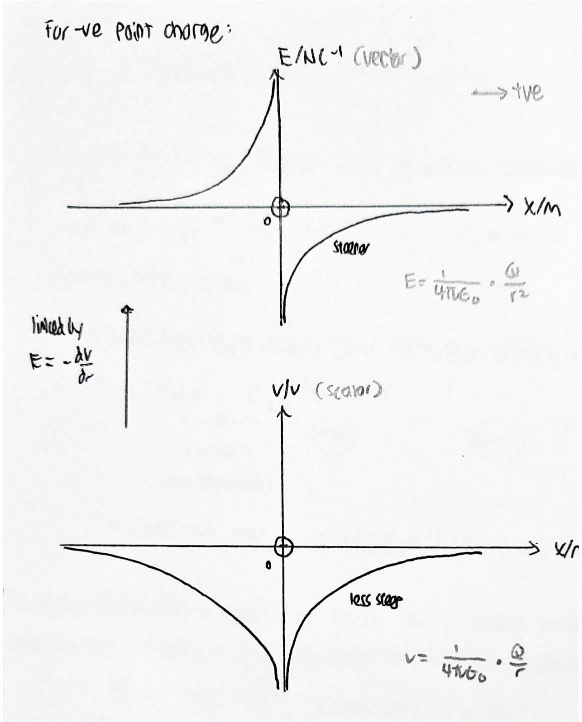
Lines cannot intersect
Equipotential lines cut field lines at right angles
Lines leave conducting surfaces at right angles
Closer field lines= greater electric field strength, vice versa
Field lines from higher potential to lower potential

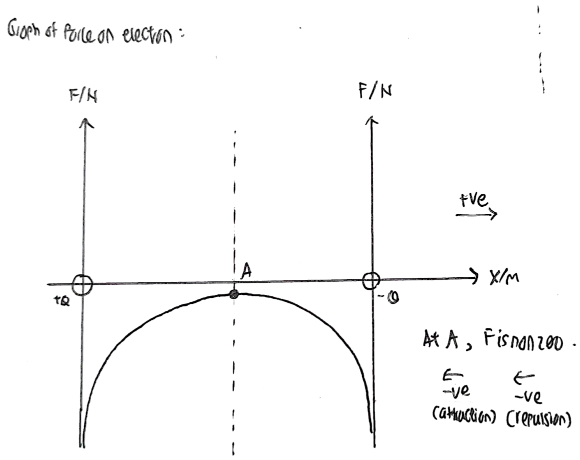


Symmetrical



Charge of one is double of the other: 2x field lines which are also closer together on charge with higher charge





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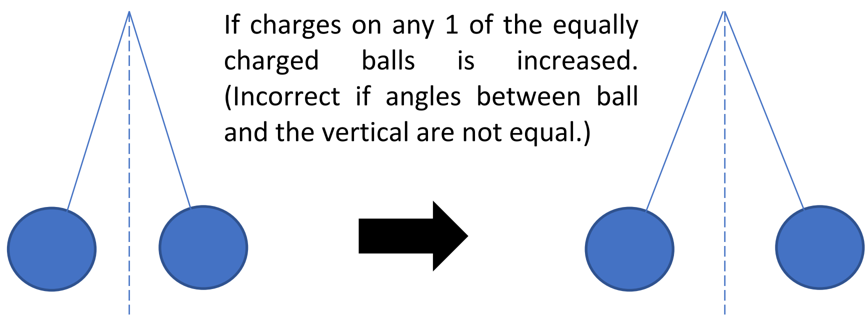
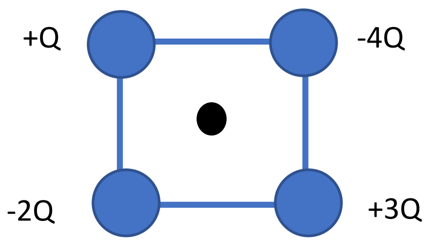
- Remember to include constants in calculations "$\frac{1}{4\pi\varepsilon_o}$" ($\approx 8.99 \times 10^9$).
- **Coulomb's law only applies for point charges separated by large distance** not if the charges are a close distance apart. In that circumstance, $F_E > \frac{Qq}{4\pi\varepsilon_o r^2}$.



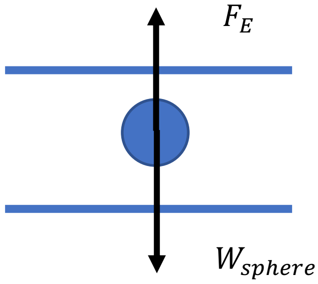
- To find electric field strength at the centre of the square efficiently,



(1) Resolve -2Q and -4Q. Net -2Q NE direction.
(2) Resolve +Q and +3Q. Net +2Q NW direction.
(3) Resolve vectors, Net $\sqrt{2}\,E_{2Q}$ N direction.
(4) Given radius of square 2m, distance from corner of square to centre= $\sqrt{2}$.
(5) Apply Efield formula.

Don't find Efield at centre of square by all 4 corners, then add up. Very tedious and slow!

- If charged sphere(s) are kept stationary by the application of p.d., implies set up is vertical and $W_{sphere} = F_E$.



- Two large horizontal metal plates are separated by a distance of 5.0mm. The lower plate is at a potential of 3.0V. What potential should be applied to the upper plate to create an electric field of strength $2000Vm^{-1}$ upwards in space between the plates?

$$E = \frac{V}{d}$$
$$2000 = \frac{V}{5.0 \times 10^{-3}}$$
$$V = 10V$$

Is the answer 13V or -7.0V?
Answer: -7.0V because upwards implies bottom plate is at a higher potential than upper plate.

- Concepts used to solve problems:
  (i) **Applied with WEP**: Loss in KE from moving from a point of lower to higher potential/ Loss in KE from moving opposite the direction of electric field lines = Gain in EPE
  (ii) **Applied with Circular Motion**: $\frac{mv^2}{r} = F_E = \frac{Qq}{4\pi\varepsilon_o r^2}$

Extension:

The distance between two equipotential surfaces, represented by the lines, indicates how rapidly the potential changes. The smallest distances correspond to the location of the greatest rate of change and therefore to the largest values of the electric field.