

## Quantum Physics

Particle nature	Wave nature
Microscopic systems	Macroscopic systems
Absorption, emission	Diffraction, interference

Definition:

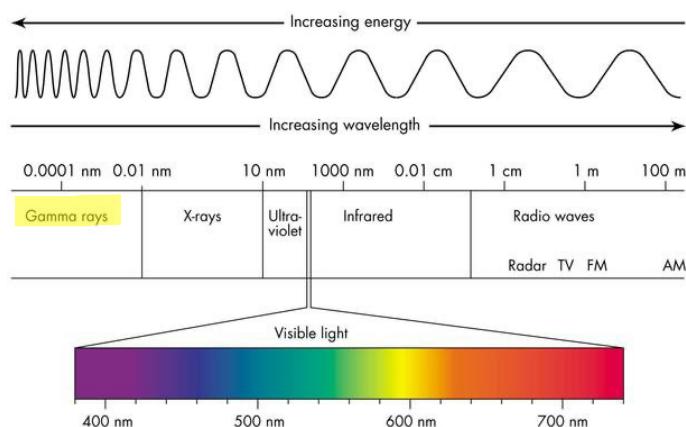
- A photon is defined as a quantum of electromagnetic energy, associated with electromagnetic radiation. Its energy, E is given by  $hf$ , where  $h$  is the planck constant and  $f$  is the frequency of electromagnetic radiation. (2m)
- The photoelectric effect refers to the emission of electrons from a metal surface when the surface is irradiated with electromagnetic radiation of a high enough frequency.
- Stopping potential is the value of potential difference between A and B, where the detected photocurrent just becomes 0.
- Threshold frequency is the lowest frequency that will eject electrons from a particular surface.
- Work function  $\phi$  is defined as the minimum energy required to eject an electron from the surface of a metal.
- Emission line spectrum is a series of distinct coloured lines against a dark background.
- Absorption line spectrum is a series of distinct dark lines against a continuous spectrum.

Formulas:

- Energy of a photon,  $E = hf$ , where  $h = 6.63 \times 10^{-34} J s$
- $1eV = 1.6 \times 10^{-19} J$
- Considering,  $v = c = f\lambda$ , energy for photon =  $\frac{hc}{\lambda}$
- Total energy of monochromatic beam containing N photons =  $Nhf$
- Power of radiation =  $\left(\frac{N}{t}\right) (hf)$
- Intensity of radiation =  $\frac{P}{A} = \left(\frac{N}{t}\right) \left(\frac{hf}{A}\right)$
- $hf = \phi + \frac{1}{2}mv_{maximum}^2 = \phi + eV_{stopping}$
- $p = \frac{E}{c} \rightarrow$  substitute  $E = hf \rightarrow \lambda = \frac{h}{p}$
- $hf = \Delta E = E_f - E_i$
- $\Delta p \Delta x \geq h$

## General Pointers:

- EM Spectrum

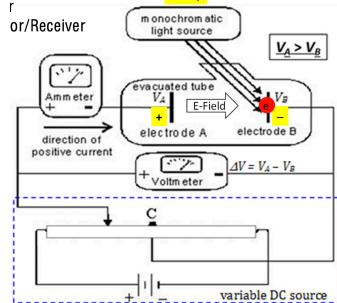
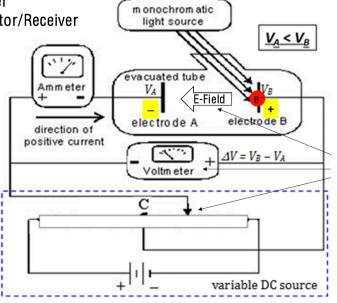
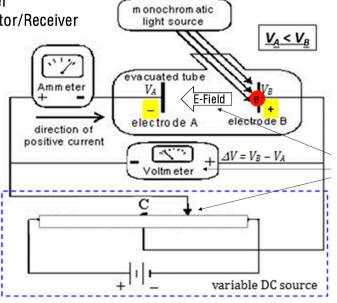
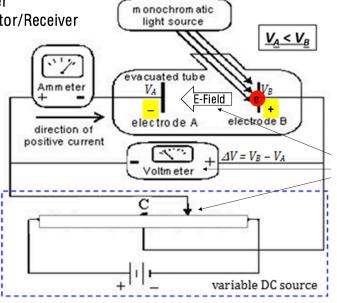


- Experimental Deductions

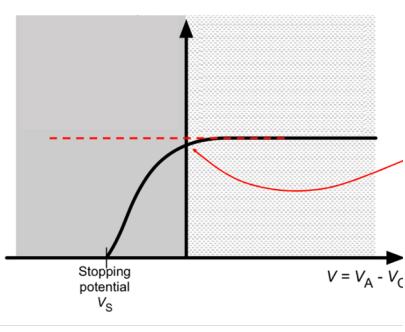
Experiment	Deduction
Young's double slit experiment	Wave nature of light
Electrons are emitted from metal surface using UV light	Photoelectric effect
Various gas can produce spectral lines	Existence of energy levels
X-rays can produce diffraction pattern through solid crystals (De-Broglie $\lambda$ )	Wave particle duality
Alpha particles are reflected through large angles by gold atoms	Rutherford's discovery of small in size, yet massive central positively charged nucleus in atom

- Particularly when question wants you to "show" (e.g. Show that wavelength of a particle is  $1.06 \times 10^{-12}\text{m}$ ), **show intermediate working** (i.e. 4sf or more), else no credit.
- Particle usually won't achieve speed of light or come close to it. So, check your answer if it does.
- N.B. Photons are massless and have momentum  $E = pc$
- If qn provide you with data in eV, then obviously it'll be in eV.
- But if qn provide you with pd and you multiply by  $1.60 \times 10^{-19}$ , your calculated value will be in J already.

## Photoelectric Effect:

$V_A > V_B$	$V_A = V_B$	$V_A < V_B$	$V_A$ at critical negative value wrt $V_B$
 <p>When light shines on electrode B, <b>photoelectrons may be emitted</b> by the metal surface.</p> <p>p.d. across the electrodes<sup>1</sup> causes <b>E-field to be set up from A to B.</b> All photoelectrons will be <b>attracted</b> towards A and <b>accelerate</b> towards electrode A, generating a photocurrent.</p>	 <p>When light shines on electrode B, <b>photoelectrons may be emitted</b> by the metal surface.</p> <p>Ejected photoelectrons from electrode B (cathode) may reach electrode A (anode), producing a current in the circuit. For this case, there is no E-field generated to bias motion of photoelectrons.</p>	 <p>p.d. across the electrodes causes <b>E-field to be set up from B to A.</b> Photoelectrons experience <b>repulsive force</b>, <b>decelerating</b> their motion towards A. loss in kinetic energy = work done against electric field <math>\Delta E_K = e\Delta V</math> Some electrons may have sufficient KE to reach A.</p>	 <p>For most energetic electron, max KE= <math>\frac{1}{2}mv_{maximum}^2 = eV_{stopping}</math> No current is generated.</p>

Photocurrent, I



Photocurrent at  $\Delta V = 0$  will not be maximum

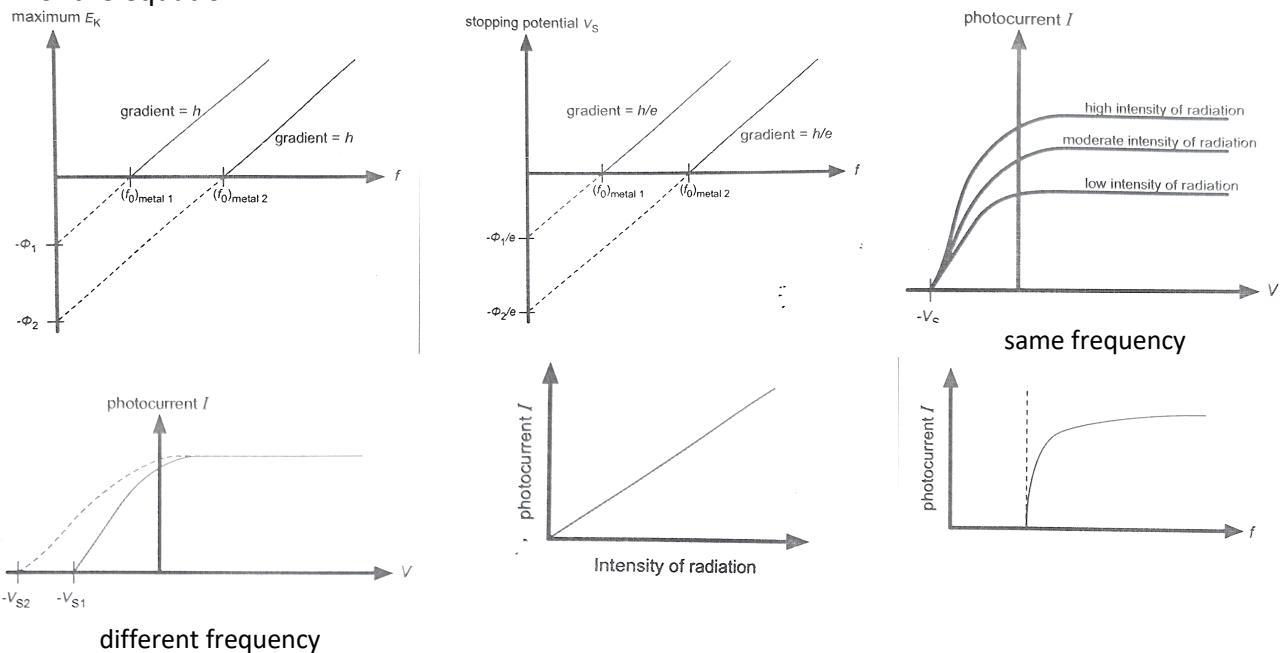
Different free electrons from electrode B require different amounts of energy to escape. Thus, photoelectrons are emitted from electrode B with different amounts of KE. Some photoelectrons may not have sufficient KE to reach electrode A.

<sup>1</sup> varied by adjusting the variable DC source

- Emission of photoelectrons is almost instantaneous
- Photo-electron interaction is one-to-one: energy of a single photon cannot be shared
- Electrons are emitted when the frequency of light is above threshold frequency or below threshold wavelength, which depends on characteristic of metal and is independent of light intensity.
- Maximum KE of ejected electrons **independent of light intensity**, but **dependent on frequency of light used**.
- Rate at which electrons are ejected or photocurrent produced is proportional to the intensity of light.

Higher intensity (f constant) → increasing no of photon incident per unit area and time → more electrons to be liberated in 1-to-1 interactions → increasing photocurrent

Use of  $hf = \phi + \frac{1}{2}mv_{maximum}^2$  to plot graphs, by manipulating variables of the equation:



Pointers:

- Directly proportional condition means  $y \propto x$  (*i.e.*  $y = x$ )

Example: Which one of the following statements about photoelectric effect is correct?

A) Max KE independent of intensity, but photoelectric current proportional to intensity

B) Max KE proportional to intensity, but photoelectric current independent of intensity

C) Both max KE and photoelectric current are proportional to intensity

D) Both max KE and photoelectric current are independent of intensity

Explanation:  $hf = \phi + \frac{1}{2}mv_{maximum}^2 \rightarrow KE_{maximum} = hf - \phi$

Since intensity,  $I = \frac{Nhf}{At}$ ,  $KE_{maximum} = \frac{IAt}{N} - \phi$  not proportional to intensity

- Describe how photoelectric effect provides evidence for particulate nature of EM radiation.

In the photoelectric experiment, electrons are observed to be emitted only when the frequency of light is above some minimum value  $f_0$ , called the **threshold frequency**. If the frequency is below  $f_0$ , no electrons are emitted regardless of how intense the light is, which contradicts classical wave theory that predicts that if sufficiently intense light is used, electrons can absorb enough energy to escape. In the particulate nature of EM theory, the interaction between electron and photon is **one-to-one**. This absorption of the photon leads to gain in energy for the electron to be emitted.

Another observation is the emission of photoelectrons is almost **instantaneous** which again can only be explained considering that the electron interacts with a discrete photon of energy. Emission occurs when the photon is of sufficient energy when interacts with the electron, allowing that electron to gain that amount of energy to escape almost instantaneously.

It is also observed that the **maximum KE** of the emitted electrons is independent of the intensity of the light, but dependent on the frequency of the light used. Electrons that are emitted are found to carry with them a certain amount of KE. Since some of the energy imparted by a photon is used to do work to liberate the electron (work function), the rest must manifest as the **max KE** of emitted electrons.

- Explain condition for electrons to be emitted from surface of metal.

The energy of the photons of EM radiation,  $E=hf$ , is lost in one-to-one interaction to the electrons. For electrons to gain energy to be emitted from the metal, KE must be greater than the work function of the metal.

- Explain why emitted photoelectrons have different energies.

The interaction between photons and electrons is one-to-one. Energy of photons is proportional to its frequency. The remaining energy from the photon after freeing the electron becomes the KE of photoelectrons. Different amount of energy may be required to remove the electrons from the metal surface (the work function only specifies the minimum energy required). Hence, photoelectrons have different energies.

- Explain in terms of the energy changes of the emitted electrons, why there is a minimum pd to reduce the current to zero.

The emitted electrons possess a maximum KE = energy of photon – work function. A minimum pd is required such that the loss of KE to electric potential energy is sufficient to stop even the electrons with the highest KE from reaching the other electrode. All other electrons would have lower KE and would be stopped, giving rise to zero current.

- Explain why the current does not continue to increase for positive values of V (after a certain point).

For positive values of V, all emitted electrons regardless of KE are accelerated to the other electrode. At steady state, the rate of flow of charge is dependent on the number of electrons per unit time in the circuit, and the photocurrent is dependent on the intensity of light and not the pd across the electrodes.

- Suggest why ratio  $\frac{\text{number of electrons emitted per second}}{\text{number of photons incident per second}}$  may not be = 1?

Not all incident photons cause ejection of electrons as some electrons that the photon is incident on may require energy higher than work function. OR

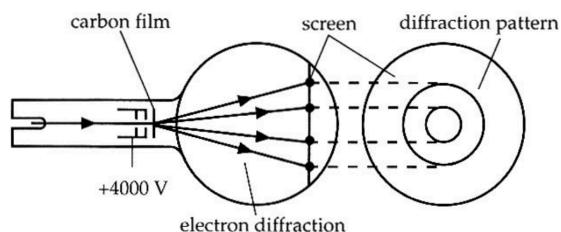
Some of the photons could also be reflected from the metal surface and may not contribute to emission in electrons.

#### Wave-Particle Duality (de Broglie wavelength):

$$\lambda = \frac{h}{p}$$

Also, from superposition:

$$\theta = \frac{\lambda}{b}$$



- Tip to aid solving  $\frac{1}{2}mv^2 = \frac{p^2}{2m}$
- As h value is small, mass of particle must be very small in order for wavelength to be noticeable.

Uncertainty of momentum:

$$\frac{\Delta p}{p} = \frac{\Delta m}{m} \times \frac{\Delta v}{v}$$

Since no uncertainty is associated with mass,

$$\Delta p = \frac{\Delta v}{v} \times p$$

Heisenberg Uncertainty Principle<sup>2</sup>:

$$\Delta p \Delta x \geq h$$

- Inherent trade-off for narrowing down particle's position is the loss of some information about its momentum.
- Applies only for simultaneous determination of quantities in the **same direction** (e.g. particles passing thorough slit)<sup>3</sup>



Pointers:

Solution:

- In the lungs, there are any sacs of air called alveoli. The average diameter of one of these sacs is 0.250mm (**note:  $\Delta x$** ). Consider an oxygen molecule of mass  $5.30 \times 10^{-26} kg$  trapped within a sac. What is the order of magnitude of the minimum uncertainty in the velocity of this oxygen molecule.

$$\begin{aligned} \Delta p \Delta x &\geq h \\ \frac{\Delta p}{p} &= \frac{\Delta m}{m} + \frac{\Delta v}{v} \\ \frac{\Delta v}{v} (mv) \Delta x &\geq h \\ \Delta v &\geq \frac{h}{m \Delta x} \\ \Delta v &\geq \frac{6.63 \times 10^{-34}}{5.3 \times 10^{-26} \times 0.250 \times 10^{-3}} \\ \Delta v &\geq 5.00 \times 10^{-5} ms^{-1} \end{aligned}$$

- The  $\Delta p$  or  $\Delta x$  that you get is like the smallest possible value of  $p$  and  $x$  that you can get.**

Example: Determine the minimum KE of an electron confined within an atom of diameter  $5.0 \times 10^{-11} m$ .

$$\min KE = \frac{p_{\text{minimum}}^2}{2m} = \frac{(\Delta p)^2}{2m} = \frac{\left(\frac{h}{\Delta x}\right)^2}{2m} = \frac{\left(\frac{6.63 \times 10^{-34}}{5 \times 10^{-11}}\right)^2}{2(9.11 \times 10^{-31})} = \underline{\hspace{2cm}} \text{Joules}$$

<sup>2</sup> concerned mainly with order of magnitude of uncertainty

<sup>3</sup> uncertainties can be along **same** horizontal/ vertical direction but more meaningful to consider vertical direction

## Bohr's Model:

$$hf = \Delta E = E_f - E_i$$

- Electron transits to **higher** energy level – photon **absorbed** ( $\Delta E > 0$ )
- Electron transit to **lower** energy level – photon **emitted** ( $\Delta E < 0$ )

Emission Spectra	Absorption Line Spectra
Atoms <b>releasing</b> photons at certain wavelengths causes <b>spikes</b> in the spectra	Light has been <b>absorbed</b> by the atom thus you see a <b>dip</b> in the spectrum
<p>Collision of accelerated charges → energy transfer to atoms → excitation → excited atoms unstable (due to EM interaction between atoms in the gas) → deexcite to ground state by emitting photons</p> <ul style="list-style-type: none"> <li>- Lyman (UV) series</li> <li>- Balmer (visible) series</li> <li>- Paschen (infrared) series</li> </ul> <p>N.B. Ultimately you have to refer to the actual energy levels to determine which transitions will give the visibility range.</p>	<p>Continuous spectrum source emit electromagnetic radiation (photons) directed to cool gas at low pressure where atoms mostly at ground state → gas atoms absorb photons with frequencies corresponding to energies they need to transit from lower to high energy state → excitation</p> <p>When de-excite → photons emitted <b>in all directions</b> → intensity of corresponding wavelength in continuous spectrum through the grating is comparatively lower → formation of "dark lines"</p>
<p><b>HYDROGEN SPECTRUM</b></p> <p><b>Emission Spectrum</b></p> <p>400nm      500nm      600nm      700nm</p>	<p>Produced by transition between energy states in atoms → distinct lines in both spectra same wavelength for both atoms</p> <p>More emission lines than absorption lines</p> <ul style="list-style-type: none"> <li>- Emission line = <math>nC2^4</math></li> <li>- Absorption line = <math>n-1^5</math></li> </ul>

<sup>4</sup> excited states can deexcite to **various states below it**

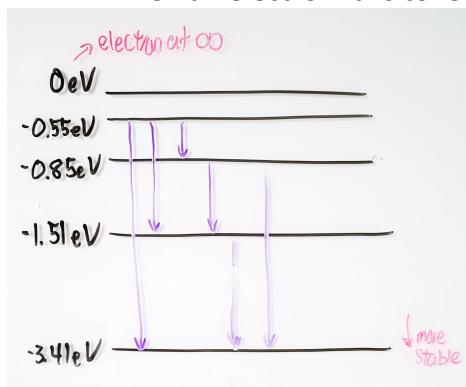
<sup>5</sup> electron in ground state can be promoted to any number of excited states **above it**

Pointers:

- If question specifically ask for the number of e.g. visible lines, cannot use formula. Need to calculate out wavelength of each line and count.
- When you ionise an atom, it does not emit light. Upon ionisation, electron is removed from the atom. For emission of light, electrons needs to be excited to a higher energy state then de-excited.
- There is a difference between type of collisions/ interactions:

① Photon-particle	② Particle-particle
one-to-one	may not be one-to-one
That is, the absorption of the photon will cause the atom to transit to a state with energy difference from ground state <b>equal</b> to energy of photon.	Collision of the particle (e.g. electron) with the atom will cause atom will transit to a state with energy difference <b>equal to or less than</b> energy of electron.
If no energy state transition of atom that matches that of the energy of photon, photon will <b>not interact</b> with the atom.	If atom transit to state with energy difference <b>less than</b> energy of electron, there will <b>KE of scattered electron</b> .

- When an electron falls to -3.41 level, it emits visible light.



2 takeaways:

- **DO NOT draw energy transition from  $\infty$ .** In doing so, you'll suggest every electron at  $\infty$  can deexcite which is not true.
- **Half-life of excited electrons is very short.** As a result, when deexcitation occurs, there will not be a case where an electron deexcites from -0.55eV to -0.85eV then continues to deexcite to -1.51eV

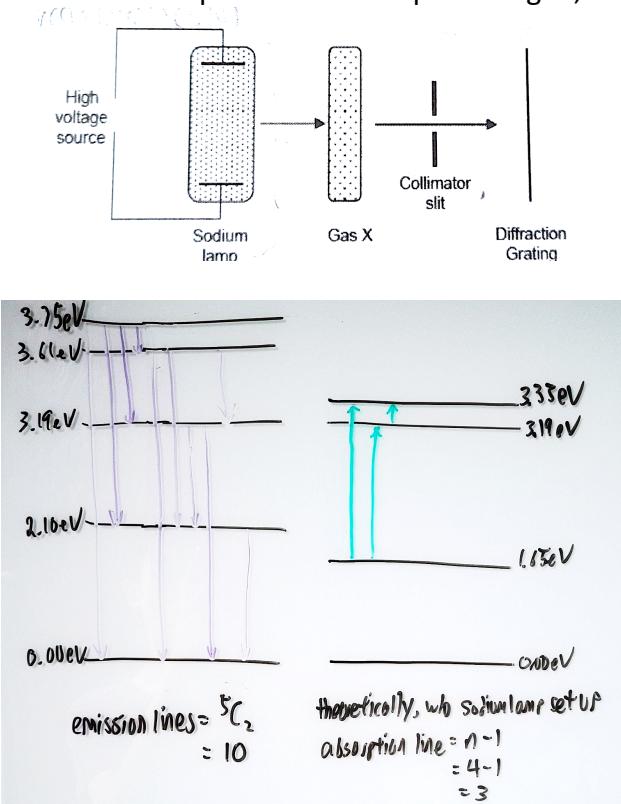
- When atom X is excited to energy level E2 from ground state E0 by collision with another atom, which of the following must be true? (Ans: C)

- A) Electrons in X lose energy just after collision  
 B) Atom X emits radiation  $E_2 - E_0$   
 C) Colliding atom loses energy to excite x  
 D) All the above

Analysis:

- A) May not be true  
 B) May not be true + may deexcite to E1  
 C) Only answer that is definitely true

- When experimental set up is changed, results change.



**Energy released:**

$$3.61 - 2.10 = 1.51$$

$$3.61 - 0.00 = 3.61$$

$$3.19 - 2.10 = 1.09$$

$$3.19 - 0.00 = 3.19$$

$$2.10 - 0.00 = 2.10$$

$$3.61 - 3.19 = 0.42$$

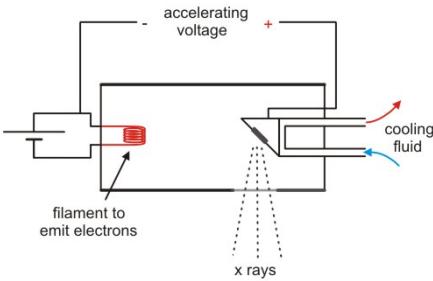
How many spectra lines are observed? (4 or 8 or 9 or 10) (Ans: 8 or 9)

Takeaways:

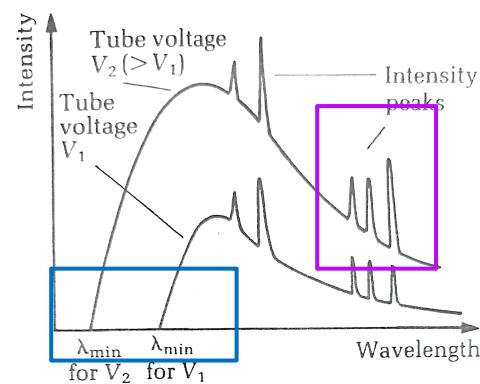
- One-one interaction
- Not pure absorption spectra set up as continuous spectrum source replaced with sodium lamp

**NOTE THE ERATA IN THIS PAGE FOR ABSORPTION SPECTRA!!** Refer to physical notes for update.

## X-ray Spectra (Continuous Bremsstrahlung Spectrum+ Characteristic Peaks):



N.B. Intensity is proportional to  $\frac{N}{t}$  not  $f$



### Explanation for broad, almost continuous spectra:

Bremsstrahlung (/braking) radiation, which is emitted when high energy external electrons coming close to the nucleus decelerate, accelerate or deflect. This energy loss in terms of photons can be any amount of energy less than the maximum kinetic energy of the electrons, thereby forming continuous spectra.

### Explanation for higher intensity of broad spectra with higher tube voltage:

With higher tube voltage there will be greater number of electrons that can be removed and accelerated from the filament. Also, generally higher KE of the electrons undergoing deceleration should be able to produce more photons.

### Explanation for lower $\lambda_{min}$ of higher tube voltage:

Higher tube voltage accelerates the electrons through a larger potential, giving them greater maximum KE, thus emitting photons with higher energy when they completely decelerate. This leads to lower  $\lambda_{min}$ .

### Explanation for sharp cut off at $\lambda_{min}$ :

Maximum possible braking radiation (smallest wavelength) emitted is when electrons are completely stopped when they reach the metal target by emitting a single photon. The cut-off at  $\lambda_{min}$  corresponds to the maximum KE of the incident electrons. There is a sharp cut-off (zero intensity) at this wavelength because no incident electrons possess more energy to emit higher energy photons. ( $E_k = eV \rightarrow hf_{maximum} = eV \rightarrow \lambda_{minimum} = \frac{hc}{eV}$ )

### Explanation for peaks at same wavelengths regardless of voltage:

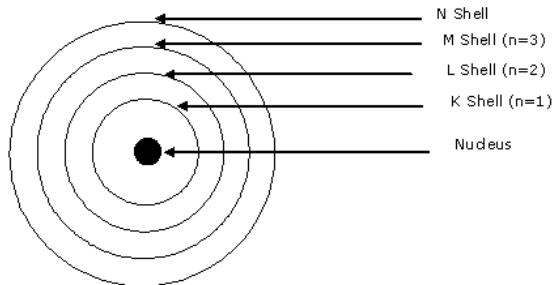
The peak wavelengths are dependent on the energy transition of other higher energy shell electrons to lower energy shells, in which discrete quanta of energy are emitted, depending on the energy level structure of the metal. This is independent of applied voltage.

### Explanation for intensity peaks at certain frequencies/ wavelengths:

The higher energy incident electrons colliding with the metal target give the innermost shell electrons sufficient energy to be removed from the metal or transited to higher energy levels. The peaks correspond to the characteristic X-rays that are emitted when higher energy level electrons transit to lower energy shells.

### X-ray Spectra (X-ray production):

- Metal target is required because transitions in the K shell caused by bombarding electrons at high speeds can only be generated with conducting materials.
- An accelerated incident electron collides with an electron from the innermost shell, kicking it out of the k shell. The atom is excited, due to the vacancy in the K-shell. An electron from the higher energy shells transits to the vacancy in the K-shell, emitting a photon.  $hf = \Delta E = E_{\text{higher energy shell}} - E_{\text{vacant shell}}$



N.B.  $K_\alpha$  line: L to K shell,  $K_\beta$  line: M to K shell,  $L_\alpha$  line: M to L shell,  $L_\beta$  line: N to L shell

### Pointers:

- **Depending on the number of electrons that an atom has, lower shells might be completely filled already. In that case, electrons would have to be excited to higher energy levels.**  
For example, in the case of copper (29 electrons), K, L, M shells are completely filled. To produce  $K_\alpha$  line, K-shell electron must be removed to higher energy shell or be completely free (i.e. at least N-shell).
- Atomic number will affect characteristics peaks, whose wavelength are independent of voltage across the tube.
- Minimum wavelength is dependent on accelerating pd and independent of atomic number of target in X-ray tube.
- Some of energy of bombarding electrons may go to producing heat in target material
- If a thicker block is used in place of a thinner foil of the same material, one may see lower intensity of characteristic peaks due to more energy of electrons lost to work function when electrons knock into deeper lying atomic electrons.
- When accelerating voltage doubles, intensity of X-ray beam may not double.

$$eV (\times 2) = hf (\neq \times 2) + \emptyset$$

$$I (\neq \times 2) = \frac{Nhf}{At}$$