Superposition

Principle of Superposition: When two or more waves of the same type meet at a point in space, the resultant displacement of the waves at any point is the vector sum of the displacement due to each wave acting independently at that point.

→ cannot add intensity of waves up only can add displacement

Stationary Waves:

Stationary waves: Waves whose waveform does not advance and there is no translation of energy. the amplitude of the waves varies according to the position from zero at the nodes to maximum at the antinodes.

Condition for stationary waves:

1. 2 progressive waves of same frequency and speed
2. travelling in opp direction twd one another
3. having similar or identical amplitude
4. being unpolarised or polarised along the same axes are superposed

Comparison of stationary and progressive waves:

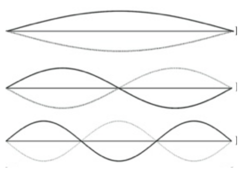
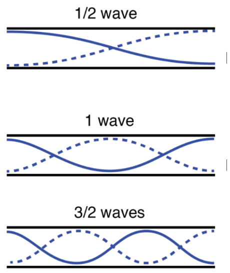
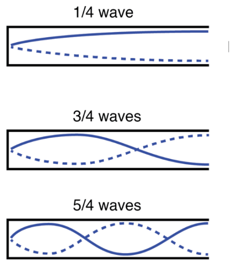
	Stationary waves	Progressive waves
Waveform	Does not advance	Advance with velocity of wave
Energy	No transfer of energy	Energy is transferred in direction of propagation of wave
Phase	Points between adjacent nodes are in phase (though may not have same velocity and displacement from equilibrium), points in adjacent segments are antiphase.	All points within one wavelength have different phases
Amplitude	Varies form 0 at nodes to maximum at antinodes	Same for all particles in wave
Wavelength	2 x distance between adjacent notes/ anti-nodes	Distance between adjacent particles in phase
Frequency	All particles vibrate in SHM with same f (amplitude at nodes = 0)	All particles vibrate in SHM with same f

To form stationary waves: (1) Generator (2) Reflector (3) Incident and reflected waves superimpose

Types of boundaries:

1. Soft boundary
   - Allow for oscillation, displacement at boundary
   - Reflected waves not inverted
2. Hard boundary
   - Does not allow for oscillation, 0 displacement at boundary
   - Reflected waves inverted (phase change = π)

Scenarios of stationary waves:

<table>
<tr><th rowspan="2"></th><th colspan="2">With only N/ AN</th><th colspan="2">With N and AN</th></tr>
<tr><th>String</th><th>Air Column: Open Pipe</th><th>Air Column: Closed Pipe</th><th>Microwave</th></tr>
<tr><td>Formula for wavelength</td><td colspan="2">$\lambda = \frac{2L}{n}$</td><td colspan="2">$\lambda = \frac{4L}{2n-1}$</td></tr>
<tr><td>Nodes and anti-nodes</td><td>Both ends are considered nodes; though end connected to oscillator not a true node</td><td>Both ends are considered displacement anti-nodes</td><td>Closed end is considered displacement node</td><td>Reflector position is a node</td></tr>
<tr><td>Sketch (include graphs of dotted and solid line)</td><td></td><td></td><td></td><td>similar to closed pipe</td></tr>
<tr><td>Nomenclature</td><td colspan="2">(n-1)th overtone
nth harmonic</td><td colspan="2">(n-1)th overtone
(2n-1)th harmonic</td></tr>
</table>

N.B. Displacement anti-node= pressure node, displacement node= pressure antinode for longitudinal waves

Need to identify questions that needs you to employ end correction:

- Length of air column (open pipe)= L + 2c
- Length of air column (closed pipe)= L + c

Interference:

Coherence: Sources of waves are coherent if they have a constant phase difference with respect to time.

Interference: Superposing/ overlapping of two or more waves to produce regions of maxima and minima in space, according to the principle of superposition.

Path difference: The extra distance that one wave travels compared to another wave.

Constructive interference: The interference that occurs when two or more waves meet at a point such that the resultant displacement is **greater** than the largest individual displacement.

Destructive interference: The interference that occurs when two or more waves meet at a point such that the resultant displacement is **less** than the largest individual displacement.

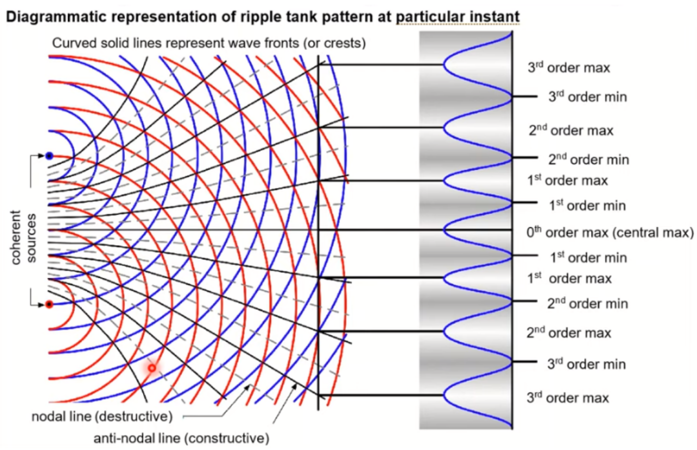
- Coherence implies sources have **same frequency and wavelength**. The reverse may not be true.

- Examples:

Coherent sources	Not coherent sources
Diffracted light through two slits Two speakers fed by the same source Two dippers attached to one vibrator	Two lasers Two candles Two filament lamps

Explanation: Filament lamps do not produce coherent light, they produce light of short coherence length and the phases changes frequently and randomly. Hence, two filament light will not produce observable interference pattern.

- Condition for **observable** interference pattern:
  1. Waves must **overlap to produce regions of minima and maxima**
  2. Sources must be **coherent**
  3. The waves must have the **same amplitude or approximately the same amplitude**
  4. The waves must be **unpolarised or with the same plane of polarisation** (for transverse waves).



- Path difference $\Delta = |S_1P - S_2P|$ (Units: λ)
  May have to make use of Pythagoras theorem to calculate path difference

- Constructive and Destructive interference

	Constructive interference (maxima)	Destructive interference (minima)
	When two waves that arrive at the same point are in phase + occur along anti-nodal lines.	When two waves that arrive at the same point are in anti-phase + occur along nodal lines.
Coherent sources S1, S2 emit waves in phase	$\Delta = n\lambda$	$\Delta = (n+\frac{1}{2})\lambda$
Coherent sources S1, S2 emit waves in anti-phase	$\Delta = (n+\frac{1}{2})\lambda$	$\Delta = n\lambda$

Where n= 0,1,2... representing nth order of maximum/ minimum

- For interference fringes, fringes become more or less spread out depending on whether nodal lines appear at a larger or smaller angle.
  If separation between point sources decreases, nodal line appears at larger angle, fringes become more spread out.

If <u>wavelength of waves decreases</u>, nodal lines appear at <u>smaller angle</u>, fringes become <u>less spread out</u>.

<u>Diffraction</u>:

Diffraction is the bending or spreading out of waves when they travel through a small opening or when they pass around a small obstacle.

The Rayleigh criterion for the limit of resolution states that two images are just resolvable when the centre of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other. This means that the two images will be distinguishable as 2 separate objects. When this happens the angular separation between the two patterns is equal to the angle between the central maximum and the first minimum of one of the two patterns. (Modified August 2021)

Fringe separation is the distance between adjacent bright fringes.

- When wavelength of water waves is very much smaller or larger than the slit width, negligible diffraction is observed. **Only when wavelength is comparable (in order of magnitude) to the slit width, significant spreading of water waves is observed**.

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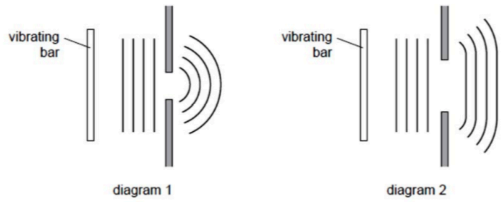
15. Diagram 1 shows a ripple tank experiment in which plane waves are diffracted through a narrow slit in a metal sheet.

Diagram 2 shows the same tank with a slit of greater width.

In each case, the pattern of the waves incident on the slit and the emergent pattern are shown.



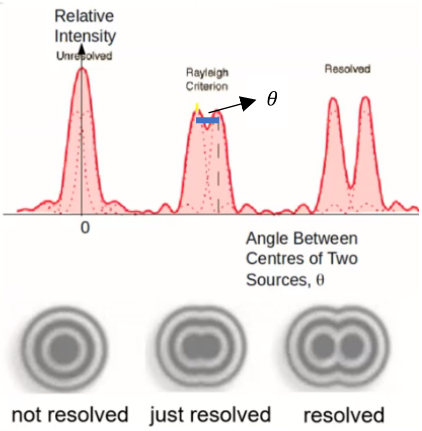
Which action would cause the waves in diagram 1 to produce an emergent pattern closer to that shown in diagram 2?

A Increasing the frequency of vibration of the bar.
B Increasing the speed of the waves by making the water in the tank deeper.
C Reducing the amplitude of vibration of the bar.
D Reducing the length of the vibrating bar.

Choose B over A because when **gap width is <u>larger</u> than the wavelength**, the wave passes through the gap and **<u>does not spread out much</u> on the other side**. When the **gap size is <u>smaller</u> than the wavelength**, **<u>more</u> diffraction** occurs.
Since diagram 2 shows relatively smaller extent of diffraction, width must be larger than wavelength. For width to be larger than wavelength for diagram 1, wavelength must decrease. **Ans: A**

Note: θ to be in radian, unless you use $sin\theta$

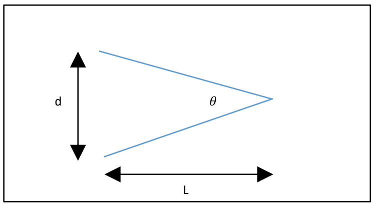
- Rayleigh criterion: $\theta \approx \frac{\lambda}{b}$, where b is the diameter of the aperture



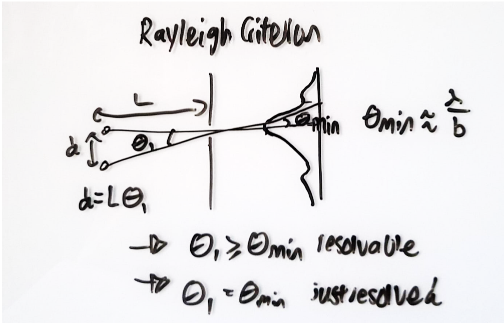
- **Increase resolving power means decreasing θ**

- Aperture of large diameter and the use of a small wavelength of electromagnetic wave is necessary to be able to discriminate two point objects that are close together. Other factors: viewing distance.

- Should also know formula: $d = L\theta$.



- Relationship between the two equations:



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2. Two blue dots of 5 mm diameter are projected on a screen. The distance between the two dots is 7 mm. A student who is facing the screen takes a few steps backwards and stops walking when she could just resolve the two dots.

The student wishes to still resolve the two dots by standing further back from the screen. What should she do?

A darken the room to increase the size of the pupil of the eye. better able to resolve $d \uparrow = L\theta \downarrow$ (for EYE)

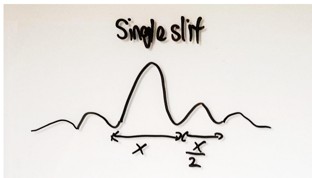
B change the colour of the two dots from blue to red. increase wavelength, $\uparrow \theta \approx \frac{\lambda}{b}$

C make the blue dots slightly brighter. nope

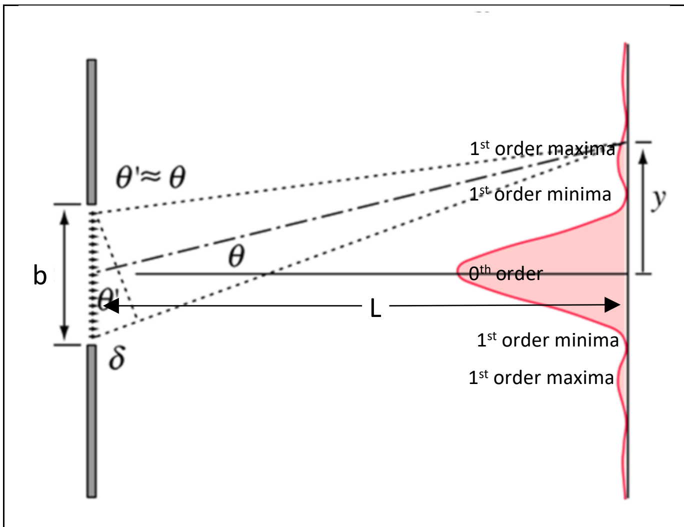
D make the two dots closer. decrease d value, $d = L\theta \downarrow$ don't want θ to decrease further

$d = L\theta$. Given L increase, θ will decrease.

$\theta \approx \frac{\lambda}{b}$ To be able to resolve smaller θ as above, there has to be changes made to λ and b to decrease θ. **Ans: A**



1. Single slit diffraction



For a rectangular opening:

$$tan\theta = \frac{y}{L}$$

$$y = \frac{\lambda}{b}L$$

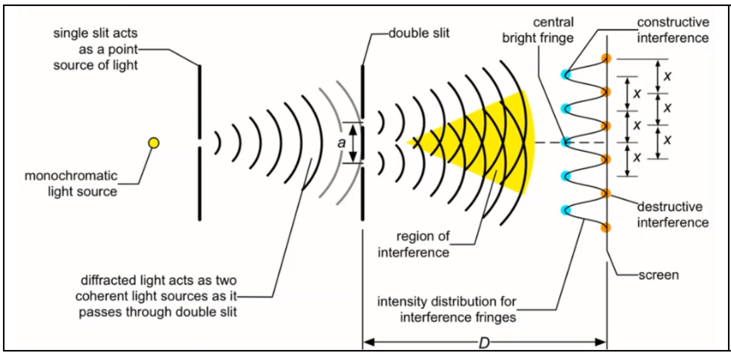
$sin\theta = m\frac{\lambda}{b}$, where m=$\pm1, \pm2$ … (order) for positions of **minima**. (defining equation for 1st order)

Note that $tan\theta \approx sin\theta \approx \theta$.

For a circular aperture:

$$\theta \approx \frac{\lambda}{b}$$

2. Double slit diffraction (Young's double slit experiment)

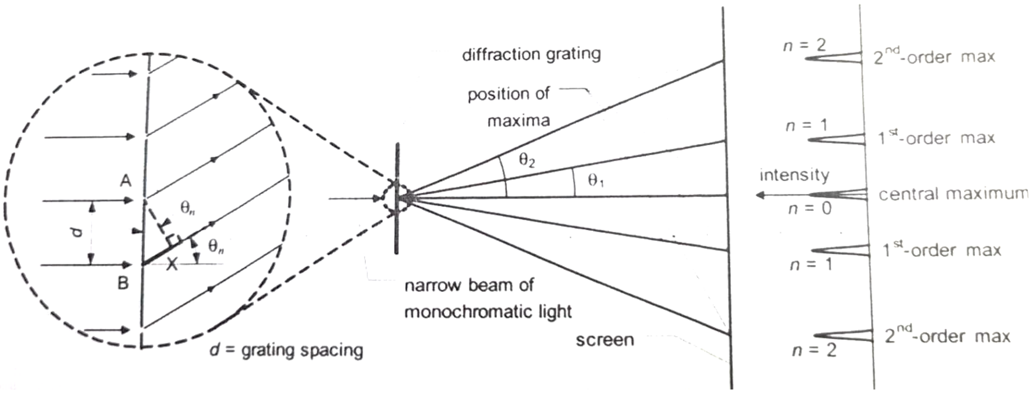


Fringe separation x $= \frac{\lambda D}{a}$

N.B. Linked to interference

- For **small angles**, <u>fringe separation is equal for adjacent maxima</u>.
- For **larger angles**, as in a ripple tank, the <u>separation between adjacent maxima increases as the order increases</u>.

3. Multiple slit diffraction (Diffraction grating)



$dsin\theta = n\lambda$, where n is the order (e.g.1,2,3,4…) for positions of maxima.

Usually, number of slits per mm/cm/m will be given and d=1/N, where N is number of slits per meter. **Convert accordingly**. (be very careful cannot convert $\frac{1m}{10\ slits/mm}$ to $\frac{1m}{10 \times 10^{-3}\ slits/mm}$, should be $\frac{1m}{10 \times 10^{3}\ slits/mm}$)

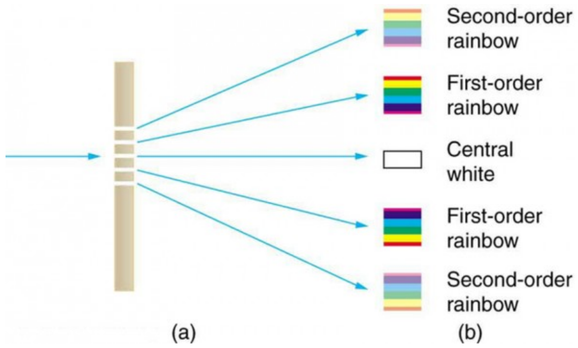
Note that as you go to higher order diffraction, the intensity of interference pattern is modulated by a diffraction envelop.

To find maximum number of maxima that can be observed, $dsin\theta = n\lambda$, maximise $sin\theta$ to find highest order. Highest order x2 + 1 gives you maximum number of maxima that you can observe.

Using higher order diffracted light is better than lower order diffracted light to find wavelength is better, because of the smaller uncertainty of angular positions and smaller uncertainty of wavelength of light.

Reasonable estimates:

- For interference of light, slit width 0.2mm, slit operation 0.5mm, slit screen distance 1m, wave length of light 600nm.
- Take speed of sound as $300ms^{-1}$, speed of microwave = $3.0 \times 10^{8}ms^{-1}$
- Wavelength of visible light ROYGBIV: (700nm) ROY (600nm) G (500nm) BIV (400nm).



Diffraction pattern of white light: Higher λ always further from central maxima

Other examples

- Explain why missing peaks are observed in the double slit interference intensity pattern (2m). Also, given that for double slit: wavelength= 633nm, slit separation 1.2mm, slit screen distance 2.5m, calculate slit width b of each slit (2m).

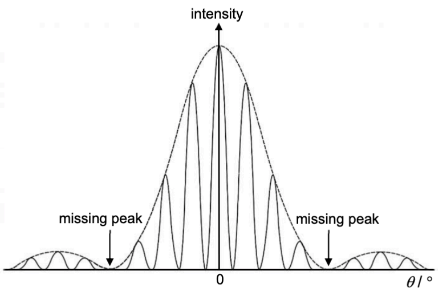


Fig. 4.2

1. The missing peaks occur when the diffraction minima of the single slit diffraction pattern coincide with the interference maxima of the double slit interference pattern.

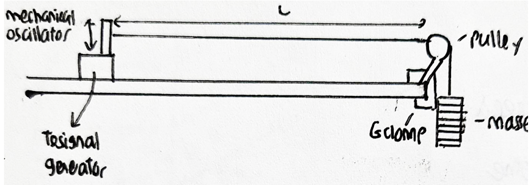
2. single slit: $b\sin\theta = m\lambda$
   double slit: $a\sin\theta = n\lambda$
   Missing peaks occur when
   $\sin\theta = \frac{m\lambda}{b} = \frac{n\lambda}{a} \Rightarrow \frac{b}{a} = \frac{m}{n}$
   For $m = 1$ and $n = 4$, $b = \frac{1}{4}a$
   $b = \frac{1}{4}(1.2\times10^{-3}) = 0.30\times10^{-3}$ m $= 0.30$ mm

- State and explain the changes to interference pattern when the slit widths of double slits is increased (2m).

There will be an increase in the intensities of the interference fringes as more light passes through the wider slits.

AND one of the following:
There will be fewer interference fringes within the narrower single slit diffraction envelope as the degree of diffraction is lesser resulting in less interference.

Missing peaks are nearer to the central maximum because the first minima of the diffraction pattern is closer to central bright fringe.



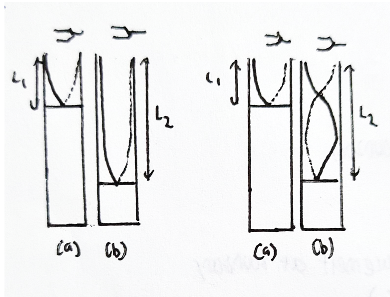
Appendix: Set up for stationary waves

1. String

A string is held taut by connecting one end to a weight over a pulley while the other end to a vibrator/ oscillator.
A signal generator is connected to the oscillator is used to vary the frequency of vibrations
The progressive wave produced travels towards the pulley which acts as a fixed end.
The incident wave superposes with the reflected wave to form a stationary wave.

2. Air Column (measuring speed of sound)

A tuning fork of frequency f is struck and held over the top of a tube filled with water. It continuously transmits sound waves of frequency f into the tube.



The water level is lowered gradually by releasing water from a tap at the bottom until the first instance when a loud resonant note is heard. Note the length of air column L1 as shown in (a).

Using resonance tube, changing frequency of sound

Fill the tube with water again. Repeat for different tuning forks. Note the new length

General equation from resonance lengths:

$$L = \frac{\lambda}{4} = \frac{v}{4}\frac{1}{f}$$

Plot a graph of L against $\frac{1}{f}$ gives the speed of sound in air.

Using resonance tube, changing length of air column

Lower the water level further until the next resonant note is heard at position (b). Note the new length L2.
2
The resonance length corresponds to successive overtones.

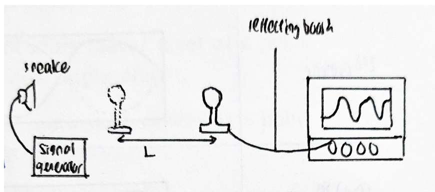
$$L_1 = \frac{\lambda}{4}, L_2 = \frac{3\lambda}{4}$$

$$L_2 - L_1 = \frac{\lambda}{2}$$

Speed of sound in air= $f\lambda$ = 2f($L_2 - L_1$)

3. CRO

The signal generator, connected to a loudspeaker, generates sound waves towards the reflecting board. The reflected waves superpose with the incident waves to form stationary waves.



A microphone connected to the CRO is moved between the loudspeaker and the reflector. The amplitude of the waveform on the CRO increases to a max at position a and then to successive maxima at B. (minima can be used also)

Since distance between 2 successive maximum is $\frac{\lambda}{2}$, and frequency can be determined from signal generator, speed of sound can be calculated by $v = f\lambda$