# Algorithms & Data Structures III: Prim's Algorithm

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# Plan

- MST
- Prim's Algorithm

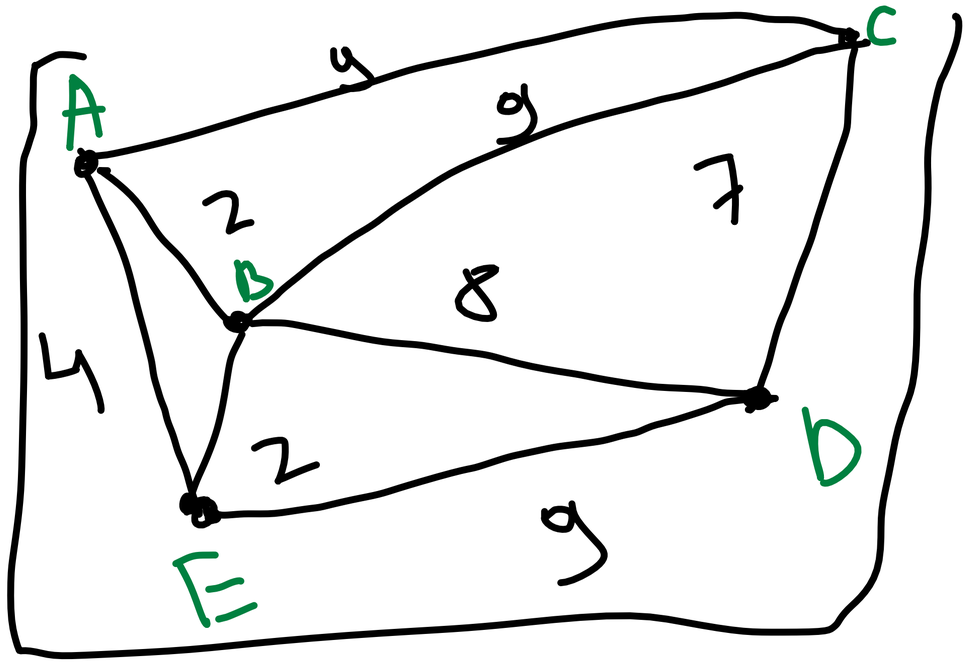
# Definitions

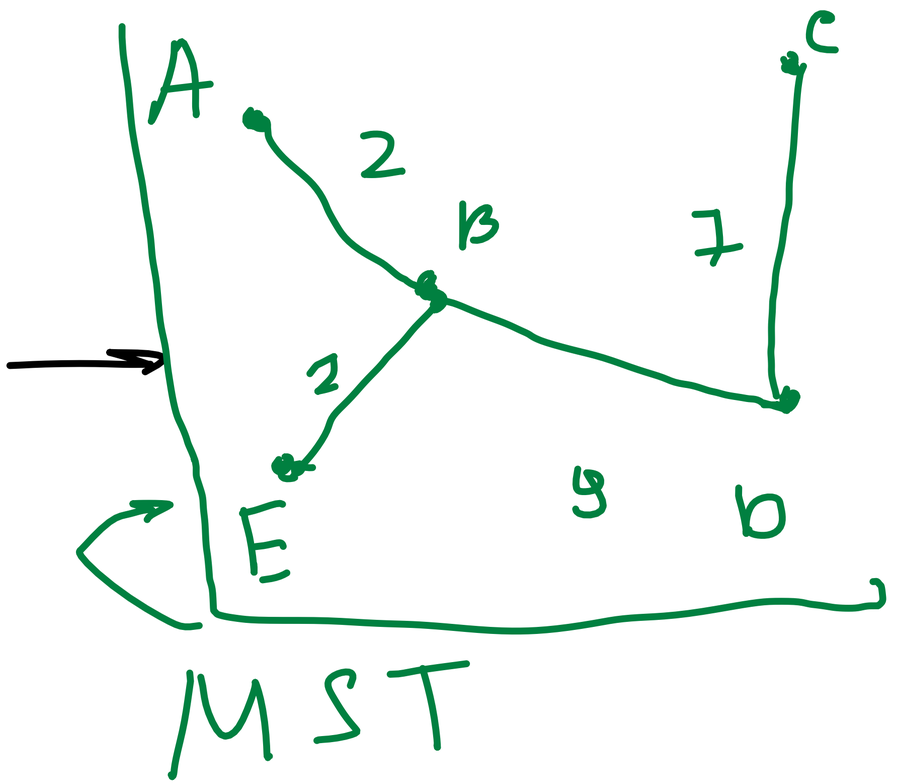
- Tree is connected acyclic graph
- Spanning tree of a graph G is a subset of edges of G that form a tree and include all vertices of G
- Minimum Spanning tree problem:

Given a graph $G = (V, E)$, edges $W: E \to \mathbb{R}$, find a spanning tree T with min weight

$\sum_{e \in T} w(e) \to \min$

G




MST

# Naive Solution

Go through all possible subgraph trees contain all nodes and choose a tree with min sum.

# Prim's Algorithm

- d[i] - distance from i-th node to builded (MST)
- p[i] - pred. of i-th node, the lightest edge connect i-th node to MST
- w(i,j) - weight of (i,j)
- Q - a priority queue, key d[i]
- T - a set of edges from MST

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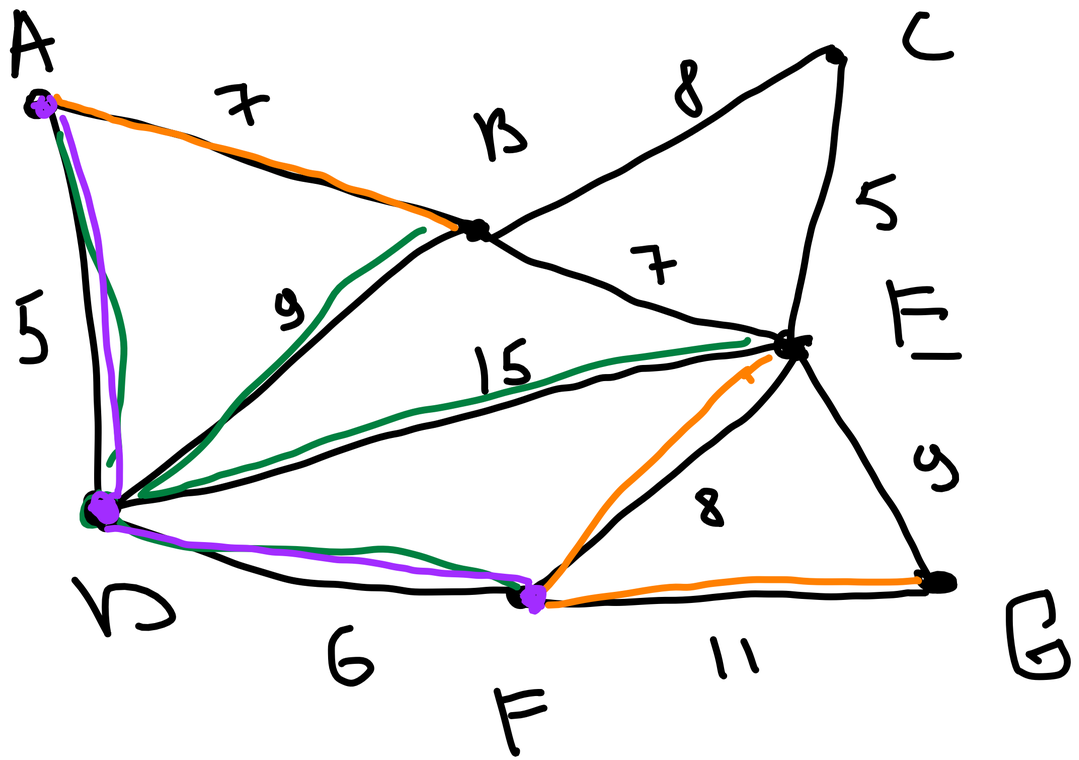
```
T ← []
for noole in V:
  d[noole] ← ∞
  p[noole] ← nil
d[1] ← 0
Q ← V
v ← extractMin(Q)
```

O(n)

## Pseudocode

```
While Q is not empty:
    for u in adj[v]:
        if u ∈ Q && w(u,v) < d[u]:
            d[u] ← w(u,v)
            p[u] ← v
    v ← extractMin(Q)
    T ← T + (p[v], v)
```




| Choosed Nodes | Edges | Not Choosed Nodes |
|---|---|---|
| { } | | {A, ..., G} |
| {D} | (D,A) = 5 | {A, B, C, E, F, G} |
| | (D,B) = 9 | |
| | (D,F) = 6 | |
| | (D,E) = 15 | |
| {A, D} | (A,B) = 7 | |
| | (D,B) = 9 | |
| | (D,F) = 6 | |
| | (D,E) = 15 | |

# Time complexity

1) on array → $O(V^2)$

2) heap → $O(E \log V)$