Lab 4

Def Variabilă aleatoare

$X : \Omega \to \mathbb{R}$ $\quad \{\omega \in \Omega \,/\, X(\omega) \le x\} \in K$

$\forall x \in \mathbb{R}$

σ alg pe $\mathbb{R}$

$$(\Omega, K, P) \xrightarrow{X} (\mathbb{R}, \mathcal{B}_{\mathbb{R}})$$

X s.n. var. aleatoare dacă X este o fcț măsurabilă

Def $\forall A \in \mathcal{B}_{\mathbb{R}}$; $X^{-1}(A) \in K$

$\mathcal{B}_{\mathbb{R}}$ este fm din mulț tuturor intervalelor din $\mathbb{R}$

$$X^{-1}((-\infty, a]) \in K \Leftrightarrow \{\underbrace{\omega \in \Omega \,/\, X(\omega) \in (-\infty, a]}_{X(\omega) \le a}\}$$

$\{X(\omega) / \omega \in \Omega\} \to$ cel mult numărabilă
X este v.a. discretă
$\to$ infinită nenumărabilă
X este v.a. cont.

V.a. discrete

$$X : \begin{pmatrix} \overset{X(\omega_1)}{\overset{\parallel}{x_1}} & \overset{X(\omega_2)}{\overset{\parallel}{x_2}} & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix}$$

Repartiția v.a. X

$x_i \in \bar{\mathbb{R}}$ $\qquad \sum_{i=1}^{n} p_i = 1$
$p_i > 0$

Exp $\quad X : \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \leftarrow$ monedă

$X : \begin{pmatrix} 0 & 1 \\ p & 1-p \end{pmatrix} \quad p \in [0,1]$

$$P(x=0)=\frac{1}{2}$$

Exc

$$x:\begin{pmatrix}-2 & -1 & 0 & 1 & 2\\ 2p & 4p & p & 2p & p\end{pmatrix},\ p\in\mathbb{R}$$

1) $p=?$

$$10p=1 \Rightarrow p=0.1 \Rightarrow x:\begin{pmatrix}-2 & -1 & 0 & 1 & 2\\ 0.2 & 0.4 & 0.1 & 0.2 & 0.1\end{pmatrix}$$

2) $P(x\leq -0{,}75)=P\big((x=-2)\cup(x=-1)\big)=$

$$=\frac{2}{10}+\frac{4}{10}=\frac{6}{10}=0.6$$

$$P(x>0)=\frac{2}{10}+\frac{1}{10}=\frac{3}{10}$$

3) $P(x<\pi-1 \mid x>0.2)=\dfrac{P\big((x<\pi-1)\cap(x>0.2)\big)}{P(x>0.2)}=$

$$=\frac{\frac{2}{10}+\frac{1}{10}}{\frac{2}{10}+\frac{1}{10}}=1$$

$F: \mathbb{R} \to [0,1] \qquad F(x) = P(X \leq x)$

$$F(x) = \begin{cases} 0, & x < -2 \\ 0.2, & -2 \leq x < -1 \\ 0.6, & -1 \leq x < 0 \\ 0.7, & 0 \leq x < 1 \\ 0.9; & 1 \leq x < 2 \\ 1; & 2 \leq x \end{cases}$$

c.d.f.