/

28.02.2018

LFA

- Seminar -

Automate finite deterministe (DFA)
nedeterministe (NFA)

Def DFA: $A = (Q, \Sigma, q_0, \delta, F)$, unde

Q, Σ, F finite, nevide

Q mulț de stări

Σ = alfabetul de intrare

$q_0 \in Q$ stare inițială

$F \subseteq Q$ mulț de stări finale

$\delta : Q \times \Sigma \to Q$

Def NFA: $A = (Q, \Sigma, \delta, q_0, F)$

$\delta : Q \times \Sigma \to 2^Q = P(Q)$

Limbajul acceptat de un DFA (Not A)

$L(A) = \{ \cancel{2} w \mid w \in \Sigma^*, \delta(q_0, w) \in F \}$

Lbj acc de un NFA (Not A)

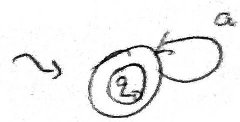
$L(A) = \{ w \mid w \in \Sigma^*, \delta(q_0, w) \cap F \neq \emptyset \}$

Exc

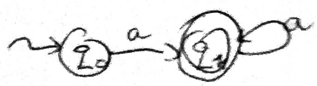
$L_{-1} = \{ a^n \mid n \geq 0 \} = a^*$

$Q = \{q_0\}$

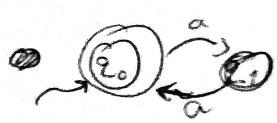
$F = \{q_0\}$



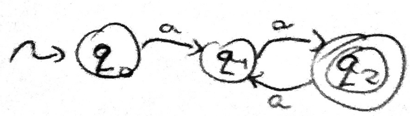
$L_0 = \{ a^n \mid n \geq 1 \} = a^+$



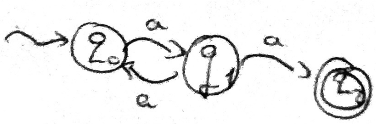
$L_1 = \{ a^{2n} \mid n \geq 0 \}$



$L_2 = \{ a^{2n} \mid n \geq 1 \}$

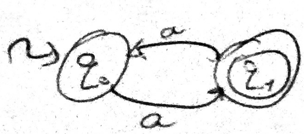


DFA

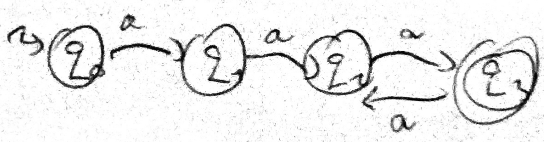


NFA

$L_3 = \{ a^{2n+1} \mid n \geq 0 \}$



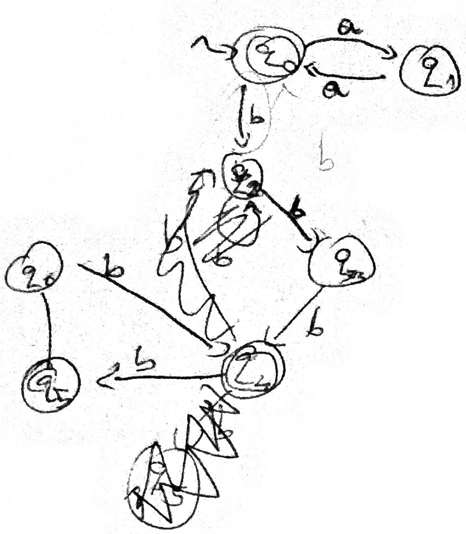
$L_4 = \{ a^{2n+1} \mid n \geq 1 \}$

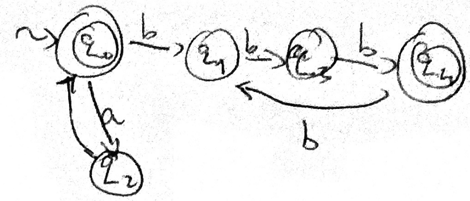


DFA

NFA

$L_5 = \{a^{2n} b^{3m} \mid n \geq 0,\ m \geq 0\}$





$\delta(q_0, aabbb) = \delta(\delta(q_0, a), abbb) =$

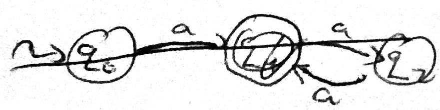
$= \delta(q_2, abbb) = \delta(\delta(q_2, a), bbb) =$

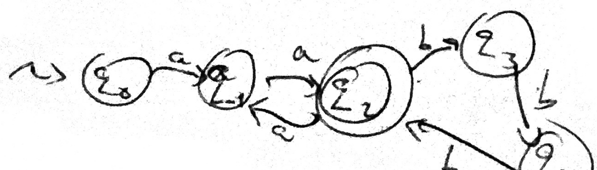
$= \delta(q_0, bbb) = \delta(\delta(q_0, b), bb) =$

$= \delta(q_1, bb) = \delta(\delta(q_1), b) =$

$= \delta(q_3, b) = q_4 \Rightarrow$ aabbb este acceptat

$L_6 = \{a^{2n} b^{3m} \mid n \geq 1,\ m \geq 0\}$

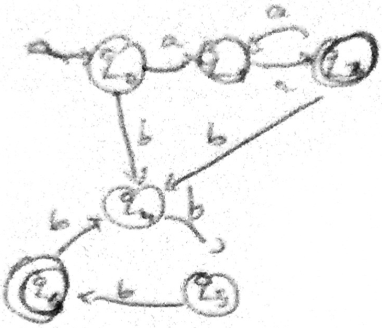




~~$\delta(a^2b^3$~~

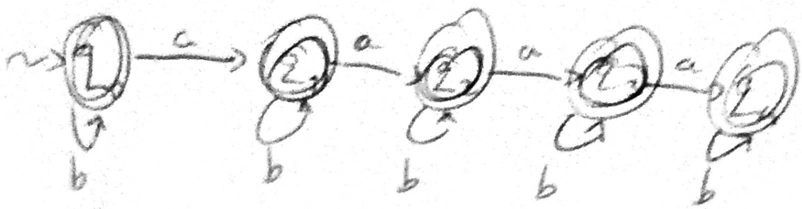
$\delta(q_0, a^2b^3) = \delta(q_2, b^3) = q_2$

$L_7 = \{a^n b^m \mid n \geq 0, m \geq 1\}$



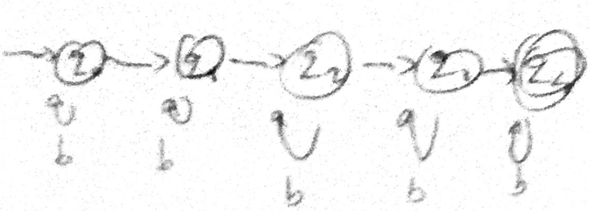
$L_8 = \{w \mid w \in \{a, b\}^*, |w_a| \leq 4\}$

↳ nr. ap. lui a in w

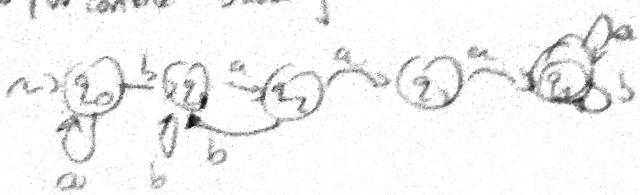


$F = \{q_0, q_1, \dots, q_4\}$

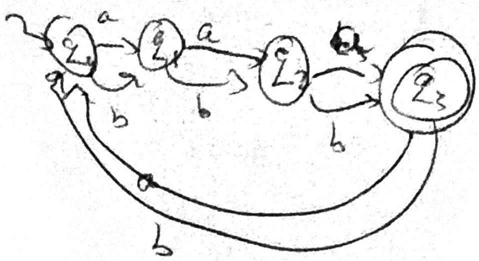
$L_9 = \{w \mid w \in \{a,b\}^*,$ ~~w contine~~ $|w_a| = 4\}$



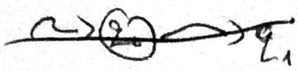
$L_{10} = \{w \mid w$ contine "baaa"$\}$



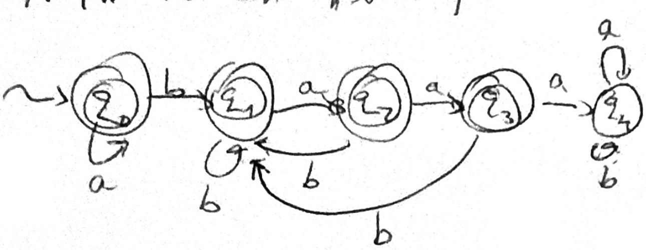
$L_1' = \{w \mid |w| \equiv 3 \pmod{4}\}$



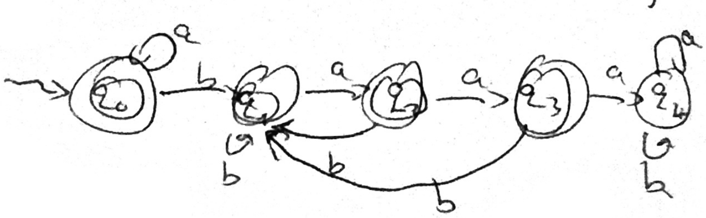
~~$L_{12} = \{w \mid w$ se term. in „baab"$\}$~~



$L_{13} = \{w \mid w$ nu cont „baaa"$\}$



$L_{14} = \{w \mid$ între 2 b-uri, $\exists$ cel puțin 3 de a$\}$



NFA:

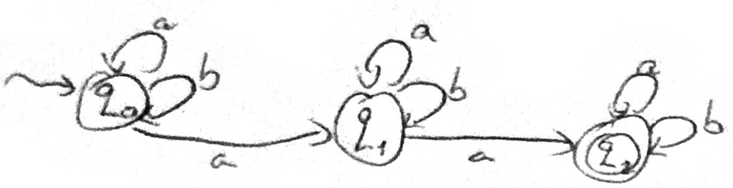
LFA

– Seminar –

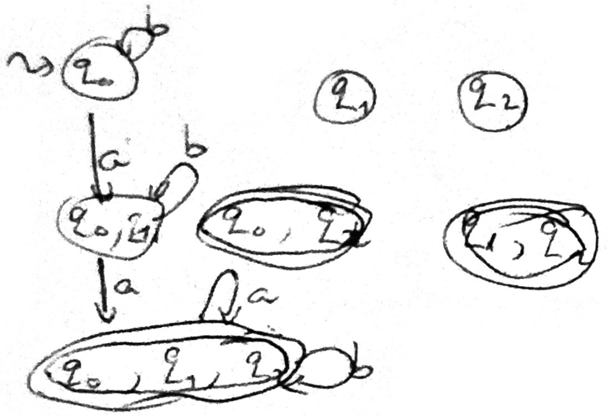
14.03.2018

Th: Orice limbaj L care este acceptat de un NFA este acceptat și de un DFA.

① NFA:



$\Downarrow$ DFA



$(q_0, bba) \vdash (q_0, ba) \vdash (q_0, a) \vdash \{(q_0, \lambda), (q_1, \lambda)\}$

$\{q_0, q_1\} \cap F = \emptyset \Rightarrow$ „bba" nu e acceptat.

$(q_0, babbaba) \vdash (q_0, abbaba) \vdash \{(q_0, bbaba), (q_1, bbaba)\} \vdash$

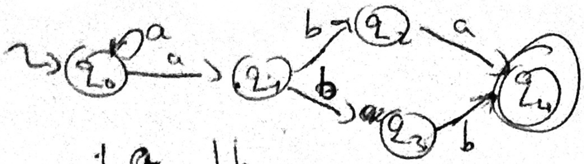
$\vdash \{(q_0, baba), (q_1, baba)\} \vdash \{(q_0, aba), (q_1, aba)\} \vdash$

$\vdash \{(q_0, ba), (q_1, ba), (q_1, ba), (q_2, ba)\} \vdash \{(q_0, a), (q_1, a), (q_1, a), (q_2, a)\} \vdash$

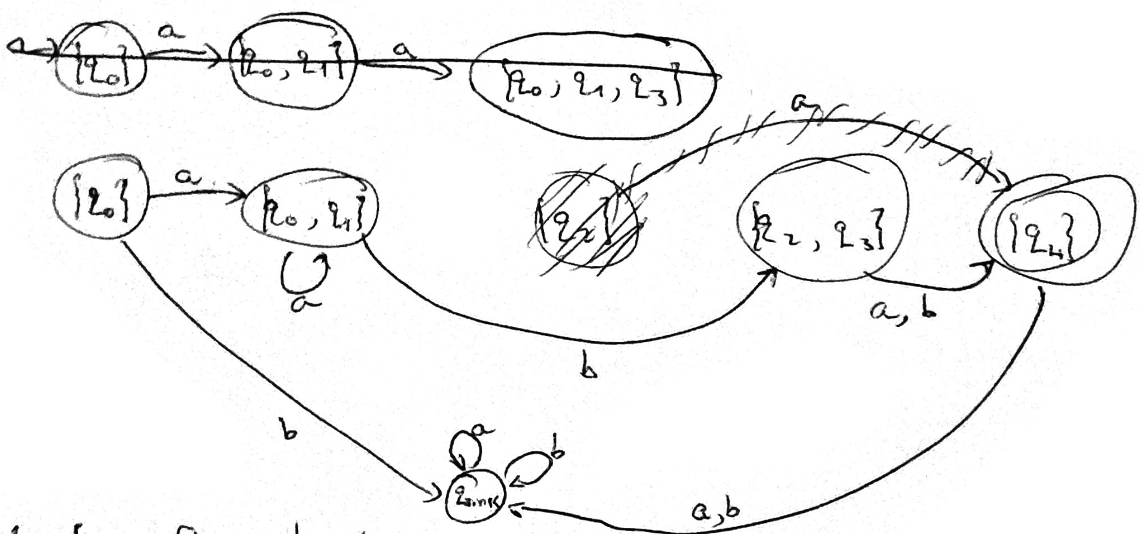
$\vdash \{(q_0, \lambda), (q_1, \lambda), (q_1, \lambda), (q_2, \lambda), (q_1, \lambda), (q_2, \lambda), (q_2, \lambda)\}$

$\{q_0, q_1, q_2\} \cap F \neq \emptyset \Rightarrow$ cuv e acceptat

② NFA

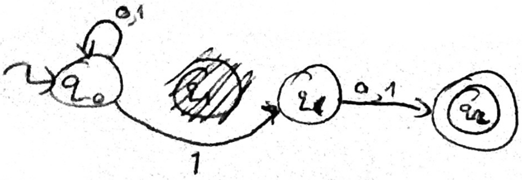


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\emptyset$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_2, q_3\}$
~~$\{q_0, q_1, q_3\}$~~	~~$\{q_0, q_1\}$~~	~~$\{q_1, q_4\}$~~
~~$\{q_2\}$~~	~~$\{q_4\}$~~	~~$\emptyset$~~
$\{q_2, q_3\}$	$\{q_4\}$	$\{q_4\}$



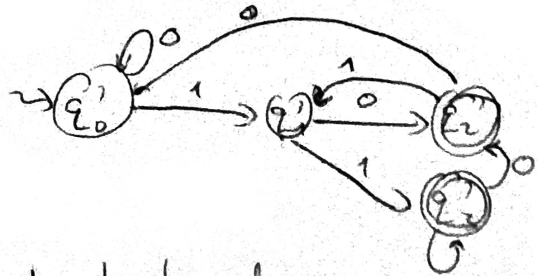
③ ~~L = { 0~~ Poz 2 de la final e 1.

NFA:



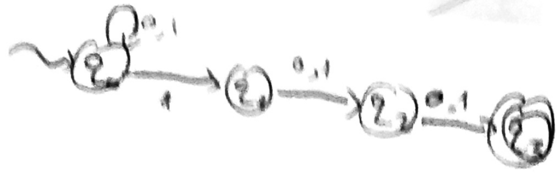
DFA:

		0	1
q_0'	$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_1'	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
q_2'	$\{q_0, q_2\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_3'	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$

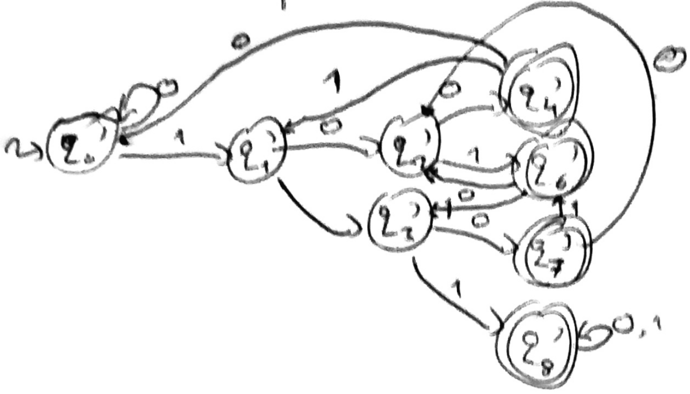


Nr de stări ale DFA-ului este 4

NFA:



		0	1
q_0'	$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_1'	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
q_2'	$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
q_3'	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
q_4'	$\{q_0, q_3\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_6'	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
q_7'	$\{q_0, q_2, q_3\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
q_8'	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$



Fie L mulț cuvintelor din $\{0,1\}^*$ în care pe a n-a poziție de la final avem 1. Atunci NFA:

DFA-ul va avea maxim 2^n stări.

Dem (intuitivă): Din q_0, orice aș alege mă întorc în q_0 => Orice stare conține q_0

=> DFA-ul conține maxim 2^n stări.

Pp că DFA-ul conține mai puțin de 2 stări $\xRightarrow{\text{P. Dirichlet}}$ ∃ o stare q în care vom ajunge atând 2 cuv. diferite $\left.\begin{matrix} a_1 a_2 \dots a_n \\ b_1 b_2 \dots b_n \end{matrix}\right\} \Rightarrow \exists\, i \in \overline{1,n}$ a.î. $a_i \neq b_i$, ...

Luăm $\begin{cases} a_p = 1 \\ b_p = 0 \end{cases}$. Dacă $p=1 \Rightarrow a_1 = 1$

$b_1 = 0$

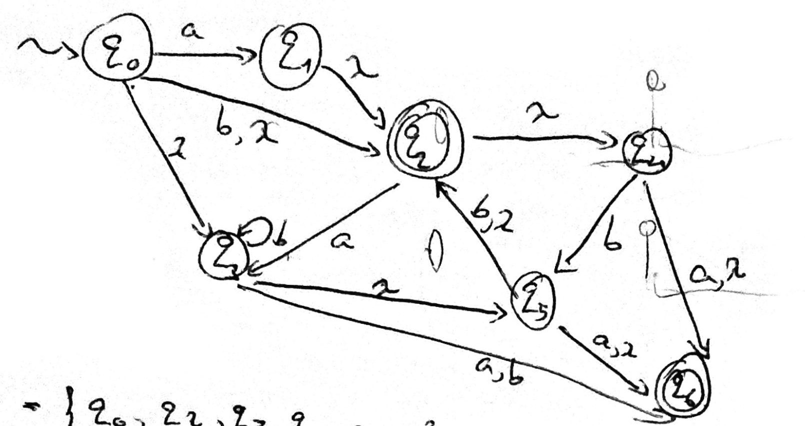
Pt a, 2 este stare finală
Pt b, 2 nu e stare finală $\}$ abs

Dacă $p > 1 \Rightarrow$ Fie p starea în care ajung citind $p-1$ de 0 la final

$a_p a_{p+1} \ldots a_n \underbrace{00\ldots0}_{p-1} \to$ este acc

$b_p b_{p+1} \ldots b_n \underbrace{0\ldots0}_{p-1} \to$ nu este acc

λ-NFA:



$\langle q_0 \rangle = \{q_0, q_2, q_3, q_4, q_6, q_5\}$

$\langle q_1 \rangle = \{q_1, q_2, q_4, q_6\}$

$\langle q_2 \rangle = \{q_2, q_4, q_6\}$

$\langle q_3 \rangle = \{q_5, q_2, q_6, q_3, q_4\}$

$\langle q_4 \rangle = \{q_4, q_6\}$

$\langle q_5 \rangle = \{q_2, q_6, q_5, q_4\}$

$\langle q_6 \rangle = \{q_6\}$

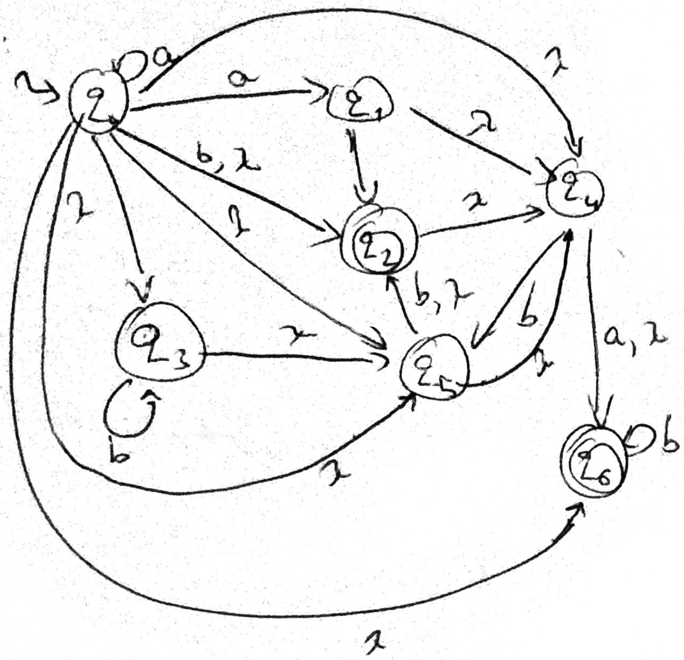
~~$\delta(\langle q_0 \rangle$~~

LFA

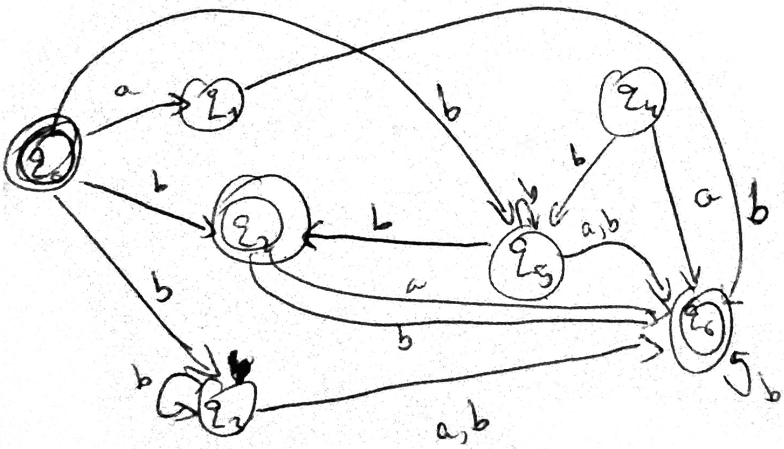
-Seminar-

1. Transformare λ-NFA în DFA

λ-NFA:



NFA:



2. Lema de pompare.

Fie L un limbaj regulat. Atunci $\exists\, p \in \mathbb{N}$ a.î. $\forall s \in L, |s| \geq p$

s se ~~de~~compune $s = xyz$ cu $\begin{cases} |xy| \leq p \\ |y| \geq 1 \\ xy^iz \in L, \forall i \geq 0 \end{cases}$

strategie

1) Pp că L e regulat

2) Aplic lema

3) Găsesc un cuv. convenabil s, $|s| \geq p$ care nu poate fi pompat $\Rightarrow$ nu e din lbg. $\cancel{\times}$

Spuneți dacă lbg este sau nu regulat. Dacă da, constr un DFA care să îl accepte, dacă nu, dem fol lema de ~~b~~ pompare.

1) $L = \{a^n b c^n \mid n \geq 0\}$

Pp abs că L e regulat $\overset{LP}{\Rightarrow} \exists\, p \in \mathbb{N}$ a.î. $\forall s \in L$, $s = xyz$ cu $\begin{cases} |xy| \leq p \\ |y| \geq 1 \\ xy^iz \in L, \forall i \end{cases}$

Fie $s = a^p b c^p$

$|s| = 2p+1 \geq p \Rightarrow$ ~~$\times$~~ ~~Fie~~

$|xy| \leq p \Rightarrow xy$ cu numai a ur:

$x = a^q$
$y = a^r$
$z = a^{p-q-r} b c^p$

$x = a^q$
$y = a^r$
$z = a^{p-q-r} b c^p$

$\Bigg\}$ Fie $i = 0$
$\Rightarrow xy^0z = xz = a^{p-r} b c^p$ cu $r > 0 \Rightarrow$
$\Rightarrow p - r < p \Rightarrow xy^0z \notin L$ $\cancel{\times}$

$L = \{a^i b^j \mid i > j\}$ nu e reg

Pp obs că L e reg $\Rightarrow$ $\exists p$ a.î. $\forall s \in L$, $s = xyz$, $|s| \geq p$ $\begin{cases} |xy| \leq p \\ |y| \geq 1 \\ xy^iz \in L \ \forall i \geq 0 \end{cases}$

$|s| = 2p - 1 \geq p$

$s = a^p b^{p-1} \Rightarrow xy$ ct doar a-uri $\Rightarrow$

$\left.\begin{array}{l} x = a^q \\ y = a^r \\ z = a^{p-r-q} b^{p-1} \end{array}\right| \Rightarrow i = 0 \Rightarrow a^{p-r} b^{p-1} \in L$

$|y| \geq 1 \Rightarrow r \geq 1 \Rightarrow p - r \leq p - 1$
$\left.\begin{array}{l} \\ p - r > p - 1 \end{array}\right| \lightning$

3. $L = \{a^i b^j \mid i < j\}$

Pp obs că L este reg $\Rightarrow$ $\exists p \in \mathbb{N}$ a.î. $\forall s \in L$ cu $|s| \geq p$, $s = xyz$ cu

$s = a^p b^{p+1}$, $|s| = 2p + 1 \geq p \Rightarrow xy$ va conține doar a-uri

$\left.\begin{array}{l} x = a^q \\ y = a^r \\ z = a^{p-q-r} b^{p+1} \end{array}\right|$ Pt $i = 2$ $\Rightarrow a^{p+r} b^{p+1}$

$|y| \geq 1 \Rightarrow r \geq 1 \Rightarrow p + r \geq p + 1$
$\left.\begin{array}{l} \\ p + r < p + 1 \end{array}\right| \lightning$

$s \notin L$

4. $L = \{a^i b^j \mid i \neq j\}$ nu e reg

Fie $s = a^k b^{2k}$, $k = p!$, p dat de lemă

$|s| = 3k \geq p \Rightarrow xy^i z = a^{k + r(i-1)} b^{2k}$

$r \mid k \Rightarrow \exists m$ a.î. $k = m \cdot r$.

$m \cdot r + r(i-1) = 2k \Rightarrow i = m+1$

Fie $i = m+1$, $xy^{m+1}z = a^{2k} b^{2k} \notin L$; abs

<u>Dem alternativă</u>

L regulat $\Leftrightarrow \overline{L}$ regulat

$\overline{L} \cap \{a^i b^j \mid i, j \geq 0\} = \{a^i b^i \mid i\}$ abs

reg   nereg

5. $L = \{a^p \mid p = \text{prim}\}$

Pp abs că e reg $\Rightarrow \exists n \in \mathbb{N}^*$ a.î. $\forall s \in L$ cu $|s| \geq n$ avem $s = xyz$

$$\begin{cases} |xy| \leq n \\ |y| \geq 1 \\ xy^i z \in L \end{cases}$$

Fie q cel mai mic nr prim $\geq n$. şi $s = a^p \in L$, p prim

$|s| = p \geq n \Rightarrow$ ~~$s = xyz$~~ Fie $\begin{cases} x = a^q \\ y = a^r \\ z = a^{p-q-r} \end{cases}$

$xy^i z = a^{q + ir + p - q - r} = a^{(i-1)r + p}$

Fie $i = p+1 \Rightarrow xy^i z = a^{prp} = a^{p(r+1)} \Rightarrow p(r+1)$ prim | $p \mid p(r+1) \Rightarrow p(r+1)$ neprim $\Rightarrow xy^{p+1}z \notin L$ abs

p prim, r+1

6 $L = \{ ww \mid w \in \{0,1\}^* \}$ nu e reg.

Fie $s = 0^p 1 0^p 1 \in L$

Pp obs L reg $\Rightarrow \exists\, p \in \mathbb{N}$ a.î. $\forall s \in L, |s| \geq p$, avem $s = xyz$ cu

$$\begin{cases} |xy| \leq p \\ |y| \geq 1 \\ xy^i z \in L, \forall i \in \mathbb{N} \end{cases}$$

$|s| = 2(p+1) \geq p$

$|xy| \leq p \Rightarrow xy$ contine doar $0 \Rightarrow$ $\left.\begin{array}{l} x = 0^q \\ y = 0^r,\ r \geq 1 \\ z = 0^{p-q-r} 1 0^p 1 \end{array}\right\} \Rightarrow$

$\Rightarrow \left.\begin{array}{l} xy^0 z = 0^{p-r} 1 0^p 1 \in L \\ r \geq 1 \Rightarrow p - r \neq p \end{array}\right\}$ abs

18.04.2018

LFA

-Seminar-

Temă (ssoc limbajele nu sunt reg)

1. $L = \{www \mid w \in \{0,1\}^*\}$
2. $L = \{a^{2k+1} b^{l+2} a^{2k-2} \mid k \geq 2, l \geq 0\}$
3. $L = \{a^n b^2 a^{2n-1} \mid n \geq 1\}$
4. $L = \{wcw^2 \mid w \in \{0,1\}^*\}$
5. $L = \{a^{2n^2+n+4} \mid n \geq 1\}$
6. $L = \{a^m b^r \mid m \leq r \text{ sau } m \geq 2r\}$
7. $L = \{a^i b^j a^k \mid i+j \neq k\}$
8. ~~L=~~ ştiind că $\{a^i b^i \mid i \geq 0\}$ nu e regulat, ssoc (fără lema de pompare) că $\{a^n b a^n \mid n \geq 1\}$ nu ~~eg~~ e regulat.

Ex1: $L = \{a^i b^j \mid gcd(i,j) = 1\}$ nu e reg

Pp ~~că~~ că L e reg $\xrightarrow{L\ Pomp}$ Fie n dat de lemă.

Fie $s = a^{p-1} b^p$, $p > n$ prim

~~...~~ $gcd(p-1, p) = 1 \Rightarrow s \in L$

S.E.1 $L1 \cup L2$ regulat, $L1$ finit $\} \overset{?}{\Rightarrow} L2$ regulat

obs: Orice lbj finit e regulat

$L_2 = \underbrace{(L_1 \cup L_2)}_{\text{regulat}} - \underbrace{(L_1 - L_2)}_{\text{finit} \Rightarrow \text{regulat}} \Big\} \Rightarrow L_2$ regulat

S.E.2. $L_1 \cup L_2$ regulat, L_1 regulat $\} \overset{?}{\Rightarrow} L_2$ regulat

Dacă $L_1 = \{0,1\}^*$ regulat
L_2 un lbj nereg peste alfab $\{0,1\}$ $\} \Rightarrow$
$\Rightarrow L_2 \subseteq L_1 \Rightarrow L_1 \cup L_2 = L_1$ regulat $\Rightarrow$ Nu, L_2 nu trb să fie reg.
($L_2 = \{0^i 1^i \mid i \geq 0\}$ nu e reg)

S.E.3: L^* regulat $\overset{?}{\Rightarrow} L$ regulat

$L = \{0^i 1^i \mid i \geq 0\} \cup \{0,1\}$
$L^* = \{0,1\}^*$ regulat

Temă de gândire: ce se înt ~~când $L_1 \cup L_2$, L_1 nereg~~ cu $L_1 \cup L_2$ dacă $L_1 =$ ~~nereg~~ nereg.

S.E.4: L regulat, L' neregulat, $L \cap L'$ nereg $\} \overset{?}{\Rightarrow} L \cup L'$ nereg

$L = \{0,1\}^*$ regulat
$L \subseteq L' = \{0^{i^2} \mid i \geq 0\}$ nereg

$L \cap L' = L'$
$L \cup L' = L$

S.E.5: L regulat, $L' \subseteq L \overset{?}{\Rightarrow} L'$ ~~rega~~ regulat?

NU: $L' = \{0^{p^2} \mid p \geq 0\} \subseteq \underbrace{\{0,1\}^*}_{reg}$

Expresii reg:

S.n familia expresiilor regulate peste Σ și se not cu $RE(\Sigma)$ mult de cuvinte peste alfab $\Sigma = \{(,), *, \cdot, +, \emptyset, \lambda\}$ def recursiv astfel.

i) $\emptyset, \lambda \in RE$, $a \in RE$, $\forall a \in \Sigma$

ii) $e_1, e_2 \in RE \Rightarrow e_1 + e_2 \in RE$

iii) $e_1, e_2 \in RE \Rightarrow e_1 \cdot e_2 \in RE$

iv) $e_1 \in RE \Rightarrow (e_1)^* \in RE$

Transfm exp reg în automate finite

exp reg $\rightarrow$ limbaj $\rightarrow$ automat

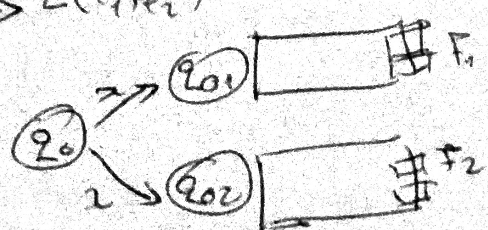
$\emptyset \rightarrow \emptyset \longrightarrow Q = \{q_0\}$, $F = \emptyset$

$\lambda \rightarrow \{\lambda\} \rightarrow Q = \{q_0\} = F$

$\Sigma \ni a \rightarrow \{a\}; a \in \Sigma \rightarrow Q = \{q_0, q_1\}$, $F = \{q_1\}$, $\delta(q_0, a) = q_1$

$e_1 + e_2 \Rightarrow L(e_1 + e_2) = L(e_1) \cup L(e_2)$

aut



$e_1 \cdot e_2 \to L(e_1 \cdot e_2) = L(e_1) \cdot L(e_2)$

$Q = Q_1 \cup Q_2$

$F = F_2$

$\delta = \delta_1 \cup \delta_2 \cup \{\delta(q_f, \lambda) = q_{02}\}$ $\forall q_f \in F_1$

$(e_1)^* \to L((e_1)^*) = (L(e_1))^*$

$Q = Q_1 \cup \{q_0\}$

$F = \{q_0\}$

$\delta = \delta \cup \{\delta(q_0, \lambda) = q_{01}\} \cup$

$\cup \{\delta(q_f, \lambda) = q_0\}$; $\forall q_f \in F_1$

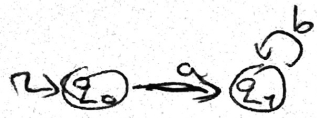
<u>SE.6</u> Construiți automate finite pt RE:

a) $L = \{ab^n\} \cup \{ba^m\}$; $n, m \geq 0$

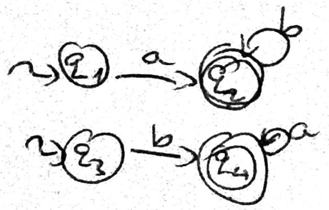
b) $\{a,b\}^* c \; a(ab)^*$

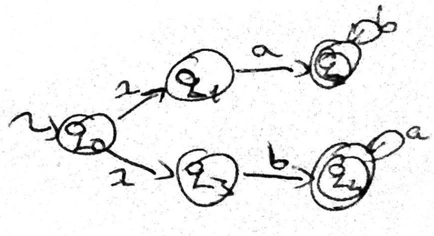
c) $(abb)^* \cdot (ab + ba)$

d) $((a^2)^* + (b^3)^*)^*$

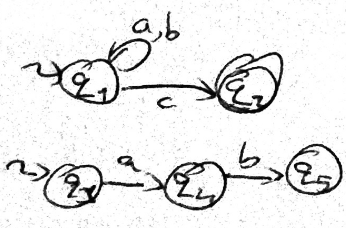


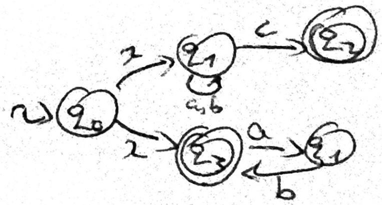
a)



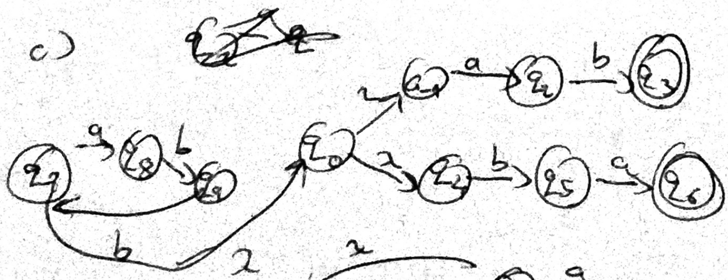


b)

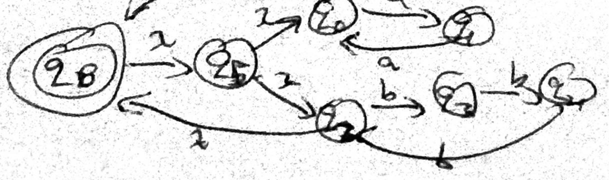




c)



d)



Tabel: # coloane = #Q

linii (complete) = $(\#Q)^2$

stările automatului vor fi num de la q_1

Prima col: $R^0_{ij} = \begin{cases} \{a \in \Sigma \mid \delta(q_i, a) = q_j\}, & i \neq j \\ \{\lambda\} \cup \{a \in \Sigma \mid \delta(q_i, a) = q_j\}; & i = j \end{cases}$

celelalte col:

$$R^{k+1}_{ij} = R^k_{ij} \cup R^k_{i\,k+1} (R^k_{k+1\,k+1})^* R^k_{k+1,j}$$

expresia reg: $\bigcup_{f \in F} R^{|Q|}_{1f}$

Formule:

1) $e \cdot \emptyset = \emptyset$, $\emptyset \cdot e = \emptyset$
2) $e \cdot \lambda = e$, $\lambda . e = e$
3) $e^* e = e^+$; $e e^* = e^+$
4) $\{e_1, e_2\}^* = (e_1 + e_2)^* \overset{(1)}{=} (e_1^* e_2^*)^*$
5) $e_1(e_2 + e_3) = (e_1 \cdot e_2) + (e_1 \cdot e_3)$
   $(e_1 + e_2) e_3 = (e_1 e_3) + (e_2 e_3)$

2 stări => 2 col și 4 lin (complete)

R^{k+1}_{ij}	k+1 = 0	k+1 = 1	
11	$1 + \lambda$	1^*	
12	0		
21	$\emptyset$		
22	λ		

$R^1_{11} = R^0_{11} \cup R^0_{11} (R^0_{11})^* R^0_{11}$

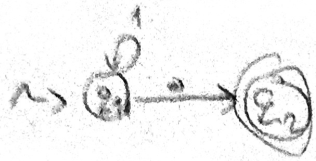
$= (1+\lambda) \cup (1+\lambda) \cdot (1+\lambda)^* (1+\lambda)$

$= (1+\lambda) \cup (1+\lambda)^*$

$= (1+\lambda)^* = \ldots = 1^*$

LFA

- Seminar -



R^{k+1}_{pq}	$k+1=0$	$k+1=1$	$k+1=2$
11	$1+\lambda$	1^*	1^*
12	0	1^*0	1^*0
21	$\emptyset$	$\emptyset$	$\emptyset$
22	λ	λ	λ

$$R^0_{pq} = \begin{cases} \{a \in \Sigma \mid \delta(q_p, a) = q_q\}, & p \neq q \\ \{\lambda\} \cup \{a \in \Sigma \mid \delta(q_p, a) = q_q\}, & p = q \end{cases}$$

$$R^0_{11} = \{1\} \cup \{\lambda\} = 1+\lambda$$
$p=q=1$

$$\boxed{Exp = \bigcup_{f \in F} R^{|Q|}_{1f} \qquad f = 2,\ |Q| = 2}$$

$Lbg = 1^*0$

$$\boxed{R^{k+1}_{pq} = R^k_{pq} \cup \left(R^k_{p\,k+1} \cdot (R^k_{k+1,k+1})^* \cdot R^k_{k+1,q}\right)}$$

$$R^2_{11} = R^1_{11} + R^1_{12}(R^1_{22})^* R^1_{21} =$$
$$= 1^* + 1^*0\lambda^*\emptyset = 1^*$$

$$R^2_{12} = R^1_{12} \cup R^1_{12}(R^1_{22})^* R^1_{22} =$$
$$= 1^*0 + 1^*0\lambda^*\lambda = 1^*0 + 1^*0 = 1^*0$$

$$R^2_{21} = R^1_{21} \cup R^1_{22}(R^1_{22})^* \cdot R^1_{21} = \emptyset$$

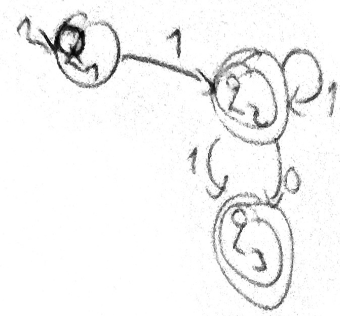
$$R^2_{22} = R^1_{22} + R^1_{22}(R^1_{22})^* R^1_{22} =$$
$$= \lambda \cup \lambda\lambda^*\lambda = \lambda$$

$$R^1_{11} = R^0_{11} \cup \left(R^0_{11} \cdot (R^0_{11})^* \cdot R^0_{11}\right) =$$
$$= R^0_{11} + (R^0_{11})^* = \cancel{(R^0_{11})^* + (1+\lambda)^*}$$
$$= (R^0_{11})^* = (1+\lambda)^* = 1^*$$

$$R^1_{12} = R^0_{12} + \left(R^0_{11} \cdot (R^0_{11})^* R^0_{12}\right) =$$
$$= 0 + (1+\lambda)^*0 = 0 + 1^*0 = 1^*0$$

$$R^1_{22} = R^0_{22} + \left(R^0_{21} \cdot (R^0_{11})^* \cdot R^0_{12}\right) =$$
$$= R^0_{22} + R^0_{21}(R^0_{11})^* R^0_{12} = \lambda + \emptyset = \lambda$$

ij	k+1=0	k+1=1	k+1=2
11	λ	λ	λ
12	1	1	1^+
13	$\emptyset$	$\emptyset$	1^+0
21	$\emptyset$	$\emptyset$	$\emptyset$
22	$1+\lambda$	$1+\lambda$	1^*
23	0	0	1^*0
31	$\emptyset$	$\emptyset$	$\emptyset$
32	1	1	1^*
33	λ	λ	$\lambda+1^*0$



$$R^1_{11} = R^0_{11} + R^0_{11}(R^0_{11})^* R^0_{11} = \lambda$$

$$R^1_{12} = R^0_{12} + R^0_{11} \cdot (R^0_{11})^* R^0_{12} = 1 + \lambda\lambda^* 1 = 1$$

$$R^1_{13} = R^0_{13} + R^0_{11} \cdot (R^0_{11})^* R^0_{13} = \emptyset$$

$$R^1_{21} = R^0_{21} + R^0_{21} \cdot (R^0_{11})^* R^0_{11} = \emptyset$$

$$R^1_{22} = R^0_{22} + R^0_{21} \cdot (R^0_{11})^* R^0_{12} = (1+\lambda) + \emptyset = 1+\lambda$$

$$R^1_{23} = R^0_{23} + R^0_{21} (R^0_{11})^* R^0_{13} = 0$$

$$R^1_{31} = R^0_{31} + R^0_{31} (R^0_{11})^* R^0_{11} = \emptyset$$

$$R^1_{32} = R^0_{32} + R^0_{31} (R^0_{11})^* R^0_{12} = 1$$

$$R^1_{33} = R^0_{33} + R^0_{31} (R^0_{11})^* R^0_{13} = \lambda + \emptyset \dots = \lambda$$

$$R^2_{11} = R^1_{11} + R^1_{12} \cdot (R^1_{22})^* R^1_{21} = \lambda$$

$$R^2_{12} = R^1_{12} + R^1_{12} \cdot (R^1_{22})^* R^1_{22} = 1^+$$

$$R^2_{13} = R^1_{13} + R^1_{12} (R^1_{22})^* R^1_{23} = \emptyset + 1(1+\lambda)^* 0 = 1^+0$$

$$R^2_{21} = R^1_{21} + R^1_{22} (R^1_{22})^* R^1_{21} = \emptyset$$

$$R^2_{22} = R^1_{22} + R^1_{22} (R^1_{22})^* R^1_{22} = 1^*$$

$$R^2_{23} = R^1_{23} + R^1_{22} (R^1_{22})^* R^1_{23} = 1^*0$$

$$R^2_{31} = R^1_{31} + R^1_{32} (R^1_{22})^* R^1_{21} = \emptyset$$

$R^2_{32} = R^1_{32} + R^1_{32}(R^1_{22})^* R^1_{22} = 1^+$

$R^2_{33} = R^1_{33} + R^1_{32}(R^1_{22})^* R^1_{23} = \lambda + 1^+0$

$\cancel{Exp = R^2_{12} \cup R^2_{13}}$

$R^3_{12}, R^3_{13} = ?$

$R^3_{12} = R^2_{12} + R^2_{13}(R^2_{33})^* R^2_{32} = 1^+ + 1^+0(\lambda + 1^+0)^* 1^+ = 1^+ + (1^+0)^+ 1^+ = (1^+0)^* 1^+$

$R^3_{13} = R^2_{13} + R^2_{13}(R_{33})^* R^2_{33} = (1^+0)^+$

$Exp = R^3_{12} + R^3_{13} = (1^+0)^* 1^+ + (1^+0)^+ \cancel{= (1^+0)^*}$

Minimizare DFA

pt o stare finală și una nefinală:

$p \in F$, $q \in Q \setminus F$ } marchez perechea $A[p,q] = 1$, $A[q,p] = 1$ } altfel 0

pt fiecare pereche construiesc o listă goală pt $\forall (p,q)$ memorată

Dacă $\exists a \in \Sigma$ a.î. $(\delta(p,a), \delta(q,a))$ e marcată, at. marchez (p,q) și marchez toate perechile de stări din lista lui (p,q). Altfel ~~pt $\forall a \in \Sigma$~~

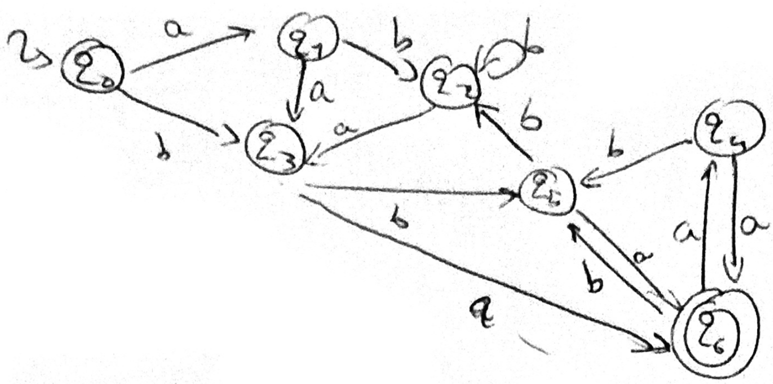
Altfel, $\forall a \in \Sigma$ pun (p,q) la $(\delta(p,a), \delta(q,a))$

$(0,3)$: $\delta(q_0, a) = q_1 \notin F$, $\delta(q_3, a) = q_6 \in F$ } $A[1,6] = 1 \Rightarrow A[0,3] = 1$

$(0,4)$: $\delta(q_0, a) = q_1$, $\delta(q_4, a) = q_6$, $A[1,6] = 1$ } $\Rightarrow A[0,4] = 1$

$(1,2)$: $\delta(q_1, a) = q_3$, $\delta(q_1, b) = q_2$, $\delta(q_2, a) = q_3$, $\delta(q_2, b) = q_2$

$(0,5)$: $\delta(q_0, a) = q_1$, $\delta(q_5, a) = q_6$, $A[1,6] = 1$ | $\Rightarrow A[0,5] = 1$



A[p,q]

	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

$(0,1)$: $\delta(q_0,a) = q_1 \notin F$

$\delta(q_1,a) = q_3 \notin F$

Pun $(0,1)$ în lista lui $(1,3)$

$\delta(q_0,b) = q_3$

$\delta(q_1,b) = q_2$ | pun $(0,1)$ la $(2,3)$

$(2,3)$ nemarcată

$(0,2)$: $\delta(q_0,a) = q_1$

$\delta(q_2,a) = q_3$ | pun $(0,2)$ la $(1,3)$

$\delta(q_0,b) = q_3$

$\delta(q_2,b) = q_2$ | pun $(0,2)$ la $(2,3)$

LFA

-Seminar-

16.05.2018

$L_6 = \{a^n b^n \mid n \geq 0\}$

$S \to aSb$

$S \to \varepsilon$

$L_7 = \{a^n b^n \mid n \geq 1\}$

$S \to aAb$

$A \to aAb$

$A \to \varepsilon$

$L_5 = \{a^{2n} b^{3m}, n \geq 0, m \geq 0\}$

$S \to aaS \mid Sbbb \mid \lambda$

$a^4 b^3$

$S \to aaS \Rightarrow aaaaS \Rightarrow a^4 S b^3 \Rightarrow a^4 b^3$

$L_8 = \{a^n b^{2n} \mid n \geq 1\}$

$S \to aSb^2 \mid ab^2$

$S \to aSb^2 \Rightarrow a^2 S b^4 \Rightarrow a^3 b^6$

$L_9 = \{a^{2n} b^k c^{3n} \mid n \geq 0, k \geq 1\}$

$S \to aaSccc \mid A$

$A \to bA \mid b$

$L_{10} = \{a^{2n} b^k c^{3n} \mid n \geq 1, k \geq 1\}$

$S \to A \mid a^2 S c^3$

$A \to a^2 B c^3$

$B \to b \mid bB$

$L_{11} = \{a^{2n} c^{3n} b^k \mid n, k \geq 1\}$

$S \to AB$

$A \to a^2 A c^3 \mid a^2 c^3$

$B \to bB \mid b$

$a^4 c^6 b^3$: $S \Rightarrow AB \Rightarrow a^2 A c^3 B \Rightarrow a^4 c^6 B \Rightarrow a^4 c^6 b^2 B \Rightarrow a^4 c^6 b^3$

$L_{12} = \{a^n b^k c^{2k} d^n \mid n \geq 1, k \geq 0\}$

$S \to aAd \mid aSd$

$A \to bAcc \mid \lambda$

$a^2 b^2 c^4 d^2$: $S \Rightarrow aSd \Rightarrow a^2 A d^2 \Rightarrow a^2 b A c^2 d^2 \Rightarrow a^2 b^2 A c^4 d^2 \Rightarrow a^2 b^2 c^4 d^2 \Rightarrow$

$\Rightarrow a^2 b^2 c^4 d^2$

$L_{13} = \{a^n b^{2n} c^k d^m e^{3k} \mid n, k, m \geq 0\}$

$S \to AB$

$A \to aAbb \mid \lambda$

$B \to cBe^3 \mid C$

$C \to dC \mid \lambda$

~~L_{14}~~

$L_{15} = \{a^n b^m c^k \mid n+m = k\}$

$S \to aSc \mid B \mid \lambda$

$B \to bBc \mid \lambda$

$S \to 0A \mid 1B$

$A \to 0AA \mid 1S \mid 1$

$B \to 1BB \mid 0S \mid 0$

De este ambiguo?

$S \Rightarrow 0A \Rightarrow 00AA \Rightarrow 001SA \Rightarrow 0011BA \Rightarrow 00111BBA \Rightarrow$
$\Rightarrow 001110BA \Rightarrow 0011100A \Rightarrow 00111001$

$S \Rightarrow 0A \Rightarrow 00AA \Rightarrow 001A \Rightarrow 0011S \Rightarrow 00111B \Rightarrow$
$\Rightarrow 001110S \Rightarrow 0011100A \Rightarrow 00111001$

~~$S \to aSa \mid aBa$~~
~~$B \to bB \mid b \mid b$~~

~~$L_{17} =$~~

19. $L_{19} = \{ w \mid w[0] = w[-1] \}$

$S \to \alpha S \alpha \mid A$ $\quad \alpha, \beta \in \Sigma$

$A \to A\beta \mid \lambda$

$S \to 0S0 \mid 1S1 \mid 0A0 \mid 1A1$
$A \to A0 \mid A1 \mid \lambda$

20. $L_{20} \{ w \mid |w| \equiv 1 (\bmod 2) \}$

$S \to 0A \mid 1A$
$A \to 00A \mid 01A \mid 10A \mid 11A \mid \lambda$

21. $L_{21} = \{ w \mid |w| \equiv 1 (\bmod 2)$ și 0 e în mijloc

$S \to 0S1 \mid 0S0 \mid 1S0 \mid 1S1 \mid 0$

Transformare în f.n. normală Chomsky

$S \to a$
$S \to AB$

$S \to \overset{(1)}{aBcDeF} \mid \overset{(2)}{HF} \mid \overset{(3)}{HBc}$
$B \to \overset{(4)}{b} \mid \overset{(5)}{\lambda}$
$D \to \overset{(6)}{d} \mid \overset{(7)}{\lambda}$
$F \to \overset{(8)}{G}$
$G \to \overset{(9)}{f} \mid \overset{(10)}{g}$
$H \to \overset{(11)}{\lambda}$

Pasul 1: Elimin λ-prod

i) Dacă neterminalul are o λ-producție și nu mai am alte producții
elimin $H \to \lambda$
(2) devine $S \to F$ (12)
(3) devine $S \to Bc$ (13)

ii) Dacă neterminalul are și alte producții
(5) elimin
(13) devine $S \to Bc$ (14)
$S \to c$ (15)
(1) devine $S \to aBcDeF$ (16)
$S \to aBceF$ (17)
$S \to acDeF$ (18)
$S \to aceF$ (19)

Pasul 2: Elimin redenumiri

$F \to G$ devine $F \to f$ (20)

$G \to f \mid g$ devine $F \to g$ (21)

Neterminalul G nu mai apare în membrul drept al unei producții

=> pot elimina producțiile (9) și (10)

$S \to F$ (12) dev. $S \to f$ (22)

$F \to f/g$ $\quad$ $S \to g$ (23)

Pasul 3: Adăugăm neterminale noi pt terminalele care apar

(16): $S \to X_a B X_c D X_e F$

$X_a \to a$

$X_c \to c$

$X_e \to e$

Pasul 4: Adăugăm neterminale noi pt a "sparge" cuv. de lun >2

(24): $S \to X_a Y_1$, $Y_1 \to B X_c D X_e F$

$Y_1 \to B Y_2$ $\quad$ $Y_2 \to X_c D X_e F$

/

~~Limbaje independente de context.~~

LFA

–Seminar–

18.05.2018

<u>Lema de pompare</u>: Fie L un limbaj independent de context.

Atunci $\exists$ $p \in \mathbb{N}$ a.î. $\forall s \in L$, $|s| \geq p$, s se descompune $s = uvwxy$

cu 1) $|vx| \geq 1$

2) $|vwx| \leq p$

3) $uv^i wx^i y \in L$, $\forall i \geq 0$

<u>Exc</u>

1. $L_1 = \{a^{2^n} \mid n \geq 0\}$ este indep. de context?

Pp obs că L_1 este indep de context.

Fie p dat de lema de pompare

Fie $s = a^{2^p}$ => $s = uvwxy$: $\begin{cases} |v| + |x| \geq 1 \\ |vwx| \leq p \\ uv^i wx^i y \in L, \forall i \geq 0 \end{cases}$

$2^p = |uvwxy| < |uv^2 wx^2 y| = |uvwxy| + |v| + |x| \leq 2^p + p < 2^p + 2^p = 2^{p+1}$ =>

=> $uv^2wx^2y \notin L$ $\lightning$

2. $L_2 = \{a^n b^j c^k \mid k = jn\}$, $\Sigma = \{a, b, c\}$

Sere L_2 nu e indep de ctxt.

Pp obs. că e indep => Fie p din lemă

Fie $s = a^p b^p c^{p^2} \in L$, $|s| = 2p + p^2 \geq p$

=> $|v| + |x| \geq 1$

$|vwx| \leq p$

$uv^i wx^i y \in L$, $\forall i \in \mathbb{N}$

I $vwx = a^r$

II $vwx = a^q b^r$

III $vwx = b^r$

IV $vwx = b^q c^r$

V $vwx = c^r$

I $vwx = a^q \Leftrightarrow v = a^h, x = a^t, w = a^{r-q-t}$ $\Rightarrow uv^2wx^2y \in L = a^{p+q+t} b^p c^{p^2} \in L \Rightarrow$

$\Leftrightarrow p^2 = p(p+(q+t))$

$q+t \geq 1$

$i=0: a^{p-q-t} b^p c^{p^2} \in L$

$p^2 = p(p-(q+t))$ $\cancel{\infty}$ $\underbrace{}_{<p}$

~~p = q+t~~

II $vwx = a^q b^r$ $v = a^s, w = a^t, x = a^{q-t}$

$v = a^s, w = a^{h-s} b^f, x = b^{r-f}$

$v = a^h$

---

3. $L = \{ww^R a^{|w|} \mid w \in \{a,b\}^*\}$ nu e indep de context.

Pp abs că este CFL. Fie p din lemă

Fie $w = a^p b^p$, $s = a^p b^p c^p a^p a^{2p} \in L$

$|w| = 2p$

$\overset{LP}{\Rightarrow} s = uvxyz$: 1) $|vy| \geq 1$

2) $|vxy| < p$

3) $uv^i x y^i z \in L, \forall i \geq 0$

Cont II.

Cazul I:

$$\underbrace{a\_\_ab\_}_{uvxy}\underbrace{\_ba\_\_a}_{z}$$

$q=0$: $|w| = 2p - |xy| < |w^R| < 2p \Rightarrow w w^R a^{|w|} \notin L$

Cazul II

$$\underbrace{a\_\_ab\_\_\_}_{u}\underbrace{\_bba\_\_a}_{vxyz}$$

$q=0$: $|a^{|w'|}| = 2p - |vy|$

Push-down automaton

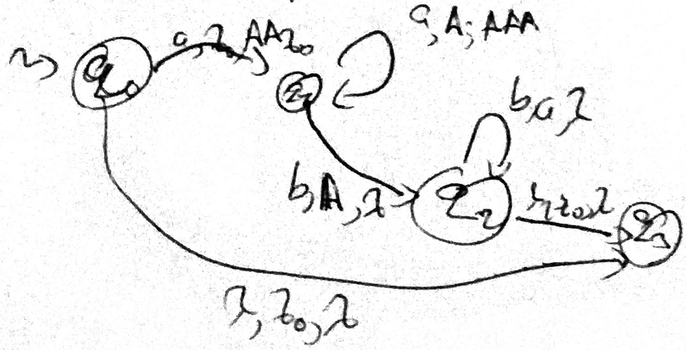
1. $PDA = (Q, \Sigma, F, \delta, q_0, \Gamma, Z_0)$

   Γ = alfab. st

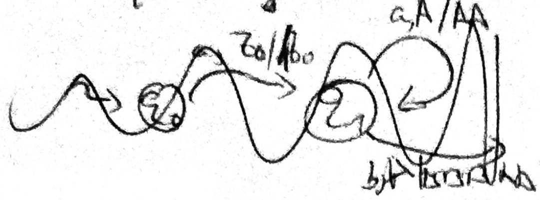
   Σ = alfab

   Z_0 = simbol initial din stivă

$L = \{a^n b^{2n} / n \in \mathbb{N}^*\}$



$L_2 = \{a^{2n} b^k_3 c^{3k} d^n \mid k, n \geq 1\}$



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