Lab 8

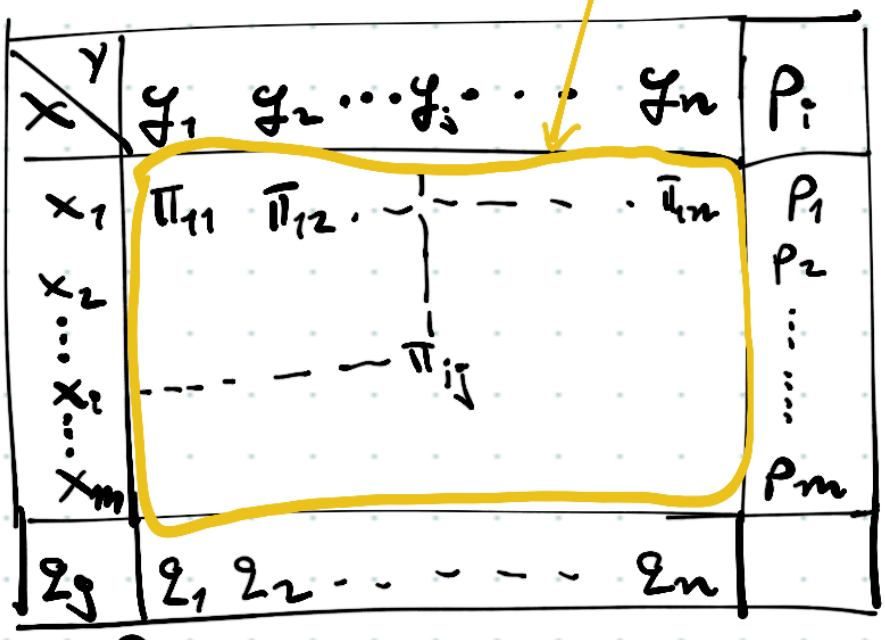
V. A. bidimensionale (vectori de V. A.)

I Cazul discret

Fie X, Y 2 v.a.d.

$Z = (X, Y)$

Repartiția comună a v.a. X, Y



$$\pi_{ij} = P(Z = (x_i, y_j)) = P(X = x_i, Y = y_j)$$

$$X : \begin{pmatrix} x_1 & x_2 & \dots & x_m \\ p_1 & p_2 & \dots & p_m \end{pmatrix}$$

$$Y : \begin{pmatrix} y_1 & y_2 & \dots & y_n \\ q_1 & q_2 & \dots & q_n \end{pmatrix}$$

X, Y ind $\Leftrightarrow \pi_{ij} = p_i q_j \; ; \forall i,j$

Ex

	$X \backslash Y$	-1	0	2	p
		-2	0	4	
-3	-1	$1/24$	$1/8$	$1/12$	$1/4$
6	2	$1/12$	$1/4$	$1/6$	$1/2$
9	3	$1/24$	$1/8$	$1/12$	$1/4$
	q	$1/6$	$1/2$	$1/3$	1

$$\pi_{13} = p_1 - \pi_{11} - \pi_{12} = \frac{1}{4} - \frac{1}{24} - \frac{1}{8} = \frac{6-1-3}{24} = \frac{1}{12}$$

$$\pi_{21} = p_2 - \pi_{22} - \pi_{23} = \frac{1}{12}$$

$$\pi_{31} = q_1 - \pi_{11} - \pi_{21} = \frac{1}{24}$$

$$\pi_{32} = p_3 - \pi_{31} - \pi_{33} = \frac{1}{8}$$

$$q_2 = \pi_{32} + \pi_{22} + \pi_{32} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$$

$$q_3 = 1 - q_1 - q_2 = 1 - \frac{1}{6} - \frac{1}{2} = \frac{1}{3}$$

$$X : \begin{pmatrix} -1 & 2 & 3 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix} \qquad Y : \begin{pmatrix} -1 & 0 & 2 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \end{pmatrix}$$

$$E(X) = -\frac{1}{4} + 1 + \frac{3}{4} = \frac{3}{2}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$X^2 : \begin{pmatrix} -1 & 4 & 9 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

$$E(X) = \frac{3}{2} \qquad E(Y) = \frac{1}{2}$$

$$Var(X) = \frac{9}{4} \qquad Var(Y) = \frac{5}{4}$$

Obs $Cov(X,Y) = E(XY) - E(X)E(Y)$

$$\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}} \in [-1, 1]$$

$$X \cdot Y : \begin{pmatrix} 1 & 0 & -2 & -2 & 0 & 4 & -3 & 0 & 6 \\ \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{1}{12} & \frac{1}{4} & \frac{1}{6} & \frac{1}{24} & \frac{1}{8} & \frac{1}{12} \end{pmatrix}$$

$$X \cdot Y : \begin{pmatrix} -3 & -2 & 0 & 1 & 4 & 6 \\ \frac{1}{24} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

$$X|(Y=0) : \begin{pmatrix} -1 & 2 & 3 \\ a & b & c \end{pmatrix}$$

$$a = P(X=-1 \mid Y=0) = \frac{P((X=-1) \cap (Y=0))}{P(Y=0)} =$$

$$= \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$$

Proprietăți:

1. $Cov(X,X) = Var(X)$
2. $Cov(X,Y) = Cov(Y,X)$
3. $Cov(aX+bY, cX+dY) = a \cdot c \cdot Cov(X,X) + a \cdot d \cdot Cov(X,Y) + b \cdot c \cdot Cov(Y,X) + b \cdot d \cdot Cov(Y,Y) =$

$$= a \cdot c \cdot \mathrm{Var}(X) + b \cdot d \cdot \mathrm{Var}(Y) + (ad + bc)\,\mathrm{Cov}(X, Y)$$

TEMĂ

1. $X \mid (Y = 0)$
   $Y \mid (X = -1)$
2. $\mathrm{Cov}(3X + 2Y, X + Y)$