Tema

26/P62

Not A - evenimentul să extragem o bilă neagră din urna separată

$P(A) = \frac{1}{15}$

B - evenimentul să extrag o bilă neagră

C - even să fie aleasă urna I

D - ev să fie aleasă urna II

$P(C) = p$

$P(D) = 1-p$

$P(B|C) = \frac{3}{5}$ $\qquad$ $P(B|D) = \frac{2}{5}$

c) D fm. partiție } B ∈ același s.p. } ⇒

FPT ⇒ $P(B) = P(B|C)P(C) + P(B|D) \cdot P(D) \Rightarrow$

$\Rightarrow P(B) = \frac{3}{5} \cdot p + \frac{2}{5}(1-p) \Rightarrow$

$\Rightarrow P(B) = \frac{3p}{5} + \frac{2-2p}{5} = \frac{2+p}{5}$

$P(B) = P(A)$

$P(A) = \frac{7}{15}$

⇒

$\Rightarrow \frac{2+p}{5} = \frac{7}{15} \Rightarrow 2+p = \frac{7}{3} \Rightarrow p = \frac{7}{3} - 2 \Rightarrow$

$\Rightarrow$ $p = \frac{1}{3}$

9/P60

Not A - ev. de a alege cele 2 bujii stricate

Nr total perechi: $\sum_{i=1}^{5} i = \frac{5 \cdot 4}{2} = 10$

Nr perechi „bune": 1

$\Rightarrow P(A) = \frac{1}{10} \Rightarrow \boxed{P(A) = 0{,}1}$

41/p131

$$f(x) = e^{-2|x|}\text{; } x \in \mathbb{R} \quad \text{dens pr.} \Rightarrow$$

$$\Rightarrow P(X < x) = \int_{-\infty}^{x} f(t)\,dt \Rightarrow$$

$$\Rightarrow P(X < n) = \int_{-\infty}^{n} f(t)\,dt \quad (1)$$

$$P(X < -n) = \int_{-\infty}^{-n} f(t)\,dt =$$

$$= \int_{-\infty}^{n} f(t)\,dt - \int_{-n}^{n} f(t)\,dt =$$

$$= \int_{-\infty}^{n} f(t)\,dt - \left(\int_{-n}^{0} f(t)\,dt - \int_{0}^{n} f(t)\,dt \right) =$$

$$= \int_{-\infty}^{n} f(t)\,dt - 2\int_{-n}^{0} f(t)\,dt \Rightarrow$$

$$\Rightarrow P(x < -n) = \int_{-\infty}^{n} f(t)\,dt - 2\int_{-n}^{0} f(t)\,dt \quad (2)$$

$$2\int_{-n}^{0} f(t)\,dt = 2\int_{-n}^{0} e^{-2|t|}\,dt = 2\int_{-n}^{0} e^{-2} e^{|t|}\,dt =$$

$$= 2e^{-2}\int_{-n}^{0} e^{|t|}\,dt \overset{n>0}{=\!=} 2e^{-2}\int_{-n}^{0} e^{-t}\,dt =$$

$$= \frac{2}{e^2}\cdot(-e^{-t})\Big|_{-n}^{0} = \frac{2}{e^2}\left(-e^{0} - (-e^{-(-n)})\right) =$$

$$= \frac{2}{e^2}\left(-1 + e^{n}\right) = \frac{2(e^{n}-1)}{e^2} \quad (3)$$

$$P(|x| < n) = P(-n < x < n) \Rightarrow$$

$$\Rightarrow P(|x| < n) = P((x < n) \cap (x > -n))$$

$$P(-n < x < n) = 1 - P((x < -n) \cup (x > n)) =$$

$x < -n,\ x > n$ incomp $\Rightarrow$

$\Rightarrow P((x < -n) \cup (x > n)) = P(x \leq -n) + P(x > n)$

$\Rightarrow P((x < -n) \cup (x > n)) = P(x \leq -n) + 1 - P(x \leq n) \Rightarrow$

$\Rightarrow P(|x| < n) = 1 - P((x < -n) \cup (x > n)) =$

$= 1 - (P(x < -n) + 1 - P(x \leq n)) =$

$= \cancel{1} - \cancel{1} + P(x < n) - P(x < -n) \Rightarrow$

$\Rightarrow P(|x| < n) = P(x < n) - P(x \leq -n)$

(1) $\Rightarrow P(x < n) = \int_{-\infty}^{n} f(t)dt$

(2) $\Rightarrow P(x \leq -n) = \int_{-\infty}^{n} f(t)dt - 2\int_{-n}^{0} f(t)dt$

$\Rightarrow$

$\Rightarrow P(|x| < n) = \cancel{\int_{-\infty}^{n} f(t)dt} - \cancel{\int_{-\infty}^{n} f(t)dt} + 2\int_{-n}^{0} f(t)dt$

$\Rightarrow P(|x| < n) = 2\int_{-n}^{0} f(t)dt$

(3) $\Rightarrow 2\int_{-n}^{0} f(t)dt = \frac{2(e^n - 1)}{e^2}$

$\Rightarrow$

$$\Rightarrow \quad P(|x|<n) = \frac{2(e^n-1)}{e^2}$$

$$\left.\begin{array}{l} \frac{2(e^n-1)}{e^2} \text{ str} \nearrow \text{ pe } (0,\infty) \\ n>0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \frac{2(e^n-1)}{e^2} > \frac{2(e^0-1)}{e^2} = 0 \Rightarrow$$

$$\Rightarrow P(|x|<n) > 0 \quad \text{margine inf.}$$

11/P202

$$X \in H(\underset{\text{pop}}{N}, \underset{\text{extr}}{n}, \underset{\text{total bune}}{p})$$

$$Var(X) = n\frac{p}{N} \cdot \frac{N-p}{N} \cdot \frac{N-n}{N-1}$$

20/P290

$$f(x,\theta)=\begin{cases}\frac{2}{\theta^2}(\theta-x), & 0\le x\le\theta\\ 0, & \text{în rest}\end{cases}$$

$\theta>0$

f densitate de repartiție

a) $E(X)=?$ $\quad Var(X)=?$

f densitate $\Rightarrow f(x)\ge 0;\ \forall x\in\mathbb{R}$

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx=1$

$f(x)\ge 0;\ \forall x\in\mathbb{R}$ evident.

$$\int_{-\infty}^{\infty} x f(x) = \int_0^{\theta} x\frac{2}{\theta^2}(\theta-x)\,dx=$$

$$=\frac{2}{\theta^2}\int_0^{\theta} x(\theta-x)dx=\frac{2}{\theta^2}\int_0^{\theta}\theta x-x^2\,dx=$$

$$=\frac{2}{\theta^2}\cdot\theta\int_0^{\theta} x\,dx-\frac{2}{\theta^2}\int_0^{\theta} x^2dx=$$

$$= \frac{2}{\theta} \cdot \frac{x^2}{2} \Big|_0^{\theta} - \frac{2}{\theta^2} \frac{x^3}{3} \Big|_0^{\theta} =$$

$$= \frac{2}{\theta} x^2 \left(\frac{1}{2} - \frac{1}{\theta} \cdot \frac{x}{3} \right) \Big|_0^{\theta} =$$

$$= \frac{2}{\theta} \theta^2 \left(\frac{1}{2} - \frac{1}{\cancel{\theta}} \cdot \frac{\cancel{\theta}}{3} \right) - 0 =$$

$$= 2\theta \left(\frac{1}{2} - \frac{1}{3} \right) =$$

$$= \frac{\theta}{3}$$

$$Var(x) = \sqrt{E((x-m)^2)} = \sqrt{E((x+\frac{\theta}{3})^2)}$$

$$E\left(\left(x - \frac{\theta}{3}\right)^2\right) = \int_{-\infty}^{\infty} \left(x - \frac{\theta}{3}\right)^2 \frac{2}{\theta^2} (\theta - x)\, dx =$$

$$= \int_0^{\theta} \left(x - \frac{\theta}{3}\right)^2 \frac{2}{\theta^2} (\theta - x)\, dx =$$

$$= \int_0^{\theta} \left(x^2 - \frac{2x\theta}{3} + \frac{\theta^2}{9}\right) \frac{2}{\theta^2} (\theta - x)\, dx =$$

$$= \int_0^\theta \left(\frac{2x^2}{\theta^2} - \frac{4x\cancel{\theta}}{3\theta^{\cancel{2}}} + \frac{2\cancel{\theta^2}}{9\cancel{\theta^2}}\right)(\theta - x)\,dx =$$

$$= \int_0^\theta \left(\frac{2}{\theta^2}x^2 - \frac{4}{3\theta}x + \frac{2}{9}\right)(\theta - x)\,dx =$$

$$= \int_0^\theta \frac{2}{\theta}x^2 - \frac{4}{3}x + \frac{2\theta}{9} - \frac{2}{\theta^2}x^3 + \frac{4}{3\theta}x^2 - \frac{2}{9}x \, dx =$$

$$= \int_0^\theta -\frac{2}{\theta^2}x^3 + x^2\left(\frac{2}{\theta}+\frac{4}{3\theta}\right) + x\left(-\frac{4}{3}-\frac{2}{9}\right) + \frac{2\theta}{9}\,dx =$$

$$= -\frac{2}{\theta^2}\int_0^\theta x^3\,dx + \frac{10}{3\theta}\int_0^\theta x^2\,dx - \frac{14}{9}\int_0^\theta x\,dx + \frac{2\theta}{9}\int 1\,dx =$$

$$= -\frac{2}{\theta^2}\frac{x^4}{4}\Big|_0^\theta + \frac{10}{3\theta}\frac{x^3}{3}\Big|_0^\theta - \frac{14}{9}\frac{x^2}{2}\Big|_0^\theta + \frac{2\theta}{9}x\Big|_0^\theta =$$

$$= -\frac{\theta^4}{2\theta^2} + \frac{10\theta^3}{9\theta} - \frac{7\theta^2}{9} + \frac{2\theta^2}{9} =$$

$$= -\frac{\theta^2}{2} + \frac{10\theta^2}{9} - \frac{7\theta^2}{9} + \frac{2\theta^2}{9} =$$

$$= -\frac{9\theta^2}{18} + \frac{20\theta^2}{18} - \frac{14\theta^2}{18} + \frac{4\theta^2}{18} =$$

$$= \frac{\theta^2}{18}$$

b) $\alpha(\hat{\theta}) = M \Rightarrow$

$$\left.\begin{aligned} &\Rightarrow \alpha(\hat{\theta}) = \frac{1}{n}\sum_{j=1}^{n} x_j \\ &\alpha(\hat{\theta}) = \int_{-\infty}^{\infty} x f(x,\hat{\theta})\,dx = \frac{\hat{\theta}}{3} \ (\text{din } a)) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \quad \frac{1}{n}\sum_{j=1}^{n} x_j = \frac{\hat{\theta}}{3} \Rightarrow \hat{\theta} = \frac{3}{n}\sum_{j=1}^{n} x_j$$

$E(\hat{\theta}) = \theta \Rightarrow \hat{\theta}$ nedeplasat

$Var(\hat{\theta}) = \dfrac{\theta^2}{2n} \Rightarrow$

$$\left.\begin{aligned} &\Rightarrow \lim_{n\to\infty} Var(\hat{\theta}) = 0 \\ &E(\hat{\theta}) = \theta \Rightarrow \lim_{n\to\infty} E(\hat{\theta}) = \theta \end{aligned}\right\} \Rightarrow$$

$\Rightarrow \hat{\theta}$ consistent.

18/P222

$$\begin{cases} \text{Moivre-Laplace} \\ P\left(\sqrt{\frac{n}{pq}}\,|x_n - p| \le \beta\right) \simeq 2\Phi(\beta) - 1 \end{cases}$$

cu $q = 1 - p$

$|x_n - p| \le 10^{-3}$

$$P\left(\sqrt{\frac{n}{pq}}\,|x_n - p| \le \beta\right) = 0{,}95 \overset{M-L}{\Longrightarrow}$$

$$\Rightarrow \quad 2\Phi(\beta) - 1 = 0{,}95 \Rightarrow \Phi(\beta) = 0{,}975 \Rightarrow$$

$$\Rightarrow \beta = 1{,}96 \Rightarrow$$

$$\sqrt{\frac{n}{pq}}\,|x_n - p| \le 1{,}96 \Rightarrow \sqrt{\frac{n}{pq}} \cdot 10^{-3} = 1{,}96 \Rightarrow$$

$$\Rightarrow \sqrt{\frac{n}{pq}} = 1960 \Rightarrow n = 1960^2 (1-p)p$$