Curs 7

VALIDITATEA ALGORITMULUI LR(1):

$G = (N, \Sigma, S, P)$ g.i.c.

$G' = (N \cup \{S'\}, \Sigma \cup \{\$\}, S', P \cup \{S' \to S\})$.

Def: Spunem că $A \to \alpha . \beta; a$ este validă pt. prefixul viabil γ dacă:

(i) $\exists\ S \overset{*}{\underset{d}{\Rightarrow}} \delta A w \underset{d}{\Rightarrow} \delta \alpha \beta w \quad (w \in \Sigma^*)$

(ii) $\delta\alpha = \gamma$

(iii) $a = First(w\$)$

Lemă: Fie $A \to . \theta B \mu ; a \in goto(I_0, \gamma)$ unde γ e prefixul viabil. Atunci, pt. $\forall\ B \to \alpha \in P$, $B \to . \alpha ; First(\mu, a) \in goto(I_0, \gamma\theta)$.

DEM: Aplicăm definiția pt. closure și goto.

Th 1: AFD bazat pe funcția goto obținută cu alg. LR(1), pt. gramatica G recunoaște mulțimea prefixelor viabile ale lui G (G').

Dem: Fie $A = (C_{G'}, N \cup \Sigma, goto, I_0, F)$, unde $C_{G'} = \{I_0, I_1, \dots, I_n\}$ mt. canonică LR(1).

$I_0 = closure(\{S' \to . S ; \$\})$, $F = \{ J \in C_{G'} \mid \exists\ A \to \alpha . ; a \in J\}$.

Arătăm că pt. $\forall\ A \neq S'$.

$A \to \alpha . \beta ; a$ goto (I_0, γ) ddacă

$A \to \alpha . \beta ; a$ validă pt. prefixul viabil γ

"$\Rightarrow$" Inducție după $n = |\gamma|$.

$n = 0$. Rez. că $\gamma = \lambda$, $goto(I_0, \lambda) = I_0 = closure(\{S' \to . S ; \$\})$ deci $\alpha = \lambda$ și $\exists$ producțiile:

$S \to A_1 \alpha_1 \in P \qquad S \to . A_1 \alpha_1 ; \$ \in I_0$

$A_1 \to A_2 \alpha_2 \in P \qquad A_1 \to . A_2 \alpha_2 ; b_2 \in I_0, \ b_2 \in First(\alpha_1$

$\vdots$

$A_{k-1} \to A_k \alpha_k \in P$ $\quad A_{k-1} \to .A_k \alpha_k ; b_k \in I_0$, $b_k \in First(\alpha_{k-1} b_{k-1})$

$A_k \to \beta_{k+1} \in P$ $\quad A_k \to .\beta_{k+1} ; b_{k+1} \in I_0$, $b_{k+1} \in First(\alpha_k b_k) \subseteq$
$\subseteq First(\alpha_k \ldots \alpha_1 \$)$

$A_k \to \beta_{k+1} ; b_{k+1} = A \to .\beta ; a$ $\quad (\beta = \beta_{k+1}, A_k = A, a = b_{k+1})$

Fie $\alpha_i' \in \Sigma^*$, $\alpha_i \overset{*}{\underset{d}{\Rightarrow}} \alpha_i'$ și $b_{k+1} = First(\alpha_k' \ldots \alpha_1', \$)$

Avem:

$S \underset{d}{\Rightarrow} A_1 \alpha_1 \overset{*}{\underset{d}{\Rightarrow}} A_1 \alpha_1' \Rightarrow A_2 \alpha_2 \alpha_1' \overset{*}{\underset{d}{\Rightarrow}} A_2 \alpha_2' \alpha_1' \Rightarrow \ldots$
$\Rightarrow A_k \alpha_k' \ldots \alpha_1' \Rightarrow \beta_{k+1} \alpha_k' \ldots \alpha_1'$,

adică (i) $S \overset{*}{\underset{d}{\Rightarrow}} A \alpha_k' \ldots \alpha_1' \underset{d}{\Rightarrow} \beta \alpha_k' \ldots \alpha_1'$

(ii) $\gamma = \lambda$

(iii) $a = b_{k+1} = First(\alpha_k', \ldots \alpha_1', \$)$,

adică $A \to .\beta ; a$ validă pt. prefixul viabil γ.

$\underline{\underline{n \to n+1}}$, $|\gamma| = n+1$, $\gamma = X_1 \ldots X_n X_{n+1}$ și $A \to \alpha.\beta ; a \in$
$goto(I_0, X_1 \ldots X_n X_{n+1})$.

Fie $I_t = goto(I_0, X_1, \ldots X_n)$
$I_j = goto(I_t, X_{n+1}) = goto(I_0, \gamma)$.

Avem $A \to \alpha.\beta ; a \in goto(I_t, X_{n+1})$.

Rez. că $\exists$ $B \to \alpha'.X_{n+1}\beta' ; b \in I_t$ și $A \to \alpha.\beta ; a \in$
$closure(\{B \to \alpha' X_{n+1}.\beta' ; b\})$.

Avem cazurile:

a) $A \to \alpha.\beta ; a = B \to \alpha' X_{n+1} . \beta' ; b$ adică $A = B$;
$\alpha = \alpha' X_{n+1}$; $\beta = \beta'$ și $a = b$.

Deoarece $B \to \alpha'.X_{n+1}\beta' ; b \in I_t = goto(I_0, X_1 \ldots X_n)$
cf. ip. de ind. $\Rightarrow$ $B \to \alpha'.X_{n+1}\beta' ; b$ validă pt. prefix. viabil $X_1 \ldots X_n$, adică:

i) $\exists$ $S \overset{*}{\underset{d}{\Rightarrow}} \theta B w \underset{d}{\Rightarrow} \theta \alpha' X_{n+1} \beta' w$

ii) $X_1 \ldots X_n = \theta \alpha'$

iii) $b = First(w \$)$

$A \to \alpha.\beta ; a \in$ goto (I_0, γ) ddacă

$A \to \alpha.\beta ; a$ validă pt. prefixul viabil γ.

$A \neq S'$

deci (i) $S \underset{d}{\overset{*}{\Rightarrow}} \theta A w \Rightarrow \theta \alpha' X_{n+1} \beta' = \theta \alpha \beta w$

(ii) $X_1 \ldots X_{n+1} = \theta \alpha' X_{n+1} = \theta \alpha$

(iii) $a = First(w\$)$

atunci $A \to \alpha.\beta ; a$ validă pt. prefixul viabil γ

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b) $\alpha = \lambda ; \beta' = B_1 \alpha_1$

$B_1 \to B_2 \alpha_2 \in P$, $B_1 \to .B_2 \alpha_2 ; b_2 \in I_j$, $b_2 \in First(\alpha_1 b)$

$B_{k-1} \to B_k \alpha_k \in P$, $B_k \to .\beta_{k+1} ; b_{k+1} \in I_j$, $b_{k+1} \in First(\alpha_k b_k) \subseteq \ldots \subseteq First(\alpha_k \ldots \alpha_1, b)$

$A \to \alpha.\beta ; a = B_k \to .\beta_{k+1} ; b_{k+1}$

adică $A = B_k$, $\alpha = \lambda$, $\beta = \beta_{k+1}$, $a = b_{k+1} \in First(\alpha_k \ldots \alpha_1 b)$

Fie $\alpha_i' \in \Sigma^*$ a.î. $\alpha_i \underset{d}{\overset{*}{\Rightarrow}} \alpha_i'$ și $b_{k+1} = First(\alpha_k' \ldots \alpha_1' b)$

Rescriem i), ii), iii)

(i') $S \underset{d}{\overset{*}{\Rightarrow}} \theta B w \Rightarrow \theta \alpha' X_{n+1} \beta' w = \theta \alpha' X_{n+1} B_1 \alpha_1 w \underset{d}{\overset{*}{\Rightarrow}} \theta \alpha' X_{n+1} B_1 \alpha_1' w \underset{d}{\Rightarrow} \theta \alpha' X_{n+1} B_2 \alpha_2 \alpha_1' w \Rightarrow \ldots \Rightarrow \theta \alpha' X_{n+1} B_k \alpha_k' \ldots \alpha_1' w \Rightarrow \theta \alpha' X_{n+1} \beta_{k+1} \alpha'_k \ldots \alpha_1' w$

(ii') $X_1 \ldots X_{n+1} = \theta \alpha' X_{n+1}$

iii') $a = b_{k+1} = First(\alpha'_k \ldots \alpha_1' w\$)$

Deci $B_k \to .\beta_{k+1} ; b_{k+1} = A \to .\beta ; a$ validă pt. pr. viabil $\gamma = X_1 \ldots X_{n+1}$

"⇐" Fie $A \to \alpha.\beta; a$ validă pt. pr. viabil γ

Atunci:

(i) $\exists\ S \overset{n}{\underset{d}{\Rightarrow}} \theta A w \Rightarrow \theta\alpha\beta w$

(ii) $\gamma = \theta\alpha$

(iii) $a = First(w\$)$

Arăt că $(\forall) A \to \delta \in P$, $A \to \delta; First(w\$) \in goto(I_0, \gamma)$

Ind după n. Pt. $n = 0$, $\gamma = \lambda$ deci $goto(I_0, \gamma) = I_0 =$ closure $(\{S \to .S; \$\})$

Rez. că $A = S$ și $S \to .\delta; \$ \in I_0$,
$w = \lambda$
$\theta = \lambda$

Pp. că $n \to n+1$. $S \overset{n}{\underset{d}{\Rightarrow}} \theta_1 A_1 w_1 \Rightarrow \theta_1\theta_2 A w_2 w_1$

$\theta_1\theta_2 = \theta$, $w_2 w_1 = w$

$A_1 \to \theta_2 A w_2 \in P$. Cf. ip de ind $A_1 \to .\theta_2 A w_2; First(w_1\$) \in goto(I_0, \theta_1)$

$A_1 \to \theta_2. A w_2; First(w_1\$) \in goto(I_0; \theta_1\theta_2)$

deci $A \to .\delta; First(w_2 w_1 \$) \in goto(I_0, \theta_1\theta_2)$, pt.

$(\forall) A \to \delta \in P$ adică $A \to .\delta; First(w\$) \in goto(I_0, \theta)$

Din $S \overset{*}{\underset{d}{\Rightarrow}} \theta A w$, $A \to \alpha\beta \in P$ rez. $A \to .\alpha\beta;$
$First(w\$) \in goto(I_0, \theta)$ deci $A \to \alpha.\beta;$
$First(w\$) \in goto(I_0, \theta\alpha)$, $A \to \alpha.\beta$,
$a \in goto(I_0, \gamma)$.

Th2: Gram. G este LR(1) ⟺ tabela action (def. cu alg. LR(1)) nu are intrări multiple.

Dem: "⇒" Fie G de tip LR(1). Pp. că action are intrări multiple. Atunci:

a) $\exists\ I_j \in C_G'$ pt. care: $A \to \alpha.a\beta; b \in I_j$, goto (I_j, a)

$= I_k$, action $[j, a]$ = shift k, $a \in \Sigma$

action $[j, a]$ = reduce $B \to \gamma$

adică $B \to \gamma.; a \in I_j$

Fie $\gamma_1 \in (N \cup \Sigma)^*$ a.î. $I_j = goto(I_0, \gamma_1)$. Cf. th. 1,

$A \to \alpha . a\beta; b$ și $B \to \gamma., a$ valide pt. pr. viabil γ_1

Atunci:

(i) $\exists\ S \overset{*}{\underset{d}{\Rightarrow}} \theta A w \Rightarrow \theta \alpha a \beta w$

(ii) $\gamma_1 = \theta\alpha$

(iii) $b = First(w\$)$

(i') $\exists\ S \overset{*}{\underset{d}{\Rightarrow}} \theta' B w' \Rightarrow \theta' \gamma w'$

(ii') $\gamma_1 = \theta'\gamma$

(iii') $a = First(w'\$)$

Deci: $S \overset{*}{\underset{d}{\Rightarrow}} \theta' B w' \Rightarrow \theta' \gamma w' = \gamma_1 w'$

$S \overset{*}{\underset{d}{\Rightarrow}} \theta A w \underset{d}{\Rightarrow} \theta \alpha a \beta w = \gamma_1 a \beta w'$

$First(w') = First(a\beta w') = \{a\}$

Deoarece G este $LR(1)$, $\theta = \theta'$, $A = B$, $w = a\beta w$, adică

$a = \lambda$ $\cancel{\alpha}$

b) $\exists\ I_j \in C_{G'}$, $a \in \Sigma \cup \{\$\}$ pt. care action $[j, a]$ = reduce $A \to \alpha$, action $[j, a]$ = reduce $B \to \beta$

$A \to \alpha \neq B \to \beta$

deci $A \to \alpha.; a \in I_j = goto(I_0, \gamma)$

$B \to \beta.; a \in I_j$

"$\Leftarrow$" Pp. că action nu are intrări multiple.

Pp. prin RA că G nu este $LR(1)$.

Atunci: $S \overset{*}{\underset{d}{\Rightarrow}} \alpha A w \underset{d}{\Rightarrow} \alpha\beta w$

$S \overset{*}{\underset{d}{\Rightarrow}} \gamma B x \underset{d}{\Rightarrow} \alpha \beta y$

a.î. $First(w) = First(y)$ și atunci $\alpha \neq \gamma$, $A \neq B$, $x \neq y$

sunt adevărate.

Fie $B \to \delta$ a.î. $\alpha\beta y = \gamma\delta x$. Avem cazurile:

a) $x = y$. Rezultă $\alpha\beta = \gamma\delta$.

Din $S \underset{d}{\overset{*}{\Rightarrow}} \alpha A w$ rez. că $A \to .\beta$; $First(w\$) \in goto(I_0, \alpha)$.

Din $S \underset{d}{\overset{*}{\Rightarrow}} \gamma B x$, rez. $B \to .\delta$; $First(x\$) \in goto(I_0, \gamma)$.

Deoarece $x = y$, avem $First(x) = First(y)$ deci $First(x\$) = First(w\$)$.

Avem $A \to \beta.$; $First(w\$) \in goto(I_0, \alpha\beta)$.

$B \to \delta.$; $First(x\$) \in goto(I_0, \gamma\delta)$.

Fie $I_K = goto(I_0, \gamma\delta) = goto(I_0, \alpha\beta)$

Fie $a = First(x\$) = First(w\$)$.

Atunci $action[K, a] = reduce\ A \to \beta$

$= reduce\ B \to \delta$ (intrări multiple)

02. Aprilie 2019

Seminar 4

EX1: Alg. Earley

$S' \to S$

$G \begin{cases} S \to aABb \\ A \to BbS \mid B \\ B \to \lambda \end{cases}$

$w = \underset{1\,2\,3\,4\,5}{ababb}$, $n = 5$

$S_0 \begin{cases} S' \to .S, 0 \\ S \to .aABb, 0 \end{cases}$

S_1, $w_1 = 'a'$

$S \to a.ABb, 0$

$A \to .BbS, 1$

$A \to .B, 1$

$S \to aA.Bb, 0$

$B \to ., 1$

$A \to B.bS, 1$

$A \to B., 1$

$S \to aAB.b, 0$