Lab 5

Prop fct de repartitie

$F(x) = P(X \leq x)$

1) $P(a < X \leq b) = F(b) - F(a)$

$P(a \leq X < b) = F(b) - F(a) - P(X = b)$

$P(a \leq X \leq b) = F(b) - F(a) + P(X = a)$

$P(a \leq X < b) = F(b) - F(a) - P(X = b) + P(X = a)$

Op cu v.a. discrete indep

$X: \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix}$ X, Y ind.

$Y: \begin{pmatrix} y_1 & y_2 & \dots & y_k \\ q_1 & q_2 & \dots & q_k \end{pmatrix}$

$P((X \leq x) \cap (Y \leq y)) = P(X \leq x) \cdot P(Y \leq y)$

$\forall x, y \in \mathbb{R}$

$\{X(\omega) \mid \omega \in \Omega\}$ → cel mult num DISCRETĂ

↳ infin. nenum CONTINUĂ

$$c \dot{\pm} X: \begin{pmatrix} c \dot{\pm} x_1 & c \dot{\pm} x_2 \cdots c \dot{\pm} x_n \\ p_1 & p_2 \cdots p_n \end{pmatrix}$$

$$X^{\alpha}: \begin{pmatrix} x_1^{\alpha} & x_2^{\alpha} \cdots x_n^{\alpha} \\ p_1 & p_2 \cdots p_n \end{pmatrix}$$

$$x \dot{\pm} y: \begin{pmatrix} x_1 \dot{\pm} y_1 & x_1 \dot{\pm} y_2 \cdots x_i \dot{\pm} y_k \; x_i \dot{\pm} y_1 \cdots x_n \dot{\pm} y_k \\ p_1 q_1 & p_1 q_2 \cdots p_1 q_k \; p_2 q_1 \cdots p_n q_k \end{pmatrix}$$

Media și dispersia

$$\underbrace{E(X) = \sum_{i=1}^{n} p_i \cdot x_i}_{\in \mathbb{R}} \qquad \underbrace{Var(X) = E\left((X - E(X))^2\right)}_{\geq 0}$$

Proprietăți

1) $E(c) = c$ $\quad Var(c) = 0$

2) $E(cX) = cE(X)$ $\quad Var(cX) = c^2 Var(X)$

3) $E(X \pm Y) = E(X) \pm E(Y)$ $\quad Var(X \pm Y) = Var(X) + Var(Y)$ (Dacă X, Y ind)

4) $E(X \cdot Y) = E(X) \cdot E(Y)$ (Dacă X, Y ind)

1) Sp 2) $\Rightarrow E(aX + bY) = aE(X) + bE(Y)$

2) Sp 3) $\Rightarrow Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$

Exc

1) $X : \begin{pmatrix} -5 & 0 & 5 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$ $\quad Y : \begin{pmatrix} 0 & 3 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

X, Y indep

$$X-Y: \begin{pmatrix} -5 & -10 & 0 & -5 & 5 & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{9} & \frac{2}{9} & \frac{1}{18} & \frac{1}{9} \end{pmatrix}$$

$$x-y: \begin{pmatrix} -10 & -5 & 0 & 5 \\ \frac{6}{18} & \frac{7}{18} & \frac{4}{18} & \frac{1}{18} \end{pmatrix}$$

$$XY: \begin{pmatrix} 0 & -25 & 0 & 0 & 0 & 25 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{9} & \frac{2}{9} & \frac{1}{18} & \frac{1}{9} \end{pmatrix}$$

$$x\,y: \begin{pmatrix} -25 & 0 & 25 \\ \frac{6}{18} & \frac{10}{18} & \frac{2}{18} \end{pmatrix}$$

$$5xy: \begin{pmatrix} -125 & 0 & 125 \\ \frac{6}{18} & \frac{10}{18} & \frac{2}{18} \end{pmatrix}$$

2) $E(X), E(Y), E(X-Y), E(5xy)$

$$E(X) = -\frac{5}{2} + 0 + \frac{5}{6} = -\frac{10}{6}$$

$$E(Y) = \frac{10}{3}$$

$$E(x-y) = E(x) - E(y) = -\frac{10}{6} - \frac{10}{3} = -\frac{30}{6}$$

$$E(5xy) = 5E(x)E(y) = 5\left(-\frac{10}{6}\right)\frac{10}{3} = -\frac{500}{18}$$

$$Var(x) = E(x^2) - E(x)^2$$

$$E(X^2) = \frac{25}{2} + \frac{25}{6} = \frac{100}{6}$$

$$X^2 : \begin{pmatrix} 0 & 25 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \qquad Y^2 : \begin{pmatrix} 0 & 25 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$Var(Y) = E(Y^2) - E(Y)^2 = \frac{50}{3} - \frac{100}{9} = \frac{50}{9}$$

$$Var(X-Y) = Var(X) + Var(Y) = \frac{700}{36}$$

$$Var(5XY) = 25\,Var(XY)$$

$$Var(XY) = E((XY)^2) - E(XY)^2$$

BONUS

$$X : \begin{pmatrix} 1 & 2 & 3 & \cdots & n & \cdots \\ \frac{1}{2} & \left(\frac{1}{2}\right)^2 & \left(\frac{1}{2}\right)^3 & \cdots & \left(\frac{1}{2}\right)^n & \cdots \end{pmatrix}$$

a) Vf. buna definire

b) $E(X) = ?$ $\quad Var(X) = ?$

c) Constr o altă V.A. cu o inf. numărabilă de valori: