Review problems for Midterm #2

1 Find the dual of the following linear programming problem.

$$\begin{array}{l} \text{Maximize } z = 3x_1 + x_2 \\ \text{subject to} \\ \begin{array}{rcrcl} x_1 & + & 3x_2 & \geq & 10 \\ 3x_1 & + & 2x_2 & = & 20 \\ x_1 & & & \leq & 5 \\ x_1, & & x_2 & \geq & 0 \end{array} \end{array}$$

Answer: First I would convert the problem to standard form:

$$\begin{array}{l} \text{Maximize } z = 3x_1 + x_2 \\ \text{subject to} \\ \begin{array}{rcrcl} -x_1 & - & 3x_2 & \leq & -10 \\ 3x_1 & + & 2x_2 & \leq & 20 \\ -3x_1 & - & 2x_2 & \leq & -20 \\ x_1 & & & \leq & 5 \\ x_1, & & x_2 & \geq & 0 \end{array} \end{array}$$

Then the dual is easy to find:

$$\begin{array}{l} \text{Minimize } z' = -10w_1 + 20w_2 - 20w_3 + 5w_4 \\ \text{subject to} \\ \begin{array}{rcrcrcrcl} -w_1 & + & 3w_2 & - & 3w_3 & + & w_4 & \geq & 3 \\ -3w_1 & + & 2w_2 & + & -2w_3 & & & \geq & 1 \\ w_1, & & w_2, & & w_3, & & w_4 & \geq & 0 \end{array} \end{array}$$

2 Consider the following linear programming problem

$$\begin{array}{l} \text{Maximize } z = 15x_1 + 4x_2 \\ \text{subject to} \\ \begin{array}{rcrcl} 5x_1 & + & 2x_2 & \leq & 10 \\ x_1 & & & \leq & \frac{3}{2} \\ 4x_1 & + & 4x_2 & \leq & 14 \\ x_1, & & x_2 & \geq & 0 \end{array} \end{array}$$

(a) Find the dual of this problem.

(b) Use the fact that $\begin{bmatrix} \frac{3}{2} & \frac{5}{4} \end{bmatrix}^T$ is an optimal solution to the primal problem and the Principle of Complementary Slackness to find an optimal solution to the dual problem. (No points will be given for solving the dual problem by any other method.)

Answer: Part (a):

$$\begin{array}{l} \text{Minimize } z' = 10w_1 + \frac{3}{2}w_2 + 14w_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} 5w_1 & + & w_2 & + & 4w_3 & \geq & 15 \\ 2w_1 & & & + & 4w_3 & \geq & 4 \\ w_1, & & w_2, & & w_3, & \geq & 0 \end{array} \end{array}$$

For part (b), we start by plugging the optimal solution for the primal problem into the constraints there, and get:

$$\begin{array}{rcrcll} 5\left(\frac{3}{2}\right) & + & 2\left(\frac{5}{4}\right) & = & 10 & \text{inequality tight, doesn't tell us anything} \\ \left(\frac{3}{2}\right) & & & = & \frac{3}{2} & \text{inequality tight, doesn't tell us anything} \\ 4\left(\frac{3}{2}\right) & + & 4\left(\frac{5}{4}\right) & = & 11 & \text{inequality has slack, so the third dual variable, } w_3\text{, must be 0} \end{array}$$

So all that told us is that $w_3 = 0$. Substituting this back into the dual problem constraints, we get

$$\begin{array}{rcrcl} 5w_1 & + & w_2 & \geq & 15 \\ 2w_1 & & & \geq & 4 \end{array}$$

We also know that x_1 and x_2 (in the optimal solution to the primal problem) are nonzero. This means, again by the Principle of Complementary Slackness, that the first and second constraints in the dual problem must be tight. This gives us enough information to solve for w_1 and w_2. Starting with the second constraint, which since it must be tight, is $2w_1 = 4$, we get $w_1 = 2$. Then we plug this into the first constraint to see that $10 + w_2 = 15$, so $w_2 = 5$.

$\boxed{3}$ Consider the following primal problem

$$\begin{array}{rcrcrcl} \multicolumn{7}{l}{\text{Maximize } z = x_1 + 3x_2 + 5x_3} \\ \multicolumn{7}{l}{\text{subject to}} \\ 2x_1 & - & 5x_2 & + & x_3 & \leq & 3 \\ x_1 & + & 4x_2 & & & \leq & 5 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array}$$

and its dual

$$\begin{array}{rcrcl} \multicolumn{5}{l}{\text{Minimize } z' = 3w_1 + 5w_2} \\ \multicolumn{5}{l}{\text{subject to}} \\ 2w_1 & + & w_2 & \geq & 1 \\ -5w_1 & + & 4w_2 & \geq & 3 \\ w_1 & & & \geq & 5 \\ w_1, & & w_2 & \geq & 0 \end{array}$$

Show that $\underline{x} = \begin{bmatrix} 0 \\ \frac{5}{4} \\ \frac{37}{4} \end{bmatrix}$ is an optimal solution to the primal problem and that $\underline{w} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ is an optimal solution to the dual problem. (Hint: do **not** attempt to solve either problem. Instead use a theorem from section 3.2.)

Solution: The applicable theorem from section 3.2 is the following, which is an easy consequence of the Weak Duality Theorem:

Theorem 3.6: If $\underline{x}$ and $\underline{w}$ are feasible solutions to the primal problem and dual problem, respectively, and if they give the same objective values, then they are both optimal solutions to their respective problems.

To use this theorem you have to check it's two hypotheses, which are

(1) $\underline{x}$ and $\underline{w}$ are both feasible,

(2) $\underline{x}$ and $\underline{w}$ give the same objective value

Both of these can be checked quite easily. For (1), we just have to plug the solutions into the constraints:

$$\begin{array}{rcrcrcrcl} 2\,(0) & - & 5\left(\frac{5}{4}\right) & + & \frac{37}{4} & = & 3 & \leq & 3 \\ 0 & + & 4\left(\frac{5}{4}\right) & & & = & 5 & \leq & 5 \\ 0, & & \frac{5}{4}, & & \frac{37}{4} & & & \geq & 0 \end{array}$$

$$\begin{array}{rcrcrcl} 2(5) & + & 7 & = & 17 & \geq & 1 \\ -5(5) & + & 4(7) & = & 3 & \geq & 3 \\ 5 & & & = & 5 & \geq & 5 \\ 5, & & 7 & & & \geq & 0 \end{array}$$

So both solutions are feasible. Now we check the objective function values:

$$\begin{array}{rcl} 0 + 3\left(\frac{5}{4}\right) + 5\left(\frac{37}{4}\right) & = & 50, \\ 3(5) + 5(7) & = & 50. \end{array}$$

Therefore we have checked both hypotheses of the theorem, so it applies to tell us that both $\underline{x}$ and $\underline{w}$ are optimal solutions.

$\boxed{4}$ Use the dual simplex method to restore feasibility in the following tableau.

	x_1	x_2	u_1	u_2	u_3	u_4	
x_2	0	1	-2	0	3	0	2
x_1	1	0	1	0	2	0	1
u_2	0	0	-4	1	5	0	0
u_4	0	0	1	0	-5	1	-1
	0	0	2	0	4	0	5

Answer: There is only one negative entry in the right-most column, so we know that it's basic variable – u_4 – must depart. Then there is only one negative entry in the u_4 row, the entry corresponding to u_3, so u_3 enters. After pivoting, we get

	x_1	x_2	u_1	u_2	u_3	u_4	
x_2	0	1	$-\frac{7}{5}$	0	0	$\frac{3}{5}$	$\frac{7}{5}$
x_1	1	0	$\frac{7}{5}$	0	0	$\frac{2}{5}$	$\frac{3}{5}$
u_2	0	0	-3	1	0	1	-1
u_3	0	0	$-\frac{1}{5}$	0	1	$-\frac{1}{5}$	$\frac{1}{5}$
	0	0	$\frac{14}{5}$	0	0	$\frac{4}{5}$	$\frac{21}{5}$

But we're not done! There is a new negative entry in the right-most column, so we need to apply Dual Simplex again. This time u_2 departs and u_1 enters, and after pivoting, we get

	x_1	x_2	u_1	u_2	u_3	u_4	
x_2	0	1	0	$-\frac{7}{15}$	0	$\frac{2}{15}$	$\frac{28}{15}$
x_1	1	0	0	$\frac{7}{15}$	0	$\frac{13}{15}$	$\frac{2}{15}$
u_1	0	0	1	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$
u_3	0	0	0	$-\frac{1}{15}$	1	$-\frac{4}{15}$	$\frac{4}{15}$
	0	0	0	$\frac{14}{15}$	0	$\frac{26}{15}$	$\frac{49}{15}$

Now we're done.

$\boxed{5}$ Consider again our saw mill problem, which corresponds to the linear programming problem

$$\begin{array}{l} \text{Maximize } z = 120x_1 + 100x_2 \\ \text{subject to} \\ \begin{array}{rcrcl} 2x_1 & + & 2x_2 & \leq & 8 \\ 5x_2 & + & 3x_2 & \leq & 15 \\ x_1, & & x_2, & \geq & 0 \end{array} \end{array}$$

To solve it we introduce slack variables u_1 and u_2 to get the following initial tableau

		120	100	0	0	
$\underline{c}_B$		x_1	x_2	u_1	u_2	
0	u_1	2	2	1	0	8
0	u_2	5	3	0	1	15
		-120	-100	0	0	0

Then we do two iterations of the Simplex Method and get the following final tableau

		120	100	0	0	
$\underline{c}_B$		x_1	x_2	u_1	u_2	
100	x_2	0	1	$\frac{5}{4}$	$-\frac{1}{2}$	$\frac{5}{2}$
120	x_1	1	0	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{2}$
		0	0	35	10	430

(a) Suppose that we now decrease b_2 from 15 to 10. Find an optimal solution for this new problem. (No points will be given for starting again from scratch! The point of this problem is for you to use the techniques from section 3.6.)

(a) Suppose that instead we increase c_1 from 120 to 200. Find an optimal solution for this problem. (Again, no points will be given for starting over.)

Answer: For (a) we have to update the right-most column and the objective value of our final tableau and then possibly do a few iterations of the Dual Simplex Method to restore feasibility. The formula for updating the right-most column is that we add to it the following column vector:

$$\Delta b_2 B^{-1} \underline{e}_2,$$

where

$$\begin{array}{rcl} \Delta b_2 & = & \text{the change in } b_2, \\ B^{-1} & = & \text{the matrix consisting of the columns in the } \textit{final} \text{ tableau corresponding} \\ & & \text{to basic variables in the } \textit{initial} \text{ tableau,} \\ \underline{e}_2 & = & \text{the } 2^{nd} \text{ elementary basis vector.} \end{array}$$

In our case, this means that

$$\Delta b_2 = -5,$$

$$B^{-1} = \begin{bmatrix} \frac{5}{4} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{1}{2} \end{bmatrix},$$

$$\underline{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

So we add

$$-5 \begin{bmatrix} \frac{5}{4} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ -\frac{5}{2} \end{bmatrix}$$

to the right-most column of the final tableau.

In addition, we have now have to update the objective value of this tableau to take into account the changes we have made to the right-most column. The formula here is

$$\underline{c}_B^T \underline{x}_B,$$

where $\underline{x}_B$ denote the right-most column. Since our $\underline{x}_B$ is given by

$$\underline{x}_B = \begin{bmatrix} \frac{5}{2} \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} \frac{5}{2} \\ -\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix},$$

The new objective value is

$$\begin{bmatrix} 100 & 120 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix} = 380.$$

Making these replacements, we get the tableau

		120	100	0	0	
$\underline{c}_B$		x_1	x_2	u_1	u_2	
100	x_2	0	1	$\frac{5}{4}$	$-\frac{1}{2}$	5
120	x_1	1	0	$-\frac{3}{4}$	$\frac{1}{2}$	-1
		0	0	35	10	380

Since there is now a negative entry in the right-most column, this tableau represents an infeasible solution. We must apply the Dual Simplex Method to restore feasibility. Our departing variable will be x_1 and our entering variable will be u_1. After pivoting, we have

	x_1	x_2	u_1	u_2	
x_2	$\frac{5}{3}$	1	0	$\frac{1}{3}$	$\frac{10}{3}$
u_1	$-\frac{4}{3}$	0	1	$-\frac{2}{3}$	$\frac{4}{3}$
	$\frac{140}{3}$	0	0	$\frac{100}{3}$	$\frac{1000}{3}$

This tableau represents the feasible solution $x_1 = 0$, $x_2 = 10/3$, with a profit of $1000/3$.

Part (b) is similar, only this time we update the last row instead of the last column. The formula here is that the jth entry in the last row is equal to

$$\underline{c}_B^T \underline{t}_j - c_j,$$

where

$$\begin{aligned} \underline{c}_B &= \text{the entries of the cost vector corresponding to the basic variables,} \\ \underline{t}_j &= \text{the } j\text{th column, without the objective row entry,} \\ c_j &= \text{the entry of the cost vector corresponding to the } j\text{th variable.} \end{aligned}$$

(Remember that all references to $\underline{c}$ really mean "the changed $\underline{c}$.")

In our case we have

$$\begin{aligned} \underline{c}_B &= \begin{bmatrix} 100 \\ 200 \end{bmatrix} \\ \underline{t}_1 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \underline{t}_2 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \underline{t}_3 &= \begin{bmatrix} \frac{5}{4} \\ -\frac{3}{4} \end{bmatrix} \\ \underline{t}_4 &= \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \\ c_1 &= 200 \\ c_2 &= 100 \\ c_3 &= 0 \\ c_4 &= 0 \end{aligned}$$

So we get

$$\begin{aligned} \text{new first entry in objective row} &= \begin{bmatrix} 100 & 200 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 200 = 0 \\ \text{new second entry in objective row} &= \begin{bmatrix} 100 & 200 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 100 = 0 \\ \text{new third entry in objective row} &= \begin{bmatrix} 100 & 200 \end{bmatrix} \begin{bmatrix} \frac{5}{4} \\ -\frac{3}{4} \end{bmatrix} - 0 = -25 \\ \text{new fourth entry in objective row} &= \begin{bmatrix} 100 & 200 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} - 0 = 50 \end{aligned}$$

We also have to recompute the new objective value. It is given by the dot product of the right-most column with $\underline{c}_B$. In our case it is

$$\begin{bmatrix} 100 & 200 \end{bmatrix} \begin{bmatrix} \frac{5}{2} \\ \frac{3}{2} \end{bmatrix} = 550.$$

Finally, the updated tableau is shown below.

		200	100	0	0	
$\underline{c}_B$		x_1	x_2	u_1	u_2	
100	x_2	0	1	$\frac{5}{4}$	$-\frac{1}{2}$	$\frac{5}{2}$
200	x_1	1	0	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{2}$
		0	0	-25	50	550

Notice that this tableau corresponds to a feasible solution but does not satisfy the Optimality Condition, so we need to apply the Simplex Method to get an optimal solution out of it. Our entering variable is u_1 and our departing variable is x_2 and after pivoting we obtain the following tableau.

		200	100	0	0	
$\underline{c}_B$		x_1	x_2	u_1	u_2	
0	u_1	0	$\frac{4}{5}$	1	$-\frac{2}{5}$	2
200	x_1	1	$\frac{3}{5}$	0	$\frac{1}{5}$	3
		0	20	0	40	600

$\boxed{6}$ Suppose that there are three jobs to be assigned and we have three workers available, and so we want to match each of these three workers to exactly one of the three jobs. A point scale has been set up rating the value of assigning a particular worker to a particular job, which is presented in the following chart:

Worker	Job 1	Job 2	Job 3
1	6	9	5
2	5	5	5
3	7	3	7

Formulate, but **do not solve**, an integer linear programming problem whose solution would reveal how to assign each worker to a job in such a way to maximize the total number of value points.

Answer: First we need to introduce variables. I will set

$$x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ is assigned job } j, \\ 0 & \text{otherwise.} \end{cases}$$

We want to maximize the total value of the assignment, which is

$$z = 9x_{11} + 9x_{12} + 5x_{13} + 5x_{21} + 5x_{22} + 5x_{23} + 7x_{31} + 3x_{32} + 7x_{33}.$$

Then our constraints are that each person can work only one job and each job can be worked by only one person. For example, the first person can work only one job means

$$x_{11} + x_{12} + x_{13} = 1,$$

and the fact that the second job can be worked by only one person means

$$x_{12} + x_{22} + x_{32} = 1.$$

Putting this all together gives the following integer programming problem.

$$\begin{array}{rcl}
\multicolumn{3}{l}{\text{Maximize } z = 9x_{11} + 9x_{12} + 5x_{13} + 5x_{21} + 5x_{22} + 5x_{23} + 7x_{31} + 3x_{32} + 7x_{33}} \\
\multicolumn{3}{l}{\text{subject to}} \\
x_{11} + x_{12} + x_{13} & = & 1 \\
x_{21} + x_{22} + x_{23} & = & 1 \\
x_{31} + x_{32} + x_{33} & = & 1 \\
x_{11} + x_{21} + x_{31} & = & 1 \\
x_{12} + x_{22} + x_{32} & = & 1 \\
x_{13} + x_{23} + x_{33} & = & 1 \\
x_{ij} & \leq & 1 \\
x_{ij} & \geq & 0 \text{ integral}
\end{array}$$

$\boxed{7}$ Solve the following integer programming problem using the Cutting Plane Method.

$$\begin{array}{rcrcl}
\multicolumn{5}{l}{\text{Maximize } z = x + 4y} \\
\multicolumn{5}{l}{\text{subject to}} \\
x & + & 6y & \leq & 36 \\
3x & + & 8y & \leq & 60 \\
x, & & y, & \geq & 0, \text{ integral.}
\end{array}$$

Answer: First we solve the problem using the simplex method and forgetting about the integer requirement. The initial tableau is

	x	y	u_1	u_2	
u_1	1	6	1	0	36
u_2	3	8	0	1	60
	-1	-4	0	0	0

After one iteration:

	x	y	u_1	u_2	
y	$\frac{1}{6}$	1	$\frac{1}{6}$	0	6
u_2	$\frac{5}{3}$	0	$-\frac{4}{3}$	1	12
	$-\frac{1}{3}$	0	$\frac{2}{3}$	0	24

After another iteration we have the final tableau:

	x	y	u_1	u_2	
y	0	1	$\frac{3}{10}$	$-\frac{1}{10}$	$\frac{24}{5}$
x	1	0	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{36}{5}$
	0	0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{132}{5}$

Now we choose one of the constraints with a fractional right-hand-side and round it down. Let's choose the second constraint in this tableau, which corresponds to

$$x - \frac{4}{5}u_1 + \frac{3}{5}u_2 = \frac{36}{5}.$$

Then we round everything and replace the "=" by an "≤" (remind yourself why we are allowed to do this!):

$$\lfloor 1 \rfloor x + \lfloor -\frac{4}{5} \rfloor u_1 + \lfloor \frac{3}{5} \rfloor u_2 \leq \lfloor \frac{36}{5} \rfloor.$$

This gives

$$x - u_1 \leq 7.$$

Now we have to add a new slack variable and put this constraint into our tableau. So we add in u_3 and our constraint is

$$x - u_1 + u_3 = 7.$$

After adding it to our tableau, we have

	x	y	u_1	u_2	u_3	
y	0	1	$\frac{3}{10}$	$-\frac{1}{10}$	0	$\frac{24}{5}$
x	1	0	$-\frac{4}{5}$	$\frac{3}{5}$	0	$\frac{36}{5}$
u_3	1	0	-1	0	1	7
	0	0	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{132}{5}$

Now we have to clean up the column corresponding to x, because x is a basic variable. To do this we subtract the x row from the u_3 row, leaving

	x	y	u_1	u_2	u_3	
y	0	1	$\frac{3}{10}$	$-\frac{1}{10}$	0	$\frac{24}{5}$
x	1	0	$-\frac{4}{5}$	$\frac{3}{5}$	0	$\frac{36}{5}$
u_3	0	0	$\frac{1}{5}$	$-\frac{3}{5}$	1	$-\frac{1}{5}$
	0	0	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{132}{5}$

Now we need to restore feasibility with the Dual Simplex Method. The departing variable is u_3 and the entering variable is u_2. After pivoting, we have

	x	y	u_1	u_2	u_3	
y	0	1	$\frac{4}{15}$	0	$-\frac{1}{6}$	$\frac{29}{6}$
x	1	0	$-\frac{3}{5}$	0	1	7
u_2	0	0	$-\frac{1}{3}$	1	$-\frac{5}{3}$	$\frac{1}{3}$
	0	0	$\frac{7}{15}$	0	$\frac{1}{3}$	$\frac{79}{3}$

And still we have fractions, so we repeat. We choose the first constraint,

$$y + \frac{4}{15}u_1 - \frac{1}{6}u_3 = \frac{29}{6},$$

round it down to give

$$y - u_3 \leq 4,$$

add a new slack variable to get

$$y - u_3 + u_4 = 4,$$

and add it to our tableau:

	x	y	u_1	u_2	u_3	u_4	
y	0	1	$\frac{4}{15}$	0	$-\frac{1}{6}$	0	$\frac{29}{6}$
x	1	0	$-\frac{3}{5}$	0	1	0	7
u_2	0	0	$-\frac{1}{3}$	1	$-\frac{5}{3}$	0	$\frac{1}{3}$
u_4	0	1	0	0	-1	1	4
	0	0	$\frac{7}{15}$	0	$\frac{1}{3}$	0	$\frac{79}{3}$

clean up the y column:

	x	y	u_1	u_2	u_3	u_4	
y	0	1	$\frac{4}{15}$	0	$-\frac{1}{6}$	0	$\frac{29}{6}$
x	1	0	$-\frac{3}{5}$	0	1	0	7
u_2	0	0	$-\frac{1}{3}$	1	$-\frac{5}{3}$	0	$\frac{1}{3}$
u_4	0	0	$-\frac{4}{15}$	0	$-\frac{5}{6}$	1	$-\frac{5}{6}$
	0	0	$\frac{7}{15}$	0	$\frac{1}{3}$	0	$\frac{79}{3}$

Now we need to apply the Dual Simplex Method to restore feasibility. The departing variable will be u_4, but we have to compute ratios to figure out the entering variable. The ratio for the u_1 column is

$$\frac{\frac{7}{15}}{-\frac{4}{15}} = -\frac{7}{4},$$

and the ratio for the u_3 column is

$$\frac{\frac{1}{3}}{\frac{-5}{6}} = -\frac{2}{5}.$$

Since we want to choose the least negative ratio, we will have u_3 enter. After pivoting, we get

	x	y	u_1	u_2	u_3	u_4	
y	0	1	$\frac{8}{25}$	0	0	$-\frac{1}{5}$	5
x	1	0	$-\frac{23}{25}$	0	0	$\frac{6}{5}$	6
u_2	0	0	$\frac{1}{5}$	1	0	-2	2
u_3	0	0	$\frac{8}{25}$	0	1	$-\frac{6}{5}$	1
	0	0	$\frac{9}{25}$	0	0	$\frac{2}{5}$	26

Finally, we have an integer solution, and we can stop! The optimal solution is $x = 6$ and $y = 5$, which gives an optimal objective value of 26.

$\boxed{8}$ Solve the following integer linear programming problem using the Branch and Bound Method.

$$\begin{array}{l} \text{Maximize } z = 2x_1 + 2x_2 + 3x_3 \\ \text{subject to} \\ \begin{array}{ccccccc} 2x_1 & + & 3x_2 & + & 2x_3 & \leq & 5 \\ x_1, & & x_2, & & x_3 & \geq & 0, \text{ integral.} \end{array} \end{array}$$

Answer: Coming soon.