Seminar 5

Funcții primitive recursive

Funcții pr. rec. de bază

$$succ(x) = x+1$$
$$null(x) = 0$$
$$p_i^n(x_1, \dots, x_n) = x_i ; \ 1 \le i \le n$$

Compunerea funcțiilor

Fie $f: \mathbb{N}^k \to \mathbb{N}$; $g_1, \dots g_k : \mathbb{N}^n \to \mathbb{N}$

$h: \mathbb{N}^n \to \mathbb{N}$, $h(x_1, \dots, x_n) = f(g_1(x_1, \dots x_n), \dots$
$g_k(x_1, \dots x_n))$

Dacă f, g pr. rec., atunci h pr. rec.

Not $[f \circ (g_1, \dots, g_k)](x_1, \dots, x_n)$

Recursie primitivă

$h: \mathbb{N} \to \mathbb{N}$, $g: \mathbb{N}^2 \to \mathbb{N}$

$h(0) = k$, $h(t+1) = g(t, h(t))$

Dacă g pr. rec., at. h pr. rec

Recursie generală

Fie $f: \mathbb{N}^n \to \mathbb{N}$, $g: \mathbb{N}^{n+2} \to \mathbb{N}$ și

$h: \mathbb{N}^{n+1} \to \mathbb{N}$,

$h(x_1, \dots, x_n, 0) = f(x_1, \dots, x_n)$

$h(x_1, \dots, x_n, t+1) = g(t, h(x_1, \dots x_n, t), x_1, \dots)$

Dacă f și g pr. rec., at. h pr. rec

Ex Ssac urm fct sunt pr. rec

a) $const_m(x) = m$, $m \in \mathbb{N}$ dat

b) $sum(x, y) = x + y$

c) $mult(x, y) = xy$

d) $fact(x) = x!$

e) $exp(x,y) = x^y$

f) $pred(x) = \begin{cases} x-1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

g) $sub(x,y) = x \dot{-} y = \begin{cases} x-y, & x \geq y \\ 0, & x < y \end{cases}$

h) $diff(x,y) = |x-y|$

i) $eq_0(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0 \end{cases}$

Rez a) $const_m(x) = succ(succ(\dots succ(null(x))\dots))$

b) $sum(x,0) = x = p_1^1(x)$

$sum(x,y+1) = succ(sum(x,y)) =$

$(g(y, sum(x,y), x))$

$= succ(p_2^3(y, sum(x,y), x))$

$g = succ \circ p_2^3$

c) $mult(x,0) = null(x)$

$mult(x,y+1) = mult(x,y) + x = sum(mult(x,y), x) =$

$= sum(p_2^3(y, mult(x,y), x), p_3^3(y, mult(x,y), x))$

$= [sum \circ (p_3^2, p_2^3)](y, mult(x,y), x)$

d) $fact(0) = 1$

$fact(x+1) = mult(fact(x), x+1) =$

$= mult(fact(x), succ(x))$

e) $exp(x, 0) = 1, \forall x \in \mathbb{N}$

$exp(x, y+1) = mult(exp(x,y), x)$

f) $pred(0) = 0$

$pred(x+1) = p_1^2(x, pred(x))$

g) $sub(x, 0) = x = p_1^1(x)$

$sub(x, y+1) = pred(sub(x,y))$

h) $diff(x, 0) = x = p_1^1(x)$

$diff(x, y) = sum(sub(x,y), sub(y,x))$

i) $eq_0(0) = 1$

$eq_0(x+1) = const_0(x) = const_0 \circ p_1^2(x, eq_0(x))$

Ex

j) $eq_m(x) = \begin{cases} 1, \text{ dacă } x = m \\ 0, \text{ dacă } x \neq m \end{cases}$

k) $eq_S(x) = \begin{cases} 1, \text{ dacă } x \in S \\ 0, \text{ dacă } x \notin S \end{cases}$

$S \subseteq \mathbb{N}; \; S = \{a_1, a_2, \ldots, a_p\}$

l) $E(x) = \begin{cases} 0, x \text{ par} \\ 1, x \text{ impar} \end{cases}$

m) $H(x) = \begin{cases} \frac{x}{2}, x \text{ par} \\ \frac{x-1}{2}, x \text{ impar} \end{cases}$

n) $f(0) = 0, \; f(1) = 1, \; f(2) = 2^2, \; f(3) = 3^{3^3}$

$f(x) = x^{x^{\cdot^{\cdot^{\cdot^x}}}}$ (x de x ori)

Rez

j) $eq_m(x) = eq_0(diff(x, m))$

k) $eq_S(x) = eq_{a_1}(x) + eq_{a_2}(x) + \ldots + eq_{a_p}(x)$

l) $E(0) = 0$

$E(x+1) = 1 - E(x) = sub(1, E(x))$

m) $H(0) = 0$

$H(x+1) = H(x) + E(x)$

n) $g(x,y) = x^{x^{\cdots^{x}}}$ (y times)

$f(x) = g(x,x)$

$g(x,0) = 0$

$g(x,y+1) = exp(x, g(x,y))$

rec generală $\Rightarrow$ g rec $\Rightarrow$ f pr rec

exp pr rec

Ex 1) Să se scrie o MTD_2 care recunoaște

$L_1 = \{a^n b^n c^n \mid n \geq 0\}$

2) Să se scrie o MTD_2 care rec

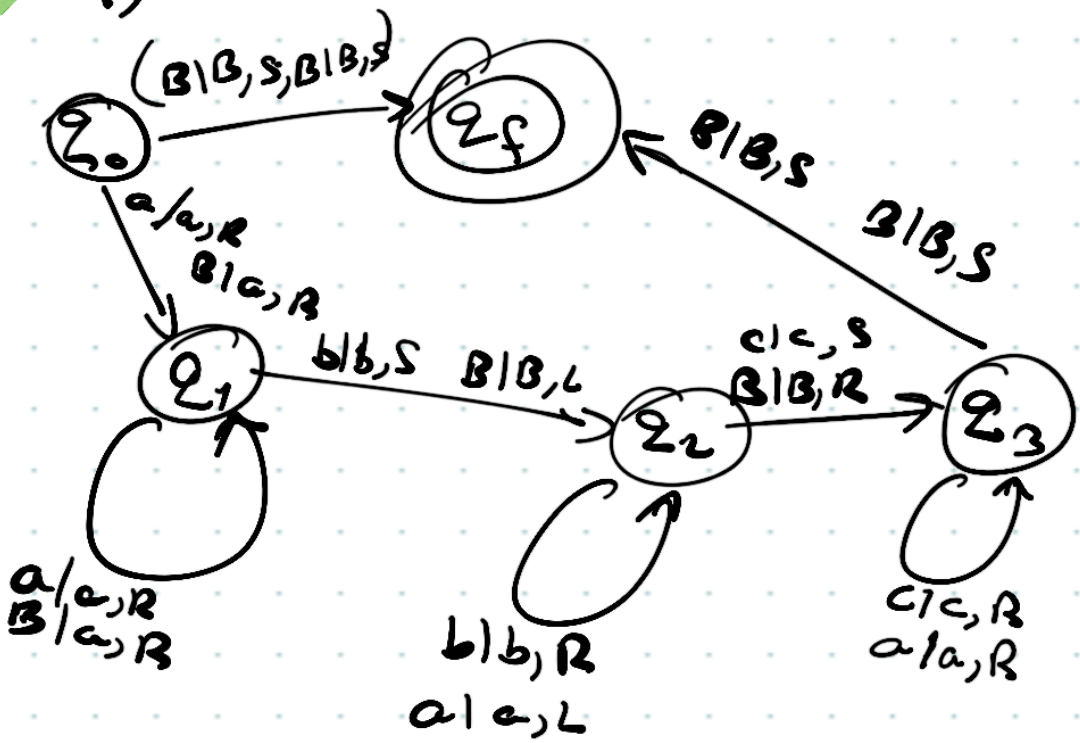
$L_2 = \{a b^{i_1} a b^{i_2} a \ldots a b^{i_k} \mid 0 \leq i_1 \leq i_2 \leq \ldots \leq i_k, k \geq 1\}$ fixat

$(x|Y, D; x'|Y', D')$

$D, D' \in \{L, R, S\}$

↑ staționar

Rez 1)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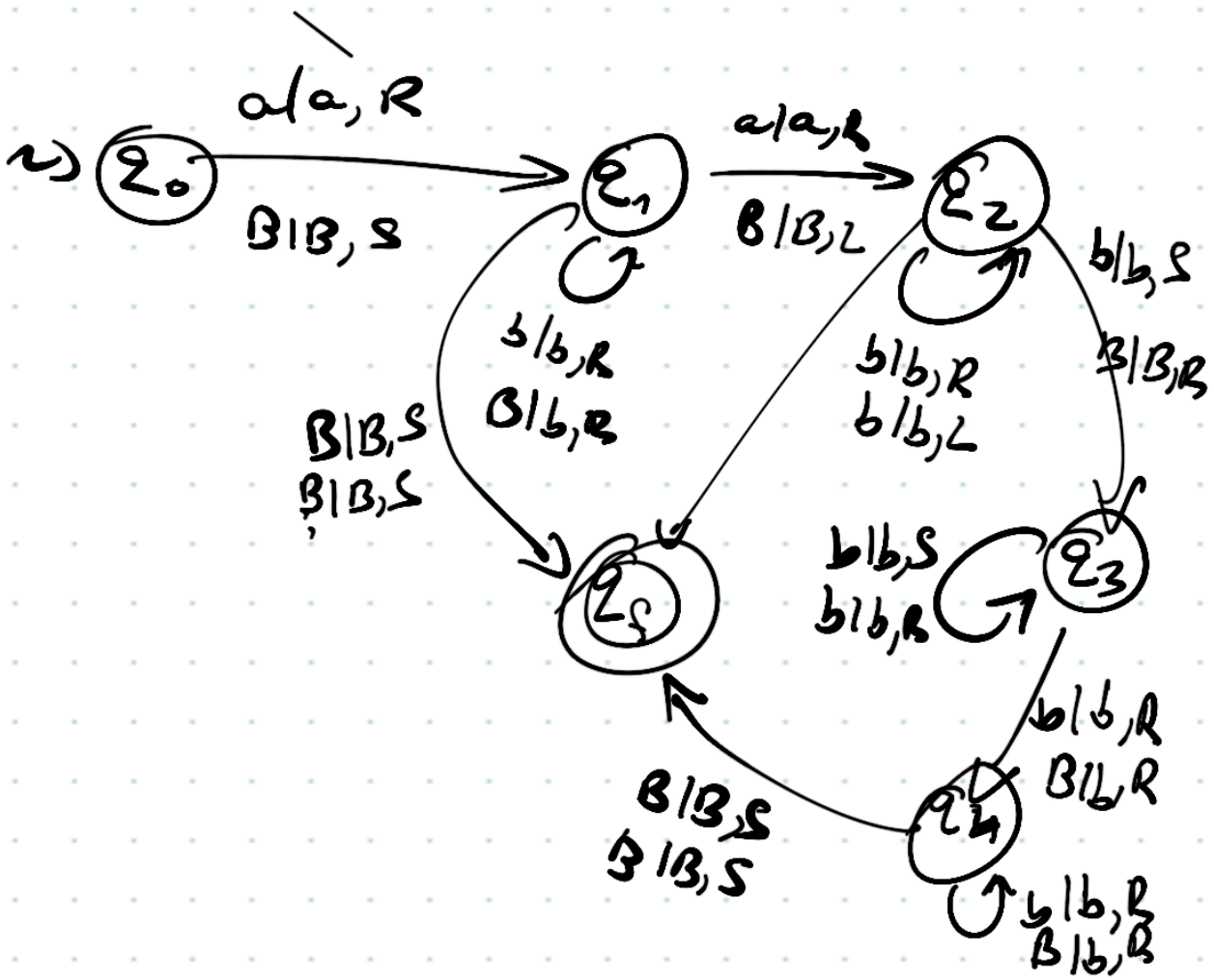
2)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