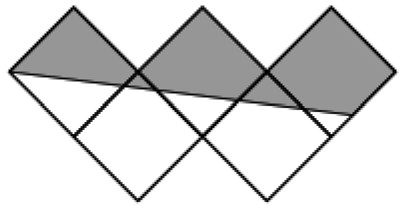
1. This W is cut into two pieces of equal area by the straight line. The straight line intersects the right edge of the W at a point. Exactly what ratio does this point split the lengths of the right edge into?



2. Ezra rolls 3 fair dice. The sum of the three numbers is equal to n with probability p_n. What is the maximum value of p_n?

3. Find the number of polynomials which satisfy $p(0) = 0$ and $p(z^2 + 1) = p(z)^2 + 1$.

4. Evaluate the sum

$$\frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + ... + \frac{1}{2020^2-1}.$$

5. Find the last two digits of $\left(\sum_{n=1}^{2019} n!\right)^{2019}$.

6. There are 2019 lockers in a gym, numbered from 1 to 2019, and they all start closed. There are also 2013 students. On the first day, student #1 toggles the status of lockers $1, 2, 3, ..., 2019$. (Since they were all closed, they are now all open.) On the second day, student #2 toggles the status of lockers $2, 4, 6, ..., 2018$. On the ith day, student i toggles the status of lockers $i, 2i, 3i, ...$ How many lockers are open after all 2019 students have passed through?

7. Find the smallest integer n for which there exist $n \geq 6000$ distinct pairs $(x_1, y_1), ..., (x_n, y_n)$ of positive integers with $1 \leq x_i, y_i \leq 2019$ for $i = 1, 2, ..., n$ such that for any indices $r, s \in \{1, 2, ..., n\}$ (not necessarily distinct), there exists an index $t \in \{1, 2, ..., n\}$ such that 2019 divides $x_r + x_s - x_t$ and $y_r + y_s - y_t$.

8. Consider a 8×8 grid. Several of the tiles have tokens on them and there can only be one token on any tile. Call any tile *Alan* if there exists an adjacent tile with a token on it. What is the minimum number of tokens needed so that all tiles of the chessboard are *Alan*?
(Note: tiles are considered adjacent if and only if they share a side).