1. Let P be a right hexagonal prism. Haveesh drills a square-shaped tunnel through the interior of the hexagonal bases of P. Then, Leo uses a sharpie to triangulate each face of P. Let b_0, b_1, b_2 be the number of vertices, edges, and triangles (respectively) of P. Compute $b_0 - b_1 + b_2$.

2. How many lattice points are there in the interior of the parallelogram with vertices $(0,0)$, $(8,2)$, $(2,6)$ and $(10,8)$?

3. In how many ways can seven people be seated in a circle if two arrangements are considered the same whenever each person has the same neighbors (not necessarily on the same side)?

4. What is the least integer n such that any n pairwise relatively prime integers greater than 1 and less than 2019 contains at least one prime number?

5. Let $f = 1/x$ and $g = (x-1)/x$. Let G be the set of functions which can be written as a finite composition of the functions f and g. For example, $f \circ f \circ f \in G$ and $f \circ g \circ f \in G$. Find the number of elements in G.

6. Compute the infinite product
$$\prod_{n=0}^{\infty} \frac{2017^{2^n}}{2017^{2^n}+1}.$$

7. There is a proper k-coloring of a graph $G(V,E)$ if there is a map $\phi : V \to [k]$ such that whenever $ij \in E$, $\phi(i) \neq \phi(j)$. Let $\chi(G)$ be the smallest positive integer k such that there is a proper k-coloring of G. Let $K(V,E)$ be the graph where $V = \binom{[2019]}{10}$ and $E = \{\{A,B\} : A, B \in V \text{ and } A \cap B = \emptyset\}$. Compute $\chi(K)$.

8. Compute the following sum
$$\sum_{k=1}^{71} \left\lfloor \frac{k^2+k}{71} \right\rfloor.$$

9. Let p_n denote the nth prime number. Compute
$$\lim_{n\to\infty} \frac{p_n}{\sqrt[n]{p_1 p_2 \ldots p_n}}.$$

10. Let K be a field with $|K| = 2017^{2017}$. A polynomial $f \in K[x]$ is *thicc* if every irreducible factor of f has multiplicity at least 2. Let P be the number of monic thicc polynomials of degree 301. Compute the maximum integer k such that $2017^k | P$.

11. Let Ω be the circumcircle of a 13-14-15 triangle XYZ and ω be the incircle of the triangle. For every $A \in \Omega$, there is a triangle ABC having Ω for circumcircle and ω for incircle. Let $\mathcal{L}$ be the locus of the centroid of $\triangle ABC$ as A traverses Ω. Compute the area that $\mathcal{L}$ encloses.

12. Let $S = [10]$. Let G be the union of all closed line segments joining any two elements of $S \times S$ along a vertical or horizontal line, or along a line with slope ± 1. Determine the combined total of the number of (non-degenerate) triangles and rectangles for which all edges are subsets of G.