Theorem 1 (Product Rule)**.** *For processes X_t, Y_t, we have that*

$$d(X_tY_t) = Y_t dX_t + X_t dY_t + d\langle X, Y\rangle_t.$$

Theorem 2 (One-dimensional Ito's Lemma)**.** *Let $X_t = X_0 + M_t + A_t$ be a continuous semimartingale. Then for $f \in C^2(\mathbb{R})$, we have*

$$f(X_t) = f(X_0) + \int_0^t f'(X_s)\, dM_s + \int_0^t f'(X_s)\, dA_s + \frac{1}{2}\int_0^t f''(X_s)\, d\langle M\rangle_s$$

Theorem 3 (Multi-dimensional Ito's Lemma)**.** *Let $X = (X^1, \ldots, X^d) = (X_0^1 + X_t^1 + A_t^1, \ldots, X_0^d + X_t^d + A_t^d)$ be a vector continuos semimartingales. Let $f \in C^2(\mathbb{R}^d)$. Then, for all t, we have that*

$$f(X_t) = f(X_0) + \sum_{i=1}^d \int_0^t \frac{\partial f}{\partial x^i}(X_s)\, dM_s^i + \sum_{i=1}^d \int_0^t \frac{\partial f}{\partial x^i}(X_s)\, dA_s^i + \frac{1}{2}\sum_{i,j=1}^d \int_0^t \frac{\partial^2 f}{\partial x^i \partial x^j}(X_s)\, d\langle M^i, M^j\rangle_s.$$

Theorem 4 (Levy)**.** *If $M \in \mathscr{M}_c^2$ satisfies $\langle M\rangle_t = t$ and $M_0 = 0$, then M is a standard Brownian motion.*

Theorem 5 (Danbis-Dubis-Schwarz, DDS)**.** *Let $M \in \mathscr{M}_c^2$ and $\lim_{t\to\infty}\langle M\rangle_t = \infty$ almost surely. Let $\tau_s := \inf\{t \geq 0 : \langle M\rangle_t = s\}$. Then $B_s := M_{\tau_s}$ is a standard Brownian motion. Also $M_t = B_{\langle M\rangle_t}$ for $t \geq 0$.*

Theorem 6 (Girsanov's Theorem)**.** *Given $(B^{(1)}, \ldots, B^{(m)})$ a m-dimensional standard Brownian motion and $X^{(1)}, \ldots, X^{(m)} \in \mathscr{L}$, define*

$$Z_t(X) := \exp\left(\sum_{i=1}^m \int_0^t X_s^{(i)}\, dB_s^{(i)} - \frac{1}{2}\sum_{i=1}^m \left(X_s^{(i)}\right)^2 ds\right).$$

If $(Z_t(X))_{t\in[0,T]}$ is a martingale, then:

$$(W_t^{(1)}, \ldots, W_t^{(m)}) := \left(B_t^{(1)} - \int_0^t X_s^{(1)}\, ds, \ldots, B_t^{(m)} - \int_0^t X_s^{(m)}\, ds\right)$$

is a standard Brownian motion on $[0,T]$ under $\tilde{\mathbb{P}}$ where

$$\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} = Z_T(X).$$

Theorem 7 (Novikov's Condition)**.** *If $\mathbb{E}\left[e^{\frac{1}{2}\sum_{i=1}^m \int_0^T (X_s^{(i)})^2\, ds}\right] < \infty$, then $Z(X)$ is a martingale on $[0,T]$.*

Theorem 8 (Strong Uniqueness)**.** *If for all $R, T < \infty$, there exists $C := C(R,T)$ such that*

$$|b(t,x) - b(t,y)| \leq C|x-y|$$
$$|\sigma(t,x) - \sigma(t,y)| \leq C|x-y|$$

for all $-R \leq x, y \leq R$ and $t \in [0,T]$, then strong uniqueness holds for

$$dX_t = b(t, X_t)\, dt + \sigma(t, X_t)\, dB_t, \quad X_0 = x_0.$$

Lemma 1 (Gronwell's Lemma). *A function $g : [0, \infty) \to \mathbb{R}$ such that*

$$0 \le g(t) \le \alpha + \beta \int_0^t g(s)\, ds, \quad t \ge 0$$

Then $g(t) \le \alpha e^{\beta t}$, $t \ge 0$. If $\alpha = 0$ then $g \equiv 0$.

Theorem 9 (Yamada-Watanabe). *Assume there is K such that*

$$\begin{aligned} |b(t,x) - b(t,y)| &\le K|x-y| \\ |\sigma(t,x) - \sigma(t,y)| &\le h(|x-y|) \end{aligned}$$

for all t, x, y where h is nondecreasing, $h(0) = 0$ and $\int_0^1 \frac{da}{h(a)^2} = \infty$. Then strong uniqueness holds.

Theorem 10 (Strong Existence). *If there is $C < \infty$ such that*

$$\begin{aligned} |b(t,x) - b(t,y)| &\le C|x-y| \\ |\sigma(t,x) - \sigma(t,y)| &\le C|x-y| \\ |b(t,x)|^2 + |\sigma(t,x)|^2 &\le C(1 + |x|^2) \end{aligned}$$

for all t, x, y then strong existence holds.