Problem 1: Multiples of 3 or 5

Published on Friday, 5th October 2001, 06:00 pm / Solved by 970416 / Difficulty rating: 5%

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get $3, 5, 6$ and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

Problem 2: Even Fibonacci Numbers

Published on Friday, 19th October 2001, 06:00 pm / Solved by 774724 / Difficulty rating: 5%

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be: $$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$$

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

Problem 3: Largest Prime Factor

Published on Friday, 2nd November 2001, 06:00 pm / Solved by 558284 / Difficulty rating: 5%

The prime factors of 13195 are $5, 7, 13$ and 29.

What is the largest prime factor of the number 600851475143?

Problem 4: Largest Palindrome Product

Published on Friday, 16th November 2001, 06:00 pm / Solved by 494370 / Difficulty rating: 5%

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is $9009 = 91 \times 99$.

Find the largest palindrome made from the product of two 3-digit numbers.

Problem 5: Smallest Multiple

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is _evenly divisible_ by all of the numbers from 1 to 20?

Problem 6: Sum Square Difference

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + ... + 10^2 = 385.$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + ... + 10)^2 = 55^2 = 3025.$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Problem 7: 10001st Prime

By listing the first six prime numbers: $2, 3, 5, 7, 11$, and 13, we can see that the 6th prime is 13.

What is the $10\,001$st prime number?

Problem 8: Largest Product in a Series

The four adjacent digits in the 1000-digit number that have the greatest product are $9 \times 9 \times 8 \times 9 = 5832$.

```
7316717653133062491922511967442657474235534919493496983520312774506326239578318016984801869478851843
8586156078911294949545950173795833195285320880551112540698747158523863050715693290963295227443043557
6689664895044524452316173185640309871112172238311362229893423380308135336276614282806444486645238749
3035890729629049156044077239071381051585930796086670172427121883998797908792274921901699720888093776
6572733300105336788122023542180975125454059475224352584907711670556013604839586446706324415722155397
5369781797784617406495514929086256932197846862248283972241375657056057490261407972968652414535100474
8216637048440319989000889524345065854122758866688116427171479924442928230863465674813919123162824586
1786645835912456652947654568284891288314260769004224219022671055626321111093705442175069416589604080
7198403850962455444362981230987879927244284909188845801561660979191338754992005240636899125607176060
5886116467109405077541002256983155200055935729725716362695618826704282524836008232575304207529634500
```

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

Problem 9: Special Pythagorean Triplet

A Pythagorean triplet is a set of three natural numbers, $a \lt b \lt c$, for which, $$a^2 + b^2 = c^2.$$

For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

There exists exactly one Pythagorean triplet for which $a + b + c = 1000$.
Find the product abc.

Problem 10: Summation of Primes

The sum of the primes below 10 is $2 + 3 + 5 + 7 = 17$.

Find the sum of all the primes below two million.

Problem 11: Largest Product in a Grid

In the 20×20 grid below, four numbers along a diagonal line have been marked in red.

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48
```

The product of these numbers is $26 \times 63 \times 78 \times 14 = 1788696$.

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the 20×20 grid?

Problem 12: Highly Divisible Triangular Number

The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. The first ten terms would be: $$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \dots$$

Let us list the factors of the first seven triangle numbers:

$$\begin{align} \mathbf 1 &\colon 1\\ \mathbf 3 &\colon 1,3\\ \mathbf 6 &\colon 1,2,3,6\\ \mathbf{10} &\colon 1,2,5,10\\ \mathbf{15} &\colon 1,3,5,15\\ \mathbf{21} &\colon 1,3,7,21\\ \mathbf{28} &\colon 1,2,4,7,14,28 \end{align}$$

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

Problem 13: Large Sum

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

```
37107287533902102798797998220837590246510135740250
46376937677490009712648124896970078050417018260538
74324986199524741059474233309513058123726617309629
91942213363574161572522430563301811072406154908250
23067588207539346171171980310421047513778063246676
89261670696623363820136378418383684178734361726757
28112879812849979408065481931592621691275889832738
44274228917432520321923589422876796487670272189318
47451445736001306439091167216856844588711603153276
70386486105843025439939619828917593665686757934951
62176457141856560629502157223196586755079324193331
64906352462741904929101432445813822663347944758178
92575867718337217661963751590579239728245598838407
58203565325359399008402633568948830189458628227828
80181199384826282014278194139940567587151170094390
35398664372827112653829987240784473053190104293586
86515506006295864861532075273371959191420517255829
71693888707715466499115593487603532921714970056938
54370070576826684624621495650076471787294438377604
53282654108756828443191190634694037855217779295145
36123272525000296071075082563815656710885258350721
45876576172410976447339110607218265236877223636045
17423706905851860660448207621209813287860733969412
81142660418086830619328460811191061556940512689692
51934325451728388641918047049293215058642563049483
62467221648435076201727918039944693004732956340691
15732444386908125794514089057706229429197107928209
55037687525678773091862540744969844508330393682126
18336384825330154686196124348767681297534375946515
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8038628759287849020152168555482871720121925776954
7818283375799310361474035685644909552709786479758
1672632010043689784255353992093183744149780686098
4840309812907779179908821879532736447567559084803
8708698755139271185451707854416185242432069315033
5995940689575653678210749269665376763262354472102
6979395067965269474259770973916669376304263398708
4105268470829908521139942736573411618276031500127
6537860736150108570091499395125570281987460043753
5829035317434717326932123578154982629742552737307
9495375976510530594696067683156574377167401875275
8890280257173322961917666871381993181104877019027
2526768027607800301367868099252546340106163286652
3627021854049770558562994658063623799314074625596
2407448690823117497779236546625724692332281091714
9143028819710328859780666976089293863828502533340
3441306557801612781592181500556186883646842009047
2305308117281643048762379196984248725503663878458
1148769693215490281042402013833512446218144177347
6378329949063625966649858761822122522551248676453
6772018697169854431241957240991395900895231005882
9554825530026352078153229679624948164195386821877
7608532713228572311042480345612486769706450799523
3777424253541129168427686553892620502491032657296
2370191327572567528565324825826546309220705859652
2979886027225833191312637514734199488953476574550
1849570145487928898485682772607771372140379887971
3829820378303147352772158034814451349137322665138
3482954382919991818027891652243102739225112286953
4095795306640523263253804410005965493915987959363
2974615218550237130764225512118369380358038858490
4169811622207297718615823667842468915799353296192
6246795719440126904387710727504810239089552359745
2318970677254791506150550495392297953090112996751
8618808822587531452958409925120382900940777077567
1130673970830472448381653387350234084564705807730
8295917476714036319800818712901187549131054712658
9762333104481838626951545633492636657289756340050
4284628018351707052783183942588214552122725125032
5512160354698120058176216521282765275169129689778
3223819573432933994643750190783694576588335239988
7550616496518477518073816883786109152735792970133
6217784275219262340194239963916804498399317331273
3292418570714734956691667468763466091503591467750
9951867143023521962889489010242332511691361962662
7326746080059154747183079839286853520694694454072
7684182252467441716151403642798227334805555621481
9714261791034259864720451689398942217982608807685
8778364618279934631376775430780936333018982642090
1048802521674670883215120158883543223812876952786
7132961247478246453863699300904931036361976387803
6218407357239979422340623539380833965132740801111
6662789198148808779794187687614423003098449085141
6066182629368283676474477923918033511098906979071
8578694408955299065364044742557608365997664579509
6602439640990538960712019821997604759949019723029
6491398268003297315603712004137790378556608508925

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16730939319872750275468906903707539413042652315011
94809377245048795150954100921645863754710598436791
78639167021187492431995700641917969777599028300699
15368713711936614952811305876380278410754449733078
40789923115535562561142322423255033685442488917353
44889911501440648020369068063960672322193204149535
41503128880339536053299340368006977710650566631954
81234880673210146739058568557934581403627822703280
82616570773948327592232845941706525094512325230608
22918802058777319719839450180888072429661980811197
77158542502016545090413245809786882778948721859617
72107838435069186155435662884062257473692284509516
20849603980134001723930671666823555245252804609722
53503534226472524250874054075591789781264330331690
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Problem 14: Longest Collatz Sequence

Published on Friday, 5th April 2002, 06:00 pm / Solved by 233116 / Difficulty rating: 5%

The following iterative sequence is defined for the set of positive integers:

$n \to n/2$ (n is even)
$n \to 3n + 1$ (n is odd)

Using the rule above and starting with 13, we generate the following sequence: $$13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1.$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.
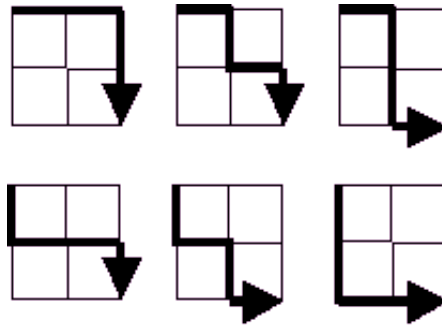
Which starting number, under one million, produces the longest chain?

NOTE: Once the chain starts the terms are allowed to go above one million.

Problem 15: Lattice Paths

Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.

How many such routes are there through a 20×20 grid?

Problem 16: Power Digit Sum

$2^{15} = 32768$ and the sum of its digits is $3 + 2 + 7 + 6 + 8 = 26$.

What is the sum of the digits of the number 2^{1000}?

Problem 17: Number Letter Counts

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are $3 + 3 + 5 + 4 + 4 = 19$ letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

Problem 18: Maximum Path Sum I

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

<pre>
      3
     7 4
    2 4 6
   8 5 9 3
</pre>

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom of the triangle below:

<pre>
                        75
                      95 64
                    17 47 82
                  18 35 87 10
                20 04 82 47 65
              19 01 23 75 03 34
            88 02 77 73 07 63 67
          99 65 04 28 06 16 70 92
        41 41 26 56 83 40 80 70 33
      41 48 72 33 47 32 37 16 94 29
    53 71 44 65 25 43 91 52 97 51 14
  70 11 33 28 77 73 17 78 39 68 17 57
91 71 52 38 17 14 91 43 58 50 27 29 48
63 66 04 68 89 53 67 30 73 16 69 87 40 31
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23
</pre>

NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, **Problem 67**, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

Problem 19: Counting Sundays

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.
- Thirty days has September,
  April, June and November.
  All the rest have thirty-one,
  Saving February alone,
  Which has twenty-eight, rain or shine.
  And on leap years, twenty-nine.
- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

Problem 20: Factorial Digit Sum

$n!$ means $n \times (n - 1) \times \cdots \times 3 \times 2 \times 1$.

For example, $10! = 10 \times 9 \times \cdots \times 3 \times 2 \times 1 = 3628800$,
and the sum of the digits in the number $10!$ is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$.

Find the sum of the digits in the number $100!$.

Problem 21: Amicable Numbers

Let $d(n)$ be defined as the sum of proper divisors of n (numbers less than n which divide evenly into n).
If $d(a) = b$ and $d(b) = a$, where $a \ne b$, then a and b are an amicable pair and each of a and b are called amicable numbers.

For example, the proper divisors of 220 are $1, 2, 4, 5, 10, 11, 20, 22, 44, 55$ and 110; therefore $d(220) = 284$. The proper divisors of 284 are $1, 2, 4, 71$ and 142; so $d(284) = 220$.

Evaluate the sum of all the amicable numbers under 10000.

Problem 22: Names Scores

Published on Friday, 19th July 2002, 06:00 pm / Solved by 139254 / Difficulty rating: 5%

Using **names.txt** (right click and 'Save Link/Target As...'), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth $3 + 15 + 12 + 9 + 14 = 53$, is the 938th name in the list. So, COLIN would obtain a score of $938 \times 53 = 49714$.

What is the total of all the name scores in the file?

Problem 23: Non-Abundant Sums

Published on Friday, 2nd August 2002, 06:00 pm / Solved by 108326 / Difficulty rating: 5%

A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be $1 + 2 + 4 + 7 + 14 = 28$, which means that 28 is a perfect number.

A number n is called deficient if the sum of its proper divisors is less than n and it is called abundant if this sum exceeds n.

As 12 is the smallest abundant number, $1 + 2 + 3 + 4 + 6 = 16$, the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

Problem 24: Lexicographic Permutations

Published on Friday, 16th August 2002, 06:00 pm / Solved by 118903 / Difficulty rating: 5%

A permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

<div align="center">012 021 102 120 201 210</div>

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

Problem 25: 1000-digit Fibonacci Number

The Fibonacci sequence is defined by the recurrence relation:

$F_n = F_{n - 1} + F_{n - 2}$, where $F_1 = 1$ and $F_2 = 1$.

Hence the first 12 terms will be:

$$\begin{align} F_1 &= 1\\ F_2 &= 1\\ F_3 &= 2\\ F_4 &= 3\\ F_5 &= 5\\ F_6 &= 8\\ F_7 &= 13\\ F_8 &= 21\\ F_9 &= 34\\ F_{10} &= 55\\ F_{11} &= 89\\ F_{12} &= 144 \end{align}$$

The 12th term, F_{12}, is the first term to contain three digits.

What is the index of the first term in the Fibonacci sequence to contain 1000 digits?

Problem 26: Reciprocal Cycles

A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$$\begin{align} 1/2 &= 0.5\\ 1/3 &=0.(3)\\ 1/4 &=0.25\\ 1/5 &= 0.2\\ 1/6 &= 0.1(6)\\ 1/7 &= 0.(142857)\\ 1/8 &= 0.125\\ 1/9 &= 0.(1)\\ 1/10 &= 0.1 \end{align}$$

Where $0.1(6)$ means $0.166666\cdots$, and has a 1-digit recurring cycle. It can be seen that $1/7$ has a 6-digit recurring cycle.

Find the value of $d \lt 1000$ for which $1/d$ contains the longest recurring cycle in its decimal fraction part.

Problem 27: Quadratic Primes

Euler discovered the remarkable quadratic formula:

$$n^2 + n + 41$$

It turns out that the formula will produce 40 primes for the consecutive integer values $0 \le n \le 39$. However, when $n = 40, 40^2 + 40 + 41 = 40(40 + 1) + 41$ is divisible by 41, and certainly when $n = 41, 41^2 + 41 + 41$ is clearly divisible by 41.

The incredible formula $n^2 - 79n + 1601$ was discovered, which produces 80 primes for the consecutive values $0 \le n \le 79$. The product of the coefficients, -79 and 1601, is -126479.

Considering quadratics of the form:

$n^2 + an + b$, where $|a| < 1000$ and $|b| \le 1000$

where $|n|$ is the modulus/absolute value of n
e.g. $|11| = 11$ and $|-4| = 4$

Find the product of the coefficients, a and b, for the quadratic expression that produces the maximum number of primes for consecutive values of n, starting with $n = 0$.

Problem 28: Number Spiral Diagonals

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

```
21 22 23 24 25
20  7  8  9 10
19  6  1  2 11
18  5  4  3 12
17 16 15 14 13
```

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals in a 1001 by 1001 spiral formed in the same way?

Problem 29: Distinct Powers

Consider all integer combinations of a^b for $2 \le a \le 5$ and $2 \le b \le 5$:

$$\begin{matrix} 2^2=4, &2^3=8, &2^4=16, &2^5=32\\ 3^2=9, &3^3=27, &3^4=81, &3^5=243\\ 4^2=16, &4^3=64, &4^4=256, &4^5=1024\\ 5^2=25, &5^3=125, &5^4=625, &5^5=3125 \end{matrix}$$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms: $$4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125.$$

How many distinct terms are in the sequence generated by a^b for $2 \le a \le 100$ and $2 \le b \le 100$?

Problem 30: Digit Fifth Powers

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits: $$\begin{align} 1634 &= 1^4 + 6^4 + 3^4 + 4^4\\ 8208 &= 8^4 + 2^4 + 0^4 + 8^4\\ 9474 &= 9^4 + 4^4 + 7^4 + 4^4 \end{align}$$

As $1 = 1^4$ is not a sum it is not included.

The sum of these numbers is $1634 + 8208 + 9474 = 19316$.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

Problem 31: Coin Sums

In the United Kingdom the currency is made up of pound (£) and pence (p). There are eight coins in general circulation:

   1p, 2p, 5p, 10p, 20p, 50p, £1 (100p), and £2 (200p).

It is possible to make £2 in the following way:

   1×£1 + 1×50p + 2×20p + 1×5p + 1×2p + 3×1p

How many different ways can £2 be made using any number of coins?

Problem 32: Pandigital Products

We shall say that an n-digit number is pandigital if it makes use of all the digits 1 to n exactly once; for example, the 5-digit number, 15234, is 1 through 5 pandigital.

The product 7254 is unusual, as the identity, $39 \times 186 = 7254$, containing multiplicand, multiplier, and product is 1 through 9 pandigital.

Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through 9 pandigital.

HINT: Some products can be obtained in more than one way so be sure to only include it once in your sum.

Problem 33: Digit Cancelling Fractions

The fraction $49/98$ is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that $49/98 = 4/8$, which is correct, is obtained by cancelling the 9s.

We shall consider fractions like, $30/50 = 3/5$, to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.

Problem 34: Digit Factorials

145 is a curious number, as $1! + 4! + 5! = 1 + 24 + 120 = 145$.

Find the sum of all numbers which are equal to the sum of the factorial of their digits.

Note: As $1! = 1$ and $2! = 2$ are not sums they are not included.

Problem 35: Circular Primes

The number, 197, is called a circular prime because all rotations of the digits: 197, 971, and 719, are themselves prime.

There are thirteen such primes below 100: $2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79$, and 97.

How many circular primes are there below one million?

Problem 36: Double-base Palindromes

The decimal number, $585 = 1001001001_2$ (binary), is palindromic in both bases.

Find the sum of all numbers, less than one million, which are palindromic in base 10 and base 2.

(Please note that the palindromic number, in either base, may not include leading zeros.)

Problem 37: Truncatable Primes

The number 3797 has an interesting property. Being prime itself, it is possible to continuously remove digits from left to right, and remain prime at each stage: 3797, 797, 97, and 7. Similarly we can work from right to left: 3797, 379, 37, and 3.

Find the sum of the only eleven primes that are both truncatable from left to right and right to left.

NOTE: 2, 3, 5, and 7 are not considered to be truncatable primes.

Problem 38: Pandigital Multiples

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Take the number 192 and multiply it by each of 1, 2, and 3:

$$\begin{align} 192 \times 1 &= 192\\ 192 \times 2 &= 384\\ 192 \times 3 &= 576 \end{align}$$

By concatenating each product we get the 1 to 9 pandigital, 192384576. We will call 192384576 the concatenated product of 192 and $(1,2,3)$.

The same can be achieved by starting with 9 and multiplying by 1, 2, 3, 4, and 5, giving the pandigital, 918273645, which is the concatenated product of 9 and $(1,2,3,4,5)$.

What is the largest 1 to 9 pandigital 9-digit number that can be formed as the concatenated product of an integer with $(1,2, \dots, n)$ where $n \gt 1$?

Problem 39: Integer Right Triangles

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If p is the perimeter of a right angle triangle with integral length sides, $\{a, b, c\}$, there are exactly three solutions for $p = 120$.

$\{20,48,52\}$, $\{24,45,51\}$, $\{30,40,50\}$

For which value of $p \le 1000$, is the number of solutions maximised?

Problem 40: Champernowne's Constant

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An irrational decimal fraction is created by concatenating the positive integers:
$$0.12345678910{\color{red}\mathbf 1}112131415161718192021\cdots$$

It can be seen that the 12th digit of the fractional part is 1.

If d_n represents the nth digit of the fractional part, find the value of the following expression.
$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

Problem 41: Pandigital Prime

We shall say that an n-digit number is pandigital if it makes use of all the digits 1 to n exactly once. For example, 2143 is a 4-digit pandigital and is also prime.

What is the largest n-digit pandigital prime that exists?

Problem 42: Coded Triangle Numbers

The nth term of the sequence of triangle numbers is given by, $t_n = \frac12n(n+1)$; so the first ten triangle numbers are: $$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \dots$$

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is $19 + 11 + 25 = 55 = t_{10}$. If the word value is a triangle number then we shall call the word a triangle word.

Using **words.txt** (right click and 'Save Link/Target As...'), a 16K text file containing nearly two-thousand common English words, how many are triangle words?

Problem 43: Sub-string Divisibility

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property.

Let d_1 be the 1st digit, d_2 be the 2nd digit, and so on. In this way, we note the following:

- $d_2d_3d_4=406$ is divisible by 2
- $d_3d_4d_5=063$ is divisible by 3
- $d_4d_5d_6=635$ is divisible by 5
- $d_5d_6d_7=357$ is divisible by 7
- $d_6d_7d_8=572$ is divisible by 11
- $d_7d_8d_9=728$ is divisible by 13
- $d_8d_9d_{10}=289$ is divisible by 17

Find the sum of all 0 to 9 pandigital numbers with this property.

Problem 44: Pentagon Numbers

Pentagonal numbers are generated by the formula, $P_n=n(3n-1)/2$. The first ten pentagonal numbers are: $$1, 5, 12, 22, 35, 51, 70, 92, 117, 145, \dots$$

It can be seen that $P_4 + P_7 = 22 + 70 = 92 = P_8$. However, their difference, $70 - 22 = 48$, is not pentagonal.

Find the pair of pentagonal numbers, P_j and P_k, for which their sum and difference are pentagonal and $D = |P_k - P_j|$ is minimised; what is the value of D?

Problem 45: Triangular, Pentagonal, and Hexagonal

Triangle, pentagonal, and hexagonal numbers are generated by the following formulae:

Triangle	$T_n=n(n+1)/2$	$1, 3, 6, 10, 15, \dots$
Pentagonal	$P_n=n(3n - 1)/2$	$1, 5, 12, 22, 35, \dots$
Hexagonal	$H_n=n(2n - 1)$	$1, 6, 15, 28, 45, \dots$

It can be verified that $T_{285} = P_{165} = H_{143} = 40755$.

Find the next triangle number that is also pentagonal and hexagonal.

Problem 46: Goldbach's Other Conjecture

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$$\begin{align} 9 &= 7 + 2 \times 1^2\\ 15 &= 7 + 2 \times 2^2\\ 21 &= 3 + 2 \times 3^2\\ 25 &= 7 + 2 \times 3^2\\ 27 &= 19 + 2 \times 2^2\\ 33 &= 31 + 2 \times 1^2 \end{align}$$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

Problem 47: Distinct Primes Factors

The first two consecutive numbers to have two distinct prime factors are:

$$\begin{align} 14 &= 2 \times 7\\ 15 &= 3 \times 5. \end{align}$$

The first three consecutive numbers to have three distinct prime factors are:

$$\begin{align} 644 &= 2^2 \times 7 \times 23\\ 645 &= 3 \times 5 \times 43\\ 646 &= 2 \times 17 \times 19. \end{align}$$

Find the first four consecutive integers to have four distinct prime factors each. What is the first of these numbers?

Problem 48: Self Powers

The series, $1^1 + 2^2 + 3^3 + \cdots + 10^{10} = 10405071317$.

Find the last ten digits of the series, $1^1 + 2^2 + 3^3 + \cdots + 1000^{1000}$.

Problem 49: Prime Permutations

The arithmetic sequence, $1487, 4817, 8147$, in which each of the terms increases by 3330, is unusual in two ways: (i) each of the three terms are prime, and, (ii) each of the 4-digit numbers are permutations of one another.

There are no arithmetic sequences made up of three 1-, 2-, or 3-digit primes, exhibiting this property, but there is one other 4-digit increasing sequence.

What 12-digit number do you form by concatenating the three terms in this sequence?

Problem 50: Consecutive Prime Sum

The prime 41, can be written as the sum of six consecutive primes:

$$41 = 2 + 3 + 5 + 7 + 11 + 13.$$

This is the longest sum of consecutive primes that adds to a prime below one-hundred.

The longest sum of consecutive primes below one-thousand that adds to a prime, contains 21 terms, and is equal to 953.

Which prime, below one-million, can be written as the sum of the most consecutive primes?