(1) Consider an embedded system with an ADC sample rate of 80 kHz. Give the type and parameters for an antialiasing filter that passes all frequencies up to 8 kHz. with minimal distortion in the passband.

(2) Given a DT system described by the difference equation

$$y[n+2] + \frac{1}{2} y[n+1] = x[n+2]$$

Find the step response of the system by any valid method.

Soln using Z transform.

$$z^2 Y(z) + \frac{1}{2} z Y(z) = z^2 X(z)$$

$$Y(z) = \frac{z^2}{z(z+\frac{1}{2})} X(z) = \frac{z}{z+\frac{1}{2}} X(z)$$

$$X(z) = \frac{z}{z-1} \quad \text{thus} \quad Y(z) = \frac{z^2}{(z+\frac{1}{2})(z-1)}$$

$$\frac{Y(z)}{z} = \frac{A}{z+\frac{1}{2}} + \frac{B}{z-1} = \frac{z}{(z+\frac{1}{2})(z-1)}$$

$$A = \left.\frac{z}{z-1}\right|_{z=-1/2} = \frac{-1/2}{-1/2-1} = \frac{-1/2}{-3/2} = \frac{1}{3}$$

$$B = \left.\frac{z}{z+\frac{1}{2}}\right|_{z=1} = \frac{1}{1+1/2} = \frac{2}{3}$$

$$Y(z) = \frac{Az}{z+1/2} + \frac{Bz}{z-1} \qquad y[n] = \frac{1}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{2}{3} u[n]$$

3 Given LCCDE

$$\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = x(t)$$

a) Find the TF of the system

Soln: Take Laplace transform.

$$s^3Y(s) + 7s^2Y(s) + 4sY(s) + Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3+7s^2+4s+1}$$

b) Determine if the system is Stable.

Soln: By Routh-Hurwitz

$$\frac{7\cdot 4 - 1}{7} = \frac{27}{7}$$

s^3	1	4	0
s^2	7	1	
s	$\frac{27}{7}$	0	
1	1		

No sign change ∴ stable.

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4 Given TF for DT System $H(z) = \dfrac{z^2+z-1}{z^2-3z+2}$

Find corresponding LCCDE in recursive form.

Soln: $$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2+z-1}{z^2-3z+2}$$

$$(z^2-3z+2)Y(z) = (z^2+z-1)X(z)$$

$$Y(z) - 3z^{-1}Y(z) + 2z^{-2}Y(z) = X(z) + z^{-1}X(z) - z^{-2}X(z)$$

$$y[n] - 3y[n-1] + 2y[n-2] = x[n] + x[n-1] - x[n-2]$$

$$y[n] = 3y[n-1] - 2y[n-2]\ x[n] + x[n-1] - x[n-2]$$

(5) Given impulse $h(t) = u(-t) + e^{-t}u(t)$

Find Laplace transform, includ ROC, if it exists. If it does not exist. State why.

Soln: we note this is Not a causal signal.

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt = \int_{-\infty}^{0} e^{-st}\,dt + \int_{0}^{\infty} e^{-t}e^{-st}\,dt$$

$$= -\frac{1}{s}e^{-st}\Big|_{-\infty}^{0} + \frac{-1}{s+1}e^{-(s+1)t}\Big|_{0}^{\infty}$$

$$= -\frac{1}{s} + \lim_{T\to-\infty}\frac{1}{s}e^{-sT} + \lim_{T\to\infty} -\frac{1}{s+1}e^{-(s+1)T} + \frac{1}{s+1}$$

$$= -\frac{1}{s} + \underset{Re\{s\}<0}{0} + \underset{Re\{s\}>-1}{0} + \frac{1}{s+1}$$

$$= -\frac{1}{s} + \frac{1}{s+1} \quad -1 < Re(s) < 0$$

$$= \frac{-s-1+s}{s(s+1)} = \frac{-1}{s(s+1)} \quad -1 < Re(s) < 0.$$

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(6) Give TF $H(s) = \dfrac{s}{s^3 + 5s^2 + s - 1}$

Find FR. $H(j\omega)$ if it exists. If it does Not exist state why.

Soln: Check stability: since sign change, system is unstable and FR does NOT exist.

(7) Compare to Find FR of TF = $\dfrac{s}{s^3+5s^2+s+1}$

Solution: Check stability

s^3	1	1
s^2	5	1
s	4/5	0
s^0	1	

No sign change: stable.

$$H(j\omega) = H(s)\Big|_{s=j\omega} = \frac{j\omega}{(j\omega)^3+5(j\omega)^2+(j\omega)+1}$$

$$= \frac{j\omega}{-j\omega^3-5\omega^2+j\omega+1}$$

$$= \frac{j\omega}{1-5\omega^2+j\omega(1-\omega^2)}$$

What is Gain @ DC: at $\omega=0$ $|H(j\omega)| = \dfrac{|0|}{|1|} = 0$

What is response to input $x(t)=\cos(100t)$?

$$y(t) = |H(j100)| \cos(100t + \angle H(100))$$

$$|H(j100)| = \frac{|j100|}{|1-5(100)^2+j(100-100^3)|}$$

$$\angle H(j100) = \tan^{-1}\left(\frac{100}{0}\right) - \tan^{-1}\left(\frac{100(1-100^2)}{1-5\cdot 100^2}\right)$$

(8) Given $X(z) = \dfrac{z}{z+1/2} - \dfrac{3z}{z-3/4}$ $\quad |z| > 3/4$ (5)

Find $x[n]$ using definition of inverse Z transform.

soln: write $X(z) = \dfrac{z(z-3/4) - 3z(z+1/2)}{(z+1/2)(z-3/4)}$

$$= \frac{z^2 - \frac{3}{4}z - 3z^2 - \frac{3}{2}z}{(z+1/2)(z-3/4)} = \frac{-2z^2 - \frac{9}{4}z}{(z+1/2)(z-3/4)}$$

since ROC corresponds to causal signal.

$$\underset{n \geq 0}{x[n]} = \frac{1}{2\pi j}\oint X(z) z^{n-1}\, dz$$

$$= \frac{1}{2\pi j}\oint \frac{(-2z^2 - \frac{9}{4}z) z^n}{z(z+1/2)(z-3/4)}\, dz$$

$$= \frac{1}{2\pi j}\oint \frac{(-2z - 9/4) z^n}{(z+1/2)(z-3/4)}\, dz$$

$$= \frac{1}{2\pi j} \cdot 2\pi j \,[K_1 + K_2]$$

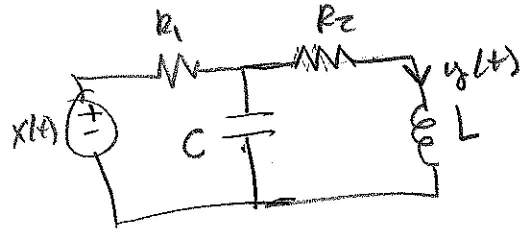
$$K_1 = \left.\frac{(-2z - 9/4) z^n}{z - 3/4}\right|_{z=-1/2} = \frac{(1 - 9/4)(-\frac{1}{2})^n}{-5/4} = \frac{-5/4}{-5/4} = 1\left(-\tfrac{1}{2}\right)^n$$

$$K_2 = \left.\frac{(-2z - 9/4) z^n}{z + 1/2}\right|_{z=3/4} = \frac{(-\frac{3}{2} - \frac{9}{4})(\frac{3}{4})^n}{\frac{3}{4} + \frac{1}{2}} = \frac{-\frac{15}{4}}{\frac{5}{4}}\left(\tfrac{3}{4}\right)^n$$

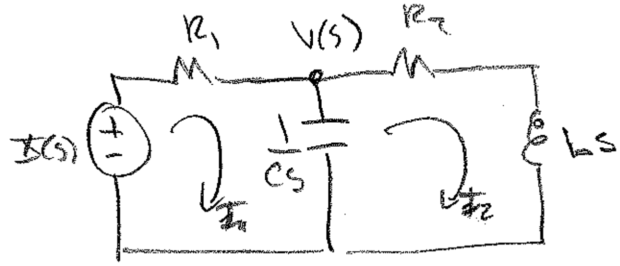
$$= -3\left(\tfrac{3}{4}\right)^n$$

Thus $x[n] = \left(-\tfrac{1}{2}\right)^n u[n] - 3\left(\tfrac{3}{4}\right)^n u[n]$

(9) Given circuit.



Find TF of system.



Soln:

$X(s) = R_1 I_1(s) + V(s)$ (Eq 1)

$V(s) = R_2 I_2(s) + Ls\, I_2(s)$ (Eq 2)

$V(s) = \frac{1}{Cs}(I_1(s) - I_2(s))$ (Eq 3)

3 Eq + 3 unknowns ($V(s), I_1(s), I_2(s)$)

$Y(s) = I_2(s)$.

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(10) Given LCCDE

$$\frac{d^3y}{dt^3} + \frac{2d^2y}{dt^2} + 4\frac{dy}{dt} + 5y(t) = x(t)$$

Find State Space Description.

Soln Let $q_1(t) = y(t)$ $q_2(t) = \frac{dy}{dt}$ $q_3 = \frac{d^2y}{dt^2}$

Then $\dot{q}_3 + 2q_3 + 4q_2 + 5q_1 = x$

or $\dot{q}_3 = -5q_1 - 4q_2 - 2q_3 + x$

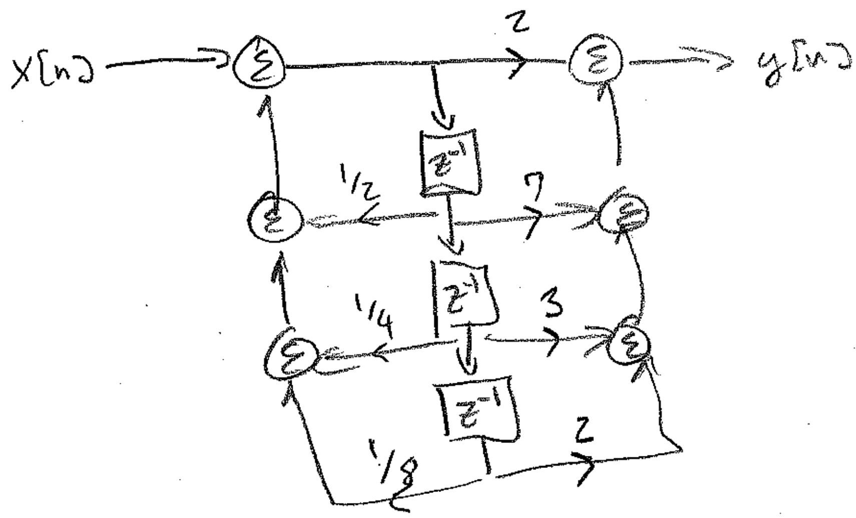
$\dot{q}_2 = q_3$

$\dot{q}_1 = q_2$

Thus $\dot{q} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -4 & -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x$

$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} q$

(11) Given BD.



Find TF $H(z)$

Soln:

$$Q(z) = X(z) + \tfrac{1}{2} z^{-1} Q(z) + \tfrac{1}{4} z^{-2} Q(z) + \tfrac{1}{8} z^{-3} Q(z)$$

$$Y(z) = 2Q(z) + 7z^{-1}Q(z) + 3z^{-2}Q(z) + 2z^{-3}Q(z)$$

$$\frac{Q(z)}{X(z)} = \frac{1}{1 - \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$\frac{Y(z)}{Q(z)} = \frac{2 + 7z^{-1} + 3z^{-2} + 2z^{-3}}{1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{Q(z)} \cdot \frac{Q(z)}{X(z)}$$

$$= \frac{2 + 7z^{-1} + 3z^{-2} + 2z^{-3}}{1 - \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$= \frac{2z^3 + 7z^2 + 3z + 2}{z^3 - \frac{1}{2}z^2 - \frac{1}{4}z - \frac{1}{8}}$$