CHUNG CHENG HIGH SCHOOL (MAIN)
Sec 3 Additional Mathematics 2024
Weighted Assessment 1

Chapter 1: Quadratic Functions
Chapter 2: Equations and Inequalities

30

Name: SOLUTIO ( ) Date: ______________

Class: ______________

Duration: **45 minutes** Parent's Signature: ______________

INSTRUCTIONS:
Answer all questions.
Omission of any essential working will result in loss of marks.
The number of marks is given in the brackets [ ] at the end of each question or part question.

1 Solve the simultaneous equations

$$y = 4 - 2x,$$
$$y^2 - 2x^2 = 16.$$

[4]

$y = 4 - 2x$ --- (1)

$y^2 - 2x^2 = 16$ --- (2)

Subst. (1) into (2):

$(4-2x)^2 - 2x^2 = 16$

$16 - 16x + 4x^2 - 2x^2 = 16$

$2x^2 - 16x = 0$

$2x(x-8) = 0$

$x = 0$ or $x = 8$

$y = 4 \qquad y = -12$

$\therefore x = 0$ and $y = 4$

or

$x = 8$ and $y = -12$

2 **(a)** Find the x-intercepts of the graph of $y=2(3-x)(x+1)$. [1]

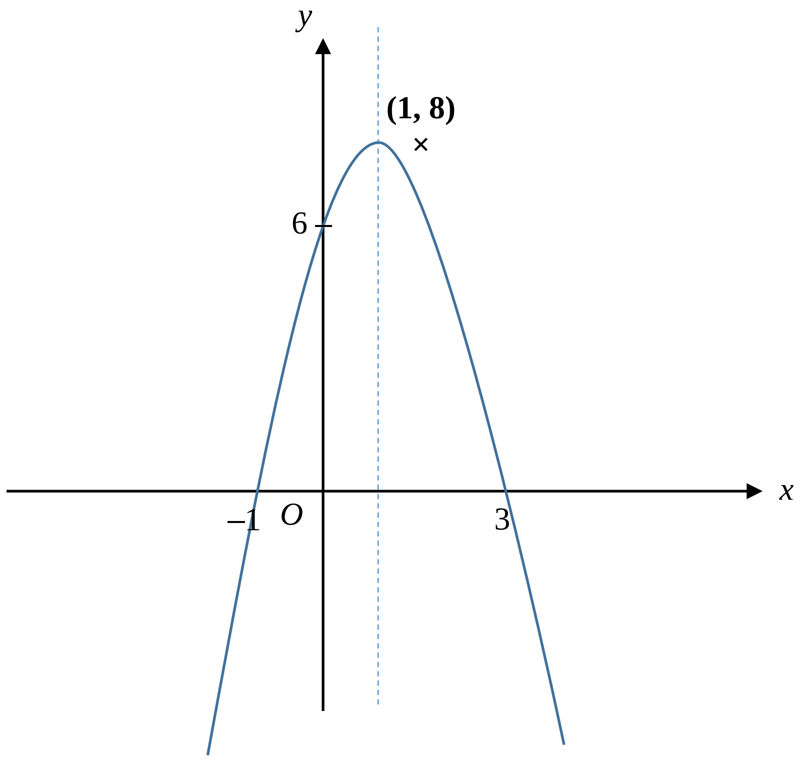
x-

$$2(3-x)(x+1)=0$$
$$(3-x)(x+1)=0$$
$$x=3 \text{ or } x=-1$$

(b) Sketch the graph of $y=2(3-x)(x+1)$ on the axes below.

Indicate clearly the values where the graph crosses the x- and y- axes. [2]

y-intercept:
When $x=0$,
$y=2(3)(1)=6$



(c) Another graph with equation $y=a(x+1)(x-3)$, where $a>0$, is given.

A line drawn passes through both the turning point of the graph of $y=2(3-x)(x+1)$ and the turning point of this graph.

(i) Write down the equation of the line. [1]

$$x=\frac{-1+3}{2}$$
$$x=1$$

(ii) Explain your answer in **part (i)**.

Since the new graph has the **same x-intercepts**, they have the same line of symmetry which passes through their turning point.

……………………………………………………………………………………[1]

3 Find the range of values of m for which the equation $x^2-2x+4m-5=0$ has real and distinct roots. [3]

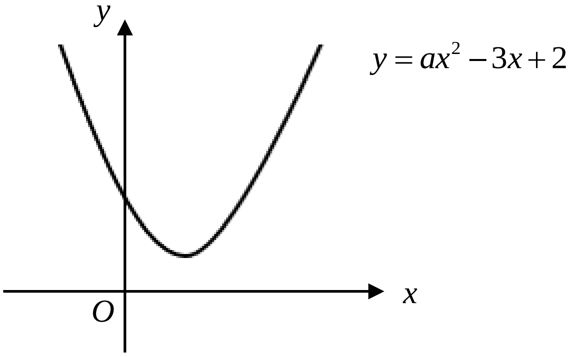
$x^2-2x+4m-5=0$

$a=1,\ b=-2,\ c=4m-5$

For real and distinct roots,

$$b^2-4ac>0$$
$$(-2)^2-4(1)(4m-5)>0$$
$$4-16m+20>0$$
$$-16m>-24$$
$$m<\frac{3}{2}$$

4 The graph of $y=ax^2-3x+2$, where a is a constant, is given below.



(a) What condition(s) must apply to the constant a? [1]

$a>0$ and

$$b^2-4ac<0$$
$$(-3)^2-4a(2)<0$$
$$9-8a<0$$
$$-8a<-9$$
$$a>\frac{9}{8}$$

The same graph can also be described by the equation $y=a(x-h)^2+k$.

(b) Without solving for h and k, explain why $0 < k < 2$.

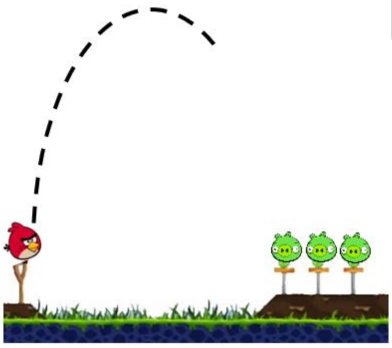
Since the turning point of the curve is above the x-axis, $k > 0$.

Since the turning point of the curve is below the y-intercept $= 2$, $k < 2$.

Therefore, $0 < k < 2$. [2]

5 Angry Birds was once a popular game related to projectile motion.

The picture on the right shows an Angry Bird being launched into the air using a catapult. Three pigs are placed at least 10 metres horizontally away from the catapult.



The path that the Angry Bird follows can be modelled by the equation

$$y = -\frac{1}{10}x^2 + \frac{9}{5}x + \frac{6}{5},$$

where x metres is the horizontal distance from the catapult and y metres is the corresponding height measured from the ground.

(i) Find the height of the Angry Bird just before it was launched into the air. [1]

Let $x = 0$,

$$y = \frac{6}{5}$$

The height of the Angry Bird is 1.2m.

(ii) Express $y = -\frac{1}{10}x^2 + \frac{9}{5}x + \frac{6}{5}$ in the form $y = a(x-h)^2 + k$, where a, h and k are constants. [3]

$$\begin{aligned}
y &= -\frac{1}{10}x^2 + \frac{9}{5}x + \frac{6}{5} \\
&= -\frac{1}{10}\left(x^2 - 18x\right) + \frac{6}{5} \\
&= -\frac{1}{10}\left(x^2 - 18x + 9^2 - 9^2\right) + \frac{6}{5} \\
&= -\frac{1}{10}\left[(x-9)^2 - 81\right] + \frac{6}{5} \\
&= -\frac{1}{10}(x-9)^2 + \frac{81}{10} + \frac{6}{5} \\
&= -\frac{1}{10}(x-9)^2 + \frac{93}{10}
\end{aligned}$$

(iii) Hence, write down the greatest height reached by the Angry Bird and the corresponding horizontal distance travelled. [2]

Greatest height = $\frac{93}{10}$m or $9\frac{3}{10}$m or 9.3m

Corresponding horizontal distance travelled = 9m

The Angry Bird hits one of the basic pigs at a height of 2m from the ground.

(iv) Find the horizontal distance of this basic pig from the Angry Bird just before it was launched into the air. [4]

$$-\frac{1}{10}(x-9)^2+\frac{93}{10}=2$$

$$-\frac{1}{10}(x-9)^2=-\frac{73}{10}$$

$$(x-9)^2=73$$

$$x-9=\pm\sqrt{73}$$

$$x=9\pm\sqrt{73}$$

$$=17.5 \text{ (3 s.f.) or } 0.456 \text{ (3 s.f.) (rej.} \because x\geq 10)$$

The basic pig is at a horizontal distance of 17.5 m from the Angry Bird.

Alternative:

$$-\frac{1}{10}x^2+\frac{9}{5}x+\frac{6}{5}=2$$

$$-\frac{1}{10}x^2+\frac{9}{5}x-\frac{4}{5}=0$$

$$x=\frac{-\frac{9}{5}\pm\sqrt{\left(\frac{9}{5}\right)^2-4\left(-\frac{1}{10}\right)\left(-\frac{4}{5}\right)}}{2\left(-\frac{1}{10}\right)}$$

$$=\frac{-\frac{9}{5}\pm\sqrt{\frac{73}{23}}}{-\frac{1}{5}}$$

$$x=17.5 \text{ (3 sf) or } 0.456 \text{ (3 sf) (rej.} \because x\geq 10)$$

$\therefore$ The basic pig is at a horizontal distance of **17.5 m** from the Angry Bird.

6 The equation of a curve is $y = px^2 + (2p+2)x + p$, where p is a constant.

Find the range of values of p for which the line $y = 2x - 3$ does not intersect the curve. [3]

$$px^2 + (2p+2)x + p = 2x - 3$$
$$px^2 + 2px + p + 3 = 0$$

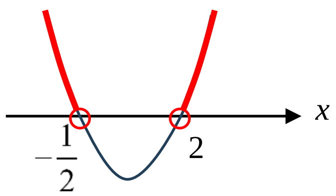
For the line not to intersect the curve,

$$b^2 - 4ac < 0$$
$$(2p)^2 - 4(p)(p+3) < 0$$
$$4p^2 - 4p^2 - 12p < 0$$
$$p > 0$$

7 Find the range of values of x for which

$$2x^2 - 3x > 2.$$ [2]

$$2x^2 - 3x - 2 > 0$$
$$(2x+1)(x-2) > 0$$



$$x < -\frac{1}{2} \text{ or } x > 2$$