Simulation of an Induction Machine with Machine Variables

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***Abstract*—The induction machine is a workhorse of the power industry whether it may be through the generation of power or the production of mechanical torque, they are undeniably one of the most mature electromechanical machines in this present day. It is therefore, imperative to establish methods of analysis that utilize modern understanding of the physical world through concepts such as Newtons and Faradays Law to establish the relationship between flux linkage and mechanical torque. A 3 horsepower and a 7 horsepower machine are then modeled in Simulink to obtain graphical representation of the aforementioned relationship which simplifies the analysis of the machine. Parameters are then modified to observe the impact on the induction machines, as will be demonstrated in the results.**

I. Introduction

In this project, induction machines are modeled using machine variables to establish relationships between currents, flux linkages, torque, and voltages in MATLAB's Simulink environment. Two machines have been selected for analysis in order to observe the difference in the amount of inrush current that must energize the windings and subsequently how the inertia of a machine influences the time to reach steady state operation.

II. Motivation

Induction machines are widely used to convert electric power to mechanical work. They are also used as the controlled driver motor in vehicles, air conditioning systems, and wind turbines [1]. On a smaller scale, induction motors are used in household appliances such as refrigerators, ACs, laundry machines, etc. Understanding these basic machines is important since they are the basis for more complicated induction machines. It is also important to understand how to model equations and devices using Simulink since it helps simplify the analysis and provides a graphical view of the results rather than just numerical results.

III. Methodology

A 3 HP and a 7HP induction machines are modeled, each with their respectives parameters, listed in Table I. Where $V_{rms(l-l)}$ is the RMS Line to Line Voltage applied to stator, ω_B represents that base speed of excitation on the machine, r_s is the per phase resistance of the stator winding, X'_{lr} is the referred rotor leakage reactance, r'_r is the per phase referred rotor winding resistance, X_{ls} represents the stator leakage reactance, X_M is the magnetized mutual reactance, J is the inertia of the machine and B is the rotor damping coefficient.

TABLE I
Machine Parameters

Parameters	3 HP	7 HP	Units
$V_{rms(l-l)}$	220	220	V
Poles	4	4	-
Base Speed ω_B	377	377	rad/s
r_s	0.45	0.3	Ω
X'_{lr}	0.75	0.27	Ω
r'_r	0.8	0.15	Ω
X_{ls}	0.75	0.57	Ω
X_M	27	20	Ω
J	0.09	0.25	$kg\text{-}m^2$
B	10×10^{-6}	10×10^{-6}	kg/s

In an induction machine, the electrical variables such as voltage, current, resistance, and flux linkage are related through Faradays Law, shown in Equations 1 and 2.

$$\vec{v}_{abcs} = r_s \vec{i}_{abcs} + \frac{d\vec{\lambda}_{abcs}}{dt} \tag{1}$$

$$\vec{v}_{abcr} = r'_r \vec{i'}_{abcr} + \frac{d\vec{\lambda'}_{abcr}}{dt} \tag{2}$$

For a magnetically linear machine, the flux linkage is expressed in Equation 3, where the interaction between inductances and currents are shown.

$$\begin{bmatrix} \vec{\lambda}_{abcs} \\ \vec{\lambda'}_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L'_{sr} \\ {L'_{sr}}^T & L'_r \end{bmatrix} \begin{bmatrix} \vec{i}_{abcs} \\ \vec{i'}_{abcr} \end{bmatrix} \tag{3}$$

Torque represents the energy that was transferred from the electrical side to the mechanical side through the coupling field, which is found in terms of stator and rotor currents, magnetic inductance, number of poles and angular displacement of rotor.

$$T_e = \left(\frac{P}{2}\right) L_{ms}\Big\{ \sin\theta_r \Big[i_{as}(i'_{ar} - \frac{1}{2}i'_{br} - \frac{1}{2}i'_{cr}) + i_{bs}(i'_{br} - \frac{1}{2}i'_{ar} - \frac{1}{2}i'_{cr}) + i_{cs}(i'_{cr} - \frac{1}{2}i'_{br} - \frac{1}{2}i'_{ar}) \Big] + \frac{\sqrt{3}}{2}\cos\theta_r \Big[i_{as}(i'_{br} - i'_{cr}) + i_{bs}(i'_{cr} - i'_{ar}) + i_{cs}(i'_{ar} - i'_{br}) \Big] \Big\} \tag{4}$$

Newton's Law is then applied to find an equation for the angular displacement and velocity in terms of electric torque, number of poles, inertia and damping as shown in Equation 5.

$$T_e = J\left(\frac{2}{P}\right)\frac{d\omega_r}{dt} + B\omega_r + T_L \tag{5}$$

In the Experimental Model section, how these equations were specifically manipulated to produce the proper results is explained.

IV. Experimental Model

In Figure 1, a Simulink block diagram is presented with six submodules of interest: Stator Faraday's Law, Rotor Faraday's Law, State Space Equations, Power Calculations, Newton's Law and Export Data to Workspace.

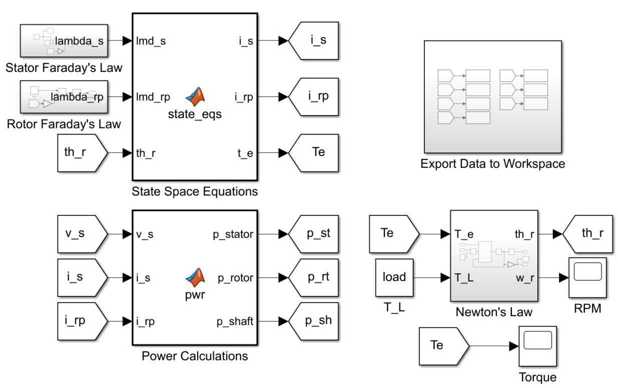

Fig. 1. Simulink block diagram used to model an induction motor

Stator Faraday's Law utilizes Equation 1 to calculate the flux linkages from the stator side $\vec{\lambda}_{abcs}$ by rearranging it to the following form shown in Equation 6.

$$\vec{\lambda}_{abcs} = \int \vec{v}_{abcs} - r_s\vec{i}_{abcs}\ dt \tag{6}$$

Rotor Faraday's Law manipulates Equation 2 to the form shown in Equation 7 to calculate $\vec{\lambda'}_{abcr}$.

$$\vec{\lambda'}_{abcr} = \int \vec{v}_{abcr} - r'_r\vec{i'}_{abcr}\ dt \tag{7}$$

The output from both Stator and Rotor Faraday's Law are inputted into the State Space Equations submodule, which contains the following: Matrix 8 where the diagonal elements is the summation of the stator leakage and mutual inductance and the off diagonal elements are negative half of the mutual inductance,

$$L_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \tag{8}$$

Matrix 9 where the diagonal elements is the summation of the referred rotor leakage and mutual inductance and the off diagonal elements are negative half of the mutual inductance,

$$L'_r = \begin{bmatrix} L'_{lr} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L'_{lr} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L'_{lr} + L_{ms} \end{bmatrix} \tag{9}$$

and the Matrix 10, a function of θ_r that governs the interaction that occurs in the air gap. With $ph = \frac{2\pi}{3}$, the matrix L'_{sr} can be represented in the following form.

$$L'_{sr} = L_{ms} \begin{bmatrix} \cos\theta_r & \cos\Big(\theta_r + ph\Big) & \cos\Big(\theta_r - ph\Big) \\ \cos\Big(\theta_r - ph\Big) & \cos\theta_r & \cos\Big(\theta_r + ph\Big) \\ \cos\Big(\theta_r + ph\Big) & \cos\Big(\theta_r - ph\Big) & \cos\theta_r \end{bmatrix} \tag{10}$$

By rearranging Equation 3, currents $\vec{i}_{abcs}$ and $\vec{i'}_{abcr}$ can be calculated in the new form presented by Equation 11.

$$\begin{bmatrix} \vec{i}_{abcs} \\ \vec{i'}_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L'_{sr} \\ {L'_{sr}}^T & L'_r \end{bmatrix}^{-1} \begin{bmatrix} \vec{\lambda}_{abcs} \\ \vec{\lambda'}_{abcr} \end{bmatrix} \tag{11}$$

Newton's Law submodule takes as input the electromagnetic torque & loaded torque and outputs the angular displacement of the rotor and the angular velocity in terms of revolutions per minute. Where the angular displacement of the rotor is given by Equations 12 and 13.

$$\omega_r = \frac{P}{2J}\int (\tau_e - \tau_L - B\omega_r)\ dt \tag{12}$$

$$\theta_r = \int \omega_r\ dt \tag{13}$$

The Power Calculation block takes in the stator voltage, stator currents and referred rotor currents. From which the power supplied to the stator is given by Equation 14, the power loss in the stator windings is given by Equation 15 and the power loss in the rotor windings is given by Equation 16. Finally the power in the shaft is calculated with Equation 17.

$$P_{Supply} = \vec{v}_s \cdot \vec{i}_s \tag{14}$$

$$P_{Stator} = \sum r_s \vec{i}_s^{\,2} \tag{15}$$

$$P_{Rotor} = \sum r_r' \vec{i}_r'^{\,2} \tag{16}$$

$$P_{Shaft} = P_{Supply} - P_{Stator} - P_{Rotor} \tag{17}$$

In the Export Data to Workspace block, stator & rotor currents, electromagnetic torque, angular acceleration, angular velocity, supplied power, power supplied to the rotor and the shaft power are exported to the workspace for graphical representation and further analysis.

V. Results and Discussions

A. 3 HP Machine

Figure 2 shows seven different graphs: all three stator currents, all three rotor currents, and electromagnetic torque. The three phase stator currents have a peak value (in-rush current) of 102.4 A and begin to stabilize around 0.45 s. The three phase rotor currents have a peak value of 100 A as well but they stabilize at around 0.35 s. These currents stabilize when the torque approaches zero.

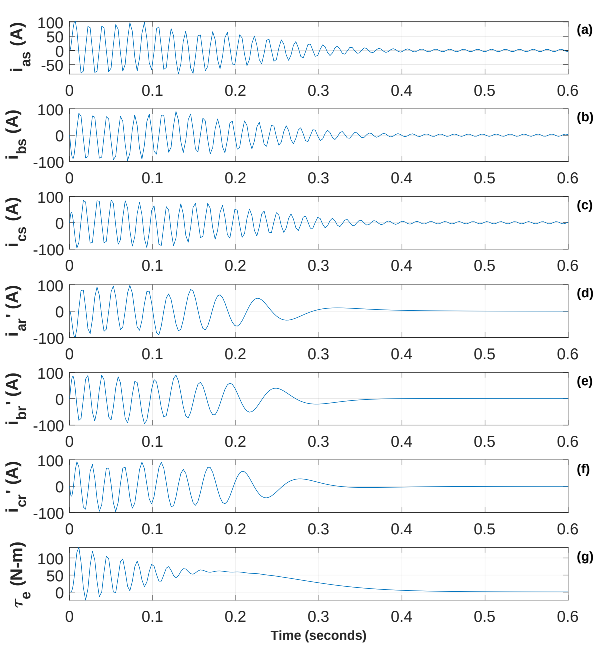

Fig. 2. Machine variables during free acceleration of a 3-hp induction motor

Figure 3 shows the plot for Electromagnetic Torque vs rotor speed. This graph follows a damping oscillation, with the maximum value being 131.05 N-m and the minimum value being -23.53 N-m, until it reaches 2000 rad/s. From that point, Torque decays as the rotor speed increases. The decrease, however, is shallow which means it requires a significant decrease in rotor speed to load the machine.

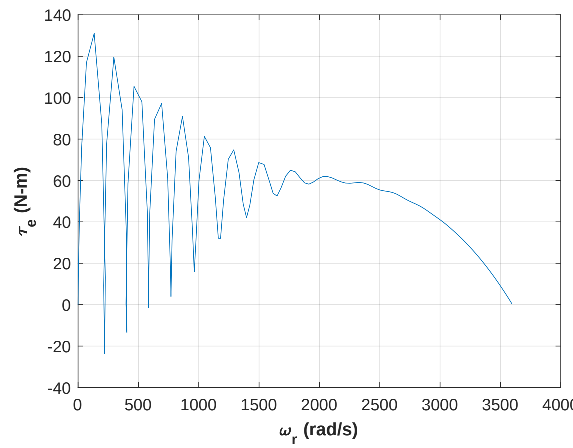

Fig. 3. Torque vs rotor speed for 3-hp induction motor

The plots for rotor, stator, and shaft power vs are shown in Figure 4. Stator power starts as the greatest out of the three, followed by rotor power and shaft power. However, stator and rotor power are decreasing while shaft power increases, follwoing the Law of Conservation. At around 2.5 s all three of them start converging and approach zero.

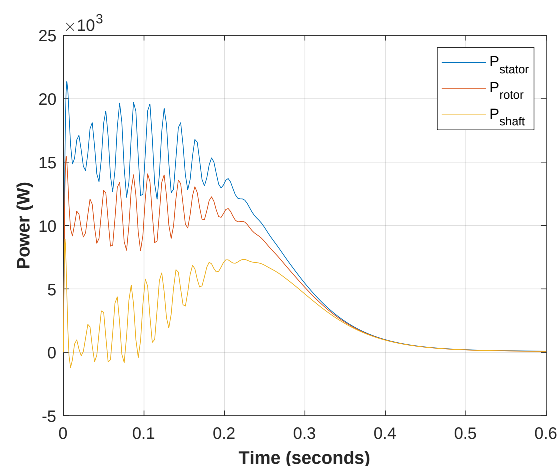

Fig. 4. Stator, rotor, and shaft power for 3-hp induction motor

B. 7 HP Machine

In Figure 5, three quantities can be appreciated stator currents, rotor currents and the torque. The three phase currents from the stator have a peak value of 228.44 A, and begin to stabilize around 0.75 s, which is roughly when the torque is approximating zero.

In Figure 6, the electromagnetic torque vs. the angular speed in RPM is presented, where a favorable relationship between

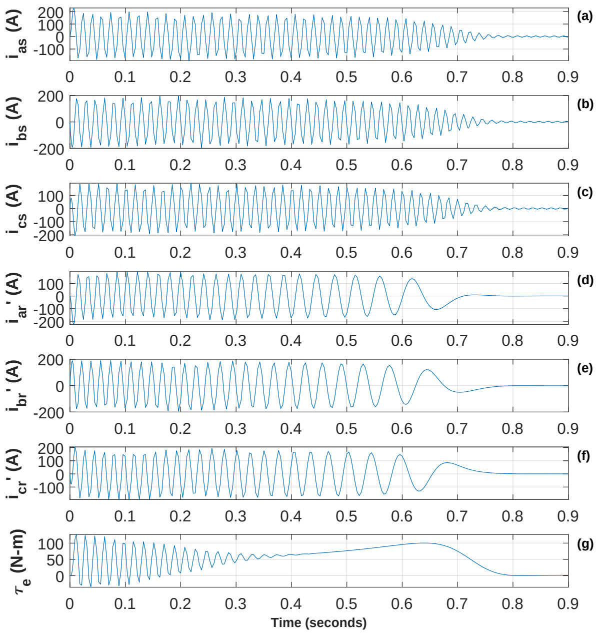

Fig. 5. Machine variables during free acceleration of a 7-hp induction motor

torque and speed can be observed. The slope begins to get steeper as the speed increases, which allows for less slippage to occur. Therefore, it is easier to load the machine without veering off from the unloaded speed.

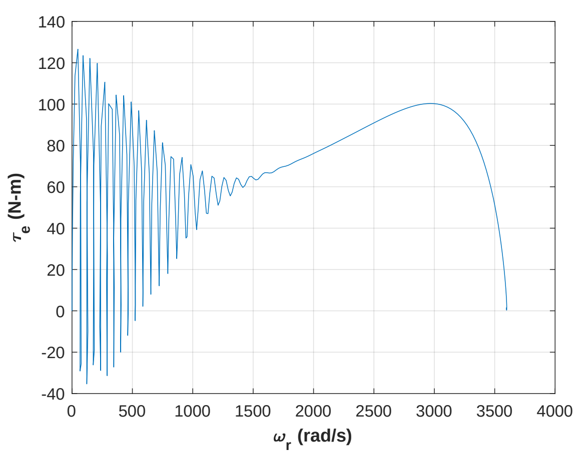

Fig. 6. Torque vs rotor speed for 7-hp induction machine

In Figure 7, the power supplied, the power present in the rotor and the power that is transferred to the shaft is presented. The shaft power is converted to mechanical torque, from the graph it is apparent that as the shaft power begins to increase the stator and rotor power begin to decrease because of the Law of Conservation. The power that is being consumed from time zero to approximately 0.75 s, is what is necessary to energize the windings in both the stator and rotor, only then can power be transferred to the shaft for the given objective of the machine.

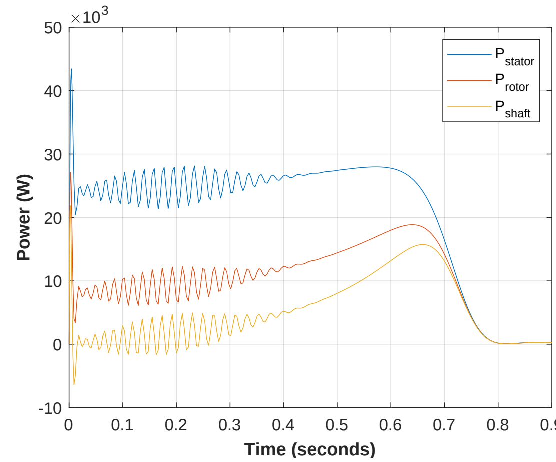

Fig. 7. Stator, rotor, and shaft power for 7-hp induction motor

C. Comparison

There is nearly a 123% increase in the peak current of the 7 HP machine from 228.49 A compared to the peak current of 3 HP machine at around 102.35 A. An understandable phenomenon due to the 5.22 kW power rating of the 7 HP being just about 2 times the 3 HP 2.23 kW power rating.

Comparing the torque vs. speed plot, the 3 HP machine will be less desirable because in order to meet a higher torque load the speed of the machine must decrease significantly for example for a 45 N-m load the speed would have to drop around 600 rad/s. Whereas the 7 HP machine has a much steeper slope only requiring a drop in 100 rad/s to meet the same load of 45 N-m.

The 3 HP machine has a minimum torque of -23.53 N-m and a maximum torque of 131.1 N-m. Meanwhile, the 7 HP machine has a minimum torque of -35.35 N-m and a maximum torque of 126.5 N-m. As for time to reach steady state, in the context of this paper, will be defined with the following equation:

$$t_{steady} = \mid \dot{\omega_r} \mid < tolerance \tag{18}$$

With a tolerance of 0.3 for the 3 HP machine the time to reach steady state was 0.6 s, at which the torque had a value of 0.43 N-m with an approximate overshoot of 30300% this value might seem outrageous but considering the rather large tolerance used for this machine it is understandable that such a value is observed because steady state only occurs towards the end of the simulation where the torque is almost at zero.

With a tolerance of 5×10^{-3}, the 7 HP machine took about 0.87 s for angular velocity to reach about zero, at the same time the electric torque had a value of 1.44 N-m. The overshoot from steady steady state for torque is roughly 8720%, because the speed goes below the tolerance sooner.

This difference in steady state torque values could potentially be due to the power ratings (and therefore the consumed currents) of the machines.

VI. Conclusion

An induction motor was modeled using machine variables to establish relationships between flux linkages, currents, and torque. Equations 1 and 2 show the differential equations obtained from Faradays Law. With them, flux linkages were found and then equation 3 was used to find stator and rotor currents. Once these values were known, electric torque, stator power, rotor power, and shaft power could be found using equations 4, 15, 16, and 17 respectively. All equations were modeled in MATLAB's Simulink environment to provide graphical results and therefore simplify the analysis of the induction motor.

Two different induction machines were simulated: a $2.23\ kW$ (3-hp) and a $5.22\ kW$ (7-hp). One noticeable difference between them is the amount of current the 7-hp machine consumes ($228.49\ A$) compared to that of the 3-hp machine ($102.35\ A$). This is mainly due to the power rating of them; the 7-hp power rating nearly doubles the 3-hp power rating. Steady state values of electric torque for the 3-hp and 7-hp machines were $0.43\ N$-m and $1.44\ N$-m respectively. One last noticeable difference is how much rotor speed should decrease to achieve load requirements for the machine. By comparing Figures 4 and 6, it was determined that the 7-hp, thanks to its steeper slope in the Torque vs rotor speed plot, requires a smaller drop in speed to achieve a higher torque load, making it a more desirable machine.

References

[1] P. C. Krause, O. Wasynczuk, S. D. Sudhoff, and S. Pekarek, *Analysis of Electric Machinery and Drive Systems*. Wiley, 2013.