Simulation of Induction Machine in DQ Frame with a Steady State Analysis

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***Abstract*—This paper highlights the results and key information learned from the project given on basic induction machines. This project models the dynamic and steady state characteristics of a 3-horsepower (HP) induction machine which is analyzed in the dq stationary reference frame. These characteristics include the torque vs. speed, input power, and shaft power of the machine during these two analyses. One of the main points of this project is to be able to compare these measured characteristics of the same induction machine under a steady state and dynamic simulation and understand the differences in these two analysis techniques. Furthermore, this same machine will be analyzed with a 25% stall torque applied to the load to observe the input power and shaft power compared between the steady state and dynamic analyses. To simulate and analyze this machine for both types of analyses, a Simulink model was created and simulated, which was driven by a governing MATLAB script.**

***Index Terms*—Induction machine, electromagnetic torque, stator, rotor, shaft, stall torque, steady state analysis, dynamic analysis**

I. Introduction

Induction machines are used in everyday-life throughout the world for many commercial and recreational applications such as ceiling fans, pumps, mills, and even renewable wind generation. The induction machine was originally invented in 1887 by Nikola Tesla. Since Tesla invented the induction machine, it has expanded to everyday global use.

An induction machine can use two different types of rotors: a cage rotor or a wound rotor. The cage rotor consists of bars arranged around its edge with the bars being connected on both ends by a continuous ring. The wound rotor has a similar design, but the bars are replaced by copper windings, comparable to those in the stator, and typically also are connected by a slip ring [1]. Induction machines can have singly fed or doubly fed windings. Singly fed windings only have one external power connection to the windings, compared to doubly fed machines, which have external connections to both the armature and the field magnet windings of the machine. The machine that was modeled for this project was singly fed, so there was no external voltage applied to the rotor of the machine.

This project models the dynamic and steady state characteristics of a 3 HP induction machine, observed in the dq stationary reference frame. First, the torque vs. speed curve for this machine will be plotted for steady state and dynamic simulations of this machine and be compared to each other. Then, this machine will be loaded to 25% of its stall torque, which will be measured from the torque vs. speed curve in the steady state analysis of this machine. From here, the input power and output shaft power of the machine will be observed and compared between the dynamic and steady state analyses. A Simulink model was created and simulated to complete these analyses and was driven by a governing MATLAB script.

II. Methodology and Equations

All the given parameters from this project's requirements for the 3 HP induction machine are listed in Table I. In this table, $V_{rms(l-l)}$ is the RMS Line-to-Line Voltage applied to stator, ω_B represents the base speed of excitation on the machine, r_s represents the per-phase resistance of the stator winding, X'_{lr} represents the referred rotor leakage reactance, r'_r represents the per phase referred rotor winding resistance, X_{ls} represents the stator leakage reactance, X_M represents the magnetized mutual reactance, J represents the inertia of the machine and B represents the rotor damping coefficient. All of the other variables necessary for calculations and analysis of this machine can be calculated using these given variables in equations presented later in this paper.

In order to simulate and observe this 3 HP machine, several equations needed to be solved for. For simplification of the equations necessary for the steady state and dynamic analyses, the abc machine variables were referred to the dq stationary reference frame (the reference frame speed was set to zero).

TABLE I
Machine Parameters

Parameters	3 HP	Units
$V_{rms(l-l)}$	220	V
Poles	4	-
Base Speed ω_B	377	rad/s
r_s	0.45	Ω
X'_{lr}	0.75	Ω
r'_r	0.8	Ω
X_{ls}	0.75	Ω
X_M	27	Ω
J	0.09	$kg\text{-}m^2$
B	10×10^{-6}	kg/s

Doing so effectively eliminated the time-varying inductances in the voltage equations and led to simplifying many of the equations that are involved with analyzing an induction machine. The equations solved for involved several matrix calculations that solved for the stator and referred rotor current values. Also, electromagnetic torque was solved for by using an equation involving stator currents, rotor currents, and rotor angle values.

Using the dq stationary reference frame to solve the necessary equations of this machine, the given line-to-line (rms) voltages of the stator were converted to line-to-neutral values by multiplying the given voltage values by the square root of two (to convert the voltage to a peak value) and by dividing this value by the square root of three (to convert the voltage to a line-to-neutral value). The referred rotor voltages were set to zero because this induction machine was not designed to have both an applied rotor voltage and an applied stator current. In other words, this induction machine was singly fed. In order to convert the applied stator voltages to dq variables in the stationary reference frame, the transformation equation of (1) was applied:

$$\vec{K}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{1}$$

As a result, the three-phase voltages were converted to two constant voltage variables.

A. Equations for Induction Machine Steady State Analysis

The equations in this section describe a balanced steady state operation of an induction machine. The voltages from the stator and rotor side are described by (2) and (3). Variables with the $\tilde{x}$ marker must be treated as phasor values.

$$\tilde{V}_{as} = \left(r_s + j\frac{\omega_e}{\omega_b}X_{ls}\right)\tilde{I}_{as} + j\frac{\omega_e}{\omega_b}X_M(\tilde{I}_{as} + \tilde{I}'_{ar}) \tag{2}$$

$$\frac{\tilde{V}'_{ar}}{s} = \left(\frac{r'_r}{s} + j\frac{\omega_e}{\omega_b}X'_{lr}\right)\tilde{I}'_{ar} + j\frac{\omega_e}{\omega_b}X_M(\tilde{I}_{as} + \tilde{I}'_{ar}) \tag{3}$$

The slip ratio is defined in (4), where ω_r is the rotor angular velocity, ω_e is the excitation frequency. The calculation of the rotor angular velocity is presented in (5), where P is the number of poles, J is the inertia of the machine, T_e is the mechanical torque produced, T_L is the load torque placed on the machine and B is the damping coefficient to approximate a practical machine.

$$s = \frac{\omega_e - \omega_r}{\omega_e} \tag{4}$$

$$\omega_r = \frac{P}{2J}\int T_e \; dt \tag{5}$$

Considering the machine is singly fed, there is no excitation on the rotor and the excitation frequency is the same as the base frequency. The following assumptions and simplifications are given in (6):

$$\begin{aligned} V'_{ar} &= 0 \\ \omega_e &= \omega_b \\ X_{ss} &= X_{ls} + X_M \\ X'_{rr} &= X'_{lr} + X_M \end{aligned} \tag{6}$$

Therefore, taking into account the the simplifications and assumptions in (6), Equations (2) and (3) can be reduced to:

$$\tilde{V}_{as} = \left(r_s + jX_{ss}\right)\tilde{I}_{as} + jX_M\tilde{I}'_{ar} \tag{7}$$

$$\frac{\tilde{V}'_{ar}}{s} = \left(\frac{r'_r}{s} + jX'_{rr}\right)\tilde{I}'_{ar} + jX_M\tilde{I}_{as} \tag{8}$$

Solving for $\tilde{I}_{as}$, (8) can then be reformulated allowing the expression to be reduced to the following:

$$\tilde{I}_{as} = -\frac{\frac{r'_r}{s} + jX'_{rr}}{jX_M}\tilde{I}'_{ar} \tag{9}$$

From this point (9) is inserted into (2) thus resulting in the following formulation:

$$\tilde{V}_{as} = \left[\frac{-\left(r_s + jX_{ss}\right)\left(\frac{r'_r}{s} + jX'_{rr}\right)}{jX_M} + jX_M\right]\tilde{I}'_{ar} \tag{10}$$

Now to solve for $\tilde{I}'_{ar}$, (11) shows the necessary manipulation:

$$\tilde{I}'_{ar} = \frac{\tilde{V}_{as}}{\left[\frac{-\left(r_s + jX_{ss}\right)\left(\frac{r'_r}{s} + jX'_{rr}\right)}{jX_M} + jX_M\right]} \tag{11}$$

At this point $\tilde{I}_{as}$ can then be calculated by plugging the value for $\tilde{I}'_{ar}$ into (9), this is done for phase b and c of the machine. In the case of singly fed induction machine and taking into consideration the simplification in (6), torque is expressed in the following manner:

$$T_e = 3\left(\frac{P}{2}\right)\left(\frac{X_M}{\omega_b}\right)Re[j\tilde{I}*_{as}\,\tilde{I}'_{ar}] \tag{12}$$

B. Equations for Induction Machine Dynamic Analysis

After obtaining the qd0 voltage variables by transforming the abc voltage variables into the dq stationary reference frame, the stator and referred rotor currents needed to be calculated to simulate this induction machine. Equations (13) and (14) display Faradays law, which was used to calculate the stator and referred rotor currents of the induction machine:

$$\vec{v}_{qd0s} = r_s\vec{i}_{qd0s} + \omega\vec{\lambda}_{dqs} + \frac{d\vec{\lambda}_{qd0s}}{dt} \tag{13}$$

$$\vec{v}_{qd0r'} = r'_r\vec{i'}_{qd0r} + (\omega - \omega_r)\vec{\lambda'}_{dqs} + \frac{d\vec{\lambda'}_{qd0r}}{dt} \tag{14}$$

After rearranging the Faraday's law equations of (13) and (14) for the qd0 stator and rotor variables, the stator and rotor currents can be solved for in terms of flux linkage using the matrix equation of (15):

$$\begin{bmatrix} \vec{\lambda}_{qd0s} \\ \vec{\lambda'}_{qd0r} \end{bmatrix} = \begin{bmatrix} K_s L_s K_s^{-1} & K_s L'_{sr} K_r^{-1} \\ K_r {L'_{sr}}^T K_s^{-1} & K_r L'_r K_r^{-1} \end{bmatrix} \begin{bmatrix} \vec{i}_{qd0s} \\ \vec{i'}_{qd0r} \end{bmatrix} \quad (15)$$

The matrix variables of (15) can be calculated using (16), (17), and (18):

$$K_s L_s K_s^{-1} = \begin{bmatrix} L_{ls} + L_{ms} & 0 & 0 \\ 0 & L_{ls} + L_{ms} & 0 \\ 0 & 0 & L_{ls} + L_{ms} \end{bmatrix} \quad (16)$$

$$K_r L'_r K_r^{-1} = \begin{bmatrix} L'_{lr} + L_{ms} & 0 & 0 \\ 0 & L'_{lr} + L_{ms} & 0 \\ 0 & 0 & L'_{lr} + L_{ms} \end{bmatrix} \quad (17)$$

$$K_s L'_{sr} K_r^{-1} = K_r {L'_{sr}}^T K_s^{-1} = \begin{bmatrix} L_{ms} & 0 & 0 \\ 0 & L_{ms} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (18)$$

In all of these equations, the subscripts denoted s and r refer to the stator and rotor variables respectively. All referred variables are denoted with the apostrophe symbol, ('). For the sake of this project, the rotor variables are referred to the stator windings. L_{ls} and $L_{lr'}$ are the leakage inductances for the stator and rotor windings respectively. L_M is the mutual inductance between the rotor and the stator. The electromagnetic torque T_e can be found using (19):

$$T_e = \frac{3}{2}\frac{P}{2}(\vec{\lambda}_{ds}\vec{i}_{qs} - \vec{\lambda}_{qs}\vec{i}_{ds}) \quad (19)$$

The input power can be calculated using the qd0 variables by (20):

$$P_{Input} = \frac{3}{2}(v_{qs}i_{qs} + v_{ds}i_{ds} + 2v_{0s}i_{0s}) \quad (20)$$

In this project, all losses of the induction machine except for the copper losses in the stator and rotor are neglected. These copper losses are calculated using (21) and (22) for the stator and rotor respectively:

$$P_{LossStator} = \frac{3}{2}(i_{qs}^2 r_s + i_{ds}^2 r_s + 2i_{0s}^2 r_s) \quad (21)$$

$$P_{LossRotor} = \frac{3}{2}(i_{qr'}^2 r'_r + i_{dr'}^2 r'_r + 2i_{0r'}^2 r'_r) \quad (22)$$

Therefore, the output power at the shaft can be calculated by subtracting the copper losses of the stator and rotor by the input power of the machine as seen in (23):

$$P_{Shaft} = P_{Input} - P_{LossStator} - P_{LossRotor} \quad (23)$$

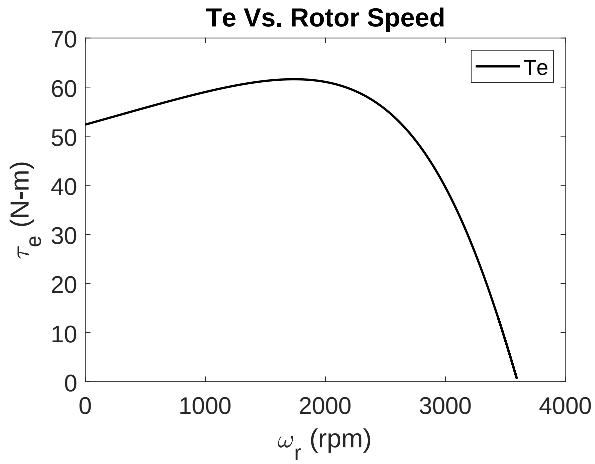

Fig. 1. Steady State Model Torque vs. Speed Plot

All of these equations were implemented using matrix calculations in a MATLAB function block.

III. Results and Discussion

A. Steady State Analysis

The steady state model of the 3 HP induction machine is formulated to calculate the stall torque with a time step of $1\ \mu s$. Stall torque is defined as the maximum electromagnetic torque, when the speed of the machine is at zero. The torque vs. speed plot is presented in Fig. 1, from which the stall torque was determined to be 52.36 N-m. This value is then used to load the dynamic model at 25% of it. A key point to mention is that steady state model abstracts the transient, so that they are not present in the model as seen in Fig. 1.

The steady state model of the induction machine is then presented with a load of 25% of the stall torque at time 0.5 seconds which is 13.09 N-m. The input power given to the stator of the induction machine is given with the equation (24) from which Fig. 2 is produced.

$$P_{Input} = v_{as}i_{as} + v_{bs}i_{bs} + v_{cs}i_{cs} \quad (24)$$

After the transient have passed and the power begins to stabilize at around 0.78 seconds, the average value of power is 3172 W. Considering that this is a 2.23 kW machine, this rather large value might be a phenomenon that is not taken into account in the steady state model. It is worth mentioning that Fig. 2 begins at time 0.5 seconds. The mechanical power after the losses in the stator winding, rotor winding and windage & friction losses is given by equation (25) from which Fig. 3 displays.

$$P_{shaft} = \frac{2}{P}\omega_r \tau_e \quad (25)$$

The power provided to shaft begins to stabilize at around 0.78 seconds to a value of 2355 W, this value is more reasonable considering the rating of the machine. It must be noted that Fig. 3 begins at time 0.5 seconds.

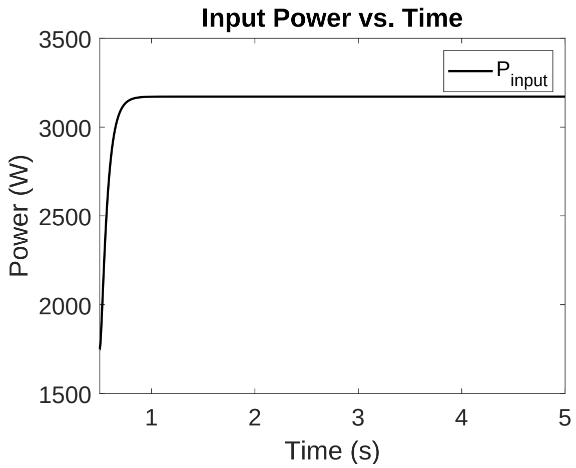

Fig. 2. Steady State Model Input Power Plot

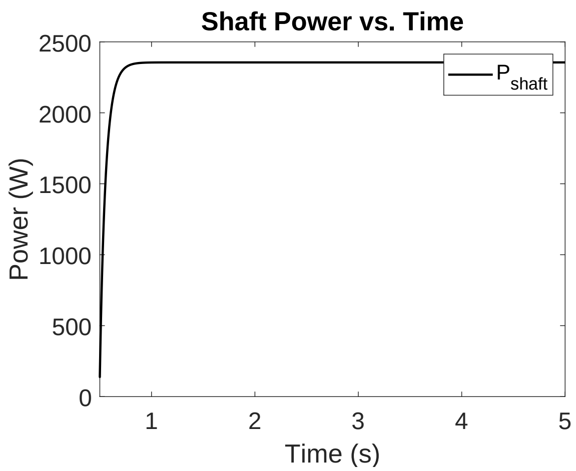

Fig. 3. Steady State Model Shaft Power Plot

B. Dynamic Analysis

In the beginning of this dynamic analysis, the torque vs. speed curve was plotted for this induction machine. This plot can be seen in Fig. 4.

In the previous section, the torque vs. speed curve of the steady state analysis of the induction machine shown in (1) was used to determine the stall torque value of the machine. This stall torque was used to develop a dynamic simulation of the same machine to analyze the load torque characteristics of this machine. For this dynamic machine analysis, the load torque was set to zero for the first 0.5 seconds of the simulation, and then was increased to 25% of the stall torque for the rest of the simulation. Delaying the application of the load torque helped ensure that the effects of transient conditions did not skew the results of the simulation. This load torque value came out to be $13.09N$-m. The simulation was set to last 5 seconds long at a time step of 1 μs.

The input power waveform for the simulated dynamic model of the induction machine studied in this project is displayed in Fig. 5. It is important to note that the power value was obtained

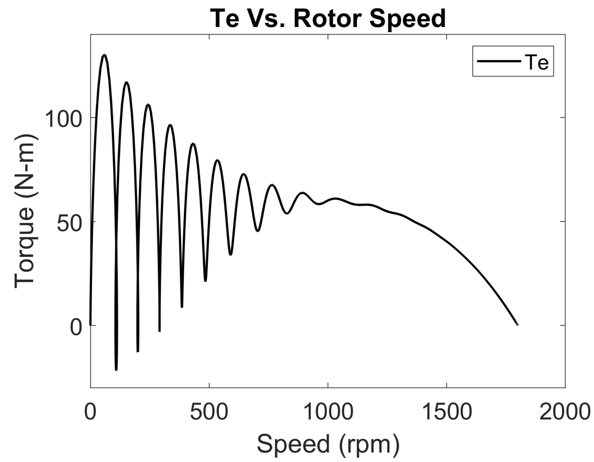

Fig. 4. Dynamic Model Torque vs. Speed Plot

starting around 1 second in order to avoid the transient power oscillations involved in this machine. The input power is about $2551\ W$.

The shaft power waveform for the simulated dynamic model of the induction machine studied in this project is displayed in Fig. 6. It is important to note that this figure starts at 1 second in order to avoid the transient power oscillations involved in this machine. The shaft power was measured to be about $2355\ W$.

C. Steady State and Dynamic Analysis Comparison

The steady state model gives a rough estimate of the characteristics of this induction machine, but has limited capabilities. A difference is noticeable in the torque vs. speed plot for the steady state model given in Fig. 1 and the dynamic model given in Fig. 4. There are no transients present in the steady state model; in the dynamic model there are plenty of transient oscillations when the energizing of the windings.

The input power for the steady state model is given in Fig. 2 and measures about $3172\ W$. The input power for the dynamic

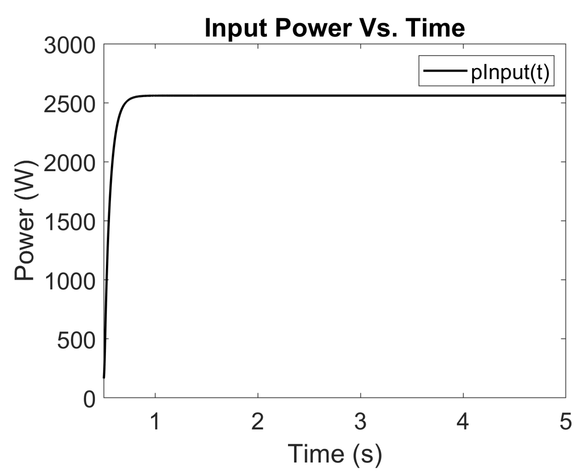

Fig. 5. Dynamic Model Input Power Plot

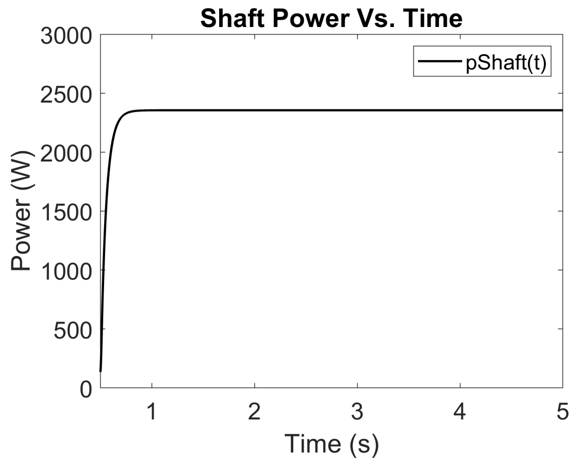

Fig. 6. Dynamic Model Shaft Power Plot

model given in Fig. 5 is about $2551\ W$, that is a difference of 24% between the values. Clearly there is some detail that is not being accounted for in the steady state model. The shaft power for the steady state model presented in Fig. 3 has a value of $2355\ W$ and the dynamic model has a value of $2355\ W$ which can be appreciated in Fig. 6, there absolutely no difference in their calculation, so the key information such as angular velocity of the rotor and the torque produced still holds.

IV. Conclusion

In conclusion, from this analysis, the team members learned that there two approaches to modeling a machine depending on degree of the detail is required from the model. If the purpose is to control an electromagnetic machine, then it is imperative to know the transient events which are only displayed in the dynamic model. Clearly as stated in the previous section, there are some discrepancies in both the models, the dynamic model might have some overhead but it stays true to the governing physics of the system.

References

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