Arbitrary Reference Frame Analysis

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***Abstract*—The purpose of this project is to look at variables in several reference frames. In this project, two blocks will be developed to allow the transform from three-phases, *abc*, variables to direct-quadrature-zero, *qdz*, variables and the inverse transform from *qdz* variables to *abc* variables respectively. This project will examine the functionality of these transforms by observing waveforms generated for phase voltages and currents as well as the waveform generated for the power of each system.**

I. Introduction

In this project, variables are analyzed in several reference frames. These reference frames consist of a stationary, synchronous and jump & run reference frame. The transformed values are further examined to provide an understanding of the functionality of said transforms. Figures 3.9-1, 3.9-2, and 3.9-3 from the Analysis of Electric Machinery and Drive Systems [1] textbook are reproduced using simulations created on MATLAB-Simulink. These systems portrayed in the aforementioned figures are then unbalanced by the scaling of the b-phase. The inverse transforms of these systems are also examined in this project.

II. Motivation

It is beneficial to transform state variables in the three-phases *abc* domain to the direct-quadrature-zero domain *qdz*, because while preserving important information of the waveform it makes it considerably easier for analysis of the signals. Once in this simplified stage, simple calculations can be done on these transformed values, which can then be effortlessly transformed back to the three-phase domain for implementation. In the case of three-phase synchronous machines the transformation allows for the stator and rotor quantities to be represented in a single rotating reference frame to eliminate time-varying inductances.

III. Formulation of Transforms

As aforementioned in the previous section, to reduce the complexity of the differential equations in question a change of variables is called for. At first each change of variable was considered unique and therefore treated separately. After subsequent research in 1965, it was discovered that they were in reality just one transformation. Therefore, eliminating all rotor position-dependent mutual inductances by allowing machine variables to be referred from a frame of reference rotating at an arbitrary angular velocity. Thus, giving rise to what is now known as the arbitrary reference frame.

The forward transformation is given by Matrix 1, where the first row corresponds to the quadrature component, the second row to the direct component and the last row to the zero component. Taking the three phase components to the direct-quadrature-zero domain.

$$\vec{K}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{1}$$

The inverse transformation is given in Matrix 2, where the first row corresponds to the a component, the second row to the b component and the last row to the c component. From there the three-phase (*abc*) components can be recalculated to bring it back from the qdz domain.

$$\vec{K}_s^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \tag{2}$$

With these two transformations, given the proper angular velocity, transforming from the *abc* domain to the *qdz* domain is a very simple process that allows for easier analysis of an induction or synchronous machine.

IV. Experimental Model

Using the equations in 3 the three phase voltage sources are generated and simulated in Simulink, where v_{as} is the a phase voltage, v_{bs} is the b phase voltage, and v_{cs} is the c phase voltage.

$$\begin{aligned} v_{as} &= \sqrt{2}V_s\cos\left(\omega_e t\right) \\ v_{bs} &= \sqrt{2}V_s\cos\left(\omega_e t - \frac{2\pi}{3}\right) \\ v_{cs} &= \sqrt{2}V_s\cos\left(\omega_e t + \frac{2\pi}{3}\right) \end{aligned} \tag{3}$$

The parameters are then derived from Equations 4 - 6 using the values defined in Table I, where Z_s is the impendence, τ is the time constant, and α is a phase delay.

$$Z_s = r_s + j\omega_e L_s \tag{4}$$

$$\tau = \frac{L_s}{r_s} \tag{5}$$

$$\alpha = \tan^{-1}\frac{\omega_e L_s}{r_s} \tag{6}$$

TABLE I
CONSTANTS

Variable	Value	Unit
V_s	$10/\sqrt{2}$	V
r_s	0.216	Ω
ω_e	377	rad/s
$\omega_e L_s$	1.09	Ω

The currents are modeled by the equations in 7, where i_{as} is the a phase current, i_{bs} is the b phase current, and i_{cs} is the c phase current.

$$\begin{aligned} i_{as} =& \frac{\sqrt{2}V_s}{|Z_s|}\Big[-e^{-t/\tau}\cos\alpha + \cos(\omega_e t - \alpha)\Big] \\ i_{bs} =& \frac{\sqrt{2}V_s}{|Z_s|}\Big[-e^{-t/\tau}\cos\Big(\alpha + \frac{2\pi}{3}\Big) + \cos\Big(\omega_e t - \alpha - \frac{2\pi}{3}\Big)\Big] \\ i_{cs} =& \frac{\sqrt{2}V_s}{|Z_s|}\Big[-e^{-t/\tau}\cos\Big(\alpha - \frac{2\pi}{3}\Big) + \cos\Big(\omega_e t - \alpha + \frac{2\pi}{3}\Big)\Big] \end{aligned} \tag{7}$$

The completed model is shown in Figure 1, the subsystem three phase values outputs two $[3\times1]$ column vectors $\vec{v}_{abc}$ and $\vec{i}_{abc}$. The *scenario* constant is selected from the script file that is used to instantiate the variables from which the *case selector* subsystem then outputs the corresponding ω, angular velocity, for either the stationary, synchronous or the jump & run case. The *forwards transformation* subsystem takes in $\vec{v}_{abc}$, $\vec{i}_{abc}$, ω, and t as inputs and returns two $[3 \times 1]$ column vectors $\vec{v}_{qdz}$ and $\vec{i}_{qdz}$. The *inverse transformation* subsystem takes in $\vec{v}_{qdz}$, $\vec{i}_{qdz}$, ω, and t as inputs and returns two $[3\times1]$ column vectors $\vec{v}_{abc}$ and $\vec{i}_{abc}$. The *abc power* subsystem returns the power for the three phase components and the *qdz power* returns the power for the direct-quadrature-zero components.

V. Results and Discussions

A. Balanced System

In Figure 2, the balanced *abc* phase voltage and currents are displayed. In Figure 1, a stationary reference frame is utilized to transform the *qdz* values variables to *qdz* variables. In the stationary reference frame $\omega = 0$ and $i^s_{qs} = i_{as}$. In the synchronously rotating reference frame where $\omega = \omega_e$, the electric transients are represented by an exponentially decaying balanced set varying at ω_e, and the constant amplitude

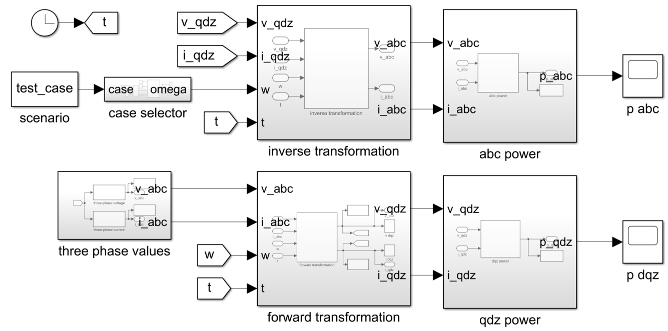

Fig. 1. Simulation of an arbitrary reference frame transformation

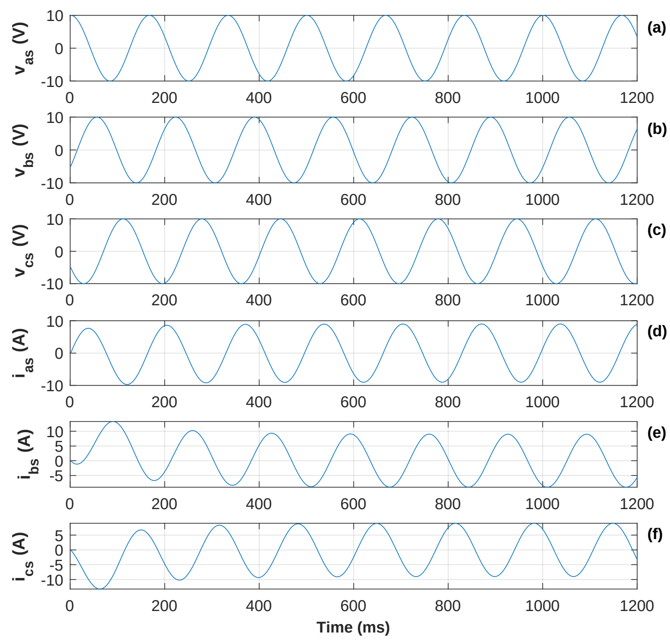

Fig. 2. Three phase voltages and currents of a balanced system

balanced set becomes constant. In the textbook Figures 3.9-1 - 3.9-3 [1], the speed of the reference frame is switched from its original value of -377 rad/s to zero and then ramped to 377 rad/s. In the synchronous reference frame, the actual variables are observed from various reference frames by the nature of the jump then run reference frame, a nature that identifies the movement from one reference from to the next by first jumping, then running.

B. Unbalanced System

To create an unbalanced system, the b phase voltage is scaled to 60% of the original value. In Figure 3 the b phase voltage is scaled to six. The wave forms provided from the textbook in Figures 3.9-1 - 3.9-3 [1], were reproduced with the scaled b phase voltage. As can be seen in Figures 7 - 9, the wave forms are similar, but not all the same as the balanced system. In the stationary reference frame, the voltage waveform for instantaneous electric power begins to oscillate at 200 ms and remains this way. In the synchronous reference

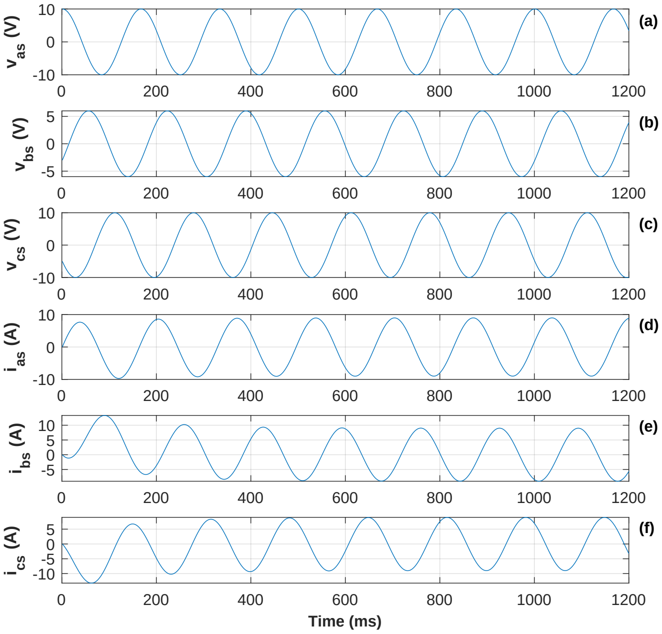

Fig. 3. Three phase voltages and currents of an unbalanced system

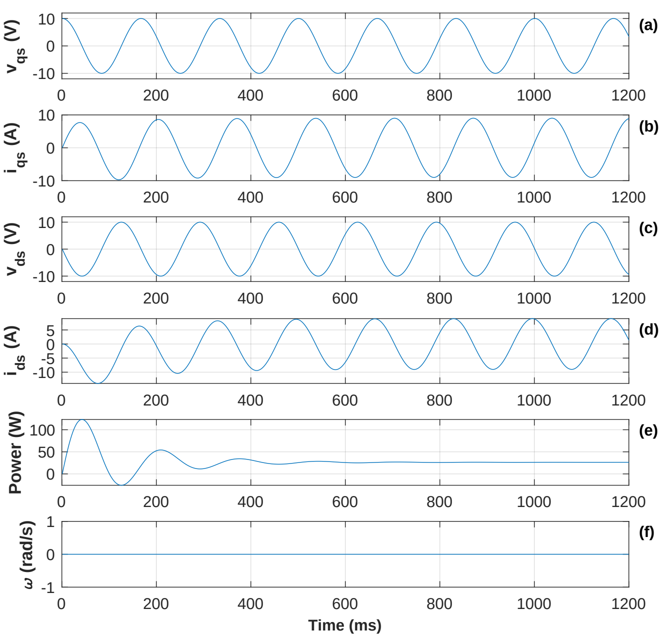

Fig. 4. Stationary reference frame of a balanced three phase system

frame of the unbalanced system, the voltage and power outputs differ significantly. The instantaneous electric power output is identical to that of the stationary reference frame and the voltage outputs create a steady oscillation. In the jump and run reference frame, the output of voltage in the q and d phase begin to differ from the balanced system at approximately 900 ms. These differences in output are as they should be. Due to an imbalance in the *abc* reference frame in the b phase voltage, the output voltage of q and d do not reach a point at which

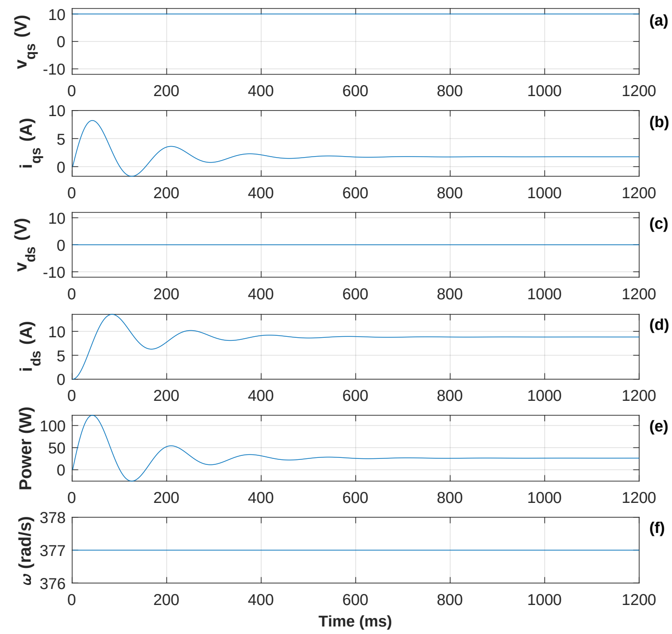

Fig. 5. Synchronous reference frame of a balanced three phase system

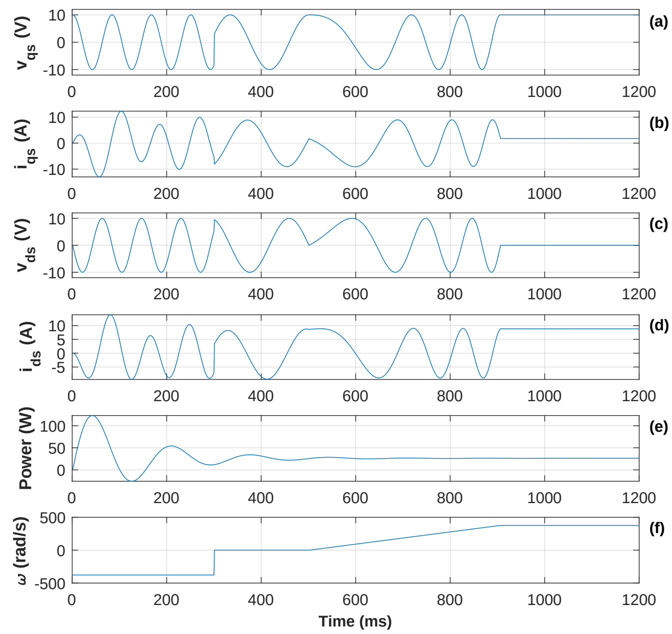

Fig. 6. Jump and Run reference frame of a balanced three phase system

they remain constant.

C. Inverse Transformations

To achieve transformation from the *qdz* variables to the *abc* variables it is necessary to use an inverse transformation. In equation 2 the inverse transformation block is represented. This concept is tested by allowing the *qdz* variables to be used as inputs into the inverse transform block seen in Figure 1 to produce the corresponding *abc* variables. As verification of the

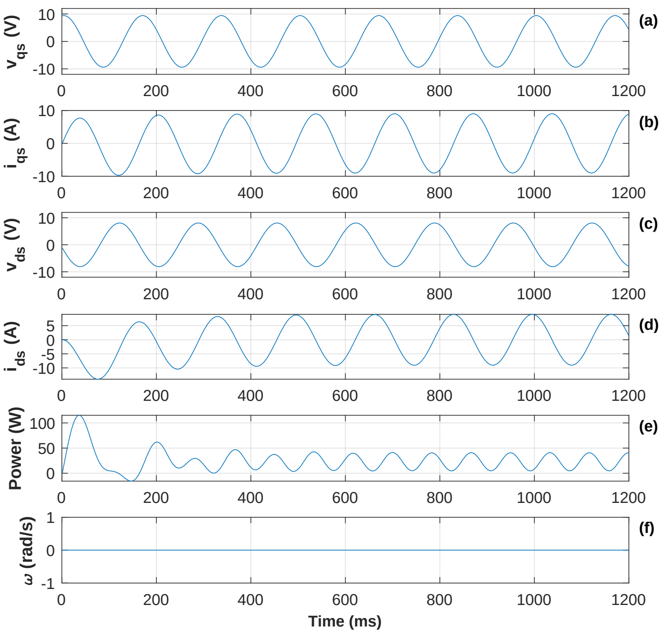

Fig. 7. Stationary reference frame of a unbalanced three phase system

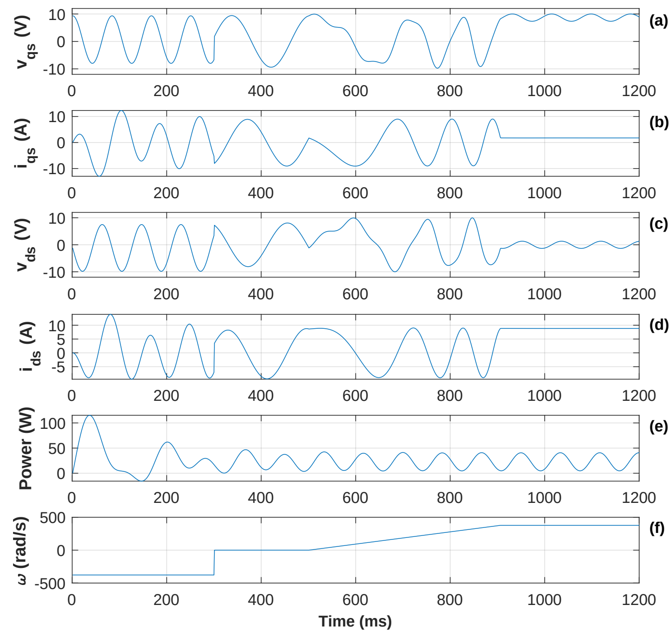

Fig. 9. Jump and Run reference frame of a unbalanced three phase system

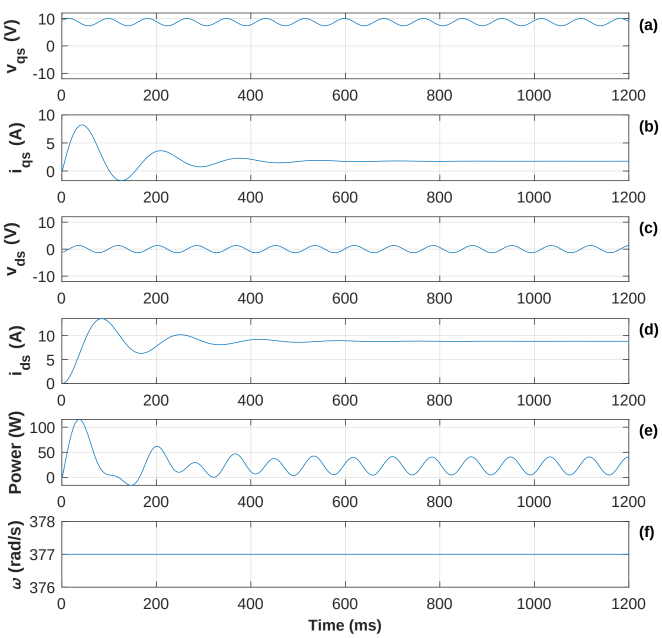

Fig. 8. Synchronous reference frame of a unbalanced three phase system

success of the inverse transformation block, the instantaneous electric power was verified for each reference frame. In Figure 10, the power output of the stationary, synchronous, and jump and run reference frames after undergoing inverse transformation are displayed. Based on the inverse transformation's electric power outputs per balanced and unbalanced reference frame in Figures 10 and 11, the inverse transformation block correctly transfers the *qdz* variables back to *abc* variables. In Figures 12 and 13, the currents of phase *a* can be observed per reference frame in the unbalanced and balanced system. Here it is seen that the imbalance of the systems had no significant effect to the system and most importantly, that the inverse transformation block was able to produce the correct output of *abc* variables as one phase is an accurate representation of all other phases in a three-phase system due to symmetry.

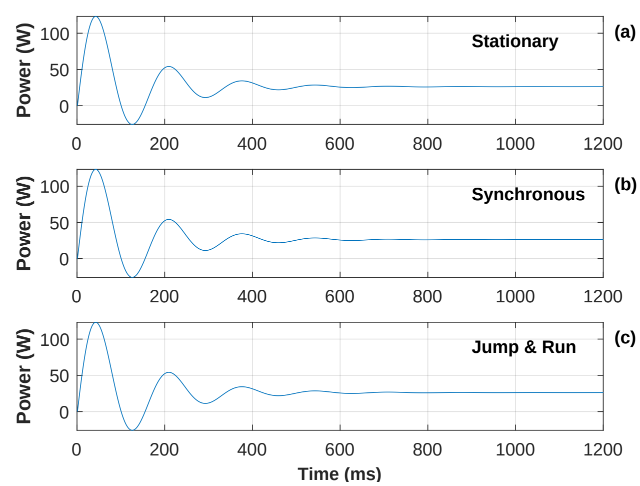

Fig. 10. Three phase power of a balanced three phase system for a stationary, synchronous and jump & run reference frame

VI. Conclusion

This project provided a thorough exploration of the reference frame theory. The state of the electric system is independent of the frame of reference form which it is observed. Although the variables produce different wave forms per

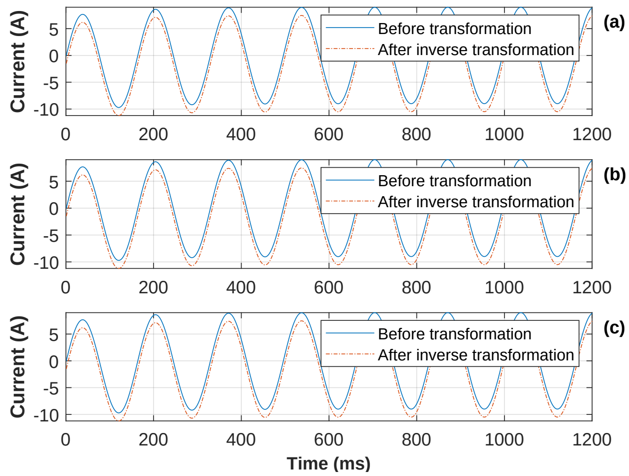

Fig. 11. Phase *a* of a balanced three phase system for a stationary, synchronous and jump & run reference frame before and after the inverse transformation with an offset of 1.5 A

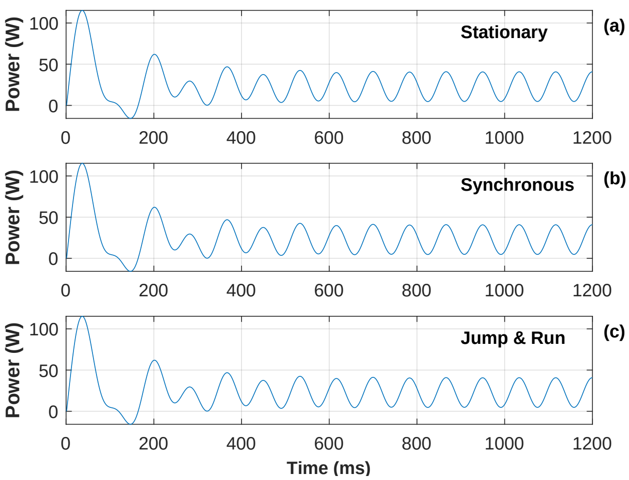

Fig. 12. Three phase power of an unbalanced three phase system for a stationary, synchronous and jump & run reference frame

frame, the power remains the same in each reference frame both balanced and unbalanced. Each reference frame provided a different perspective of which to observe the outputs of the *q* and *d* phase voltage and current variables. The balanced system of variables was created using forward transformation from *abc* to *qdz*. The production of the outputs in Figures 4 - 6 supports this claim.

As the project moved to the observation of an unbalanced system, the effects of the change in *b* phase voltage became apparent in the output of the *qdz* variables in each reference frame. As in the balanced system, the power output per reference frame remained the same through out, however, the individual current and voltage outputs differed per reference frame. If a closer look is taken into the wave outputs, it is seen that the amplitude in both balanced and unbalanced remains the same.

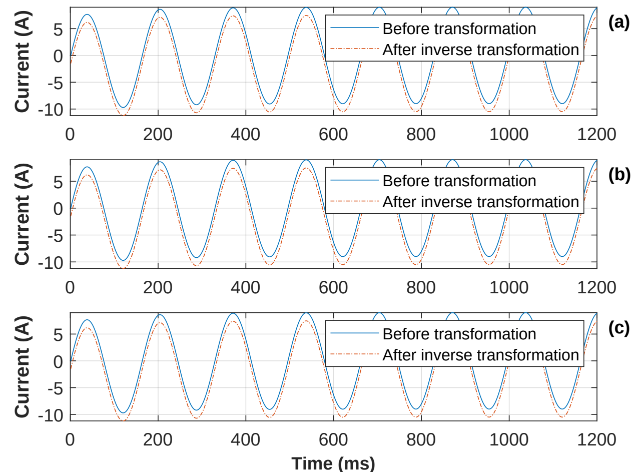

Fig. 13. Phase *a* of an unbalanced three phase system for a stationary, synchronous and jump & run reference frame before and after the inverse transformation with an offset of 1.5 A

This project allowed students to verify the reference frame theory and its forward and inverse transformations through the reproduction of outputs. In reproducing the output wave forms, a visual aid was created to provide a better understanding of the effects of balanced and unbalanced systems as they relate to changing reference frames. Although the effects are seen in the nature and shape of the output, the conclusion can be drawn that an imbalance in a system does not affect the system's ability to maintain one instantaneous electric power waveform throughout each reference frame.

References

[1] P. C. Krause, O. Wasynczuk, S. D. Sudhoff, and S. Pekarek, *Analysis of Electric Machinery and Drive Systems*. Wiley, 2013.