Linear Machine Modeling and Simulation with Electromechanical Energy Conversion

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***Abstract*—Using the electromechanical energy conversion principle, equations are formulated to model the underlying dynamics of a linear machine, that utilizes a magnetic field to transfer its energy to the moving member. Plots from the Analysis of Electric Machinery and Drive Systems are reproduced with a degree of accuracy. Then parameter such as resistance, mass and force are varied to better understand what the behavior of the machine.**

I. Introduction

In this project we were asked to model graphs of linear machines to get a better understanding of how the machine works. We were given parameters to follow in order to replicate the graphs. Using the parameters and solving for dynamic systems equations on MATLAB/Simulink, we were able to reproduce the graphs. Once we had the graphs, we changed the parameters in order to see how a change of force or velocity for instance, would impact the overall system. This helped us understand what each variable did to the machine and the effects it had on the system.

A. Design of the Linear Machine

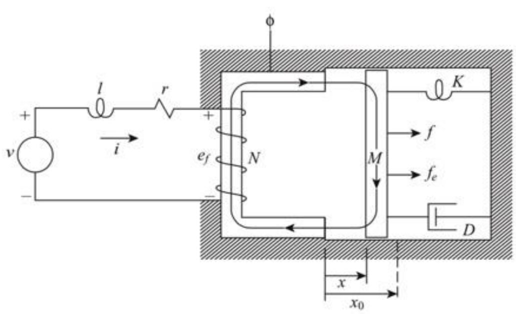

Figure 1 Electromechanical system with magnetic field

Symbol	*Quantity*	*Unit*
v	Voltage	V
i	Current	I
l	Inductance	H
r	Resistance	Ω
e_f	Induced Voltage	V
N	Number of Turns	
M	Mass	Kg
K	Spring	N/m
F	External Force	N
f_e	Electromagnet Force	N
D	Damping Coefficient	N·s/m
x	Ending Position	mm
x_o	Initial Position	mm
Φ	Flux in the System	WB

PARAMETERS	**VALUE**
l	0
M	0.055 kg
D	4 N·s/m
R	10 Ω
x_o	3 mm
K	2667 N/m
k	$6.293\cdot10^{-5}$ H·m

B. Electromechanical Energy Conservation

An electromechanical system is a system where electrical systems and mechanical systems combine. The two systems interact through a coupling field that is created from the electromagnetic and electrostatic fields that are common for both systems. To analyze the systems, one must account for the types of energy within the system. W_E represents the total electrical energy for the system; while W_M represents the total mechanical energy. Both energy totals are comprised of the energy that is transferred to the coupling field, and losses within the system. They total combination is best described as:

$$W_E = W_e + W_{eL} + W_{eS} \quad (1.1)$$

$$W_M = W_m + W_{mL} + W_{mS} \quad (1.2)$$

Where W_e is the energy transferred to the coupling field, W_{eL} is the energy loss through resistance related to the system, and W_{eS} is the energy stored in the system. The same subscripts apply to the mechanical system. Rather than analyzing these components separately, we analyze the total effects within the coupling field. Where W_F is the total energy of the field, comprised of $W_f + W_{fL}$. W_f is the energy stored in the coupling field and W_{fL} is the energy lost within the field. For the purpose of this simulation, W_{fL} is assumed to be null. Therefore, the total field energy is the total transferred electrical energy plus the total transferred mechanical energy.

$$W_f = W_e + W_m \quad (1.3)$$

To determine the quantities of the W_e and W_m one must examine the equations of Faraday's Law and Newton's Law: The voltage equation describes the effects for the electrical system, e_f being the voltage drop across the coupling field.

$$V = ri + l\frac{di}{dt} + e_f \quad (1.4)$$

Newton's law describes the effects of the mechanical system. Note that *f* is the external mechanical force, M is the mass of the movable component of the system, D is the dampening coefficient, K is the spring constant, and f_e is the electromagnetic force.

$$f = M\ddot{x} + D\dot{x} + K(x - x_0) - f_e \quad (1.5)$$

From the above equations one can solve for the desired variables and determine both W_e and W_m to be represented as the following equations:

$$W_e = \int e_f i\, dt \quad (1.6)$$

$$W_m = -\int f_e \frac{dx}{dt} dt \quad (1.7)$$

When expressing energy, it is convenient to neglect the losses, which the losses are normally so insignificant that neglecting the losses does not cause a noticeable error. Also it is convenient to hold the mechanical displacement constant. Holding the mechanical displacement causes *dx* to be zero; therefore, the mechanical energy is considered to be zero. This allows field energy to be modeled as:

$$W_f = \int e_f i\, dt \quad (1.8)$$

To receive a more detailed output representation, one needs to incorporate the electrical responses with the mechanical. Taking the equation given for newton's law, all variables except for f_e go to zero because the mechanical displacement is still held constant, where:

$$f_e = \frac{-i^2 k}{2x^2} \quad (1.9)$$

The graphical representation of the system output is referred to as the $\lambda - i$ curve. The area to the right of the curve is called co-energy, and is defined as:

$$W_c = \int \lambda\, di \quad (1.10)$$

Co-energy can also be described using:

$$W_c = \lambda i - W_f \quad (1.11)$$

Co-energy is used in calculations as a convenient way of describing electromagnetic force. However, for a linear magnetic system$W_c = W_f$. Therefore, we can calculate and determine that W_f can be described as:

$$W_f = \int e_f i\, dt - \int f_e \frac{dx}{dt} dt = \frac{1}{2}\lambda i = \frac{1}{2}L(x)i^2 \quad (1.12)$$

As complicated as the derivations for electromechanical energy conversions appear, they are still the preferred method of modeling compared to the derivations from Maxwell's equations. Using the conversion method allows researchers to quickly and accurately describe a machine using mathematical models that do not depends on the calculations of the curls in the magnetic field.

II. Results

A. *Plots in Figure 1.3 – 11*

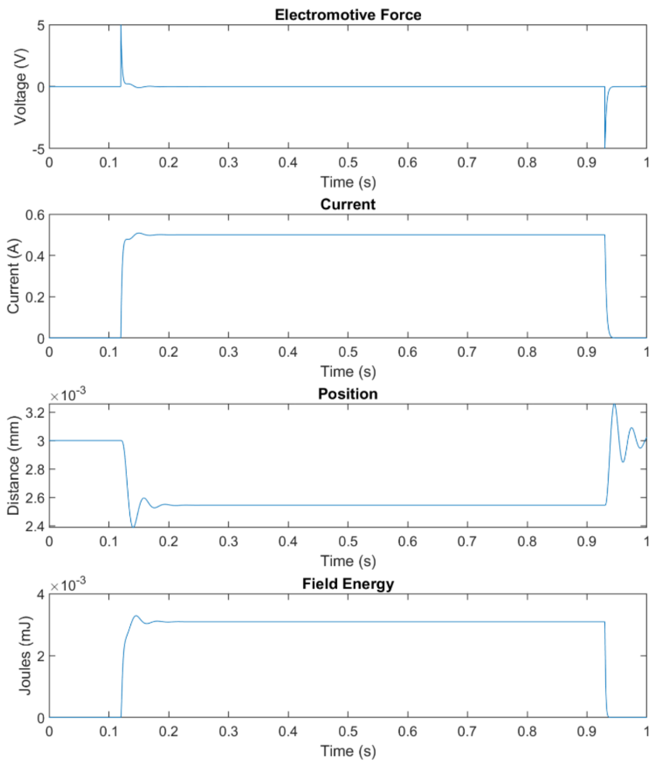

Figure 2 Dynamic performance of the electromechanical system shown in Figure 1, during step changes in the source voltage

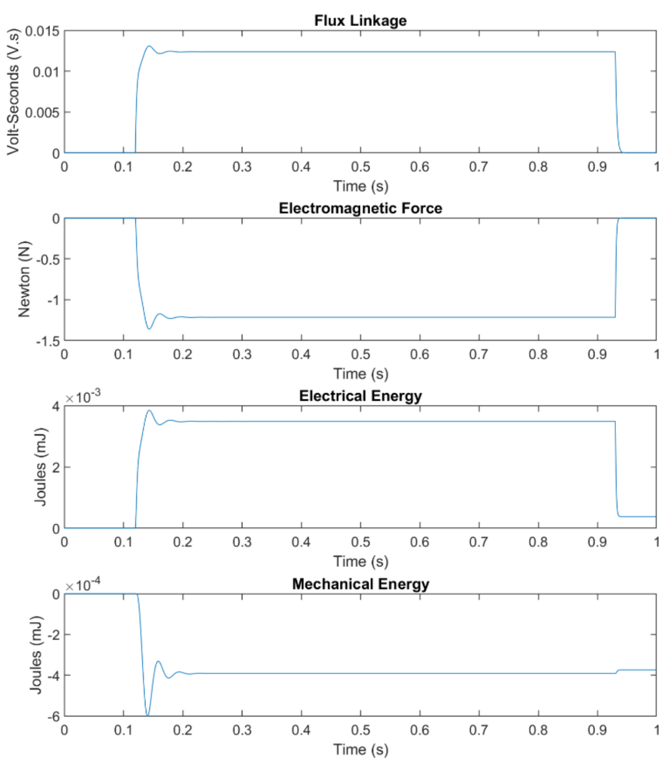

Figure 3 Continuation of dynamic performance of the electromechanical system shown in Figure 1, during step changes in the source voltage

B. *Plots in Figure 1.3 – 12*

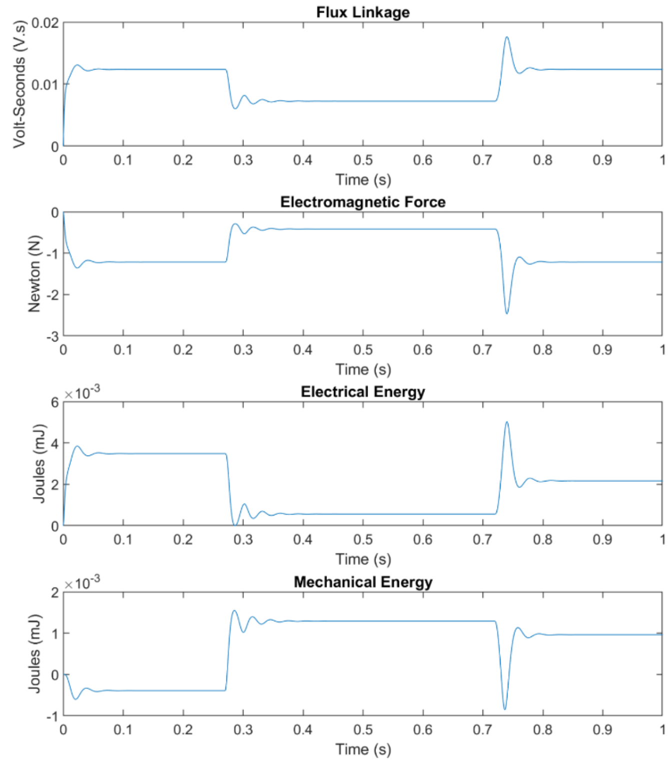

Figure 4 Dynamic performance of the electromechanical system shown in Figure 1, during step changes in the applied force

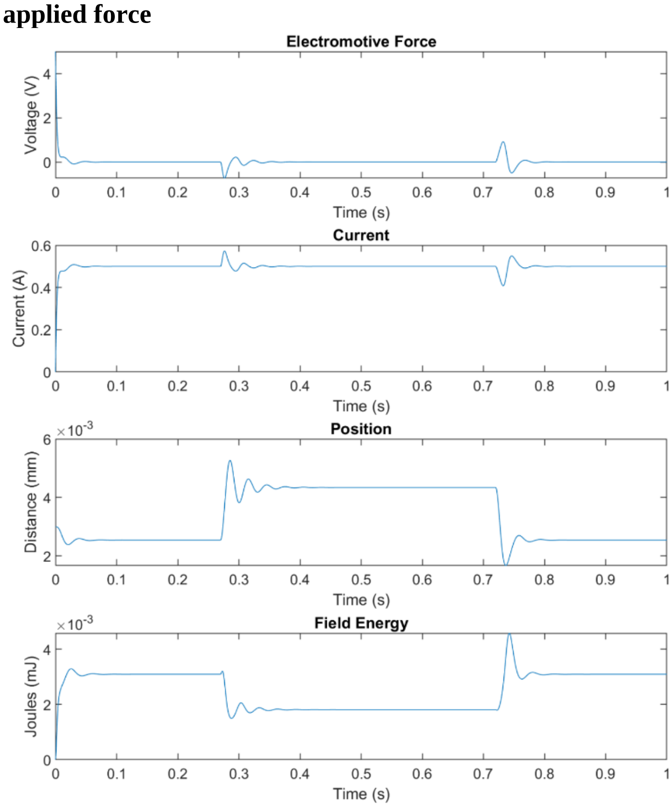

Figure 5 Continuation dynamic performance of the electromechanical system shown in Figure 1, during step changes in the applied force

C. *Varying Resistance*

Resistance was varied to see how the resistivity of a material changes the amount of current circulating through the coil, thus affecting flux linkage in the system. With an applied voltage of $v_o = 5\ V$ for a duration of $0.81\ s$, an applied force of $F_o = 4$ for a duration of $0.45\ s$ and with the mass of the moving member held constant at $M = 0.055\ kg$.

As the resistance is increased, the current is expected to decrease because in the following equation describing Faraday's Law:

$$v - R * i = e_f \qquad (1.13)$$

There is clear relationship between the electromotive force and the applied voltage. The following plots show the electromotive force (e_f), winding current (i), electromagnetic force (f_e) and velocity (v) when resistance from 1 Ω, 10 Ω to 100 Ω.

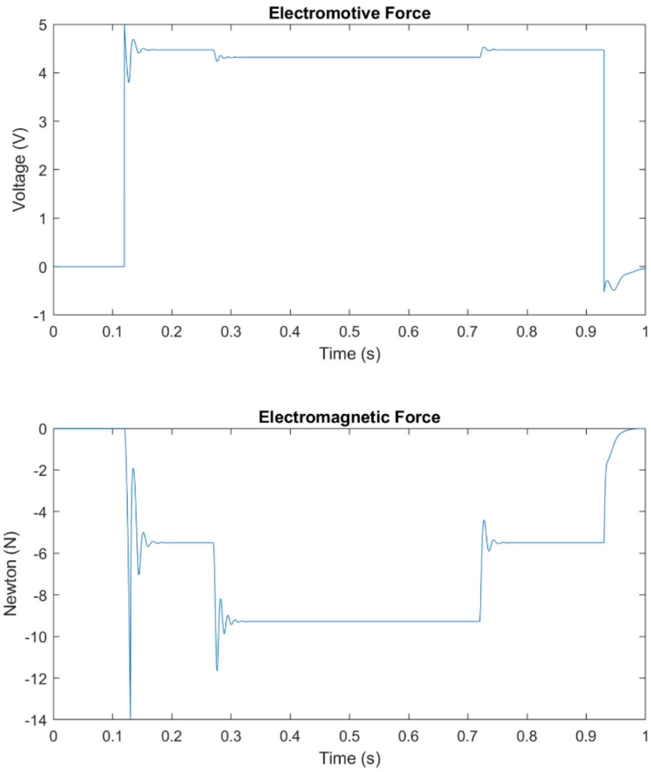

Figure 6 Resistance set to 1 Ω

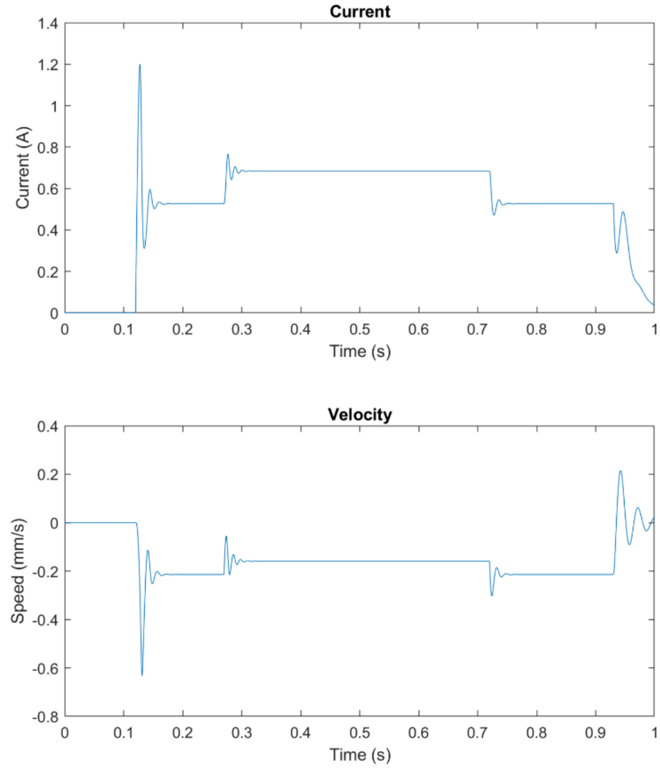

Figure 7 Continuation resistance set to 1 Ω

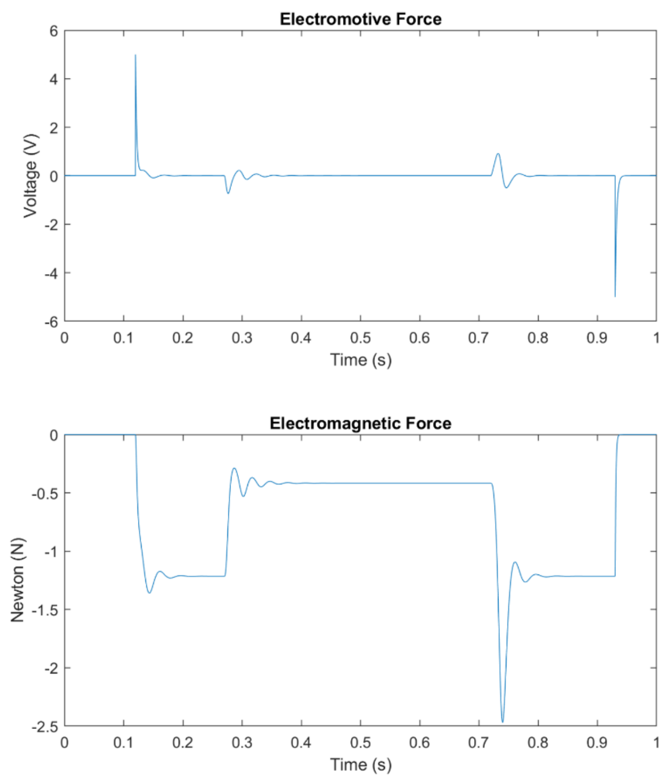

Figure 8 Resistance set to 10 Ω

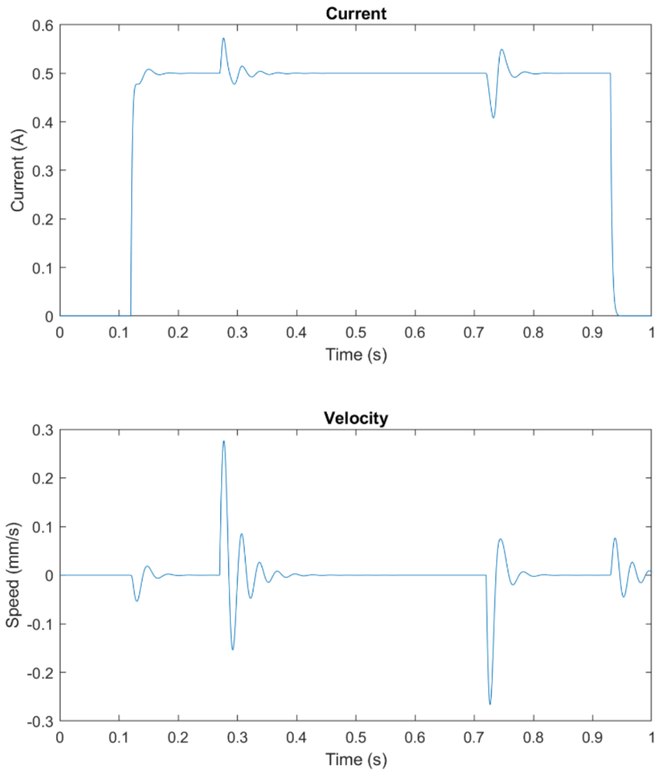

Figure 9 Continuation resistance set to 10 Ω

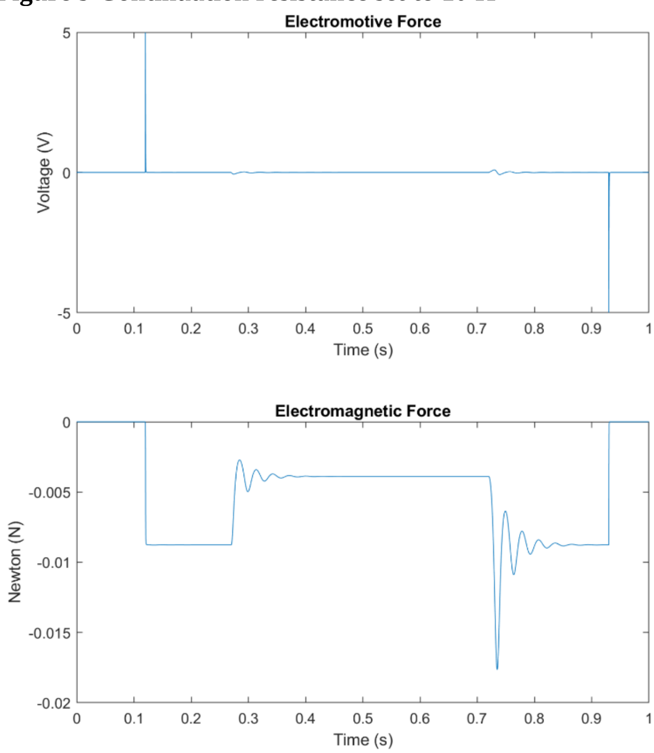

Figure 10 Resistance set to 100 Ω

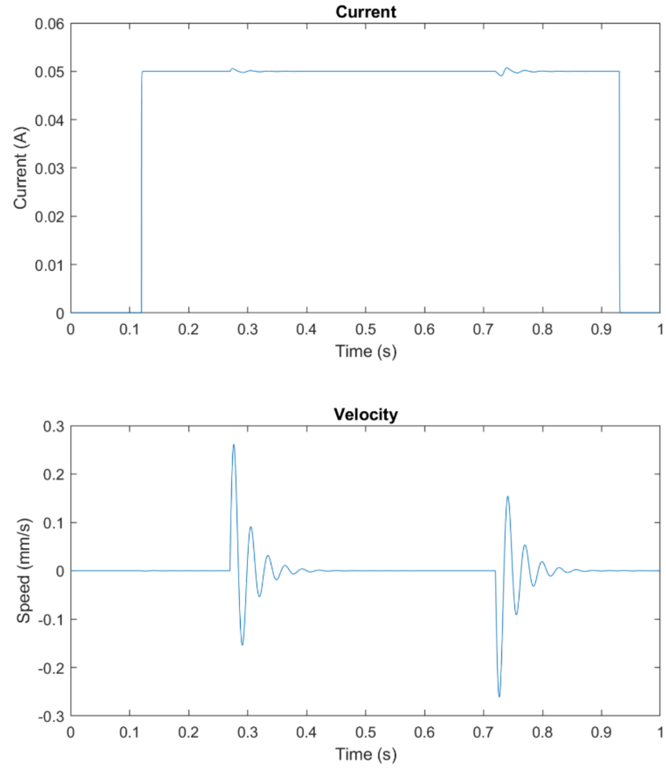

Figure 11 Continuation resistance set to 100 Ω

The e_f in Figure 6 stays around $4.5\,V$ for the duration of the voltage pulse which very different from the e_f from Figure 8 where the only response seen is the dynamics. When the resistance is increased to $100\,\Omega$, the dynamics for the electromotive force are considerably less, and the peaks are a lot sharper. In Figure 1, the current has a sudden spike passing $1\,A$ as soon as the voltage is applied on the system, in Figure 8 a slight oscillation is seen when the external force is applied and turned off. In Figure 11, there are is a significant decrease in the amount of winding current as the max value is now around $0.05\,A$ with barely an oscillation response when the external force is applied.

D. *Varying Force*

The external force applied was varied to see how the moving member reacts to different types of load placed on it, which would then affect how much the electromagnetic force can move the member. It is expected that as the force is increased, the velocity should increase as well. This is because Newton's Equation:

$$F = M * \frac{d^2x}{dt^2} + D * \frac{dx}{dt} + K * (x - x_o) - f_e \qquad (1.14)$$

Where the force and velocity are a tightly coupled system. With an applied voltage of $v_o = 5\,V$ for a duration of $0.81\,s$, a constant resistance of $R = 10\,\Omega$ and with the mass of the moving member held constant at $M = 0.055\,kg$.The applied force of F_o with a pulse duration of $0.45\,s$, was varied. The following plots show the electromotive force (e_f), winding current (i), electromagnetic force (f_e) and velocity (v) when force is increased from $0.4\,N$, $4\,N$ to $40\,N$.

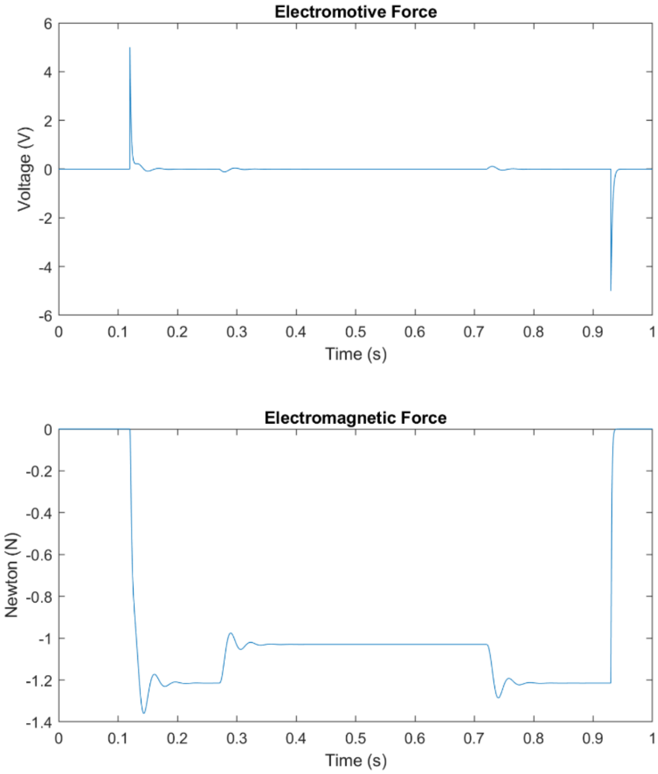

Figure 12 Force set to 0.4 N

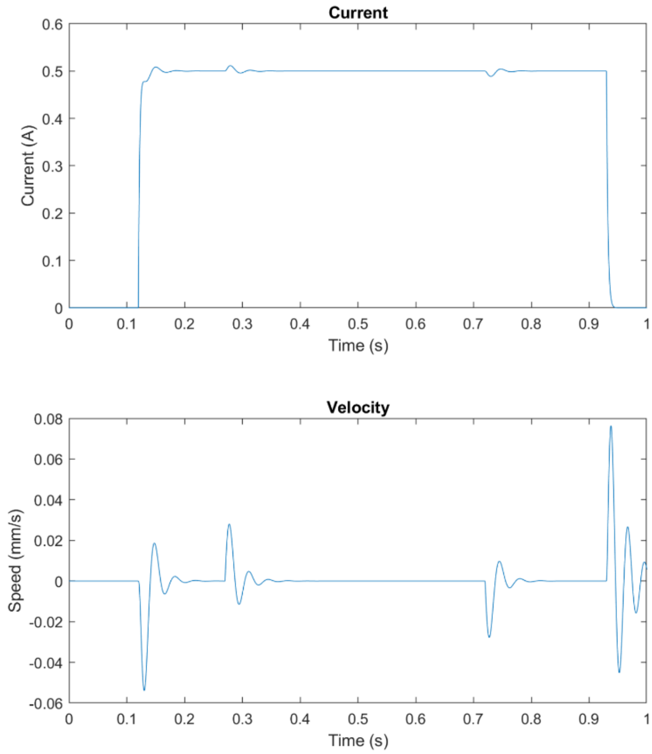

Figure 13 Continuation Force set to 0.4 N

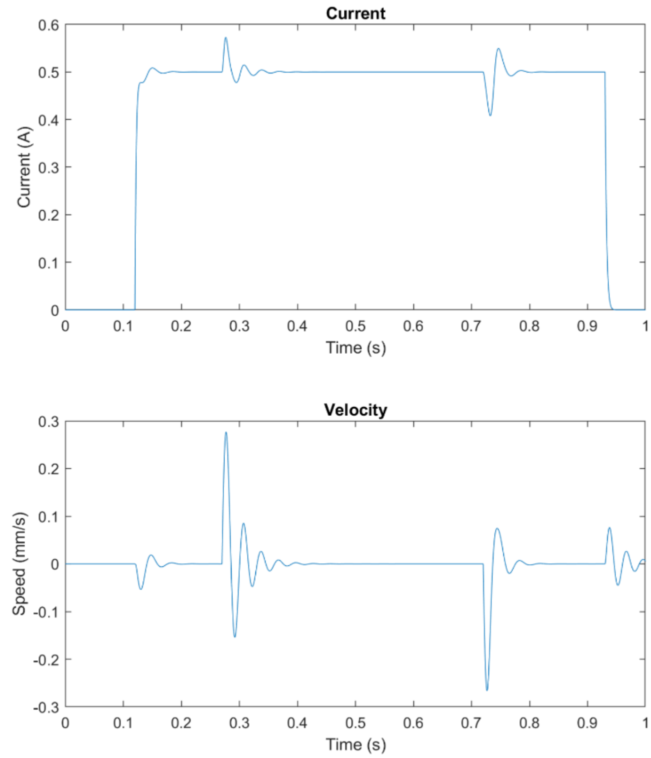

Figure 14 Force set to 4 N

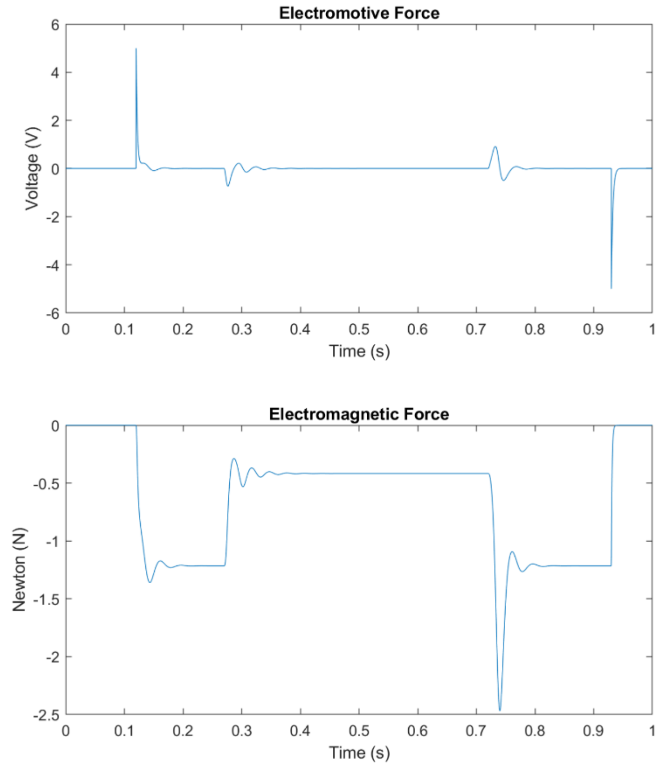

Figure 15 Continuation Force set to 4 N

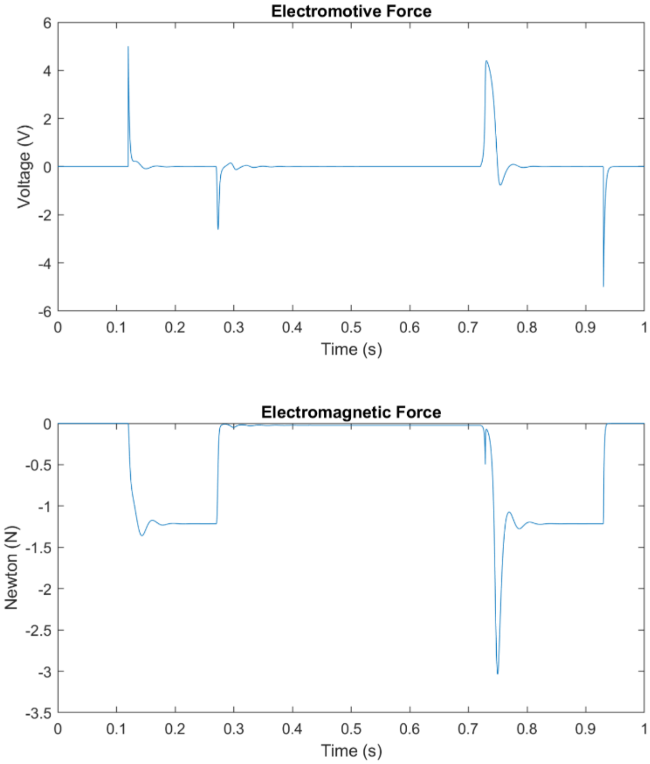

Figure 16 Force set to 40 N

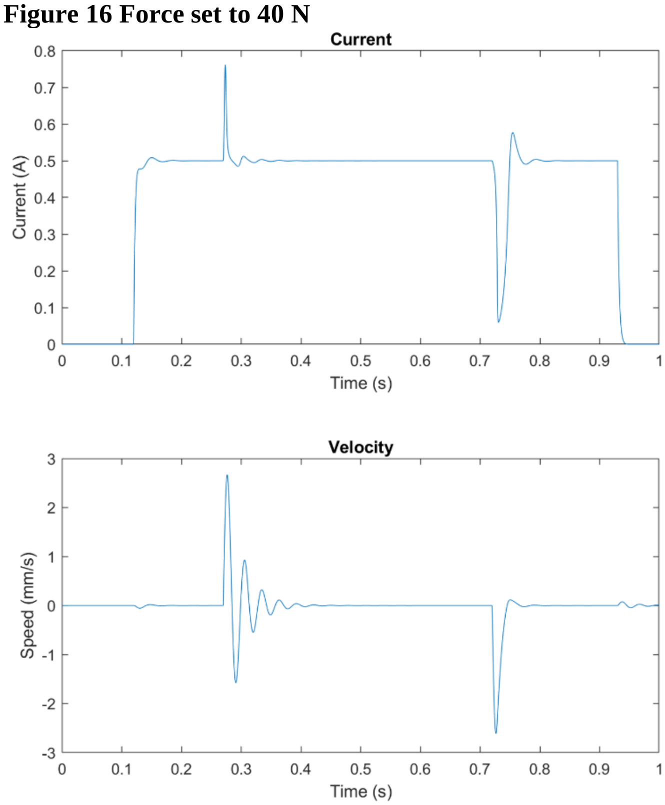

Figure 17 Continuation Force set to 40 N

As seen in Figure 16, the e_f experiences considerably larger peaks when the external force is applied to the moving member. Figure 16 also shows how the machine is trying to put a force on the system to move it, but the force applied is to great that the electromagnetic force cannot overcome it. In Figure 13, the velocity of the system is very small in the order of magnitude of 10^{-3}, as the force increases to $40\ N$ the velocity has a peak value of around $2.7\frac{mm}{s}$.

E. Varying Mass

The mass of the moving member was increased from $0.0055\ kg$, $0.055\ kg$, to $0.55\ kg$. Greater oscillations are expected to occur when the mass is the greatest. Moving members with higher mass have a higher inertia making it more difficult to slow down. Therefore, less massive moving members will then have less inertia. Due, to the tightly coupled nature of the linear machine and from Newton's Law, it is also expected that the velocity of the member will change as the mass varies. The following plots show the electromotive force (e_f), winding current (i), electromagnetic force (f_e) and velocity (v):

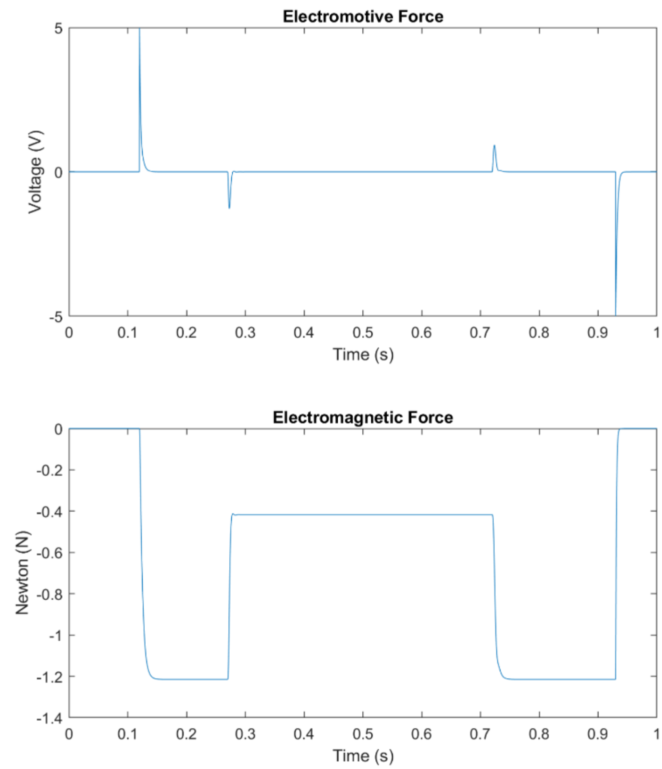

Figure 18 Mass set to 0.0055 kg

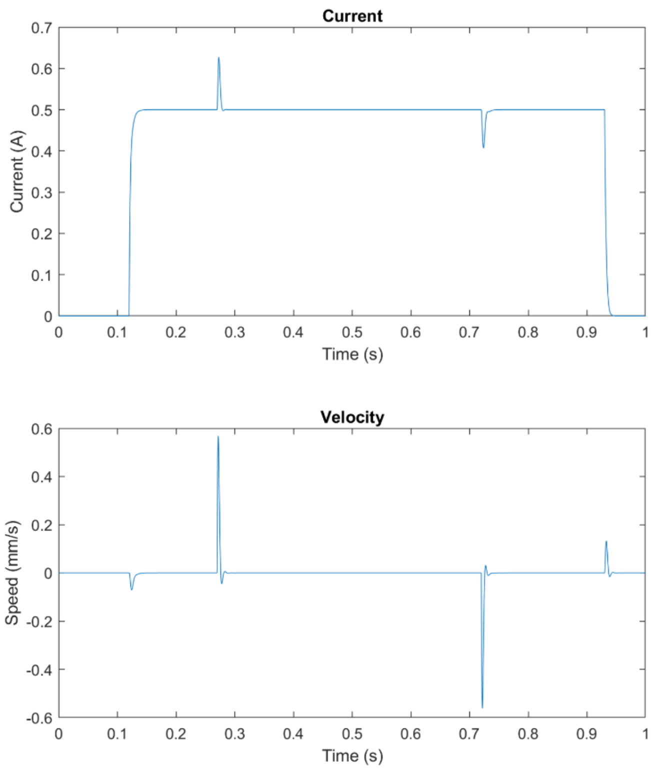

Figure 19 Continuation Mass set to 0.0055 kg

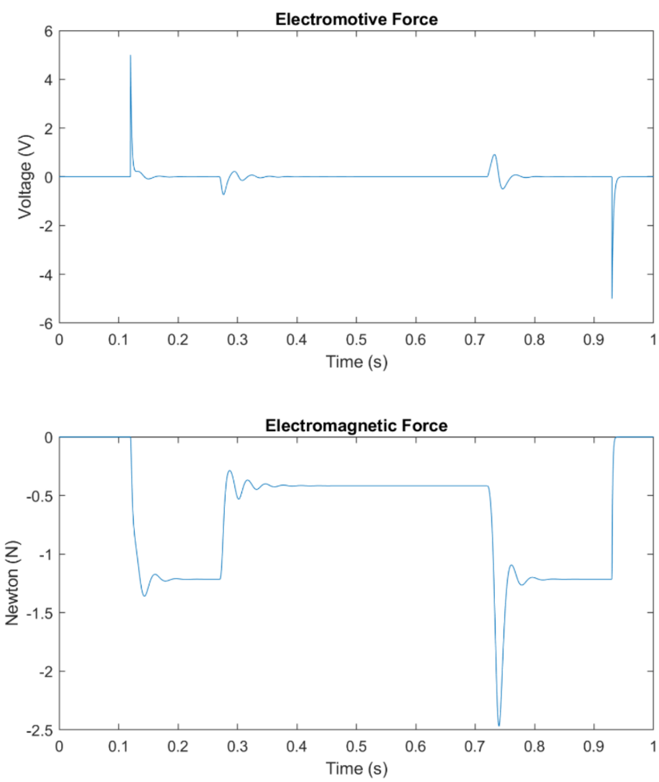

Figure 20 Mass set to 0.055 kg

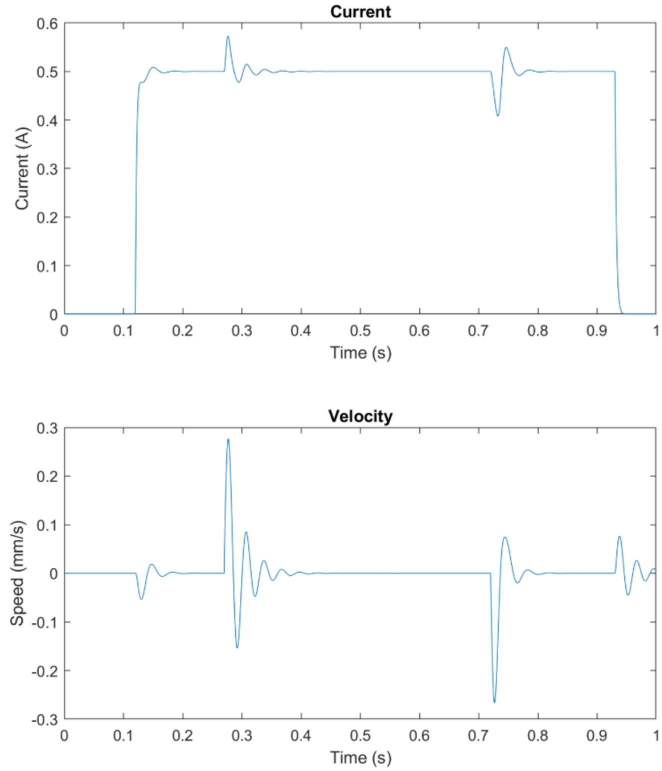

Figure 21 Continuation Mass set to 0.055 kg

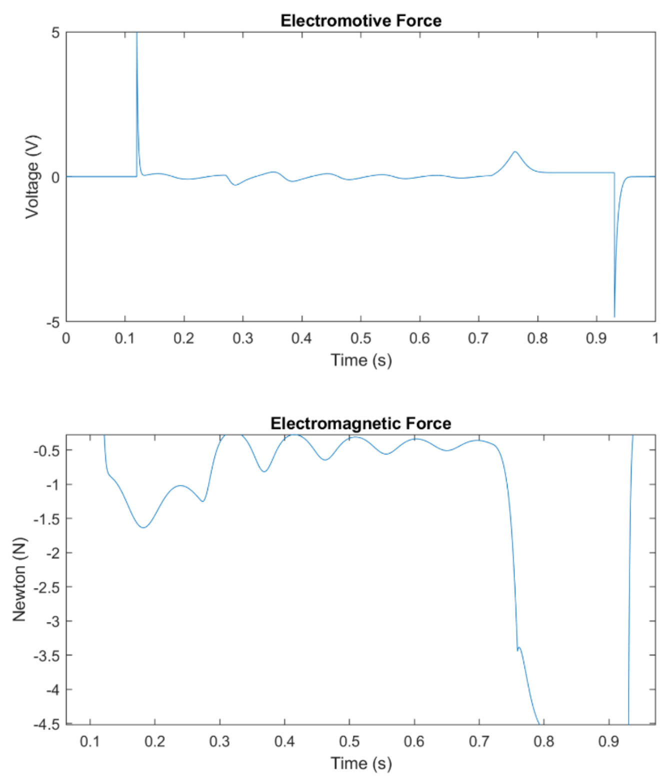

Figure 22 Mass set to 0.55 kg

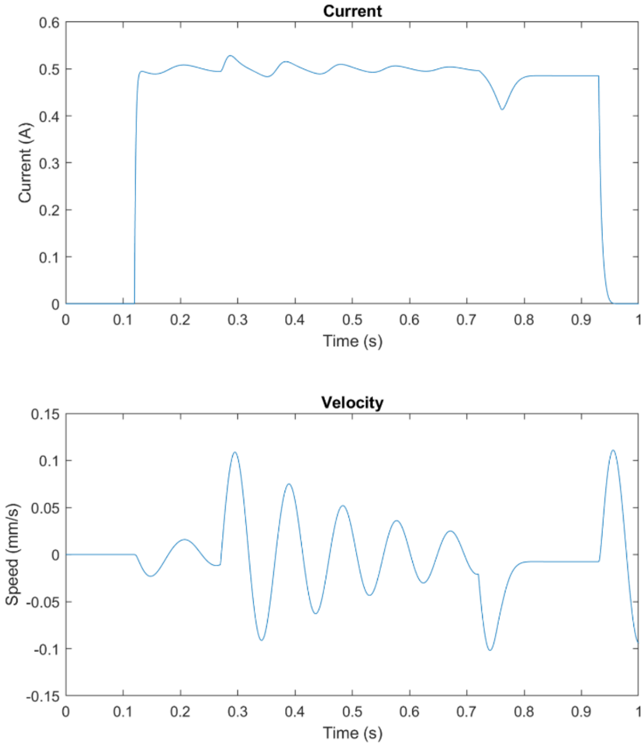

Figure 23 Continuation Mass set to 0.55 kg

In Figure 19, with $M = 0.0055\,kg$, there is barely no oscillations present in the system because there is less inertia so there is less energy stored in the member, therefore less energy is required to slow it down. On the other hand, in Figure 23 when the mass is heavy more energy is stored as inertia thus requiring more energy to bring it to a halt.

III. Conclusion

After analyzing the results of the simulation, the students better comprehend the effects that minor changes enact on the system. Seeing the changes of individual parameters also helped the students understand how the machine operates as a whole. For example, varying the mass by simply by a factor of ten either up or down, caused a dramatic change in the electrical energy needed to complete the same process. The same effect is noted in the stabilization time of the moving member when the mass is increase, the greater the mass the longer it takes for the member to reach a stable position. Alternately the lower the mass the quicker the member reached stability. Understanding these effects on the system allowed the students to better appreciate the intricacies that go into designing and building a stable, efficient machine.

References

[1] P. C. Krause, *Analysis of electric machinery and drive systems*. Wiley.