1、基于简化条件，推导纵向质心运动方程简化公式

关键步骤：（1）速度坐标系下的动力学方程

$$
\begin{aligned}
m\begin{bmatrix} \dot{v} \\ v\dot{\theta}\cos\sigma \\ -v\dot{\sigma} \end{bmatrix} &= \boldsymbol{R}_x[-\nu]\left(\left(\boldsymbol{F}_P\right)_a+\left(\boldsymbol{F}_A\right)_a+m\left(\boldsymbol{g}\right)_a-m\left(\boldsymbol{\omega}_E\right)_a\times\left(\left(\boldsymbol{\omega}_E\right)_a\times\left(\boldsymbol{r}\right)_a\right)-2m\left(\boldsymbol{\omega}_E\right)_a\times\left(\boldsymbol{v}_l\right)_a\right) \\
&= F_P\boldsymbol{R}_x[-\nu]\boldsymbol{R}_{ab}\begin{bmatrix} \cos\left(\delta_\psi\right)\cos\left(\delta_\varphi\right) \\ \sin\left(\delta_\varphi\right) \\ -\cos\left(\delta_\varphi\right)\sin\left(\delta_\psi\right) \end{bmatrix}+\boldsymbol{R}_x[-\nu]\begin{bmatrix} -C_D q S_M \\ C_L q S_M \\ C_C q S_M \end{bmatrix}+m\boldsymbol{R}_x[-\nu]\boldsymbol{R}_{al}\left(\frac{g_r-g_\phi\tan\phi}{r}\begin{bmatrix} r_{olx}+r_{lx} \\ r_{oly}+r_{ly} \\ r_{olz}+r_{lz} \end{bmatrix}+\frac{g_\phi}{\omega_E\cos\phi}\begin{bmatrix} \omega_{Elx} \\ \omega_{Ely} \\ \omega_{Elz} \end{bmatrix}\right) \\
&\quad -m\boldsymbol{R}_x[-\nu]\boldsymbol{R}_{al}\begin{bmatrix} \omega_{Elx}^2-\omega_E^2 & \omega_{Elx}\omega_{Ely} & \omega_{Elx}\omega_{Elz} \\ \omega_{Elx}\omega_{Ely} & \omega_{Ely}^2-\omega_E^2 & \omega_{Ely}\omega_{Elz} \\ \omega_{Elx}\omega_{Elz} & \omega_{Ely}\omega_{Elz} & \omega_{Elz}^2-\omega_E^2 \end{bmatrix}\begin{bmatrix} r_{olx}+r_{lx} \\ r_{oly}+r_{ly} \\ r_{olz}+r_{lz} \end{bmatrix}-2m\boldsymbol{R}_x[-\nu]\boldsymbol{R}_{al}\begin{bmatrix} 0 & -\omega_{Elz} & \omega_{Ely} \\ \omega_{Elz} & 0 & -\omega_{Elx} \\ -\omega_{Ely} & \omega_{Elx} & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}
\end{aligned}
$$

（2）引力假设：

（a）地球视为均质球体，忽略地球扁率及引力分量 g_ϕ 的影响，且服从平方反比定律

（b）引力加速度只有沿 y 轴的分量

（3）地球自转假设：忽略地球旋转的影响，即忽略哥式加速度和牵连加速度

（4）小角度假设：

（a）欧拉角以欧拉角 $\alpha,\beta,\gamma,\psi,\sigma,\nu,\theta-\varphi$ 以及控制量 $\delta_\varphi,\delta_\psi$ 均为小量，正弦取其角度，余弦取为 1；

（b）出现这些角度值之间的乘积时，作为二阶以上项略去

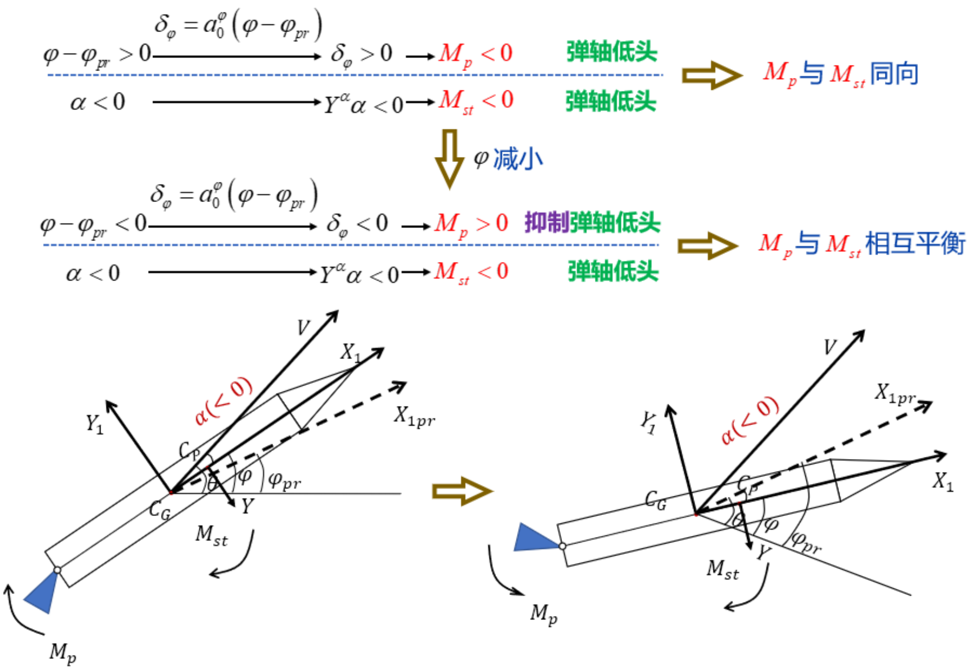
（5）力矩瞬时平衡假设

（6）简化形式

$$
\begin{aligned}
&\dot{v}=\frac{F_P-D}{m}+g\sin\left(\theta\right) \\
&\dot{\theta}=\frac{\left(F_P+CY^\alpha\right)\alpha}{mv}+\frac{g}{v}\cos\left(\theta\right) \\
&\dot{x}=v\cos\left(\theta\right) \\
&\dot{y}=v\sin\left(\theta\right) \\
&m=m_0-\dot{m}t
\end{aligned}
$$

2、画出纵向平面内的静不稳定火箭大气层内转弯受力图，并分析静不稳定火箭的（转弯过程）“三轴”变化规律。

关键步骤：



（1）注意程序段转弯的实际情况

（2）注意平衡的动态过程

3、推导关机方程与各变量的偏导关系

关键步骤：

$$\Delta L\left[\boldsymbol{X}(t_k),t_k\right]=\dot{L}\left[\overline{\boldsymbol{X}}(\overline{t_k}),\overline{t_k}\right]\left(t_k-\overline{t_k}\right)+\delta L\left[\boldsymbol{X}(t_k),t_k\right]$$

$$\delta L\left[\boldsymbol{X}(t_k),t_k\right]\approx\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\overline{t_k}}\delta\boldsymbol{X}(t_k)+\frac{1}{2}\delta\boldsymbol{X}(t_k)^T\left.\frac{\partial^2 L}{\partial \boldsymbol{X}^2}\right|_{\overline{t_k}}\delta\boldsymbol{X}(t_k)$$

$$\frac{\partial L}{\partial \boldsymbol{X}_\theta}=\begin{bmatrix}\frac{\partial L}{\partial r_x} & \frac{\partial L}{\partial r_y} & \frac{\partial L}{\partial r_z} & \frac{\partial L}{\partial V} & \frac{\partial L}{\partial \theta}\end{bmatrix}^{\mathrm{T}}$$

$$\frac{\partial^2 L}{\partial {\boldsymbol{X}_\theta}^2}=\begin{bmatrix}L_{r_xr_x} & L_{r_xr_y} & L_{r_xr_z} & L_{r_xV} & L_{r_x\theta}\\ L_{r_yr_x} & L_{r_yr_y} & L_{r_yr_z} & L_{r_yV} & L_{r_y\theta}\\ L_{r_zr_x} & L_{r_zr_y} & L_{r_zr_z} & L_{r_zV} & L_{r_z\theta}\\ L_{Vr_x} & L_{Vr_y} & L_{Vr_z} & L_{VV} & L_{V\theta}\\ L_{\theta r_x} & L_{\theta r_y} & L_{\theta r_z} & L_{\theta V} & L_{\theta\theta}\end{bmatrix}$$

$$L_{r_xr_x}=\frac{\partial^2 L}{\partial {r_x}^2}$$

$$L_{r_xr_y}=L_{r_yr_x}=\frac{\partial^2 L}{\partial r_x\partial r_y}$$

$$L_{r_xV}=L_{Vr_x}=\frac{\partial^2 L}{\partial V\partial r_x}$$

$$L_{r_x\theta}=L_{\theta r_x}=\frac{\partial^2 L}{\partial r_x\partial\theta}$$

$$L_{VV}=\frac{\partial^2 L}{\partial V^2}$$

$$L_{\theta\theta}=\frac{\partial^2 L}{\partial\theta^2}$$

$$L_{V\theta}=L_{\theta V}=\frac{\partial^2 L}{\partial V\partial\theta}$$

对射程偏差系数的计算和分析表明，在二阶射程偏差系数中，以下两个偏导数对落点射程偏差影响最大：

$$\frac{\partial^2 L}{\partial\theta^2}\quad\frac{\partial^2 L}{\partial V\partial\theta}$$

4、线性化动力学方程（发惯系和速度系下）

关键步骤：

（1）发惯系下：

（a）列写实际状态方程

$$
\begin{aligned}
\dot{r}_x &= v_x \\
\dot{r}_y &= v_y \\
\dot{r}_z &= v_z \\
\dot{v}_x &= \frac{1}{m}\cos(\psi_L)\cos(\varphi_L)F_p + \frac{1}{m}F_{Ax} + g_x \\
\dot{v}_y &= \frac{1}{m}\cos(\psi_L)\sin(\varphi_L)F_p + \frac{1}{m}F_{Ay} + g_y \\
\dot{v}_z &= -\frac{1}{m}\sin(\psi_L)F_p + \frac{1}{m}F_{Az} + g_z
\end{aligned}
$$

（b）列写标称状态方程

$$
\begin{aligned}
\dot{\bar{r}}_x &= \bar{v}_x \\
\dot{\bar{r}}_y &= \bar{v}_y \\
\dot{\bar{r}}_z &= \bar{v}_z \\
\dot{\bar{v}}_x &= \frac{1}{m}\cos(\bar{\psi}_L)\cos(\bar{\varphi}_L)\bar{F}_p + \bar{g}_x + \bar{f}_x \\
\dot{\bar{v}}_y &= \frac{1}{m}\cos(\bar{\psi}_L)\sin(\bar{\varphi}_L)\bar{F}_p + \bar{g}_y + \bar{f}_y \\
\dot{\bar{v}}_z &= -\frac{1}{m}\sin(\bar{\psi}_L)\bar{F}_p + \bar{g}_z + \bar{f}_z
\end{aligned}
$$

$$
\begin{bmatrix} \bar{f}_x & \bar{f}_y & \bar{f}_z \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \frac{1}{m}\bar{F}_{Ax} & \frac{1}{m}\bar{F}_{Ay} & \frac{1}{m}\bar{F}_{Az} \end{bmatrix}^{\mathrm{T}}
$$

（c）实际状态方程与标称状态方程相减，并经过泰勒一阶展开，得到线性化摄动方程

$$\delta \dot{r}_x = \delta v_x$$
$$\delta \dot{r}_y = \delta v_y$$
$$\delta \dot{r}_z = \delta v_z$$
$$\delta \dot{v}_x = \frac{\partial g_x}{\partial r_x}\delta r_x + \frac{\partial g_x}{\partial r_y}\delta r_y + \frac{\partial g_x}{\partial r_z}\delta r_z - \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right)\delta\varphi_L - \frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right)\delta\psi_L + f_x$$
$$\delta \dot{v}_y = \frac{\partial g_y}{\partial r_x}\delta r_x + \frac{\partial g_y}{\partial r_y}\delta r_y + \frac{\partial g_y}{\partial r_z}\delta r_z + \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right)\delta\varphi_L - \frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right)\delta\psi_L + f_y$$
$$\delta \dot{v}_z = \frac{\partial g_z}{\partial r_x}\delta r_x + \frac{\partial g_z}{\partial r_y}\delta r_y + \frac{\partial g_z}{\partial r_z}\delta r_z - \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\delta\psi_L + f_z$$

（d）列写为紧缩形势可以得到

$$\delta\dot{\boldsymbol{X}}(t) = \boldsymbol{A}(t)\delta\boldsymbol{X}(t) + \boldsymbol{B}(t)\delta\boldsymbol{U}(t) + \boldsymbol{C}(t)\boldsymbol{f}(t)$$

$$\boldsymbol{A}(t) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial g_x}{\partial r_x} & \frac{\partial g_x}{\partial r_y} & \frac{\partial g_x}{\partial r_z} & 0 & 0 & 0 \\ \frac{\partial g_y}{\partial r_x} & \frac{\partial g_y}{\partial r_y} & \frac{\partial g_y}{\partial r_z} & 0 & 0 & 0 \\ \frac{\partial g_z}{\partial r_x} & \frac{\partial g_z}{\partial r_y} & \frac{\partial g_z}{\partial r_z} & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{B}(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right) & -\frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right) \\ \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right) & -\frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right) \\ 0 & -\frac{F_p}{m}\cos\left(\bar{\psi}_L\right) \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

（2）速度系下推导过程与惯性系下相同

$$\begin{aligned}
&\dot{v}=\frac{F_P-D}{m}+g\sin\theta\\
&\dot{\theta}=\frac{F_P\left(\delta_\varphi+\alpha\right)+C_L q S_M}{mv}+\frac{g}{v}\cos\theta\\
&\dot{y}=v\sin\theta\\
&\dot{x}=v\cos\theta\\
&\alpha=A\left(\varphi_{pr}-\theta\right)
\end{aligned}$$

$$\begin{aligned}
&\dot{\bar{v}}=\frac{F_P}{m}+\bar{g}\sin\bar{\theta}\\
&\dot{\bar{\theta}}=\frac{1}{m\bar{v}}F_P\left(\bar{\delta}_\varphi+\bar{\alpha}\right)+\frac{\bar{g}}{\bar{v}}\cos\bar{\theta}\\
&\dot{\bar{y}}=\bar{v}\sin\bar{\theta}\\
&\dot{\bar{x}}=\bar{v}\cos\bar{\theta}\\
&\bar{\alpha}=A\left(\varphi_{pr}-\bar{\theta}\right)
\end{aligned}$$

$$\begin{aligned}
&\delta\dot{v}=\left[\frac{\partial g_x}{\partial x}\delta x+\frac{\partial g_x}{\partial y}\delta y\right]\sin\bar{\theta}+\bar{g}\cos\bar{\theta}\delta\theta+f_x\\
&\delta\dot{\theta}=\left[\frac{\partial g_x}{\partial x}\delta x+\frac{\partial g_x}{\partial y}\delta y\right]\frac{\cos\bar{\theta}}{v}-\left(\frac{\bar{g}}{\bar{v}}\sin\bar{\theta}+\frac{F_P}{m\bar{v}}A\right)\delta\theta-\frac{1}{\bar{v}^2}\left(\bar{g}\cos\bar{\theta}+\frac{F_P\left(\bar{\delta}_\varphi+A\left(\varphi_{pr}-\bar{\theta}\right)\right)}{m}\right)\delta v+\frac{F_P}{m\bar{v}}\delta(\delta_\varphi)+f_y\\
&\delta\dot{y}=\bar{v}\cos\bar{\theta}\delta\theta+\sin\bar{\theta}\delta v\\
&\delta\dot{x}=-\bar{v}\sin\bar{\theta}\delta\theta+\cos\bar{\theta}\delta v
\end{aligned}$$

注意：1. 在制导过程中，认为控制参数为姿态角 δ_φ，而非攻角 α（作业中均判正确，计算无误即可，但在考试中要注意准确性）。

2. 干扰力部分不可省略；

5、分析横向导引方程的原理

关键步骤：（1）推导落点横向偏差

$$\Delta L\left[\boldsymbol{X}(t_k),t_k\right]=\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]\Delta t_k+\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}\delta\boldsymbol{X}(t_k)=0$$

$$\Delta H\left[\boldsymbol{X}(t_k),t_k\right]=\dot{H}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]\Delta t_k+\left.\frac{\partial H}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}\delta\boldsymbol{X}(t_k)$$

$$\Delta t_k=-\frac{1}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}\delta\boldsymbol{X}(t_k)$$

$$\Delta H\left[\boldsymbol{X}(t_k),t_k\right]=\left[\left.\frac{\partial H}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}-\frac{\dot{H}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}\right]\delta\boldsymbol{X}(t_k)=0$$

（2）预测关机时间偏差

$$\begin{aligned}\Delta L\left[\boldsymbol{X}(t_k),t_k\right]&=\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]\Delta t_k+\delta L\left[\boldsymbol{X}(t_k),t_k\right]\\&=\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]\Delta t_k-\boldsymbol{\Lambda}_{s2}^T(t)\delta\boldsymbol{X}(t)\Delta t_k+\boldsymbol{\Lambda}_{s1}^T(t)\delta\boldsymbol{X}(t)\end{aligned}$$

$$\Delta t_k=-\frac{\boldsymbol{\Lambda}_{s1}^T(t)}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]-\boldsymbol{\Lambda}_{s2}^T(t)\delta\boldsymbol{X}(t)}\delta\boldsymbol{X}(t)$$

（3）预测落点横向偏差

$$\Delta H\left[\boldsymbol{X}(t_k),t_k\right]=\left[\left.\frac{\partial H}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}-\frac{\dot{H}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}\right]\delta\boldsymbol{X}(t_k)=0$$

$$\boldsymbol{\Lambda}_h(t_k)=\left[\left.\frac{\partial H}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}-\frac{\dot{H}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\bar{t}_k}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Lambda}_h(t)=\boldsymbol{\Lambda}_{h1}(t)-\boldsymbol{\Lambda}_{h2}(t)\Delta t_k$$

$$\Delta t_k=-\frac{\boldsymbol{\Lambda}_{s1}^T(t)}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]-\boldsymbol{\Lambda}_{s2}^T(t)\delta\boldsymbol{X}(t)}\delta\boldsymbol{X}(t)$$

（4）横向导引策略

$$\Delta H\left[\boldsymbol{X}(t_k),t_k\right]=\boldsymbol{\Lambda}_n^{\mathrm{T}}(t_k)\delta\boldsymbol{X}(t_k)=\boldsymbol{\Lambda}_n^{\mathrm{T}}(t)\delta\boldsymbol{X}(t)$$

$$\delta\psi_L(t)=\boldsymbol{K}_\psi\Delta H$$

注意：作业中此处还没有讲到伴随状态量的求解，故作业中不做要求，但考试中需要完整做答。

6. 推导使用导航信息的摄动制导算法（关机方程，法向导引）

关键步骤：

（1）射程关机方程为：

$$\Delta L\left[\boldsymbol{X}(t_k),t_k\right]=\dot{L}\left[\overline{\boldsymbol{X}}(\overline{t}_k),\overline{t}_k\right]\Delta t_k+\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\overline{t}_k}\delta\boldsymbol{X}(t_k)=0$$

（2）定义伴随系统为：

$$\dot{\boldsymbol{\Lambda}}_s(t)=-\boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{\Lambda}_s(t)$$

$$\boldsymbol{\Lambda}_s(t_k)=\left(\left.\frac{\partial L}{\partial \boldsymbol{X}}\right|_{\overline{t}_k}\right)^{\mathrm{T}}=\begin{bmatrix}\left.\frac{\partial L}{\partial r_x}\right|_{\overline{t}_k} & \left.\frac{\partial L}{\partial r_y}\right|_{\overline{t}_k} & \left.\frac{\partial L}{\partial r_z}\right|_{\overline{t}_k} & \left.\frac{\partial L}{\partial v_x}\right|_{\overline{t}_k} & \left.\frac{\partial L}{\partial v_y}\right|_{\overline{t}_k} & \left.\frac{\partial L}{\partial v_z}\right|_{\overline{t}_k}\end{bmatrix}^{\mathrm{T}}$$

解得：

$$\boldsymbol{\Lambda}_s(t)=\Phi\left(t,t_k\right)\boldsymbol{\Lambda}_s(t_k)_k$$

（3）泰勒展开获得：

$$\begin{aligned}\boldsymbol{\Lambda}_s(t)&=\Phi\left(t,t_k\right)\boldsymbol{\Lambda}_s(t_k)\\&=\left(\Phi\left(t,\overline{t}_k\right)+\left.\frac{d\Phi\left(t,t_k\right)}{dt_k}\right|_{t_k=\overline{t}_k}\Delta t_k\right)\boldsymbol{\Lambda}_s(t_k)\\&=\Phi\left(t,\overline{t}_k\right)\boldsymbol{\Lambda}_s(t_k)-\left(-\Phi\left(t,\overline{t}_k\right)\boldsymbol{A}^{\mathrm{T}}(\overline{t}_k)\boldsymbol{\Lambda}_s(t_k)\right)\Delta t_k\end{aligned}$$

（4）将 $\boldsymbol{\Lambda}_s(t)$ 近似为两部分求解

$$\boldsymbol{\Lambda}_s(t)=\boldsymbol{\Lambda}_{s1}(t)-\boldsymbol{\Lambda}_{s2}(t)\Delta t_k$$

$$\begin{aligned}\boldsymbol{\Lambda}_{s1}(t)&=\Phi\left(t,\overline{t}_k\right)\boldsymbol{\Lambda}_s(t_k)\\\boldsymbol{\Lambda}_{s1}(\overline{t}_k)&=\boldsymbol{\Lambda}_s(t_k)\end{aligned}$$

$$\boldsymbol{\Lambda}_{s2}(t)=-\Phi\left(t,\overline{t}_k\right)\boldsymbol{A}^{\mathrm{T}}(\overline{t}_k)\boldsymbol{\Lambda}_s(t_k)=-\Phi\left(t,\overline{t}_k\right)\boldsymbol{A}^{\mathrm{T}}(\overline{t}_k)\boldsymbol{\Lambda}_{s1}(\overline{t}_k)=\Phi\left(t,\overline{t}_k\right)\dot{\boldsymbol{\Lambda}}_{s1}(\overline{t}_k)$$

$$\boldsymbol{\Lambda}_{s2}(\overline{t}_k)=\dot{\boldsymbol{\Lambda}}_{s1}(\overline{t}_k)$$

（5）得到关机时间偏差

$$\begin{aligned}\Delta L\left[\boldsymbol{X}(t_k),t_k\right]&=\dot{L}\left[\overline{\boldsymbol{X}}(\overline{t}_k),\overline{t}_k\right]\Delta t_k+\delta L\left[\boldsymbol{X}(t_k),\overline{t}_k\right]\\&=\dot{L}\left[\overline{\boldsymbol{X}}(\overline{t}_k),\overline{t}_k\right]\Delta t_k-\boldsymbol{\Lambda}_{s2}^{T}(t)\delta\boldsymbol{X}(t)\Delta t_k+\boldsymbol{\Lambda}_{s1}^{T}(t)\delta\boldsymbol{X}(t)\\&=0\end{aligned}$$

$$\Delta t_k=-\frac{\boldsymbol{\Lambda}_{s1}^{T}(t)}{\dot{L}\left[\overline{\boldsymbol{X}}(\overline{t}_k),\overline{t}_k\right]-\boldsymbol{\Lambda}_{s2}^{T}(t)\delta\boldsymbol{X}(t)}\delta\boldsymbol{X}(t)$$

（6）推导关机时刻速度倾角偏差

$$\Delta\theta\left[\boldsymbol{X}(t_k),t_k\right]=\left[\left.\frac{\partial\theta}{\partial\boldsymbol{X}}\right|_{\bar{t}_k}-\frac{\dot{\theta}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}\left.\frac{\partial L}{\partial\boldsymbol{X}}\right|_{\bar{t}_k}\right]\delta\boldsymbol{X}(t_k)$$

$$\boldsymbol{\Lambda}_n(t_k)=\left[\left.\frac{\partial\theta}{\partial\boldsymbol{X}}\right|_{\bar{t}_k}-\frac{\dot{\theta}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}{\dot{L}\left[\bar{\boldsymbol{X}}(\bar{t}_k),\bar{t}_k\right]}\left.\frac{\partial L}{\partial\boldsymbol{X}}\right|_{\bar{t}_k}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Lambda}_n(t)=\boldsymbol{\Lambda}_{n1}(t)-\boldsymbol{\Lambda}_{n2}(t)\Delta t_k$$

$$\Delta\theta\left[\boldsymbol{X}(t_k),t_k\right]=\boldsymbol{\Lambda}_n^{\mathrm{T}}(t_k)\delta\boldsymbol{X}(t_k)=\boldsymbol{\Lambda}_n^{\mathrm{T}}(t)\delta\boldsymbol{X}(t)$$

（7）选择控制律并设计控制方程

$$\delta\varphi_L(t)=\boldsymbol{K}_\varphi\Delta\theta$$

注意：要求推导过程流畅严谨，逻辑清晰（不可有缺漏或跳跃）。

7. 在无导航信息解算条件下，推导使用惯性平台的摄动制导算法（关机方程，法向导引）

关键步骤：

（1）构建全量线性系统

$$
\begin{aligned}
&\dot{r}_x = v_x \\
&\dot{r}_y = v_y \\
&\dot{r}_z = v_z \\
&\dot{v}_x = \frac{\partial g_x}{\partial r_x} r_x + \frac{\partial g_x}{\partial r_y} r_y + \frac{\partial g_x}{\partial r_z} r_z + a_{mx} + \tilde{g}_x \\
&\dot{v}_y = \frac{\partial g_y}{\partial r_x} r_x + \frac{\partial g_y}{\partial r_y} r_y + \frac{\partial g_y}{\partial r_z} r_z + a_{my} + \tilde{g}_y \\
&\dot{v}_z = \frac{\partial g_z}{\partial r_x} r_x + \frac{\partial g_z}{\partial r_y} r_y + \frac{\partial g_z}{\partial r_z} r_z + a_{mz} + \tilde{g}_z
\end{aligned}
$$

$$
\begin{aligned}
&\tilde{g}_x = \overline{g}_x - \frac{\partial g_x}{\partial r_x} \overline{r}_x - \frac{\partial g_x}{\partial r_y} \overline{r}_y - \frac{\partial g_x}{\partial r_z} \overline{r}_z \\
&\tilde{g}_y = \overline{g}_y - \frac{\partial g_y}{\partial r_x} \overline{r}_x - \frac{\partial g_y}{\partial r_y} \overline{r}_y - \frac{\partial g_y}{\partial r_z} \overline{r}_z \\
&\tilde{g}_z = \overline{g}_z - \frac{\partial g_z}{\partial r_x} \overline{r}_x - \frac{\partial g_z}{\partial r_y} \overline{r}_y - \frac{\partial g_z}{\partial r_z} \overline{r}_z
\end{aligned}
$$

定义状态量为：

$$
\boldsymbol{X} = \begin{bmatrix} r_x & r_y & r_z & v_x & v_y & v_z \end{bmatrix}^{\mathrm{T}}
$$

定义控制量为：

$$
\boldsymbol{U} = \begin{bmatrix} a_{mx} + \tilde{g}_x & a_{my} + \tilde{g}_y & a_{mz} + \tilde{g}_z \end{bmatrix}^{\mathrm{T}}
$$

（2）基于无导航情况下的 ΔL 求解关机时间偏差 Δt_k

参照 PPT 得到

$$
\Delta t_k \approx \frac{\left.\dfrac{\partial L}{\partial t_k}\right|_{\overline{t}_k} - \boldsymbol{\Lambda}_{s1}^T(0)\boldsymbol{X}(0) - \int_0^t \boldsymbol{\Lambda}_{s1}^T(\tau)\boldsymbol{B}(\tau)\boldsymbol{U}(\tau)\mathrm{d}\tau - \int_t^{\overline{t}_k} \boldsymbol{\Lambda}_{s1}^T(\tau)\boldsymbol{B}(\tau)\overline{\boldsymbol{U}}(\tau)\mathrm{d}\tau}{\left.\dfrac{\partial L}{\partial t_k}\right|_{\overline{t}_k} - \boldsymbol{\Lambda}_{s2}^T(0)\boldsymbol{X}(0) - \int_0^t \boldsymbol{\Lambda}_{s2}^T(\tau)\boldsymbol{B}(\tau)\boldsymbol{U}(\tau)\mathrm{d}\tau - \int_t^{\overline{t}_k} \boldsymbol{\Lambda}_{s2}^T(\tau)\boldsymbol{B}(\tau)\overline{\boldsymbol{U}}(\tau)\mathrm{d}\tau}
$$

（3）干扰抑制原理的应用

在标称轨迹附近，将质心运动方程进行一阶线性展开，得到：

$$\delta\dot{r}_x = \delta v_x$$
$$\delta\dot{r}_y = \delta v_y$$
$$\delta\dot{r}_z = \delta v_z$$
$$\delta\dot{v}_x = \frac{\partial g_x}{\partial r_x}\delta r_x + \frac{\partial g_x}{\partial r_y}\delta r_y + \frac{\partial g_x}{\partial r_z}\delta r_z - \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right)\delta\varphi_L - \frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right)\delta\psi_L + a_{mx} - \frac{1}{m}\cos\left(\psi_L\right)\cos\left(\varphi_L\right)F_p$$
$$\delta\dot{v}_y = \frac{\partial g_y}{\partial r_x}\delta r_x + \frac{\partial g_y}{\partial r_y}\delta r_y + \frac{\partial g_y}{\partial r_z}\delta r_z + \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right)\delta\varphi_L - \frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right)\delta\psi_L + a_{my} - \frac{1}{m}\cos\left(\psi_L\right)\sin\left(\varphi_L\right)F_p$$
$$\delta\dot{v}_z = \frac{\partial g_z}{\partial r_x}\delta r_x + \frac{\partial g_z}{\partial r_y}\delta r_y + \frac{\partial g_z}{\partial r_z}\delta r_z - \frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\delta\psi_L + a_{mz} + \frac{1}{m}\sin\left(\psi_L\right)F_p$$

干扰力部分为：

$$f_x = \frac{1}{m}F_{Ax} = a_{mx} - \frac{1}{m}\cos\left(\psi_L\right)\cos\left(\varphi_L\right)F_p$$
$$f_y = \frac{1}{m}F_{Ay} = a_{my} - \frac{1}{m}\cos\left(\psi_L\right)\sin\left(\varphi_L\right)F_p$$
$$f_z = \frac{1}{m}F_{Az} = a_{mz} + \frac{1}{m}\sin\left(\psi_L\right)F_p$$

根据干扰抑制定理得到：

$$\delta\varphi_L(t) = -\left[\boldsymbol{\Lambda}_n^{\mathrm{T}}(t)\boldsymbol{B}_n(t)\right]^{-1}\boldsymbol{\Lambda}_n^{\mathrm{T}}(t)\boldsymbol{C}_n(t)\boldsymbol{f}(t)$$

$$\begin{aligned}\delta\varphi_L(t) = -&\left[-\Lambda_{n4}(t)\frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right) - \Lambda_{n5}(t)\frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right)\right]^{-1}\times\\
&[\Lambda_{n4}(t)\left(a_{mx} - \frac{1}{m}\cos\left(\psi_L\right)\cos\left(\varphi_L\right)F_p\right)\\
&+\Lambda_{n5}(t)\left(a_{my} - \frac{1}{m}\cos\left(\psi_L\right)\sin\left(\varphi_L\right)F_p\right)\\
&+\Lambda_{n6}(t)\left(a_{mz} + \frac{1}{m}\sin\left(\psi_L\right)F_p\right)]\end{aligned}$$

$$\delta\psi_L(t) = -\left[\boldsymbol{\Lambda}_h^{\mathrm{T}}(t)\boldsymbol{B}_h(t)\right]^{-1}\boldsymbol{\Lambda}_h^{\mathrm{T}}(t)\boldsymbol{C}_h(t)\boldsymbol{f}(t)$$

$$\begin{aligned}\delta\psi_L(t) = -&\left[-\Lambda_{n4}(t)\frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\cos\left(\bar{\varphi}_L\right) - \Lambda_{n5}(t)\frac{F_p}{m}\sin\left(\bar{\psi}_L\right)\sin\left(\bar{\varphi}_L\right) - \Lambda_{n6}(t)\frac{F_p}{m}\cos\left(\bar{\psi}_L\right)\right]^{-1}\times\\
&[\Lambda_{n4}(t)\left(a_{mx} - \frac{1}{m}\cos\left(\psi_L\right)\cos\left(\varphi_L\right)F_p\right)\\
&+\Lambda_{n5}(t)\left(a_{my} - \frac{1}{m}\cos\left(\psi_L\right)\sin\left(\varphi_L\right)F_p\right)\\
&+\Lambda_{n6}(t)\left(a_{mz} + \frac{1}{m}\sin\left(\psi_L\right)F_p\right)]\end{aligned}$$

注意：要求推导过程流畅严谨，逻辑清晰（不可有缺漏或跳跃）。

8. 推导满足下面条件的显式制导算法（要求写出三通道制导指令规律）

已知：1）当前时刻 t_0 的位置和速度：$\boldsymbol{x}(t_0)=\boldsymbol{x}_0$，$\dot{\boldsymbol{x}}(t_0)=\dot{\boldsymbol{x}}_0$

2）以推力加速度为控制量：$\ddot{\boldsymbol{x}}(t)=\boldsymbol{g}(\boldsymbol{r})+\boldsymbol{a}_P(t)$

3）终端约束包括：终端位置、终端速度与终端加速度

终端 t_{f} 时刻，到达目标位置和速度：$\boldsymbol{x}(t_{\mathrm{f}})=\boldsymbol{x}_{\mathrm{f}}$，$\dot{\boldsymbol{x}}(t_{\mathrm{f}})=\dot{\boldsymbol{x}}_{\mathrm{f}}$

终端 t_{f} 时刻，推力方向：$\dfrac{\boldsymbol{a}_{Px}(t_{\mathrm{f}})}{|\boldsymbol{a}_P(t_{\mathrm{f}})|}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}^{\mathrm{T}}$

关键步骤：

（1）对于 y 轴和 z 轴，终端约束包括位置，速度和加速度：

以 y 轴为例，（a）构建多项式：

$$\ddot{y}(t)=c_{y0}+c_{y1}(t_{\mathrm{f}}-t)+c_{y2}(t_{\mathrm{f}}-t)^2$$

$$c_{y0}=\ddot{y}_{\mathrm{f}}=g_y(t_{\mathrm{f}})$$

（b）积分计算

$$\dot{y}_{\mathrm{f}}-\dot{y}_0=\int_{t_0}^{t_{\mathrm{f}}}\ddot{y}(t)\mathrm{d}t=c_{y0}t_{\mathrm{go}}+\frac{1}{2}c_{y1}t_{\mathrm{go}}^2+\frac{1}{3}c_{y2}t_{\mathrm{go}}^3$$

$$y_{\mathrm{f}}-y_0-\dot{y}_0t_{\mathrm{go}}=\int_{t_0}^{t_{\mathrm{f}}}\left[\int_{t_0}^{t}\ddot{y}(s)\mathrm{d}s\right]\mathrm{d}t=\frac{1}{2}c_{y0}t_{\mathrm{go}}^2+\frac{1}{3}c_{y1}t_{\mathrm{go}}^3+\frac{1}{4}c_{y2}t_{\mathrm{go}}^4$$

（c）反解 E 矩阵得到参数 c_1 和 c_2

$$\dot{y}_{\mathrm{f}}-\dot{y}_0-c_{y0}t_{go}=\frac{1}{2}c_{y1}t_{go}^2+\frac{1}{3}c_{y2}t_{go}^3$$

$$y_{\mathrm{f}}-y_0-\dot{y}_0t_{\mathrm{go}}-\frac{1}{2}c_{y0}t_{go}^2=\frac{1}{3}c_{y1}t_{go}^3+\frac{1}{4}c_{y2}t_{go}^4$$

$$\begin{bmatrix}\dot{y}_{\mathrm{f}}-\dot{y}_0-c_0t_{go}\\ y_{\mathrm{f}}-y_0-\dot{y}_0t_{\mathrm{go}}-\frac{1}{2}c_0t_{go}^2\end{bmatrix}=\begin{bmatrix}\frac{1}{2}t_{go}^2 & \frac{1}{3}t_{go}^3\\ \frac{1}{3}t_{go}^3 & \frac{1}{4}t_{go}^4\end{bmatrix}\begin{bmatrix}c_{y1}\\ c_{y2}\end{bmatrix}$$

$$E=\begin{bmatrix}\frac{1}{2}t_{go}^2 & \frac{1}{3}t_{go}^3\\ \frac{1}{3}t_{go}^3 & \frac{1}{4}t_{go}^4\end{bmatrix}^{-1}=\begin{bmatrix}\dfrac{18}{t_{go}^2} & -\dfrac{24}{t_{go}^3}\\ -\dfrac{24}{t_{go}^3} & -\dfrac{36}{t_{go}^4}\end{bmatrix}$$

$$\begin{bmatrix} c_{y1} \\ c_{y2} \end{bmatrix} = E\begin{bmatrix} \dot{y}_{\mathrm{f}} - \dot{y}_0 - c_0 t_{go} \\ y_{\mathrm{f}} - y_0 - \dot{y}_0 t_{\mathrm{go}} - \frac{1}{2}c_0 t_{go}^2 \end{bmatrix} = \begin{bmatrix} \frac{18}{t_{go}^2}\left(\dot{y}_{\mathrm{f}} - \dot{y}_0 - c_0 t_{go}\right) - \frac{24}{t_{go}^3}\left(y_{\mathrm{f}} - y_0 - \dot{y}_0 t_{\mathrm{go}} - \frac{1}{2}c_{y0} t_{go}^2\right) \\ -\frac{24}{t_{go}^3}\left(\dot{y}_{\mathrm{f}} - \dot{y}_0 - c_0 t_{go}\right) - \frac{36}{t_{go}^4}\left(y_{\mathrm{f}} - y_0 - \dot{y}_0 t_{\mathrm{go}} - \frac{1}{2}c_{y0} t_{go}^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{24}{t_{go}^3}\left(y_{\mathrm{f}} - y_0\right) + \frac{6}{t_{go}^2}\left(3\dot{y}_{\mathrm{f}} + \dot{y}_0\right) - \frac{6c_{y0}}{t_{go}} \\ \frac{36}{t_{go}^4}\left(y_{\mathrm{f}} - y_0\right) - \frac{12}{t_{go}^3}\left(2\dot{y}_{\mathrm{f}} + \dot{y}_0\right) + \frac{6c_{y0}}{t_{go}} \end{bmatrix}$$

（d）最终得到推力加速度指令为

$$a_y(t) = \ddot{y}(t) - g_y(t) = g_y(t_{\mathrm{f}}) - 6\left(\dot{y}_{\mathrm{f}} + \dot{y}_0\right)/t_{\mathrm{go}} + 12\left(y_{\mathrm{f}} - y_0\right)/t_{\mathrm{go}}^2 - g_y(t)$$

（e）z 轴指令同理可得：

$$a_z(t) = \ddot{z}(t) - g_z(t) = g_z(t_{\mathrm{f}}) - 6\left(\dot{z}_{\mathrm{f}} + \dot{z}_0\right)/t_{\mathrm{go}} + 12\left(z_{\mathrm{f}} - z_0\right)/t_{\mathrm{go}}^2 - g_z(t)$$

（1）对于 x 轴，终端约束包括位置，速度：

（a）构建多项式：

$$\ddot{x}(t) = c_{x1} + c_{x2}\left(t_{\mathrm{f}} - t\right)$$

（b）积分计算

$$\dot{x}_{\mathrm{f}} - \dot{x}_0 = \int_{t_0}^{t_{\mathrm{f}}} \ddot{x}(t)\mathrm{d}t = c_{x1}t_{\mathrm{go}} + \frac{1}{2}c_{x2}t_{\mathrm{go}}^2$$

$$x_{\mathrm{f}} - x_0 - \dot{x}_0 t_{\mathrm{go}} = \int_{t_0}^{t_{\mathrm{f}}}\left[\int_{t_0}^{t}\ddot{x}(s)\mathrm{d}s\right]\mathrm{d}t = \frac{1}{2}c_{x1}t_{go}^2 + \frac{1}{3}c_{x2}t_{go}^3$$

（c）反解 E 矩阵得到参数 c_1 和 c_2

$$\dot{x}_{\mathrm{f}} - \dot{x}_0 = c_{x1}t_{\mathrm{go}} + \frac{1}{2}c_{x2}t_{\mathrm{go}}^2$$

$$x_{\mathrm{f}} - x_0 - \dot{x}_0 t_{\mathrm{go}} = \frac{1}{2}c_{x1}t_{go}^2 + \frac{1}{3}c_{x2}t_{go}^3$$

$$\begin{bmatrix} \dot{x}_{\mathrm{f}} - \dot{x}_0 \\ x_{\mathrm{f}} - x_0 - \dot{x}_0 t_{\mathrm{go}} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2}t_{\mathrm{go}}^2 \\ \frac{1}{2}t_{\mathrm{go}}^2 & \frac{1}{3}t_{\mathrm{go}}^3 \end{bmatrix}\begin{bmatrix} c_{x1} \\ c_{y2} \end{bmatrix}$$

$$E=\begin{bmatrix}1 & \frac{1}{2}t_{\mathrm{go}}^2\\ \frac{1}{2}t_{\mathrm{go}}^2 & \frac{1}{3}t_{\mathrm{go}}^3\end{bmatrix}^{-1}=\begin{bmatrix}\frac{4}{t_{\mathrm{go}}} & -\frac{6}{t_{\mathrm{go}}^2}\\ -\frac{6}{t_{\mathrm{go}}^2} & \frac{12}{t_{\mathrm{go}}^3}\end{bmatrix}$$

$$\begin{bmatrix}c_{x1}\\ c_{x2}\end{bmatrix}=E\begin{bmatrix}\dot{x}_{\mathrm{f}}-\dot{x}_0\\ x_{\mathrm{f}}-x_0-\dot{x}_0t_{\mathrm{go}}\end{bmatrix}=\begin{bmatrix}-\frac{6}{t_{\mathrm{go}}^2}\left(x_{\mathrm{f}}-x_0\right)+\frac{1}{t_{\mathrm{go}}}\left(4\dot{x}_{\mathrm{f}}+2\dot{x}_0\right)\\ \frac{12}{t_{\mathrm{go}}^3}\left(x_{\mathrm{f}}-x_0\right)-\frac{6}{t_{\mathrm{go}}^2}\left(\dot{x}_{\mathrm{f}}+\dot{x}_0\right)\end{bmatrix}$$

（d）最终得到推力加速度指令为

$$a_x(t)=\ddot{x}(t)-g_x=-\left(2\dot{x}_{\mathrm{f}}+4\dot{x}_0\right)/t_{\mathrm{go}}+6\left(x_{\mathrm{f}}-x_0\right)/t_{\mathrm{go}}^2-g_x(t)$$

注意：1. x轴需要单独分类讨论；

2. 题目中已知量为终端推力方向，终端推力大小为未知量，不可以拿未知量来进行制导计算；

3. y 轴终端加速度为 $g_y(t_{\mathrm{f}})$，推导制导指令时不可以等同于时间函数 $g_y(t)$；

z轴同理

9、推导偏航与滚转通道姿态运动的线性化模型

关键步骤：

（1）列写原方程

$$\begin{cases} -mv\dot{\sigma} = -mg\sin\theta\sin\sigma - F_P\cos\alpha\sin\beta - C_z^{\beta}qS_M\beta - \dfrac{F_P}{2}\delta_{\psi} + F_z \\ J_y\dot{\omega}_y + \left(J_x - J_z\right)\omega_z\omega_x = -m_y^{\beta}qS_M l_k\beta - \dfrac{m_y^{\omega_y}qS_M l_k^2}{v}\omega_y - \dfrac{F_P}{2}\left(x_c - x_g\right)\delta_{\psi} + M_y \end{cases}$$

$$J_x\dot{\omega}_x + \left(J_z - J_y\right)\omega_y\omega_z = -\frac{m_x^{\omega_x}qS_M l_k^2}{v}\omega_x - F_P z_c\delta_{\gamma} + M_x$$

（2）小角度假设

$$-mv\Delta\dot{\sigma} = -mg\sin(\theta_0 + \Delta\theta)\Delta\sigma - F_P\cos\alpha_0\Delta\beta - C_z^{\beta}qS_M\Delta\beta - \frac{F_P}{2}\delta_{\psi} + F_z$$

$$J_y\left(\Delta\ddot{\psi} + \dot{\varphi}_0\Delta\dot{\gamma}\right) + \left(J_x - J_z\right)\dot{\varphi}(\Delta\dot{\gamma} - \dot{\varphi}_0\Delta\psi) = -m_y^{\beta}qS_M l_k\Delta\beta - \frac{m_y^{\omega_y}qS_M l_k^2}{v}\left(\Delta\dot{\psi} + \dot{\varphi}_0\Delta\gamma\right) - \frac{F_P}{2}\left(x_c - x_g\right)\delta_{\psi} + M_y$$

$$J_x\left(\Delta\ddot{\gamma} - \dot{\varphi}_0\Delta\dot{\psi}\right) + \left(J_z - J_y\right)\dot{\varphi}(\Delta\dot{\psi} + \dot{\varphi}_0\Delta\gamma) = -\frac{m_x^{\omega_x}qS_M l_k^2}{v}\left(\Delta\dot{\gamma} - \dot{\varphi}_0\Delta\psi\right) - F_P z_c\delta_{\gamma} + M_x$$

（3）最终结果

$$\Delta\dot{\sigma} = \left(\frac{F_P\cos\alpha_0}{mv} + \frac{C_z^{\beta}qS_M}{mv}\right)\Delta\beta + \frac{g\sin\theta_0}{v}\Delta\sigma + \frac{F_P}{2mv}\Delta\delta_{\psi} - \frac{F_z}{mv} = c_1\Delta\beta + c_2\Delta\sigma + c_3\Delta\delta_{\psi} + \bar{F}_z$$

$$\Delta\ddot{\psi} + \frac{m_y^{\omega_y}qS_M l_k^2}{J_y v}\Delta\dot{\psi} + \frac{m_y^{\beta}qS_M l_k}{J_y}\Delta\beta + \frac{F_P}{2J_y}\left(x_c - x_g\right)\Delta\delta_{\psi} = \frac{M_y}{J_y}$$

$$\Delta\ddot{\gamma} + \frac{m_x^{\omega_x}qS_M l_k^2}{J_x v}\Delta\dot{\gamma} + \frac{F_P z_c}{J_x}\Delta\delta_{\gamma} = \frac{M_x}{J_x}$$

$$\psi = \sigma + \beta$$

注意：1. 偏航通道原方程中，推力分量进行了修正（作业中不扣分）

10、推导偏航与滚转通道姿态运动的传递函数

说明：

$$\psi = \sigma + \beta$$

$$\Delta\dot{\sigma} = \left(\frac{F_P}{mv} + \frac{C_z^\beta q S_M}{mv}\right)\Delta\beta + \frac{g\sin\theta_0}{v}\Delta\sigma + \frac{F_P}{2mv}\Delta\delta_\psi - \frac{F_z}{mv} = c_1\Delta\beta + c_2\Delta\sigma + c_3\Delta\delta_\psi + \bar{F}_z$$

$$\Delta\ddot{\psi} + \frac{m_y^{\omega_y} q S_M l_k^2}{J_y v}\Delta\dot{\psi} + \frac{m_y^\beta q S_M l_k}{J_y}\Delta\beta + \frac{F_P}{2J_y}\left(x_c - x_g\right)\Delta\delta_\psi = \frac{M_y}{J_y}$$

$$\Delta\ddot{\gamma} + \frac{m_x^{\omega_x} q S_M l_k^2}{J_x v}\Delta\dot{\gamma} + \frac{F_P z_c}{J_x}\Delta\delta_\gamma = \frac{M_x}{J_x}$$

$$\psi = \sigma + \beta$$

$$\Delta\dot{\sigma} = \left(\frac{F_P}{mv} + \frac{C_z^\beta q S_M}{mv}\right)\Delta\beta + \frac{g\sin\theta_0}{v}\Delta\sigma + \frac{F_P}{2mv}\Delta\delta_\psi - \frac{F_z}{mv} = c_1\Delta\beta + c_2\Delta\sigma + c_3\Delta\delta_\psi + \bar{F}_z$$

$$\Delta\ddot{\psi} = -\frac{m_y^{\omega_y} q S_M l_k^2}{J_y v}\Delta\dot{\psi} - \frac{m_y^\beta q S_M l_k}{J_y}\left(\Delta\psi - \Delta\sigma\right) - \frac{F_P}{2J_y}\left(x_c - x_g\right)\Delta\delta_\psi + \frac{M_y}{J_y}$$

$$\Delta\ddot{\gamma} = -\frac{m_x^{\omega_x} q S_M l_k^2}{J_x v}\Delta\dot{\gamma} - \frac{F_P z_c}{J_x}\Delta\delta_\gamma + \frac{M_x}{J_x}$$

$$\begin{aligned}
&\Delta\dot{\sigma} = \left(c_2 - c_1\right)\Delta\sigma + c_1\Delta\psi + c_3\Delta\delta_\psi + \bar{F}_z \\
&\Delta\dot{\psi} = \Delta\psi \\
&\Delta\ddot{\psi} = b_2\Delta\sigma - b_2\Delta\psi - b_1\Delta\dot{\psi} - b_3\Delta\delta_\psi + \bar{M}_y \\
&\Delta\ddot{\gamma} = -d_1\Delta\dot{\gamma} - d_2\Delta\delta_\gamma + \bar{M}_x
\end{aligned}$$

忽略干扰力和力矩，则俯仰通道的状态方程可表示为：

$$\begin{bmatrix}\Delta\dot{\sigma}\\ \Delta\dot{\psi}\\ \Delta\ddot{\psi}\end{bmatrix} = \begin{bmatrix} c_2 - c_1 & c_1 & 0\\ 0 & 0 & 1\\ b_2 & -b_2 & -b_1\end{bmatrix}\begin{bmatrix}\Delta\sigma\\ \Delta\psi\\ \Delta\dot{\psi}\end{bmatrix} + \begin{bmatrix} c_3\\ 0\\ -b_3\end{bmatrix}\Delta\delta_\psi$$

$$\begin{bmatrix}\Delta\dot{\gamma}\\ \Delta\ddot{\gamma}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -d_1\end{bmatrix}\begin{bmatrix}\Delta\gamma\\ \Delta\dot{\gamma}\end{bmatrix} + \begin{bmatrix}0\\ -d_2\end{bmatrix}\mathit{\Delta}\delta_\gamma = 0$$

$$\begin{bmatrix}\dfrac{\Delta\sigma(s)}{\Delta\delta_\psi(s)}\\[2ex] \dfrac{\Delta\psi(s)}{\Delta\delta_\psi(s)}\\[2ex] \dfrac{\Delta\dot{\psi}(s)}{\Delta\delta_\psi(s)}\end{bmatrix} = (sI - A)^{-1}B = \begin{bmatrix}\dfrac{c_3 s^2 + b_1 c_3 s + b_2 c_3 - b_3 c_1}{s^3 + \left(b_1 + c_1 - c_2\right)s^2 + \left[b_2 + b_1\left(c_1 - c_2\right)\right]s - b_2 c_2}\\[2ex] \dfrac{-\left[b_3 s + b_3\left(c_1 - c_2\right) - b_2 c_3\right]}{s^3 + \left(b_1 + c_1 - c_2\right)s^2 + \left[b_2 + b_1\left(c_1 - c_2\right)\right]s - b_2 c_2}\\[2ex] \dfrac{-s\left[b_3 s + b_3\left(c_1 - c_2\right) - b_2 c_3\right]}{s^3 + \left(b_1 + c_1 - c_2\right)s^2 + \left[b_2 + b_1\left(c_1 - c_2\right)\right]s - b_2 c_2}\end{bmatrix} := \begin{bmatrix}W_{\delta_\psi}^{\sigma}(s)\\ W_{\delta_\psi}^{\psi}(s)\\ W_{\delta_\psi}^{\dot{\psi}}(s)\end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\Delta\gamma(s)}{\Delta\delta_{\gamma}(s)} \\ \dfrac{\Delta\dot{\gamma}(s)}{\Delta\delta_{\gamma}(s)} \end{bmatrix} = (sI - A)^{-1}B = \begin{bmatrix} -\dfrac{d_3}{s^2 + sd_1} \\ -\dfrac{d_3}{s + d_1} \end{bmatrix}$$

11、根据发动机布局求解相关问题

关键步骤：

（1）冗余度为 1

定义发动机中心到火箭截面圆心的距离为 z_c，物理舵偏的推导与等效舵偏的定义分别为：

$$\begin{aligned} M_x &= -4pz_c\sin\delta_\gamma = -pz_c\left(\delta_I+\delta_{II}+\delta_{III}+\delta_{IV}\right) \\ M_y &= -4\times\frac{\sqrt{2}}{2}p\left(x_c-x_g\right)\sin\delta_\psi = \frac{\sqrt{2}}{2}p\left(x_c-x_g\right)\left(\delta_I-\delta_{II}-\delta_{III}+\delta_{IV}\right) \\ M_z &= -4\times\frac{\sqrt{2}}{2}p\left(x_c-x_g\right)\sin\delta_\varphi = \frac{\sqrt{2}}{2}p\left(x_c-x_g\right)\left(\delta_I+\delta_{II}-\delta_{III}-\delta_{IV}\right) \end{aligned}$$

辅助方程设为：

$$\delta_I+\delta_{III}=\delta_{II}+\delta_{IV}$$

经小量假设得到控制方程为：

$$\begin{aligned} \delta_I &= \delta_\gamma-\delta_\psi-\delta_\varphi \\ \delta_{II} &= \delta_\gamma+\delta_\psi-\delta_\varphi \\ \delta_{III} &= \delta_\gamma+\delta_\psi+\delta_\varphi \\ \delta_{IV} &= \delta_\gamma-\delta_\psi+\delta_\varphi \end{aligned}$$

（2）冗余度为 0

定义发动机中心到火箭截面圆心的距离为 z_c，物理舵偏的推导与等效舵偏的定义分别为：

$$\begin{aligned} M_x &= -3pz_c\sin\delta_\gamma = -pz_c\left(\delta_I+\delta_{II}+\delta_{III}\right) \\ M_y &= -\left(p+2\times\frac{1}{2}p\right)\left(x_c-x_g\right)\sin\delta_\psi = \left(x_c-x_g\right)\left(\frac{1}{2}\delta_I+\frac{1}{2}p\delta_{II}-p\delta_{III}\right) \\ M_z &= -2\times\frac{\sqrt{3}}{2}p\left(x_c-x_g\right)\sin\delta_\varphi = \left(x_c-x_g\right)\left(\frac{\sqrt{3}}{2}p\delta_I-\frac{\sqrt{3}}{2}p\delta_{II}\right) \end{aligned}$$

无需辅助方程；

经小量假设得到控制方程为：

$$
\begin{aligned}
\delta_I &= \delta_\gamma - \frac{2}{3}\delta_\psi - \delta_\varphi \\
\delta_{II} &= \delta_\gamma - \frac{2}{3}\delta_\psi + \delta_\varphi \\
\delta_{III} &= \delta_\gamma + \frac{4}{3}\delta_\psi
\end{aligned}
$$

（3）冗余度为 9，故障后冗余度为 1

定义火箭建模截面圆心到芯级发动机摆动轴的距离为 c_1（即芯级发动机中心到火箭截面圆心的距离为 $\sqrt{2}c_1$），火箭建模截面圆心到助推发动机的距离为 c_2，推力大小分别为 p_1 和 p_2，那么物理舵偏的推导与等效舵偏的定义分别为：

$$
\begin{aligned}
M_x &= c_1 p_1\left(\delta_1-\delta_2-\delta_3+\delta_4\right)+c_2 p_2\left(\delta_{I2}+\delta_{II1}-\delta_{III2}-\delta_{IV1}\right) \\
&= -\left(2p_1c_1+4p_2c_2\right)\sin\delta_\gamma \\
M_y &= -\left(x_c-x_g\right)\left[p_1\left(\delta_2+\delta_4\right)+p_2\left(\delta_{I2}+\delta_{II2}+\delta_{III2}+\delta_{IV2}\right)\right] \\
&= -\left(p_1+2p_2\right)\left(x_c-x_g\right)\sin\delta_\psi \\
M_z &= -\left(x_c-x_g\right)\left[p_1\left(\delta_1+\delta_3\right)+p_2\left(\delta_{I1}+\delta_{II1}+\delta_{III1}+\delta_{IV1}\right)\right] \\
&= -\left(p_1+2p_2\right)\left(x_c-x_g\right)\sin\delta_\varphi
\end{aligned}
$$

辅助方程设为：

$$\delta_I+\delta_{II}=\delta_{III}+\delta_{IV}$$

将辅助方程代入方程组，并认为助推器发动机的 8 个控制角为已知量，经小量假设得到：

$$
\begin{aligned}
p_1\left(\delta_1-\delta_3\right) &= -\left(p_1+2\frac{p_2c_2}{c_1}\right)\delta_\gamma-\frac{1}{2}\frac{c_2p_2}{c_1}\left(\delta_{I2}^*+\delta_{II1}^*-\delta_{III2}^*-\delta_{IV1}^*\right)=M_x^* \\
p_1\left(\delta_2+\delta_4\right) &= \left(p_1+2p_2\right)\delta_\psi-p_2\left(\delta_{I2}^*+\delta_{II2}^*+\delta_{III2}^*+\delta_{IV2}^*\right)=M_y^* \\
p_1\left(\delta_1+\delta_3\right) &= \left(p_1+2p_2\right)\delta_\varphi-p_2\left(\delta_{I1}^*+\delta_{II1}^*+\delta_{III1}^*+\delta_{IV1}^*\right)=M_z^*
\end{aligned}
$$

根据已知量 M_x^*,M_y^*,M_z^*，得到控制方程为：

$$
\begin{aligned}
\delta_1 &= \frac{M_x^*+M_z^*}{2p_1},\delta_2=\frac{M_y^*-M_x^*}{2p_1} \\
\delta_3 &= \frac{M_z^*-M_x^*}{2p_1},\delta_4=\frac{M_x^*+M_y^*}{2p_1}
\end{aligned}
$$

注意：引入不同的辅助方程，定义不同的等效舵偏，将得出不一样的推导结果