

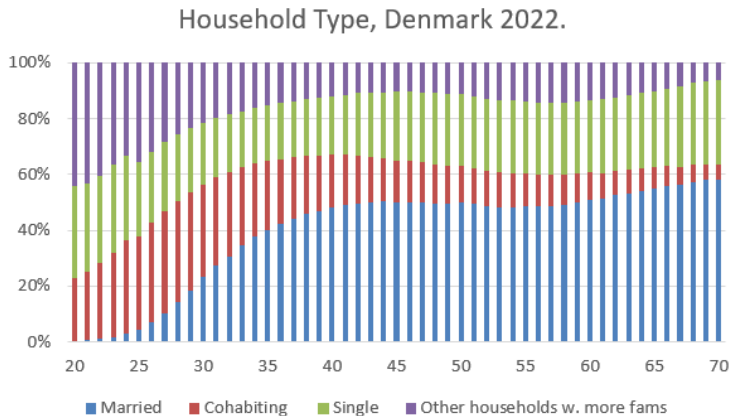
Models of Household Behavior

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Empirical Motivation

- Many people live in couples



Introduction

- **Unitary model** until now
The couple acted as one unit

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The couple acted as one unit
- There are ∞ many ways of modeling household decisions
Some large and some small differences
- I will focus on some main types of models
Give an idea of the main similarities and differences

Introduction

- **Unitary model** until now
The couple acted as one unit
- There are ∞ many ways of modeling household decisions
Some large and some small differences
- I will focus on some main types of models
Give an idea of the main similarities and differences
- Remember: Models are ***abstractions***, hopefully usefull...
- Notation on dynamic models differ from Chiappori and Mazzocco (2017)
Based on lecture note

Outline

- 1 Static Models
 - Setting
 - Unitary Model
 - Non-cooperative
 - Cooperative: Collective
- 2 Dynamic Models
 - Full Commitment
 - Limited Commitment

Production Technology

- **Superscript:** individual (1,2), **subscript:** element
- **Private** goods ($h = 1, \dots, n$) produced as

$$q_h = q_h^1 + q_h^2 = f_h(x_h, d_h) \quad (1)$$

where

x_h : market goods inputs

$d_h = (d_h^1, d_h^2)$: time inputs

Production Technology

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- **Public** goods ($k = 1, \dots, N$) produced as

$$Q_k = F_k(X_k, D_k) \quad (2)$$

where

X_k : market goods inputs

$D_k = (D_k^1, D_k^2)$: time inputs

Preferences: Utility and Felicity Function

- Individual *utility* function

$$U^i(Q, q^1, q^2, l^1, l^2)$$

where

l^i is leisure time

$T^i = h^i + l^i + \sum_{k=1}^N D_k^i + \sum_{h=1}^n d_n^i$ is available time

h^i is hours worked

Preferences: Utility and Felicity Function

- **Caring preferences:**

Care not about the allocation of partner but only their welfare:

$$U^i(Q, q^1, q^2, l^1, l^2) = W^i(u^1(Q, q^1, l^1), u^2(Q, q^2, l^2))$$

where

$u^i(Q, q^i, l^i)$ is called the *felicity* function

Preferences: Utility and Felicity Function

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where

$u^i(Q, q^i, l^i)$ is called the *felicity* function

- **Egotistic preferences:**

Care not about the partner:

$$U^i(Q, q^1, q^2, l^1, l^2) = F^i(u^i(Q, q^i, l^i), a)$$

where a can contain marital status etc.

Budget Constraint

• Budget constraint

$$p' \left(\sum_{k=1}^N X_k + \sum_{h=1}^n x_n \right) = \sum_{i=1}^2 (y^i + w^i h_i) \quad (3)$$

where

p is vector of market prices

y^i is non-market income.

w^i is wage rate

Budget Constraint

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- Can be written as in Chiappori and Mazzocco (2017)

$$p' \left(\sum_{k=1}^N X_k + \sum_{h=1}^n x_n \right) + \sum_{i=1}^2 w^i (l^i + \sum_{k=1}^N D_k^i + \sum_{h=1}^n d_n^i) = \underbrace{\sum_{i=1}^2 (y^i + w^i T^i)}_{Y \text{ (pot. inc.)}}$$

(note that $T^i = h^i + l^i + \sum_{k=1}^N D_k^i + \sum_{h=1}^n d_n^i$, they miss l^i on p. 989)

- **Income pooling:** non-labor income, y^i , enters identically for both

Unitary Model

- **Unitary model**, households solve

(conditional on *potential* income, $Y = \sum_{i=1}^2 (y^i + w^i T^i)$)

$$\max_{X, x, l^1, l^2, d^1, d^2, D^1, D^2} U^H(Q, q, l^1, l^2)$$

s.t.

$$Q_k = F_k(X_k, D_k), \quad k = 1, \dots, N$$

$$q_h = q_h^1 + q_h^2 = f_h(x_h, d_h), \quad h = 1, \dots, n$$

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where $U^H(Q, q, l^1, l^2)$ is *some* household-level utility function

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- **Rationalized** via

Samuelson's welfare index

Becker's rotten kid (we skip)

Transferable Utility (TU)

Unitary Motivation: Samuelson's welfare index

- Samuelson's welfare index

$$U^H(Q, q, l^1, l^2) = \max_{q_1, q_2} W(u^1(Q, q^1, l^1), u^2(Q, q^2, l^2))$$

s.t.

$$q = q_1 + q_2$$

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- Example** could be

$$W(u^1(Q, q^1, l^1), u^2(Q, q^2, l^2)) = \lambda u^1(Q, q^1, l^1) + (1 - \lambda) u^2(Q, q^2, l^2)$$

where

λ is a constant weight on each member's utility; "power"

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where

λ is a constant weight on each member's utility; “power”

- **Arbitrary** that households should have some $W()$... but this example is a special form of the “collective model” below [nice]

Unitary Motivation: Transferable Utility

- **TU:** If there exists a Pareto frontier,
such that a *cardinal transformation*, $k()$ gives

$$k(u^1(Q, q^1, l^1)) + k(u^2(Q, q^2, l^2)) = K(p, w, Y)$$

→ utility possibility frontier has a slope of -1 ,

$$k(u^1(Q, q^1, l^1)) = K(p, w, Y) - k(u^2(Q, q^2, l^2))$$

- **Then** we can describe the optimization problem using $U^H(Q, q, l^1, l^2)$

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- **Then** we can describe the optimization problem using $U^H(Q, q, l^1, l^2)$
- **Example:** Conditional on Y ,

$$U^{m*}(\bar{u}^f) = \max_{c^m} U^m(c^m) = \sqrt{c^m}$$

s.t.

$$Y = c^m + c^f$$

$$\bar{u}^f = U^f(c^f) = \sqrt{c^f}$$

gives $U^{m*}(\bar{u}^f)^2 = \text{constant}(Y) - (\bar{u}^f)^2$.

Unitary Model: Not Consistent with Data

- **Two testable implications**

1. Income pooling (source of non-labor income does not matter for behavior)
2. Slutsky symmetry (commodity prices affect members' demand similarly)

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- **Two testable implications**

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- **Almost always rejected** see Chiappori and Mazzocco (2017, p. 1022)

- **Alternatives** have been proposed

Non-cooperative

Cooperative (collective)

Non-Cooperative

- Non-cooperative models:**

A game with two players

Nash equilibrium

$$\max_{Q^1, q^1, l^1} u^1(Q^1 + Q^2, q^1, q^2, l^1, l^2)$$

s. t.

$$PQ^1 + p'q^1 = Y^1$$

and

$$\max_{Q^2, q^2, l^2} u^2(Q^1 + Q^2, q^1, q^2, l^1, l^2)$$

s. t.

$$PQ^2 + p'q^2 = Y^2$$

- Generally not efficient:** Partner's gains not internalized.

Cooperative: Collective

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Typically formulated as

$$\max_{X, x, l^1, l^2, d^1, d^2, D^1, D^2} \lambda(z) u^1(Q, q^1, l^1) + (1 - \lambda(z)) u^2(Q, q^2, l^2)$$

s.t.

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- **Distribution factors:** z

Anything that affect power, such as p, w, y .

Cannot be endogenous: Over-investment in power \rightarrow inefficient.

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- **Unitary model** is nested, $\lambda(z) = \text{constant} \rightarrow$ unitary model

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Dynamic Models

- All comes down to how the bargaining weight is updated.
- My slides combine Chiappori and Mazzocco (2017) with Theloudis, Velilla, Chiappori, Giménez-Nadal and Molina (2022) and own lecture note

General Setup: Choices

- Period $t = 0$: Individuals A and B become a couple
- Periods $t > 0$: As a couple, they decide on
 - private consumption, c_t^A and c_t^B (and thus savings, a_t)
 - labor supply, $l_t^A, l_t^B \in \{0, 0.75, 1\}$
 - whether to split up (no re-partnering for simplicity)
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- **Inter-temporal budget** constraint of couple

$$a_t + c_t^A + c_t^B = Ra_{t-1} + w^A l_t^A + w^B l_t^B$$

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- **Example throughout**, but apply more generally.

General Setup: Utility

- **Individual utility** is for $j \in \{A, B\}$

$$u^j(c_t^j, l_t^j)$$

- **Household utility** is weighted sum

$$U(c_t^A, c_t^B, l_t^A, l_t^B; \psi_t, \mu_t) = \mu_t u^A(c_t^A, l_t^A) + (1 - \mu_t) u^B(c_t^B, l_t^B) + \psi_t$$

where match quality/“love” is

$$\psi_t = \psi_{t-1} + \varepsilon_t, \varepsilon \sim iid \mathcal{N}(0, \sigma_\varepsilon^2)$$

and μ_t is the **bargaining power** of agent A

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- How μ_t is determined defines the different types of models:

Unitary: $\mu_t = \mu$ is a **constant number**

Full commitment: $\mu_t = \mu_0(Z)$ is a **function** (of states known i $t = 0$)

No commitment: μ_t is updated in each period

Limited commitment: $\mu_t = \mu_t(\bullet, \mu_{t-1})$ is a function of past power

General Setup: Recursive Formulation

- **Outside option:** Value of being single

$$V_{j,t}^s(a_{t-1}) = \max_{c_t^j, l_t^j} u^j(c_t^j, l_t^j) + \beta V_{j,t+1}^s(a_t)$$

s.t.

$$a_t = Ra_{t-1} + w^j l_t^j - c_t^j$$

where I do not allow for re-partnering ($V_{j,t}^{m \rightarrow s} = V_{j,t}^s$).

- **Non-cooperation** could be outside option

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- **Non-cooperation** could be outside option
- **Partnership dissolution:**

Share κ_1 of wealth is transferred to agent A and $\kappa_2 = 1 - \kappa_1$ to agent B.

$$a_t^A = \kappa_1 a_t \text{ and } a_t^B = \kappa_2 a_t$$

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- **Value of being in a couple**

depends on what we assume about the *bargaining process*.

Unitary Model

- **Constant** bargaining power, $\mu_t = \mu$.

Unitary Model

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- **Value of a couple** is

$$W_t(a_{t-1}, \psi_t) = \max_{c_t^A, c_t^B, l_t^A, l_t^B} U(c_t^A, c_t^B, l_t^A, l_t^B, \psi_t; \mu) + \beta \tilde{W}_{t+1}(a_t, \psi_t)$$

the *expected* (w.r.t. ψ_{t+1}) continuation value is

$$\begin{aligned} & \tilde{W}_{t+1}(a_t, \psi_t) \\ &= \mathbb{E}_t[\max\{W_{t+1}(a_t, \psi_{t+1}); \underbrace{\mu V_{A,t+1}^s(a_t^A) + (1 - \mu) V_{B,t+1}^s(a_t^B)}_{\text{weighted value of singlehood}}\}] \end{aligned}$$

Commitment Models

- **Endogenously determined μ_t**

FC: Full commitment, μ_t is a **constant function**

We will see in Bruze, Svarer and Weiss (2015)

NC: No commitment, μ_t updated **every period**

We will just discuss today

LC: Limited commitment, μ_t updated **sometimes** → function of past power

We will see in several papers + code

Full Commitment

- Bargaining power function is determined and agreed upon **at beginning of partnership**
- Bargaining power is thus a *constant function*

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- Z_t : **Known at beginning** of partnership, at $t = 0$!
 - Assume that couples can **commit** to this bargaining power function
 - Will e.g. not request more bargaining power from (changes in) something not in Z_t
 - If time-varying elements in Z_t : Assuming perfect foresight

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 - If time-varying elements in Z_t : Assuming perfect foresight
- **How is this function determined?**
We will come back to this in a few slides

Full Commitment

- **Value of being a couple** is then

$$W_t(a_{t-1}, \psi_t) = \max_{c_t^A, c_t^B, l_t^A, l_t^B} U(c_t^A, c_t^B, l_t^A, l_t^B; \psi_t, \mu_0(Z_t)) + \beta \tilde{W}_{t+1}(a_t, \psi_t)$$

where *expected continuation* value is

$$\tilde{W}_{t+1}(a_t, \psi_t) = \mathbb{E}_t[\max\{W_{t+1}(a_t, \psi_{t+1})$$

$$; \underbrace{\mu_0(Z_{t+1}) V_{A,t+1}^s(a_t^A) + (1 - \mu_0(Z_{t+1})) V_{B,t+1}^s(a_t^B)}_{\text{weighted value of singlehood}}\}]$$

Full Commitment

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- **Transferable utility:** Household jointly decide to divorce if

$$W_t(a_t) < \mu_0(Z_t) V_{A,t}^s(a_t^A) + (1 - \mu_0(Z_t)) V_{B,t}^s(a_t^B)$$

- No constraints on individual members' utilities

Full Commitment: Determining Bargaining Power

- How could $\mu_0(Z_t)$ be determined?

Full Commitment: Determining Bargaining Power

- How could $\mu_0(Z_t)$ be determined?
- **Idea 1: Nash-bargaining** at the point of partnership formation

$$\mu_0(Z) = \arg \max_{\mu \in [0,1]} \left(\mu W_0(a_{-1}) - V_{A,0}^s(\lambda a_{-1}) \right)^{0.5} \\ \times \left((1 - \mu) W_0(a_{-1}) - V_{B,0}^s((1 - \lambda) a_{-1}) \right)^{0.5}$$

- μ_0 “non-parametric” constant function of e.g. $Z = (a_{-1}, w_0^A, w_0^B)$

Full Commitment: Determining Bargaining Power

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- μ_0 “non-parametric” constant function of e.g. $Z = (a_{-1}, w_0^A, w_0^B)$
- **Idea 2: Assume a functional form** (Bruze, Svarer and Weiss, 2015)

$$\mu_0(w_t^A / w_t^B) = \frac{\exp(\alpha_0 + \alpha_1 w_t^A / w_t^B)}{1 + \exp(\alpha_0 + \alpha_1 w_t^A / w_t^B)}$$

and estimate parameters α_0 and α_1 using data.

[perfect foresight assumption on wages]

- If $\alpha_1 = 0$: Similar to the unitary model.

No- and Limited Commitment

- My definition of “No commitment” is different from that of Mazzocco (2007)
 - I will call his setup “Limited commitment” (as is standard now)
- They are closely related: Both **do not assume transferable utility**
 - Only differ in how the bargaining power is updated dynamically

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- They are closely related: Both **do not assume transferable utility**
 - Only differ in how the bargaining power is updated dynamically
- We thus need to check **individual “participation” constraints**:
Is it optimal for each agent to be part of the couple without receiving any utility from the other partner
- **We need to define a new objects** for this purpose:
The value of agent j from being in the couple if μ_{t-1} is the bargaining power coming into period t

Notation: Partnership Status and Transitions

- All values are **individual**
- **Four** transition possibilities

from\to	married	single
married	$V_{j,t}^{m \rightarrow m}(\mathcal{S}_t, \mu_{t-1})$	$V_{j,t}^{m \rightarrow s}(\mathcal{S}_t, \mu_{t-1})$
single	$V_{j,t}^{s \rightarrow m}(\mathcal{S}_t)$	$V_{j,t}^{s \rightarrow s}(\mathcal{S}_t)$

Notation: Partnership Status and Transitions

- All values are **individual**
- **Four** transition possibilities

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- Value of **starting** in a state

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- When singlehood is absorbing (and no div. costs): $V_{j,t}^s = V_{j,t}^{m \rightarrow s}$.

Recursive Formulation

- **Individual value of choice** c_t^j, l_t^j while remaining in couple is

$$v_{j,t}^{m \rightarrow m}(c_t^j, l_t^j; a_{t-1}, \psi_t, \mu) = u^j(c_t^j, l_t^j) + \psi_t + \beta \mathbb{E}_t[V_{j,t}^m(a_t^j, \psi_{t+1}, \mu)]$$

where μ is *some* bargaining power, we will discuss in great detail.

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1. **Conditional on remaining together, optimal choices are**

$$\begin{aligned} \tilde{c}_t^A(\mu), \tilde{c}_t^B(\mu), \tilde{l}_t^A(\mu), \tilde{l}_t^B(\mu) = \arg \max_{c_t^A, c_t^B, l_t^A, l_t^B} & \mu v_{A,t}^{m \rightarrow m}(c_t^A, l_t^A; a_{t-1}, \psi_t, \mu) \\ & + (1 - \mu) v_{B,t}^{m \rightarrow m}(c_t^B, l_t^B; a_{t-1}, \psi_t, \mu) \end{aligned}$$

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2. **Marital surplus for agent j is**

$$S_t^j(\mu) = \underbrace{v_{j,t}^{m \rightarrow m}(\tilde{c}_t^j(\mu), \tilde{l}_t^j(\mu); a_{t-1}, \psi_t, \mu)}_{V_{j,t}^{m \rightarrow m}(a_{t-1}, \psi_t, \mu)} - v_{j,t}^{m \rightarrow s}(a_{t-1}^j)$$

Limited Commitment: Bargaining Process

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There are 3 cases

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 - 3.2 If $S_t^A(\mu_{t-1}) < 0$ and $S_t^B(\mu_{t-1}) < 0$ then they divorce, $D_t^* = 1$.
 - 3.3 If e.g. $S_t^A(\mu_{t-1}) < 0$ (member A not happy with current division)
They re-bargain to find a new potential distribution both can accept
(point 4 on next slide)

Limited Commitment: Updating Bargaining Weight

4. Let $\tilde{\mu}^A : S_t^A(\tilde{\mu}^A) = 0$ be the allocation that makes A (barely) want to remain together.

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$$\mu_t = \mu_t^* = \tilde{\mu}^A$$

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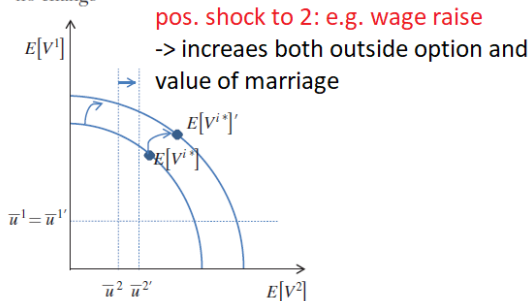
- Combined the co-state evolves as

$$\mu_t = \mu_t^*(S_t, \mu_{t-1}) = \begin{cases} \mu_{t-1} & \text{if } S_t^A(S_t, \mu_{t-1}) \geq 0 \text{ and } S_t^B(S_t, \mu_{t-1}) \geq 0 \\ \tilde{\mu}^A & \text{if } S_t^A(S_t, \mu_{t-1}) < 0 \text{ and } S_t^B(S_t, \tilde{\mu}^A) \geq 0 \\ \tilde{\mu}^B & \text{if } S_t^A(S_t, \tilde{\mu}^B) \geq 0 \text{ and } S_t^B(S_t, \mu_{t-1}) < 0 \\ \emptyset & \text{else} \end{cases} \quad (4)$$

Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **small**.

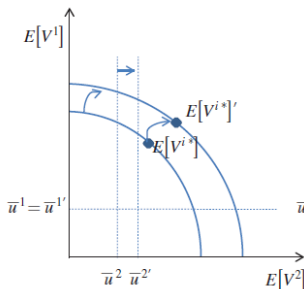
Panel A. Marriage
no change



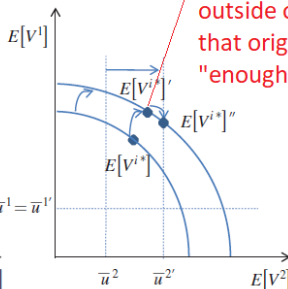
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **medium**.

Panel A. Marriage
no change



Panel B. Marriage
with renegotiation

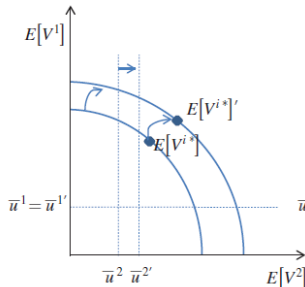


Pos. shock to 2: Increases
outside option so much
that original "power" not
"enough" to stay

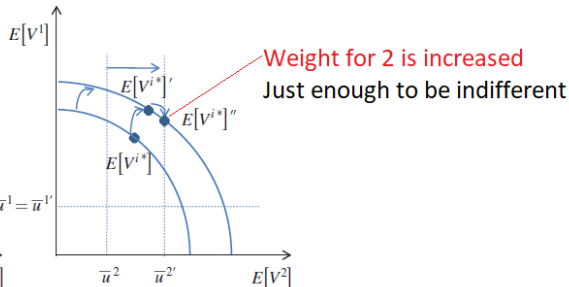
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **medium**.

Panel A. Marriage
no change



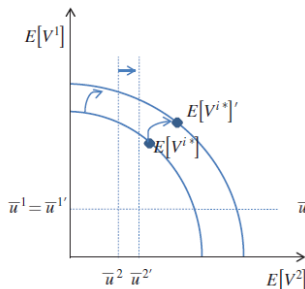
Panel B. Marriage
with renegotiation



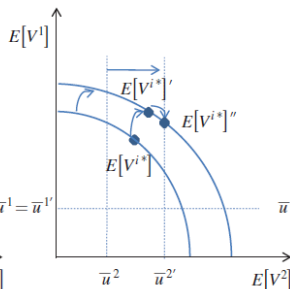
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **large**.

Panel A. Marriage
no change

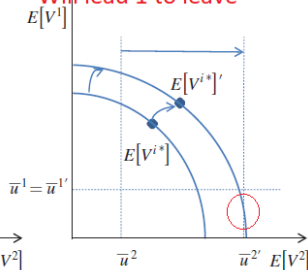


Panel B. Marriage
with renegotiation



Panel C. Divorce

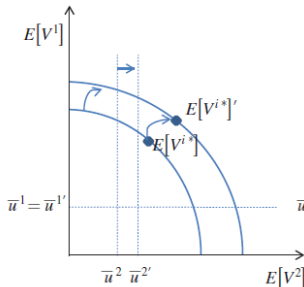
Power needed to keep 2
Will lead 1 to leave



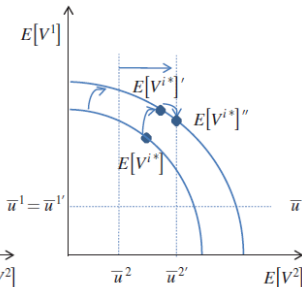
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **large**.

Panel A. Marriage
no change

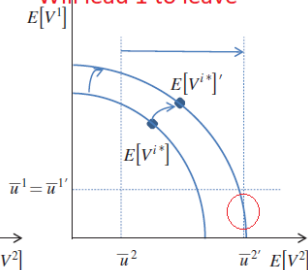


Panel B. Marriage
with renegotiation



Panel C. Divorce

Power needed to keep 2
Will lead 1 to leave



- See also figures 2 and 3 in lecture note.

No Commitment

- **Limited commitment:** Bargaining power updated if individual participation constraints violated at current bargaining position, μ_{t-1} ,

$$\mu_t = \mu_t^*(a_{t-1}, \psi_t, \mu_{t-1})$$

No Commitment

- **Limited commitment:** Bargaining power updated if individual participation constraints violated at current bargaining position, μ_{t-1} ,

$$\mu_t = \mu_t^*(a_{t-1}, \psi_t, \mu_{t-1})$$

- **No commitment:** Bargaining power updated in *all periods*,

$$\mu_t = \mu_t^*(a_{t-1}, \psi_t)$$

replace step 3 with e.g. [instead of the discussion before]

$$\mu_t = \arg \max_{\tilde{\mu}} S_t^A(\tilde{\mu})^{0.5} S_t^B(\tilde{\mu})^{0.5}$$

- **We focus on limited commitment** in the code

What about initial bargaining power, μ_0 , then?

Could be found through Nash bargaining :)

Next Time

- **Next time:**

Divorce Laws, Savings and Labor Supply.

- **Literature:**

Voena (2015): "Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?"

- **Read** before lecture

- **Reading guide:**

Section 0: Introduction. Key

Section 1: US divorce law. Key.

Section 2: Model. *Key*, but complex. Get the idea.

Under unilateral divorce: limited commitment model.

Section 3: Data and RF motivation. Get the overall results/motivation.

Section 4: Structural Estimation: Read fast.

Section 5: Counterfactual simulations. Key.

References I

- BRUZE, G., M. SVARER AND Y. WEISS (2015): "The Dynamics of Marriage and Divorce," *Journal of Labor Economics*, 33(1), 123–170.
- CHIAPPORI, P.-A. AND M. MAZZOCCO (2017): "Static and Intertemporal Household Decisions," *Journal of Economic Literature*, 55(3), 985–1045.
- MAZZOCCO, M. (2007): "Household intertemporal behaviour: A collective characterization and a test of commitment," *The Review of Economic Studies*, 74(3), 857–895.
- THELOUDIS, A., J. VELILLA, P. A. CHIAPPORI, J. I. GIMÉNEZ-NADAL AND J. A. MOLINA (2022): "Commitment and the Dynamics of Household Labor Supply," Discussion paper.
- VOENA, A. (2015): "Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?," *American Economic Review*, 105(8), 2295–2332.