Read Chapter #1

The idea is to read Chapter 1 of the textbook by Daniel Solow. The assignment is to read it in a particular way. It may take 3 hours to get it done, but you will learn something in those three hours, and you will start to develop a very important skill.

Get a copy of Chapter 1, “The Truth of it All.” Get out your notebook or some paper or write directly on a blank PDF. You will need to turn in your notes so I can have a look at them. Go somewhere quiet, where you won’t be interrupted for a while. Silence your phone so you aren’t disturbed. Don’t listen to music that will distract you, and make sure there is no video going where you can see it or hear it.

Put the notebook or paper right in front of you. Put the textbook itself a bit farther away. Make note of the time that you start reading in your notebook, maybe in the left margin. Read the first paragraph of the chapter, then write one or two sentences in your notes which capture the main idea(s) of the paragraph.

Continue reading and writing a sentence summarizing each paragraph. I believe that if you are not writing, you are probably not thinking as hard as you need to. Read slowly. If you run into a word you don’t know, look it up. If you really don’t know it, write the definition in your notebook. It is OK to spend 15 minutes on each page of the book. Really. It is not a goal of the course to learn how to read faster. The goal is to learn how to get more out of the time you spend reading. If you stop to take a break, note the time that you stopped and the time you start again so you can calculate the total time.

When you read Section 1.1, comment on the goals he lays out. Do you have the same goals? Different? How?

When you read Section 1.2, there are a few vocabulary words in bold, be sure to learn those. There is a very important example about when you might call your friend a liar. Personally, I don’t think this is about your friend being a liar; if you study hard and don’t get a good grade, then your friend was just wrong, not trying to deceive you. Still, the point is that when trying to prove that A implies B, the only way that things can go wrong is a situation in which A is true but B is false. Read and reflect on this example multiple times. Write a sentence or two to demonstrate that you understand this example. Use the additional examples he gives to understand better. If you are left with questions, please write them in your notes, perhaps highlight them, so that I can respond to them when I read your notes.

Examples 1, 2, 3, and 4 illustrate how to do some of the exercises.

Note that solutions of the exercises marked with W are available online at http://higheredbcs.wiley.com/legacy/college/solow/1118164024/sm/sm.pdf Resist the urge to turn your brain off and just read the solutions. That is not what they are there for!

Please do the following exercises and write solutions in your notebook:

1.2

1.4

1.7

1.8 “Not lose” is the same as “win” in this problem.

1.12 This is all about avoiding a situation in which A is true and B is false.

1.16 Answer the question yourself, then compare your answer with the online solution.

1.18 I suggest that you try various choices for n and x on a calculator and see what you find. Write out clearly what A and B are, and how you have made A true but B false. For part (b), look for agreement on 5 decimal places, not just 3, and explain how you found an answer to part (b). After doing 1.18, check your answer to 1.16.

At the end, tally up how much time you have spent on reading this chapter. Write this number at the beginning of your notes in the upper left corner.

Read Chapter #2, The Forward-Backward Method

Read Chapter 2 in the book by Daniel Solow, about the forward-backward method for finding a proof that statement A implies statement B.

As you read the chapter, write short summaries of each paragraph, so that your notes provide a short version of the ideas in the chapter. Specific things to do are listed below to help you check them off.

1. Keep track of the time it takes you to read the chapter.
2. Read Proposition 1 and make sure you understand what it is saying. Apparently the area of a right triangle is not always equal to the hypotenuse squared divided by 4.
3. When reading Section 2.1, recall our work on vector sums and dot product, where we were able to write down where we need to start, leave a lot of space, and then write down where we need to end. At first, Solow is working up from the bottom, from statement B back to B1 back to B2

   For working backward, he introduces the idea of a "key question," which is a way to put your finger on what is needed to know that B is true. The key question needs to be specific enough to be helpful to the problem at hand, but general enough that it make sense to someone who is not immersed in the details of the problem. It may help to think of formulating the key question as an internet search, since most people have experience with writing searches that are not too specific and not too general at the same time.
4. When starting Section 2.2, use some space in your notes to write out A and A1 at the top, leave 10 lines of space, and write out B2, B1, and B at the bottom. Fill in steps as you read the section.
5. When reading Section 2.3, use the numbering of Table 2.1 to list out the statements that are made in each of the four proofs of Proposition 1, in the order that they are made. If a statement is not actually made, don't write the corresponding label. This will help to illustrate the order in which the proofs are written and what steps are left out.
6. At the end of the chapter there is an illustration of a maze. Work through the maze from A to B and count how many dead ends there are on the way to B. Work through the maze from B to A and count how many dead ends there are. Note that some of the dead ends are different, depending which way you are going.
7. Do exercise 2.5.
8. Do exercise 2.7.
9. Do exercise 2.11.
10. Do exercise 2.14a and 2.15b.
11. Do exercise 2.19.
12. Do exercise 2.24.
13. Do exercise 2.30.
14. Do exercise 2.37, by writing out the steps in order from A to B.
15. At the end, tally up how much time you have spent on this chapter. Write this number in the upper left corner of your notes.

Read Chapter #3, On Definitions and Mathematical Terminology

Read Chapter 3 in the book by Daniel Solow. This chapter is about definitions and how to use them in the forward and backward process. Fortunately, this does not seem to be a complicated chapter, so work through it and make sure you get the message. Follow these instructions carefully.

1. Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material. You do not need to paraphrase each paragraph, but I would encourage you to write down things that you learn, things that surprise you, or things that you need to remember.

2. When reading Section 3.1, pay close attention to "if and only if" because it comes up often in proofs, and you need to know how to show an "if and only if" statement.

3. In Section 3.1, there is a page and a half on overlapping notation. This is a good discussion.

4. In Section 3.2, read the analysis of proof for Proposition 3 and then write down the steps A, A1, B2, B1 in order to make your own proof of the result. Explain why each step in the proof can be taken.

5. Make yourself a glossary (vocabulary list) for the terms converse, inverse, contrapositive, proposition, theorem, lemma, corollary, axiom, so you learn them well. In Section 3.3, the author says that a proposition is a true statement that you are trying to prove. I would emphasize that we don't call it a proposition until we know that it can be proven, and maybe we are trying to understand the proof. Something we are simply trying to prove should be a called a conjecture; it might not turn out to be true!

6. Do Exercise 3.2. You do not need to write proofs, only pose the key question, answer it abstractly, and rephrase your answer in terms used in the problem.

7. Do Exercise 3.5 parts c and d. This will be a bit challenging because it requires you to look up two new definitions and interpret them. That's an excellent skill, so practice it.

8. Do Exercise 3.12.

9. Do Exercise 3.15.

10. Do Exercise 3.19. Note that definitions are not the kind of previous knowledge that we are looking for here. Look for the two previous implications that are used here.

11. Do Exercise 3.21.

12. Do Exercise 3.27.

13. At the end, tally up how much time you have spent on this chapter. Write this number in your notes.

Reading assignment #4

Due on Tuesday, September 26. 20 points

Read Chapter 4 in the book by Daniel Solow. It is about showing that there is an "object" with a "certain property" such that "something happens." We have already done a number of proofs of this general form.

From the class survey, I am reminded that people like to take notes in different ways. Do what works for you, but make sure that your notes show that you read each section and that you found and understood the main messages there.

Also, from the class survey, people would like to work on something related to the reading, so we'll start with something from this reading on Tuesday. Good idea!

Specific requirements

- Read Section 4.1 and take notes. Then, look back through the Even and Odd activity and the Vector Sum and Dot Product activity and list by number all of the exercises that are of the form "Show that there is an object with a certain property such that something happens." I've posted previous activities on the Syllabus section on Canvas. Note: Showing that n is even means showing that there exists an integer k for which $n = 2k$.

- The existence part of the Division Algorithm is of the form described in this chapter. Write it out following the general pattern that there is an "object' with a "certain property" such that "something that happens," in that order. Hint: The last thing to write is $n = qk + r$.

- In Section 4.3, Proposition 5 assumes that m is even. Suppose instead that m is odd, and show that $m^2 + n^2 - 1$ is a multiple of 4.

- Do exercise 4.2.

- Do exercise 4.9. In each case, explain how you found the object.

- Do exercise 4.11. There are two objects getting constructed here, k and x. Where do their values come from? Under what condition could you produce an additional rational root?

- Do exercise 4.13. This is an excellent project with several parts. Work hard on it.

- Do exercise 4.16.

- Do exercise 4.22.

- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook. Bring your notebook to class and turn it in for grading.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.

Reading assignment #5

Due on Tuesday, October 3. 20 points

Read Chapter 5 in the book by Daniel Solow. It is about showing that something happens "for all" objects with a certain property. We have already done a number of proofs of this general form.

Pay particular attention to the beginning of the chapter, about set theory. We will soon start doing set theory activities in class.

Specific requirements

- Read Section 5.1 slowly and make sure to think through every sentence. There is a lot of new content in just a few pages. Set theory is super important, and this is a very nice introduction to certain aspects of it that we will spend a lot of time with. Make sure that your notes reflect the time you spend and your understanding of the material.
- Read Section 5.2. It has an extended discussion of using the forward-backward method to do a proof. After you have read it, write out the statements in order and using the labels **A, A1, A2, . . . , B6, B5, . . . B2** so that it is clear that you understand exactly how the proof works. I think this will help make it clearer to you, also. Because only half of the proof is being done here, start with the definitions of sets S and T, which is part **A** of the proof, write **A1:** as "Let x be an element of S." and end with **B2:** S is a subset of T. In **A1:**, the word "Let" means that a specific object is being brought into existence, with a specific property, for you to work with.
- Do exercise 5.2.
- Do exercise 5.6.
- Do exercise 5.7.
- Do exercise 5.14. Following the chapter, first identify the objects, the certain property, and the something that happens in the for-all statements. Then do a nice job explaining what is right or wrong about a, b, c, d, and e.
- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook. Bring your notebook to class and turn it in for grading.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.

Reading assignment Chapter 7

Due on Tuesday, October 17. 20 points

Read Chapter 7 in the book by Daniel Solow. This chapter is about nested quantifiers. We have actually been working with these from day 1 of the course, but now you will write things out more explicitly.

Here is an example. When you proved that the sum of two even numbers is even, you proved that:

- For all integers m and n, there exists an integer k such that $m + n = 2k$.

In order to prove this, you used the choose method (described in Chapter 5) to fix particular values of m and n and then wrote a model proof which worked for all m and n. As part of that model proof, you constructed the new integer k, which follows the construction method described in Chapter 4. There are many theorems following the general pattern "For all objects a, there exists an object b for which something depending on a and b happens.' Note that in the example, a is the pair m, n and b is the integer k. Also, note that b pretty much always depends on a.

There are also theorems following the pattern "There exists an object a such that for all objects b, something depending on a and b happens. These work differently. Here you have to do the construction of a in a way that it will work for all b simultaneously, then you fix an object b and write a model proof that will work for all b. The difference here is the order in which the nested quantifiers occur. Note that here b does not depend on a, and a cannot be chosen for any one particular b but needs to work for all b.

Specific requirements

- As you read Section 7.1, outline how you would use the "construction" and the "choose" methods to prove S1, S2, S3, and S4, following the model above.
- Similarly, when reading Section 7.2, outline how you would show that a function is onto.
- Do exercise 7.2.
- Do exercise 7.4. Instead of doing it exactly as stated, instead create five examples of (x, y) pairs that satisfy the criteria in part a and part b, and then answer part c. This is why we don't fuss too much about the general pattern "there exists a such that there exists b for which hsomething depending on a and b happens."
- Do exercise 7.7. Instead of doing it exactly as stated, follow the model at the beginning of this assignment to explain how to use the construction method and the choose method to do a, b, and c.
- Show that for all $a > 0$, there exists an integer $n > 0$ such that $\frac{1}{n} < a$. While doing so, mention which part uses the "construction" method and which part uses the "choose" method.
- Do exercise 7.18. I suggest that for every z in $\mathbb{R}$, you show that there exists an x in $\mathbb{R}$ for which $f(g(x)) = z$. That leaves the letter y available, and you'll want to use it.
- Do exercise 7.19. Fortunately, you can construct x.
- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook. Bring your notebook to class and turn it in for grading.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.

Reading assignment Chapter 8

Due on Tuesday, October 24. 20 points

Read Chapter 8 in the book by Daniel Solow. This chapter is about negation of logical statements, especially of statements containing quantifiers. This just takes some practice and you'll be good at it. The key text is steps 1, 2, 3 at the top of page 95.

Specific requirements

- Write the NOT of this statement: "For all $x \in \mathbb{R}$, $\ln(x) < 14$.
- Write the NOT of this statement: "For all $a \in [2,5]$, there exists $b \in [2,5]$ such that $a < b$.
- Write the NOT of this statement: "There exists $a \in A$ such that for all $b \in B$, $f(b) < a$.
- Write the NOT of this statement: "For all $\varepsilon > 0$, there exists $\delta > 0$ such that for all x with $|x-a| < \delta$, $|f(x) - L| < \varepsilon$.
- Write the NOT of this statement: "For all $a \in \mathbb{R}$, for all $\varepsilon > 0$, there exists $\delta > 0$ such that for all x with $|x - a| < \delta$, $|f(x) - f(a)| < \varepsilon$.
- Do exercise 8.2. Note that taking the NOT of "For all a and for all b, something happens" becomes "There exists a and there exists b such that, NOT something happens."
- Do exercise 8.3. For part (a), note that "There is no integer n with ..." is how you write English for the logical statement "NOT (there exists an integer n with ...)". So it's easy to negate. For all parts, note that you are just going to put NOT in front of the bold word, and then write the NOT of what comes after "if and only if".
- Do exercise 8.7. Clearly identify the logical statements A and B in each case, and then clearly state NOT B and NOT A. Leave out statements such as k is an integer; these are like the fabric of reality, not to be negated in these exercises. Make clear what you will work forward from and what you will work backward from. Note that you do not have to do the proofs, but if you see how to do them, you might as well do them.
- Is this statement true? "For all $a \in [2,5]$, there exists $b \in [2,5]$ such that $a < b$. If so, explain. If not, find a counterexample.
- Is this statement true? "For all $a \in (2,5)$, there exists $b \in (2,5)$ such that $a < b$. If so, explain. If not, find a counterexample.
- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook. Bring your notebook to class and turn it in for grading.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.

Reading assignment Chapter 9

Due on Tuesday, October 31. 20 points

Read Chapter 9 in the book by Daniel Solow. It is about proof by contradiction. We have seen a few examples of this in class, in this form: If you want to prove that the logical statement P is true, pretend for a minute that $\neg P$ is true, and make a series of logical deductions that lead to a statement you know is false. Then you know that $\neg P$ is false, and so P is true.

Chapter 9 is mostly about proving implications like $P \to Q$. Recall that in a direct proof, you suppose that P is true and make a series of logical deductions to show that Q is true. We have discussed the contrapositive method in class, where you suppose that $\neg Q$ is true and make a series of deductions to show that $\neg P$ is true. In both cases, you are trying to show that it cannot happen that P is true and $\neg Q$ is true at the same time. One way to look at proof by contradiction is that you pretend for a minute that $P \wedge \neg Q$ is true and make a series of logical deductions that lead to a false statement, so you know that $P \wedge \neg Q$ is false. The beauty of this method is that you have two statements to work forward from: P and $\neg Q$. The downside is that you can't work backward; you are trying to argue toward a false statement, and you don't know for sure what that is.

Note that in doing a proof of $P \to Q$ by contradiction, you will need to negate $\neg Q$, and when Q has quantifiers you will need to be extra careful.

Specific requirements

- Write a defintion of what a "contradiction" is from your reading of the chapter.
- In Section 9.4, please completely rewrite the proof of proposition 14 in your own words and with your own structure. The next two pages have an analysis of proof, but instead of reading that, work through the proof on your own and make sense of it. Be patient, get it done.
- Do exercise 9.2. In every case, explicitly write out P and Q and then P and $\neg Q$ to answer the question.
- Prove the result in exercise 9.3. Identify P and Q and $\neg Q$ and work from P and $\neg Q$ to arrive at a false statement. Ignore (a) and (b).
- Do exercise 9.7 in this way. Identify P and Q and describe how you would use the "construct" and "choose" methods to do a direct proof. Then, write out $\neg Q$ and describe how you would do a proof by contradiction.
- Do exercise 9.11 as a proof by contradiction.
- Do exercise 9.15 as a proof by contradiction.
- Do exercise 9.23 by once again identifying P, Q, and $\neg Q$ and then reading the proof.
- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook. Bring your notebook to class and turn it in for grading.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.

Reading assignment Chapter 11

Due on Tuesday, November 21. 20 points

Read Chapter 11 in the book by Daniel Solow. It is about showing uniqueness. We have seen an example already in the Division Algorithm, when you showed that given an integer $k > 0$ and an integer n, there are unique integers q and r with $0 \leq r < k$ so that $n = kq + r$. You used the Direct Uniqueness Method: you supposed that you could also write $n = kq_2 + r_2$ with $0 \leq r_2 < k$ and showed that $q = q_2$ and $r = r_2$. There is also an Indirect Uniqueness Method, where you would pretend for a minute that $q \neq q_2$ or that $r \neq r_2$ and argue to a false statement, so that you know this is fantasyland.

Specific requirements

- Do exercise 11.2. The notation may be confusing. In part (a), your goal is to show that x^* equals y^*. How could you do that? It might be easier to call the numbers x_1^* and x_2^*. In part (b), the function f is given and fixed. You might want to think of the problem as showing that $G_1 = G_2$. In part (c), p and q play the role of a.

- Do exercise 11.5. The answer to part (c) is "specialization" which was covered in Chapter 6, which we did not read. Please explain what specialization means in the context of this question. It's not a hard concept.

- Do exercise 11.6. Draw a relevant picture. Rewrite the proof so that each step is on a different line, and give a justification for each step. Explain which uniqueness method is being used.

- Do exercise 11.7. Draw a relevant picture. Rewrite the proof so that each step is on a different line, and give a justification for each step. Explain which uniqueness method is being used.

- Do exercise 11.9.

- Do exercise 11.11.

- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook. Bring your notebook to class and turn it in for grading.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.

Read Chapter #12, Mathematical induction

Read Chapter 12 in the book by Daniel Solow. This chapter is about induction. It is well written and will hopefully help you understand induction much better. Pay special attention to the introduction of strong induction in sections 12.2 and 12.3.

Specific requirements

- Write useful notes as you read the chapter, and turn those in.
- Do exercise 12.2, and please do a good job on it.
- Do exercise 12.6.
- Do exercise 12.10.
- Do exercise 12.21. In addition to doing what the book asks, rewrite the proof and justify each step of the proof. That is, give a reason that each step of the proof is true, especially with the string of equalities and inequalities. This is going to take some work. Roll up your sleeves and get it done.
- Do exercise 12.22. This will also take some real work. In addition to answering the questions in the problem, please answer this question:

  d. Could we use $n = 1$ as the base case, and save ourselves the trouble of checking the base case for $n = 2$?
- Do exercise 12.23. Part (a) is asking about the sentence "Let x_0 be a real number." which is called the Choose Method in Chapter 5.

  The point x_* is called a *fixed point* of the function, and the inequality involving α means that the fixed point is *attractive*. The point of the result is that if you apply the function f over and over again, the values converge quickly to x_*.

  Before you do 12.23, you might enjoy playing this little game. On a calculator, calculate the square root of a number like 20, then take the square root again, and again, and again, and see what happens. Then start with a number like 0.02 and take the square root again and again and again. You could also do this with cosine, or with sine, or with exp. These functions may or may not have a fixed point, and the fixed points may all be different.

  If you took Math 3370, Differential Equations, you may have seen the result that Picard iteration has an attractive fixed point, and that is how you show existence of solutions of differential equations.
- At the end, tally up how much time you have spent on this chapter. Write this number in your notebook.

General comments

Set yourself up in a place where you won't be disturbed. Read slowly, and write notes in your own words that reflect your understanding of the material.